

2.17 The data file *collegetown* contains observations on 500 single-family houses sold in Baton Rouge, Louisiana, during 2009–2013. The data include sale price (in thousands of dollars), *PRICE*, and total interior area of the house in hundreds of square feet, *SQFT*.

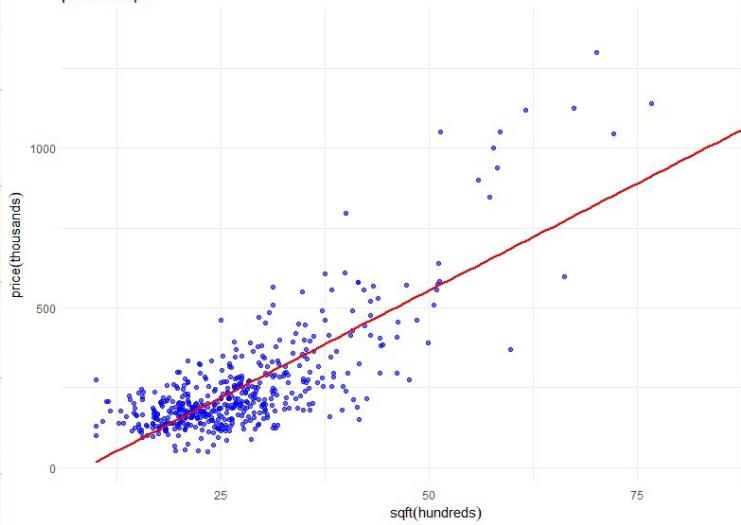
- Plot house price against house size in a scatter diagram.

94 CHAPTER 2 The Simple Linear Regression Model

- Estimate the linear regression model $PRICE = \beta_1 + \beta_2 SQFT + e$. Interpret the estimates. Draw a sketch of the fitted line.
- Estimate the quadratic regression model $PRICE = \alpha_1 + \alpha_2 SQFT^2 + e$. Compute the marginal effect of an additional 100 square feet of living area in a home with 2000 square feet of living space.
- Graph the fitted curve for the model in part (c). On the graph, sketch the line that is tangent to the curve for a 2000-square-foot house.
- For the model in part (c), compute the elasticity of *PRICE* with respect to *SQFT* for a home with 2000 square feet of living space.
- For the regressions in (b) and (c), compute the least squares residuals and plot them against *SQFT*. Do any of our assumptions appear violated?
- One basis for choosing between these two specifications is how well the data are fit by the model. Compare the sum of squared residuals (*SSE*) from the models in (b) and (c). Which model has a lower *SSE*? How does having a lower *SSE* indicate a “better-fitting” model?

a.b.

price & sqrt



```
> cat("Intercept (Alpha):", coef(tab)[1], "\n")
Intercept (Alpha): -115.4236
> cat("Slope (Beta):", coef(tab)[2], "\n")
Slope (Beta): 13.40294
> print(results$summary)
```

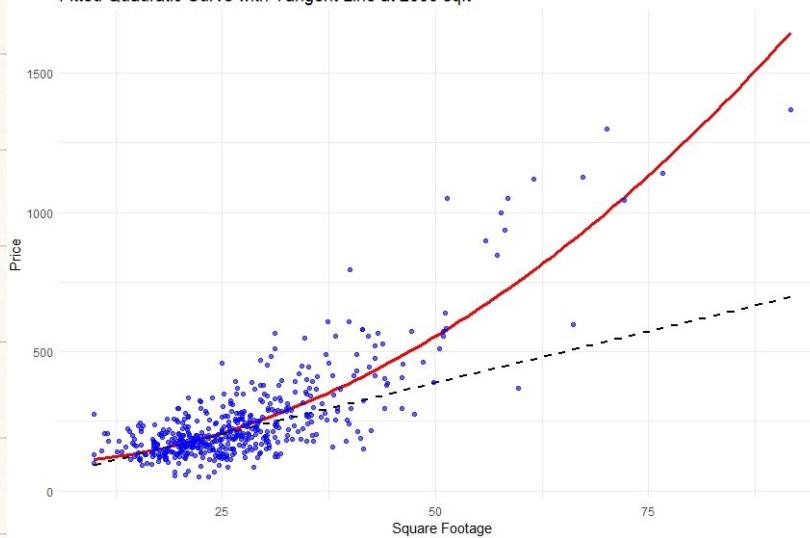
c.

```
> # Output the marginal effect
> cat("The marginal effect of an additional 100 square feet at 2000 square feet is:", marginal_effect_2000, "\n")
The marginal effect of an additional 100 square feet at 2000 square feet is: 7.38076
```

(4)

d.

