intro and tutorial for Gaussian multi-frequency VLBI analyses

(GaMVAs)

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github: https://github.com/HWJeong1122/gamvas

Download & Python dependencies

Download: github page (https://github.com/HWJeong1122/gamvas)
then, add gamvas directory to PYTHONPATH, e.g.,

```
sys.path.append('/directory/to/gamvas/') # in python
or, export PYTHONPATH='/directory/to/gamvas/:$PYTHONPATH' # in ~/.bashrc
```

numpy (pip install numpy)

pandas (pip install pandas)

matplotlib (pip install matplotlib)

uncertainties (pip install uncertainties)

scipy (pip install scipy)

sklearn (pip install sklearn)

astropy (pip install astropy)

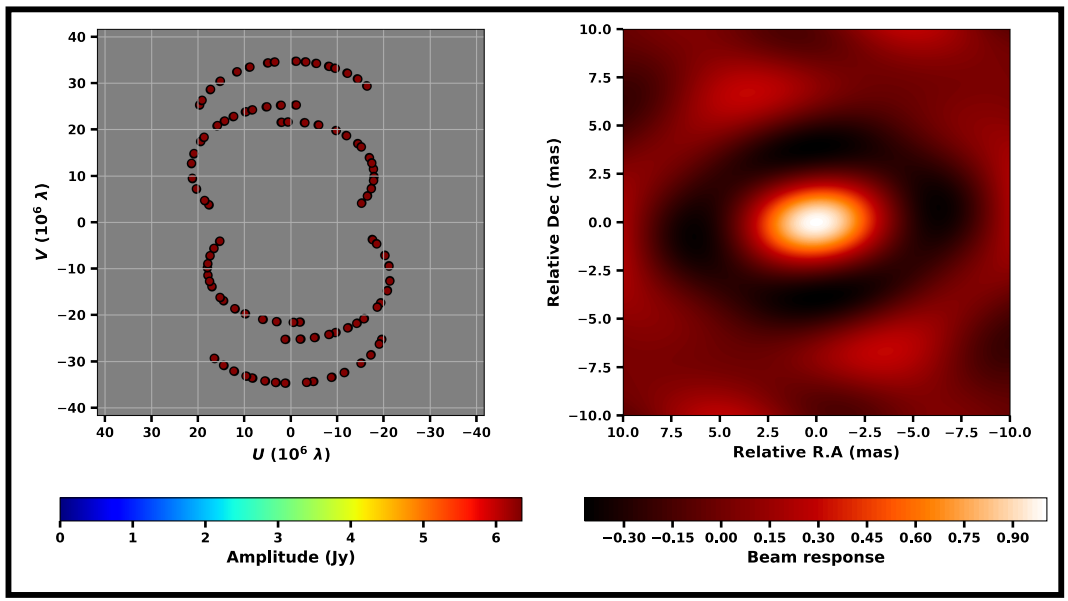
dynesty (pip install dynesty)

Intro

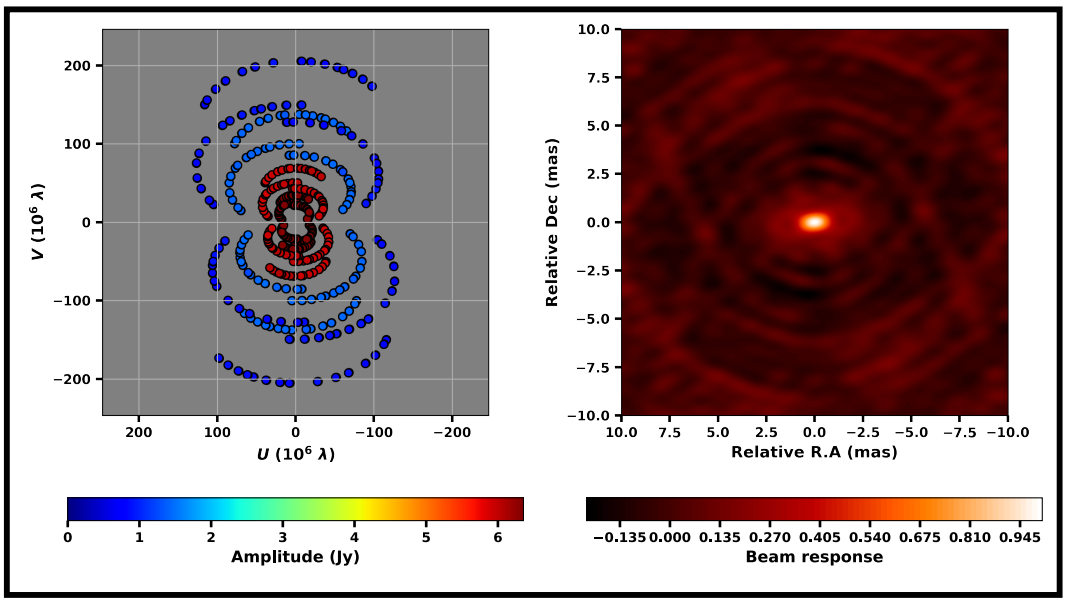
* Project code: p20st02j, p20st02k

** Source: 3C 345 (1641+399)

single-frequency



multi-frequency



Aims

1. Effective geometrical modeling for interferometers consisting small number of antenna (improved uv–coverage)

2. Use of closure quantities (not supported in Difmap)

3. Direct spectral analysis through co-identified Gaussians

Assumptions

1. Gaussian approximation for AGN jets

2. Consistent jet geometry within radio frequencies

3. Negligible core-shift effect within angular resolution & frequency range

GaMVAs follows Bayesian approach via nested sampling

MCMC: estimate likelihood (& therefore, posterior distribution) directly

Nested sampling: estimate evidence based on (remaining) prior volume

$$\underbrace{p(\theta \mid x, H)}_{\text{probability}} = \frac{\overbrace{p(x \mid \theta, H)}^{\text{likelihood}}\ \overbrace{p(\theta \mid H)}^{\text{prior distribution}}}{\underbrace{p(x \mid H)}_{\substack{\text{evidence} \\ \text{(or, marginal likelihood)}}}}$$

x: observed data
θ: parameters
H: model

Implemetation: DYNESTY (documentation, for more details, see J S. Speagle 2020 (ADS))

maximizing evidence, $Z \equiv \int_{\Omega_\theta} \mathscr{L}(\theta)\,\pi(\theta)\,\mathrm{d}\theta = \int_0^1 \mathscr{L}(X)\,\mathrm{d}X$

stopping criteria: $\Delta \ln \hat{Z}_i \equiv \ln(\hat{Z}_i + \Delta\hat{Z}_i) - \ln(\hat{Z}_i) \lesssim \mathscr{L}^{\max} \hat{X}_i$

