4.4 The general manager of a large engineering firm wants to know whether the experience of technical artists influences their work quality. A random sample of 50 artists is selected. Using years of work experience (*EXPER*) and a performance rating (*RATING*, on a 100-point scale), two models are estimated by least squares. The estimates and standard errors are as follows:

Model 1:

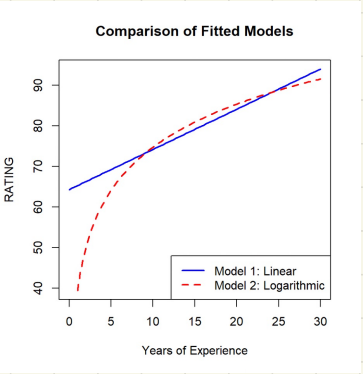
$$\widehat{RATING} = 64.289 + 0.990EXPER \quad N = 50 \quad R^2 = 0.3793$$
$$(se) \qquad (2.422) \quad (0.183)$$

Model 2:

$$\widehat{RATING} = 39.464 + 15.312\ln(EXPER) \quad N = 46 \quad R^2 = 0.6414$$
$$(se) \qquad (4.198) \quad (1.727)$$

a. Sketch the fitted values from Model 1 for *EXPER* = 0 to 30 years.
b. Sketch the fitted values from Model 2 against *EXPER* = 1 to 30 years. Explain why the four artists with no experience are not used in the estimation of Model 2.
c. Using Model 1, compute the marginal effect on *RATING* of another year of experience for (i) an artist with 10 years of experience and (ii) an artist with 20 years of experience.
d. Using Model 2, compute the marginal effect on *RATING* of another year of experience for (i) an artist with 10 years of experience and (ii) an artist with 20 years of experience.
e. Which of the two models fits the data better? Estimation of Model 1 using just the technical artists with some experience yields $R^2 = 0.4858$.
f. Do you find Model 1 or Model 2 more reasonable, or plausible, based on economic reasoning? Explain.

a. b.



ln(EXPER) 要求 EXPER > 0

ln(0) 沒有定義

⇒有 4 名不包含在內

c.

EXPER = 10 marginal effect = $\frac{\partial RATING}{\partial EXPER}$ = 0.99

EXPER = 20 marginal effect = 0.99

d.

$\frac{\partial RATING}{\partial EXPER} = 15.312 \times \frac{1}{EXPER}$

EXPER = 10 15.312/10 = 15.312

EXPER = 20 15.312/20 = 0.7656

e. model 2 R^2 > model 1 R^2

model 2 better

f. model 2 較合理，考慮邊際報酬遞減現象

4.28 The file *wa-wheat.dat* contains observations on wheat yield in Western Australian shires. There are 48 annual observations for the years 1950–1997. For the Northampton shire, consider the following four equations:

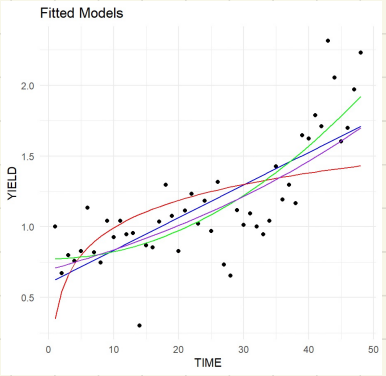
$$YIELD_t = \beta_0 + \beta_1 TIME + e_t$$
$$YIELD_t = \alpha_0 + \alpha_1 \ln(TIME) + e_t$$
$$YIELD_t = \gamma_0 + \gamma_1 TIME^2 + e_t$$
$$\ln(YIELD_t) = \phi_0 + \phi_1 TIME + e_t$$

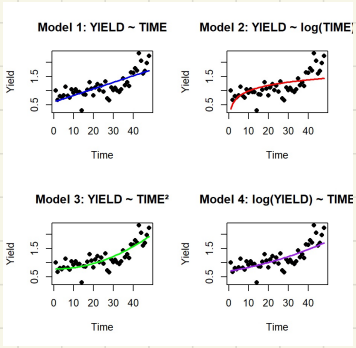
a. Estimate each of the four equations. Taking into consideration (i) plots of the fitted equations, (ii) plots of the residuals, (iii) error normality tests, and (iii) values for R^2, which equation do you think is preferable? Explain.

