5.6.

$n = 63$.

$\beta = \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}$

a.

① $H_0: \beta_2 = 0$

$H_a: \beta_2 \neq 0$

② $\alpha = 5\%$

③ test statistic: $t_0 = \frac{\hat{\beta}_2 - \beta_2}{SE(\hat{\beta}_2)} \sim t_{(63-4)}$

④ $RR = \{|t_0| \geq t_{0.05}(59) = 1.96\}$

⑤ $t_0 = \frac{3-0}{\sqrt{5}} \doteq 1.732 \notin RR$

⑥ Don't reject H_0, 樣本證據無足夠資訊指出 β_2 重要於 0

b.

$Var(c'b) = c' \times Cov(\beta) \times c$

$= [1, 2] \times \begin{bmatrix} 3 & -2 \\ -2 & 4 \end{bmatrix} \times \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$= [-1, 6] \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$= [11]$

① $H_0: \beta_1 + 2\beta_2 = 5$

$H_a: \beta_1 + 2\beta_2 \neq 5$

② $\alpha = 0.05$

③ test statistic: $t_0 = \dfrac{\hat{\beta}_1 + 2\hat{\beta}_2 - (\beta_1 + 2\beta_2)}{SE(\hat{\beta}_1 + 2\hat{\beta}_2)} \overset{H_0}{\sim} t_{0.025}(39)$

④ RR: $\{|t_0| \geq t_{0.025}(39) = 1.96\}$

⑤ $t_0 = \dfrac{(2 + 2\times 3) - 5}{\sqrt{7}} = 0.9449$

⑥ Reject H_0 樣本證據無足夠資訊指出 $\beta_1 + 2\beta_2 = 5$

c. $Var(\hat{\beta}_1 - \hat{\beta}_2 + \hat{\beta}_3) = \begin{bmatrix} 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 3 & -2 & 1 \\ -2 & 4 & 0 \\ 1 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$

$= 16$

① $H_0: \beta_1 - \beta_2 + \beta_3 = 4$

$H_a: \beta_1 - \beta_2 + \beta_3 \neq 4$

② $\alpha = 0.05$

③ test statistic: $t_0 = \dfrac{\hat{\beta}_1 - \hat{\beta}_2 + \hat{\beta}_3 - (\beta_1 - \beta_2 + \beta_3)}{SE(\hat{\beta}_1 - \hat{\beta}_2 + \hat{\beta}_3)} \overset{H_0}{\sim} t_{0.025}(39)$

④ $RR = \{|t_0| \geq t_{0.025}(39) = 1.96\}$

⑤ $t_0 = \dfrac{-4}{\sqrt{16}} = -1$

⑥ Reject H_0 樣本證據無足夠資訊指出 Diet

$\beta_1 - \beta_2 + 3 = 4$