11.28 Supply and demand curves as traditionally drawn in economics principles classes have price (P) on the vertical axis and quantity (Q) on the horizontal axis.

a. Rewrite the truffle demand and supply equations in (11.11) and (11.12) with price P on the left-hand side. What are the anticipated signs of the parameters in this rewritten system of equations?

$$\text{Demand: } Q_i = \alpha_1 + \overset{(-)}{\alpha_2} P_i + \overset{(+)}{\alpha_3} PS_i + \overset{(+)}{\alpha_4} DI_i + e_{di} \qquad (11.11)$$

$$\text{Supply: } Q_i = \beta_1 + \underset{(+)}{\beta_2} P_i + \underset{(-)}{\beta_3} PF_i + e_{si} \qquad (11.12)$$

Rewrite demand fcn:

$$P_i = \frac{Q_i - \alpha_1 - \alpha_3 PS_i - \alpha_4 DI_i - e_{di}}{\alpha_2}$$

$$P_i = -\frac{\alpha_1}{\alpha_2} + \underbrace{\frac{1}{\alpha_2}}_{(-)} Q_i - \underbrace{\frac{\alpha_3}{\alpha_2}}_{(+)} PS_i - \underbrace{\frac{\alpha_4}{\alpha_2}}_{(+)} DI_i - \frac{1}{\alpha_2} e_{di}$$

Rewrite Supply fcn:

$$P_i = \frac{Q_i - \beta_1 - \beta_3 PF_i - e_{si}}{\beta_2}$$

$$P_i = -\frac{\beta_1}{\beta_2} + \underbrace{\frac{1}{\beta_2}}_{(+)} Q_i - \underbrace{\frac{\beta_3}{\beta_2}}_{(+)} PF_i - \frac{1}{\beta_2} e_{si}$$

b. Using the data in the file *truffles*, estimate the supply and demand equations that you have formulated in (a) using two-stage least squares. Are the signs correct? Are the estimated coefficients significantly different from zero?

```
Call:
ivreg(formula = p ~ q + ps + di | ps + di + pf, data = truffles)

Residuals:
    Min      1Q  Median      3Q     Max
-39.661  -6.781   2.410   8.320  20.251

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  -11.428     13.592  -0.841  0.40810
q             -2.671      1.175  -2.273  0.03154 *
ps             3.461      1.116   3.103  0.00458 **
di            13.390      2.747   4.875 4.68e-05 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 13.17 on 26 degrees of freedom
Multiple R-Squared: 0.5567,     Adjusted R-squared: 0.5056
Wald test: 17.37 on 3 and 26 DF,  p-value: 2.137e-06
```

correct

All are significant at $\alpha = 5\%$.

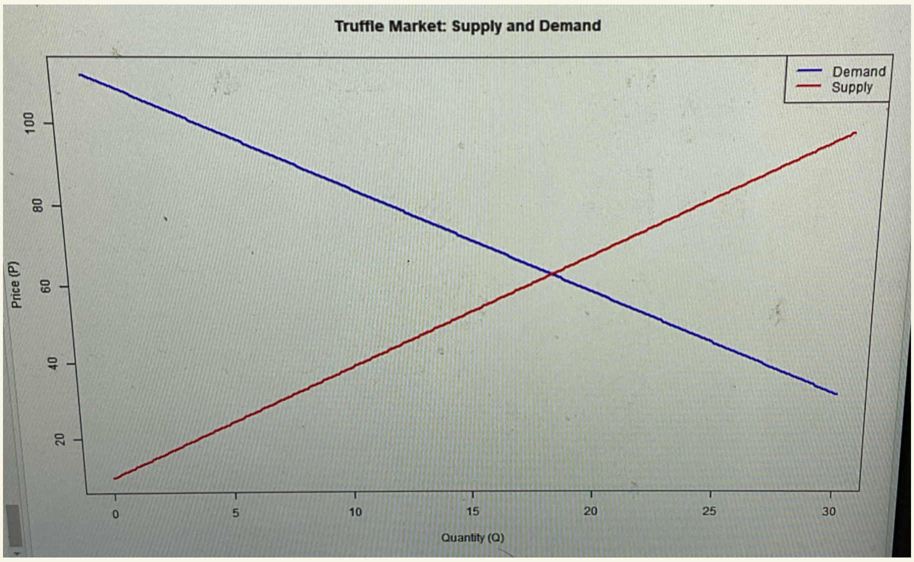
```
Call:
ivreg(formula = p ~ q + pf | pf + ps + di, data = truffles)

Residuals:
     Min       1Q   Median       3Q      Max
-9.7983 -2.3440 -0.6281  2.4350 11.1600

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) -58.7982     5.8592  -10.04 1.32e-10 ***
q             2.9367     0.2158   13.61 1.32e-13 ***
pf            2.9585     0.1560   18.97  < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 4.399 on 27 degrees of freedom
Multiple R-Squared: 0.9486,     Adjusted R-squared: 0.9448
Wald test: 232.7 on 2 and 27 DF,  p-value: < 2.2e-16

>
```

