1. Using a little bit of algebra, prove that (4.2) is equivalent to (4.3). In other words, the logistic function representation and logit representation for the logistic regression model are equivalent.

$$p(X) = \frac{e^{\beta_0+\beta_1 X}}{1+e^{\beta_0+\beta_1 X}}. \qquad (4.2)$$

$$\frac{p(X)}{1-p(X)} = e^{\beta_0+\beta_1 X}. \qquad (4.3)$$

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$$p(x) = \frac{1}{1+e^{-(\beta_0+\beta_1 x)}}$$

Let $t = e^{-(\beta_0+\beta_1 x)} \Rightarrow p(x) = \frac{1}{1+t}$

$$\frac{p(x)}{1-p(x)} = \frac{\frac{1}{1+t}}{1-\frac{1}{1+t}} = \frac{\frac{1}{1+t}}{\frac{t}{1+t}} = \frac{1}{t} = e^{\beta_0+\beta_1 x} \ ※$$