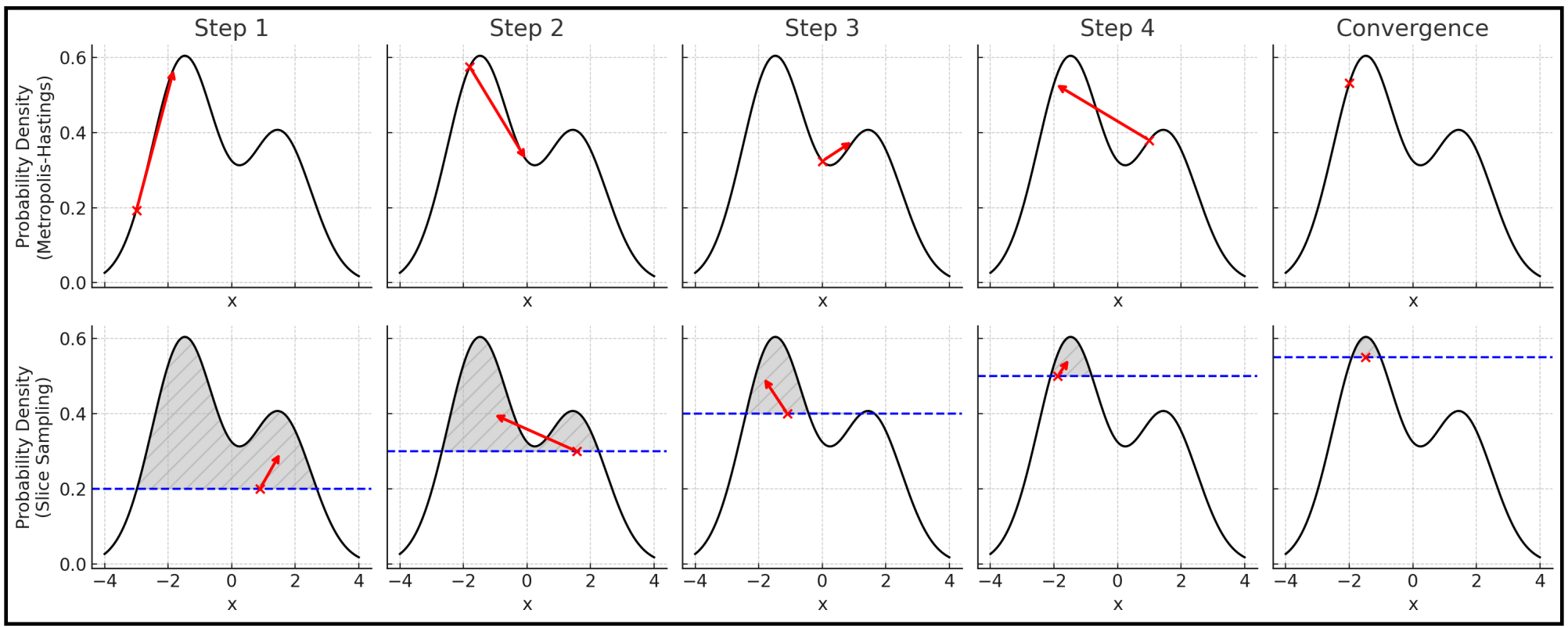
ˆ **(hat)** denotes remaining quantities

Intro // Methodologies

'Slice sampling' vs. 'random walk'

Nested sampling Metropolis-Hastings (MCMC)

* An example for 1-D problem

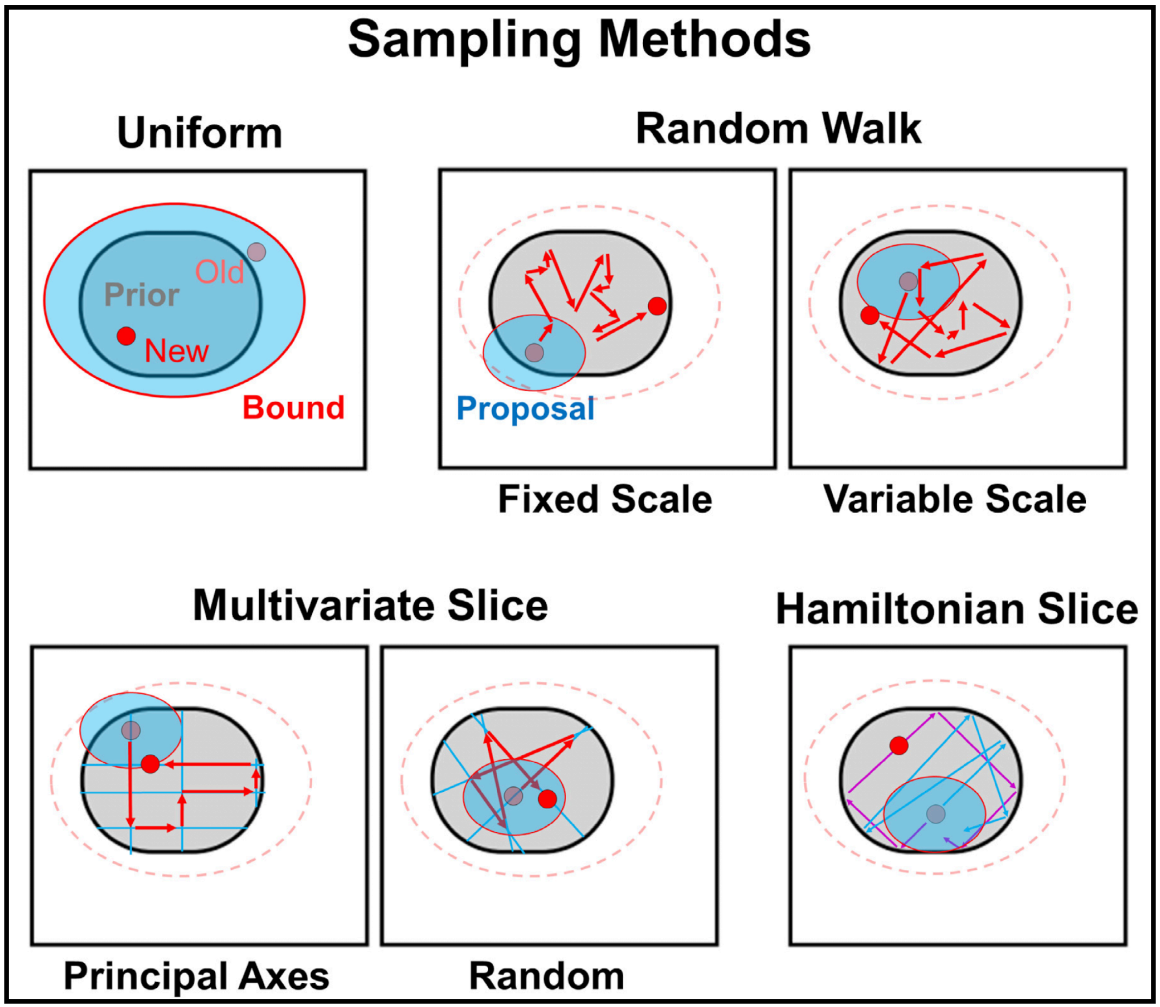


** Blue dashed line denotes auxiliary variable (a threshold)

'Slice sampling' vs. 'random walk'

Nested sampling Metropolis-Hastings (MCMC)