Fitted Quadratic Curve with Tangent Line at 2000 sqft

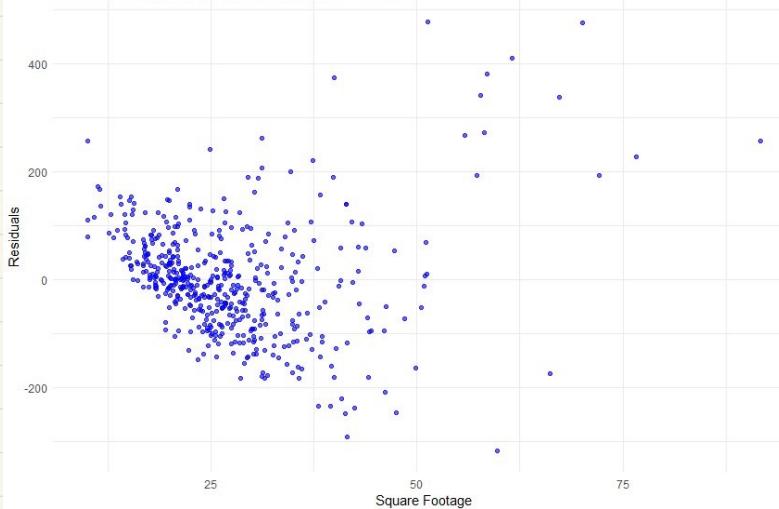


e.

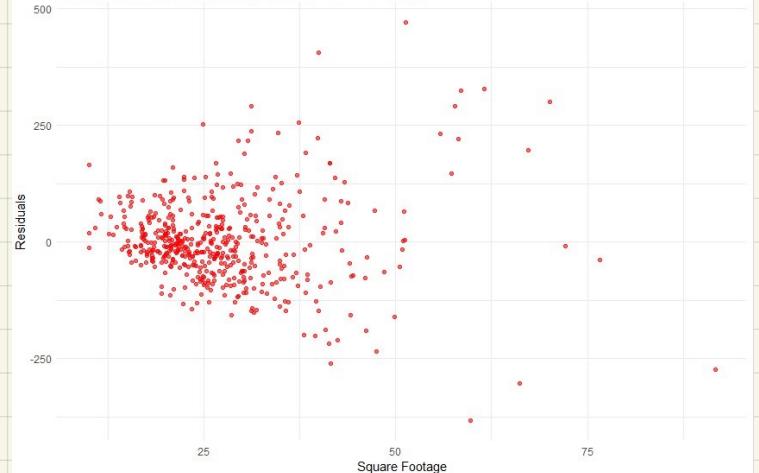
```
> # Output the result  
> cat("The elasticity of PRICE with respect to SQFT at 2000 square feet is:", elasticity_at_2000, "\n")  
The elasticity of PRICE with respect to SQFT at 2000 square feet is: 0.8819511
```

f.

Residuals for Linear Regression (Price ~ SQFT)



Residuals for Quadratic Regression (Price ~ SQFT^2)



由圖可知 $SQFT \uparrow$, 殘差 \uparrow ,
不符合常態方差假設

g.

```
> # 4. 顯示結果  
> cat("Model 1 (Linear) Residual Sum of Squares (RSS):", rss1, "\n")  
Model 1 (Linear) Residual Sum of Squares (RSS): 5262847  
> cat("Model 2 (Quadratic) Residual Sum of Squares (RSS):", rss2, "\n")  
Model 2 (Quadratic) Residual Sum of Squares (RSS): 4222356
```

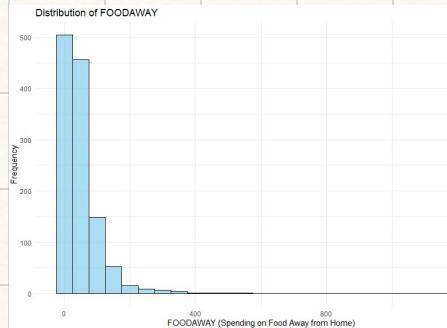
model in part (c) 較佳

$SSE(c) < SSE(b)$

2.25 Consumer expenditure data from 2013 are contained in the file *cex5_small*. [Note: *cex5* is a larger version with more observations and variables.] Data are on three-person households consisting of a husband and wife, plus one other member, with incomes between \$1000 per month to \$20,000 per month. *FOODAWAY* is past quarter's food away from home expenditure per month per person, in dollars, and *INCOME* is household monthly income during past year, in \$100 units.

