Identifying Network Ties from Panel Data: Theory and an application to tax competition

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Working paper, 2023

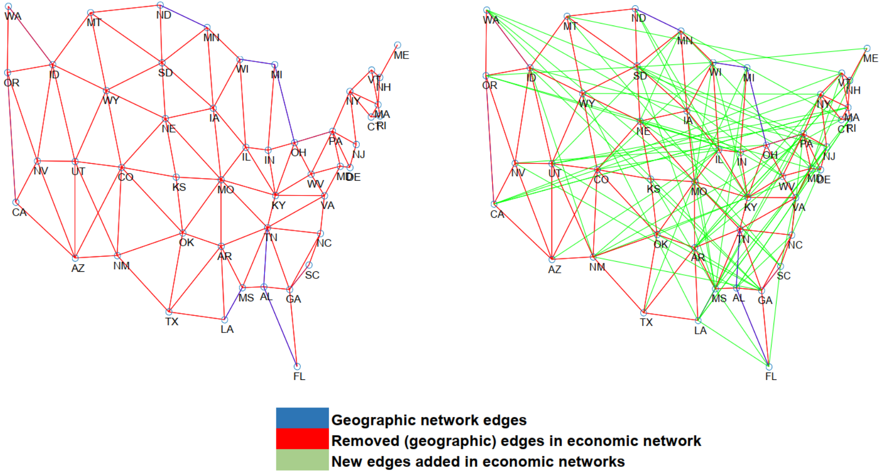
Presenter: Haoyang Li

Why this paper

- Detailed elucidation of the identification of unknown network
- Estimation method (GMM) for small samples (T=5)
- Comparisons with geographic networks

Geographic VS Economic network

Figure 1B: Network Graph of US States, Identified Economic Neighbors



Motivation

- Behaviour is shaped by social interactions between agents: Educational test scores, technology adoption, firms' networks, ties between jurisdictions, etc.
- Information on social ties missing or too hard to collect: Postulated or elicited networks remain imperfect solutions (Chandrasekhar and Lewis, 2011; De Paula, 2017)
- Validation of social ties infeasible.

This paper:
Deriving sufficient conditions for global identification of the entire structure without information on the network ties.

Identification – Set up

The model:

$$y_{it} = \rho_0 \sum_{j=1}^{N} W_{0,ij} y_{jt} + \beta_0 x_{it} + \gamma_0 \sum_{j=1}^{N} W_{0,ij} x_{jt} + \epsilon_{it}. \tag{1}$$

Or in matrix notation:

$$y_t = \rho_0 W_0 y_t + \beta_0 x_t + \gamma_0 W_0 x_t + \epsilon_t \tag{2}$$

Assumptions:

- $\mathbb{E}(\epsilon_t | x_t) = 0$ or $\mathbb{E}(\epsilon_t | z_t) = 0$
- W_0 predetermined and constant
- N fixed and possibly large

Parameters: $\theta_0 = (W_0, \rho_0, \beta_0, \gamma_0)$.

Identification – Reduced form

$$y_t = \Pi_0 x_t + \upsilon_t, \tag{3}$$

where

$$\Pi_0 = (I - \rho_0 W_0)^{-1}(\beta_0 I + \gamma_0 W_0),$$
$$\upsilon_t = (I - \rho_0 W_0)^{-1}\epsilon_t.$$

Identification strategy:
How changes in x_{it} reverberate through the system and impact y_t.
Summarized by entries of Π_0.

Identification – Six assumptions

Assumptions underpinning main identification results:

(**A1**) $(W_0)_{ii} = 0, i = 1, 2, ..., N$.
Now parameter vector $\theta = (W_{12}, ..., W_{N,N-1}, \rho, \beta, \gamma)' \in \mathbb{R}^m$, where $m = N(N-1) + 3$.

(A2) $\sum_{j=1}^{N} |\rho_0 (W_0)_{ij}| < 1, \ ||W_0|| < C, \ |\rho_0| < 1$.
Implications:

- maximum eigenvalues of $\rho_0 W_0$ is less than 1.
- $(I - \rho_0 W_0)$ is non-singular.
- $(I - \rho_0 W_0) = \sum_{j=0}^{\infty} (\rho_0 W_0)^j$ is appropriate.

