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Dynamical Assignments of Distributed Autonomous Robotic Systems to Manufacturing Targets Considering Environmental Feedbacks

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Abstract— **Distributed autonomous robotic units are dynamically assigned to manufacturing targets in a model of a complex manufacturing environment such that an objective function like the total profit is optimized. The introduced control of the robotic units is based on a self-organization approach motivated by the selection of modes in pattern formation of physical, chemical or biological systems. This approach is even resistant against sudden changes like breakdowns of some robotic units. In the present work, it will be shown how environmental feedbacks like time delays from manoevring around obstacles can be respected such that, in opposite to many other approaches, feasibility of the solutions can be guaranteed.**

I. Introduction

Future manufacturing concepts require a large amount of flexibility and therefore distributed architectures. As prototype and model example for such flexible manufacturing systems the assignment of distributed autonomous robotic units to manufacturing targets [17], [10], [9], [8], [15] is considered in the following.

In the considered model, autonomous robots are requested at various locations to fulfill certain tasks [17]. These locations are so-called manufacturing targets and are positioned in a two- or three-dimensional manufacturing environment. In order to obtain a flexible, fault resistant and robust algorithm, the dynamic target assignment and movement control is done by a self-organization approach [12], [17], which basic principles are well known in complex physical and chemical systems [4], [11], but yet not adapted to the engineering fields in a mathematical rigorous way. In case of a surplus of robots, just as many units as there are targets will be assigned with regard to the most suitable placement of the robotic units or smallest working costs they cause. The remaining spare units stay alert in case one of the working robots breaks down.

In [10] a behaviour based approach [1] motivated from pedestrian models [7], [6] is used to navigate the robotic units in combination with the dynamical selection of manufacturing targets based on the coupled selection equations [12]. This allowed first for more efficient paths of the robots and second for the inclusion of collision avoidance in the navigation of the robots.

More complex situations were considered in [15], where for each up-coming task two robotic units of different types have to move to that manufacturing location to work on the task in cooperation. In the language of combinatorial optimization this means that the underlying optimization problem of the robot-target assignment is the $\mathcal{NP}$-hard three-index assignment problem instead of the two-index assignment problem used in the previous papers. In [9], the communication fault tolerance of such distributed and self-organized multi-robot systems where the control is based on coupled selection equations is demonstrated while an experimental realization and verification was done in [8], [2].

In the present work, this self-organization approach is extended to environmental feedbacks which changes the coupled selection equations [12] to a non-autonomous dynamical system. Nevertheless, feasible solutions can be guaranteed, i.e. there are no spurious states or local minima in the solution for the assignment of robots to manufacturing targets.

II. Dynamical Assignments of Robots to Manufacturing Targets

A special selection mechanism is required to select suitable robots for each target and vice versa in a cost effective, fault resistant and robust way. Traditionally, combinatorial problems are solved with integer algorithms (see e.g. [3]). However, dynamical system approaches [16] often work by gradually emerging or evolving of the final solution and are therefore in particular useful for situtation where fault-resistance and sudden changes of the environment has to be considered. In the following the specific dynamical system approach of coupled selection equations [12] is used which can also be applied to $\mathcal{NP}$-hard three-index or higher-index assignment problems.

Considering robot-target assignments with underlying two-index assignment problems, each target has to be served by one and only one robotic unit. The aim is to maximize the total winnings w, i.e., the decision variable $x \in \mathbf{R}^{n\times n}$ has to be found for

$$w = \sum_{i,j} w_{ij} x_{ij} \to \max \qquad (1)$$

such that $x = (x_{ij})$ is a permutation matrix, or in other words fulfills $x_{ij} \in \{0,1\} \quad \forall i,j$ and

$$\sum_i x_{ij} = 1 \quad \forall j \qquad (2)$$

$$\sum_j x_{ij} = 1 \quad \forall i. \tag{3}$$

Each robot is thereby represented by the first index i and each target by the second index j. The winnings w_{ij} represent the individual preference of each robot i working at manufacturing target j.

