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HW0331

5.6.

$\beta_2 = 0$; $H_0 : \beta_2 = 0$ vs $H_1 : \beta_2 \neq 0$.

$$t = \frac{\beta_2}{se(\beta_2)} = \frac{3}{\sqrt{4}} = 1.5 < t_{(0.975,60)} = 2.00$$

$\Rightarrow$ We cannot reject H_0

$$Cov(b_1, b_2, b_3) = \begin{bmatrix} Var(b_1) & Cov(b_1 b_2) & Cov(b_1, b_3) \\ Cov(b_1, b_2) & Var(b_2) & Cov(b_2, b_3) \\ Cov(b_1 b_3) & Cov(b_2, b_3) & Var(b_3) \end{bmatrix} = \begin{bmatrix} 3 & -2 & 1 \\ -2 & 4 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

b) H_0 $\beta_1 + 2\beta_2 = 5$

H_1 $\beta_1 + 2\beta_2 \neq 5$

$$t = \frac{(\beta_1 + 2\beta_2) - 5}{se(\beta_1 + 2\beta_2)} = \frac{3}{\sqrt{11}} = 0.9045 < t \rightarrow \text{We cannot reject } H_0$$

c) $H_0 : \beta_1 - \beta_2 + \beta_3 = 4$; $H_0 : \beta_1 - \beta_2 + \beta_3 \neq 4$

$$t = \frac{(\beta_1 - \beta_2 + \beta_3) - 4}{se(\beta_1 - \beta_2 + \beta_3)} = \frac{-6}{\sqrt{16}} = -1.5.$$

$t < t_{cr} = 2 \rightarrow$ We cannot reject H_0.

5.13.a

Beta1: If Bill departs from Carnegie at 6:30 AM under ideal conditions (no red lights or trains), his commute is expected to take approximately 20.87 minutes.

Beta2: Should Bill leave later than 6:30 AM, his travel time is projected to increase by around 3.7 minutes for every additional 10 minutes delay, assuming that the frequency of red lights and trains stays the same.

Beta3: With the departure time and train encounters held constant, every red light is anticipated to add roughly 1.52 minutes to his commute.

Beta4: Similarly, if his departure time and the occurrence of red lights remain unchanged, each train encountered is estimated to add about 3.02 minutes to his travel time.

5.13.b

```
> confint(mod1, level = 0.95)
                 2.5 %    97.5 %
(Intercept) 17.5694018 24.170871
depart       0.2989851  0.437265
reds         1.1574748  1.886411
trains       1.7748867  4.272505
```

5.13.c

The computed t-statistic is

$t = (\beta_3 - 2)/se(\beta_2) = -2.584$, which is lower than -1.651.

Thus, we conclude that the expected delay attributable to each red light is under 2 minutes.

5.13.d

The t-statistic is

$t = (\beta_4 - 3)/se(\beta_4) = 0.037$, a value less than 1.651.

Therefore, we cannot reject the null hypothesis that β_4 is equal to 3 minutes.

5.13.e

With

$t = (\beta_3 - 1/3)/se(\beta_2) = 0.991$, which is below the critical value of 1.651,

we do not have enough evidence to reject the null hypothesis that a 30-minute delay in departure results in an increase of at least 10 minutes in the expected travel time.

5.13.f

The t-statistic calculated as

$t = (\beta_4 - 3\beta_3)/se(\beta_4 - 3\beta_3) = -1.825027$ falls below -1.651.

This result leads us to reject the null hypothesis (that β_4 is greater than $3\beta_3$), indicating that the expected delay caused by a train is less than three times the delay from a red light.

5.13.g

$H_0: \beta_1 + 30\beta_2 + 6\beta_3 + \beta_4 \leq 45$

To test this, we compute the test statistic:

$t = (b_1 + 30b_2 + 6b_3 + b_4) / SE(b_1 + 30b_2 + 6b_3 + b_4) = -1.725964$

Since the calculated t-value (-1.725964) is less than the critical value (1.651), we fail to reject the null hypothesis H_0.

5.13.h

$H_0: \beta_1 + 30\beta_2 + 6\beta_3 + \beta_4 \geq 45$

In this case, if the test statistic $t > -1.651$, we fail to reject H_0.

However, since $t < -1.651$, we reject the null hypothesis.

This means there's sufficient evidence to conclude that Bill takes less than 45 minutes to get to the meeting.

