6. Suppose we collect data for a group of students in a statistics class with variables $X_1 =$ hours studied, $X_2 =$ undergrad GPA, and $Y =$ receive an A. We fit a logistic regression and produce estimated coefficient, $\hat{\beta}_0 = -6, \hat{\beta}_1 = 0.05, \hat{\beta}_2 = 1$.

(a) Estimate the probability that a student who studies for 40 h and has an undergrad GPA of 3.5 gets an A in the class.

(b) How many hours would the student in part (a) need to study to have a 50 % chance of getting an A in the class?

得到A的機率

$$\Rightarrow P(Y=1|X_1, X_2) = \frac{1}{1+\exp(-(\beta_0+\beta_1 X_1+\beta_2 X_2))}$$

1. $X_1 = 40, X_2 = 3.5$

$$P(Y=1) = \frac{1}{1+\exp(-(-6+0.05\times 40+1\times 3.5))} = \frac{1}{1+\exp(0.5)}$$

$$\approx 0.377 \#$$

2. h: Hours used for studies.

$$P(Y=1) = 0.5 = \frac{1}{1+\exp(-(-6+0.05\times h+1\times 3.5))}$$

$$\Rightarrow \exp(-(-6+0.05h+3.5)) = 1$$

$$\Rightarrow -6+0.05h+3.5 = 0, \quad 0.05h = 2.5$$

$$\therefore h = 50 \#$$