HW1028_02.

(4.17) $p_k(x) = \dfrac{\pi_k \frac{1}{\sqrt{2\pi}\sigma} \exp(-\frac{1}{2\sigma^2}(x-\mu_k)^2)}{\sum_{l=1}^{k} \pi_l \frac{1}{\sqrt{2\pi}\sigma} \exp(-\frac{1}{2\sigma^2}(x-\mu_l)^2)}$

(4.18) $\delta_k(x) = x \cdot \frac{\mu_k}{\sigma^2} - \frac{\mu_k^2}{2\sigma^2} + \log(\pi_k)$

log是單調函數，∴最大化 $p_k(x)$ 也等於最大化 $\log(p_k(x))$

$\log(p_k(x)) = \log(\pi_k \frac{1}{\sqrt{2\pi}\sigma} \exp(-\frac{1}{2\sigma^2}(x-\mu_k)^2)) - \log(\sum_{l=1}^{k} \pi_l \frac{1}{\sqrt{2\pi}\sigma} \exp(-\frac{1}{2\sigma^2}(x-\mu_l)^2))$

$\log(\pi_k) - \frac{(x-\mu_k)^2}{2\sigma^2} = \log(\pi_k) - \frac{x^2 - 2x\mu_k + \mu_k^2}{2\sigma^2} = x \cdot \frac{\mu_k}{\sigma^2} - \frac{\mu_k^2}{2\sigma^2} + \log(\pi_k) - \frac{x^2}{2\sigma^2} = \delta_k(x)$

∴ 最大化 $\log(p_k(x))$ = 最大化 $\delta_k(x)$.