1. Using a little bit of algebra, prove that (4.2) is equivalent to (4.3). In other words, the logistic function representation and logit representation for the logistic regression model are equivalent.

$$p(X) = \frac{e^{\beta_0+\beta_1 X}}{1+e^{\beta_0+\beta_1 X}}. \tag{4.2}$$

$$\frac{p(X)}{1-p(X)} = e^{\beta_0+\beta_1 X}. \tag{4.3}$$

$$\longrightarrow 1 - p(X) = 1 - \frac{e^{\beta_0+\beta_1 X}}{1+e^{\beta_0+\beta_1 X}} = \frac{1}{1+e^{\beta_0+\beta_1 X}}$$

$$\therefore \frac{p(X)}{1-p(X)} = p(X) \times \frac{1}{1-p(X)}$$

$$= \frac{e^{\beta_0+\beta_1 X}}{1+e^{\beta_0+\beta_1 X}} \times \left(1+e^{\beta_0+\beta_1 X}\right)$$

$$= e^{\beta_0+\beta_1 X} \quad \#$$