HW0519_313707002_羅芷羚

Q15.6

f.

f. Column (5) contains the random effects estimates. Which coefficients, apart from the intercepts, show the most difference from the fixed effects estimates? Use the Hausman test statistic (15.36) to test whether there are significant differences between the random effects estimates and the fixed effects estimates in column (3) (Why that one?). Based on the test results, is random effects estimation in this model appropriate?

(f) ① $EXPER^2$ 係數差了 $\frac{0.0023}{0.0012} = 1.92$ 倍.

② Hausman test $t_j = \dfrac{\hat{\beta}_{FE} - \hat{\beta}_{RE}}{\sqrt{Var(\hat{\beta}_{FE}) - Var(\hat{\beta}_{RE})}}$

$t_{EXPER} = \dfrac{0.0575 - 0.0986}{\sqrt{0.033^2 - 0.022^2}} = -1.67$ $\qquad$ $t_{SOUTH} = \dfrac{-0.3261 - (-0.2326)}{\sqrt{0.1258^2 - 0.0317^2}} = -0.77$

$t_{EXPER^2} = \dfrac{-0.0012 - (-0.0023)}{\sqrt{0.0011^2 - 0.0007^2}} = 1.296$ $\qquad$ $t_{EXPER} = \dfrac{0.0822 - 0.1027}{\sqrt{0.0312^2 - 0.0245^2}} \; -1.06$

只有EXPER的係數在10%顯著水準上有顯著差異 → random effects estimation is appropriate.

Q15.17

b. $\widehat{LIQUORD}_{it} = 0.9690 + 0.2658 INCOMED_{it}$

In part (a), the 95% confidence interval for the coefficient is very wide and includes zero, indicating no statistically significant effect. In contrast, under the Random Effects specification in part (b), the standard error falls sharply to 0.00701, yielding a much tighter 95% confidence interval of [0.0127, 0.0404], which excludes zero. Consequently, the RE model provides strong evidence that a one-thousand-dollar increase in income is associated with an approximately $27 increase in annual liquor expenditure.

```
plm(formula = liquor ~ income, data = pdat, model = "random")

Balanced Panel: n = 40, T = 3, N = 120

Effects:
                 var std.dev share
idiosyncratic 0.9640  0.9819 0.571
individual    0.7251  0.8515 0.429
theta: 0.4459

Residuals:
     Min.   1st Qu.    Median   3rd Qu.      Max.
-2.263634 -0.697383  0.078697  0.552680  2.225798

Coefficients:
             Estimate Std. Error z-value  Pr(>|z|)
(Intercept) 0.9690324  0.5210052  1.8599 0.0628957 .
income      0.0265755  0.0070126  3.7897 0.0001508 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

```
> c(`2.5 %` = ci_low, `97.5 %` = ci_high)
 2.5 %.income 97.5 %.income
   0.01283111    0.04031983
```

c. Since the p-value is far below 0.05, we reject the null hypothesis of no individual random effects.

```
> plmtest(liquor ~ income, data = pdat,
+         type = "bp",        # Breusch-Pagan
+         effect = "individual")

        Lagrange Multiplier Test - (Breusch-Pagan)

data:  liquor ~ income
chisq = 20.68, df = 1, p-value = 5.429e-06
alternative hypothesis: significant effects
```

d. $LIQ\hat{U}ORD_{it} = 0.9163 + 0.0207income + 0.0066INCOMED_{it}$

Since γ is not significantly different from zero, there is no evidence that the individual random effects c_i are correlated with $INCOMEM_{it}$.

