

```
In [1]: import pandas as pd
import numpy as np
import sklearn
import matplotlib.pyplot as plt
from sklearn.model_selection \
import train_test_split
import seaborn as sns
from sklearn.linear_model import LogisticRegression
from sklearn.preprocessing import StandardScaler, LabelEncoder
```

Question#1.1 load the data into Pandas dataframe and add a column "color". For each class 0, this should contain "green" and for each class 1 it should contain "red"

```
In [2]: data=pd.read_csv("data_banknote_authentication.csv")
color=[]
for i in range(len(data)):
    if data["class"][i]==0:
        color.append("green")
    else:
        color.append("red")
data["color"]=color
print(data)
```

	variance	skewness	curtosis	entropy	class	color
0	3.62160	8.66610	-2.8073	-0.44699	0	green
1	4.54590	8.16740	-2.4586	-1.46210	0	green
2	3.86600	-2.63830	1.9242	0.10645	0	green
3	3.45660	9.52280	-4.0112	-3.59440	0	green
4	0.32924	-4.45520	4.5718	-0.98880	0	green
...
1367	0.40614	1.34920	-1.4501	-0.55949	1	red
1368	-1.38870	-4.87730	6.4774	0.34179	1	red
1369	-3.75030	-13.45860	17.5932	-2.77710	1	red
1370	-3.56370	-8.38270	12.3930	-1.28230	1	red
1371	-2.54190	-0.65804	2.6842	1.19520	1	red

[1372 rows x 6 columns]

Question#1.2 for each class and for each feature f1, f2, f3, f4, compute its mean $\mu()$ and standard deviation $\sigma()$. Round the results to 2 decimal places and summarize them in a table as shown below:

```
In [3]: #f1
f1_mu=[]
f1_mu.append(round(data.loc[(data["class"]==0),:]["variance"].mean(),2))
f1_mu.append(round(data.loc[(data["class"]==1),:]["variance"].mean(),2))
f1_mu.append(round(data["variance"].mean(),2))
f1_std=[]
f1_std.append(round(data.loc[(data["class"]==0),:]["variance"].std(),2))
f1_std.append(round(data.loc[(data["class"]==1),:]["variance"].std(),2))
f1_std.append(round(data["variance"].std(),2))

#f2
f2_mu=[]
f2_mu.append(round(data.loc[(data["class"]==0),:]["skewness"].mean(),2))
f2_mu.append(round(data.loc[(data["class"]==1),:]["skewness"].mean(),2))
```

```

f2_mu.append(round(data["skewness"].mean(),2))
f2_std=[]
f2_std.append(round(data.loc[(data["class"]==0),:]["skewness"].std(),2))
f2_std.append(round(data.loc[(data["class"]==1),:]["skewness"].std(),2))
f2_std.append(round(data["skewness"].std(),2))

#f3
f3_mu=[]
f3_mu.append(round(data.loc[(data["class"]==0),:]["curtosis"].mean(),2))
f3_mu.append(round(data.loc[(data["class"]==1),:]["curtosis"].mean(),2))
f3_mu.append(round(data["curtosis"].mean(),2))
f3_std=[]
f3_std.append(round(data.loc[(data["class"]==0),:]["curtosis"].std(),2))
f3_std.append(round(data.loc[(data["class"]==1),:]["curtosis"].std(),2))
f3_std.append(round(data["curtosis"].std(),2))

#f4
f4_mu=[]
f4_mu.append(round(data.loc[(data["class"]==0),:]["entropy"].mean(),2))
f4_mu.append(round(data.loc[(data["class"]==1),:]["entropy"].mean(),2))
f4_mu.append(round(data["entropy"].mean(),2))
f4_std=[]
f4_std.append(round(data.loc[(data["class"]==0),:]["entropy"].std(),2))
f4_std.append(round(data.loc[(data["class"]==1),:]["entropy"].std(),2))
f4_std.append(round(data["entropy"].std(),2))

Q1_d={"Feature":pd.Series(["0","1","all"]),
      "μ(f1)":pd.Series(f1_mu),
      "σ(f1)":pd.Series(f1_std),
      "μ(f2)":pd.Series(f2_mu),
      "σ(f2)":pd.Series(f2_std),
      "μ(f3)":pd.Series(f3_mu),
      "σ(f3)":pd.Series(f3_std),
      "μ(f4)":pd.Series(f4_mu),
      "σ(f4)":pd.Series(f4_std)}
Q1_df=pd.DataFrame(Q1_d)
print(Q1_df)