The dynamics of the coupled selection equations [12] for this problem is given by

$$\frac{d}{dt}\xi_{ij} = \xi_{ij}\left(1 - \xi_{ij}^2 - \beta\sum_{i' \neq i}\xi_{i'j}^2 - \beta\sum_{j' \neq j}\xi_{ij'}^2\right) \tag{4}$$

with $\beta > 1/2$. Non-negative initial values

$$\xi_{ij}(0) := w_{ij} \qquad \forall i, j, \tag{5}$$

where $0 \leq w_{ij} \leq 1$ can be assumed without loss of generality, ensure for square matrices (ξ_{ij}) that the system will always asymptotically end in a stable solution of permutation matrices, i.e., there is one and only one non-vanishing element which is equal to 1 in each row and in each column. A proof is given in [12]. Due to this fact, for equal numbers of robots and targets, in the selection of the destination there is a one to one correspondence of robotic units to targets. In the case of a surplus of robotic units the coupled selection equations (4) ensure that there is not more than one target as destination for each of the robotic units and vice versa. The results of this heuristic dynamical system approach for assignment problems compare very well to other methods [16].

To optimize the total path length of the distributed robotic system, the initial values (5) were defined by the linear transformed EUCLIDean distances $\mathbf{d}_{ij} = \mathbf{r}_i(0) - \mathbf{g}_j$ of the robotic units at positions $\mathbf{r}_i(0)$ to the targets located at $\mathbf{g}_j$

$$\xi_{ij}(0) = a - b\frac{\|\mathbf{d}_{ij}\| - \min\limits_{i',j'}(\|\mathbf{d}_{i'j'}\|)}{\max\limits_{i',j'}(\|\mathbf{d}_{i'j'}\|) - \min\limits_{i',j'}(\|\mathbf{d}_{i'j'}\|)}, \tag{6}$$

where $a = 0.9$ and $b = 0.8$ were used for the simulations so that the initial values $\xi_{ij}(0)$ lie in the interval $[0.1, 0.9]$ which avoids the values 0.0 and 1.0 corresponding to a preselection of some of the robots to the targets. Using this transformation, targets which are located closer to the robotic units than others obtain larger initial values, i.e., a larger preference for the final selection. In addition, more sophisticated cost functions including manufacturing costs at the target's location, could be considered as well.

In [17], [10], [9], [8], [15] the abilities of the coupled selection equations to cover up with sudden changes like breakdowns of some of the robotic units were demonstrated and investigated. In addition, to this fault resistance, it will be shown in the following how environmental feedbacks like time delays from manoevring around obstacles can be respected by an extension of the equations. This is done by a control parameter $\alpha(t)$ representing environmental feedbacks. The main difficulty in doing this is to take care that the nice property of the coupled selection equations, which is very rare in this field, to always guarantee a feasible solution, is not destroyed.

Many models of complex physical systems have control parameters in the linear term of the evolution equation which dominates in a neighbourhood of the origin compared to higher order terms. This works in the present problem as first ansatz as well and was indeed used succesfully in [5], [18] but this control parameter shifts the solution so that the feasibility is only approximately fulfilled. Multiplying such an control parameter not only to the linear but also to one of the nonlinear terms allows for an external control without destroying the guarantee of feasibility of the solutions. In the context of classical constrained optimization theory this corresponds to a smooth exact penalty approach for the assignment problem [13]. Using for each of the variables ξ_{ij} a time-dependent control parameter $\alpha_{ij}(t) > 0$ being a measure for the fitness of the relation between robot i and target j at time t, one finally gets

$$\frac{d}{dt}\xi_{ij} = \kappa\xi_{ij}\left(\alpha_{ij}(t)\left(1 - \xi_{ij}^2\right) - \beta\sum_{i' \neq i}\xi_{i'j}^2 - \beta\sum_{j' \neq j}\xi_{ij'}^2\right) \tag{7}$$

with a time scaling factor κ which adjusts the change per time compared to the below defined equation of motion (9) defining the movements of the robots.