```
plm(formula = liquor ~ income + INCOMEM, data = pdat2, model = "random")

Balanced Panel: n = 40, T = 3, N = 120

Effects:
                 var std.dev share
idiosyncratic 0.9640  0.9819 0.571
individual    0.7251  0.8515 0.429
theta: 0.4459

Residuals:
     Min.   1st Qu.    Median   3rd Qu.      Max.
-2.300955 -0.703840  0.054992  0.560255  2.257325

Coefficients:
             Estimate Std. Error z-value Pr(>|z|)
(Intercept) 0.9163337  0.5524439  1.6587  0.09718 .
income      0.0207421  0.0209083  0.9921  0.32117
INCOMEM     0.0065792  0.0222048  0.2963  0.76700
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

Q15.20

d

Comparison with OLS and Fixed Effects ,most coefficients in the random-effects model closely match those from the pooled OLS and fixed-effects specifications. This similarity suggests that, for the majority of regressors, there is little correlation with unobserved school-level heterogeneity.

That Breusch–Pagan Lagrange Multiplier test (with df = 1) is again overwhelmingly significant, so we reject the null of no random effect. In other words, there is highly significant school-level heterogeneity in reading scores, which confirms that a random-effects specification is warranted over simple pooled OLS.

```
plm(formula = readscore ~ small + aide + tchexper + boy + white_asian +
    freelunch, data = pdata, model = "random")

Unbalanced Panel: n = 79, T = 34-137, N = 5766

Effects:
                 var std.dev share
idiosyncratic 751.43   27.41 0.829
individual    155.31   12.46 0.171
theta:
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
 0.6470  0.7225  0.7523  0.7541  0.7831  0.8153

Residuals:
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
-97.483 -17.236  -3.282   0.037  12.803 192.346

Coefficients:
              Estimate Std. Error  z-value  Pr(>|z|)
(Intercept) 436.126774   2.064782 211.2217 < 2.2e-16 ***
small         6.458722   0.912548   7.0777 1.466e-12 ***
aide          0.992146   0.881159   1.1260    0.2602
tchexper      0.302679   0.070292   4.3060 1.662e-05 ***
boy          -5.512081   0.727639  -7.5753 3.583e-14 ***
white_asian   7.350477   1.431376   5.1353 2.818e-07 ***
freelunch   -14.584332   0.874676 -16.6740 < 2.2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

```
        Lagrange Multiplier Test - (Breusch-Pagan)

data:  readscore ~ small + aide + tchexper + boy + white_asian + freelunch
chisq = 6677.4, df = 1, p-value < 2.2e-16
alternative hypothesis: significant effects
```

e. Since $p < 0.05$ and $13.81 > \chi^2(0.95,6)=12.59$, we reject the null of no correlation between the unobserved school effects and our regressors. Random effects is therefore inconsistent, and the fixed-effects estimator is preferred.

```
+   cat(sprintf("%-12s:  t = %5.2f,   p = %.3f\n", v, t_stat, p_val))
+ }
small        :  t =  1.15,   p = 0.252
aide         :  t =  0.13,   p = 0.898
tchexper     :  t = -1.94,   p = 0.053
white_asian  :  t =  1.22,   p = 0.223
freelunch    :  t = -0.10,   p = 0.924
```

```
        Hausman Test

data:  readscore ~ small + aide + tchexper + boy + white_asian + freelunch
chisq = 13.809, df = 6, p-value = 0.03184
alternative hypothesis: one model is inconsistent
```

f. Because the p-value is far below 0.05, we reject the null and conclude that the school-level average of income is significantly correlated with the unobserved heterogeneity. Consequently, the pure random-effects model is not appropriate.

```
Linear hypothesis test:
small_avg = 0
aide_avg = 0
tchexper_avg = 0
boy_avg = 0
white_asian_avg = 0
freelunch_avg = 0

Model 1: restricted model
Model 2: readscore ~ small + aide + tchexper + boy + white_asian + freelunch +
    small_avg + aide_avg + tchexper_avg + boy_avg + white_asian_avg +
    freelunch_avg

Note: Coefficient covariance matrix supplied.

  Res.Df Df      F  Pr(>F)
1   5695
2   5689  6 2.2541 0.03557 *
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```