5.33.a

All the estimated coefficients are statistically significant at the 1% level, except for the coefficient on $EDUC^2$, which is only significant at the 12% level.

```
Coefficients:
                  Estimate Std. Error t value Pr(>|t|)
(Intercept)      1.038e+00  2.757e-01   3.764 0.000175 ***
educ             8.954e-02  3.108e-02   2.881 0.004038 **
I(educ^2)        1.458e-03  9.242e-04   1.578 0.114855
exper            4.488e-02  7.297e-03   6.150 1.06e-09 ***
I(exper^2)      -4.680e-04  7.601e-05  -6.157 1.01e-09 ***
I(educ * exper) -1.010e-03  3.791e-04  -2.665 0.007803 **
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.4638 on 1194 degrees of freedom
Multiple R-squared:  0.3227,	Adjusted R-squared:  0.3198
F-statistic: 113.8 on 5 and 1194 DF,  p-value: < 2.2e-16
```

5.33.b

The marginal effect of education is given by:

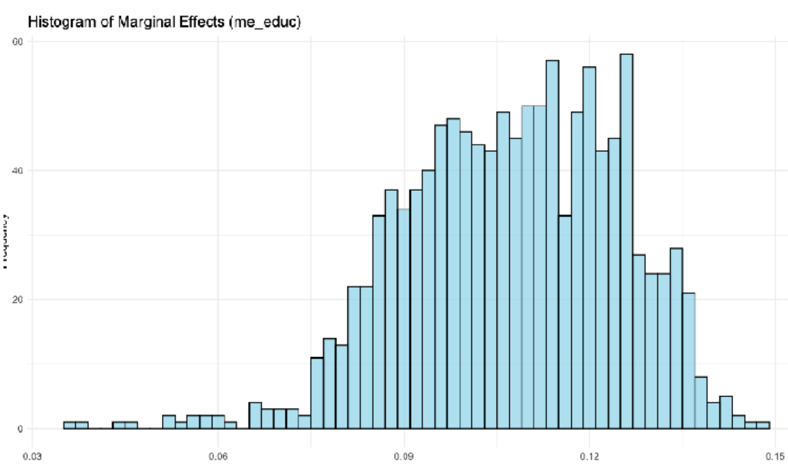
ME | Educ = $\beta_2 + 2\beta_3$ * Educ + β_6 * Expert

The estimated marginal effect is:

ME_EDUC = 0.089539 + 0.002916 * Educ – 0.001010 * Expert

This indicates that the marginal effect of education increases with higher levels of education, but decreases as experience increases.

5.33.c




```
> print(summary_stats)
        Mean    Median   Maximum    Minimum Percentile_5 Percentile_95    Std_Dev   Skewness Kurtosis
5% 0.1073483 0.1084313 0.1478705 0.03565419   0.08008187     0.1336188 0.01722176 -0.4066173 3.150502
   Jarque_Bera_Stat Jarque_Bera_p_value
5%         34.20008        3.745819e-08
```

We find that the marginal effects range from 0.036 to 0.148, with the majority falling between 0.085 and 0.13.

The 5th, 50th (median), and 95th percentiles of the marginal effects distribution are, respectively.

5.33.d

the marginal effect of experience is given by:

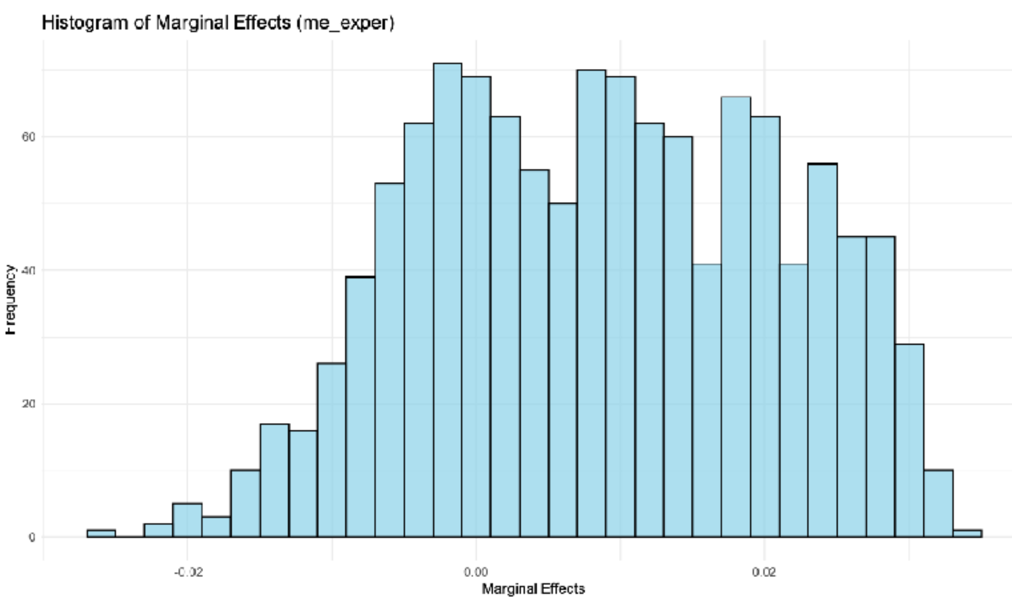
$ME \mid Exper = \beta_4 + 2\beta_5 * Exper + \beta_6 * Educ$