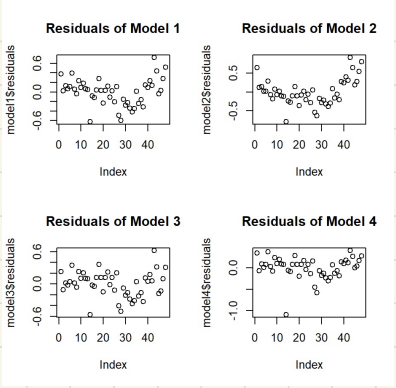
b. Interpret the coefficient of the time-related variable in your chosen specification.

c. Using your chosen specification, identify any unusual observations, based on the studentized residuals, *LEVERAGE*, *DFBETAS*, and *DFFITS*.

d. Using your chosen specification, use the observations up to 1996 to estimate the model. Construct a 95% prediction interval for *YIELD* in 1997. Does your interval contain the true value?








```
> shapiro.test(model1$residuals)

        Shapiro-Wilk normality test

data:  model1$residuals
W = 0.98236, p-value = 0.6792

> shapiro.test(model2$residuals)

        Shapiro-Wilk normality test

data:  model2$residuals
W = 0.96657, p-value = 0.1856

> shapiro.test(model3$residuals)

        Shapiro-Wilk normality test

data:  model3$residuals
W = 0.98589, p-value = 0.8266

> shapiro.test(model4$residuals)

        Shapiro-Wilk normality test

data:  model4$residuals
W = 0.86894, p-value = 7.205e-05
```

```
> r_squared
       Model        R2
1     Linear 0.5778369
2 Linear-Log 0.3385733
3  Quadratic 0.6890101
4 Log-Linear 0.5073566
```

model 3, R^2 最大且不拒绝常态假设

(b) $time^2$ 增加 1 单位，yield 增加 4.986e-04

C. Studentized Residuals (学生化残差)

unusual observations: $|std.res| > 2$

=> 14. 28. 约

k: 变数数量 (包含截距)

n: 样本数

Leverage (槓桿值，影響回歸的程度)

$lev > 2 \times k/n$

=> 45. 46. 47. 48

DFBETAS (单个变数影響回歸系数的变化程度)

$|dfb| > 2/\sqrt{n}$

=> 14. 43. 44. 48 (对 the 变数)　　6. 14 (对截距)

DFFITS (刪除該觀測值後，模型擬合值的变化程度)

$|dff| > 2 \times \sqrt{k/n}$

=> 14. 43. 48

d. [1.372403, 2.398819], Yes, the value = 2.231

4.29 Consider a model for household expenditure as a function of household income using the 2013 data from the Consumer Expenditure Survey, *cex5_small*. The data file *cex5* contains more observations. Our attention is restricted to three-person households, consisting of a husband, a wife, plus one other. In this exercise, we examine expenditures on a staple item, food. In this extended example, you are asked to compare the linear, log-log, and linear-log specifications.

a. Calculate summary statistics for the variables: *FOOD* and *INCOME*. Report for each the sample mean, median, minimum, maximum, and standard deviation. Construct histograms for both variables. Locate the variable mean and median on each histogram. Are the histograms symmetrical and "bell-shaped" curves? Is the sample mean larger than the median, or vice versa? Carry out the Jarque–Bera test for the normality of each variable.

b. Estimate the linear relationship $FOOD = \beta_1 + \beta_2 INCOME + e$. Create a scatter plot *FOOD* versus *INCOME* and include the fitted least squares line. Construct a 95% interval estimate for β_2. Have we estimated the effect of changing income on average *FOOD* relatively precisely, or not?

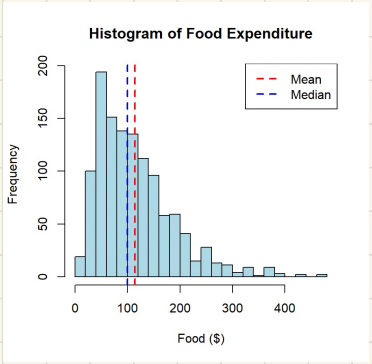
c. Obtain the least squares residuals from the regression in (b) and plot them against *INCOME*. Do you observe any patterns? Construct a residual histogram and carry out the Jarque–Bera test for normality. Is it more important for the variables *FOOD* and *INCOME* to be normally distributed, or that the random error e be normally distributed? Explain your reasoning.

d. Calculate both a point estimate and a 95% interval estimate of the elasticity of food expenditure with respect to income at $INCOME = 19, 65$, and 160, and the corresponding points on the fitted line, which you may treat as not random. Are the estimated elasticities similar or dissimilar? Do the interval estimates overlap or not? As *INCOME* increases should the income elasticity for food increase or decrease, based on Economics principles?

