

4. Suppose we estimate the regression coefficients in a linear regression model by minimizing

$$\sum_{i=1}^n \left( y_i - \beta_0 - \sum_{j=1}^p \beta_j x_{ij} \right)^2 + \lambda \sum_{j=1}^p \beta_j^2$$

for a particular value of  $\lambda$ . For parts (a) through (e), indicate which of i. through v. is correct. Justify your answer.

- (a) As we increase  $\lambda$  from 0, the training RSS will:
- i. Increase initially, and then eventually start decreasing in an inverted U shape.
  - ii. Decrease initially, and then eventually start increasing in a U shape.
  - iii. Steadily increase.
  - iv. Steadily decrease.
  - v. Remain constant.

As  $\lambda$  increases, the penalty term will grows, making the model smoother and shrinking the regression coefficients, which reduces the model's fitting ability and causes training RSS to steadily increase. (iii)

**(b) Repeat (a) for test RSS.**

As  $\lambda$  increases, the model becomes simpler, reducing overfitting, so the test RSS may initially decrease. However, if  $\lambda$  becomes too large, the model will underfit, causing test RSS to increase in a U shape. **(ii)**

**(c) Repeat (a) for variance.**

**Explanation:** As  $\lambda$  increases, the model becomes smoother and simpler, reducing its sensitivity to data fluctuations, so **variance steadily decreases**. **(iv)**.

**(d) Repeat (a) for (squared) bias.**

As  $\lambda$  increases, the model complexity decreases, causing **bias to steadily increase** because the model's ability to fit the data is reduced. **(iii)**.

**(e) Repeat (a) for the irreducible error.**

The irreducible error is due to noise and other unexplained variance in the data and is not affected by changes in  $\lambda$ . Hence, it will **remain constant**. The correct answer is **(v)**.