- Construct a histogram of *FOODAWAY* and its summary statistics. What are the mean and median values? What are the 25th and 75th percentiles?
- What are the mean and median values of *FOODAWAY* for households including a member with an advanced degree? With a college degree member? With no advanced or college degree member?
- Construct a histogram of *ln(FOODAWAY)* and its summary statistics. Explain why *FOODAWAY* and *ln(FOODAWAY)* have different numbers of observations.
- Estimate the linear regression $\ln(\text{FOODAWAY}) = \beta_1 + \beta_2 \text{INCOME} + e$. Interpret the estimated slope.
- Plot *ln(FOODAWAY)* against *INCOME*, and include the fitted line from part (d).
- Calculate the least squares residuals from the estimation in part (d). Plot them vs. *INCOME*. Do you find any unusual patterns, or do they seem completely random?

a.



Mean (均值): 49.27085
 Median (中位數): 32.555
 25th Percentile: 12.04
 75th Percentile: 67.5025

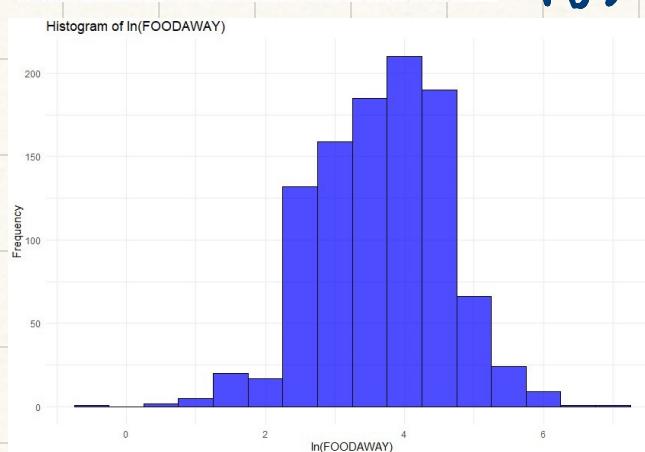
b.

	Category	Mean	Median
1	Advanced Degree	73.15494	48.15
2	College Degree	48.59718	36.11
3	No Degree	39.01017	26.02

c.

	Statistic	Value
1	Mean	3.650804
2	Median	3.686499
3	25th Percentile	3.075929
4	75th Percentile	4.279717