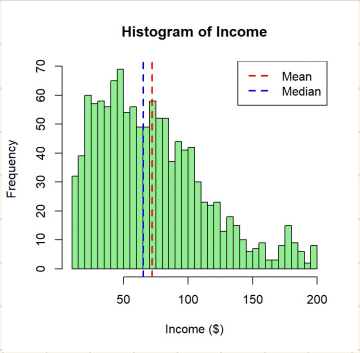
e. For expenditures on food, estimate the log-log relationship $\ln(FOOD) = \gamma_1 + \gamma_2 \ln(INCOME) + e$. Create a scatter plot for $\ln(FOOD)$ versus $\ln(INCOME)$ and include the fitted least squares line. Compare this to the plot in (b). Is the relationship more or less well-defined for the log-log model

a.

```
> print(stats)
  Variable      Mean Median   Min    Max      SD
1     Food 114.44311  99.80  9.63 476.67 72.65750
2   Income  72.14264  65.29 10.00 200.00 41.65228
```



Food & Income
→ mean > median
right-skewed
they're not symmetrical
and "bell-shaped"



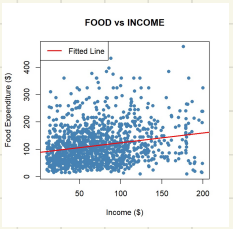

```
> jarque.bera.test(cex5_small$food)

        Jarque Bera Test

data:  cex5_small$food
X-squared = 648.65, df = 2, p-value < 2.2e-16

> jarque.bera.test(cex5_small$income)

        Jarque Bera Test

data:  cex5_small$income
X-squared = 148.21, df = 2, p-value < 2.2e-16
```

p-value < 0.05
→ 不符合常態分布

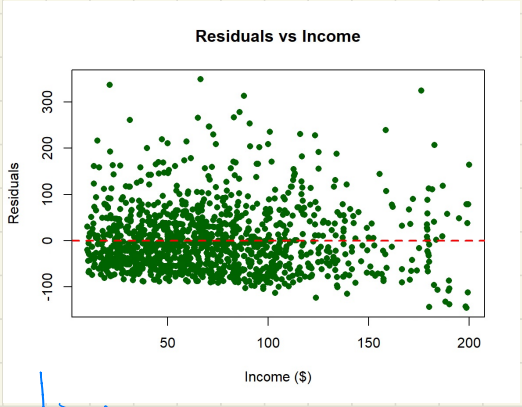
b.

CI: [0.2619215 , 0.4555452]

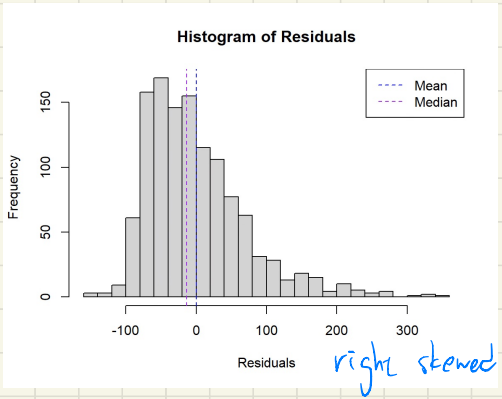
No., R^2 only 0.04228



c



↳ 未呈随机分布



right skewed

```
        Jarque Bera Test

data:  residuals
X-squared = 624.19, df = 2, p-value < 2.2e-16
```

p-value < 0.05

=> 不符合常態分布

e 的常態性較重要 => E(e)=0、常態、同變 不符合設定要求

d.

	Income	Fitted_Food	Elasticity	Lower_CI	Upper_CI
(Intercept)	19	95.38155	0.07145038	0.05217475	0.09072601
(Intercept)1	65	111.88114	0.20838756	0.15216951	0.26460562
(Intercept)2	160	145.95638	0.39319883	0.28712305	0.49927462

dissimilar

not overlap

根據經濟原理，food應隨著 income 上升花費比例減少（ε下降）

relative to the linear specification? Calculate the generalized R^2 for the log-log model and compare it to the R^2 from the linear model. Which of the models seems to fit the data better?

f. Construct a point and 95% interval estimate of the elasticity for the log-log model. Is the elasticity of food expenditure from the log-log model similar to that in part (d), or dissimilar? Provide statistical evidence for your claim.

g. Obtain the least squares residuals from the log-log model and plot them against ln(*INCOME*). Do you observe any patterns? Construct a residual histogram and carry out the Jarque–Bera test for normality. What do you conclude about the normality of the regression errors in this model?

