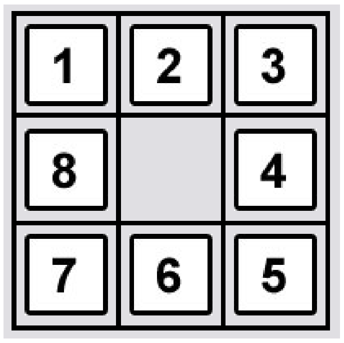

# 8-Puzzle Solver

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`https://github.com/HWFord16/CS205-Project1`

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## Introduction

The first project for the Introduction to AI course taught by Dr.Eamonn Keogh is developing a program to solve the 8-puzzle problem using three different search algorithms; Uniform Cost Search, A* search using the Misplaced Tile heuristic, and A* search using the Manhattan Distance heuristic. The 8-puzzle created in 1874 by Noyes Palmer Chapman is a 3x3 square with 9 slots, where there are 8 pieces numbered from one to eight and one empty space.[1] The starting state of the puzzle is the 8 numbered pieces scrambled in random order and the objective is to place the 8 pieces in numerical order going from left to right and top to bottom also known as the goal state.[1] This project's goal is to solve this problem efficiently with the three algorithms mentioned above. The implementation follows an object-oriented approach written in Python. This report will explore the design of the program, the three algorithm's implementations, followed by a performance comparison of the three algorithms applied to a few test cases from ranging from trivial to hard difficulties.

## Overview of Algorithms and Heuristics

### Uniform Cost Search (UCS)

This algorithm is considered an uniformed or blind search which means the search algorithm searches its problem space without any additional information to assist in finding the goal state.[2] This algorithm follows a breadth-first search approach which is considered to be a complete and optimal algorithm, but also has an additional benefit of handling various edge weights.[3] The idea is to select the cheapest node to expand to keep the overall cost (denoted as $g(n)$) to the solution depth as low as possible.[2] However, in terms of performance, this algorithm has an exponential time and space complexity of $O(b^d)$ which is quite expensive.[4] For this project, each operator's cost to move a puzzle piece is 1, so this means that the depth at which the goal state is found is equivalent the total cost of the algorithm.

[1]UC Berkeley GamesCrafters Research Group. https://gamescrafters.berkeley.edu/site-legacy-archive-sp20/games.php?puzzle=8puzzle

[2]Kendall, G. 5AIAI: Blind Searches: Blind Search Methods.https://www.cs.ucdavis.edu/~vemuri/classes/ecs170/blindsearches_files/blind_searches.htm

[3]CS440 Lectures. (n.d.). https://courses.grainger.illinois.edu/cs440/fa2021/lectures/search4.html

[4]Keogh, E. Blind Search_part2. https://www.dropbox.com/scl/fo/lucvvfzc0fi7zf3tdlvpx/AJFYrHAXs3fO1nj_OeFxwvc?dl=0&e=18&preview=2__Blind+Search_part2.pptx&rlkey=cpr5hsj0grbd3pueqm05iao21

## A-Star Search

The A* search algorithm is an informed search algorithm, also called heuristic search as it uses a heuristic function (denoted as $h(n)$) to estimate the cost remaining to reach the goal state from the current state.[5] It will estimate this cost by using the the following function; $f(n) = g(n) + h(n)$.[5] This algorithm combines the best qualities from the complete and optimal UCS algorithm and the fast and greedy Hill Climbing algorithm.[6] A* is the best search algorithm currently and is the fastest algorithm dependent on the quality of the heuristic.[6] It also has an advantage when it come to memory usage as it has polynomial space complexity of $O(b \cdot d)$.[6] For this project, the $h(n)$ value needed for A* is estimated using the Misplaced tile or Manhattan distance heuristics.