```

	Feature	$\mu(f1)$	$\sigma(f1)$	$\mu(f2)$	$\sigma(f2)$	$\mu(f3)$	$\sigma(f3)$	$\mu(f4)$	$\sigma(f4)$
0	0	2.28	2.02	4.26	5.14	0.80	3.24	-1.15	2.13
1	1	-1.87	1.88	-0.99	5.40	2.15	5.26	-1.25	2.07
2	all	0.43	2.84	1.92	5.87	1.40	4.31	-1.19	2.10

Question#1.3 examine your table. Are there any obvious patterns in the distribution of banknotes in each class

The variance and skewness of real banknotes will be better than counterfeit banknotes

Question#2.1 split your dataset X into training Xtrain and Xtesting parts (50/50 split). Using "pairplot" from seaborn package, plot pairwise relationships in Xtrain separately for class 0 and class 1. Save your results into 2 pdf files "good bills.pdf" and "fake bills.pdf"

```

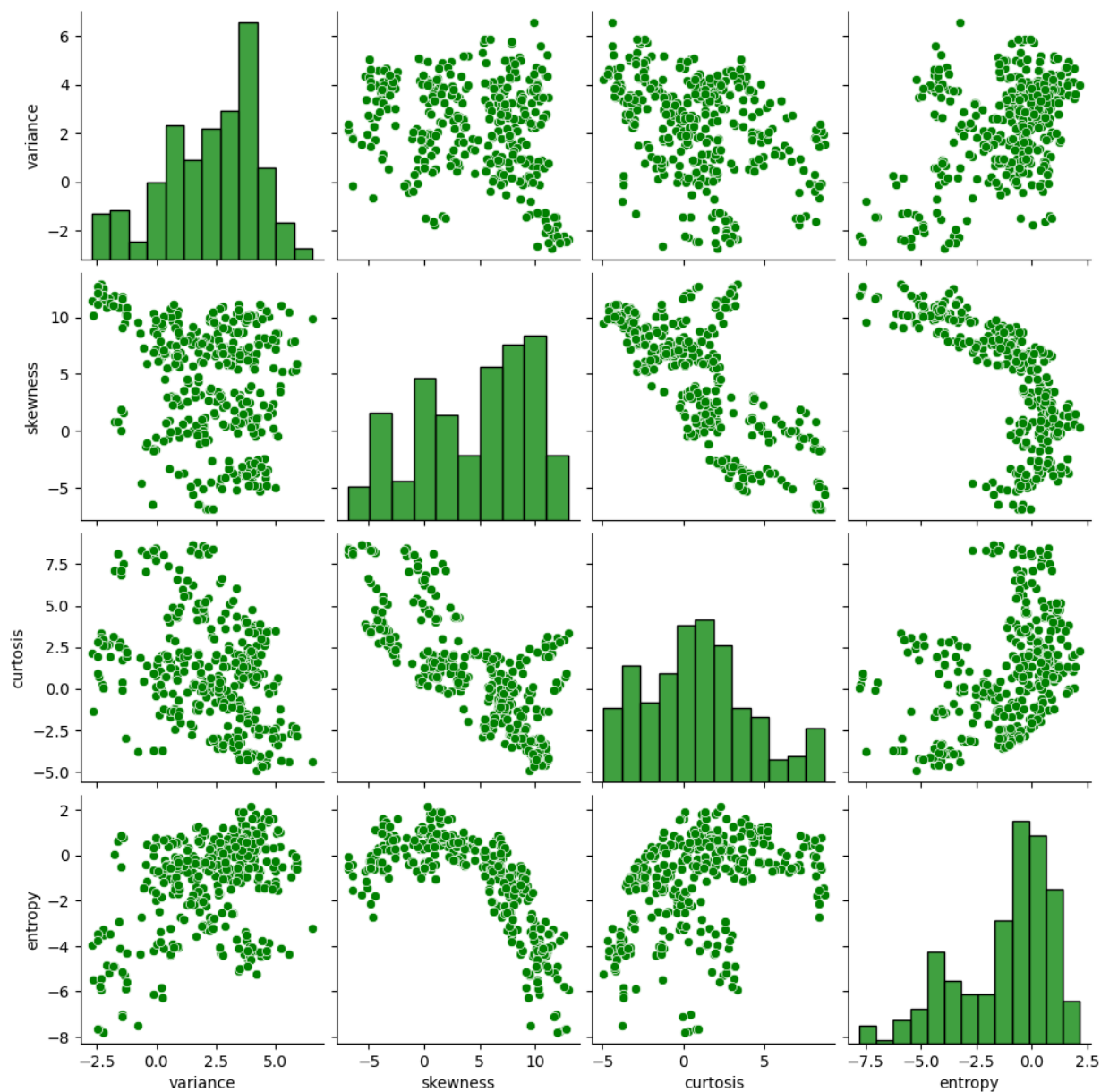
In [4]: X = data[["variance","skewness","curtosis","entropy","class"]]
y = data["class"]
X_train,X_test,y_train,y_test=train_test_split(X,y, train_size=0.5)
X_train_0=X_train.loc[(X_train["class"]==0),:]
X_train_1=X_train.loc[(X_train["class"]==1),:]

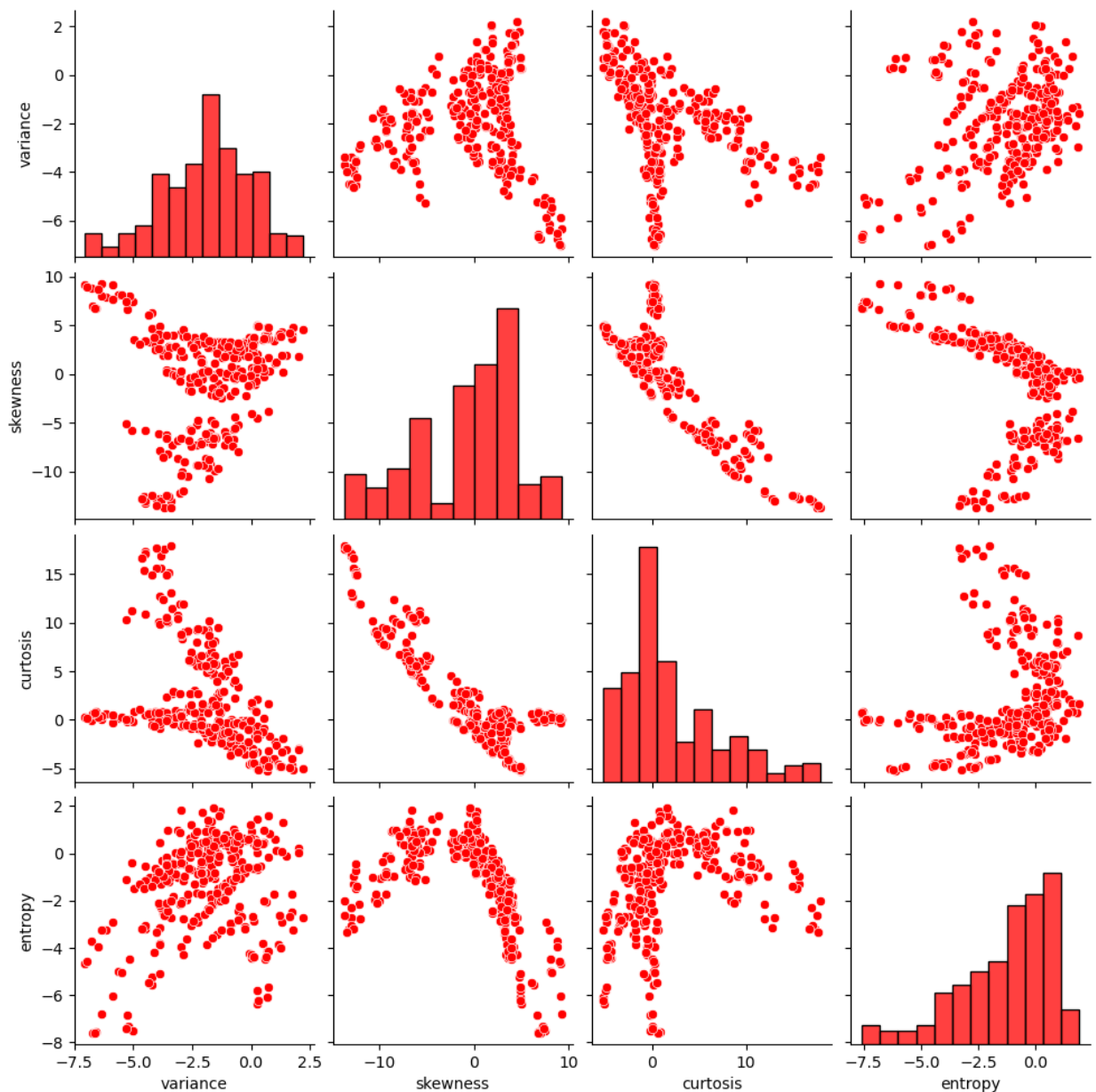
```

```

features=["variance","skewness","curtosis","entropy"]
pair_plot_bad = sns.pairplot(X_train_0[features],diag_kws=dict(color='green'),r
plt.show()
pair_plot_good = sns.pairplot(X_train_1[features],diag_kws=dict(color='red'),pl
plt.show()

