Ch 05 Q01

$$\mathrm{Var}(\alpha X+(1-\alpha)Y) = \alpha^2\sigma_X^2 + 2\alpha(1-\alpha)\mathrm{cov}(X,Y) + (1-\alpha)^2\sigma_Y^2$$

$$= \alpha^2(\underbrace{\sigma_X^2 - 2\sigma_{XY} + \sigma_Y^2}_{a\geq 0}) - 2\alpha(\underbrace{-\sigma_{XY} + \sigma_Y^2}_{b}) + \underbrace{\sigma_Y^2}_{c}$$

$$= a\alpha^2 + 2b\alpha + c$$

$$\min_\alpha \mathrm{Var}(\alpha X+(1-\alpha)Y) = \min_\alpha \left[a\left(\alpha - \frac{b}{a}\right)^2 + \frac{b^2}{a} + c\right]$$

$\alpha = \frac{b}{a}$ 時有 min $\because a \geq 0$

$$\Rightarrow \alpha = \frac{\sigma_Y^2 - \sigma_{XY}}{\sigma_X^2 - 2\sigma_{XY} + \sigma_Y^2}$$ 時, $\mathrm{Var}(\alpha X+(1-\alpha)Y)$ 有 min.