

1. Using a little bit of algebra, prove that (4.2) is equivalent to (4.3). In other words, the logistic function representation and logit representation for the logistic regression model are equivalent.

- logistic function

$$p(X) = \frac{e^{\beta_0 + \beta_1 X}}{1 + e^{\beta_0 + \beta_1 X}}. \quad (4.2)$$

$$\frac{p(X)}{1 - p(X)} = e^{\beta_0 + \beta_1 X}. \quad (4.3)$$

<proof>

$$1 - p(X) = \frac{1}{1 + e^{\beta_0 + \beta_1 X}}$$
$$\Rightarrow \frac{p(X)}{1 - p(X)} = \frac{\frac{e^{\beta_0 + \beta_1 X}}{1 + e^{\beta_0 + \beta_1 X}}}{\frac{1}{1 + e^{\beta_0 + \beta_1 X}}}$$
$$= e^{\beta_0 + \beta_1 X} \quad \#$$