c. Estimate the price elasticity of demand "at the means" using the results from (b).

$$\varepsilon_D = \frac{dQ}{dP}\cdot\frac{\bar{P}}{\bar{Q}} = \frac{1}{\delta_2}\cdot\frac{\bar{P}}{\bar{Q}} = -1.2725$$

```
> elasticity
        Q
-1.272464
>
```

d. Accurately sketch the supply and demand equations, with P on the vertical axis and Q on the horizontal axis, using the estimates from part (b). For these sketches set the values of the exogenous variables DI, PS, and PF to be $DI^* = 3.5$, $PF^* = 23$, and $PS^* = 22$.



e. What are the equilibrium values of P and Q obtained in part (d)? Calculate the predicted equilibrium values of P and Q using the estimated reduced-form equations from Table 11.2, using the same values of the exogenous variables. How well do they agree?

TABLE 11.2a Reduced Form for Quantity of Truffles (*Q*)

Variable	Coefficient	Std. Error	*t*-Statistic	Prob.
C	7.8951	3.2434	2.4342	0.0221
PS	0.6564	0.1425	4.6051	0.0001
DI	2.1672	0.7005	3.0938	0.0047
PF	−0.5070	0.1213	−4.1809	0.0003

TABLE 11.2b Reduced Form for Price of Truffles (*P*)

Variable	Coefficient	Std. Error	*t*-Statistic	Prob.
C	−32.5124	7.9842	−4.0721	0.0004
PS	1.7081	0.3509	4.8682	0.0000
DI	7.6025	1.7243	4.4089	0.0002
PF	1.3539	0.2985	4.5356	0.0001

```
> # 顯示結果
> cat("簡化形式預測的均衡數量 Q*:", round(Q_pred, 2), "\n")
簡化形式預測的均衡數量 Q*: 18.26
> cat("簡化形式預測的均衡價格 P*:", round(P_pred, 2), "\n")
簡化形式預測的均衡價格 P*: 62.81
> # 計算均衡數量 Q* (由 P需求 = P供給)
> Q_eq <- (111.579 - 9.259) / (2.937 + 2.671)  # 解聯立方程式
> P_eq <- 111.579 - 2.671 * Q_eq  # 代入需求方程式求 P
> # 顯示結構方程式預測的均衡數量 Q* 和均衡價格 P*
> cat("結構方程式預測的均衡數量 Q*:", round(Q_eq, 2), "\n")
結構方程式預測的均衡數量 Q*: 18.25
> cat("結構方程式預測的均衡價格 P*:", round(P_eq, 2), "\n")
結構方程式預測的均衡價格 P*: 62.85
>
```

f. Estimate the supply and demand equations that you have formulated in (a) using OLS. Are the signs correct? Are the estimated coefficients significantly different from zero? Compare the results to those in part (b).

demand

```
Call:
lm(formula = p ~ q + ps + di, data = truffles)

Residuals:
     Min       1Q   Median       3Q      Max
-25.0753  -2.7742  -0.4097   4.7079  17.4979

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) -13.6195     9.0872  -1.499   0.1460
q             0.1512     0.4988   0.303   0.7642
ps            1.3607     0.5940   2.291   0.0303 *
di           12.3582     1.8254   6.770 3.48e-07 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 8.814 on 26 degrees of freedom
Multiple R-squared:  0.8013,	Adjusted R-squared:  0.7784
F-statistic: 34.95 on 3 and 26 DF,  p-value: 2.842e-09

> # OLS 估計供給方程
```

supply

```
Call:
lm(formula = p ~ q + pf, data = truffles)

Residuals:
    Min      1Q  Median      3Q     Max
-8.4721 -3.3287  0.1861  2.0785 10.7513

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) -52.8763     5.0238  -10.53 4.68e-11 ***
q             2.6613     0.1712   15.54 5.42e-15 ***
pf            2.9217     0.1482   19.71  < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 4.202 on 27 degrees of freedom
Multiple R-squared:  0.9531,	Adjusted R-squared:  0.9496
F-statistic: 274.4 on 2 and 27 DF,  p-value: < 2.2e-16

>
```

