Ch2, qn1, -許傑 - 313707025

a. completes the entries in the table. Put the sums in the last row. What are the sample means $\bar{x}$ and $\bar{y}$?

x	y	x-$\bar{x}$	$(x-\bar{x})^2$	y-$\bar{y}$	(x-$\bar{x}$)(y-$\bar{y}$)
3	4	2	4	2	4
2	2	1	1	0	0
1	3	0	0	1	0
-1	1	-2	4	-1	2
0	0	-1	1	-2	2
$\sum x_i$ = 5	$\sum y_i$ = 10	$\sum x_i - \bar{x}$ = 0	$\sum(x_i-\bar{x})^2$ = 10	$\sum y_i - \bar{y}$ = 0	$\sum(x-\bar{x})(y-\bar{y})$ = 8

$\bar{x}$ = 5/5 = 1, $\bar{y}$ = 10/5 = 2

b. Calculate b1 and b2 using (2.7) and (2.8) and state their interpretation.

b2 = $\frac{\sum(x-\bar{x})(y-\bar{y})}{\sum(x_i-\bar{x})^2} = \frac{8}{10} = 0.8$; This implies that for increase in x, y will increase too by 0.8

b1= $\bar{y} - b_2\bar{x} = 2 - 0.8 \times 1 = 1.2$; this implies that when x = 0, 1.2 is the expected value of y.

c. compute... using these numerical values, show that ...

$\sum x_i{}^2$ = $3^2 + 2^2 + 1^2 + (-1)^2 + 0^2 = 0 + 4 + 1 + 1 + 0 = 15$

$\sum x_i y_i$ = $3 \times 4 + 2 \times 2 + 1 \times 3 + (-1) \times 1 + 0 \times 0 = 18$

$\sum(x-\bar{x})^2$ = $\sum x_i{}^2 - N\bar{x}^2$; $10 = 15 - 5 \times 1^2 = 15 - 5 = 10$; Verified

$\sum(x_i-\bar{x})(y_i-\bar{y})$ = $\sum x_i{}^2 - N\overline{xy}$; $8 = 18 - 5 \times 1 \times 2 = 18 - 10 = 8$; Verified

d. Use the least squares estimates from part (b) to compute the fitted values of y and complete the remainder of the table below. Put the sums in the last row. ...

x_i	y_i	$\hat{y}_i$	$\hat{e}_i$	$\hat{e}_i{}^2$	$x_i\hat{e}_i$
3	4	3.6	0.4	6	1.2
2	2	2.8	-0.8	4	-1.6
1	3	2	1	0	1
-1	1	0.4	0.6	6	-0.6
0	0	1.2	-1.2	4	0
$\sum x_i$ = 5	$\sum y_i$ = 10	$\sum \hat{y}_i$ = 10	$\sum \hat{e}_i$ = 0	$\sum \hat{e}_i{}^2$ = 3.6	$\sum x_i\hat{e}_i$ = 0

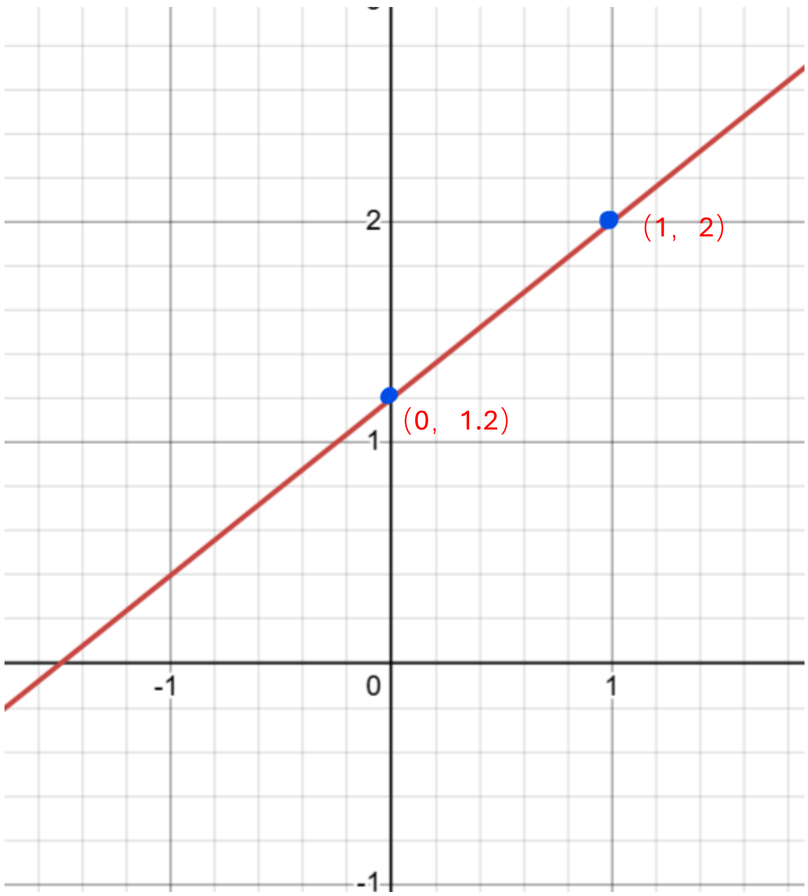
sample variance of x, and y is: $s_y^2 = \frac{\sum(y_i-y)^2}{N-1} = \frac{10}{4} = 2.5$, $s_x^2 = \frac{\sum(x_i-x)^2}{N-1} = \frac{10}{4} = 2.5$

sample covariance between x and y is: $s_{xy} = \frac{\sum(x_i-x)(y_i-y)}{N-1} = \frac{8}{4} = 2,$

sample correlation between x and y is: $r_{xy} = \frac{s_{xy}}{s_x s_y} = \frac{2}{\sqrt{2.5}\times\sqrt{2.5}} = \frac{2}{2.5} = 0.8$

The coefficient of variation is $CV_x = 100 \times \frac{s_x}{\bar{x}} = 100 \times \frac{2.5}{1} \approx 158.11$, and the median for x is from [−1,0,1,2,3], which is 1.

e. plot



f. the point of mean is $(\bar{x}, \bar{y})$ = (1, 2), which the plot passes through.

g. $\bar{y} = 2$ and $b_1 + b_2\bar{x} = 1.2 + 0.8 \times 1 = 2 = \bar{x}$, verified

h. $\bar{\hat{y}} = \frac{\sum \hat{y}_i}{N} = \frac{10}{5} = 2 = \bar{y}$, thus $\bar{\hat{y}} = \bar{y}$, verified

i. $\hat{\sigma}^2 = \frac{\sum \hat{e}_i{}^2}{N-2} = \frac{3.6}{3} = 1.2$

j. $\widehat{var}\ (b_2|x) = \frac{\hat{\sigma}^2}{x_i - \bar{x}} = \frac{1.2}{10} = 0.12$, and $se(b_2) = \sqrt{0.12} \approx 0.3464$