

**11.28** Supply and demand curves as traditionally drawn in economics principles classes have price ( $P$ ) on the vertical axis and quantity ( $Q$ ) on the horizontal axis.

- Rewrite the truffle demand and supply equations in (11.11) and (11.12) with price  $P$  on the left-hand side. What are the anticipated signs of the parameters in this rewritten system of equations?
- Using the data in the file *truffles*, estimate the supply and demand equations that you have formulated in (a) using two-stage least squares. Are the signs correct? Are the estimated coefficients significantly different from zero?
- Estimate the price elasticity of demand “at the means” using the results from (b).
- Accurately sketch the supply and demand equations, with  $P$  on the vertical axis and  $Q$  on the horizontal axis, using the estimates from part (b). For these sketches set the values of the exogenous variables  $DI$ ,  $PS$ , and  $PF$  to be  $DI^* = 3.5$ ,  $PF^* = 23$ , and  $PS^* = 22$ .
- What are the equilibrium values of  $P$  and  $Q$  obtained in part (d)? Calculate the predicted equilibrium values of  $P$  and  $Q$  using the estimated reduced-form equations from Table 11.2, using the same values of the exogenous variables. How well do they agree?
- Estimate the supply and demand equations that you have formulated in (a) using OLS. Are the signs correct? Are the estimated coefficients significantly different from zero? Compare the results to those in part (b).

a.

在需求曲線之中，價格跟數量成反比；在供給曲線之中，價格跟數量成正比

b.

Demand

supply

```
Call:
ivreg(formula = p ~ q + ps + di | ps + di + pf, data = truffles)

Residuals:
    Min       1Q   Median       3Q      Max
-39.661  -6.781   2.410   8.320  20.251

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  -11.428    13.592   -0.841  0.40810
q             -2.671     1.175   -2.273  0.03154 *
ps             3.461     1.116    3.103  0.00458 **
di            13.390     2.747   4.875  4.68e-05 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 13.17 on 26 degrees of freedom
Multiple R-Squared:  0.5567,    Adjusted R-squared:  0.5056
Wald test: 17.37 on 3 and 26 DF, p-value: 2.137e-06
```

```
Call:
ivreg(formula = p ~ q + pf | ps + di + pf, data = truffles)

Residuals:
    Min       1Q   Median       3Q      Max
-9.7983  -2.3440  -0.6281   2.4350  11.1600

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  -58.7982     5.8592  -10.04 1.32e-10 ***
q             2.9367     0.2158   13.61 1.32e-13 ***
pf            2.9585     0.1560   18.97 < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 4.399 on 27 degrees of freedom
Multiple R-Squared:  0.9486,    Adjusted R-squared:  0.9448
Wald test: 232.7 on 2 and 27 DF, p-value: < 2.2e-16
```

Demand方向是對的，價格低，需求量越多，因此是負向關係，會使價格降低，是的， $p$ -value < 0.05所以拒絕虛無假設，代表有顯著影響。

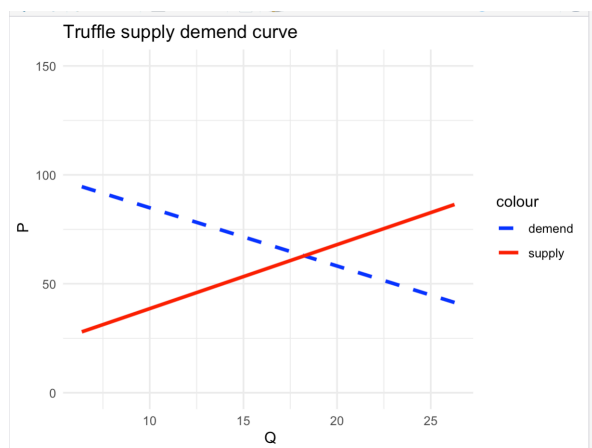
Supply方向是對的，價格越高，供給量越多，因此是同向的關係，且係數皆顯著，代表有顯著的影響。

C.

```
> elasticity <- alpha_2sls * (q_mean/p_mean)
> elasticity
      q
-0.7858767
```

彈性是負的因為價格越高，需求越低；且彈性 $<1$ ，代表價格變動引起的需求變動較小

d. 畫圖



e. 找交點的值跟

reduced-form 去估計值

```
> 
      Q_eq P_eq.(Intercept)
18.25021      62.84257
```

```
Q_reduced.(Intercept) P_reduced.(Intercept)
18.26040      62.81537
```

