6. Suppose we collect data for a group of students in a statistics class with variables X1 = hours studied, X2 = undergrad GPA, and Y = receive an A. We fit a logistic regression and produce estimated coefficient,
$\hat{\beta}_0 = -6$, $\hat{\beta}_1 = 0.05$, $\hat{\beta}_2 = 1$.
(a) Estimate the probability that a student who studies for 40 h and has an undergrad GPA of 3.5 gets an A in the class.
(b) How many hours would the student in part (a) need to study to have a 50 % chance of getting an A in the class?

$$\text{logit}\left(P(Y=1|X)\right) = \beta_0 + \beta_1 X_1 + \beta_2 X_2$$

(a) $X_1 = 40$, $X_2 = 3.5$

$$\text{logit}\left(P(Y=1|X)\right) = -6 + 0.05 \times 40 + 1 \times 3.5$$

$$= -6 + 2 + 3.5$$

$$= -0.5$$

$$P(Y=1|X) = \frac{1}{1+e^{-(-0.5)}} = \frac{1}{1+e^{0.5}} \approx 37.75\% \text{ ※}$$

(b) $P(Y=1|X) = 50\%$

$$\frac{1}{1+e^{-x}} = 0.5 \Rightarrow x = 0 = \text{logit}(0.5)$$

$$\text{logit}(0.5) = -6 + 0.05 X_1 + 1 \times 3.5 = 0$$

$$\Rightarrow 0.05 X_1 = 2.5$$

$$\Rightarrow X_1 = 50 \text{ (h)} \text{ ※}$$