The proof of the feasibility of the solutions can be transfered from the one for the coupled selection equations without control parameters [12] if one assumes $\beta > \max_{i,j}\alpha_{ij}/2$ and $\lim\limits_{t \to \infty}\alpha_{ij}(t) = \text{const.} > 0$ [13].

The control parameter could respect many types of information like availability of manufacturing targets or material supply which has to be considered for the selection process. For reasons of simple visualization of the selection processes the updated distances to the targets is considered only:

$$\alpha_{ij}(t) = a' - b'\frac{\|\mathbf{d}_{ij}(t)\|}{\max\limits_{i',j'}(\|\mathbf{d}_{i'j'}(t)\|)}, \tag{8}$$

where $a' = 1.0$ and $b' = 0.9$ were used for the simulations so that $\alpha_{ij}(t)$ lies in the interval $[0.1, 1.0]$. Alternatively, $\alpha_{ij}(t)$ could have been chosen similar to equation (6). The above choice of $\alpha_{ij}(t)$ allows for respecting time delays from manoevring around obstacles for the selection process which is demonstrated in the following.

III. Equation of Motion for the Robotic Units

In contrast to the work described in [10], [9], [8], [15], the equation of motion which will be used here to control the robots defines not the temporal changes of the velocities but those of the positions of the robots. The movement of the robots is given here as direct result of the destination vector $\mathbf{e}^0$ and replacement velocities $\mathbf{f}$ as

$$\frac{d}{dt}\mathbf{r}_i = v_i^0\mathbf{e}_i^0 + \sum_{i' \neq i}\mathbf{f}_{ii'}^{\mathrm{r}}(\mathbf{r}_{i'} - \mathbf{r}_i) + \sum_k \mathbf{f}_{ik}^{\mathrm{o}}(\mathbf{x}_k - \mathbf{r}_i). \tag{9}$$

The destination vector for each robotic unit is defined as

$$\mathbf{e}_i^0(t) = \mathbf{N}_{\gamma\delta}\left(\sum_j \xi_{ij}(t)\mathbf{N}_{\gamma'\delta'}(\mathbf{g}_j - \mathbf{r}_i(t))\right) \tag{10}$$

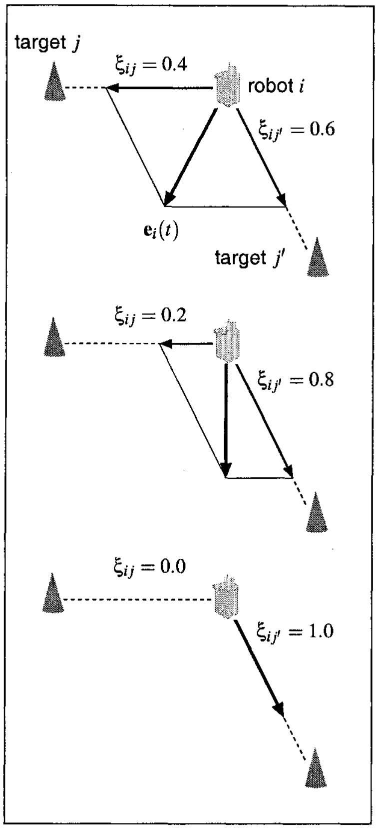

Fig. 1. Visualization of the dynamic selection process. At the beginning (upper picture) the destination vector $\mathbf{e}_i^0(t)$ points towards a linear combination of the weighted difference vectors $\xi_{ij}(t)\mathbf{N}_{\gamma'\delta'}(\mathbf{g}_j - \mathbf{r}_i)$ to the targets and after a while (lower picture), only one of the target is selected by the coupled selection equations and the destination vector points to that target only. From [14].

which is a kind of linear combination of the difference vectors of the robotic unit i to all targets. The weights $\xi_{ij}(t)$ which depict the preferences of each of the robotic units to a certain target, or respectively the preferences the targets have for being served by a specific robot. This equation in combination with the coupled selection equations (4) or (7) finally selects one and only one target for each robot i as for large times t $\xi_{ij}(t)$ tends to one for the most suitable $j = j'$ and to zero for all other $j \neq j'$. Figure 1 shows an easy example of a robot selecting one of two targets. Each robot will move to its selected target while all remaining robotic units which have not been assigned to a target (in a situation of a surplus of robots) will stop moving as all of their associated coefficients $\xi_{ij}(t)$ converge to zero.

