Dissolution Notes

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The dissolution of star clusters in external galaxies can in principle be derived empirically from the analysis of the age distribution of detected clusters. If the clusters are destroyed rapidly in their host galaxy, the number of clusters will decrease rapidly with age, in the sense that there will be many fewer old than young clusters.

1. assume (or determine) a cluster formation history, it shapes of the distribution of the observed clusters.
2. Clusters fade with age due to stellar evolution for magnitude limited samples.
3. Incompleteness, which relates the true distribution of clusters to the observed one.
4. The accuracy of the age and mass determinations, which are based on the fitting of the photometric spectral energy distributions with cluster models.

We define the number of observed clusters $N(t, M)$ as a function of ~~time~~ t and current mass M.

Under some assumptions, we have access to the 4th item, with θ the intrinsic parameters of a given cluster

$$\mathcal{P}\left(\theta = \{\,age, mass...\,\} \mid \vec{d}_i\right) = \mathcal{P}\left(\vec{d}_i \mid \theta, \sigma_{\vec{d}_i}\right)\mathcal{P}(\theta) \tag{1}$$

We can also presume that the completeness function is also known in the age-mass space as $S(t, M)$.

Our goal is to ~~describe~~ the cluster distribution $\frac{\partial^2 N}{\partial t\, \partial M}$ **~~with a set of parameters π carefully chosen.~~**

$$\begin{aligned}
\mathcal{P}\left(\pi \mid \vec{d}\right) &\propto \mathcal{P}\left(\vec{d} \mid \pi\right)\mathcal{P}(\pi) && (2)\\
&\propto \mathcal{P}(\pi)\int \mathcal{P}\left(\vec{d} \mid \vec{d}_{\cancel{0}}\right) S(\vec{d}_{\cancel{0}})\,\mathcal{P}\left(\vec{d}_{\cancel{0}} \mid \pi\right) d\vec{d}_{\cancel{0}} && (3)\\
&\propto \mathcal{P}(\pi)\int \mathcal{P}\left(\vec{d} \mid \theta\right) S(\theta)\,\mathcal{P}(\theta \mid \pi)\, d\theta && (4)
\end{aligned}$$

The later equation reconcile the parameters of the clusters into the formalism.

Hence we need to explicit $\mathcal{P}\left(\, \theta \mid \pi \,\right)$.

We can assume that we are working in the "true data" space, in which there is no uncertainty on t and M of a given clusters. The number of clusters is then the number of clusters formed subtracted from the number of destroyed clusters.

$$\begin{aligned} \mathcal{P}\left(\, t, M \mid \pi \,\right)\, dt\, dM &= \frac{\partial^2 N}{\partial t\, \partial M}\, dt\, dM & (5) \\ &\propto \dot{n}_f(t,M)\, dt\, dM - \dot{n_d}(t,M)\, dt\, dM & (6) \end{aligned}$$

$\int \dot{n}_f(t,M) dM$ is the cluster formation rate (CFR). We now need to define $\dot{n}_f$ and $\dot{n}_d$.

$$\begin{aligned} \dot{n}_f(t,M) &= \dot{n}_f(t_f, M_f)\, t^\beta\, M^\alpha & (7) \\ \dot{n}_d(t,M) &= \dot{n}_d(M_f)\, M^\gamma & (8) \end{aligned}$$

The predicted number of observable clusters is then given by

$$\begin{aligned} N(t,M) &= S(t,M)\; (\dot{n}_f(t,M)\, dt\, dM - \dot{n_d}(t,M)\, dt\, dM) & (9) \\ &= S(t,M)\; \left(\dot{n}^0_f\, t^\beta\, M^\alpha - \dot{n}^0_d\, M^\gamma\, t\right) & (10) \\ &= S(t,M)\, \dot{n}^0_f \left(t^\beta\, M^\alpha - \frac{\dot{n}^0_d}{\dot{n}^0_f}\, M^\gamma\, t\right) & (11) \\ &= S(t,M)\, \dot{n}^0_f\, t^\beta\, M^\alpha \left(1 - \frac{\dot{n}^0_d}{\dot{n}^0_f}\, M^{\gamma-\alpha}\, t^{1-\beta}\right) & (12) \end{aligned}$$

