Q1

$K=2 \Rightarrow y_i = b_1 + b_2 x_i + e_i$ $\qquad Y = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \qquad X = \begin{bmatrix} 1 & x_1 \\ 1 & x_2 \\ \vdots & \vdots \\ 1 & x_n \end{bmatrix}$

Matrix $Y = X\beta + e$

$$X'X = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ x_1 & x_2 & \cdots & x_n \end{bmatrix} \begin{bmatrix} 1 & x_1 \\ 1 & x_2 \\ \vdots & \vdots \\ 1 & x_n \end{bmatrix} = \begin{bmatrix} n & \Sigma x_i \\ \Sigma x_i & \Sigma x_i^2 \end{bmatrix}$$

$$(X'X)^{-1} = \frac{1}{n\Sigma x_i^2 - (\Sigma x_i)^2} \begin{bmatrix} \Sigma x_i^2 & -\Sigma x_i \\ -\Sigma x_i & n \end{bmatrix}$$

$$X'Y = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ x_1 & x_2 & \cdots & x_n \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} \Sigma y_i \\ \Sigma x_i y_i \end{bmatrix}$$

$$b = (X'X)^{-1}(X'Y) = \frac{1}{n\Sigma x_i^2 - (\Sigma x_i)^2} \begin{bmatrix} \Sigma x_i^2 & -\Sigma x_i \\ -\Sigma x_i & n \end{bmatrix} \begin{bmatrix} \Sigma y_i \\ \Sigma x_i y_i \end{bmatrix}$$

$$= \frac{1}{n\Sigma x_i^2 - (\Sigma x_i)^2} \begin{bmatrix} \Sigma x_i^2 \Sigma y_i - \Sigma x_i \Sigma x_i y_i \\ -\Sigma x_i \Sigma y_i + n \Sigma x_i y_i \end{bmatrix}$$

$$b_2 = \frac{-\Sigma x_i \Sigma y_i + n\Sigma x_i y_i}{n\Sigma x_i^2 - (\Sigma x_i)^2} = \frac{-n^2 \bar{x}\bar{y} + n\Sigma x_i y_i}{n\Sigma x_i^2 - n^2\bar{x}^2} = \frac{-n\bar{x}\bar{y} + \Sigma x_i y_i}{\Sigma x_i^2 - n\bar{x}^2}$$

$$= \frac{(-2n\bar{x}\bar{y} + n\bar{x}\bar{y}) + \Sigma x_i y_i}{\Sigma x_i^2 + (n\bar{x}^2 - 2n\bar{x}^2)} = \frac{\Sigma x_i y_i - \Sigma x_i \bar{y} - \bar{x}\Sigma y_i + n\bar{x}\bar{y}}{\Sigma (x_i - \bar{x})^2}$$

$$= \frac{\Sigma (x_i - \bar{x})(y_i - \bar{y})}{\Sigma (x_i - \bar{x})^2}$$

$$b_1 = \frac{\Sigma x_i^2 \Sigma y_i - \Sigma x_i \Sigma x_i y_i}{n\Sigma x_i^2 - (\Sigma x_i)^2} = \frac{\Sigma x_i^2 \Sigma y_i - n^2\bar{x}^2\bar{y} - \Sigma x_i \Sigma x_i y_i + n^2\bar{x}^2\bar{y}}{n\Sigma(x_i - \bar{x})^2}$$

$$= \frac{\Sigma x_i^2 \bar{y} - \Sigma x_i \bar{x}\bar{y} - \bar{x}\Sigma x_i y_i + \bar{x}^2 \Sigma y_i}{\Sigma(x_i - \bar{x})^2}$$

$$= \frac{\Sigma x_i^2 \bar{y} - 2\Sigma x_i \bar{x}\bar{y} + n\bar{y}\bar{x}^2 - \bar{x}\Sigma x_i y_i + \Sigma x_i \bar{x}\bar{y} + \bar{x}^2\Sigma y_i - n\bar{x}^2\bar{y}}{\Sigma (x_i - \bar{x})^2}$$

$$= \frac{\bar{y}\Sigma (x_i - \bar{x})^2 - \bar{x}\Sigma (x_i - \bar{x})(y_i - \bar{x})}{\Sigma (x_i - \bar{x})^2} = \bar{y} - b_1 X$$