```





Question#2. visually examine your results. Come up with three simple comparisons that you think may be sufficient to detect a fake bill. For example, your classifier may look like this:

```
In [5]: X_test.loc[len(data.index)]=[8,2,9,8,0]

X_test["variance_label"]=X_test["variance"].apply(lambda x: 1 if x<0 else 0)
X_test["skewness_label"]=X_test["skewness"].apply(lambda x: 1 if x<5 else 0)
X_test["kurtosis_label"]=X_test["kurtosis"].apply(lambda x: 1 if x<2.3 else 0)
X_test["entropy_label"]=X_test["entropy"].apply(lambda x: 1 if x>X_test["entropy"]

#data.to_csv("Q2.csv")

X_test=X_test.reset_index(drop=True)

Q2_label=[]
for i in range(len(X_test)):
    if (X_test["variance_label"][i]+X_test["skewness_label"][i]+X_test["kurtosis_label"][i]+X_test["entropy_label"][i])>3:
        Q2_label.append(1)
```

```

else:
    Q2_label.append(0)
X_test["Q2_label"]=Q2_label
#print(X_test)

```

	variance	skewness	curtosis	entropy	class	variance_label	\
0	2.13190	-2.04030	2.55740	-0.061652	0	0	
1	-2.32110	3.16600	-1.00020	-2.715100	1	1	
2	-2.31420	-0.68494	1.98330	-0.448290	1	1	
3	-6.28150	6.66510	0.52581	-7.010700	1	1	
4	4.15420	7.27560	-2.47660	-1.209900	0	0	
..	
682	-2.01490	3.68740	-1.93850	-3.891800	1	1	
683	-0.40804	0.54214	-0.52725	0.658600	1	1	
684	-2.00660	-6.71900	9.01620	0.099985	1	1	
685	4.17360	3.33360	-1.42440	0.604290	0	0	
686	8.00000	2.00000	9.00000	8.000000	0	0	

	skewness_label	curtosis_label	entropy_label	Q2_label
0	1	0	1	0
1	1	1	0	1
2	1	1	1	1
3	0	1	0	1
4	0	1	0	0
..
682	1	1	0	1
683	1	1	1	1
684	1	0	1	1
685	1	1	1	1
686	1	0	1	0

[687 rows x 10 columns]

Question#2.3 apply your simple classifier to Xtest and compute predicted class labels

Question#2.4 compare your predicted class labels with true labels in Xtest, compute the following:

```

In [6]: TP_Q2=X_test.loc[(X_test["Q2_label"]==0)&(X_test["class"]==0),:]["class"].count
FP_Q2=X_test.loc[(X_test["Q2_label"]==0)&(X_test["class"]==1),:]["class"].count
TN_Q2=X_test.loc[(X_test["Q2_label"]==1)&(X_test["class"]==1),:]["class"].count
FN_Q2=X_test.loc[(X_test["Q2_label"]==1)&(X_test["class"]==0),:]["class"].count
TPR_Q2=TP_Q2/(TP_Q2+FN_Q2)
TNR_Q2=TN_Q2/(TN_Q2+FP_Q2)
ACC_Q2=(TP_Q2+TN_Q2)/len(X_test)

Q2_d={"Classifier":pd.Series(["predict"]),
      "TP":pd.Series([TP_Q2]),
      "FP":pd.Series([FP_Q2]),
      "TN":pd.Series([TN_Q2]),
      "FN":pd.Series([FN_Q2]),
      "ACC":pd.Series([ACC_Q2]),
      "TPR":pd.Series([TPR_Q2]),
      "TNR":pd.Series([TNR_Q2])}
Q2_df=pd.DataFrame(Q2_d)
print(Q2_df)

```

	Classifier	TP	FP	TN	FN	ACC	TPR	TNR
0	predict	255	1	301	130	0.809316	0.662338	0.996689

Question#2.5 summarize your findings in the table as shown below:

This model I customized has a high probability of finding genuine banknotes

Question#2.6 6. does you simple classifier gives you higher accuracy on identifying "fake" bills or "real" bills? Is your accuracy better than 50% ("coin" flipping)?

