15.6 Using the NLS panel data on $N = 716$ young women, we consider only years 1987 and 1988. We are interested in the relationship between ln(*WAGE*) and experience, its square, and indicator variables for living in the south and union membership. Some estimation results are in Table 15.10.

TABLE 15.10 Estimation Results for Exercise 15.6

	(1) OLS 1987	(2) OLS 1988	(3) FE	(4) FE Robust	(5) RE
C	0.9348	0.8993	1.5468	1.5468	1.1497
	(0.2010)	(0.2407)	(0.2522)	(0.2688)	(0.1597)
EXPER	0.1270	0.1265	0.0575	0.0575	0.0986
	(0.0295)	(0.0323)	(0.0330)	(0.0328)	(0.0220)
$EXPER^2$	−0.0033	−0.0031	−0.0012	−0.0012	−0.0023
	(0.0011)	(0.0011)	(0.0011)	(0.0011)	(0.0007)
SOUTH	−0.2128	−0.2384	−0.3261	−0.3261	−0.2326
	(0.0338)	(0.0344)	(0.1258)	(0.2495)	(0.0317)
UNION	0.1445	0.1102	0.0822	0.0822	0.1027
	(0.0382)	(0.0387)	(0.0312)	(0.0367)	(0.0245)
N	716	716	1432	1432	1432

(standard errors in parentheses)

a. The OLS estimates of the ln(*WAGE*) model for each of the years 1987 and 1988 are reported in columns (1) and (2). How do the results compare? For these individual year estimations, what are you assuming about the regression parameter values across individuals (heterogeneity)?

The OLS estimates for 1987 and 1988 are very similar, indicating stable relationships between wages and the explanatory variables across the two years. However, these year-specific OLS regressions assume that all individuals have the same regression parameters, ignoring individual heterogeneity such as unobserved ability or motivation.

b. The ln(*WAGE*) equation specified as a panel data regression model is

$$\ln\left(WAGE_{it}\right) = \beta_1 + \beta_2 EXPER_{it} + \beta_3 EXPER_{it}^2 + \beta_4 SOUTH_{it} + \beta_5 UNION_{it} + \left(u_i + e_{it}\right) \quad \text{(XR15.6)}$$

Explain any differences in assumptions between this model and the models in part (a).

The panel data model in (XR15.6) differs from the OLS models in part (a) by allowing for individual-specific effects u_i, which capture unobserved heterogeneity across individuals (e.g., innate ability, motivation, education background). These effects are constant over time but vary across individuals.
In contrast, the year-by-year OLS models in part (a) assume no individual-specific unobserved effects, treating all individuals as homogeneous in each year. This means the OLS models may suffer from omitted variable bias if u_i is correlated with the regressors.

Thus, the panel data model accounts for unobserved heterogeneity, while the single-year OLS models do not.

c. Column (3) contains the estimated fixed effects model specified in part (b). Compare these estimates with the OLS estimates. Which coefficients, apart from the intercepts, show the most difference?

The coefficients on EXPER and $EXPER^2$ show the most difference between the fixed effects and OLS models. This suggests that the OLS estimates overstate the return to experience due to not accounting for unobserved individual heterogeneity.

d. The F-statistic for the null hypothesis that there are no individual differences, equation (15.20), is 11.68. What are the degrees of freedom of the F-distribution if the null hypothesis (15.19) is true? What is the 1% level of significance critical value for the test? What do you conclude about the null hypothesis.

$df_1 = 716 - 1 = 715$, $df_2 = 1432 - 716 - 4 = 712$

$F_{715,712}(0{,}01) = 1{,}19$

$11{,}68 > 1{,}19$

The calculated F-statistic (11.68) is far greater than the 1% critical value ($\approx$ 1.19). Therefore, we reject the null hypothesis that there are no individual effects. This supports the use of a fixed effects model, indicating that individual heterogeneity is important in explaining wage differences.

e. Column (4) contains the fixed effects estimates with cluster-robust standard errors. In the context of this sample, explain the different assumptions you are making when you estimate with and without cluster-robust standard errors. Compare the standard errors with those in column (3). Which ones are substantially different? Are the robust ones larger or smaller?

Estimating the fixed effects model without cluster-robust standard errors (column 3) assumes that errors are homoskedastic and uncorrelated across observations. In contrast, using cluster-robust standard errors (column 4) allows for arbitrary heteroskedasticity and within-individual autocorrelation — a more realistic assumption in panel data since we have repeated observations (1987 and 1988) for each woman.

Comparing columns (3) and (4), the robust standard errors in column (4) are generally larger, especially for the UNION and SOUTH coefficients. This reflects the fact that clustering accounts for within-individual correlation, which standard errors in column (3) ignore. Therefore, cluster-robust standard errors are more conservative and reliable in this panel context.

