

4.4 The general manager of a large engineering firm wants to know whether the experience of technical artists influences their work quality. A random sample of 50 artists is selected. Using years of work experience (*EXPER*) and a performance rating (*RATING*, on a 100-point scale), two models are estimated by least squares. The estimates and standard errors are as follows:

Model 1:

$$\widehat{RATING} = 64.289 + 0.990EXPER \quad N = 50 \quad R^2 = 0.3793$$

(se) (2.422) (0.183)

Model 2:

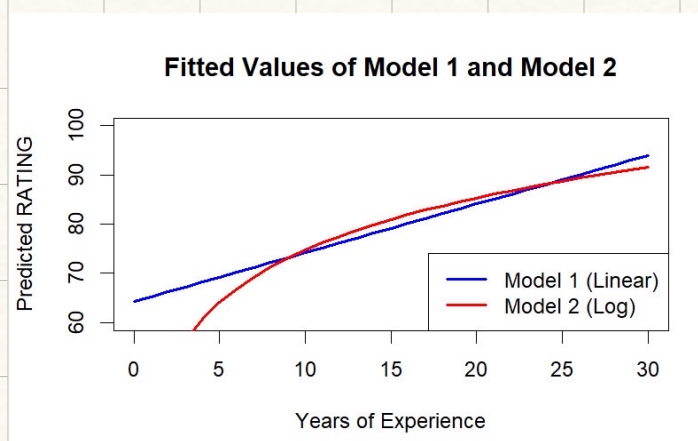
$$\widehat{RATING} = 39.464 + 15.312 \ln(EXPER) \quad N = 46 \quad R^2 = 0.6414$$

(se) (4.198) (1.727)

CHAPTER 4 Prediction, Goodness-of-Fit, and Modeling Issues

- Sketch the fitted values from Model 1 for *EXPER* = 0 to 30 years.
- Sketch the fitted values from Model 2 against *EXPER* = 1 to 30 years. Explain why the four artists with no experience are not used in the estimation of Model 2.
- Using Model 1, compute the marginal effect on *RATING* of another year of experience for (i) an artist with 10 years of experience and (ii) an artist with 20 years of experience.
- Using Model 2, compute the marginal effect on *RATING* of another year of experience for (i) an artist with 10 years of experience and (ii) an artist with 20 years of experience.
- Which of the two models fits the data better? Estimation of Model 1 using just the technical artists with some experience yields $R^2 = 0.4858$.
- Do you find Model 1 or Model 2 more reasonable, or plausible, based on economic reasoning? Explain.

(a)(b)



因為 $\ln 0$ 沒有被定義，無法計算。

(c) $\frac{d(\widehat{RATING})}{d(EXPER)} = 0.99$, 實際效果固定

The marginal effects with 10 years and 20 years of experience are equal to 0.99.

(d) $\frac{d(\widehat{RATING})}{d(EXPER)} = 15.312 \times \frac{1}{EXPER}$

The marginal effect with 10 years of experience is $15.312 \times \frac{1}{10} = 1.5312$

The marginal effect with 20 years of experience is $15.312 \times \frac{1}{20} = 0.7656$

(e) Model 1 $R^2 = 0.3793$

Model 2 $R^2 = 0.6414$

Model 1 (僅有經驗的藝術家) $R^2 = 0.4858$

由於 Model 2 $R^2 = 0.6414$ 最大，更能解釋 *RATING* 差異

(f) Model 2 更合理，因為隨著時間增加，影響遞減較符合實際情況。

4.28 The file *wa-wheat.dat* contains observations on wheat yield in Western Australian shires. There are 48 annual observations for the years 1950–1997. For the Northampton shire, consider the following four equations:

$$YIELD_t = \beta_0 + \beta_1 TIME + e_t$$

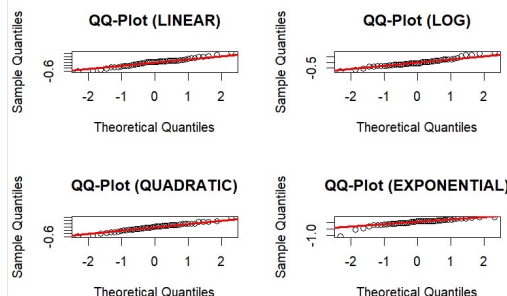
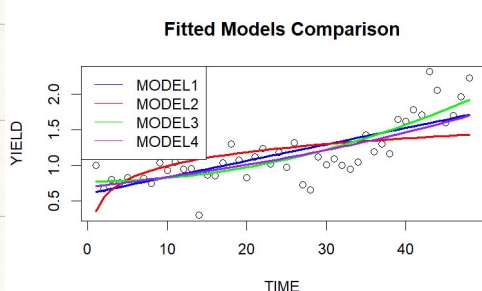
$$YIELD_t = \alpha_0 + \alpha_1 \ln(TIME) + e_t$$

$$YIELD_t = \gamma_0 + \gamma_1 TIME^2 + e_t$$

$$\ln(YIELD_t) = \phi_0 + \phi_1 TIME + e_t$$

- Estimate each of the four equations. Taking into consideration (i) plots of the fitted equations, (ii) plots of the residuals, (iii) error normality tests, and (iii) values for R^2 , which equation do you think is preferable? Explain.
- Interpret the coefficient of the time-related variable in your chosen specification.
- Using your chosen specification, identify any unusual observations, based on the studentized residuals, *LEVERAGE*, *DFBETAS*, and *DFFITs*.
- Using your chosen specification, use the observations up to 1996 to estimate the model. Construct a 95% prediction interval for *YIELD* in 1997. Does your interval contain the true value?