* An example for 2-D problem

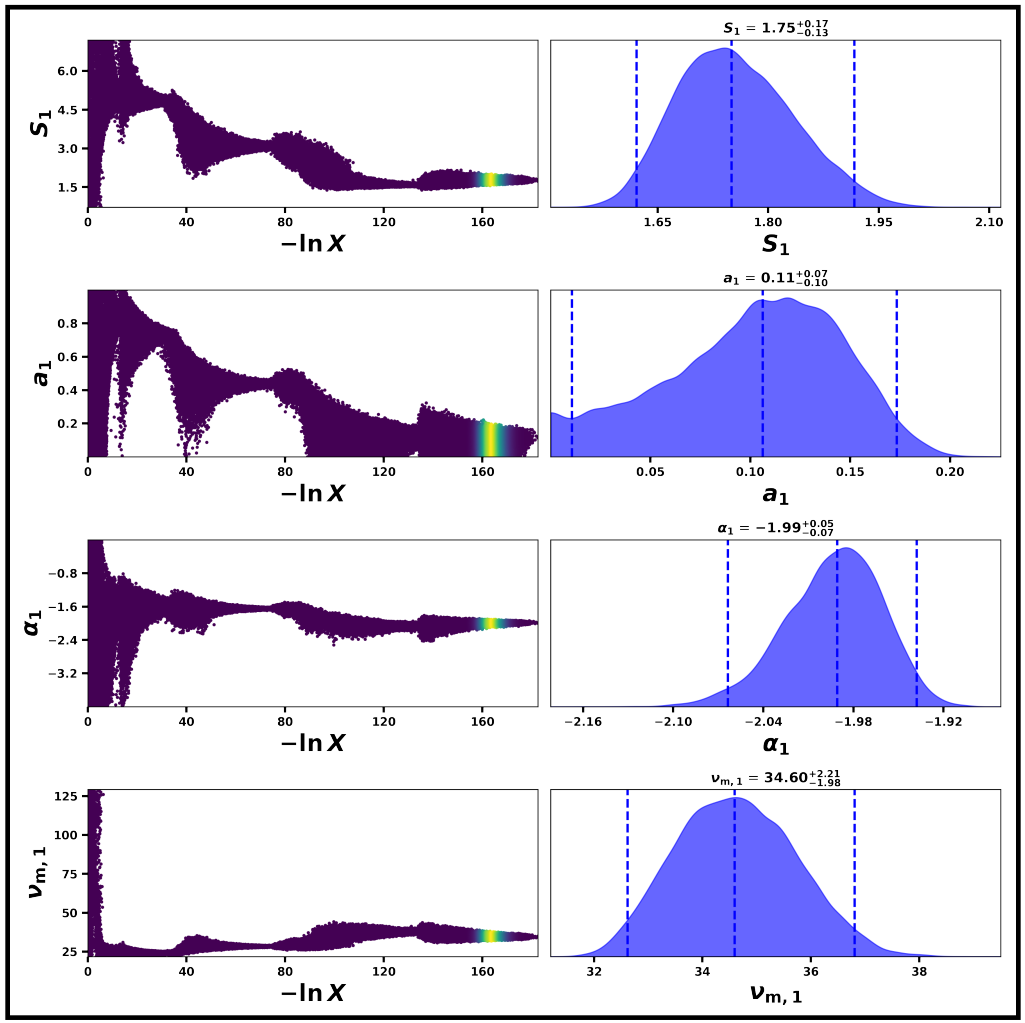


Speagle 2020

Intro // Methodologies

'Slice sampling' vs. 'random walk'

Nested sampling Metropolis-Hastings (MCMC)



Intro // visibility of a multi-frequency Gaussian

1. complex visibility in a baseline: $$\mathscr{V}(u,v) = \iint I(l,m)\, e^{-2\pi i(ul+vm)}\, dl\, dm$$

→ circular Gaussian: (single frequency) $$\mathscr{V}_m(u,v;\theta') = Se^{-2\pi a^2(u^2+v^2)} e^{2\pi i(ul_0+vm_0)}$$

$\theta' = (S,\ a,\ l_0,\ m_0)$

S : flux density [Jy]

a : angular size [mas]

l_0 : right ascension offset [mas]

m_0 : declination offset [mas]

2. Synchrotron self-absorption : (Turler+1999)

$$S(\nu; S_\mathrm{m}, \nu_\mathrm{m}, \alpha) = S_\mathrm{m} \left(\frac{\nu}{\nu_\mathrm{m}}\right)^{2.5} \times \frac{1-\exp(-\tau_\mathrm{m}(\nu/\nu_\mathrm{m})^{\alpha-2.5})}{1-e^{-\tau_\mathrm{m}}}$$

$\theta = (S,\ a,\ l_0,\ m_0,\ \alpha,\ \nu_\mathrm{m})$

α : spectral index

ν_m : turnover frequency [GHz]

ν : observing frequency [GHz]

3. Multi-frequency Gaussian :

$$\mathscr{V}_m(u_\nu, v_\nu, \nu; \theta) = S(\nu; S_\mathrm{m}, \nu_\mathrm{m}, \alpha) \times e^{-2\pi a^2(u_\nu^2+v_\nu^2)} e^{2\pi i(u_\nu l_0 + v_\nu m_0)}$$

Intro // objective function

1. Modeling script aims to minimize **Bayesian information crietria (BIC)**

$$\mathrm{BIC} = k\ln(n) - 2\ln(\mathscr{L})$$

k : degree of freedom

n : the number of data points

2. Likelihood $\mathscr{L}$ with independent normal Gaussian noise is given by

$$\mathscr{L} = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\sigma_i^2}} \exp\left(-\frac{1}{2}\left(\frac{\mathrm{model}_i - \mathrm{data}_i}{\sigma_i}\right)^2\right)$$

σ_i : uncertainty of i^{th} data

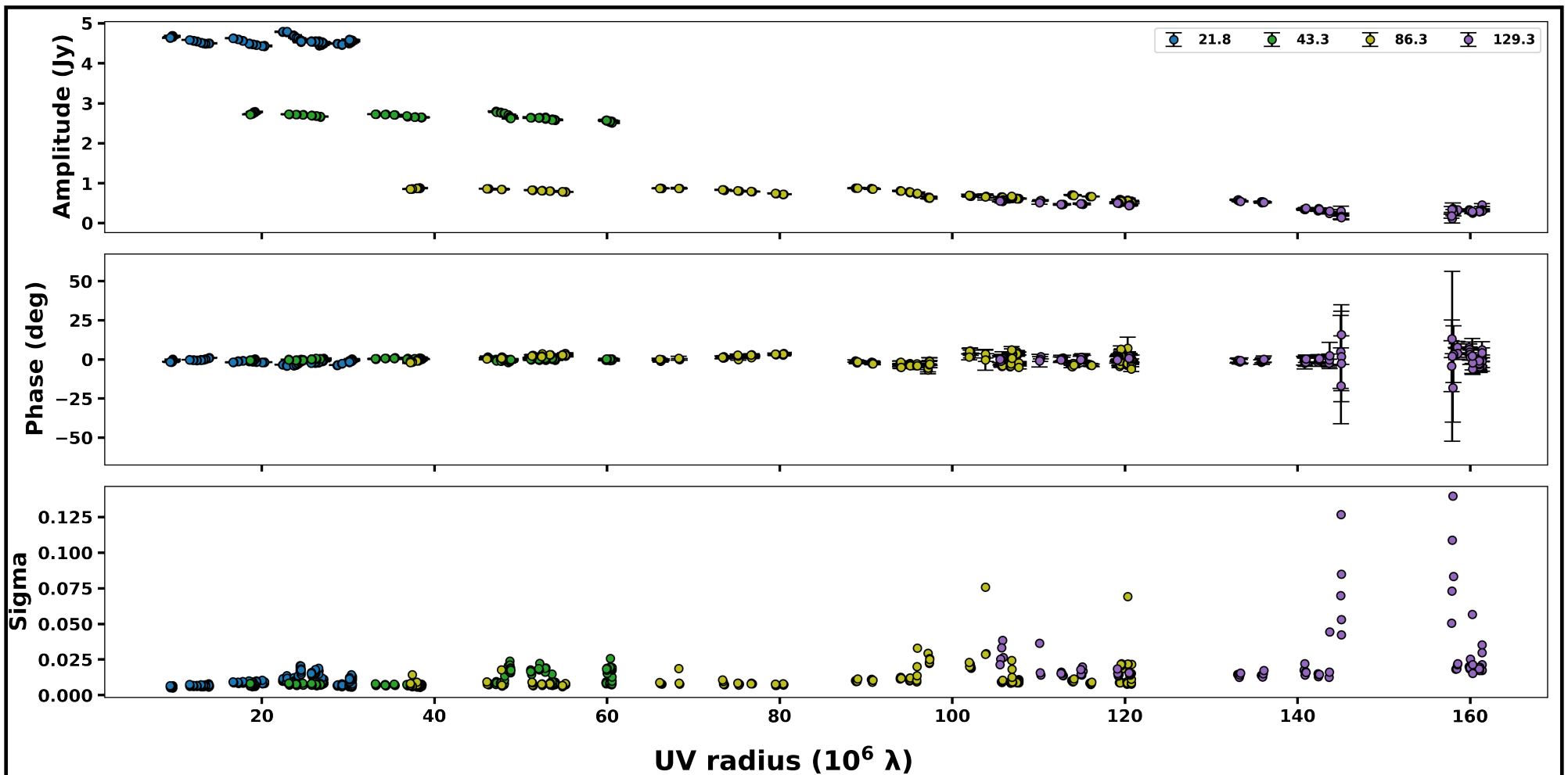
then, $$\mathrm{BIC} = k\ln(n) + \sum_{i=1}^{n}\left[\left(\frac{\mathrm{model}_i - \mathrm{data}_i}{\sigma_i}\right)^2 + \ln(2\pi\sigma_i^2)\right]$$

