

Ch 05 Q01

$$\begin{aligned}\text{Var}(dX + (1-d)Y) &= d^2 \sigma_X^2 + 2d(1-d)\text{cov}(X,Y) + (1-d)^2 \sigma_Y^2 \\&= d^2 (\underbrace{\sigma_X^2}_{a''} - 2\underbrace{\sigma_{XY}}_b + \underbrace{\sigma_Y^2}_{c''}) - 2d(-\underbrace{\sigma_{XY}}_b + \underbrace{\sigma_Y^2}_{c''}) + \underbrace{\sigma_Y^2}_{c''} \\&= ad^2 + bda + c\end{aligned}$$

$$\min_d \text{Var}(dX + (1-d)Y) = \min_d \left[a \left(d - \frac{b}{a} \right)^2 + \frac{b^2}{a} + c \right]$$

$$\alpha = \frac{b}{a} \text{ 时 } \text{有 } \min \quad \because a \geq 0$$

$$\Rightarrow \alpha = \frac{\sigma_Y^2 - \sigma_{XY}}{\sigma_X^2 - 2\sigma_{XY} + \sigma_Y^2} \text{ 时, } \text{Var}(dX + (1-d)Y) \text{ 取 } \min.$$