From my answer, My model has higher accuracy in identifying fake banknotes, while the probability of identifying real banknotes is relatively low. But both are over 50, so is higher than "coin" flipping

Question#3.1 take $k = 3, 5, 7, 9, 11$. For each k , generate X_{train} and X_{test} using 50/50 split as before. Train your k -NN classifier on X_{train} and compute its accuracy for X_{test}

```
In [7]: from sklearn.neighbors import KNeighborsClassifier
from sklearn.metrics import accuracy_score

data_Q3=pd.read_csv("data_banknote_authentication.csv")

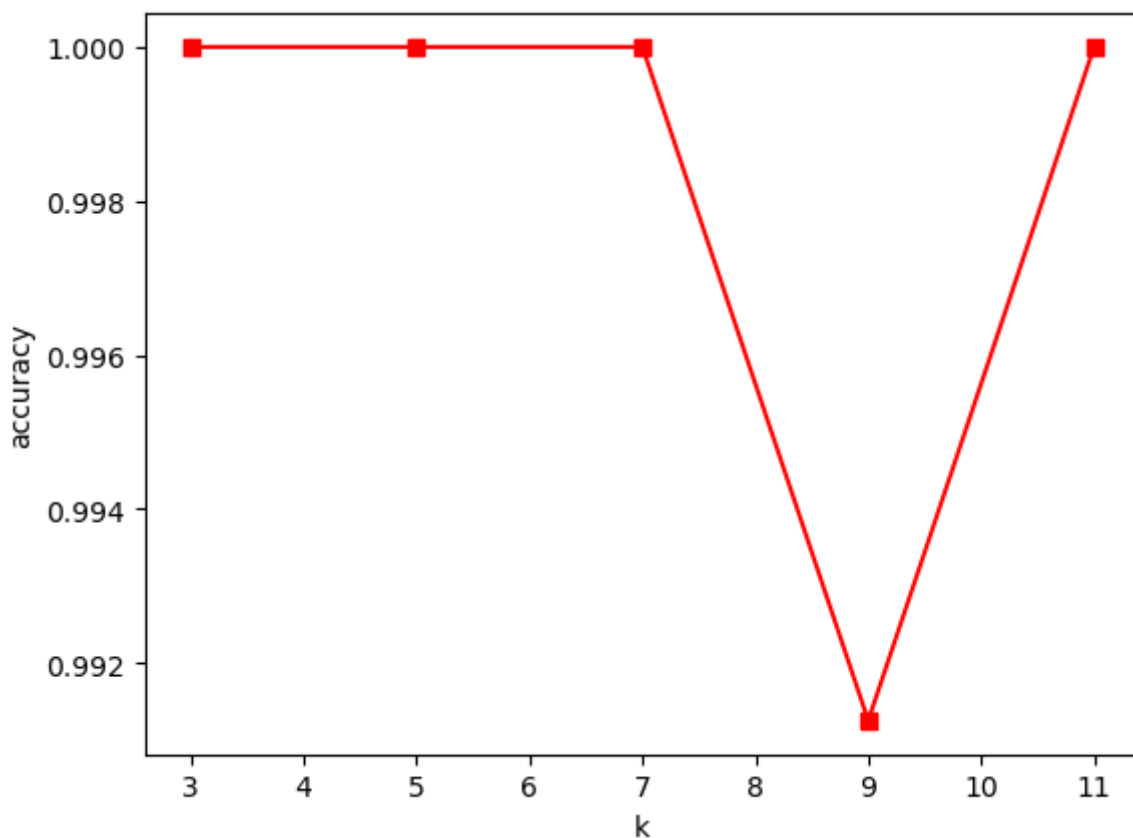
def Q3_KNN(Q3_data,n):
    Q3_X=data_Q3[["variance","skewness","curtosis","entropy"]]
    Q3_y=data_Q3["class"]
    Q3_X_train, Q3_X_test, Q3_y_train, Q3_y_test = train_test_split(Q3_X, Q3_y,
    knn = KNeighborsClassifier(n_neighbors=n)
    knn.fit(Q3_X_train, Q3_y_train)
    return accuracy_score(Q3_y_test, knn.predict(Q3_X_test))

#k=3
K_3=Q3_KNN(data_Q3,3)
K_5=Q3_KNN(data_Q3,5)
K_7=Q3_KNN(data_Q3,7)
K_9=Q3_KNN(data_Q3,9)
K_11=Q3_KNN(data_Q3,11)
print("K=3:",K_3)
print("K=5:",K_5)
print("K=7:",K_7)
print("K=9:",K_9)
print("K=11:",K_11)
```

```
K=3: 1.0
K=5: 1.0
K=7: 1.0
K=9: 0.9912536443148688
K=11: 1.0
```

