11. | (a)

$$y_1 = a_1 y_2 + \ell_1$$

$$y_2 = a_2 y_1 + \beta_1 x_1 + \beta_2 x_2 + \ell_2$$

$$\Rightarrow y_2 = a_2 (a_1 y_2 + \ell_1) + \beta_1 x_1 + \beta_2 x_2 + \ell_2$$

$$= a_1 a_2 y_2 + a_2 \ell_1 + \beta_1 x_1 + \beta_2 x_2 + \ell_2$$

$$\Rightarrow y_2 (1 - a_1 a_2) = \beta_1 x_1 + \beta_2 x_2 + (a_2 \ell_1 + \ell_2)$$

$$y_2 = \frac{\beta_1}{1 - a_1 a_2} x_1 + \frac{\beta_2}{1 - a_1 a_2} x_2 + \left[\frac{1}{1 - a_1 a_2} (a_2 \ell_1 + \ell_2) \right]$$

$$v_2 = \frac{a_2 \ell_1 + \ell_2}{1 - a_1 a_2}$$

$$Cov(y_2, \ell_1) = Cov(\pi_1 x_1 + \pi_2 x_2 + v_2, \ell_1)$$

$$= Cov(v_2, \ell_1) \neq 0.$$

11.1 (b)

又有 reduced form

$y_2 = \pi_1 x_1 + \pi_2 x_2 + v_2$ 可被一致性的估計.

(大樣本下)

11.1 (c)

y_1 is identified $(\lambda > 1)$

y_2 不一定 identified. $(0 < 1)$

11.1 (d)

11.1 (e)

$$y_{2i} = \pi_1 x_1 + \pi_2 x_2 + v_2$$

$$S(\pi_1, \pi_2) = \sum (y_{2i} - \pi_1 x_{1i} - \pi_2 x_{2i})^2$$

$$\frac{\partial S}{\partial \pi_1} = -2\sum (y_{2i} - \pi_1 x_{1i} - \pi_2 x_{2i}) x_{1i} = 0$$

$$\Rightarrow \sum y_{2i} x_{1i} - \pi_1 \sum x_{1i}^2 - \pi_2 \sum x_{1i} x_{2i} = 0.$$

$$\sum y_{2i} x_{1i} = \pi_1 \sum x_{1i}^2 + \pi_2 \sum x_{1i} x_{2i}$$

$$\frac{\partial S}{\partial \pi_2} = -2\sum (y_{2i} - \pi_1 x_{1i} - \pi_2 x_{2i}) x_{2i} = 0$$

$$\sum y_{2i} x_{2i} = \pi_1 \sum x_{1i} x_{2i} + \pi_2 \sum x_{2i}^2$$

解联立可得

$$\hat{\pi}_1 = \frac{(\sum x_{1i}^2)(\sum x_{2i} y_{2i}) - (\sum x_{1i} x_{2i})(\sum x_{1i} y_{2i})}{(\sum x_{1i}^2)(\sum x_{2i}^2) - (\sum x_{1i} x_{2i})^2}$$

$$\hat{\pi}_2 = \frac{(\sum x_{2i}^2)(\sum x_{1i} y_{2i}) - (\sum x_{1i} x_{2i})(\sum x_{2i} y_{2i})}{(\sum x_{1i}^2)(\sum x_{2i}^2) - (\sum x_{1i} x_{2i})^2}$$

11.1 (f)

$$\hat{\pi}_1 = \frac{1 \times 4 - 0}{1 \times 1 - 0} = 4$$

$$\hat{\pi}_2 = \frac{1 \times 3 - 0}{1} = 3$$

11.1 (g)

y_{2i} 中含有 v_{1i} 会导致 OLS 不一致

y_{2i} 是由外生 V x_1, x_2 构成.

$$E(y_{2i}, v_{1i}) = 0$$

$$\sum \frac{\sum y_1 y_2}{\sum y_2^2}$$

11.1 (9)

$$y_1 = a\hat{y}_2 + e_1^*$$

$$\hat{a}_1 = \frac{\sum y_1 \hat{y}_2}{\sum \hat{y}_2^2}$$ 同 (7)