## Misplaced Tile Heuristic

This heuristic is used with A* search by simply counting the number of misplaced pieces in the puzzle or pieces not matching their goal state positions as an estimate to how far the search algorithm is to the solution.[7] This $h(n)$ value is added to the $g(n)$ value which is essentially the cost spent expanding the previous nodes to reach the current node. Both of these values are summed to build the $f(n)$ value which is used to select and expand the child node from the frontier.[7]

## Manhattan Distance Heuristic

This heuristic is considered the "standard" for 2-dimensional grid problems where the operators move along the horizontal and vertical x-y axes.[8] The following equation is used to determine the distance of two points on a plane: $D(p_1, p_2) = |x1 - y1| + |x2 - y2|$.[9] The idea is to apply this distance equation to n-number of points in the problem space and sum those distances to estimate how close or far the current state's is to the goal state.[8] In terms of this project, the Manhattan distance equation is applied to each puzzle piece and the sum of those values are taken to build the heuristic $h(n) = \sum_{i=1}^{n}(|x1 - y1| + |x2 - y2|)$ value for each state.[9]

[5]The A* Algorithm: A complete guide. Datacamp. https://www.datacamp.com/tutorial/a-star-algorithm

[6]Keogh, E. Heuristic Search. https://www.dropbox.com/scl/fo/lucvvfzc0fi7zf3tdlvpx/AJFYrHAXs3f01nj_OeFxwvc?dl=0&e=18&preview=3__Heuristic+Search.pptx&rlkey=cpr5hsj0grbd3pueqm05iao21

[7]Tuccar, M. Analyzing the A* search heuristics with the solutions of the 8-Puzzle. https://www.cs.uml.edu/ecg/uploads/AIfall12/tuccar_project.pdf

[8]Heuristics. (2025). https://theory.stanford.edu/~amitp/GameProgramming/Heuristics.html

[9]Manhattan distance. https://xlinux.nist.gov/dads/HTML/manhattanDistance.html

## Design

To complete the 8-puzzle problem,, the design of the project is split between four files; main.py, search.py, node.py, and tree.py. The file containing main() serves as a UI for the users to interact with the 8-puzzle. It prompts the user to choose between the the default hard-coded puzzle or enter a custom puzzle, followed by the prompt to choose an algorithm to execute. Based on the user's algorithm choice, it calls the generic search algorithm implemented in search.py by passing boolean flag parameters related to the user's choice. The search.py file is responsible for creating the root node of the puzzle's starting state, creating the frontier, and maintaining the frontier with the child nodes via a heap priority queue. The other major half of the program contains the Node and Tree modules. The Node class serves as an efficient approach for the creation of node objects, representing various states of the puzzle and its child states to expand after applying the possible operators while executing the search algorithm. Node objects track their own state in relation to the goal state, depth, operators applied, child nodes expanded and most importantly the heuristic for the search algorithms to use. Also, the logic for the heuristic functions are implemented in this file as part of the class functions along with the node expansion routine. Once arriving at the goal state, the tree object tracking the expanded nodes is able to quickly trace the path from the goal state back to the initial state. This approach is ideal for outputting clean traces through the solution path.

## Performance

### Test Cases

All puzzle test cases were randomly generated and built with an online tool.[10]

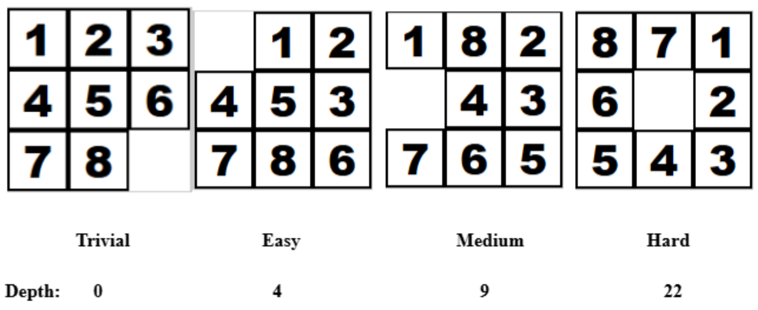


Figure 1: Test cases used to evaluate the 8-puzzle solver and their solution depths

- **NOTE:** Each result graphed below has their values scaled using log() function to compress the large range differences for each search algorithm based on puzzle difficulty.

