

5.33

$$\ln(WAGE) = \beta_1 + \beta_2 EDUC + \beta_3 EDUC^2 + \beta_4 EXPER + \beta_5 EXPER^2 + \beta_6 (EDUC \times EXPER) + e$$

a.

At what levels of significance are each of the coefficient estimates “significantly different from zero”?

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)	
(Intercept)	1.038e+00	2.757e-01	3.764	0.000175	***
educ	8.954e-02	3.108e-02	2.881	0.004038	**
educ2	1.458e-03	9.242e-04	1.578	0.114855	
exper	4.488e-02	7.297e-03	6.150	1.06e-09	***
exper2	-4.680e-04	7.601e-05	-6.157	1.01e-09	***
educ_exper	-1.010e-03	3.791e-04	-2.665	0.007803	**

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.4638 on 1194 degrees of freedom
Multiple R-squared: 0.3227, Adjusted R-squared: 0.3198
F-statistic: 113.8 on 5 and 1194 DF, p-value: < 2.2e-16

We can see the p-value is less than 0.01, so at 1% significant level we can conclude that the coefficient is different from zero, excepted that educ^2 is 0.1148 so at 12% significant level we can conclude that the coefficient of educ^2 is different from zero.

b.

Obtain an expression for the marginal effect $E[\ln(WAGE)|EDUC, EXPER]$.

Comment on how the estimate of this marginal effect changes as EDUC and EXPER increase.

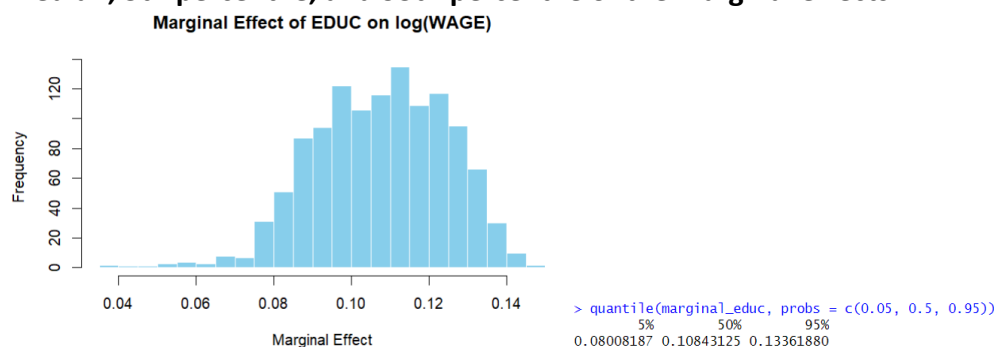
The marginal effect of educ is $\beta_2 + 2\beta_3 \cdot educ + \beta_6 \cdot exper$.

$0.08954 + 0.002916 \cdot educ - 0.001010 \cdot exper$

When education increases the wage will increase, but experience increases the wage is decreased.

c.

Evaluate the marginal effect in part (b) for all observations in the sample and construct a histogram of these effects. What have you discovered? Find the median, 5th percentile, and 95th percentile of the marginal effects.



Most individuals have a positive marginal effect of education on $\log(\text{wage})$, typically between 0.09 and 0.13. This suggests that additional years of education are generally associated with higher wages. The distribution is slightly left-skewed, and the median marginal effect is around 0.105, indicating that one more year of education increases $\log(\text{wage})$ by about 10.5% on average.

d.

Obtain an expression for the marginal effect $\text{Eln}(\text{WAGE})\text{EDUC}$, EXPER EXPER .

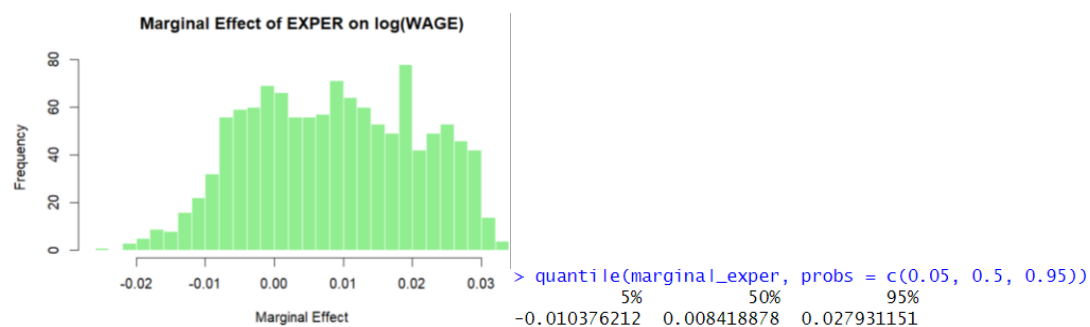
Comment on how the estimate of this marginal effect changes as EDUC and EXPER increase.