Example run // f24sl02b

```
import gamvas as gv

# Load uv-fits files (map range: 22 mas)
uvf1 = gv.load.open_fits(path=path_uvf, file=file_k, mrng=22 * gv.mas)
uvf2 = gv.load.open_fits(path=path_uvf, file=file_q, mrng=22 * gv.mas)
uvf3 = gv.load.open_fits(path=path_uvf, file=file_w, mrng=22 * gv.mas)
uvf4 = gv.load.open_fits(path=path_uvf, file=file_d, mrng=22 * gv.mas)

# select='ll': load ll-polarization, uvw='u': use uniform visibility weighting
uvf1.load_uvf(select='ll', uvw='u')
uvf2.load_uvf(select='ll', uvw='u')
uvf3.load_uvf(select='ll', uvw='u')
uvf4.load_uvf(select='ll', uvw='u')

# uv-average in 60s (weighted-average)
uvf1.uvave(uvave=60, scanlen=300)
uvf2.uvave(uvave=60, scanlen=300)
uvf3.uvave(uvave=60, scanlen=300)
uvf4.uvave(uvave=60, scanlen=300)

# set multi-frequency uvf
uvfs = [uvf1, uvf2, uvf3, uvf4]
uvall = gv.utils.set_uvf(uvfs, type='mf')

# set data terms and weights
ftype = ['amp', 'clamp', 'clphs']
fwght = [0.01, 1, 1]    # low-weight to visibility amplitude

# set the number of CPU core (for multi-processing)
ncpu = N    # to use maximum number, ncpu = os.cpu_count()
```

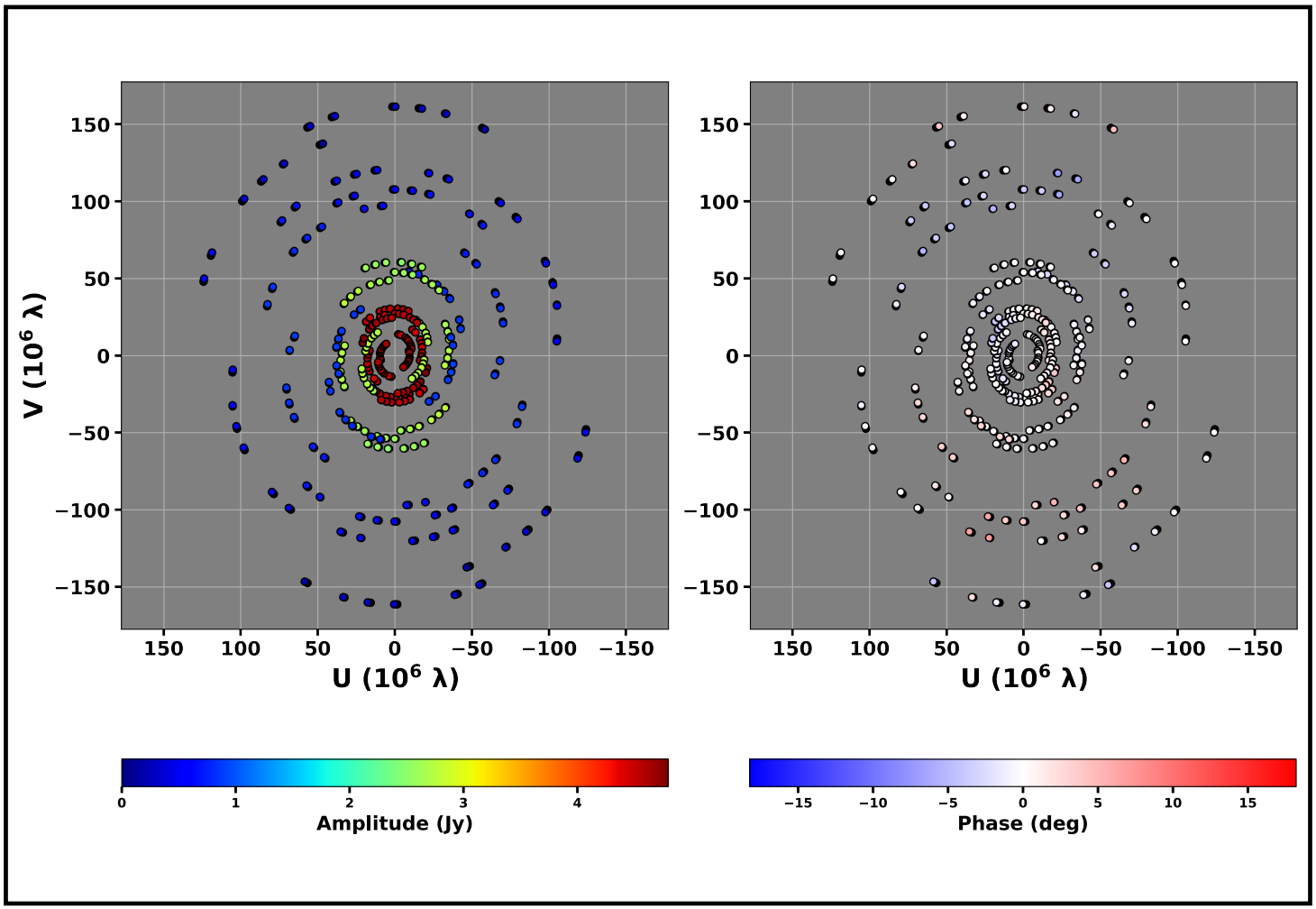
Example run // f24sl02b

```
# check visibility data
uvall.ploter.draw_radplot(uvall, plotimg=True) # save_path=, save_name=)
```