Question#3.2 plot a graph showing the accuracy. On x axis you plot k and on y-axis you plot accuracy. What is the optimal value k^* of k ?

```
In [8]: Q3_xlab = [3,5,7,9,11]
plt.plot(Q3_xlab,[K_3,K_5,K_7,K_9,K_11], 's-', color = 'r', label="ATT-RLSTM")
plt.ylabel("accuracy")
plt.xlabel("k")
plt.show()
```



Question#3.3 use the optimal value k^* to compute performance measures and summarize them in the table

```
In [9]: from sklearn.metrics import confusion_matrix
Q3_X=data_Q3[["variance","skewness","curtosis","entropy"]]
Q3_y=data_Q3["class"]
Q3_X_train, Q3_X_test, Q3_y_train, Q3_y_test = train_test_split(Q3_X, Q3_y, test_size=0.2, random_state=42)
knn = KNeighborsClassifier(n_neighbors=5)
knn.fit(Q3_X_train, Q3_y_train)

Q3_acc=accuracy_score(Q3_y_test, knn.predict(Q3_X_test))
Q3_TN, Q3_FP, Q3_FN, Q3_TP = confusion_matrix(Q3_y_test, knn.predict(Q3_X_test))
Q3_TPR=Q3_TP/(Q3_TP+Q3_FN)
Q3_TNR=Q3_TN/(Q3_TN+Q3_FP)

Q3_d={"Classifier":pd.Series(["predict"]),
      "TP":pd.Series([Q3_TP]),
      "FP":pd.Series([Q3_FP]),
      "TN":pd.Series([Q3_TN]),
      "FN":pd.Series([Q3_FN]),
      "ACC":pd.Series([Q3_acc]),
      "TPR":pd.Series([Q3_TPR]),
      "TNR":pd.Series([Q3_TNR])}
Q3_df=pd.DataFrame(Q3_d)
print(Q3_df)
```

	Classifier	TP	FP	TN	FN	ACC	TPR	TNR
0	predict	296	0	390	0	1.0	1.0	1.0

Question#3.4 is your k-NN classifier better than your simple classifier for any of the

measures from the previous table?

The correct rate of the K_NN algorithm is definitely higher than that of the simple classifier

Question#3.5 consider a bill x that contains the last 4 digits of your BUID as feature values. What is the class label predicted for this

```
In [10]: data1 = {'variance': [8],
                'skewness': [2],
                'curtosis': [9],
                'entropy': [8]}
df = pd.DataFrame(data1)
#print(type(df))
print(knn.predict(df))
```

[0]

In the assembled model, I put my BUID information on the last line of the test set. And you get a prediction of 0. And for KNN part I use the last training data to predict my BUID. the answer is same. I also got 0.