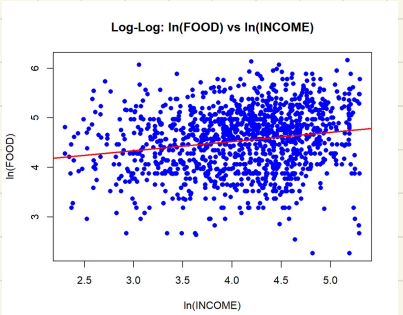
h. For expenditures on food, estimate the linear-log relationship $FOOD = \alpha_1 + \alpha_2 \ln(INCOME) + e$. Create a scatter plot for *FOOD* versus ln(*INCOME*) and include the fitted least squares line. Compare this to the plots in (b) and (e). Is this relationship more well-defined compared to the others? Compare the R^2 values. Which of the models seems to fit the data better?

i. Construct a point and 95% interval estimate of the elasticity for the linear-log model at *INCOME* = 19, 65, and 160, and the corresponding points on the fitted line, which you may treat as not random. Is the elasticity of food expenditure similar to those from the other models, or dissimilar? Provide statistical evidence for your claim.

j. Obtain the least squares residuals from the linear-log model and plot them against ln(*INCOME*). Do you observe any patterns? Construct a residual histogram and carry out the Jarque–Bera test for normality. What do you conclude about the normality of the regression errors in this model?

k. Based on this exercise, do you prefer the linear relationship model, or the log-log model or the linear-log model? Explain your reasoning.

e.



	Model	R_squared
1	Linear	0.04228120
2	Log-Log	0.03322915

R^2 都很低

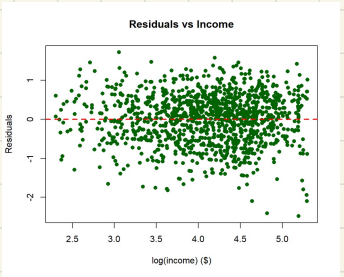
都沒有很好 fit the data

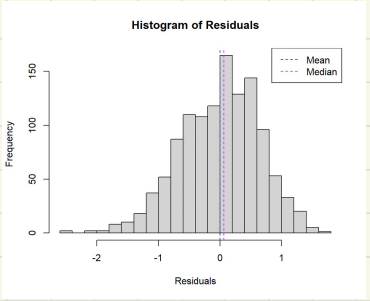
f

	Income	Fitted_Food	Elasticity	Lower_CI	Upper_CI
(Intercept)	19	75.75443	0.1863054	0.1293432	0.2432675
(Intercept)1	65	95.26313	0.1863054	0.1293432	0.2432675
(Intercept)2	160	112.67012	0.1863054	0.1293432	0.2432675

log-log model 的 E 是固定的

g.



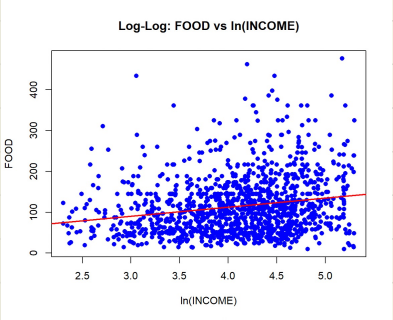



```
        Jarque Bera Test

data:  residuals
X-squared = 25.85, df = 2, p-value = 2.436e-06
```

p-value < 0.05

⇒ 不符合常態分布

h.



	Model	R_squared
1	Linear	0.04228120
2	Log-Log	0.03322915
3	Linear-Log	0.03799984

← 在兩個中間 (都不好)

Linear R^2 最高

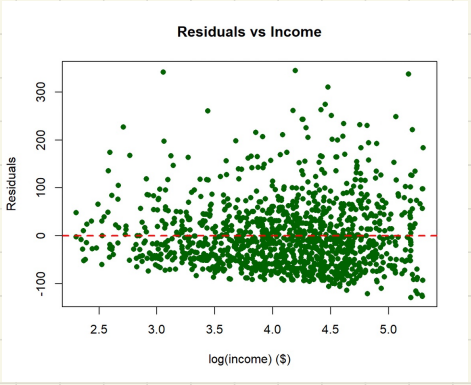
i.

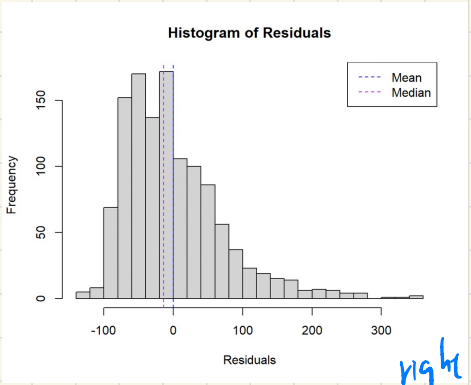
	Income	Fitted_Food	Elasticity	Lower_CI	Upper_CI
(Intercept)	19	88.89788	0.2495828	0.1784009	0.3207648
(Intercept)1	65	116.18722	0.1909624	0.1364992	0.2454256
(Intercept)2	160	136.17332	0.1629349	0.1164652	0.2094046

Income↑, E↓

⇒符合經濟原理

j.





right-skewed

```
        Jarque Bera Test

data:  residuals
X-squared = 628.07, df = 2, p-value < 2.2e-16
```

不符合常態

k. log-log model, residual 較 random.