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What makes an asset useful?

1 Paper Review

1.1 Fundamental Questions

The main limitation of the existing literature in analyzing the value in investing in multiple assets is that two fundamentally different questions are often entangled:

- How much incremental value can an investment manager derive from trading a given additional asset
- How should an investment manager go about extracting the potential incremental value inherent to trading a specific additional asset?

Naturally, if one is able to figure out how to extract value out of trading a given additional asset, then it means that the additional asset does indeed add intrinsic value to the existing pool. However, attempting to determine optimal allocation within a large pool of related assets can pose serious numerical challenges, such as ill-conditioning of the covariance matrix, capable of impairing the optimization process. In such situations, and more generally, one would be better off first determining how much incremental value can a new asset add to an existing pool, and then determining how to extract incremental value, if any.

Definition. Asset
We denote an asset as any investment resulting in a periodic stream of returns, realized or marked-to-market.

We identify an asset by its time series of returns, and we consider that two assets having identical time series of returns are identical for all investment purposes. In particular, our approach to quantifying the usefulness of an asset primarily relies on its time series of returns. Moreover, we assume all returns time series have the same sample frequency (i.e., daily, quarterly, minutely).

To avoid any confusion, we use the expression 'pool of assets' to denote a universe or set of assets, and we reserve the expressions 'portfolio of assets' and 'fund' to any specific allocation of capital across assets in a pool. Additionally, we denote 'static portfolio' any portfolio of assets whose target capital allocation does not change over time.

Usefulness of an asset can only be considered relative to a reference pool of assets

WE argue that for an asset to be useful, it needs

1. sufficiently add to the diversification of the reference pool of assets
2. returns of time series be sufficiently predictable
3. mitigate tail events
4. suitable for passive investment

We need a framework for quantifying incremental diversification respective to predictability of returns.

1.2 What exactly makes an asset useful?

No matter how high an asset's returns are, if the asset's time series of returns can be mimicked or replicated using assets the investment manager already has access to, and/or factors or benchmarks the investment manager would like to avoid exposure to, then it is fair to conclude that the new asset presents little incremental usefulness to the investment manager. Thus, the usefulness of an asset to an investment manager can be thought of as the incremental usefulness that the asset adds to a reference pool of assets and/or factors/benchmarks the investment manager trades and/or would like to avoid exposure to.

1.2.1 Incremental Diversification

Perhaps the most fundamental reason why an investment manager might want to consider broadening the pool of assets he/she trades is diversification.

Definition. Diversification

Throughout this paper we denote **diversification** as the act through which one aims to reducing the level of risk of portfolio, for the same level of expected return, by adding one or more assets.

We note however that not all new assets have the same potential for **reducing the risk of a portfolio**; some provide more diversification potential than others. We note that, as much as the actual reduction of risk incurred by trading a new asset depends on the specific allocation of capital across all assets (including the new one), in some cases, the new asset will not provide any risk reduction (for a given level of expected return) irrespective of the allocation.

An important criteria of usefulness of an asset is therefore the extent to which it can provide a reduction of risk (for a given level of expected return), which is a function of the reference pool of assets, and is independent of asset allocation.

1.2.2 Predictability of Returns

Central to most popular portfolio optimization approaches, is the need to estimate expected values of asset returns, which practitioners typically do by forecasting future asset returns [3, 4]. However, if a time series of returns is pure noise, any attempt to forecast it would be vain, and most portfolio optimization processes would fail to make use of the new asset. More precisely, the best forecast for future returns of the new asset would be the average of all past returns, which would be constant and, mostly likely, very close to 0. Hence, rational portfolio allocation procedures wouldn't allocate capital to the new asset as it will be thought to have too low a return per unit of risk. Therefore, for a new asset to be useful, its time series of returns needs to be sufficiently predictable.

1.2.3 Reduced Tail Risk

Another reason that can motivate investment managers to add a new asset to their trading universe is to mitigate the likelihood that their portfolio can undergo a significant idiosyncratic move, thereby possibly causing their investors to panic and withdraw assets. Unlike the incremental diversification requirement previously discussed, this requirement focuses on the tail (or extreme) events.

The rationale for widening the trading universe as a way of mitigating tail events is that, doing so could reduce the proportion of total exposure that is concentrated in a single asset, thereby reducing the sensitivity of the overall portfolio to idiosyncratic shocks. However, trading more assets doesn't always result in fewer tail events. If the new asset has lighter tails than, or is negatively correlated with assets in the reference pool, its idiosyncratic moves might be lighter or coincide and cancel out shocks in other assets in the reference pool so that, overall, including the new asset in the trading universe would reduce tail events.