[10]Gurkaynak, D. 8-Puzzle Solver. `https://deniz.co/8-puzzle-solver/`

## Results

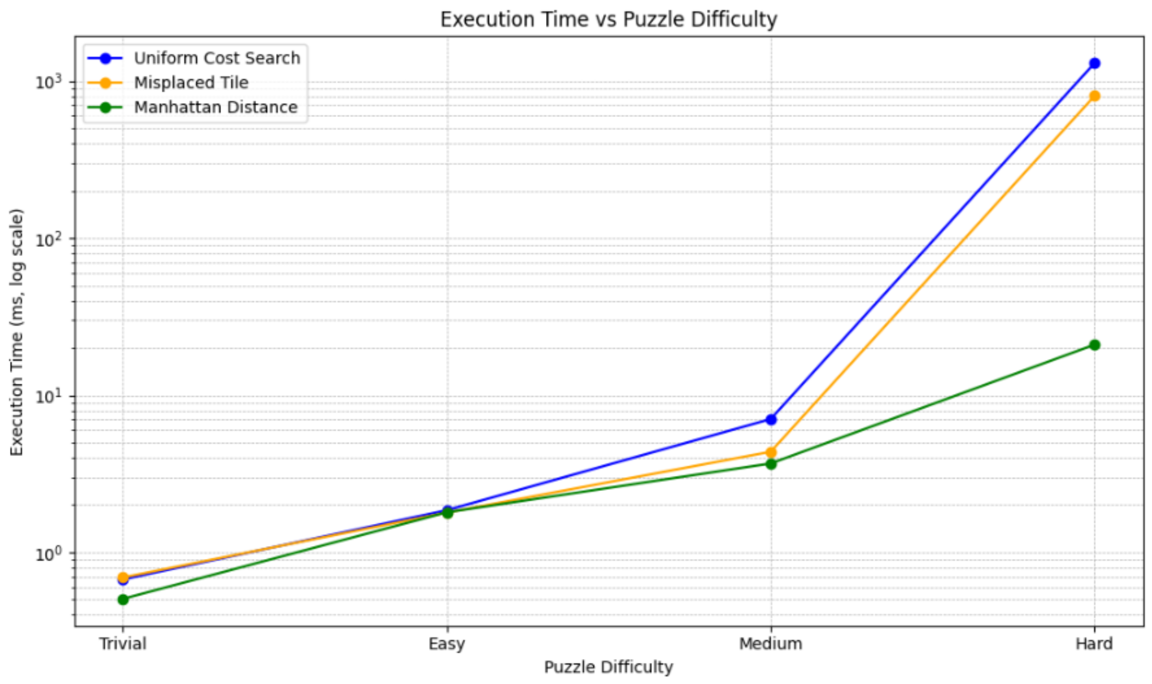


Figure 2: Runtimes by puzzle difficulty using the three search algorithms

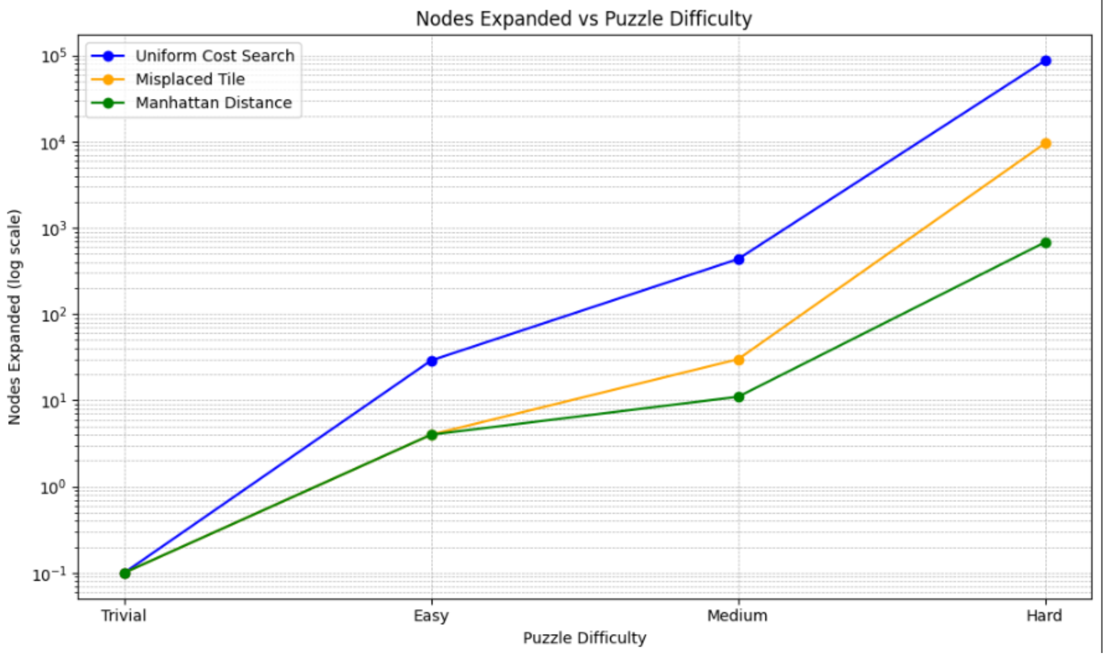