Q2 $Var(b) = \sigma^2 (x'x)^{-1} = \sigma^2 \dfrac{1}{n\Sigma x_i^2 - (\Sigma x_i)^2} \begin{bmatrix} \Sigma x_i^2 & -\Sigma x_i \\ -\Sigma x_i & n \end{bmatrix}$

$b \sim N(\beta, \sigma^2 (x'x)^{-1})$

$$= \begin{bmatrix} \dfrac{\sigma^2 \Sigma x_i^2}{n\Sigma(x_i - \bar{x})^2} & \dfrac{\sigma^2(-\bar{x})}{\Sigma(x_i - \bar{x})^2} \\ \dfrac{\sigma^2(-\bar{x})}{\Sigma(x_i - \bar{x})^2} & \dfrac{\sigma^2}{\Sigma(x_i - \bar{x})^2} \end{bmatrix}$$

So $Var(b_1 | x) = \dfrac{\sigma^2 \Sigma x_i^2}{n\Sigma(x_i - \bar{x})^2}$ $\quad Var(b_2 | x) = \dfrac{\sigma^2}{\Sigma(x_i - \bar{x})^2}$

$Cov(\beta_1, \beta_2) = \sigma^2 \dfrac{-\Sigma x_i}{n\Sigma x_i^2 - (\Sigma x_i)^2} = \dfrac{-\sigma^2 \Sigma x_i}{n\Sigma(x_i - \bar{x})^2} = \dfrac{-\sigma^2 \bar{x}}{\Sigma(x_i - \bar{x})^2}$ OK

5.3 a) $t(b_1) = \dfrac{b_1}{se(b_1)} = \dfrac{1.4515}{2.2019} = 0.659$ $\quad se(b_2) = \dfrac{b_2}{t(b_2)} = \dfrac{2.7648}{5.7103} = 0.484$

$b_3 = t(b_3)\, se(b_3) = -3.9376 \times 0.3695 = -1.4549$

$R^2 = 1 - \dfrac{SSE}{SST} = 1 - \dfrac{SSE}{(n-1) S_y} = 1 - \dfrac{46221.62}{(1200-1)\, 6.39547} = 0.0575$

$\sigma^2 = \sqrt{\dfrac{SSE}{n-K}} = \sqrt{\dfrac{46221.62}{1200-4}} = 6.217$

b)

$b_2 = 2.675$ in log form: when total expenditure increase 1%, the budget Spending on Alcho increase by $\dfrac{b_2}{100} = \dfrac{2.675}{100} = 0.02675$ percentage points

$b_3 = -1.4549$. when the number of children increase 1 child, the budget spending on Alcohol decrease by. 1.4549 percentage points

$b_4 = -0.1503$. when the age of household head increase by 1 year, the ... decrease by 0.1503 percentage points.

c) $b_4 \pm t_{(0.975, 1196)}\, se(b_4) = (-0.1503 \pm 1.96 \times 0.0235) = [-0.1964, -0.1042]$

If the age of house hold head increases by 1 year, the decrease of ... by 0.1042 to 0.1964 percentage points

e) Test. $b_3 \neq -2$

Scanned with CamScanner

$H_0: \beta_3 = -2 \quad H_1: \beta_3 \neq -2 \quad t = \frac{b_3 - 2}{Se(b_3)} = \frac{-1.4549 - 2}{0.3695} = 1.475 < 1.96$

We cannot reject H_0 so $\beta_3 \sim -2$

d) Yes, pvalue < 5%; all coefficients are 5% significant.

Excercise 5.23:

a) I expect $\beta_2 < 0$; $\beta_3 > 0$ and $\beta_4 < 0$

b) Model is

Price = 90.84669 − 0.05997 Quant + 0.11624 QUAL − 2.35458 TREND

The signs of coeffiaients are same as my expectation

If Quantity increase by 1 unit, the prices derease by 0.05997 unit

If Quality increase ______ increase by 0.11624 unit

If times ______ decrease by 2.35458 unit

c) $R^2 = 0.5097$

d) $H_0: \beta_2 \geq 0 \quad H_1: \beta_2 < 0 \quad t = \frac{\beta_2 - 0}{Se(b_2)} = -5.892 < t = -1.96$

H_0 $\beta_2 < 0$ Then statement is true

e) $H_0: \beta_3 = 0$ & $H_1: \beta_3 > 0$ $t = 0.572$ We cannot reject H_0

So Quality dose not effect to price.

f) The average annual change of price is $b_4 = -2.35458$

Since price will derease overtime (enhance productive procedure, reduce fixed cost, etc)