We describe the evolution of a given cluster mass by the addition of stellar evolution and disruption as:

$$dM/dt = (dM/dt)_{ev} + (dM/dt)_{dis} \tag{13}$$

of which the second term from Boutloukos & Lamers 2003 can be defined as:

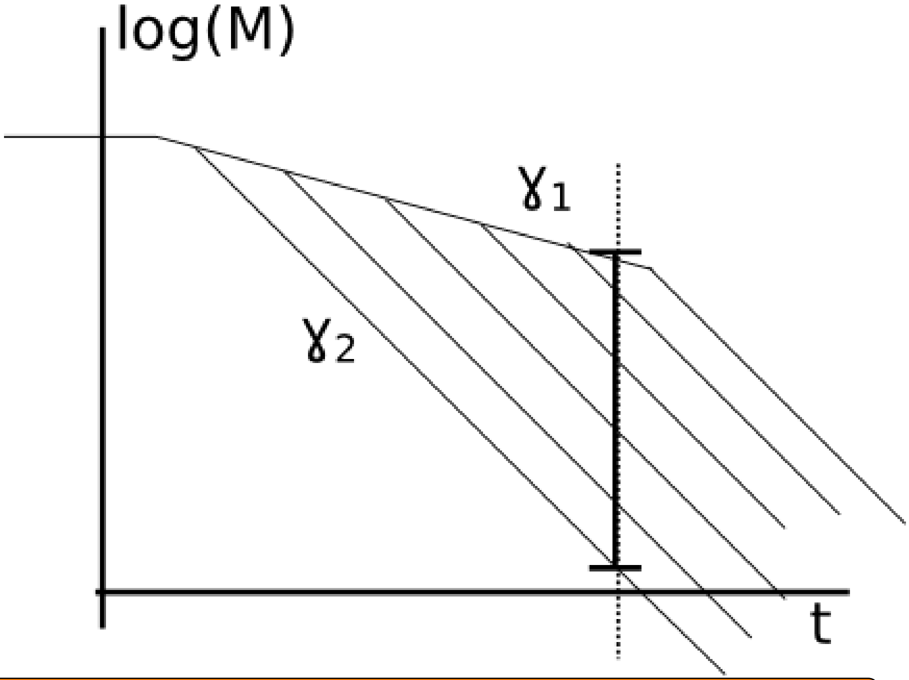
$$\begin{aligned} (dM/dt)_{dis} = -\frac{M}{t_{dis}(M)} = -\frac{1}{t_0} M^{1-\gamma} & \quad (14) \\ \gamma \approx 0.62 & \quad (15) \\ t_0 \text{ depends on the tidal field} & \quad (16) \\ M_i \text{ birth mass of the cluster} & \quad (17) \end{aligned}$$

If $\gamma = 1$, and the evolutionary mass loss could be ignored, then the mass of the cluster would decrease linearly with time until $t = t_0\, M_i$.

If $N(M,t)$ is the number of clusters of mass M and age t (unit: $M^{-1}\, yr{-}1$), it is related by conservation of the number of clusters by

$$N(M,t)dM = N(M_i,t)dM_i \tag{18}$$

Geometric look at $N(t,M)$:



We have unit problems in the following

$$
\begin{aligned}
M_{max} &= M_i\, t^{-\gamma_1} && (19)\\
M_{min} &= M_i\, t^{-\gamma_2} && (20)\\
X_{max} &= \ln\left(M_{max}/M_i\right) = -\gamma_1\, t && (21)\\
X_{min} &= \ln\left(M_{min}/M_i\right) = -\gamma_2\, t && (22)\\
& && (23)
\end{aligned}
$$

We want to find something like:

$$\mathcal{P}\left(\, M \;\mid\; M_i, t\,\right) = M_i e^{\mathcal{U}(X_{min}, X_{max})} \tag{24}$$