

CENG 223

Discrete Computational Structures

Fall 2024-2025

Take Home Exam 1

Due date: 28/10/2024, Monday, 23.59

Questions

Q1. (20 pts)

a) Determine whether the following statement is a tautology, a contradiction, or neither by using a truth table. (10 pts)

$$(\neg p \wedge q) \vee ((p \wedge (p \vee q)) \wedge \neg q)$$

b) Show that following expressions are logically equivalent, give reference to every law in each step. (10 pts)

$$p \vee (\neg q \rightarrow (p \wedge r)) \equiv (p \vee q)$$

Q2. Translate the following statements into predicate logic using the predicates. (30 pts)

- $S(x)$: x is a student.
- $C(y)$: y is a course.
- $E(x, y)$: Student x is enrolled in course y .
- $P(x, y)$: Student x has passed course y .
- $R(y, z)$: Course y is a prerequisite for course z .

- a) "Every student who is enrolled in a course has passed all its prerequisites." (5 pts)
- b) "There exists a student who is enrolled in exactly one course." (5 pts)
- c) "For every course, if a student has passed it, then the student has passed all its prerequisites." (5 pts)
- d) "Some students have not passed all the courses they are enrolled in." (5 pts)
- e) "There exists a course that no student has passed." (5 pts)
- f) "Every course has at least one student who is either enrolled in it or has passed it." (5 pts)

Q3. Prove the following claims by natural deduction. Use only the natural deduction rules $\vee, \wedge, \rightarrow, \neg$, introduction and elimination. If you attempt to make use of a lemma or equivalence, you need to prove it by natural deduction too. **(25 pts)**

$$p \rightarrow (q \vee r), \neg r \wedge \neg s, q \rightarrow s \vdash \neg p$$

Q4. (25 pts)

Assume the following denotions;

- $P(x)$: x is a philosopher.
- $S(x)$: x is a scientist.
- $K(x)$: x knows everything.

The following premises are given;

- Some philosophers are scientists.
- All philosophers know everything.

with these premises we can claim "Some scientists know everything.". Based on this scenario, answer the following questions:

- a) Translate the two premises and the claim into predicate logic. (10 pts)
- b) Prove the claim using natural deduction rules. (15 pts)

Specifications

- You are expected to **write your answers in LaTeX format**. Handwritten solutions will not be accepted.
- Please do not skip the calculation steps. Show every step of your work.
- **There is no late policy**. If you submit your file later than the deadline, you will get zero.
- Cheating is forbidden. The violators will be punished according to the department regulations. Your files will be checked for plagiarism.
- Follow the course page on ODTUClass for any updates and clarifications. Please ask your questions on ODTUClass instead of e-mailing if they do not contain some part of the solution. If they contain, you can send an email to "haktans@ceng.metu.edu.tr".

Submission

Submissions will be done via ODTUClass. You are expected to submit a **single** PDF file named "**the1.pdf**".