Figure 3: Nodes expanded by each search algorithm based on puzzle difficulty

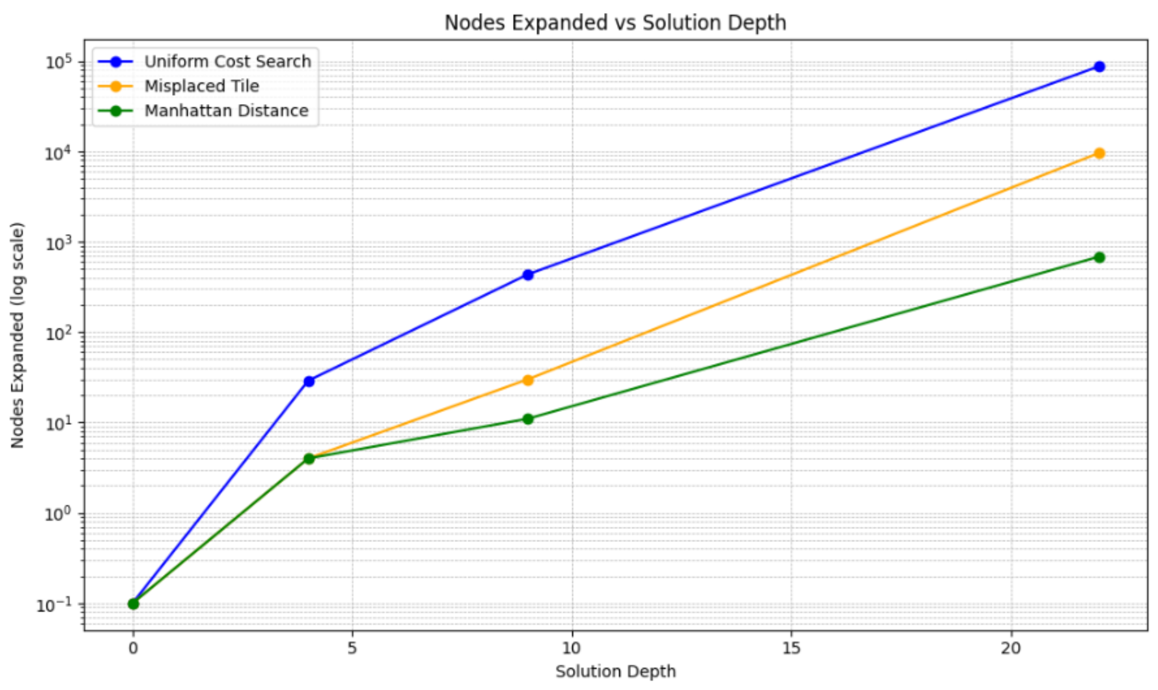


Figure 4: Maximum frontier size for each search algorithm based on puzzle difficulty

