6、31、33.

5.6 Suppose that, from a sample of 63 observations, the least squares estimates and the corresponding estimated covariance matrix are given by

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix} \qquad \widehat{\operatorname{cov}}(b_1, b_2, b_3) = \begin{bmatrix} 3 & -2 & 1 \\ -2 & 4 & 0 \\ 1 & 0 & 3 \end{bmatrix}$$

Using a 5% significance level, and an alternative hypothesis that the equality does not hold, test each of the following null hypotheses:

a. $\beta_2 = 0$

b. $\beta_1 + 2\beta_2 = 5$

c. $\beta_1 - \beta_2 + \beta_3 = 4$

$t_{0.975,\,60} = 2.0003$

$y = b_1 + b_2 X_2 + b_3 X_3$

a. $var_2 = 4$, $se(b_2) = 2$

$H_0: \beta_2 = 0$

$H_1: \beta_2 \neq 0$

$t = \frac{3-0}{2} \sim t(63-3)$

$1.5 < 2.0003$

Not reject H_0，沒有足夠的證據證明$\beta_2 \neq 0$

b. $H_0: \beta_1 + 2\beta_2 = 5$

$H_1: \beta_1 + 2\beta_2 \neq 5$

$var(b_1 + 2b_2)$

$= \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} 3 & -2 \\ -2 & 4 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$= \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} 3-4 \\ -2+8 \end{bmatrix}$

$= \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} -1 \\ 6 \end{bmatrix}$

$= 11$

$se(b_1 + 2b_2) = \sqrt{11}$

$t = \frac{2 + 2 \times 3 - 5}{\sqrt{11}} \sim t(63-3)$

$0.9045 < 2.0003$

Not to reject H_0

c.

$var(b_1 - b_2 + b_3)$

$= \begin{bmatrix} 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 3 & -2 & 1 \\ -2 & 4 & 0 \\ 1 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 3+2+1 \\ -2-4+0 \\ 1+0+3 \end{bmatrix}$

$= \begin{bmatrix} 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 6 \\ -6 \\ 4 \end{bmatrix}$

$= 6 + 6 + 4$

$= 16$

$seb(b_1 - b_2 + b_3) = \sqrt{16} = 4$

$H_0: \beta_1 - \beta_2 + \beta_3 = 4$

$H_1: \beta_1 - \beta_2 + \beta_3 \neq 4$

$t = \frac{2-3-1-4}{4} = -1.5$

$-1.5 < -2.0003$

Not to rejet H_0.