Identification – Six assumptions

Expanding expression for $\Pi(\theta_0)$:

$$\Pi(\theta_0) = (I - \rho_0 W_0)^{-1}(\beta_0 I + \gamma_0 W_0) \tag{4}$$

$$= \beta_0 I + (\rho_0\beta_0 + \gamma_0)\sum_{k=1}^{\infty} \rho_0^{k-1} W_0^k \tag{5}$$

(A3) $(\rho_0\beta_0 + \gamma_0) \neq 0$.

At least one row of W_0 sums up to a fixed and known number:
(A4) There is an i such that $\sum_j (W_0)_{ij} = 1$.
Required for identification of ρ_0 and γ_0.

Identification – Six assumptions

(A5) There exists l, k such that $(W_0^2)_{ll} \neq (W_0^2)_{kk}$, i.e. The diagonal of W_0^2 is not proportional to ι, an $N \times 1$ vector of ones.

Required for identification of ρ_0 and γ_0 when W_0 is known (Bramoullé et al., 2009). An example for identification with assumptions above:

$$\Pi_0 = \frac{1}{455}\begin{bmatrix} 275 & 310 & 0 \\ 310 & 275 & 0 \\ 0 & 0 & 182 \end{bmatrix} \Rightarrow W_0 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

From (3,3) element of Π_0: $\beta_0 = \frac{182}{455} = 0.4$.
Note that $(I - \rho_0 W_0)\Pi_0 = \beta_0 I + \gamma_0 W_0$.
From (1,1) elements of both matrices: $\frac{275}{455} - \rho_0 \frac{310}{455} = 0.4 \Rightarrow \rho_0 = 0.3$.
From (1,2) elements: $\gamma_0 = \frac{310}{455} - 0.3\frac{275}{455} = 0.5$.

(A6) y_t and x_t are observed for individuals $i = 1, 2..., N$, and instances $t = 1, 2, ..., T$, and the network W_0 does not depend on t.

Identification – Main results

Let $\lambda_{0,j}$ denote an eigenvalue of W_0 with eigenvector $v_{0,j}$ for $j = 1, 2, ..., N$. Under assumptions (A2) and (A3):

$$\begin{aligned} \Pi_0 v_{0,j} &= \beta_0 v_{0,j} + (\rho_0\beta_0 + \gamma_0) \sum_{k=1}^{\infty} \rho_0^{k-1} W_0^k v_{0,j} \\ &= \left[\beta_0 + (\rho_0\beta_0 + \gamma_0) \sum_{k=1}^{\infty} \rho_0^{k-1} \lambda_{0,j}^k\right] v_{0,j} \\ &= \frac{\beta_0 + \gamma_0\lambda_{0,j}}{1 - \rho_0\lambda_{0,j}} v_{0,j} \end{aligned} \tag{6}$$

Eigencentralities may be identified from Π_0 even when W_0 is not identified. Eigencentralities allow a mapping back to underlying models of social interactions (De Paula, 2017; Jackson et al., 2017).

Identification – Main results

$\theta_0 \in \Theta$ is **locally identified** under assumptions (A1) - (A6).
Identifying the sign of $(\rho_0\beta_0 + \gamma_0)$ is required for **global identification**.

Corollary 3. *Assume A(1)-A(6). If* $\rho_0 > 0 and (W_0)_{ij} \geq 0$*, the model is globally identified.*

Corollary 4. *Assume A(1)-A(6),* $(W_0)_{ij} \geq 0$ *and* W_0 *is irreducible. If* W_0 *has at least two real eigenvalues or* $|\rho_0| < \sqrt{2}/2$*, then the model is globally identified.*
Corollary 4 rules out cases where the network is not connected.

Identification – Extensions

Individual fixed effects:

$$y_t = \rho_0 W_0 y_t + \beta_0 x_t + \gamma_0 W_0 x_t + \alpha^* + \epsilon_t \tag{7}$$

Common shocks:

$$y_t = \rho_0 W_0 y_t + \beta_0 x_t + \gamma_0 W_0 x_t + \alpha_t \iota + \epsilon_t \tag{8}$$

(A4') $\sum_j^N (W_0)_{ij} = 1$ for all $i = 1, 2, ..., N$. (row sum normalization)

Let $H = \frac{1}{N}\iota\iota'$,

$$(I - \rho_0 W_0)^{-1}\alpha_t \iota = \frac{\alpha_t}{1-\rho_0}\iota \Rightarrow (I - H)(I - \rho_0 W_0)^{-1}\alpha_t \iota = 0$$

Identification – Extensions

More entensions:

- Multivariate covariates: if at least one $W_x = \gamma_0 W_0$.
- Heterogenous β_0: If at least one $\beta_{0,k}$ is homogeneous.
- Time-varying W: Gaussian kernel throughout the entire periods (Kapetanios et al., 2019).