Question#4.1 take your best value k^* . For each of the four features f_1, \dots, f_4 , generate new X_{test} and X_{train} and drop that feature from both X_{train} and X_{test} . Train your classifier on the "truncated" X_{train} and predict labels on X_{test} using just 3 remaining features. You will repeat this for 4 cases: (1) just f_1 is missing, (2) just f_2 missing, (3) just f_3 missing and (4) just f_4 is missing. Compute the accuracy for each of these scenarios.

```
In [11]: data_Q4 = pd.read_csv("data_banknote_authentication.csv")
Q4_X = data_Q4[["variance", "skewness", "curtosis", "entropy"]]
Q4_y = data_Q4["class"]
col = ["variance", "skewness", "curtosis", "entropy"]
#for i in range(len(col)):
Q4X_f1 = Q4_X.drop(["variance"], axis=1)
Q4_X_train, Q4_X_test, Q4_y_train, Q4_y_test = train_test_split(Q4X_f1, Q4_y,
Q4_knn = KNeighborsClassifier(n_neighbors=5)
Q4_knn.fit(Q4_X_train, Q4_y_train)
print("if I drop", "variance", "the accuracy is", accuracy_score(Q4_y_test, Q4_knn.
#print(Q4X_1.columns.values)

Q4X_f2 = Q4_X.drop(["skewness"], axis=1)
#print(Q4X_f2.columns.values)
Q4_X_train, Q4_X_test, Q4_y_train, Q4_y_test = train_test_split(Q4X_f2, Q4_y,
Q4_knn = KNeighborsClassifier(n_neighbors=5)
Q4_knn.fit(Q4_X_train, Q4_y_train)
print("if I drop", "skewness", "the accuracy is", accuracy_score(Q4_y_test, Q4_knn.

Q4X_f3 = Q4_X.drop(["curtosis"], axis=1)
Q4_X_train, Q4_X_test, Q4_y_train, Q4_y_test = train_test_split(Q4X_f3, Q4_y,
Q4_knn = KNeighborsClassifier(n_neighbors=5)
Q4_knn.fit(Q4_X_train, Q4_y_train)
print("if I drop", "curtosis", "the accuracy is", accuracy_score(Q4_y_test, Q4_knn.

Q4X_f4 = Q4_X.drop(["entropy"], axis=1)
Q4_X_train, Q4_X_test, Q4_y_train, Q4_y_test = train_test_split(Q4X_f4, Q4_y,
Q4_knn = KNeighborsClassifier(n_neighbors=5)
```



```
Q4_knn.fit(Q4_X_train, Q4_y_train)
print("if I drop", "entropy", "the accuracy is", accuracy_score(Q4_y_test, Q4_knn.p

if I drop variance the accuracy is 0.9548104956268222
if I drop skewness the accuracy is 0.9810495626822158
if I drop curtosis the accuracy is 0.9723032069970845
if I drop entropy the accuracy is 0.9927113702623906
```

Question#4.2 did accuracy increase in any of the 4 cases compared with accuracy when all 4 features are used?

From my point of view, when I drop the entropy, the accuracy are higher than other 4.

Question#4.3 which feature, when removed, contributed the most to loss of accuracy?

From my point of view, when I drop the variance, the accuracy are loss accuracy most obvious.

Question#4.4. which feature, when removed, contributed the least to loss of accuracy?

From my point of view, when I drop the entropy, the accuracy are loss accuracy most obvious.

Question#5.1 Use 50/50 split to generate new Xtrain and Xtest. Train your logistic regression classifier on Xtrain and compute its accuracy for Xtest

```
In [12]: Q5_X=data_Q3[["variance", "skewness", "curtosis", "entropy"]]
Q5_y=data_Q3["class"]
scaler = StandardScaler()
scaler.fit(Q5_X)
Q5_X = scaler.transform(Q5_X)
Q5_X_train, Q5_X_test, Q5_y_train, Q5_y_test = train_test_split(Q5_X, Q5_y, tes
log_reg_classifier = LogisticRegression()
log_reg_classifier.fit(Q5_X_train, Q5_y_train)
#predicted = log_reg_classifier.predict(Q5_X_test)
Q5_accuracy = log_reg_classifier.score(Q5_X_test, Q5_y_test)
print("the accuracy for x_test is", Q5_accuracy)
```

the accuracy for x_test is 0.9795918367346939

Question#5.2 summarize your performance measures in the table

```
In [13]: Q5_TN, Q5_FP, Q5_FN, Q5_TP = confusion_matrix(Q5_y_test, log_reg_classifier.pre
Q5_TPR=Q5_TP/(Q5_TP+Q5_FN)
Q5_TNR=Q5_TN/(Q5_TN+Q5_FP)

Q5_d={"Classifier":pd.Series(["predict"]),
      "TP":pd.Series([Q5_TP]),
      "FP":pd.Series([Q5_FP]),
      "TN":pd.Series([Q5_TN]),
      "FN":pd.Series([Q5_FN]),
      "ACC":pd.Series([Q5_accuracy]),
      "TPR":pd.Series([Q5_TPR]),
      "TNR":pd.Series([Q5_TNR])}
Q5_df=pd.DataFrame(Q5_d)
print(Q5_df)
```

	Classifier	TP	FP	TN	FN	ACC	TPR	TNR
0	predict	310	11	362	3	0.979592	0.990415	0.970509

Question#5.3 is your logistic regression better than your simple classifier for any of the measures from the previous table?

