Econometrics (ECON 21020) – Winter 2020

Problem Set #2

Date Due: January 27, 2020

1. Stock and Watson, Exercises 3.2, 3.3, 3.4, 3.15, 3.16

2. Let X and U denote random variables, and suppose that $E[U|X] = 1$ and that $E[X] = 2$.

   (a) What is $E[U]$? Provide a proof of your answer.

   (b) What is $E[XU]$? Provide a proof of your answer.

   (c) What is $\text{Cov}[X, U]$? Provide a proof of your answer.

   (d) Is U mean independent of X? Explain briefly.

   (e) Is U uncorrelated with X? Explain briefly.

   (f) Do you have enough information to determine whether X is independent of U? Explain briefly.

3. Let $X_1, ..., X_n$ denote an i.i.d. sample of size n from X. Consider the estimator

$$\hat{\theta}_n = \sum_{i=1}^{n} a_i X_i$$

   for some constants $a_1, \ldots, a_n$.

   (a) Show that if $\hat{\theta}_n$ is an unbiased estimator of $E[X]$, then $\sum_{i=1}^{n} a_i = 1$.

   (b) Show that $\text{Var}[\hat{\theta}_n] = \text{Var}[X] \sum_{i=1}^{n} a_i^2$.

   (c) Find $a_1, \ldots, a_n$ that minimize $\text{Var}[\hat{\theta}_n]$ subject to the constraint that $\hat{\theta}_n$ is an unbiased estimator of $E[X]$.

4. Let $X_1, ..., X_n$ denote an i.i.d. sample of size n from X. Suppose $\text{Var}[X] < \infty$. For $1 \leq i \leq n$ define $Z_i = a + bX_i$ and $Z = a + bX$ for some constants a and b.

   (a) Show that $\bar{Z}_n = a + b\bar{X}_n$ and $\hat{\sigma}_Z^2 = b^2 \hat{\sigma}_X^2$.

   (b) Prove that $\bar{Z}_n$ is an unbiased estimator of $E[Z]$.

   (c) Prove that $\bar{Z}_n$ is a consistent estimator of $E[Z]$. (Hint: Is the function $g(t) = a + bt$ continuous?)

5. Let (X, Y) denote a random vector, and let $(X_1, Y_1), ..., (X_n, Y_n)$ denote an i.i.d. sample of size n from (X, Y). Suppose $\text{Var}[X] < \infty$ and $\text{Var}[Y] < \infty$. Consider estimating $E[X]E[Y]$ using the estimator $\bar{X}_n \bar{Y}_n$.

(a) Can you prove that $\bar{X}_n \bar{Y}_n$ is an unbiased estimator of $E[X]E[Y]$? Why or why not?

(b) Show that $\bar{X}_n \bar{Y}_n$ is a consistent estimator of $E[X]E[Y]$.