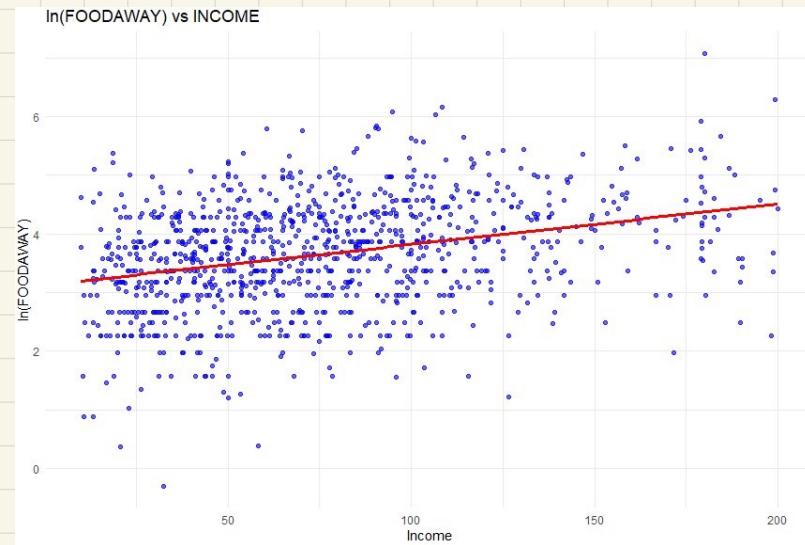
foodaway 有些為 0
 而 ln 0 無定義，為缺失值



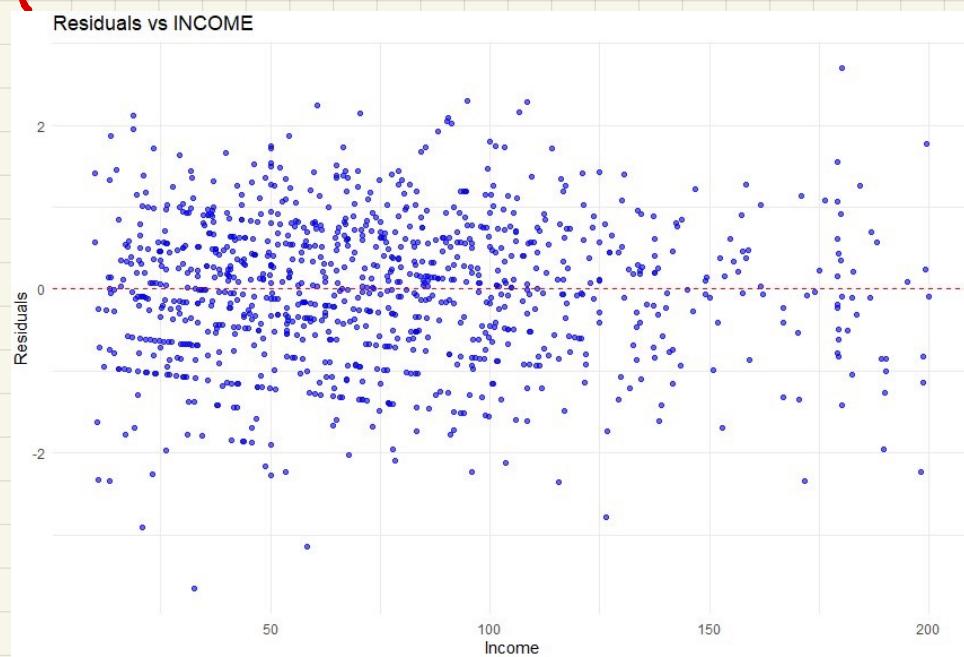
d.

```
> beta_2 <- coef(model)[2]
> cat("The slope is:", beta_2, "\n")
The slope is: 0.006901748
```

e.



f.



no unusual pattern, it seems random.

2.28 How much does education affect wage rates? The data file *cps5_small* contains 1200 observations on hourly wage rates, education, and other variables from the 2013 Current Population Survey (CPS). [Note: *cps5* is a larger version.]

- a. Obtain the summary statistics and histograms for the variables *WAGE* and *EDUC*. Discuss the data characteristics.
- b. Estimate the linear regression $WAGE = \beta_1 + \beta_2 EDUC + e$ and discuss the results.
- c. Calculate the least squares residuals and plot them against *EDUC*. Are any patterns evident? If assumptions SR1–SR5 hold, should any patterns be evident in the least squares residuals?
- d. Estimate separate regressions for males, females, blacks, and whites. Compare the results.
- e. Estimate the quadratic regression $WAGE = \alpha_1 + \alpha_2 EDUC^2 + e$ and discuss the results. Estimate the marginal effect of another year of education on wage for a person with 12 years of education and for a person with 16 years of education. Compare these values to the estimated marginal effect of education from the linear regression in part (b).
- f. Plot the fitted linear model from part (b) and the fitted values from the quadratic model from part (e) in the same graph with the data on *WAGE* and *EDUC*. Which model appears to fit the data better?

a.

