1. Using basic statistical properties of the variance, as well as single-variable calculus, derive (5.6). In other words, prove that α given by (5.6) does indeed minimize $\mathrm{Var}(\alpha X + (1-\alpha)Y)$.

$$\alpha = \frac{\sigma_Y^2 - \sigma_{XY}}{\sigma_X^2 + \sigma_Y^2 - 2\sigma_{XY}}, \tag{5.6}$$

$$\mathrm{Var}(\alpha X + (1-\alpha)Y) = \alpha^2 \mathrm{Var}(X) + (1-\alpha)^2 \mathrm{Var}(Y) + 2\alpha(1-\alpha)\mathrm{Cov}(X,Y)$$

$$= \alpha^2 \sigma_X^2 + (\alpha^2 - 2\alpha + 1)\sigma_Y^2 + (2\alpha - 2\alpha^2)\sigma_X\sigma_Y$$

$$= \alpha^2(\sigma_X^2 + \sigma_Y^2 - 2\sigma_X\sigma_Y) + \alpha(2\sigma_X\sigma_Y - 2\sigma_Y^2) + \sigma_Y^2$$

$$\Rightarrow \frac{d}{d\alpha}\mathrm{Var}(\alpha X + (1-\alpha)Y)$$

$$= 2\alpha(\sigma^2 X + \sigma^2 Y - 2\sigma_X\sigma_Y) + (2\sigma_X\sigma_Y - 2\sigma_Y^2) = 0$$

$$\therefore \cancel{2}\alpha(\sigma_X^2 + \sigma_Y^2 - 2\sigma_X\sigma_Y) = \cancel{2}(\sigma_Y^2 - \sigma_X\sigma_Y)$$

$$\Rightarrow \underline{\alpha = \frac{\sigma_Y^2 - \sigma_X\sigma_Y}{\sigma_X^2 + \sigma_Y^2 - 2\sigma_X\sigma_Y}} \ \#$$