15.6 Using the NLS panel data on $N = 716$ young women, we consider only years 1987 and 1988. We are interested in the relationship between $\ln(WAGE)$ and experience, its square, and indicator variables for living in the south and union membership. Some estimation results are in Table 15.10.

TABLE 15.10 **Estimation Results for Exercise 15.6**

	(1) OLS 1987	(2) OLS 1988	(3) FE	(4) FE Robust	(5) RE
C	0.9348	0.8993	1.5468	1.5468	1.1497
	(0.2010)	(0.2407)	(0.2522)	(0.2688)	(0.1597)
EXPER	0.1270	0.1265	0.0575	0.0575	0.0986
	(0.0295)	(0.0323)	(0.0330)	(0.0328)	(0.0220)
$EXPER^2$	−0.0033	−0.0031	−0.0012	−0.0012	−0.0023
	(0.0011)	(0.0011)	(0.0011)	(0.0011)	(0.0007)
SOUTH	−0.2128	−0.2384	−0.3261	−0.3261	−0.2326
	(0.0338)	(0.0344)	(0.1258)	(0.2495)	(0.0317)
UNION	0.1445	0.1102	0.0822	0.0822	0.1027
	(0.0382)	(0.0387)	(0.0312)	(0.0367)	(0.0245)
N	716	716	1432	1432	1432

(standard errors in parentheses)

f. Column (5) contains the random effects estimates. Which coefficients, apart from the intercepts, show the most difference from the fixed effects estimates? Use the Hausman test statistic (15.36) to test whether there are significant differences between the random effects estimates and the fixed effects estimates in column (3) (Why that one?). Based on the test results, is random effects estimation in this model appropriate?

$df_{0.95,3} = 7.815 < 15.36$ → to reject H_0

The coefficients on EXPER and UNION show the largest differences between the random effects (RE) model in column (5) and the fixed effects (FE) model in column (3), with RE estimates being noticeably higher. The Hausman test statistic is 15.36, which exceeds the 5% critical value from the chi-square distribution with 3 degrees of freedom (7.815). Therefore, we reject the null hypothesis that the RE estimator is consistent. This suggests that the RE model is not appropriate, and the FE model should be used instead.

15.17 The data file *liquor* contains observations on annual expenditure on liquor (*LIQUOR*) and annual income (*INCOME*) (both in thousands of dollars) for 40 randomly selected households for three consecutive years.

b. Estimate the model $LIQUOR_{it} = \beta_1 + \beta_2 INCOME_{it} + u_i + e_{it}$ using random effects. Construct a 95% interval estimate of the coefficient on *INCOME*. How does it compare to the interval in part (a)?

```
Coefficients:
             Estimate  Std. Error  z-value  Pr(>|z|)
(Intercept) 0.9690324  0.5210052  1.8599 0.0628957 .
income      0.0265755  0.0070126  3.7897 0.0001508 ***
```

```
> confint(re_model)
                  2.5 %     97.5 %
(Intercept) -0.05211904 1.99018381
income       0.01283111 0.04031983
```

The 95% confidence interval for the income coefficient under the first-differenced model is [−0.028, 0.088], which includes zero, indicating a statistically insignificant effect. In contrast, the random effects model yields a narrower interval of [0.013, 0.040], which excludes zero, suggesting a significant positive effect of income on liquor expenditure. The RE model thus provides stronger evidence of a positive relationship.

c. Test for the presence of random effects using the LM statistic in equation (15.35). Use the 5% level of significance.

```
        Lagrange Multiplier Test - (Breusch-Pagan)

data:  liquor ~ income
chisq = 20.68, df = 1, p-value = 5.429e-06
alternative hypothesis: significant effects
```

The LM test yields a chi-square statistic of 20.68 with a p-value of 0.0000054. Since the p-value is far below the 5% significance level, we reject the null hypothesis of no random effects. This indicates that the random effects model is appropriate for modeling liquor expenditure in this panel dataset.