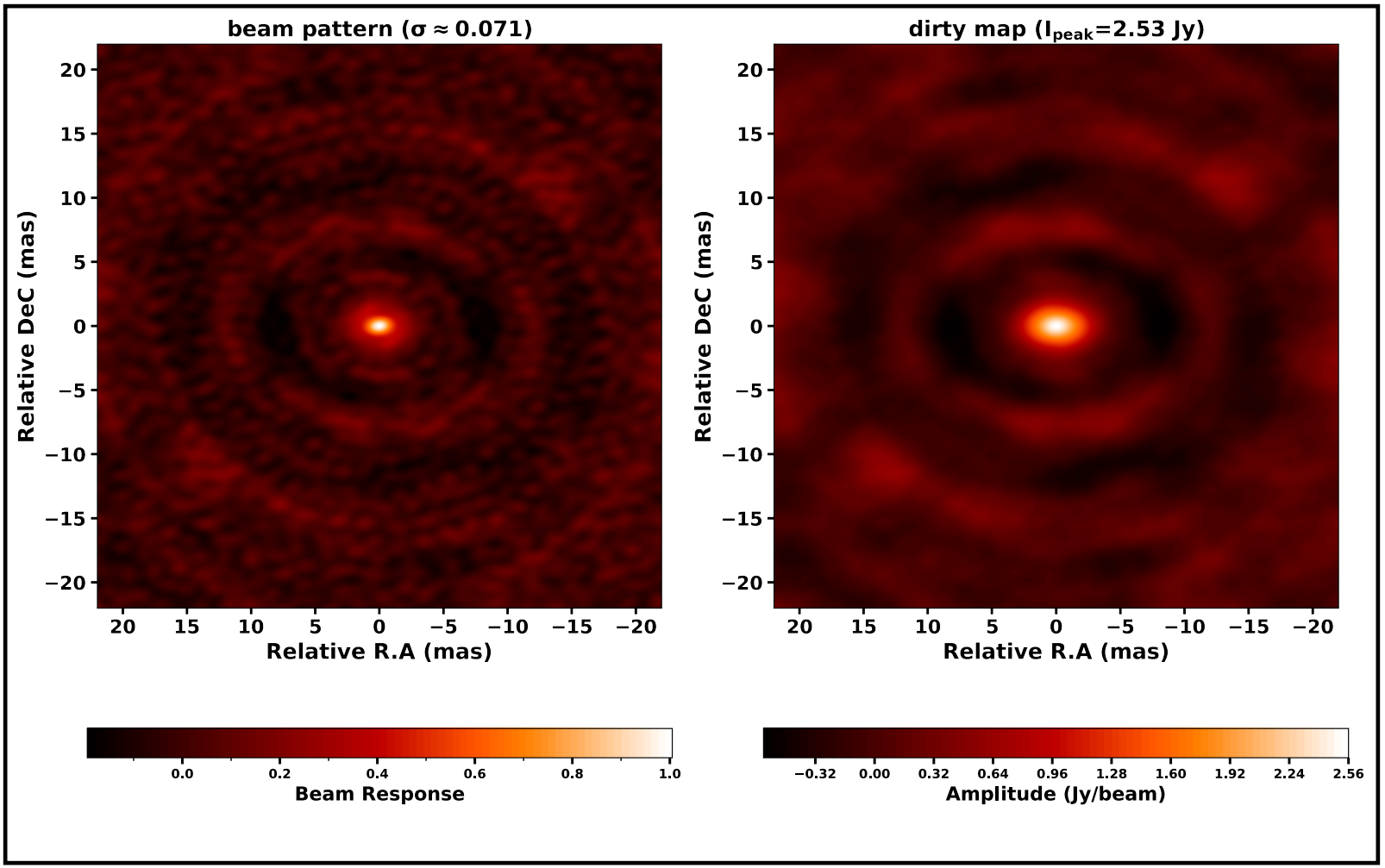
Example run // f24sl02b

```
# check visibility data
uvall.ploter.draw_uvcover(uvall, plotimg=True) # save_path=, save_name=)
```



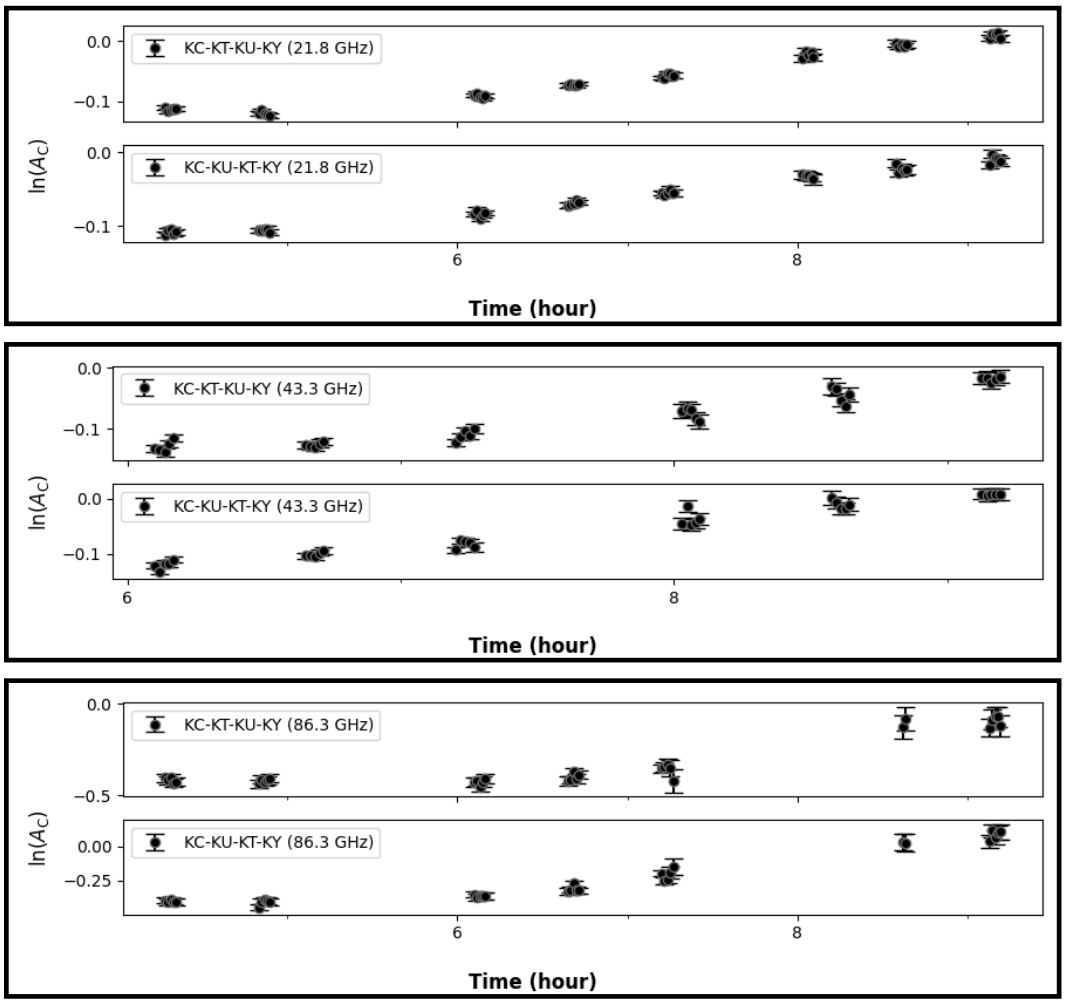
Example run // f24sl02b

```
# check visibility data
uvall.ploter.draw_dirtymap(uvall, plotimg=True) # save_path=, save_name=)
```