警告：正在使用‘POE5Rdata’這個程式套件，因此不會被安裝

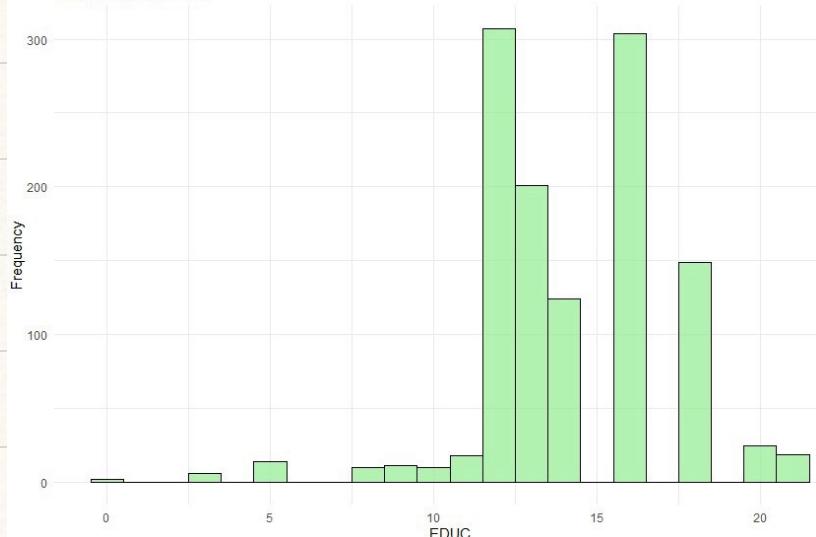
Summary Statistics for WAGE:

Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
3.94	13.00	19.30	23.64	29.80	221.10

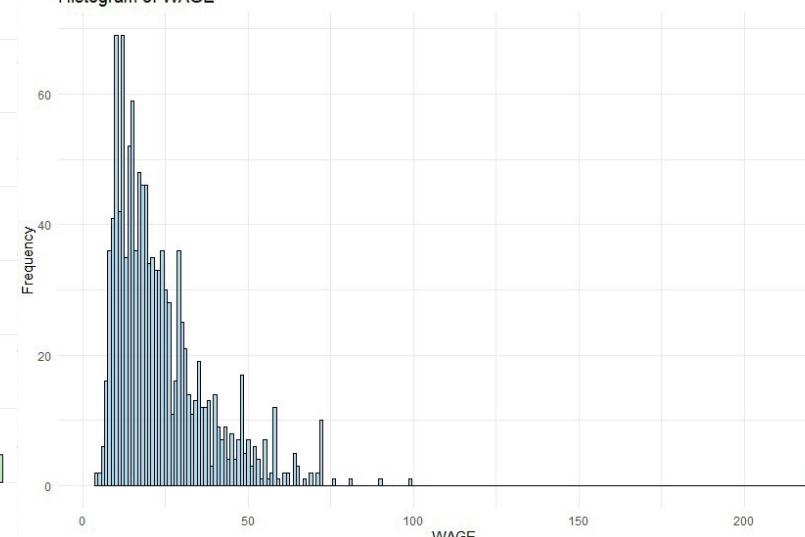
Summary Statistics for EDUC:

Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
0.0	12.0	14.0	14.2	16.0	21.0

Histogram of EDUC



Histogram of WAGE



多集中在10~20之間

大多起人工資較低
呈現右偏

b.

警告：正在使用‘POE5Rdata’這個程式套件，因此不會被安裝
截距 -10.39996
斜率 2.396761

Call:
lm(formula = wage ~ educ, data = cps5_small)

Residuals:

Min	1Q	Median	3Q	Max
-31.785	-8.381	-3.166	5.708	193.152

Coefficients:

