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1. Let $K = 2$, show that $(b_1, b_2)$ in p. 29 of slides in Ch5 reduces to the formula of $(b_1, b_2)$

1. $k=2$

$Y = X\beta + e$

$Y = (y_1 \cdots y_N)'$, $\beta = (\beta_1 \cdots \beta_K)'$, $e = (e_1 \cdots e_N)'$

$$X = \begin{bmatrix} 1 & x_{1,2} \\ 1 & x_{2,2} \\ \vdots & \vdots \\ 1 & x_{N,2} \end{bmatrix} \qquad b = (X'X)^{-1}(X'Y)$$

$$X'X = \underbrace{\begin{bmatrix} 1 & 1 & \cdots & 1 \\ x_{1,2} & x_{2,2} & \cdots & x_{N,2} \end{bmatrix}}_{2\times N} \underbrace{\begin{bmatrix} 1 & x_{1,2} \\ 1 & x_{2,2} \\ \vdots & \vdots \\ 1 & x_{N,2} \end{bmatrix}}_{N\times 2} = \underbrace{\begin{bmatrix} N & \sum_{i=1}^{N} x_{i,2} \\ \sum_{i=1}^{N} x_{i,2} & \sum_{i=1}^{N} (x_{i,2})^2 \end{bmatrix}}_{2\times 2}$$

$$X'Y = \underbrace{\begin{bmatrix} 1 & 1 & \cdots & 1 \\ x_{1,2} & x_{2,2} & \cdots & x_{N,2} \end{bmatrix}}_{2\times N} \underbrace{\begin{bmatrix} y_1 \\ \vdots \\ y_N \end{bmatrix}}_{N\times 1} = \underbrace{\begin{bmatrix} \sum_{i=1}^{N} y_i \\ \sum_{i=1}^{N} x_{i,2} \cdot y_i \end{bmatrix}}_{2\times 1}$$

$$b = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \frac{1}{N \sum_{i=1}^{N} (x_{i,2})^2 - \left(\sum_{i=1}^{N} x_{i,2}\right)^2} \underbrace{\begin{bmatrix} \sum_{i=1}^{N} (x_{i,2})^2 & -\sum_{i=1}^{N} x_{i,2} \\ -\sum_{i=1}^{N} x_{i,2} & N \end{bmatrix}}_{2\times 2} \underbrace{\begin{bmatrix} \sum_{i=1}^{N} y_i \\ \sum_{i=1}^{N} x_{i,2} \cdot y_i \end{bmatrix}}_{2\times 1}$$

$$= \frac{1}{N \sum_{i=1}^{N} x_i^2 - \left(\sum_{i=1}^{N} x_i\right)^2} \begin{bmatrix} \sum_{i=1}^{N} x_i^2 \cdot \sum_{i=1}^{N} y_i - \sum_{i=1}^{N} x_i \sum_{i=1}^{N} x_i y_i \\ -\sum_{i=1}^{N} x_i \sum_{i=1}^{N} y_i + N \sum_{i=1}^{N} x_i y_i \end{bmatrix}$$

$$b_2 = \frac{N \sum_{i=1}^{N} x_i y_i - \sum_{i=1}^{N} x_i \sum_{i=1}^{N} y_i}{N \sum_{i=1}^{N} x_i^2 - \left(\sum_{i=1}^{N} x_i\right)^2} = \frac{\sum x_i y_i - \frac{1}{N} \overbrace{\sum_{i=1}^{N} x_i}^{N\bar{x}} \overbrace{\sum_{i=1}^{N} y_i}^{N\bar{y}}}{\sum x_i^2 - \frac{1}{N} \left(\sum x_i\right)^2} = \frac{\sum x_i y_i - N\bar{x}\bar{y}}{\sum x_i^2 - N\bar{x}^2} = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (x_i - \bar{x})^2}$$

$$b_1 = \frac{\sum x_i^2 \sum y_i - \sum x_i \sum x_i y_i}{N \sum x_i^2 - \left(\sum x_i\right)^2} = \frac{\sum x_i^2 \cancel{N} \bar{y} - N\bar{x} \sum x_i y_i}{\cancel{N} \sum x_i^2 - N^{\cancel{2}} \bar{x}^2}$$

$$= \frac{\sum x_i^2 \bar{y} - \bar{x} \sum x_i y_i}{\sum (x_i - \bar{x})^2} = \frac{\sum x_i^2 \bar{y} - N\bar{x}^2 \bar{y}}{\sum (x_i - \bar{x})^2} - \bar{x}\left(\frac{\sum x_i y_i - N\bar{x}\bar{y}}{\sum (x_i - \bar{x})^2}\right) = \bar{y} - \bar{x} b_2$$