The estimated marginal effect is:

$ME\_EXPER = 0.044879 – 0.000936 * Exper – 0.001010 * Educ$

This shows that the marginal effect of experience declines both as years of experience increase and as the level of education rises.

5.33.e




```
> print(summary_stats_exper)
          Mean      Median    Maximum     Minimum Percentile_5 Percentile_95    Std_Dev    Skewness
5% 0.008651732 0.008418878 0.03398886 -0.02527874  -0.01037621    0.02793115 0.01216966 -0.04659748
   Kurtosis Jarque_Bera_Stat Jarque_Bera_p_value
5% 2.123526         38.84459        3.672862e-09
```

Although most of the marginal effects of experience are positive, their overall range spans from -0.025 to 0.034.
The 5th, 50th (median), and 95th percentiles of these marginal effects are, respectively

5.33.f

H_0: $\beta_1 + 17\beta_2 + 289\beta_3 + 8\beta_4 + 64\beta_5 + 136\beta_6 \leq \beta_1 + 16\beta_2 + 256\beta_3 + 18\beta_4 + 324\beta_5 + 288\beta_6$

Simplifying both sides by subtracting common terms, the hypothesis becomes:

H_0: $\beta_2 + 33\beta_3 - 10\beta_4 - 260\beta_5 - 152\beta_6 \leq 0$

The calculated test statistic is $t = -1.669902$, which is less than the critical value $t_a = -1.6461$.
Since $t < t_a$, we fail to reject the null hypothesis (H_0).
This means there is insufficient evidence to conclude that David's log-wage is greater.

5.33.g

The null hypothesis is:
H_0: $-\beta_2 - 33\beta_3 + 10\beta_4 + 420\beta_5 + 144\beta_6 \geq 0$

The test statistic is $t = -2.062365$, which is less than the critical value of -1.6461.
Since $t < t_a$, we reject the null hypothesis.

This provides evidence that David's log-wage is greater.

The observed difference in outcomes can be explained by diminishing returns to experience. Svetlana started with 18 years of experience, so the additional years had less impact on her log-wage. In contrast, David initially had only 8 years of experience, so the extra 8 years contributed more significantly to increasing his log-wage.

5.33.h

The marginal effect of experience is defined as:
$ME \mid Exper = \beta_4 + 2\beta_5 * Exper + \beta_6 * Educ$

For Wendy:
$ME = \beta_4 + 34\beta_5 + 12\beta_6$

For Jill:

ME = $\beta_4 + 22\beta_5 + 16\beta_6$

To test whether their marginal effects differ, we form the null hypothesis:

H_0: $\beta_4 + 34\beta_5 + 12\beta_6 = \beta_4 + 22\beta_5 + 16\beta_6$,

which simplifies to: $12\beta_5 - 4\beta_6 = 0$

The test statistic is t = -1.027304, which is greater than the critical value -1.96195.

Since t > t_cr, we fail to reject the null hypothesis.

Conclusion: There is insufficient evidence to suggest that the marginal effect of experience differs between Jill and Wendy.

5.33.i

Assuming Jill gains more experience over time but does not pursue additional education, her marginal effect of experience evolves as:

ME = $\beta_4 + 2\beta_5 * \text{Exper} + 16\beta_6$

We're interested in finding when this becomes less than 11, so we solve:

$\beta_4 + 2\beta_5 * \text{Exper} + 16\beta_6 - 11 < 0$

Based on this condition, it will take approximately 19.667 more years before Jill's marginal effect becomes negative.

A 95% confidence interval for the number of years until her marginal effect turns negative is: [15.96, 23.40]