To solve this question, we'll use the logistic regression equation:

$$P(Y = 1|X_1, X_2) = \frac{1}{1 + e^{-(\beta_0 + \beta_1 X_1 + \beta_2 X_2)}}$$

Given:

- $\beta_0 = -6$
- $\beta_1 = 0.05$
- $\beta_2 = 1$

(a) Estimate the probability that a student who studies for 40 hours and has an undergrad GPA of 3.5 gets an A.

Using $X_1 = 40$ (hours studied) and $X_2 = 3.5$ (GPA), we can substitute these values into the logistic regression equation.

1. Calculate the linear combination:

$$\text{logit} = \beta_0 + \beta_1 X_1 + \beta_2 X_2 = -6 + (0.05 \times 40) + (1 \times 3.5)$$

2. Simplify:

$$= -6 + 2 + 3.5 = -0.5$$

3. Calculate the probability:

$$P(Y = 1|X_1 = 40, X_2 = 3.5) = \frac{1}{1 + e^{0.5}}$$

4. Solve:

$$e^{0.5} \approx 1.6487$$

$$P(Y = 1) = \frac{1}{1 + 1.6487} \approx 0.3775$$

So, the probability that a student who studies for 40 hours and has a GPA of 3.5 will get an A is approximately **0.3775** or **37.75%**.

(b) How many hours would the student in part (a) need to study to have a 50% chance of getting an A?

To have a 50% chance of getting an A, the probability $P(Y = 1)$ should be 0.5.

1. Set up the equation with $P(Y = 1) = 0.5$:

$$0.5 = \frac{1}{1 + e^{-(\beta_0 + \beta_1 X_1 + \beta_2 X_2)}}$$

2. Solving for the logit when $P(Y = 1) = 0.5$:

$$0 = \beta_0 + \beta_1 X_1 + \beta_2 X_2$$

3. Substitute the values for β_0, β_1, and X_2 (GPA of 3.5):

$$-6 + 0.05 X_1 + 1 \times 3.5 = 0$$

4. Simplify:

$$-6 + 0.05 X_1 + 3.5 = 0$$

$$0.05 X_1 - 2.5 = 0$$

$$0.05 X_1 = 2.5$$

$$X_1 = \frac{2.5}{0.05} = 50$$

So, the student would need to study **50 hours** to have a 50% chance of getting an A.