Estimation

Parameter vector $\theta = (W_{12}, ..., W_{N,N-1}, \rho, \beta, \gamma)' \in \mathbb{R}^m$, where $m = N(N-1) + 3$.

OLS requires $m \ll NT \Rightarrow N \ll T$. Instead, high dimensional techniques can be used with **sparsity assumption**:
W_0 is sparse if $\tilde{m} \ll NT$, where $\tilde{m}$ is the number of non-zero elements.

Adaptive Elastic Net GMM estimator (Caner and Zhang, 2014) converges at rate:

$$\sqrt{NT/\tilde{m}} = \sqrt{NT/[dN(N-1)+K]} = O(\sqrt{T/dN})$$

Estimation – GMM Estimator

Penalized GMM objective function:

$$G_{NT}(\theta, p) \equiv g_{NT}(\theta)' M_T g_{NT}(\theta) + p_1 \sum_{\substack{i,j=1 \\ i \neq j}}^{N} |W_{i,j}| + p_2 \sum_{\substack{i,j=1 \\ i \neq j}}^{N} |W_{i,j}|^2 \qquad (9)$$

where $g_{NT}(\theta) = \sum_{t=1}^{T} [x_{1t} e_t(\theta)' \ldots x_{Nt} e_t(\theta)']'$ is an $N^2 \times 1$ vector.

L1 norm term shrinks parameters to zero.
L2 norm term works better when covariates are correlated (Zou and Zhang, 2009).

$N(N-1) + 3$ parameters: computationally expensive.

Estimation – Two step optimization

Note that for any give ρ, β, γ, the residual is linear in W:

$$e_t(\theta) = y_t - X_t\beta - W(\rho y_t + x_t\gamma) = \tilde{y}_t(\beta) - W\tilde{x}_t(\rho, \gamma).$$

This motives a two step routine:

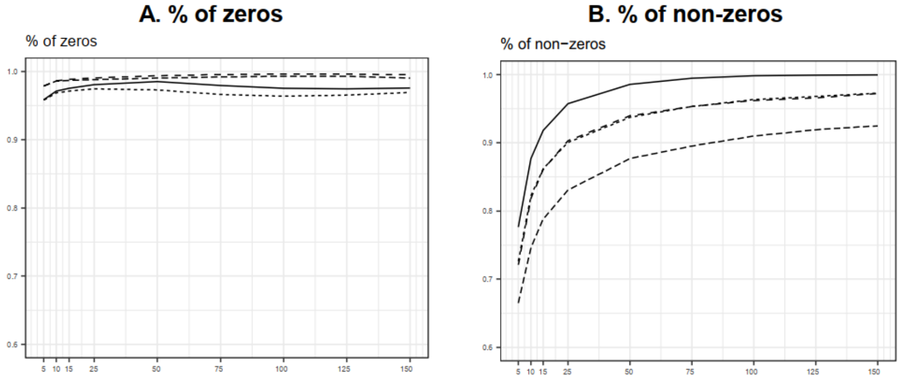
$$\min_{\theta \in \mathbf{R}^m} G_{NT}(\theta, p) = \min_{(\rho,\beta,\gamma) \in \mathbf{R}^3} \left[\min_{W_{ij} \in \mathbf{R}^{N(N-1)}} G_{NT}(\theta, p) \right]$$

A computationally efficient solution through Least Angle regression (LARS) is available for expression in brackets.

Finally, fix the support and estimate without penalization (Post-LASSO).

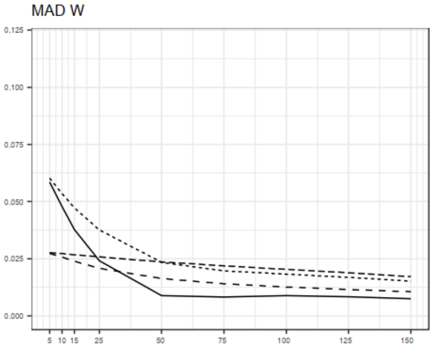
The estimator is asymptotically normal (Caner and Zhang, 2014), hence hypothesis testing and inference on θ can be conducted.