The function

$$\mathbf{N}_{\gamma\delta}(\mathbf{x}) = \frac{1}{||\mathbf{x}|| + 1/(\gamma||\mathbf{x}|| + \delta)} \cdot \mathbf{x} \tag{11}$$

with $\gamma, \delta > 0$ mainly normalizes the vector $\mathbf{x}$ but avoids a singularity at $\mathbf{x} = \mathbf{0}$.

The short range force fields $\mathbf{f}_{ii'}^{\mathrm{r,o}}(\mathbf{r}) \in \mathbf{R}^2$ (or $\mathbf{R}^3$) change the robots' directions and speeds in order to avoid collisions with other robotic units and obstacles. They are defined by

$$\mathbf{f}_{ii'}^{\mathrm{r,o}}(\mathbf{r}) = \begin{cases} (\tan g(\tilde{r}) - g(\tilde{r})) \frac{\mathbf{r}}{||\mathbf{r}||} & \text{for} \quad 0 < \tilde{r} \leq \sigma^{\mathrm{r,o}} \\ 0 & \text{for} \quad \tilde{r} > \sigma^{\mathrm{r,o}} \end{cases} \tag{12}$$

with $\tilde{r} = ||\mathbf{r}|| - d_i^{\mathrm{r}}/2 - d_{i'}^{\mathrm{r,o}}/2$, $g(\tilde{r}) = \frac{\pi}{2}\left(\frac{\tilde{r}}{\sigma^{\mathrm{r,o}}} - 1\right)$ and $||\cdot||$ is the EUCLIDean norm, around each of the other units i' at the locations $\mathbf{r}_{i'}$ with diameter $d_{i'}^{\mathrm{r}}$ and obstacles at the locations $\mathbf{x}_{i'}$ with diameter $d_{i'}^{\mathrm{o}}$. The distances maintained between robotic units and obstacles can be chosen with appropriate range parameters σ^{r} and σ^{o} (see Figure 2). The distances between robotic units and obstacles can be adjusted with appropriate range parameters σ^{r} and σ^{o}. Figure 2 shows a plot of the used short range force which is identical 0 for distances $\tilde{r} \geq \sigma^{\mathrm{r,o}}$.

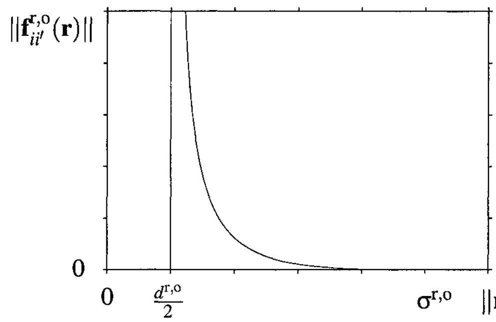

Fig. 2. Plot of the used short range force $\mathbf{f}_{ii'}^{\mathrm{r,o}}(\mathbf{r})$ defined in (12).

The use of such short range potential forces avoids unwanted stable points, i.e. local minima or spurious states where the system could get stuck in if the distances between all obstacles are pairwise large enough or in other words that there are no non-convex obstacles. These repulsive forces can be calculated with the information from infrared or ultrasonic sensors that detect the proximity to an obstacle within a given range. Many mobile robotic units are equipped with a number of such sensors around their perimeter. To avoid the stagnancy of this multiple particles system at unstable stationary points, for the numerical solution, small fluctuations are added to the equation of motion (9) near stationary points.

IV. Simulation Results

Even though many properties of the introduced dynamical system (9) with (7) can be investigated analytically [12], [13], they are shown in the present work by simulation, i.e., by integration of the equations of motion.

