

1. Using a little bit of algebra, prove that (4.2) is equivalent to (4.3).

In other words, the logistic function representation and logit representation for the logistic regression model are equivalent.

$$(4.2) \quad p(x) = \frac{e^{\beta_0 + \beta_1 x}}{1 + e^{\beta_0 + \beta_1 x}} \quad (4.3) \quad \frac{p(x)}{1 - p(x)} = e^{\beta_0 + \beta_1 x}$$

$$1 - p(x) = 1 - \frac{e^{\beta_0 + \beta_1 x}}{1 + e^{\beta_0 + \beta_1 x}} = \frac{1 + e^{\beta_0 + \beta_1 x} - e^{\beta_0 + \beta_1 x}}{1 + e^{\beta_0 + \beta_1 x}} = \frac{1}{1 + e^{\beta_0 + \beta_1 x}}$$

$$\frac{p(x)}{1 - p(x)} = \frac{e^{\beta_0 + \beta_1 x}}{1 + e^{\beta_0 + \beta_1 x}} \cdot \frac{1 + e^{\beta_0 + \beta_1 x}}{1} = e^{\beta_0 + \beta_1 x}$$

\therefore (4.2) is equivalent to (4.3)