Yes, logistic regression is better than simple classifier in terms of TPR, TNR or accuracy

Question#5.4. is your logistic regression better than your k-NN classifier (using the best k*) for any of the measures from the previous table?

From the results of my data set. KNN still has higher accuracy than logistic regression, TPR and TNR

Question#5.5. consider a bill x that contains the last 4 digits of your BUID as feature values. What is the class label predicted for this bill x by logistic regression? Is it the same label as predicted by k-NN?

```
In [14]: print(log_reg_classifier.predict(df))
         #the answer is same with knn
```

```
[0]
```

```
/Users/eamonhu/opt/anaconda3/lib/python3.9/site-packages/sklearn/base.py:443:
UserWarning: X has feature names, but LogisticRegression was fitted without fe
ature names
  warnings.warn(
```

it's same as knn reawers

Question#6.1. For each of the four features f_1, \dots, f_4 , generate new Xtrain and Xtest and drop that feature from both Xtrain and Xtest. Train your logistic regression classifier on the "truncated" Xtrain and predict labels on "truncated" Xtest using just 3 remaining features. You will repeat this for 4 cases: (1) just f_1 is missing, (2) just f_2 missing, (3) just f_3 missing and (4) just f_4 is missing. Compute the accuracy for each of these scenarios.

```
In [15]: data_Q6=pd.read_csv("data_banknote_authentication.csv")
Q6_X=data_Q6[["variance","skewness","curtosis","entropy"]]
Q6_y=data_Q6["class"]
col=["variance","skewness","curtosis","entropy"]
#for i in range(len(col)):
Q6X_f1=Q6_X.drop(["variance"],axis=1)
Q6_X_train, Q6_X_test, Q6_y_train, Q6_y_test = train_test_split(Q6X_f1, Q6_y, t
Q6_f1_log_reg_classifier = LogisticRegression()
Q6_f1_log_reg_classifier.fit(Q6_X_train, Q6_y_train)
print("if I drop","variance","the accurcy is",accuracy_score(Q6_y_test, Q6_f1_1
#print(Q4X_1.columns.values)

Q6X_f2=Q6_X.drop(["skewness"],axis=1)
Q6_X_train, Q6_X_test, Q6_y_train, Q6_y_test = train_test_split(Q6X_f2, Q6_y, t
Q6_f2_log_reg_classifier = LogisticRegression()
Q6_f2_log_reg_classifier.fit(Q6_X_train, Q6_y_train)
print("if I drop","skewness","the accurcy is",accuracy_score(Q6_y_test, Q6_f2_1
```

```

Q6X_f3=Q6_X.drop(["curtosis"],axis=1)
Q6_X_train, Q6_X_test, Q6_y_train, Q6_y_test = train_test_split(Q6X_f3, Q6_y, t
Q6_f3_log_reg_classifier = LogisticRegression()
Q6_f3_log_reg_classifier.fit(Q6_X_train, Q6_y_train)
print("if I drop", "curtosis", "the accuracy is", accuracy_score(Q6_y_test, Q6_f3_l

Q6X_f4=Q6_X.drop(["entropy"],axis=1)
Q6_X_train, Q6_X_test, Q6_y_train, Q6_y_test = train_test_split(Q6X_f4, Q6_y, t
Q6_f4_log_reg_classifier = LogisticRegression()
Q6_f4_log_reg_classifier.fit(Q6_X_train, Q6_y_train)
print("if I drop", "entropy", "the accuracy is", accuracy_score(Q6_y_test, Q6_f4_lc

if I drop variance the accuracy is 0.7973760932944607
if I drop skewness the accuracy is 0.9037900874635568
if I drop curtosis the accuracy is 0.8658892128279884
if I drop entropy the accuracy is 0.9912536443148688

```

Question6.2. did accuracy increase in any of the 4 cases compared with accuracy when all 4 features are used?

when I drop entropy the accuracy are highest in 4 features

Question#6.3. which feature, when removed, contributed the most to loss of accuracy?

From my point of view, when I drop the variance, the accuracy are loss accuracy most obvious.

Question#6.4. which feature, when removed, contributed the least to loss of accuracy?

From my point of view, when I drop the entropy, the accuracy are loss accuracy most obvious.

Question#6.5 is relative significance of features the same as you obtained using k-NN?

From the results of my data set. KNN still has higher accuracy than logistic regression, TPR and TNR