The estimated coefficient q in demand fcn is not correct (must be negative, ∵ P↑, Q↓)

11.28R

```r
#11.28(a)
demand_iv <- ivreg(p ~ q + ps + di | ps + di + pf, data = truffles)
summary(demand_iv)
supply_iv <- ivreg(p ~ q + pf | pf + ps + di, data = truffles)
summary(supply_iv)

#(b)
# 抓係數
beta_q <- coef(demand_iv)["q"]

# 計算均值
mean_p <- mean(truffles$p)
mean_q <- mean(truffles$q)

# 彈性公式
elasticity <- (1 / beta_q) * (mean_p / mean_q)
elasticity

#(d)
# 建立數量範圍
q_vals <- seq(0, 30, by = 0.1)

# 計算需求與供給對應的價格
p_demand <- 111.579 - 2.671 * q_vals
p_supply <- 9.259 + 2.937 * q_vals

# 畫圖
plot(q_vals, p_demand, type = "l", col = "blue", lwd = 2,
     ylim = range(c(p_demand, p_supply)),
     xlab = "Quantity (Q)", ylab = "Price (P)",
     main = "Truffle Market: Supply and Demand")

lines(q_vals, p_supply, col = "red", lwd = 2)
legend("topright", legend = c("Demand", "Supply"),
       col = c("blue", "red"), lty = 1, lwd = 2)

#(e)
# 定義外生變數
PS <- 22   # PS (Price of substitutes)
DI <- 3.5  # DI (Disposable income)
PF <- 23   # PF (Price of factors)

# 簡化形式方程式 (預測數量 Q)
Q_pred <- 7.8951 + 0.6564 * PS + 2.1672 * DI - 0.5070 * PF

# 簡化形式方程式 (預測價格 P)
P_pred <- -32.5124 + 1.7081 * PS + 7.6025 * DI + 1.3539 * PF

# 顯示结果
cat("簡化形式預測的均衡數量 Q*:", round(Q_pred, 2), "\n")
cat("簡化形式預測的均衡價格 P*:", round(P_pred, 2), "\n")

# 由结構方程式計算均衡數量 Q* 和均衡價格 P*
# 结構方程式的需求和供給式:
# 需求曲線: P = 111.579 - 2.671 * Q
# 供給曲線: P = 9.259 + 2.937 * Q

# 計算均衡數量 Q* (由 P需求 = P供給)
Q_eq <- (111.579 - 9.259) / (2.937 + 2.671)  # 解聯立方程式
P_eq <- 111.579 - 2.671 * Q_eq  # 代入需求方程式求 P

# 顯示结構方程式預測的均衡數量 Q* 和均衡價格 P*
cat("结構方程式預測的均衡數量 Q*:", round(Q_eq, 2), "\n")
cat("结構方程式預測的均衡價格 P*:", round(P_eq, 2), "\n")

#(f)
# OLS 估計需求方程
demand_ols <- lm(p ~ q + ps + di, data = truffles)
summary(demand_ols)

# OLS 估計供給方程
supply_ols <- lm(p ~ q + pf, data = truffles)
summary(supply_ols)
```

11.30 Example 11.3 introduces Klein's Model I. Use the data file *klein* to answer the following questions.

a. Estimate the investment function in equation (11.18) by OLS. Comment on the signs and significance of the coefficients.

```
Call:
lm(formula = i ~ p + plag + klag, data = klein)

Residuals:
     Min       1Q   Median       3Q      Max
-2.56562 -0.63169  0.03687  0.41542  1.49226

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 10.12579    5.46555   1.853 0.081374 .
p            0.47964    0.09711   4.939 0.000125 ***
plag         0.33304    0.10086   3.302 0.004212 **
klag        -0.11179    0.02673  -4.183 0.000624 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05
```

