CH05 HW0324 311707021 丁振朝

Q3

a.

1. T 值 b1 = 1.4515 / 2.2019 = 0.6592
2. Se = 2.7648 / 5.7103 = 0.4842
3. B3 係數 = 0.3695*-3.9376 = -1.4549
4. R square = 1-(SSE/SST), SST = $(6.39547)^2$*1199 = 49041.54
5. ERROR Variance = 根號 46221.62/1196 = 6.217

b.

- b2 = 2.7648： When total household expenditure increases by 1%, the proportion of the budget spent on alcohol **increases** by 2.7648% on average.
- b3 = 0.3695： When the number of children in the household (NK) increases by 1, the proportion of the budget spent on alcohol **increases** by 0.3695% on average.
- b4 = -0.1503： When the age of the household head (AGE) increases by 1 year, the proportion of the budget spent on alcohol **decreases** by 0.1503% on average.

c.

95%interval=−0.1503±1.96×0.0235=[−0.1964,−0.1042] for b4

The effect of age on alcohol expenditure falls between -0.1964 and -0.1042. Since the entire interval is negative, it confirms that an increase in age is significantly associated with a decrease in the proportion of the household budget spent on alcohol.

d.

b2,b3,b4<0.05 so they are statistically significant

e.

H0: $\beta 3 = -0.2$

H1: $\beta 3 \neq -0.2$

t= $[\beta 3-(-0.2)]/ SE3 = (-1.4515+0.2)/0.0939 = 1.475 < t(0.025,1196)=1.96$,

which **NOT** falls in the rejection region, so we cannot reject H0

Q23

a.

- $\beta 2<0$ As the quantity of cocaine sold **increases**, the price per gram typically **decreases** because buyers may receive bulk discounts.
- $\beta 3>0$ Higher purity cocaine is expected to be priced **higher** since consumers are usually willing to pay a premium for **better** quality.
- $\beta 4$ is uncertain and could be either **positive or negative**

b.

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Call:
lm(formula = price ~ quant + qual + trend, data = cocaine)

Residuals:
    Min      1Q  Median      3Q     Max 
-43.479 -12.014  -3.743  13.969  43.753 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 90.84669    8.58025  10.588 1.39e-14 ***
quant       -0.05997    0.01018  -5.892 2.85e-07 ***
qual         0.11621    0.20326   0.572   0.5700    
trend       -2.35458    1.38612  -1.699   0.0954 .  
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 20.06 on 52 degrees of freedom
Multiple R-squared:  0.5097,	Adjusted R-squared:  0.4814 
F-statistic: 18.02 on 3 and 52 DF,  p-value: 3.806e-08
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c.

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> summary(model_cocaine)$r.squared
[1] 0.50965
```

d.

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t-statistic: -5.891936
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-5.892<-1.6734，不拒絕 H0

e.

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t-statistic: 0.5716946
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0.572<1.6736，不拒絕 H0

f.

β4^ = -2.35458 indicates that, on average, the price of cocaine **decreases** by $2.35 per gram per year. This could be due to increased market competition, improvements in supply chain efficiency, or a rise in supply due to reduced enforcement efforts.

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Q_1 For $k=2$, the regression model : $Y_i = \beta_1 + \beta_2 x_i + e_i$

Let $X = \begin{bmatrix} 1 & x_1 \\ 1 & x_2 \\ \vdots & \vdots \\ 1 & x_n \end{bmatrix}$, $X'X = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ x_1 & x_2 & \cdots & x_n \end{bmatrix} \begin{bmatrix} 1 & x_1 \\ 1 & x_2 \\ \vdots & \vdots \\ 1 & x_n \end{bmatrix} = \begin{bmatrix} n & \Sigma x_i \\ \Sigma x_i & \Sigma x_i^2 \end{bmatrix}$

$(X'X)^{-1} = \dfrac{1}{n\Sigma x_i^2 - (\Sigma x_i)^2} \begin{bmatrix} \Sigma x_i^2 & -\Sigma x_i \\ -\Sigma x_i & n \end{bmatrix}$, $X'Y = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ x_1 & x_2 & \cdots & x_n \end{bmatrix} \begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_n \end{bmatrix} = \begin{bmatrix} \Sigma Y_i \\ \Sigma x_i \end{bmatrix}$

$b = [(X'X)^{-1}(X'Y)] = \dfrac{1}{n\Sigma x_i^2 - (\Sigma x_i)^2} \begin{bmatrix} \Sigma x_i^2 \Sigma Y_i - \Sigma x_i \Sigma x_i Y_i \\ -\Sigma x_i (\Sigma Y_i) + n\Sigma x_i Y_i \end{bmatrix}$

$b_2 = \dfrac{-\Sigma x_i \Sigma Y_i + n\Sigma x_i Y_i}{n\Sigma x_i^2 - (\Sigma x_i)^2} = \dfrac{-\cancel{n}^2 \bar{x}\bar{Y} + \cancel{n}\Sigma x_i Y_i}{n\Sigma x_i^2 - (n\bar{x})^2} = \dfrac{\Sigma(x_i - \bar{x})(Y_i - \bar{y})}{\Sigma(x_i - \bar{x})^2}$

$b_1 = \dfrac{\Sigma x_i^2 \Sigma Y_i - \Sigma x_i \Sigma x_i Y_i}{n\Sigma x_i^2 - (\Sigma x_i)^2} = \dfrac{\cancel{n}\Sigma x_i^2 \bar{Y} - \cancel{n}\bar{x}\Sigma x_i Y_i}{\cancel{n}\Sigma x_i^2 - (\cancel{n}\bar{x})^2}$

$= \bar{y} - b_2 \bar{x}$ #

Q_2 $Var(b) = \sigma^2 (X'X)^{-1} = \dfrac{\sigma^2}{n\Sigma x_i^2 - (\Sigma x_i)^2} \begin{bmatrix} \Sigma x_i^2 & -\Sigma x_i \\ -\Sigma x_i & n \end{bmatrix}$

$\Rightarrow \dfrac{\sigma^2}{n\Sigma x_i^2 - (n\bar{x})^2} \begin{bmatrix} \Sigma x_i^2 & -n\bar{x} \\ -n\bar{x} & n \end{bmatrix} = \begin{bmatrix} \dfrac{\sigma^2 \Sigma x_i^2}{n\Sigma x_i^2 - n\bar{x})^2} & \dfrac{-\sigma^2 \bar{x}}{\Sigma x_i^2 - n\bar{x}^2} \\ \dfrac{-\sigma^2 \bar{x}}{\Sigma x_i^2 - n\bar{x}^2} & \dfrac{-\sigma^2 \bar{x}}{(\Sigma x_i - \bar{x})^2} \end{bmatrix}$

$= \begin{bmatrix} \dfrac{\sigma^2 \Sigma x_i^2}{n(\Sigma x_i - \bar{x})^2} & \dfrac{-\sigma^2 \bar{x}}{(\Sigma x_i - \bar{x})^2} \\ \dfrac{-\sigma^2 \bar{x}}{(\Sigma x_i - \bar{x})^2} & \dfrac{\sigma^2}{(\Sigma x_i - \bar{x})^2} \end{bmatrix} = \begin{bmatrix} Var(b_1|X) & cov(b_1, b_2|X) \\ cov(b_1, b_2|X) & Var(b_2|X) \end{bmatrix}$

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