Figure A1: Simulation Results, Adaptive Elastic Net GMM

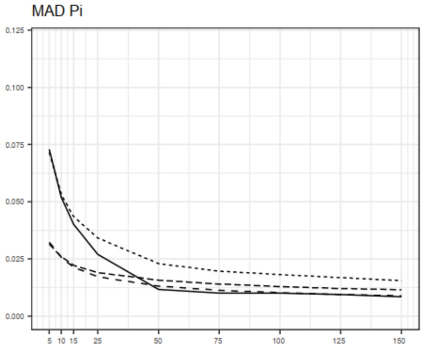


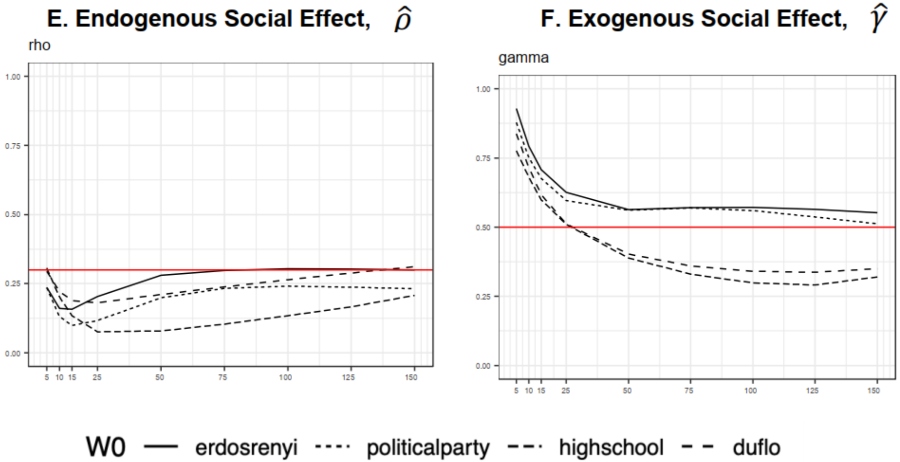
MC results

C. Mean Absolute Deviation of $\widehat{W}$



D. Mean Absolute Deviation of $\widehat{\Pi}$




E. Endogenous Social Effect, $\hat{\rho}$
rho
1.00
0.75
0.50
0.25
0.00
5 10 15 25 50 75 100 125 150
F. Exogenous Social Effect, $\hat{\gamma}$
gamma
1.00
0.75
0.50
0.25
0.00
5 10 15 25 50 75 100 125 150
W0
erdosrenyi
politicalparty
highschool
duflo

MC results

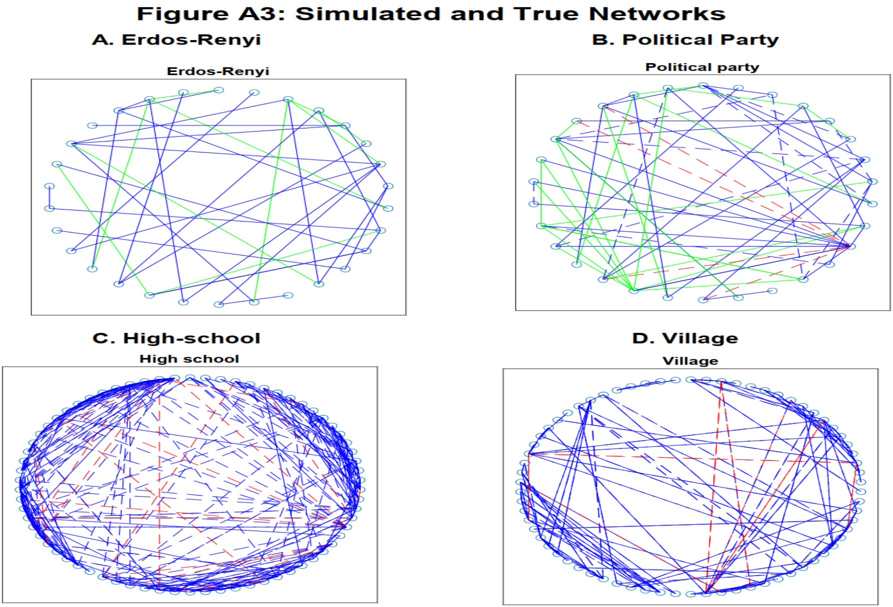


Figure A3: Simulated and True Networks
A. Erdos-Renyi
Erdos-Renyi
B. Political Party
Political party
C. High-school
High school
D. Village
Village