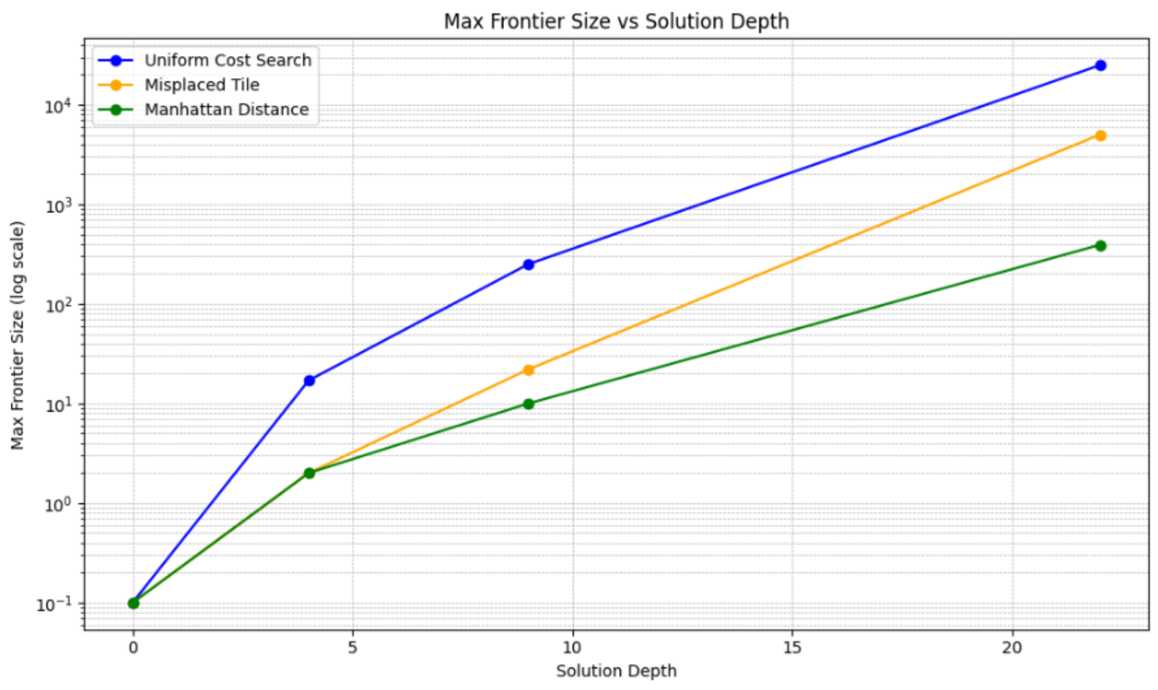
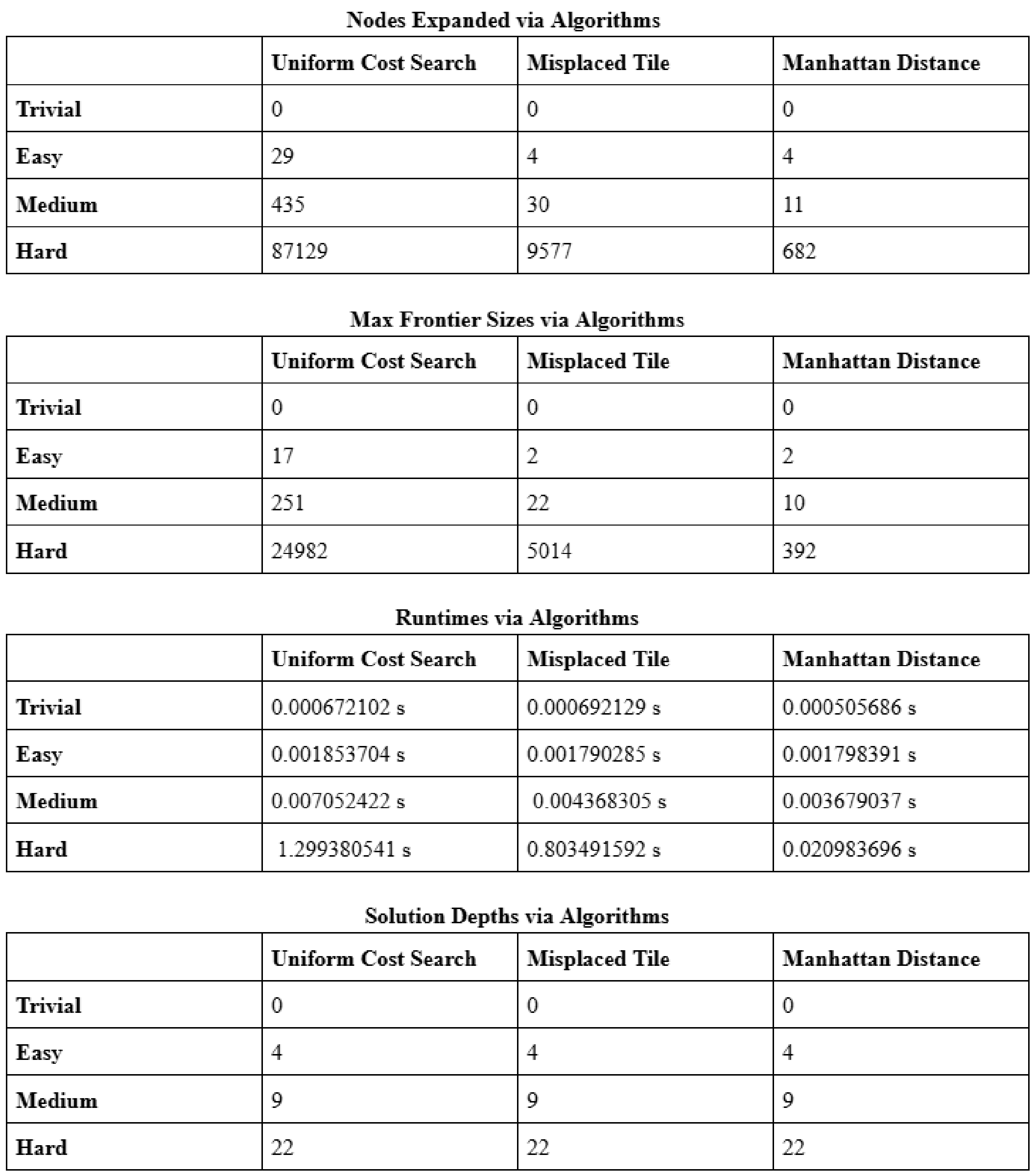