1.2.4 Suitability for passive investment

Perhaps the most wide-spread expectation one can have of an asset is that it appreciates over time. We slightly relax this requirement, and instead require of an asset that it be suitable for passive investment in order to be considered useful. In other words, it should be possible to find strategies that do not change target holdings in the asset too often —for instance buy-and-hold and short-and-hold strategies— and that perform well in the long-run.

1.2.5 Relative Importance of Usefulness Criteria

Incremental diversification and predictability of returns are primary criteria of incremental usefulness in that, if either one is not met, no investment manager will find the new asset incrementally useful. A new asset whose returns can be perfectly replicated using existing assets intuitively wouldn't add any utility to the reference pool of assets.

The alert reader might be thinking of Exchange Traded Funds (ETFs) as a counter-example. We note however that investors who prefer buying/selling ETFs over transacting in the underlying assets directly, do so for multiple reasons (e.g. lower execution costs, more favorable taxation, etc.), all of which eventually translate to a difference in net returns between the ETF and the tracked portfolio of underlying assets.

Similarly, if the new asset has returns time series that is pure noise, active investment managers would have no hope of forecasting returns, and passive investment managers would have no reason to believe that the new asset would either appreciate over time or depreciate over time –both being equally likely.

The other two criteria on the other hand are secondary in that a new asset could be useful to some investment managers, even if those criteria are not met. For instance, a new asset that exhibits strongly predictable returns can be exploited by an active investment manager, even if it isn't suitable for passive investment. Additionally, in case the new asset undergoes more or sharper idiosyncratic moves than assets in the reference pool, but otherwise has highly predictable returns and significantly diversifies the reference pool, an active investment manager might still want to consider adding it to the reference pool, and hedging the incremental tail risk with derivative products.

High predictability of returns and high incremental diversification provide a guarantee that, although buyand-hold and short-and-hold strategies on the new asset do not perform particularly well, it is possible to find a (more active) trading strategy on the new asset that both performs well and yields a returns stream that is decorrelated with existing assets.

1.3 Quantifying Incremental Diversification

In order to motivate our approach to quantifying incremental diversification, let us first make precise scenarios in which we would intuitively conclude an asset incrementally diversifies a reference pool, and cases where we would consider the new asset to be redundant. Thus, assets whose time series of returns are impossible to replicate using assets and factors in the reference pool should have the highest incremental diversification. Similarly, assets whose returns are easy to replicate using assets and factors in the reference pool should have the lowest incremental diversification.

1.3.1 Intuition

If it is easy to replicate the stream of returns of the new asset using assets and other factors in the reference pool, then the new asset doesn't add diversification to the reference pool. To affine our intuition, let us consider some concrete examples that will help derive stylized features that a suitable quantitative measure of incremental diversification should exhibit.

We start by considering a fund π whose (one-period) returns we assume are independent across time, and drawn from the same random variable r_π with mean μ and standard deviation σ. We consider another asset A, whose (one-period) returns we assume are also independent across time, and drawn from a random variable r_A whose mean μ and standard deviation

σ are the same as that of r_π, and we denote ρ the correlation between r_π and r_A. Furthermore, we denote π' the portfolio consisting of investing a fraction $0 \leq w \leq 1$ of our wealth in fund π and the rest in buying asset A . The one-period returns of the new portfolio are easily found to be independent draws across time from the random variable

$$r_{\pi'} = w r_\pi + (1-w) r_A$$

whose mean and standard deviation read

$$\mu_{\pi'} = \mathbb{E}[r_{\pi'}] = \mu$$

and

$$\begin{aligned} \sigma_{\pi'} &= \sqrt{\mathrm{Var}(r_{\pi'})} \\ &= \sigma\sqrt{1 - 2w(1-w)(1-\rho)} \\ &\leq \sigma \end{aligned}$$

where the last inequality stems from the fact that $0 \leq w \leq 1$ and $\rho \leq 1$. It is worth noting that the inequality is always strict, unless either $2w(1-w) = 0$, which corresponds to only investing in π or A, or perfect correlation.

This leads us to the most basic, yet fundamental, observation about diversification. Expected return and variance being equal, adding a new asset to a portfolio always results in a higher return per unit of risk, unless the new asset is perfectly correlated with the existing portfolio, in which case expected mean and standard deviation of returns remain unchanged. Here, we use the standard deviation of returns as a measure of risk. However, our aim is not to equate the notion of risk to the standard deviation of returns, which might fail to capture subtle tail behaviors in non-Gaussian distributions.

Definition. Let P be a reference pool of assets with returns x and let A be an asset not in P, with return r_A. We denote the portfolio that best replicates A using P as the portfolio of assets in P whose allocation, which we denote w^*, satisfies

$$\begin{aligned} w^* &= \arg\min_w \mathrm{Var}(r_A - r'_A) \\ &= \arg\min_w \mathrm{Var}(r_A - w^T x) \end{aligned} \tag{1}$$

where r_f denotes the risk-free interest rate, and

$$r'_A = w^T x + (1 - 1^T w) r_f$$

denote the return of the portfolio with allocation w across assets in P and whose excess cash (respect to net leverage) earns (resp. is funded by borrowing at the risk-free interest rate).

