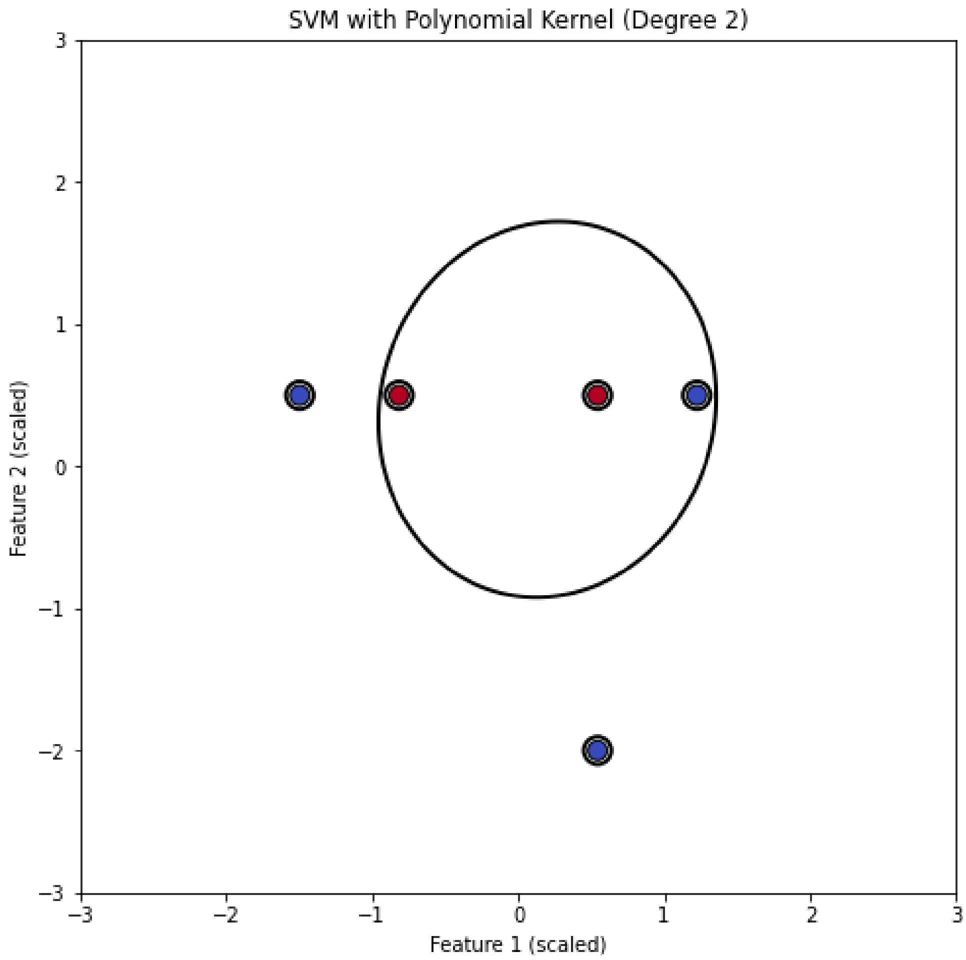
In [14]:
```python
# Import the necessary libraries
import numpy as np
import matplotlib.pyplot as plt
from sklearn.svm import SVC
from sklearn.preprocessing import StandardScaler

# Data points: two classes (+1 and -1)
X = np.array([[1, 1], [-1, 1], [2, 1], [1, -2], [-2, 1]])
y = np.array([1, 1, -1, -1, -1])

# Scale the data (standardization helps SVM performance)
scaler = StandardScaler()
X_scaled = scaler.fit_transform(X)

# Create and fit the SVM model with a polynomial kernel (degree 2)
svm_model = SVC(kernel='poly', degree=2, C=1.0)
svm_model.fit(X_scaled, y)

# Create a grid of points to plot decision boundary
xx, yy = np.meshgrid(np.linspace(-3, 3, 500), np.linspace(-3, 3, 500))
grid_points = np.c_[xx.ravel(), yy.ravel()]
grid_points_scaled = scaler.transform(grid_points)

# Predict decision boundary
Z = svm_model.decision_function(grid_points_scaled)
Z = Z.reshape(xx.shape)

# Plot the results
plt.figure(figsize=(8, 8))
plt.contour(xx, yy, Z, levels=[0], linewidths=2, colors='black')  # Decision
boundary

# Plot the data points
plt.scatter(X_scaled[:, 0], X_scaled[:, 1], c=y, cmap=plt.cm.coolwarm, s=100,
edgecolors='k')

# Highlight the support vectors
plt.scatter(svm_model.support_vectors_[:, 0], svm_model.support_vectors_[:,
1],
            s=200, facecolors='none', edgecolors='k', linewidths=2)

plt.title('SVM with Polynomial Kernel (Degree 2)')
plt.xlabel('Feature 1 (scaled)')
plt.ylabel('Feature 2 (scaled)')
plt.show()

# Extracting the support vectors from the trained SVM model
support_vectors = svm_model.support_vectors_

# Inverse transform to get back to the original scale
support_vectors_original = scaler.inverse_transform(support_vectors)

# Given Support Vectors
given_support_vectors = np.array([
    [1.22, 0.5],
    [0.54, -2.0],
    [-1.50, 0.5],
    [0.54, 0.5],
    [-0.82, 0.5]
])

# Display the calculated support vectors
```

```python
print("** Calculated Support Vectors in original coordinates:**")
for idx, sv in enumerate(support_vectors_original, start=1):
    print(f"Calculated Support-Vektor {idx}: {tuple(sv)}")

# Display the given support vectors
print("\n***** Given Support Vectors: *****")
for idx, sv in enumerate(given_support_vectors, start=1):
    print(f"Given Support-Vektor {idx}: {tuple(sv)}")



def polynomial_kernel(x, y, degree=2, coef=1):
    """Berechnet den polynomialen Kernel zwischen zwei Vektoren."""
    return (np.dot(x, y) + coef) ** degree

# Definierte Datenpunkte
data_points = np.array([[1, 1], [-1, 1], [2, 1], [1, -2], [-2, 1]])

# Berechnung der polynomialen Kernelwerte für alle Kombinationen der Datenpun
kte
kernel_matrix = np.zeros((data_points.shape[0], data_points.shape[0]))

for i in range(data_points.shape[0]):
    for j in range(data_points.shape[0]):
        kernel_matrix[i, j] = polynomial_kernel(data_points[i], data_points
[j])

# Ausgabe der Kernel-Matrix
print("\n **** Kernel-Matrix für die gegebenen Datenpunkte: ****")
print(kernel_matrix)
```




```
** Calculated Support Vectors in original coordinates:**
Calculated Support-Vektor 1: (1.9999999999999998, 1.0)
Calculated Support-Vektor 2: (1.0, -2.0)
Calculated Support-Vektor 3: (-2.0, 1.0)
Calculated Support-Vektor 4: (1.0, 1.0)
Calculated Support-Vektor 5: (-1.0, 1.0)

***** Given Support Vectors: *****
Given Support-Vektor 1: (1.22, 0.5)
Given Support-Vektor 2: (0.54, -2.0)
Given Support-Vektor 3: (-1.5, 0.5)
Given Support-Vektor 4: (0.54, 0.5)
Given Support-Vektor 5: (-0.82, 0.5)

 **** Kernel-Matrix für die gegebenen Datenpunkte: ****
[[ 9.  1. 16.  0.  0.]
 [ 1.  9.  0.  4. 16.]
 [16.  0. 36.  1.  4.]
 [ 0.  4.  1. 36.  9.]
 [ 0. 16.  4.  9. 36.]]
```