透過均衡與reduced-form去估計兩者有些微的差距但是差距不大

f.

```
Call:
lm(formula = p ~ q + ps + di, data = truffles)

Residuals:
    Min       1Q   Median       3Q      Max
-25.0753  -2.7742  -0.4097   4.7079  17.4979

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) -13.6195     9.0872  -1.499   0.1460
q             0.1512     0.4988   0.303   0.7642
ps            1.3607     0.5940   2.291   0.0303 *
di           12.3582     1.8254   6.770 3.48e-07 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 8.814 on 26 degrees of freedom
Multiple R-squared:  0.8013,    Adjusted R-squared:  0.7784
F-statistic: 34.95 on 3 and 26 DF,  p-value: 2.842e-09
```

```
Call:
lm(formula = p ~ q + pf, data = truffles)

Residuals:
    Min       1Q   Median       3Q      Max
-8.4721  -3.3287   0.1861   2.0785  10.7513

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) -52.8763     5.0238  -10.53 4.68e-11 ***
q             2.6613     0.1712  15.54 5.42e-15 ***
pf            2.9217     0.1482  19.71 < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 4.202 on 27 degrees of freedom
Multiple R-squared:  0.9531,    Adjusted R-squared:  0.9496
F-statistic: 274.4 on 2 and 27 DF,  p-value: < 2.2e-16
```

用ols估計，demand 的係數同向了，代表有內生性問題，且係數沒有顯著性，總結就是2SLS的回歸較為準確。

**11.30** Example 11.3 introduces Klein's Model I. Use the data file *klein* to answer the following questions.

- Estimate the investment function in equation (11.18) by OLS. Comment on the signs and significance of the coefficients.
- Estimate the reduced-form equation for profits,  $P_t$ , using all eight exogenous and predetermined variables as explanatory variables. Test the joint significance of all the variables except lagged profits,  $P_{t-1}$ , and lagged capital stock,  $K_{t-1}$ . Save the residuals,  $\hat{v}_t$  and compute the fitted values,  $\hat{P}_t$ .
- The Hausman test for the presence of endogenous explanatory variables is discussed in Section 10.4.1. It is implemented by adding the reduced-form residuals to the structural equation and testing their significance, that is, using OLS estimate the model

$$I_t = \beta_1 + \beta_2 P_t + \beta_3 P_{t-1} + \beta_4 K_{t-1} + \delta \hat{v}_t + e_{2t}$$

Use a  $t$ -test for the null hypothesis  $H_0: \delta = 0$  versus  $H_1: \delta \neq 0$  at the 5% level of significance. By rejecting the null hypothesis, we conclude that  $P_t$  is endogenous. What do we conclude from the test? In the context of this simultaneous equations model what result should we find?

- Obtain the 2SLS estimates of the investment equation using all eight exogenous and predetermined variables as IVs and software designed for 2SLS. Compare the estimates to the OLS estimates in part (a). Do you find any important differences?
- Estimate the second-stage model  $I_t = \beta_1 + \beta_2 \hat{P}_t + \beta_3 P_{t-1} + \beta_4 K_{t-1} + e_{2t}$  by OLS. Compare the estimates and standard errors from this estimation to those in part (d). What differences are there?
- Let the 2SLS residuals from part (e) be  $\hat{e}_{2t}$ . Regress these residuals on all the exogenous and predetermined variables. If these instruments are valid, then the  $R^2$  from this regression should be low, and none of the variables are statistically significant. The Sargan test for instrument validity is discussed in Section 10.4.3. The test statistic  $TR^2$  has a chi-square distribution with degrees of freedom equal to the number of "surplus" IVs if the surplus instruments are valid. The investment equation includes three exogenous and/or predetermined variables out of the total of eight possible. There are  $L = 5$  external instruments and  $B = 1$  right-hand side endogenous variables. Compare the value of the test statistic to the 95th percentile value from the  $\chi^2_{(4)}$  distribution. What do we conclude about the validity of the surplus instruments in this case?