Optimization problem in equation (2) is quadratic and is easily found to have solution

$$w^* = \mathrm{Cov}(x,x)^{-1}\,\mathrm{Cov}(x, r_A)$$

Denoting r^*_A the return of the best replicating portfolio, it follows that

$$\begin{aligned} \mathrm{Cov}(r_A r^*_A) &= \mathrm{Var}(r^*_A) \\ &= \mathrm{Cov}(r_A, x)\,\mathrm{Cov}(x,x)^{-1}\,\mathrm{Cov}(x, r_A) \end{aligned}$$

Additionally, the residual return of the replication — which we also refer to as tracking error —, namely

$$r_A - r^*_A$$

has variance that is found to be

$$\mathrm{Var}(r_A - r^*_A) = \mathrm{Var}(r_A) - \mathrm{Var}(r^*_A)$$

Intuitively, a low variance of residual returns of replication indicates that we are able to replicate the stream of returns of the new asset using assets in the reference pool fairly well, which in turn implies, according to our foregoing guiding principle, a low potential for incremental diversification. In other words, a suitable quantitative measure of incremental diversification should never be high when the correlation between the return of the new asset and that of its best replicating portfolio is high, or when the variance of the return of the best replicating portfolio is high relative to that of the replicated asset.

Fact. *A good quantitative measure of the incremental diversification an asset A adds to a reference pool of assets should never be high when the correlation between the new asset A and the portfolio of assets in the reference pool that best replicates A is high.*

Remark. The alert reader might be wondering what to make of trading strategies such as pairs trading, that are only possible when some assets exhibit strong relationships. We note that these strategies are only profitable if the spread between the two assets forming the pair deviates enough from its equilibrium regime, certainly enough to cover transaction costs. The more the spread deviates from its equilibrium, the bigger the profit opportunity, and therefore the more useful both assets are. At the same time, **the farther the spread deviates from its equilibrium, the harder it is to exactly replicate the returns time series of one asset using that of the other, and therefore the more incremental diversification one asset adds to the other. Thus, successful pairs trading strategies are consistent with our discussion so far.** The subtlety here is that, to be useful, such strategies as pairs trading rely on both a valid long-term equilibrium model for the relationship between asset returns, and a potential for a large short-term deviation from the equilibrium model.

Additionally, we consider undesirable any trading strategy whose return over a time period is fully determined by current and past returns of assets in the reference pool, and current and past values of other factors in the reference pool.

Fact. *Let* $\{x_t\} = \{(x_t^1, ..., x_t^n)\}$ *be the time series of asset returns and factor values of a reference pool of* n *assets and factors, and* $\{y_t\}$ *the time series of returns of a new asset A. Good quantitative measure of the incremental diversification asset A adds to the reference pool should be the lowest when returns of the new asset A can be obtained as a function of present and past values of returns and factors in the pool, that is*

$$y_t = f(x_t, ..., x_{t-m}),$$

for some function f *and memory* $m \geq 0$*.*

This fact is, however, not to say that if two fund managers trade the same universe of assets, their funds do not provide diversification to the universe of assets they trade. Diversification doesn't arise solely as a result of what a fund manager trades, but also, and perhaps more importantly, as a result of **how** he/she trades. **Stylized Fact 3 simply implies that, if a fund manager solely trades based on current and past daily returns on a certain universe of assets, and current and past daily factor values, then his daily returns shouldn't be regarded as any different from the reference universe of assets and factors.** However, a different fund manager, trading the exact same universe of assets, but using more granular data or alternative data to drive his trading decision, will produce a fund that, as an asset, diversifies the universe of assets he/she trades. Crucially, because two fund managers trade the same universe of exchange-traded assets (e.g. stocks, bonds, futures etc.) does not mean that one cannot diversify the other! This is such an important distinction that we believe it warrants a next Stylized Fact.

Fact. *A good quantitative measure of incremental diversification should allow for manager diversification. That is, two funds with identical constituents but different time-varying allocations driven by different (random) signals, should be able to diversify each other, despite their identical constituents.*

Asset returns can be scaled up and down through leverage. Thus, the scale of a time series of returns should intuitively bear no relevance on whether the corresponding asset incrementally diversifies a reference pool, or more generally is incrementally useful.

Fact. *A good quantitative measure of incremental diversification should be scale-invariant. Equivalently,6 a good quantitative measure of incremental diversification should neither depend on the standard deviations of returns time series of assets in the reference pool, nor should it depend on the standard deviation of returns of the new asset.*

Reference:

1. https://arxiv.org/pdf/1806.08444.pdf