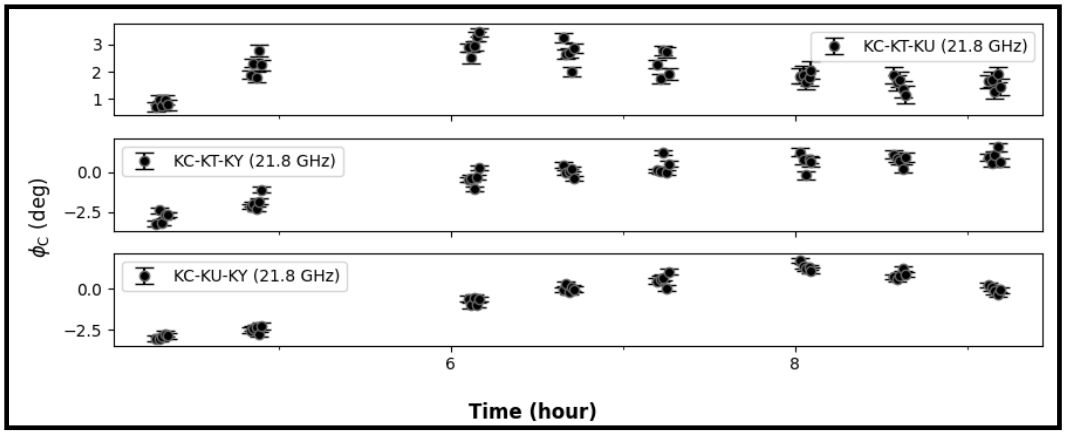
Example run // f24sl02b

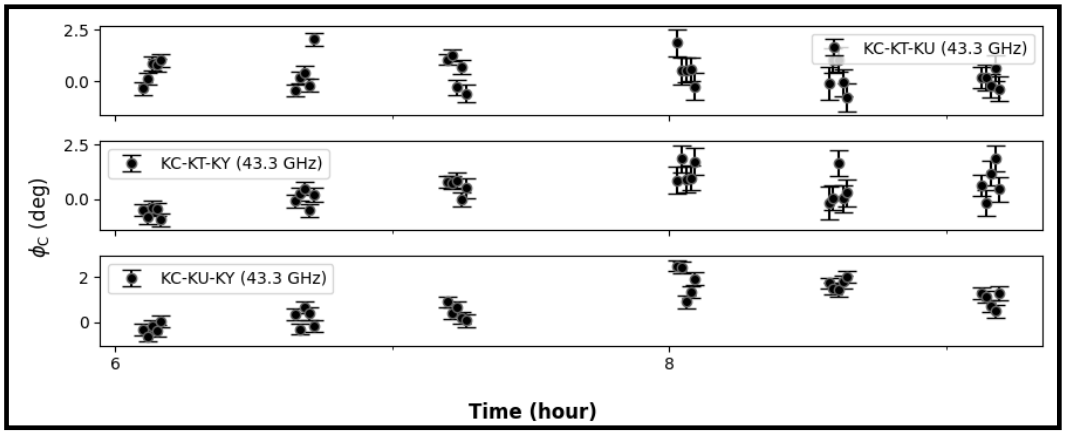
```
# check visibility data
uvall.ploter.draw_closure(type='clamp', plotimg=True) # save_path=, save_name=)
```

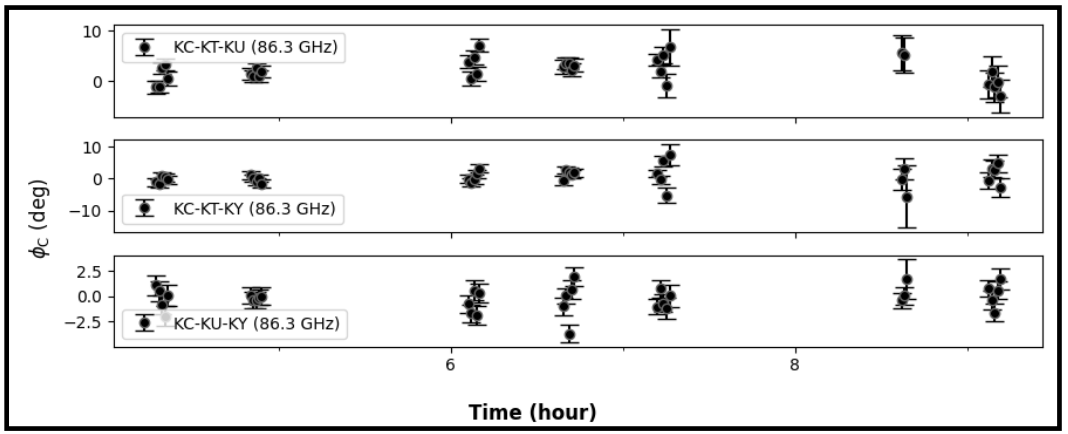


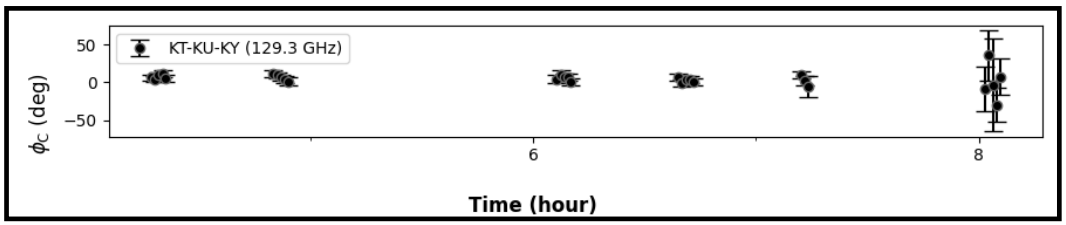
Example run // f24sl02b

```
# check visibility data
uvall.ploter.draw_closure(type='clphs', plotimg=True) # save_path=, save_name=)
```