15.17 The data file *liquor* contains observations on annual expenditure on liquor (*LIQUOR*) and annual income (*INCOME*) (both in thousands of dollars) for 40 randomly selected households for three consecutive years.

a. Create the first-differenced observations on *LIQUOR* and *INCOME*. Call these new variables *LIQUORD* and *INCOMED*. Using OLS regress *LIQUORD* on *INCOMED* without a constant term. Construct a 95% interval estimate of the coefficient.

```
Coefficients:
        Estimate Std. Error t value Pr(>|t|)
INCOMED  0.02975    0.02922   1.018    0.312
```

$\widehat{LIQUORD}_{it} = 0{,}02975\ INCOMED_{it}$

```
> confint(model)
               2.5 %     97.5 %
INCOMED -0.02841457 0.08790818
```

15.20 This exercise uses data from the STAR experiment introduced to illustrate fixed and random effects for grouped data. In the STAR experiment, children were randomly assigned within schools into three types of classes: small classes with 13–17 students, regular-sized classes with 22–25 students, and regular-sized classes with a full-time teacher aide to assist the teacher. Student scores on achievement tests were recorded as well as some information about the students, teachers, and schools. Data for the kindergarten classes are contained in the data file *star*.

a. Estimate a regression equation (with no fixed or random effects) where *READSCORE* is related to *SMALL*, *AIDE*, *TCHEXPER*, *BOY*, *WHITE_ASIAN*, and *FREELUNCH*. Discuss the results. Do students perform better in reading when they are in small classes? Does a teacher's aide improve scores? Do the students of more experienced teachers score higher on reading tests? Does the student's sex or race make a difference?

```
Coefficients:
             Estimate Std. Error t value Pr(>|t|)    
(Intercept) 437.76425    1.34622 325.180  < 2e-16 ***
small         5.82282    0.98933   5.886 4.19e-09 ***
aide          0.81784    0.95299   0.858    0.391    
tchexper      0.49247    0.06956   7.080 1.61e-12 ***
boy          -6.15642    0.79613  -7.733 1.23e-14 ***
white_asian   3.90581    0.95361   4.096 4.26e-05 ***
freelunch   -14.77134    0.89025 -16.592  < 2e-16 ***
```

The OLS regression shows that students in small classes score significantly higher in reading (+5.82 points), while teacher aides do not have a statistically significant impact. More experienced teachers are associated with better scores (+0.49 per year). Boys perform worse than girls (−6.16), and White/Asian students outperform others (+3.91). Students receiving free lunch, a proxy for low-income background, score significantly lower (−14.77), highlighting the importance of socioeconomic status in early education.

b. Reestimate the model in part (a) with school fixed effects. Compare the results with those in part (a). Have any of your conclusions changed? [*Hint*: specify *SCHID* as the cross-section identifier and *ID* as the "time" identifier.]

```
Coefficients:
              Estimate Std. Error  t-value  Pr(>|t|)    
small         6.490231   0.912962   7.1090 1.313e-12 ***
aide          0.996087   0.881693   1.1297    0.2586    
tchexper      0.285567   0.070845   4.0309 5.629e-05 ***
boy          -5.455941   0.727589  -7.4987 7.440e-14 ***
white_asian   8.028019   1.535656   5.2277 1.777e-07 ***
freelunch   -14.593572   0.880006 -16.5835 < 2.2e-16 ***
```

After including school fixed effects, the small-class effect remains strong and significant, even increasing in magnitude. This suggests that smaller class size improves reading scores within schools, not just across them. The effect of having a teacher aide remains statistically insignificant. Teacher experience still matters but its impact is reduced, likely because school-level differences in teacher experience are now controlled. Gender, race, and socioeconomic status continue to significantly influence reading scores, with low-income students consistently scoring much lower regardless of school.

c. Test for the significance of the school fixed effects. Under what conditions would we expect the inclusion of significant fixed effects to have little influence on the coefficient estimates of the remaining variables?

```
        F test for individual effects

data:  readscore ~ small + aide + tchexper + boy + white_asian + freelunch
F = 16.698, df1 = 78, df2 = 5681, p-value < 2.2e-16
alternative hypothesis: significant effects
```

The F-test for school fixed effects yields a value of 16.698 with a p-value < 2.2e-16, strongly rejecting the null hypothesis that all school effects are equal. This confirms the presence of significant school-level heterogeneity. However, if the key regressors such as small and aide vary mostly within schools rather than between schools, their coefficient estimates may remain stable even when school fixed effects are included.