HW 1022 Q1 1.

a.

X	$f_X(x)$	Y	$f_Y(y)$
1	0.19	1	0.19
2	0.32	2	0.32
3	0.31	3	0.31
4	0.18	4	0.18

b.

X	$p(X \mid Y=1)$	Y	$p(Y \mid X=1)$
1	$\frac{0.1}{0.19} = \frac{10}{19}$	1	$\frac{10}{19}$
2	$\frac{5}{19}$	2	$\frac{5}{19}$
3	$\frac{2}{19}$	3	$\frac{2}{19}$
4	$\frac{2}{19}$	4	$\frac{2}{19}$

Q2 8.

$\frac{6}{7}x^2 + \frac{6}{7}y^2 + \frac{12}{7}xy$

$p(x>y)$: $\int_0^1 \int_y^1 \frac{6}{7}(x+y)^2 \, dx \, dy = \frac{1}{2}$

$p(x+y \le 1)$: $\int_0^1 \int_0^{1-y} f(x,y) = \frac{3}{14}$

$p(x \le \frac{1}{2})$: $\int_0^1 \int_0^{\frac{1}{2}} f(x,y) = \frac{2}{7}$

b.

$f_X(x) = \int_0^1 \frac{6}{7}x^2 + \frac{6}{7}y^2 + \frac{12}{7}xy \, dy$

$= \left\{ \frac{6}{7}x^2 y + \frac{2}{7}y^3 + \frac{6}{7}xy^2 \right]_0^1 = \frac{6}{7}x^2 + \frac{2}{7} + \frac{6}{7}x$

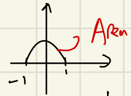
$f_Y(y) = \frac{6}{7}y^2 + \frac{2}{7} + \frac{6}{7}y$

c.

$f_{X|Y}(x|y) = \frac{f_{X,Y}(x,y)}{f_Y(y)} = \frac{3(x+y)^2}{3y^2+3y+1}$ for $0 \le x \le 1$

$f_{Y|X}(y|x) = \frac{f_{X,Y}(x,y)}{f_X(x)} = \frac{3(x+y^2)}{3x^2+3x+1}$ for $0 \le y \le 1$

HW 1022 Q3 9.



Area $= \frac{4}{3}$ $\quad -1 \le x \le 1 \quad 0 \le y \le 1-x^2$

a. $f(x,y) = \dfrac{1}{\int_{-1}^{1}(1-x^2)\,dx} = \dfrac{1}{\left[x-\frac{x^3}{3}\right]_{-1}^{1}} = \dfrac{1}{\frac{2}{3}-(-1+\frac{1}{3})} = \dfrac{1}{\frac{4}{3}} = \dfrac{3}{4}$

$f_X(x) = \int_0^{1-x^2} \frac{3}{4}\,dy = \frac{3}{4}(1-x^2)$

$f_Y(y) = \int_{-\sqrt{1-y}}^{\sqrt{1-y}} \frac{3}{4}\,dx = \frac{3}{4}(\sqrt{1-y}+\sqrt{1-y}) = \frac{3}{2}\sqrt{1-y}$

$f_{X|Y}(x|y) = \dfrac{1}{2\sqrt{1-y}} \qquad -\sqrt{1-y} \le x \le \sqrt{1-y}$

$f_{Y|X}(y|x) = \dfrac{1}{1-x^2} \qquad 0 \le y \le 1-x^2$

HW 1022 Q4 11.

$f_{U_1}(u_1) = 1 = f_{U_2}(u_2) = f_{U_3}(u_3)$

$\int_0^1\int_0^1\int_{2\sqrt{u_1u_3}}^1 1 \, du_2\, du_1\, du_3$

$U_2^2 - 4U_1U_3 > 0 \Rightarrow U_2^2 > 4U_1U_3$

$= \int_0^1\int_0^1 1-2\sqrt{u_1u_3} = \frac{1}{9}$

$P(U_2^2 > 4U_1U_3) = P(U_2 > 2\sqrt{U_1U_3}) \begin{cases} 1-2\sqrt{u_1u_3} & \text{if } 2\sqrt{u_1u_3} < 1 \\ 0 & \text{if } 2\sqrt{u_1u_3} > 1 \end{cases}$

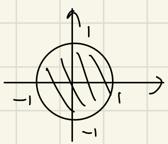
HW 1022 Q5 15.

$$f(x,y) = \int_0^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} c\sqrt{1-x^2-y^2} = 1$$

$$= \int_0^{2\pi} \int_0^1 cr\sqrt{1-r^2}\, dr\, d\theta = 1$$

$$\Rightarrow \quad C = \frac{3}{2\pi}$$

b.



c. $f(x,y) = \frac{3}{2\pi}\sqrt{1-x^2-y^2}$

$$P(x^2+y^2 \le \tfrac{1}{2}) = \int_0^{2\pi} \int_0^{\frac{1}{2}} \frac{3}{2\pi}\sqrt{1-r^2}\, r\, dr\, d\theta = \frac{2\sqrt{2}-1}{2\sqrt{2}}$$

d. $$f_X(x) = \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} f(x,y) = \frac{3}{4}(1-x^2)$$

$$f_Y(y) = \frac{3}{4}(1-y^2)$$

e.

$$f_{X|Y}(x|y) = \frac{f(x,y)}{f(y)} = \frac{2\sqrt{1-x^2-y^2}}{\pi(1-y)^2}$$

$$f_{Y|X}(y|x) = \frac{2\sqrt{1-x^2-y^2}}{\pi(1-x^2)}$$

HW 1022 Q6. 22.

$P(k) = e^{-\lambda} \frac{\lambda^k}{k!}$

$N(t_0, t_1) \sim Poisson(\lambda(t_1 - t_0))$

$N(t_1, t_2) \sim Poisson(\lambda(t_2 - t_1))$, $N(t_0, t_2) \sim Poisson(\lambda(t_2 - t_0))$

$P(N(t_0, t_1) \mid N(t_0, t_2)) = n \sim Multinomial\left(n, \frac{t_1 - t_0}{t_2 - t_0}, \frac{t_2 - t_1}{t_2 - t_0}\right)$

so $N(t_0, t_1) \mid N(t_0, t_2) = n \sim Binomial\left(n, \frac{t_1 - t_0}{t_2 - t_0}\right)$