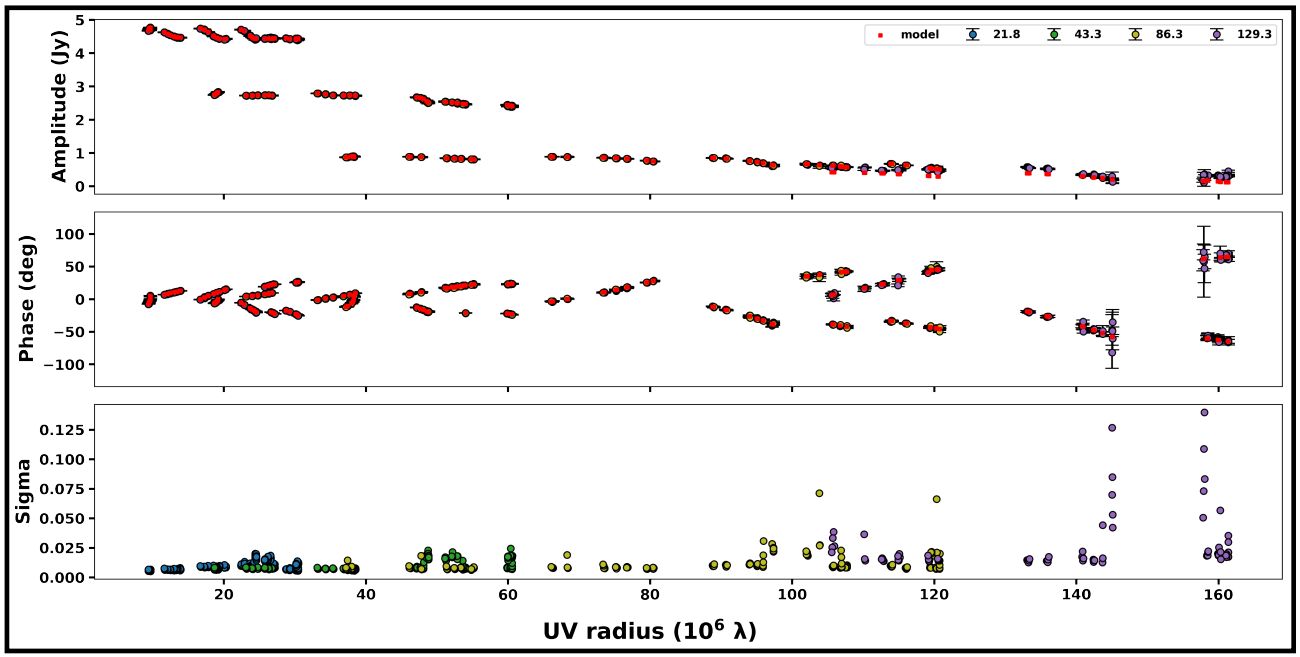


Example run // f24sl02b

```
# run model-fitting
mfu = gv.modeling.modeling(
    uvfs=uvfs, select='ll', sampler='slice', bound='multi',
    ftype=ftype, fwght=fwght, doampcal=True, dophscal=True, ncpu=ncpu,
    …
)
mfu.run()
```

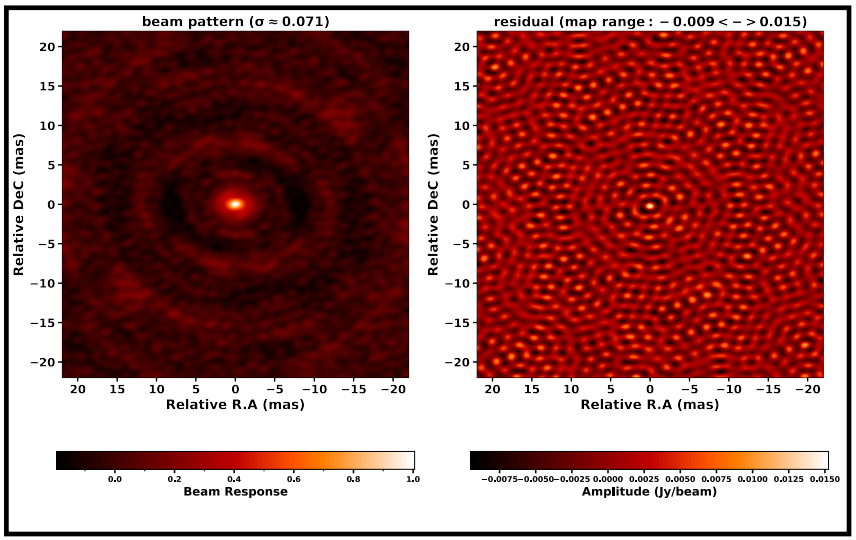
** Please see example_run.py for detailed attribute setting*

radplot



beam parttern

residual map



Example run // f24sl02b

```
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mfu = gv.modeling.modeling(
    uvfs=uvfs, select='ll', sampler='slice', bound='multi',
    ftype=ftype, fwght=fwght, doampcal=True, dophscal=True, ncpu=ncpu,
    …
)
mfu.run()
```
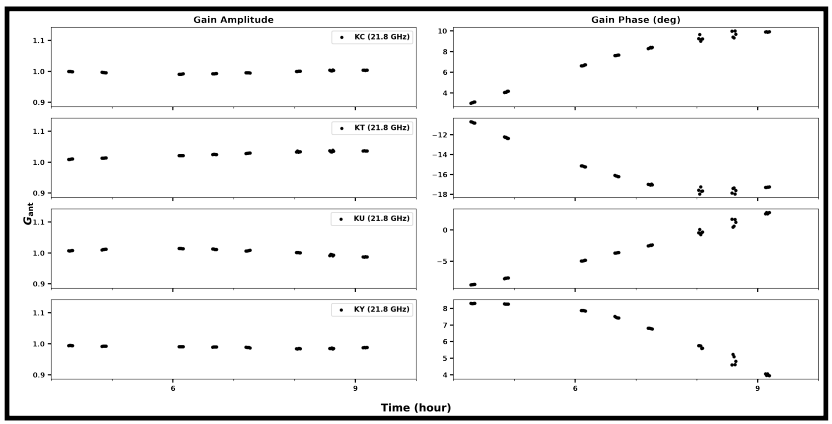
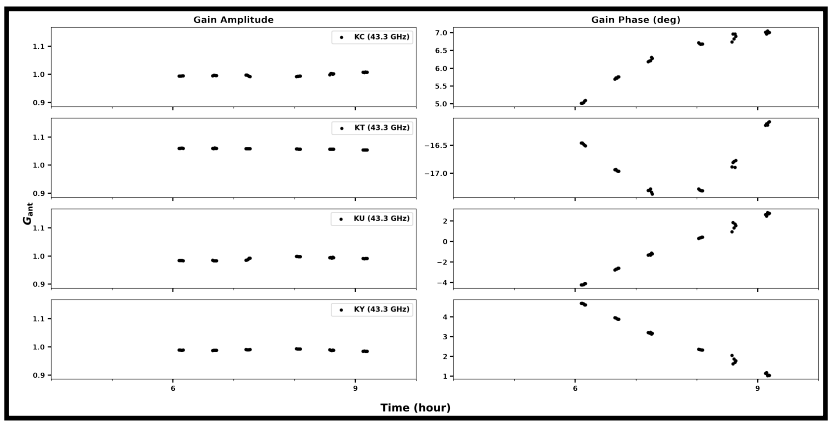

