HW 1119 Q 1

$$\mathbb{E}[X] = p \cdot 1 + (1-p) \cdot 0 = p$$

$$Var[X] = \mathbb{E}[X^2] - (\mathbb{E}[X])^2$$

$$\mathbb{E}[X^2] = p$$

$$Var[X] = p(1-p)$$

HW 1119 Q 2

$$\mathbb{E}[X] = \frac{1}{\lambda} = \int_0^\infty x \lambda e^{-\lambda x} dx$$

$$Var[X] = \frac{1}{\lambda^2} = \int_0^\infty x^2 \lambda e^{-\lambda x} dx - (\mathbb{E}[X])^2$$

HW 1119 Q 3

pdf of Gamma : $f_X(x) = \frac{\lambda^k x^{k-1} e^{-\lambda x}}{\Gamma(k)}, \quad x > 0$

$\Gamma(k) \hookrightarrow \int_0^\infty t^{k-1} e^{-t} dt$

$$\mathbb{E}[X] = \int_0^\infty x \cdot f_X(x) dx$$

$$= \frac{\lambda^k}{\Gamma(k)} \int_0^\infty x^k e^{-\lambda x} dx$$

$$= \frac{\lambda^k}{\Gamma(k)} \cdot \frac{\Gamma(k+1)}{\lambda^{k+1}} = \frac{k}{\lambda}$$

$$Var(X) = \mathbb{E}[X^2] - \mathbb{E}[X]^2$$

$$= \frac{\lambda^k}{\Gamma(k)} \int_0^\infty x^{k+1} e^{-\lambda x} dx - \mathbb{E}[X]^2 = \frac{k}{\lambda^2}$$

HW 1119 Q 4

$\mathbb{E}[X] = \mu$ ∋ because distribution is symmetric

$Var(X) = \mathbb{E}[(X-\mu)^2] = \mathbb{E}[X^2] - \mathbb{E}[X]^2$

$= \int_{-\infty}^{\infty} x^2 \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} - \mathbb{E}[X]^2$

$= \mu^2 + \sigma^2 - \mu^2 = \sigma^2$

HW 1119 Q 5

a. $E(X) = \int_0^1 x \cdot 2x = \left[\frac{2}{3}x^3\right]_0^1 = \frac{2}{3}$

b. $f(x) = 2x \quad y = x^2 \quad x = \sqrt{y} \quad dx = \frac{1}{2}y^{-\frac{1}{2}}$

$E[Y] = \int_0^1 y \cdot 2\sqrt{y} \cdot \frac{1}{2\sqrt{y}} = \frac{1}{2}$

c. $\mathbb{E}[X^2] = \int_0^1 x^2 \cdot 2x = \left[2\frac{x^4}{4}\right]_0^1 = \frac{1}{2}$

d. $Var(X) = E[X^2] - E(X)^2 = \frac{1}{2} - \left(\frac{2}{3}\right)^2 = \frac{1}{18}$

HW 11 19 Q 6

$$E(Z) = \frac{E(X) - \mu}{\sigma} = 0$$

$$Var(Z) = Var\left(\frac{X-\mu}{\sigma}\right) = \frac{1}{\sigma^2} Var(X)$$

$$= \frac{1}{\sigma^2} \sigma^2 = 1$$

Hw 11 19 Q 7

$$f(x) = 1 \qquad 1 < x < 2 \qquad \frac{1}{2} < y < 1$$

$$y = \frac{1}{x} \quad x = \frac{1}{y} \quad dx = -y^{-2}$$

$$f(y) = 1 \cdot \left(-\frac{1}{y^2}\right)$$

$$E[y] = \int_{\frac{1}{2}}^{1} y \cdot \left[-\frac{1}{y^2}\right] = \left[-\ln(y)\right]_{\frac{1}{2}}^{1}$$

$$= \ln(2) \neq \frac{1}{E[X]}$$

Hw 11 19 Q 8

$$f_X(x) = \lambda e^{-\lambda x}, \quad x \geq 0, \quad \sigma = \frac{1}{\lambda}, \; E(X) = \frac{1}{\lambda}$$

$$P(|X - E(X)| > 2\sigma) = P\left(\left|X - \frac{1}{\lambda}\right| > \frac{2}{\lambda}\right) = \int_{\frac{3}{\lambda}}^{\infty} \lambda e^{-\lambda x} = \left[-e^{-\lambda x}\right]_{\frac{3}{\lambda}}^{\infty} = e^{-3}$$

$$P(|X - E(X)| > 3\sigma) = e^{-4}$$

$$P(|X - E(X)| > 4\sigma) = e^{-5}$$

Chebyshev's :

$$P(|X - E(X)| > 2\sigma) = \frac{1}{4}$$

$$P(|X - E(X)| > 3\sigma) = \frac{1}{9}$$

$$P(|X - E(X)| > 4\sigma) = \frac{1}{16}$$

Hw1119 Q9

$$Var(X-Y) = E[(X-Y)^2] - E[X-Y]^2 = E[X^2-2XY+Y^2] - (E[X^2]-2E[XY]+E[Y^2])$$

$$Var(X) = E(X^2) - E(X)^2$$

$$Var(Y) = E(Y^2) - E(Y)^2$$

$$Cov(X,Y) = E[XY] - \mu_x\mu_y$$

$$= E[X^2] - E[X^2] + E[Y^2] - E[Y]^2$$

$$-2E[XY] + 2E[X]E[Y]$$

$$= Var[X] + Var[Y] - 2Cov(X,Y)$$

Hw1119 Q10

$$Cov(X,Y) = E(XY) - \mu_x\mu_y$$

$$Cov(X+Y, X-Y) = \underbrace{Cov(X,X)}_{Var(X)} - \underbrace{Cov(X,Y)}_{0} + \underbrace{Cov(Y,X)}_{0} - \underbrace{Cov(Y,Y)}_{Var(Y)}$$

$$= Var(X) - Var(Y) = 6^2 - 6^2 = 0$$