a.

```
Call:
lm(formula = i ~ p + plag + klag, data = klein)

Residuals:
    Min       1Q   Median       3Q      Max
-2.56562 -0.63169  0.03687  0.41542  1.49226

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 10.12579    5.46555   1.853 0.081374 .
p             0.47964    0.09711   4.939 0.000125 ***
plag          0.33304    0.10086   3.302 0.004212 **
klag         -0.11179    0.02673  -4.183 0.000624 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.009 on 17 degrees of freedom
(1 observation deleted due to missingness)
Multiple R-squared:  0.9313,    Adjusted R-squared:  0.9192
F-statistic: 76.88 on 3 and 17 DF,  p-value: 4.299e-10
```

P：本期利潤越多，投資I會越多，同向

$p_{t-1}$ ：前一期的利潤越多，投資I會越多，同向

$K_{t-1}$ ：上期資本存量越多，投資I會越少，反向

## b. reduced-form

```
Call:
lm(formula = p ~ plag + klag + w2 + g + tx + time + elag, data = klein)

Residuals:
    Min       1Q   Median       3Q      Max
-3.9067 -1.3050  0.3226  1.3613  2.8881

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  50.38442   31.63026   1.593   0.1352
plag         0.80250    0.51886   1.547   0.1459
klag        -0.21610    0.11911  -1.814   0.0928 .
w2          -0.07961    2.53382  -0.031   0.9754
g           0.43902    0.39114   1.122   0.2820
tx          -0.92310    0.43376  -2.128   0.0530 .
time         0.31941    0.77813   0.410   0.6881
elag         0.02200    0.28216   0.078   0.9390
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 2.183 on 13 degrees of freedom
Multiple R-squared:  0.8261,    Adjusted R-squared:  0.7324
F-statistic: 8.821 on 7 and 13 DF,  p-value: 0.0004481
```

## c. 進行檢定看P是不是內生變數

```
Call:
lm(formula = i ~ p + plag + klag + vhat, data = klein)

Residuals:
    Min       1Q   Median       3Q      Max
-1.04645 -0.56030  0.06189  0.25348  1.36700

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  20.27821   4.70179   4.313 0.000536 ***
p           0.15022    0.10798   1.391 0.183222
plag        0.61594    0.10147   6.070 1.62e-05 ***
klag       -0.15779    0.02252  -7.007 2.96e-06 ***
vhat        0.57451    0.14261   4.029 0.000972 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.7331 on 16 degrees of freedom
Multiple R-squared:  0.9659,    Adjusted R-squared:  0.9574
F-statistic: 113.4 on 4 and 16 DF,  p-value: 1.588e-11
```

發現vhat顯著，代表說有p有內生性

## d.

```
Call:
ivreg(formula = i ~ p + plag + klag | w2 + g + tx + time + elag +
      plag + klag, data = klein)

Residuals:
    Min       1Q   Median       3Q      Max
-3.2909 -0.8069  0.1423  0.8601  1.7956

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  20.27821   8.38325   2.419  0.02707 *
p           0.15022    0.19253   0.780  0.44598
plag        0.61594    0.18093   3.404  0.00338 **
klag       -0.15779    0.04015  -3.930  0.00108 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.307 on 17 degrees of freedom
Multiple R-Squared:  0.8849,    Adjusted R-squared:  0.8646
Wald test:  41.2 on 3 and 17 DF,  p-value: 5.148e-08
```

P在OLS是顯著的，但是在2SLS中變得不顯著了，plag klag仍然都是顯著的，係數的正負也是符合常理

e.

```
Call:
lm(formula = i ~ phat + plag + klag, data = klein)

Residuals:
    Min       1Q   Median       3Q      Max
-3.8778 -1.0029  0.3058  0.7275  2.1831

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept) 20.27821    9.97663   2.033  0.05802 .
phat         0.15022    0.22913   0.656  0.52084
plag         0.61594    0.21531   2.861  0.01083 *
klag        -0.15779    0.04778  -3.302  0.00421 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.556 on 17 degrees of freedom
Multiple R-squared:  0.837,    Adjusted R-squared:  0.8082
F-statistic: 29.09 on 3 and 17 DF,  p-value: 6.393e-07
```

用ols跑出來出來的係數跟2sls的係數是一樣的，但是標準差是不一樣的，用ivreg跑出來的才會是正確的

f.

```
[1] 0.9368218
>
>
> TR2
[1] 1.281519
```