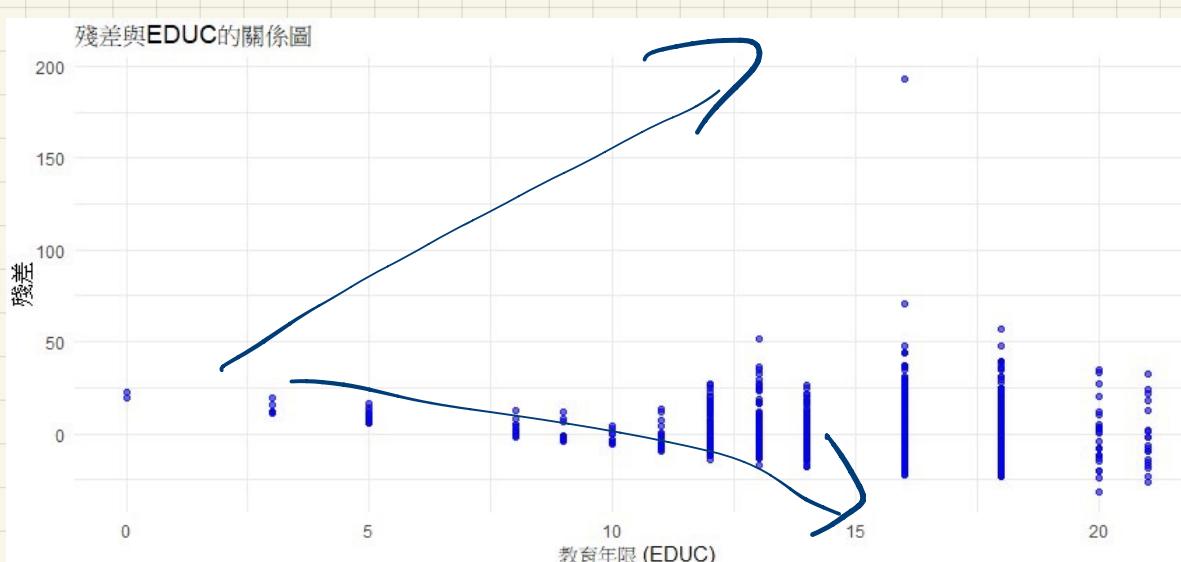
	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	-10.4000	1.9624	-5.3	1.38e-07 ***
educ	2.3968	0.1354	17.7	< 2e-16 ***

$$\text{Wage} = -10.4 + 2.4 \text{ educ}$$

$$\text{when educ} = 0 \Rightarrow \text{Wage} = -10.4$$

$$\text{educ} \uparrow 1 \text{ year} \Rightarrow \text{wage} \uparrow 2.4$$

c.



EDUC ↑, residual ↑, 不符合 Homoskedasticity (SR5)

手

男性回歸結果：

```
Call:  
lm(formula = wage ~ educ, data = filter(cps5_small, female ==  
0))
```

Residuals:

Min	1Q	Median	3Q	Max
-27.643	-9.279	-2.957	5.663	191.329

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	-8.2849	2.6738	-3.099	0.00203 ***
educ	2.3785	0.1881	12.648	< 2e-16 ***

女性回歸結果：

```
Call:  
lm(formula = wage ~ educ, data = filter(cps5_small, female ==  
1))
```

Residuals:

Min	1Q	Median	3Q	Max
-30.837	-6.971	-2.811	5.102	49.502

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	-16.6028	2.7837	-5.964	4.51e-09 ***
educ	2.6595	0.1876	14.174	< 2e-16 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

黑人回歸結果：

```
Call:  
lm(formula = wage ~ educ, data = filter(cps5_small, black ==  
1))
```

Residuals:

Min	1Q	Median	3Q	Max
-15.673	-6.719	-2.673	4.321	40.381

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	-6.2541	5.5539	-1.126	0.263
educ	1.9233	0.3983	4.829	4.79e-06 ***

白人回歸結果：

```
Call:  
lm(formula = wage ~ educ, data = filter(cps5_small, black ==  
0))
```

Residuals:

Min	1Q	Median	3Q	Max
-32.131	-8.539	-3.119	5.960	192.890

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	-10.475	2.081	-5.034	5.6e-07 ***
educ	2.418	0.143	16.902	< 2e-16 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

女性教育回報
最高(2.65)

黑人教育回報
最低(1.92)

低教育水平下

黑人薪資高：

截距為 -6.25

女性薪資低：

截距為 -16.6

e.

Residuals:

Min	1Q	Median	3Q	Max
-34.820	-8.117	-2.752	5.248	193.365

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	4.916477	1.091864	4.503	7.36e-06 ***
I(educ^2)	0.089134	0.004858	18.347	< 2e-16 ***

signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 13.45 on 1198 degrees of freedom

Multiple R-squared: 0.2194, Adjusted R-squared: 0.2187

F-statistic: 336.6 on 1 and 1198 DF, p-value: < 2.2e-16

> cat("12年教育的邊際效應:", marginal_effect_12, "\n")

12年教育的邊際效應: 2.139216

> cat("16年教育的邊際效應:", marginal_effect_16, "\n")

16年教育的邊際效應: 2.852288

> cat("線性回歸的邊際效應:", beta_linear, "\n")