Table A2: Simulation Results, Adaptive Elastic Net GMM, Alternative Network Sizes

	A. Erdos-Renyi									B. Political party								
	N = 15			N = 30			N = 50			N = 15			N = 30			N = 50		
	T=50	100	150	T=50	100	150	T=50	100	150	T=50	100	150	T=50	100	150	T=50	100	150
% True Zeroes	.945	.960	.974	.985	.975	.976	.997	.997	.991	.939	.958	.975	.973	.964	.969	.993	.993	.985
	(.016)	(.014)	(.011)	(.006)	(.006)	(.005)	(.001)	(.001)	(.003)	(.017)	(.015)	(.012)	(.007)	(.007)	(.006)	(.002)	(.002)	(.003)
% True Non-Zeroes	.973	.996	1.000	.986	.998	1.000	.993	.999	1.000	.949	.980	.993	.937	.964	.973	.974	.989	.995
	(.045)	(.017)	(.004)	(.023)	(.007)	(.004)	(.013)	(.004)	(.001)	(.063)	(.038)	(.022)	(.048)	(.037)	(.032)	(.025)	(.016)	(.012)
$MAD(\widehat{W})$	.027	.014	.008	.009	.009	.008	.004	.001	.003	.037	.024	.019	.023	.018	.015	.013	.007	.007
	(.009)	(.006)	(.004)	(.004)	(.002)	(.002)	(.002)	(.001)	(.001)	(.008)	(.005)	(.004)	(.004)	(.003)	(.002)	(.002)	(.001)	(.001)
$MAD(\widehat{\Pi})$	.030	.017	.011	.012	.010	.008	.005	.002	.003	.038	.026	.021	.023	.018	.016	.012	.007	.008
	(.008)	(.005)	(.003)	(.004)	(.002)	(.002)	(.002)	(.001)	(.001)	(.007)	(.005)	(.003)	(.004)	(.002)	(.002)	(.002)	(.001)	(.001)
$\hat{\rho}$	.270	.281	.282	.280	.304	.300	.279	.298	.309	.242	.250	.246	.199	.241	.232	.205	.250	.272
	(.070)	(.046)	(.037)	(.039)	(.030)	(.025)	(.026)	(.018)	(.017)	(.082)	(.053)	(.043)	(.056)	(.042)	(.036)	(.033)	(.023)	(.022)
$\hat{\beta}$	.409	.405	.403	.403	.402	.402	.402	.400	.400	.411	.404	.399	.404	.402	.401	.404	.400	.401
	(.043)	(.029)	(.024)	(.030)	(.020)	(.017)	(.024)	(.016)	(.013)	(.046)	(.030)	(.025)	(.031)	(.021)	(.017)	(.025)	(.016)	(.013)
$\hat{\gamma}$	.618	.549	.518	.563	.572	.552	.519	.505	.529	.593	.508	.471	.561	.560	.512	.483	.469	.513
	(.071)	(.045)	(.031)	(.043)	(.034)	(.027)	(.025)	(.019)	(.019)	(.079)	(.051)	(.037)	(.057)	(.042)	(.037)	(.028)	(.023)	(.024)

