6. Suppose we collect data for a group of students in a statistics class with variables $X_1 =$ hours studied, $X_2 =$ undergrad GPA, and $Y =$ receive an A. We fit a logistic regression and produce estimated coefficient, $\hat{\beta}_0 = -6, \hat{\beta}_1 = 0.05, \hat{\beta}_2 = 1$.

   (a) Estimate the probability that a student who studies for 40 h and has an undergrad GPA of 3.5 gets an A in the class.

Model would be like：

$$P(Y = 1|X_1, X_2) = \frac{1}{1 + e^{-(\beta_0 + \beta_1 X_1 + \beta_2 X_2)}}$$

- $\beta_0 = -6$
- $\beta_1 = 0.05$ (for X_1, hours studied)
- $\beta_2 = 1$ (for X_2, undergrad GPA)

X1 = 40 and X2 = 3.5

Logit = −6+(0.05×40)+(1×3.5) = -0.5

So, $$P(Y = 1|X_1 = 40, X_2 = 3.5) = \frac{1}{1 + e^{0.5}}$$

Calculating $e^{0.5} \approx 1.6487$:

$$P(Y = 1|X_1 = 40, X_2 = 3.5) = \frac{1}{1 + 1.6487} \approx \frac{1}{2.6487} \approx 0.3775$$

So, estimate that the probability is around **37.5%**

(b) How many hours would the student in part (a) need to study to have a 50% chance of getting an A in the class?

$P(Y = 1|X_1, X_2 = 3.5) = 0.5$

Correspond to logit = 0, so

$$\beta_0 + \beta_1 X_1 + \beta_2 X_2 = 0$$

Substitute the known values：

$$-6 + 0.05X_1 + 1 \times 3.5 = 0$$

Get：

1. $-6 + 3.5 = -2.5$
2. $0.05X_1 = 2.5$
3. $X_1 = \frac{2.5}{0.05} = 50$

Therefore, the student would need to study **50 hours** to have a 50% chance of getting an A in the class.