The marginal effect on exper is $0.04488 + 2 \cdot -0.0004680 \cdot \text{exper} - 0.001010 \cdot \text{educ}$

When the exper and educ increase, the wage will decrease.

e.

Evaluate the marginal effect in part (d) for all observations in the sample and construct a histogram of these effects. What have you discovered? Find the median, 5th percentile, and 95th percentile of the marginal effects.



The marginal effect of experience on $\log(\text{wage})$ varies widely across individuals. While most values are positive around 0.01 to 0.02, some are negative, indicating diminishing or even negative returns to additional experience for certain people. The distribution is slightly skewed left.

f.

David has 17 years of education and 8 years of experience, while Svetlana has 16 years of education and 18 years of experience. Using a 5% significance level, test the null hypothesis that Svetlana's expected log-wage is equal to or greater than David's expected log-wage, against the alternative that David's expected log-wage is greater.

David: $b_1 + 17b_2 + 17 \cdot 17b_3 + 8b_4 + 8 \cdot 8b_5 + 17 \cdot 8b_6$

Svetlana: $b_1 + 16b_2 + 16 \cdot 16b_3 + 18b_4 + 18 \cdot 18b_5 + 16 \cdot 18b_6$

Difference(s-d): $-b_2 - 33b_3 + 10b_4 + 260b_5 + 152b_6$

$H_0: s-d \geq 0$ against $h_1: s-d < 0$

t-stat = $0.03588456 / 0.02148902 = 1.669902$

the critical value is -1.645, $1.669902 > -1.645$, not reject h_0 , can not conclude that

David is greater.

g.

After eight years have passed, when David and Svetlana have had eight more years of experience, but no more education, will the test result in(f) be the same? Explain this outcome?

David: $b_1 + 17b_2 + 17 \cdot 17b_3 + 16b_4 + 16 \cdot 16b_5 + 17 \cdot 16b_6$

Svetlana: $b_1 + 16b_2 + 16 \cdot 16b_3 + 24b_4 + 24 \cdot 24b_5 + 16 \cdot 24b_6$

Difference(s-d): $-b_2 - 33b_3 + 10b_4 + 420b_5 + 144b_6$

H0: $s-d \geq 0$ against $h_1: s-d < 0$

t-stat = $-0.03091716 / 0.01499112 = -2.062365$

the critical value is -1.645, $-2.062365 < -1.645$, reject h_0 , conclude that David is greater.

The marginal effect of experience is diminishing, so when Svetlana increase its experience, the effect is lower than David.

h.

Wendy has 12 years of education and 17 years of experience, while Jill has 16 years of education and 11 years of experience. Using a 5% significance level, test the null hypothesis that their marginal effects of extra experience are equal against the alternative that they are not. State the null and alternative hypotheses in terms of the model parameters.

Wendy: $b_4 + 34b_5 + 12b_6$

Jill: $b_4 + 22b_5 + 16b_6$

Difference(w-j) = $12b_5 - 4b_6$

H0: $w-j = 0$ against $h_1: w-j \text{ is not } 0$

t-stat = $-0.001575327 / 0.001533457 = -1.027304$

the critical value is -1.96, the t-stat is greater than -1.96, we cannot reject h_0 , do not concluded that the marginal effect of experience is different from Wendy and jill.

i.

How much longer will it be before the marginal effect of experience for Jill becomes negative? Find a 95% interval estimate for this quantity.

using delta method we can get $\text{exper} = -(b_4 + 16b_6) / 2b_5$

Marginal effect turns negative at $\text{EXPER} = 19.6771$ years

se_exper 1.895713

95% CI for turning point: [15.9578 , 23.3964]