線性回歸的邊際效應: 2.396761

二次回歸隨 EDVC↑, marginal effect ↑

marginal effect = $2 \times \beta_2 \times EDVC$

↓
0.089134

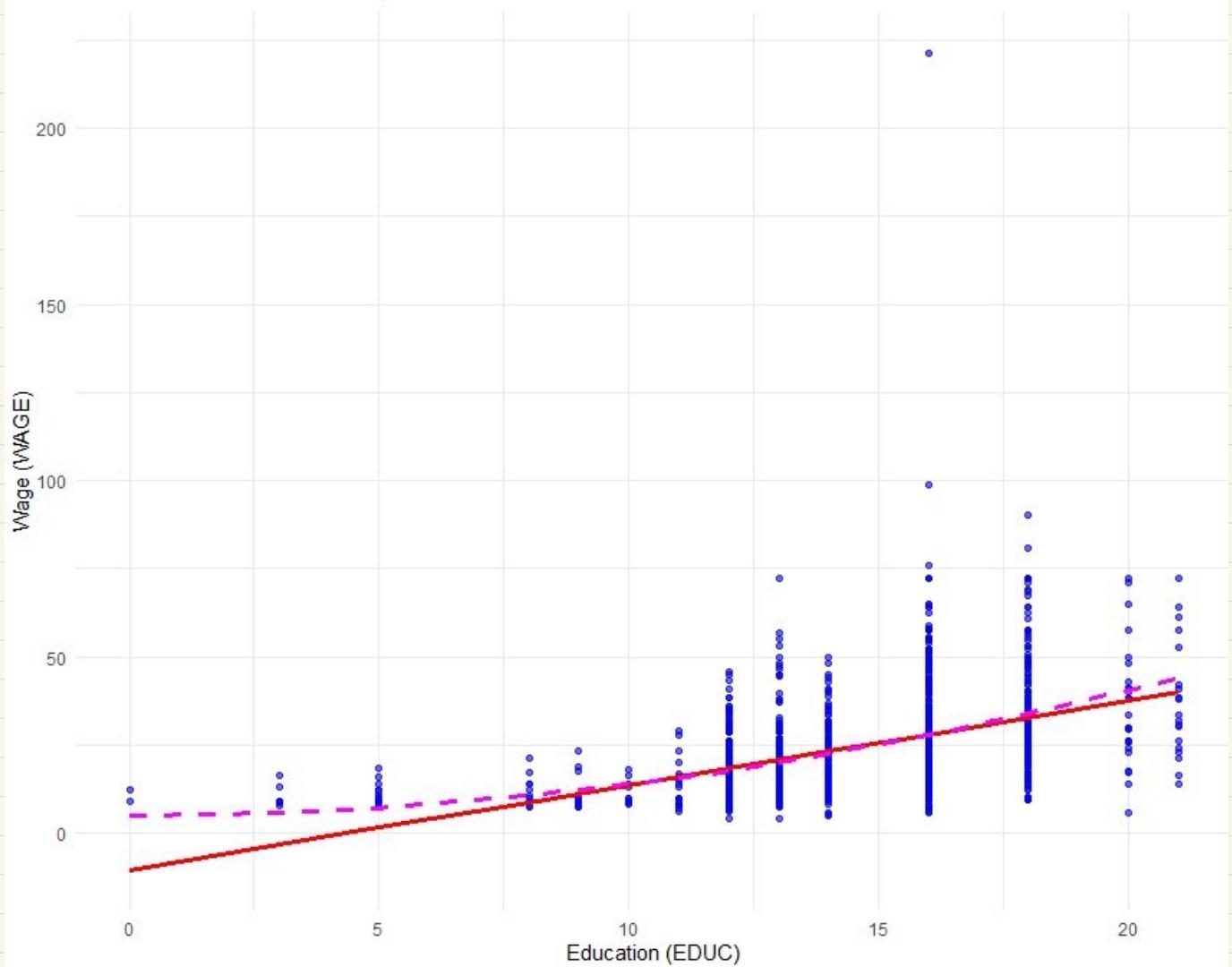
線性回歸的 marginal effect 固定為 β_2

↓

2.396761

f.

Fitted Models: Linear vs. Quadratic



由图片左方可看出二次回歸線坡度較高
右方幾乎無差異
故選擇二次回歸較佳