5.6 Suppose that, from a sample of 63 observations, the least squares estimates and the corresponding estimated covariance matrix are given by

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix} \qquad \widehat{\text{cov}}(b_1, b_2, b_3) = \begin{bmatrix} 3 & -2 & 1 \\ -2 & 4 & 0 \\ 1 & 0 & 3 \end{bmatrix}$$

Using a 5% significance level, and an alternative hypothesis that the equality does not hold, test each of the following null hypotheses:

a. $\beta_2 = 0$

$H_0: \beta_2 = 0 \qquad H_1: \beta_2 \neq 0$

$t = \frac{3-0}{\sqrt{4}} = 1.5 < t_{0.975,60} = 2.003 \Rightarrow$ not reject H_0 ※

b. $\beta_1 + 2\beta_2 = 5$

$H_0: \beta_1 + 2\beta_2 = 5 \qquad H_1: \beta_1 + 2\beta_2 \neq 5$

$se(b_1 + 2b_2) = \sqrt{3 + 2^2 \cdot 4 + 2 \cdot 2 \cdot (-2)} = \sqrt{11}$

$t = \frac{(2+6)-5}{\sqrt{11}} = 0.9045 < t_{0.975,60} = 2.003 \Rightarrow$ not reject H_0 ※

c. $\beta_1 - \beta_2 + \beta_3 = 4$

$H_0: \beta_1 - \beta_2 + \beta_3 = 4 \qquad H_1: \beta_1 - \beta_2 + \beta_3 \neq 4$

$se(b_1 - b_2 + b_3) = \sqrt{\widehat{var}(b_1) + \widehat{var}(b_2) + \widehat{var}(b_3) - 2\,cov(b_1,b_2) + 2\,cov(b_1,b_3) - 2\,cov(b_2,b_3)}$

$= \sqrt{3+4+3+4+2-0} = \sqrt{16} = 4$

$t = \frac{(2-3-1)-4}{4} = -1.5 > t_{0.025,60} = -2.003 \Rightarrow$ not reject H_0 ※