Figure 5: Max frontier size by three algorithms and their solution depths



**Nodes Expanded via Algorithms**

| | Uniform Cost Search | Misplaced Tile | Manhattan Distance |
|---|---|---|---|
| **Trivial** | 0 | 0 | 0 |
| **Easy** | 29 | 4 | 4 |
| **Medium** | 435 | 30 | 11 |
| **Hard** | 87129 | 9577 | 682 |

**Max Frontier Sizes via Algorithms**

| | Uniform Cost Search | Misplaced Tile | Manhattan Distance |
|---|---|---|---|
| **Trivial** | 0 | 0 | 0 |
| **Easy** | 17 | 2 | 2 |
| **Medium** | 251 | 22 | 10 |
| **Hard** | 24982 | 5014 | 392 |

**Runtimes via Algorithms**

| | Uniform Cost Search | Misplaced Tile | Manhattan Distance |
|---|---|---|---|
| **Trivial** | 0.000672102 s | 0.000692129 s | 0.000505686 s |
| **Easy** | 0.001853704 s | 0.001790285 s | 0.001798391 s |
| **Medium** | 0.007052422 s | 0.004368305 s | 0.003679037 s |
| **Hard** | 1.299380541 s | 0.803491592 s | 0.020983696 s |

**Solution Depths via Algorithms**

| | Uniform Cost Search | Misplaced Tile | Manhattan Distance |
|---|---|---|---|
| **Trivial** | 0 | 0 | 0 |
| **Easy** | 4 | 4 | 4 |
| **Medium** | 9 | 9 | 9 |
| **Hard** | 22 | 22 | 22 |

