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Problem 1:

$\mathcal{X}$ denotes the set of legal positions

$Next(x) = \{x' \in \mathcal{X} : x \to x'\}$: the set of legal positions That can be reached by one legal move from x

(a)

$$WIN(x) = \begin{cases} F & , \text{ if } x = (1,0,0,0,0) \\ \bigvee_{x' \in Next(x)} \neg(Win(x')) & , \text{ otherwise.} \end{cases}$$

(b) proof:

we first prove these 2 observation:

The general observation (can you prove why?) is that

(i) $\mathbf{x}$ is a winning position if and only if there exists a losing position $\mathbf{x}'$ such that $\mathbf{x} \to \mathbf{x}'$;

(ii) $\mathbf{x}$ is a losing position if and only if for all $\mathbf{x}'$ such that $\mathbf{x} \to \mathbf{x}'$, $\mathbf{x}'$ is a winning position.

proof for (i):

if there exists a losing position x', we can always do $x \to x'$

then, the aganst player will lose.

proof for (ii):

if all $x' \in Next(x)$ is False, which means no matter which x' is chousen,

the other play can never do a losing position.

now, prove the correctness of the recurenle:

denote $J(x)$: $WIN(x)$ is True, when x is a winning position

$WIN(x)$ is false, when x is a losing position.

, which means the correctness of $WIN(x)$.

Base: $x = (1,0,0,0,0)$: this means the player has to eat the poisoned square.

$\therefore J((1,0,0,0,0)) = F$

for other positions, except $x = (1,0,0,0,0)$, denote the position x.

Indution hypothesis: $\forall x' \in Next(x)$, Correctness for all $J(x')$

I. if $\exists x' \in Next(x)$, $WIN(x') = F$, so $WIN(x) = T$ (from observation (i))

II. if $\forall x' \in Next(x)$, $WIN(x') = T$, so $WIN(x) = F$. (from observation (ii))

we can find one recursive relationship for I. II:

$$WIN(x) = \neg\left(\bigwedge_{x' \in Next(x)} WIN(x')\right)$$

$$= \bigvee_{x' \in Next(x)} \left(\neg(WIN(x'))\right)$$

$\therefore J(x)$ is Correct. $\therefore$ we can prove the recurrence is correct.