2. Let $K = 2$, show that $cov(b_1, b_2)$ in p. 30 of slides in Ch5 reduces to the formula

2. $var(b) = \sigma^2 (X'X)^{-1}$

$$\sigma^2 (X'X)^{-1} = \frac{\sigma^2}{N\sum x_i^2 - (\sum x_i)^2} \begin{bmatrix} \sum x_i^2 & -\sum x_i \\ -\sum x_i & N \end{bmatrix} = \begin{array}{c} \begin{array}{cc} \quad var(b_1|x) \quad & \quad cov(b_1,b_2|x) \end{array} \\ \begin{bmatrix} \frac{\sigma^2 \sum x_i^2}{N\sum x_i^2 - (\sum x_i)^2} & \frac{-\sigma^2 \sum x_i}{N\sum x_i^2 - (\sum x_i)^2} \\ \frac{-\sigma^2 \sum x_i}{N\sum x_i^2 - (\sum x_i)^2} & \frac{\sigma^2 N}{N\sum x_i^2 - (\sum x_i)^2} \end{bmatrix} \\ \begin{array}{cc} \quad cov(b_2,b_1|x) \quad & \quad var(b_2|x) \end{array} \end{array}$$

$$var(b_1|x) = \frac{\sigma^2 \sum x_i^2}{N\sum x_i^2 - (\sum x_i)^2} = \frac{\sigma^2 \sum x_i^2}{N\sum x_i^2 - N^2\bar{x}^2} = \frac{\sigma^2 \sum x_i^2}{N(\sum x_i^2 - N\bar{x}^2)} = \sigma^2 \left[\frac{\sum x_i^2}{N(\sum x_i - \bar{x})^2}\right]$$

$$var(b_2|x) = \frac{\sigma^2 N}{N\sum x_i^2 - (\sum x_i)^2} = \frac{\sigma^2 N}{N\sum x_i^2 - N^2\bar{x}^2} = \sigma^2 \frac{1}{\sum x_i^2 - N\bar{x}^2} = \frac{\sigma^2}{\sum (x_i - \bar{x})^2}$$

$$cov(b_1, b_2|x) = \frac{-\sigma^2 \sum x_i}{N\sum x_i^2 - (\sum x_i)^2} = -\sigma^2 \frac{N\bar{x}}{N\sum x_i^2 - N^2\bar{x}^2} = \sigma^2 \left[\frac{-\bar{x}}{\sum (x_i - \bar{x})^2}\right]$$

**5.3 a. Fill in the following blank spaces that appear in this table.**

| Dependent Variable:WALC | | | | |
|---|---|---|---|---|
| Included observations:1200 | | | | |
| Variable | Coefficient | Std. Error | t-Statistic | Prob. |
| C | 1.4515 | 2.2019 | 0.6594 | 0.5099 |
| ln(TOTEXP) | 2.7648 | 0.4840 | 5.7103 | 0.0000 |
| NK | -1.4549 | 0.3695 | -3.9376 | 0.0001 |
| AGE | -0.1503 | 0.0235 | -6.4019 | 0.0000 |
| R-squared | 0.0575 | Mean dependent var | 6.19434 | |
| S.E. of regression | 6.2167 | S.D. dependent var | 6.39547 | |
| Sum squared resid | 46221.62 | | | |

**Ans.**

i. The t-statistic for $b_1$=1.4515/2.2019=0.6594

ii. The standard error for $b_2$=2.7648/5.7103=0.4840

iii. The estimate $b_3$=0.3695×(-3.9376)= -1.4549

iv. $R^2 = 1 - 46221.62/(6.39547^2 \times 1199) = 0.0575$

v. $\hat{\sigma} = [\frac{46221.62}{(1200-4)}]^{\frac{1}{2}} = 6.2167$

**b. Interpret each of the estimates $b_2$, $b_3$, and $b_4$**

**Ans.**

$\beta_2 = 2.7648$ : For every 1% increase in total household expenditure, the budget share spent on alcohol increases by approximately 0.027648 percentage points .

$\beta_3 = -1.4549$ : Each additional child in the household is associated with a decrease of approximately 1.4549 percentage points in the alcohol budget share.

$\beta_4 = -0.1503$ : For each additional year of the household head's age, the alcohol budget share decreases by approximately 0.1503 percentage points.