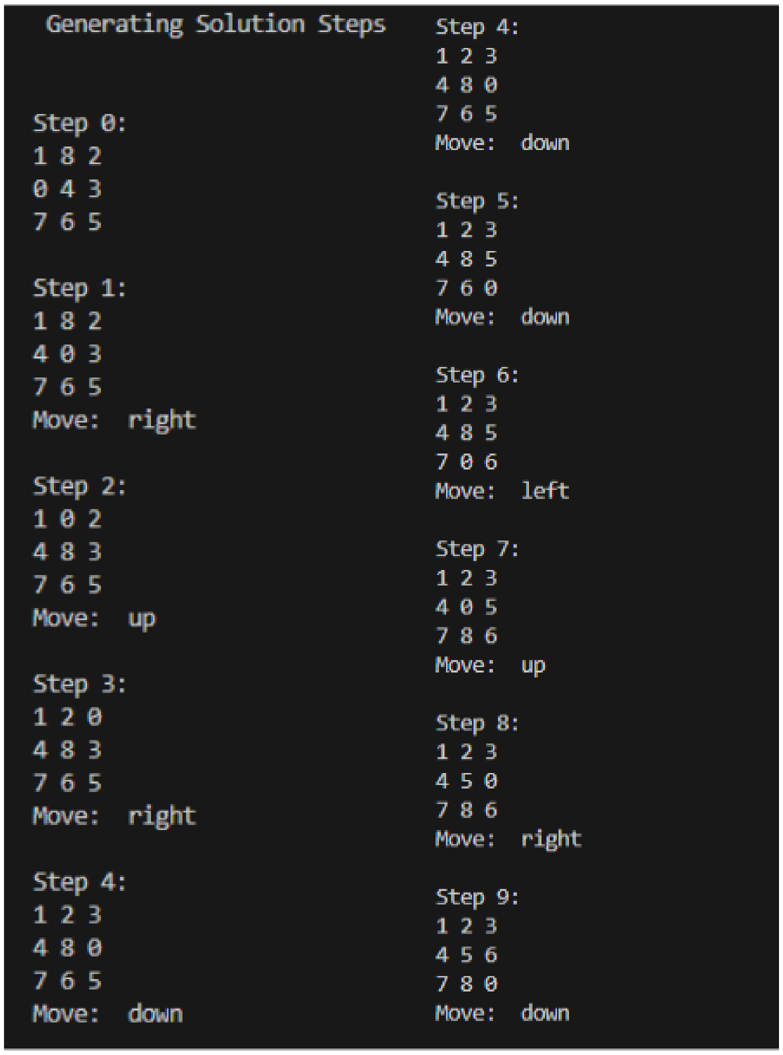
Figure 6: Tabular Values of Nodes Expanded, Max Frontier Size, Runtimes, Solution Depth

## Conclusion

After interpreting the results from running the 3 algorithms on 4 different puzzles, it's clear that the A* search algorithm is far superior than Uniform Cost Search on all fronts. When comparing A* search with different heuristics, it's evident that using the Manhattan Distance heuristic results in a better quality result for number of nodes expanded as well as Max frontier sizes. The common trend observed is that the harder the puzzle difficulty or the deeper the solution depth, the more time and space it takes to reach the goal state.

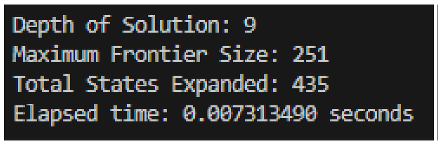
## Example Trace

The following trace is performed on the medium puzzle with a solution depth of 9.

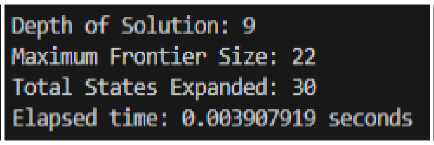


```
 Generating Solution Steps


Step 0:
1 8 2
0 4 3
7 6 5

Step 1:
1 8 2
4 0 3
7 6 5
Move:  right

Step 2:
1 0 2
4 8 3
7 6 5
Move:  up

Step 3:
1 2 0
4 8 3
7 6 5
Move:  right

Step 4:
1 2 3
4 8 0
7 6 5
Move:  down
```

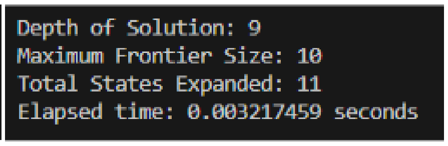
```
Step 4:
1 2 3
4 8 0
7 6 5
Move:  down

Step 5:
1 2 3
4 8 5
7 6 0
Move:  down

Step 6:
1 2 3
4 8 5
7 0 6
Move:  left

Step 7:
1 2 3
4 0 5
7 8 6
Move:  up

Step 8:
1 2 3
4 5 0
7 8 6
Move:  right

Step 9:
1 2 3
4 5 6
7 8 0
Move:  down
```



```
Depth of Solution: 9
Maximum Frontier Size: 251
Total States Expanded: 435
Elapsed time: 0.007313490 seconds
```



```
Depth of Solution: 9
Maximum Frontier Size: 22
Total States Expanded: 30
Elapsed time: 0.003907919 seconds
```



```
Depth of Solution: 9
Maximum Frontier Size: 10
Total States Expanded: 11
Elapsed time: 0.003217459 seconds
```

**Uniform Cost Search** **Misplaced Tiles** **Manhattan Distance**

Figure 7: Trace of Solution for Medium Puzzle difficulty with 3 algorithms applied