A first impression can be obtained by a simulation with 30 autonomous robots and obstacles of various size which is shown in Fig. 3.

This result is similar to those described in [10] using an equation of motion for the robots velocities and the coupled selection equations (4) but in the present paper, the considered situation is much more complex in terms of the number of robots and obstacles.

To demonstrate the effect of the time-dependent parameters $\alpha_{ij}(t)$ considering the actual situation in equation (7) for modelling an environmental feedback a simple situation is constructed with two robots competing for one target. Fig. 4 shows

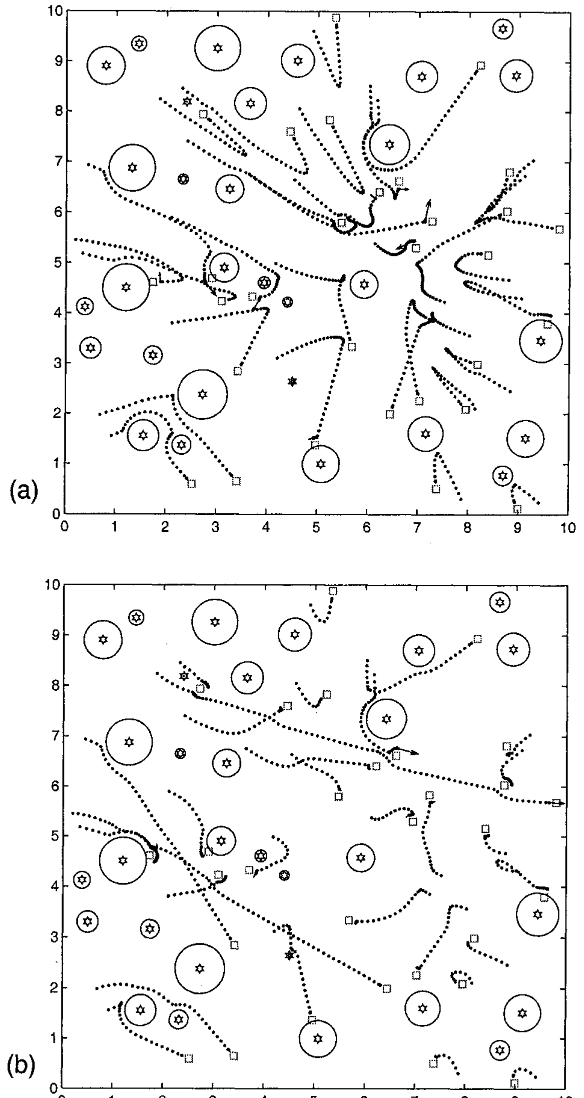

Fig. 3. Trajectories (dotted lines) of a simulation of 30 robots and obstacles of various size. The time scaling parameter κ was chosen in (a) small so that selection process of equation (7) is slow and the robots move first to a center of mass of the targets before they finally select a manufacturing target. In (b) κ was chosen large which results in a fast selection process and shorter paths. This behaviour is of advantage if spare robotic units take over the tasks of units which broke down and can be adjusted by the parameter κ.

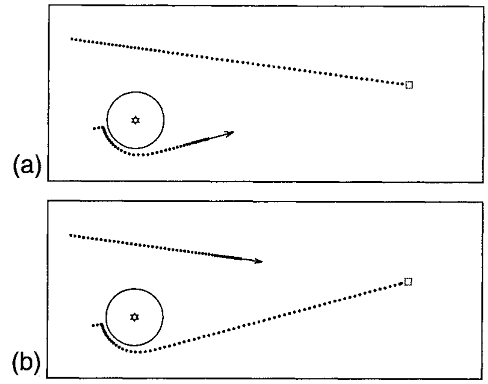

Fig. 4. Trajectories (dotted lines) of two robots to demonstrate the effect of the environmental feedback. (a) shows a simulation with feedback and (b) without feedback.