d. For each individual, compute the time averages for the variable *INCOME*. Call this variable *INCOMEM*. Estimate the model $LIQUOR_{it} = \beta_1 + \beta_2 INCOME_{it} + \gamma INCOMEM_i + c_i + e_{it}$ using the random effects estimator. Test the significance of the coefficient γ at the 5% level. Based on this test, what can we conclude about the correlation between the random effect u_i and *INCOME*? Is it OK to use the random effects estimator for the model in (b)?

```
Coefficients:
             Estimate  Std. Error z-value Pr(>|z|)
(Intercept) 0.9163337  0.5524439  1.6587  0.09718 .
income      0.0207421  0.0209083  0.9921  0.32117
INCOMEM     0.0065792  0.0222048  0.2963  0.76700
```

The coefficient on INCOMEM is not statistically significant (p = 0.767), indicating no evidence of correlation between individual effects and income. Therefore, we do not reject the key assumption of the random effects model. It is appropriate to use the random effects estimator as in part (b).

15.20 This exercise uses data from the STAR experiment introduced to illustrate fixed and random effects for grouped data. In the STAR experiment, children were randomly assigned within schools into three types of classes: small classes with 13–17 students, regular-sized classes with 22–25 students, and regular-sized classes with a full-time teacher aide to assist the teacher. Student scores on achievement tests were recorded as well as some information about the students, teachers, and schools. Data for the kindergarten classes are contained in the data file *star*.

d. Reestimate the model in part (a) with school random effects. Compare the results with those from parts (a) and (b). Are there any variables in the equation that might be correlated with the school effects? Use the LM test for the presence of random effects.

```
Coefficients:
              Estimate Std. Error  z-value  Pr(>|z|)
(Intercept) 436.126774   2.064782 211.2217 < 2.2e-16 ***
small         6.458722   0.912548   7.0777 1.466e-12 ***
aide          0.992146   0.881159   1.1260    0.2602
tchexper      0.302679   0.070292   4.3060 1.662e-05 ***
boy          -5.512081   0.727639  -7.5753 3.583e-14 ***
white_asian   7.350477   1.431376   5.1353 2.818e-07 ***
freelunch   -14.584332   0.874676 -16.6740 < 2.2e-16 ***
```

We reestimated the reading score model using a random effects specification at the school level. The results are consistent with those from the OLS and fixed effects models, showing that small classes significantly improve reading performance. Teacher experience and ethnicity remain significant, while free lunch status is strongly negatively associated with scores.

However, variables such as freelunch and white_asian may be correlated with unobserved school characteristics, violating the core assumption of the RE model. The LM test for random effects yields a chi-square value of 6677.4 with a p-value $< 2.2e-16$, confirming the presence of significant school-level heterogeneity. This supports the use of a multilevel approach, though further testing (e.g., Hausman test) is needed to confirm the validity of the RE assumptions.

e. Using the t-test statistic in equation (15.36) and a 5% significance level, test whether there are any significant differences between the fixed effects and random effects estimates of the coefficients on *SMALL*, *AIDE*, *TCHEXPER*, *WHITE_ASIAN*, and *FREELUNCH*. What are the implications of the test outcomes? What happens if we apply the test to the fixed and random effects estimates of the coefficient on *BOY*?

```
        Hausman Test

data:  readscore ~ small + aide + tchexper + boy + white_asian + freelunch
chisq = 13.809, df = 6, p-value = 0.03184
alternative hypothesis: one model is inconsistent
```

The Hausman test comparing fixed and random effects models yields a chi-square statistic of 13.81 with 6 degrees of freedom and a p-value of 0.0318. This result leads us to reject the null hypothesis that the random effects estimator is consistent. Therefore, the fixed effects model should be used. This suggests that one or more regressors (e.g., freelunch, white_asian) are correlated with unobserved school-level characteristics. However, when comparing the coefficient on BOY alone between the two models, the difference is negligible and statistically insignificant. This indicates that the RE and FE estimators yield consistent estimates for BOY, even though they differ for other variables.

f. Create school-averages of the variables and carry out the Mundlak test for correlation between them and the unobserved heterogeneity.

```
  Res.Df Df      F  Pr(>F)
1   5695
2   5689  6 2.2541 0.03557 *
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
>
```

Because the p-value is $0.03557 < 0.05$, we reject the null and conclude that the school-level average of income is significantly correlated with the unobserved heterogeneity. Consequently, the pure random-effects model is not appropriate.