Tax competition

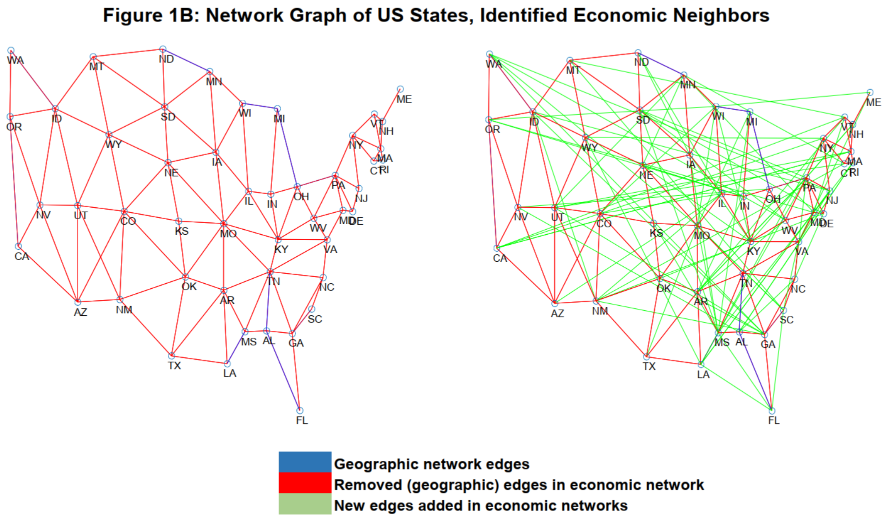


Figure 2: Out-degree Distribution

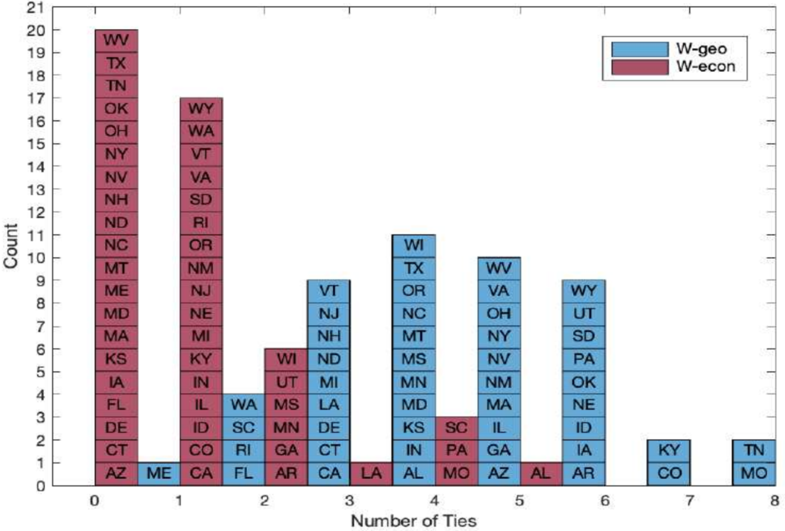


Figure 6: General Equilibrium Impacts of South Carolina Tax Rises

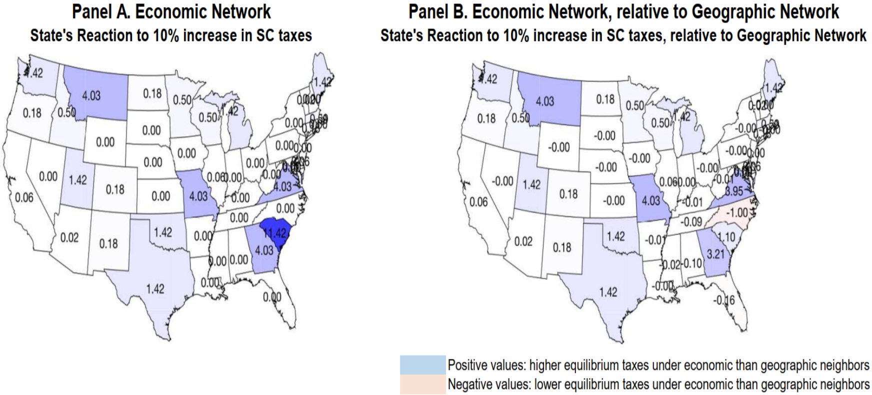


Table 4: Predicting Links to Economic Neighbors

Linear Probability Model

Dependent variable = 1 if Economic Link Between States Identified, = 0 if geographically linked

Robust standard errors in parentheses

	Distance	Economic and Demographic Homophily	Labor Mobility	Yardstick Competition	Tax Havens	Fixed Effects
	(1)	(2)	(3)	(4)	(5)	(6)
Distance	.890***	.921***	.921***	.940***	.940***	1.287***
	(.081)	(.082)	(.082)	(.091)	(.091)	(.120)
Distance sq.	-.135***	-.139***	-.139***	-.144***	-.145***	-.255***
	(.025)	(.024)	(.025)	(.027)	(.027)	(.039)
GDP Homophily		-.063	-.063	-.083	-.092	-.219
		(.078)	(.079)	(.082)	(.085)	(.348)
Demographic Homophily		-1.745***	-1.745***	-1.047*	-.960	.579
		(.552)	(.554)	(.605)	(.604)	(1.240)
Net Migration			-.033	-.020	-.185	-.039
			(.603)	(.577)	(.612)	(1.48)
Political Homophily				-.337***	-.321***	-.287*
				(.120)	(.119)	(.155)
Tax Haven					-.093**	
					(.036)	
Origin and destination FE	No	No	No	No	No	Yes
Adjusted R-squared	.664	.664	.664	.651	.657	.831
Observations	254	254	254	212	212	212

Thank you!

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