結果整體上支持Klein模型中投資與利潤及資本存量之間的理論關係，並顯示企業投資行為同時受到當期與過去利潤表現的影響，同時受到現有資本存量的抑制效果

b. Estimate the reduced-form equation for profits, P_t, using all eight exogenous and predetermined variables as explanatory variables. Test the joint significance of all the variables except lagged profits, P_{t-1}, and lagged capital stock, K_{t-1}. Save the residuals, $\hat{v}_t$ and compute the fitted values, $\hat{P}_t$.

```
Call:
lm(formula = p ~ w2 + tx + g + time + elag + plag + klag, data = klein)

Residuals:
    Min      1Q  Median      3Q     Max
-3.9067 -1.3050  0.3226  1.3613  2.8881

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 50.38442   31.63026   1.593   0.1352
w2          -0.07961    2.53382  -0.031   0.9754
tx          -0.92310    0.43376  -2.128   0.0530 .
g            0.43902    0.39114   1.122   0.2820
time         0.31941    0.77813   0.410   0.6881
elag         0.02200    0.28216   0.078   0.9390
plag         0.80250    0.51886   1.547   0.1459
klag        -0.21610    0.11911  -1.814   0.0928 .
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 2.183 on 13 degrees of freedom
  (因為不存在，1 個觀察量被刪除了)
Multiple R-squared:  0.8261,    Adjusted R-squared:  0.7324
F-statistic: 8.821 on 7 and 13 DF,  p-value: 0.0004481
```

```
> # F 檢定
> anova(model_restricted, model_reduced)
Analysis of Variance Table

Model 1: p ~ plag + klag
Model 2: p ~ w2 + tx + g + time + elag + plag + klag
  Res.Df    RSS Df Sum of Sq      F Pr(>F)
1     18 108.04
2     13  61.95  5    46.093 1.9345 0.1566
```

	year	p	pfitted	vresid
2	1921	12.4	13.255556	-0.8555556
3	1922	16.9	16.577368	0.3226319
4	1923	18.4	19.282347	-0.8823465
5	1924	19.4	20.960143	-1.5601433
6	1925	20.1	19.766509	0.3334910
7	1926	19.6	18.238731	1.3612688
8	1927	19.8	17.573065	2.2269354
9	1928	21.1	19.541720	1.5582796
10	1929	21.7	20.375101	1.3248995
11	1930	15.6	17.180415	-1.5804148
12	1931	11.4	12.705026	-1.3050261
13	1932	7.0	8.999780	-1.9997802
14	1933	11.2	9.054102	2.1458976
15	1934	12.3	12.671263	-0.3712632
16	1935	14.0	14.421338	-0.4213385
17	1936	17.6	14.711907	2.8880932
18	1937	17.3	19.796405	-2.4964049
19	1938	15.3	19.206691	-3.9066913
20	1939	19.0	17.419605	1.5803947
21	1940	21.1	20.305654	0.7943462
22	1941	23.5	22.657273	0.8427268

don't reject H_0 at $\alpha=5\%$

新增的變數並沒有顯著提升模型表現

c. The Hausman test for the presence of endogenous explanatory variables is discussed in Section 10.4.1. It is implemented by adding the reduced-form residuals to the structural equation and testing their significance, that is, using OLS estimate the model

$$I_t = \beta_1 + \beta_2 P_t + \beta_3 P_{t-1} + \beta_4 K_{t-1} + \delta \hat{v}_t + e_{2t}$$

Use a t-test for the null hypothesis $H_0: \delta = 0$ versus $H_1: \delta \neq 0$ at the 5% level of significance. By rejecting the null hypothesis, we conclude that P_t is endogenous. What do we conclude from the test? In the context of this simultaneous equations model what result should we find?

```
Call:
lm(formula = i ~ p + plag + klag + vresid, data = klein_clean)

Residuals:
     Min       1Q   Median       3Q      Max
-1.04645 -0.56030  0.06189  0.25348  1.36700

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 20.27821    4.70179   4.313 0.000536 ***
p            0.15022    0.10798   1.391 0.183222
plag         0.61594    0.10147   6.070 1.62e-05 ***
klag        -0.15779    0.02252  -7.007 2.96e-06 ***
vresid       0.57451    0.14261   4.029 0.000972 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.7331 on 16 degrees of freedom
Multiple R-squared:  0.9659,    Adjusted R-squared:  0.9574
F-statistic: 113.4 on 4 and 16 DF,  p-value: 1.588e-11
```