** Please see example_run.py for detailed attribute setting !*

complex gain
after self-calibration

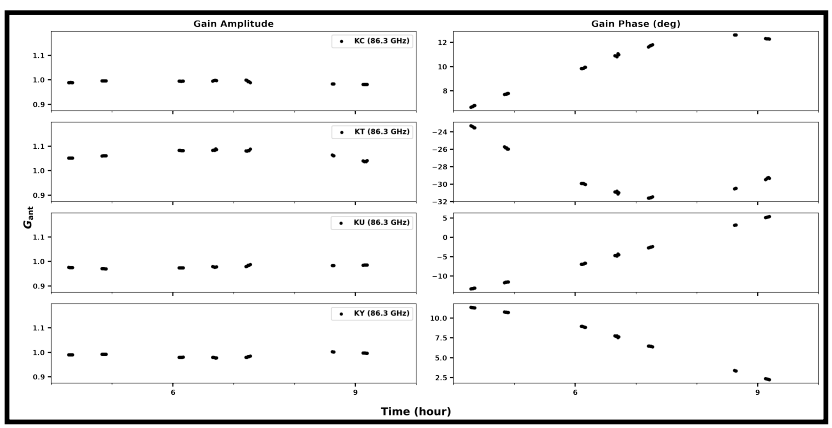
K-band



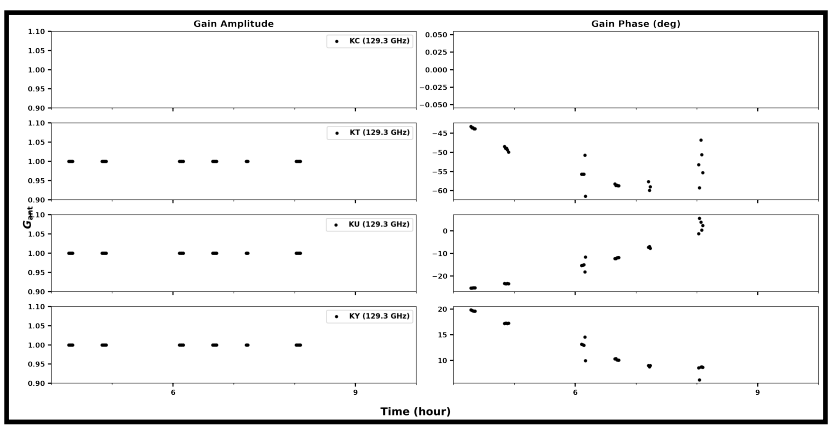
Q-band



W-band



D-band



Example run // f24sl02b

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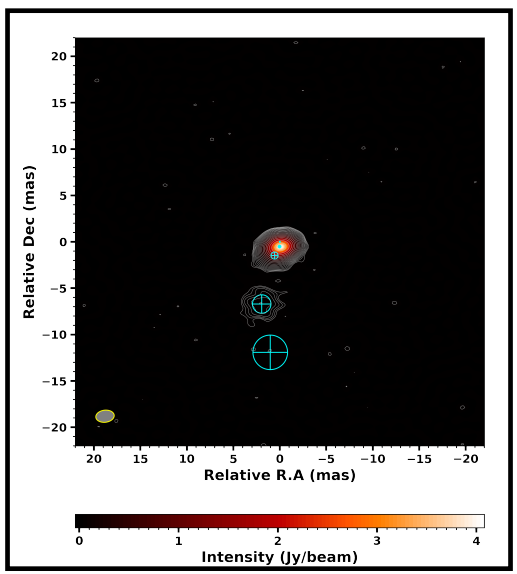
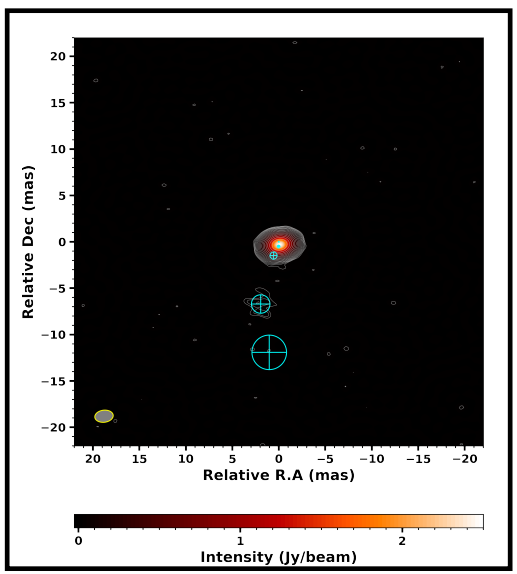
mfu = gv.modeling.modeling(
    uvfs=uvfs, select='ll', sampler='slice', bound='multi',
    ftype=ftype, fwght=fwght, doampcal=True, dophscal=True, ncpu=ncpu,
    …
)
mfu.run()
```

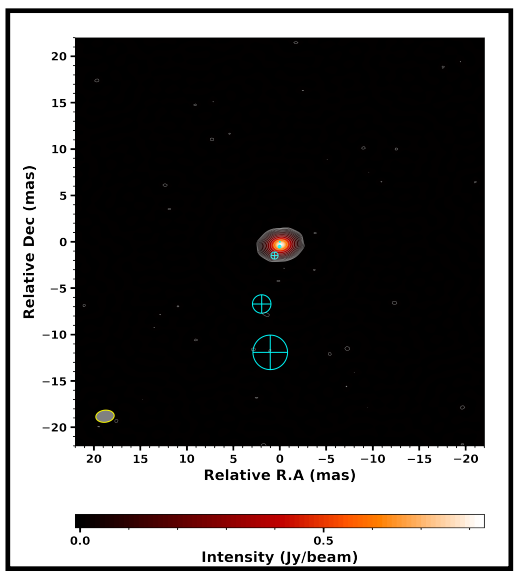
K-band



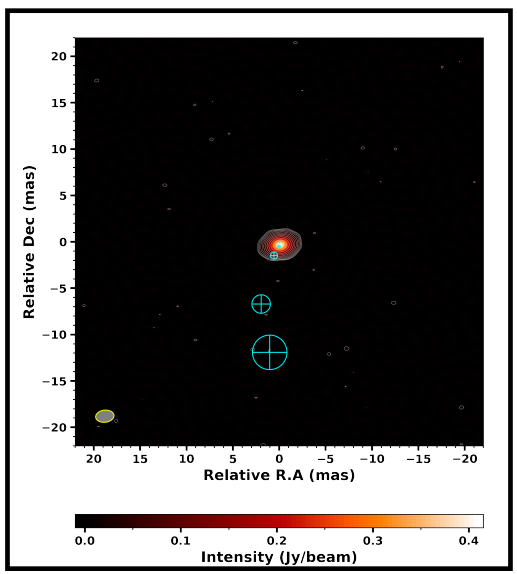
Q-band



W-band



D-band



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    …
)
mfu.run()
```
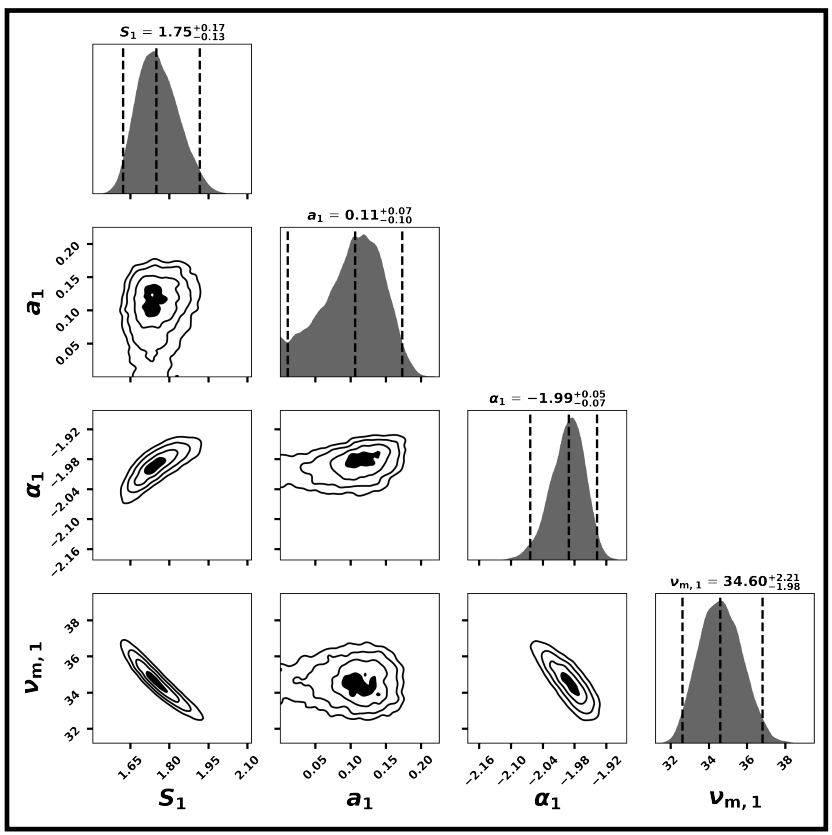

corner plot of model #1



trace plot of model #2