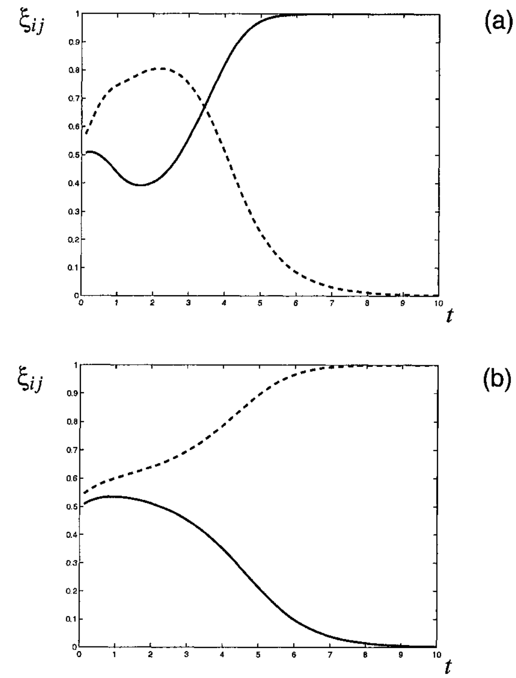

Fig. 5. Selection behaviour of $\xi_{ij}(t)$ of two robots $i = 1,2$ to one target j which corresponds to Fig. 4. (a) shows a simulation with feedback and (b) without feedback. In (b) the largest intial value wins while in (a) there is a change of the largest value after some time due to the environmental feedback.

the difference between the coupled selection equations with (a) and without (b) environmental feedback. Despite of the closer initial position of the lower robot in (a), the upper robot wins the competition to reach the target due to the time delay of the lower robot by manoevring around the obstacle. This feedback is not considered in (b) where the lower robot wins just due to its closer initial position independent of the worse position after manoevring around the obstacle. The different selection behaviour of the coupled selection equation with and without environmental feedback is shown in Fig. 5 plotting ξ_{ij} over time t.

A simple model for a flexible manufacturing environment is shown in Fig. 6. A complex situation is constructed of 25 circular obstacles in manhatten geometry and 64 manufacturing targets located at the boundaries of the obstacles representing several working positions of big machines. The 64 robotic units are initially positioned around the edges of the square working area where each of them has to serve one and only one manufacturing target.

Although this complex situation is extremely hard to solve because of its symmetric positioned robots and targets and large ratio of obstacle area to manoevring area, the proposed self-organized robotic control works well. After some time, each of the manufacturing targets is served by one and only one robotic unit and each robot serves one and only one target as it is re-

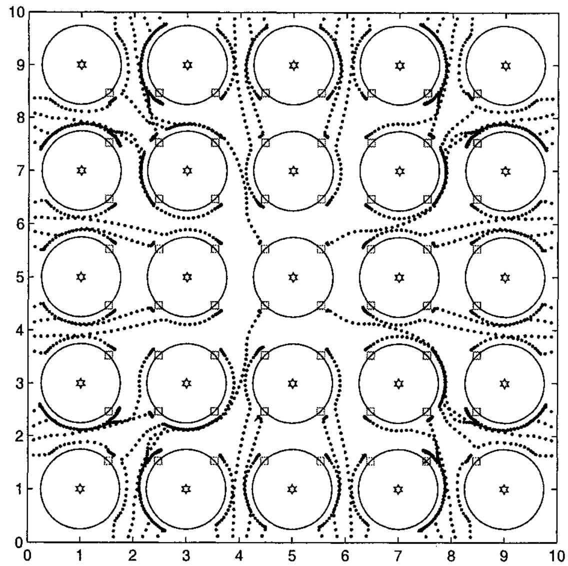

Fig. 6. Trajectories (dotted lines) of robotic units for a model of a manufacturing environment with manhatten geometry of obstacles and manufacturing targets. The simulation shows 64 robots and targets (squares) located arround the circular shaped obstacles representing working positions.