(a) i, ii, iii,



> summary(MODEL1)

```
Call:
lm(formula = NORTHAMPTON ~ TIME, data = DATA)

Residuals:
    Min       1Q   Median       3Q      Max
-0.62394 -0.17302  0.03342  0.12996  0.72050

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  0.603245   0.081858   7.369 2.55e-09 ***
TIME         0.023078   0.002908   7.935 3.69e-10 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.2791 on 46 degrees of freedom
Multiple R-squared:  0.5778,    Adjusted R-squared:  0.5687
F-statistic: 62.96 on 1 and 46 DF,  p-value: 3.689e-10
```

> summary(MODEL2)

```
Call:
lm(formula = NORTHAMPTON ~ log(TIME), data = DATA)

Residuals:
    Min       1Q   Median       3Q      Max
-0.78468 -0.20711 -0.06382  0.15447  0.91573

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  0.3510    0.1759   1.995  0.052 .
log(TIME)    0.2790    0.0575   4.852 1.44e-05 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.3494 on 46 degrees of freedom
Multiple R-squared:  0.3386,    Adjusted R-squared:  0.3242
F-statistic: 23.55 on 1 and 46 DF,  p-value: 1.44e-05
```

> summary(MODEL3)

```
Call:
lm(formula = NORTHAMPTON ~ I(TIME^2), data = DATA)

Residuals:
    Min       1Q   Median       3Q      Max
-0.56899 -0.14970  0.03119  0.12176  0.62049

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  4.737e-01   5.222e-02   9.082 < 2e-16 ***
I(TIME^2)    4.986e-04   4.939e-05  10.10 3.01e-13 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.2396 on 46 degrees of freedom
Multiple R-squared:  0.689,    Adjusted R-squared:  0.6822
F-statistic: 101.9 on 1 and 46 DF,  p-value: 3.008e-13
```

> summary(MODEL4)

```
Call:
lm(formula = log(NORTHAMPTON) ~ TIME, data = DATA)

Residuals:
    Min       1Q   Median       3Q      Max
-1.09292 -0.10049  0.07125  0.14140  0.40263

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) -0.363938   0.076192  -4.777 1.85e-05 ***
TIME         0.018632   0.002707   6.883 1.37e-08 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.2598 on 46 degrees of freedom
Multiple R-squared:  0.5074,    Adjusted R-squared:  0.4966
F-statistic: 47.37 on 1 and 46 DF,  p-value: 1.366e-08
```

I prefer model 3 because its R^2 is the biggest.

(b) From a.

$$\gamma_0 = 0.7737$$

$$\text{When Time} = 0, \text{Yield} = 0.7737$$

$$\gamma_1 = 0.0004986, \text{Yield 隨著 Time } \uparrow, \text{ 增加的速度}$$

```
> # Shapiro-wilk 正態性檢定 (p < 0.05 表示殘差不符合常態)
> shapiro.test(residuals(MODEL3))
```

Shapiro-wilk normality test

```
data: residuals(MODEL1)
W = 0.98236, p-value = 0.6792
```

```
> shapiro.test(residuals(MODEL2))
```

Shapiro-wilk normality test

```
data: residuals(MODEL2)
W = 0.96657, p-value = 0.1856
```

Shapiro-wilk normality test

```
data: residuals(MODEL3)
W = 0.98589, p-value = 0.8266
```

```
> shapiro.test(residuals(MODEL4))
```

Shapiro-wilk normality test

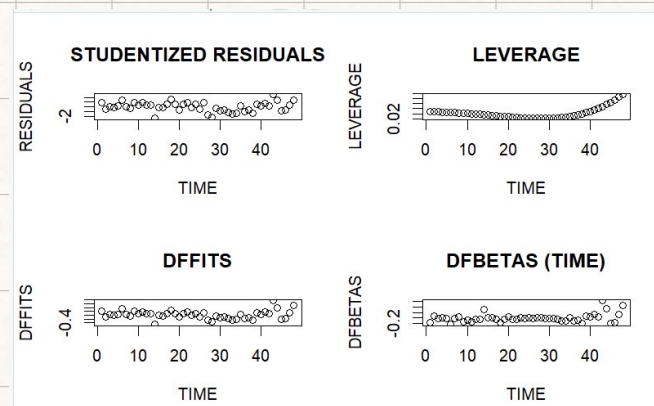
```
data: residuals(MODEL4)
W = 0.86894, p-value = 7.205e-05
```

Model 1 ~ 3 都符合 normal

Model 4, p-value < 0.05

⇒ 不符合 normal

(c)



(d)

$$CI_{\gamma_1} = [1.3724, 2.3898]$$

$$YIELD = 2.2318 \in [1.3724, 2.3898]$$

Yes.