**c. Compute a 95% interval estimate for $\beta_4$.What does this interval tell you?**

**Ans.**

-0.1503-1.96×0.0235=-0.1964 ,-0.1503+1.96×0.0235=-0.1042

The 95% interval estimate for $\beta_4$ is [-0.1964,-0.1042]

For each additional year of the household head's age, the alcohol budget share decreases by an amount between 0.1042 and 0.1964 percentage points.

**d. Are each of the coefficient estimates significant at a 5% level? Why?**

**Ans.**

Except for intercept, all coefficient estimates are significant at a 5% level since their p-values are all less than 0.05.

**e.Test the hypothesis that the addition of an extra child decreases the mean budget share of alcohol by 2 percentage points against the alternative that the decrease is not equal to 2 percentage points. Use a 5% significance level.**

**Ans.**

null hypothesis : $H_0 = -2$

Alternative hypothesis : $H_1 \neq -2$

test statistic $t = \dfrac{-1.4549 - (-2)}{0.3695} = 1.4752$

We fail to reject $H_0$ since $|1.4752| < 1.96$

Each additional child in the household is leads to a decline in the alcohol budget share that is different from two percentage points.

5.23 **a. What signs would you expect on the cofficients $\beta_2$, $\beta_3$, and $\beta_4$?**

**Ans.**

$\beta_2$(QUANT):The expected sign of $\beta_2$ is **negative**, because larger quantities in a single sale are typically associated with lower prices per gram.

$\beta_3$ (QUAL):The expected sign of $\beta_3$ is **positive**, since higher purity (quality) cocaine should command a higher price.

$\beta_4$ (TREND):The expected sign of $\beta_4$ is **uncertain.** If supply increases over time or law enforcement becomes more effective, the market price might decline. If demand increases over time, the market price might rise.

**b. Use your computer software to estimate the equation. Report the results and interpret the coefficient estimates. Have the signs turned out as you expected?**

**Ans.**

$\widehat{PRICE} = 90.8467 - 0.06QUANT + 0.1162QUAL - 2.3546TREND$

The estimated values for $\beta_2$ = -0.0600 Every additional gram sold in a transaction ,the price per gram decreases by about \$0.06. This matches our expectation of a negative sign.

The estimated values for $\beta_3$ = 0.01162 Every additional number of grams of cocaine in a given sale, the price per gram increases by about \$0.1162. This matches our expectation of a positive sign.

The estimated values for $\beta_4$ = - 2.3546 Every additional year, the price per gram decreases by about \$2.3546.This imply supply increases more effective then demand.

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Residuals:
    Min      1Q  Median      3Q     Max 
-43.479 -12.014  -3.743  13.969  43.753 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 90.84669    8.58025  10.588 1.39e-14 ***
quant       -0.05997    0.01018  -5.892 2.85e-07 ***
qual         0.11621    0.20326   0.572   0.5700    
trend       -2.35458    1.38612  -1.699   0.0954 .  
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**c.What proportion of variation in cocaine price is explained jointly by variation in quantity, quality, and time?**

**Ans.**

$R^2 = 0.5097$

**d.It is claimed that the greater the number of sales, the higher the risk of getting caught. Thus, sellers are willing to accept a lower price if they can make sales in larger quantities. Set up $H_0$ and $H_1$ that would be appropriate to test this hypothesis. Carry out the hypothesis test.**

**Ans.**

null hypothesis: $H_0 : \beta_2 \geq 0$

Alternative hypothesis: $H_0 : \beta_2 < 0$

test statistic $= -5.892$ , $|-5.892| > 1.675 = t_{(0.95,52)}$

Reject null hypothesis, means that an increase in the quantity of cocaine sold in a given transaction is associated with a decrease in the price per gram.

**e. Test the hypothesis that the quality of cocaine has no influence on expected price against the alternative that a premium is paid for better-quality cocaine.**

**Ans.**

null hypothesis: $H_0 : \beta_3 \leq 0$

Alternative hypothesis: $H_0 : \beta_3 > 0$

test statistic $= 0.572$ , $|0.572| < 1.675 = t_{(0.95,52)}$

We can't reject null hypothesis, means that a premium isn't paid for better-quality cocaine.

**f. What is the average annual change in the cocaine price? Can you suggest why price might be changing in this direction**

**Ans.**

The average annual change in the price of cocaine is a decrease of approximately $2.35 per gram per year over the period from 1984 to 1991. In other words, each year the expected price of cocaine falls by about $2.35 on average.

I think he possible reason are increased supply or technological improvements.

If more cocaine entered the market during this period, possibly due to expanded production or trafficking, prices would naturally fall due to higher availability and if technological improvements cocaine became easier or cheaper to produce and transport, producers may have been willing to accept lower prices while still maintaining profitability.