quired in the constraints of the two-index robot-target assignment problem. The distances of the dots along each trajectory gives information about the velocity of the corresponding robot on its tour. It has to be remarked that the specific geometry of obstacles and manufacturing targets shown in Fig. 6 could lead in exceptional unfavourable cases to traps due to a robot standing stationary at one of the targets to fulfill his task and creating together with the obstacle nearby a two-body non-convex obstacle. Nevertheless, this disadvantage basically found in behavioural force approaches for non-convex obstacles could be overcame by replacing the behavioural force approach by traditional path planning methods. The combination of other path planning approaches with the coupled selection equations (7) might therefore be a challenging topic for future research. Anyhow, it has to be stressed out that the key point here is the selection process for the robot-target assignment which even can respect environmental feedbacks for the decision process where feasibility of solutions can be guaranteed. The evolution of the preference matrix $(\xi_{ij}) \in \mathbf{R}^{64\times 64}$ corresponding to the trajectories of Fig. 6 represents the decision process and is shown in Fig. 7 as four snap-shots. The final state is a permutation matrix, i.e. feasible solution, representing the assignment of the robotic units to the manufacturing targets.

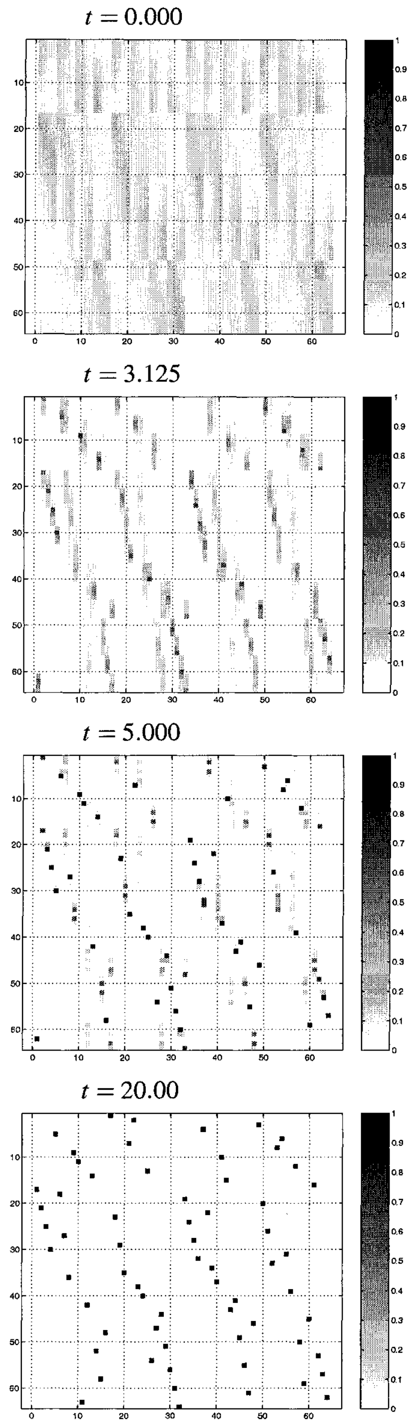

Fig. 7. Preference matrix $(\xi_{ij}) \in \mathbf{R}^{64\times 64}$ at times $t = 0.000$, $t = 3.125$, $t = 5.000$ and $t = 20.00$ corresponding to the trajectories shown in Figure 6.

V. Conclusions and Outlook

The presented dynamical system approach based on self-organizing principles of complex physical systems allows for the construction of flexible and failure-resistant algorithms. A model of flexible manufacturing, the two-index robot-target assignment problem was used for demonstration. Earlier approaches proposed by the author in previous work were simplified in terms of the equation of motion for the robotic units and extended to environmental feedbacks in the selection process of the coupled selection process. The remarkable key point here is that this is done without producing non-feasible solutions of the system.

Future work will consider time-dependent manufacturing targets with changing demands to be able to treat multi-assignments in the tour of time which models a full manufacturing line for distributed systems. In this context co-operations of the robotic units at the targets have to be considered in addition, which leads to robot control with underlying $\mathcal{NP}$-hard three-index or higher-index assignment problems.

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