p-value $= 0.000972 < 0.05$

$\therefore$ reject H_0

P_t is endogenous

d. Obtain the 2SLS estimates of the investment equation using all eight exogenous and predetermined variables as IVs and software designed for 2SLS. Compare the estimates to the OLS estimates in part (a). Do you find any important differences?

```
Call:
ivreg(formula = i ~ p + plag + klag | w2 + tx + g + time + elag +
    plag + klag, data = klein_clean)

Residuals:
    Min      1Q  Median      3Q     Max
-3.2909 -0.8069  0.1423  0.8601  1.7956

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 20.27821    8.38325   2.419  0.02707 *
p            0.15022    0.19253   0.780  0.44598
plag         0.61594    0.18093   3.404  0.00338 **
klag        -0.15779    0.04015  -3.930  0.00108 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.307 on 17 degrees of freedom
Multiple R-Squared: 0.8849,    Adjusted R-squared: 0.8646
Wald test:  41.2 on 3 and 17 DF,  p-value: 5.148e-08
```

P_t的係數顯著低於OLS模型，且在2SLS不再具統計顯著性，OLS可能因內生性問題高估了利潤對投資的影響.

e. Estimate the second-stage model $I_t = \beta_1 + \beta_2 \hat{P}_t + \beta_3 P_{t-1} + \beta_4 K_{t-1} + e_{2t}$ by OLS. Compare the estimates and standard errors from this estimation to those in part (d). What differences are there?

```
Call:
lm(formula = i ~ pfitted + plag + klag, data = klein_clean)

Residuals:
    Min      1Q  Median      3Q     Max 
-3.8778 -1.0029  0.3058  0.7275  2.1831 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)   
(Intercept) 20.27821    9.97663   2.033  0.05802 . 
pfitted      0.15022    0.22913   0.656  0.52084   
plag         0.61594    0.21531   2.861  0.01083 * 
klag        -0.15779    0.04778  -3.302  0.00421 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.556 on 17 degrees of freedom
Multiple R-squared:  0.837,	Adjusted R-squared:  0.8082 
F-statistic: 29.09 on 3 and 17 DF,  p-value: 6.393e-07
```

估計值一致
標準誤↑

f. Let the 2SLS residuals from part (e) be $\hat{e}_{2t}$. Regress these residuals on all the exogenous and predetermined variables. If these instruments are valid, then the R^2 from this regression should be low, and none of the variables are statistically significant. The Sargan test for instrument validity is discussed in Section 10.4.3. The test statistic TR^2 has a chi-square distribution with degrees of freedom equal to the number of "surplus" IVs if the surplus instruments are valid. The investment equation includes three exogenous and/or predetermined variables out of the total of eight possible. There are $L = 5$ external instruments and $B = 1$ right-hand side endogenous variables. Compare the value of the test statistic to the 95th percentile value from the $\chi^2_{(4)}$ distribution. What do we conclude about the validity of the surplus instruments in this case?

```
Call:
lm(formula = e2hat ~ w2 + tx + g + time + elag + plag + klag, 
    data = klein_clean)

Residuals:
    Min      1Q  Median      3Q     Max 
-3.4087 -0.8799  0.2702  1.0011  2.4987 

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept)  7.671103  24.976416   0.307    0.764
w2          -0.704649   2.000800  -0.352    0.730
tx          -0.022846   0.342512  -0.067    0.948
g            0.034277   0.308861   0.111    0.913
time         0.283921   0.614439   0.462    0.652
elag        -0.116046   0.222807  -0.521    0.611
plag         0.189896   0.409708   0.463    0.651
klag        -0.002262   0.094056  -0.024    0.981

Residual standard error: 1.724 on 13 degrees of freedom
Multiple R-squared:  0.06102,	Adjusted R-squared:  -0.4446 
F-statistic: 0.1207 on 7 and 13 DF,  p-value: 0.9953
```

$R^2 = 0.06102$

樣本數 $T = 21$

Sargan test

$TR^2 = 21 \times 0.06102 = 1.2814$

$\chi^2_{0.95,4} = 9.488$

$\because TR^2 = 1.2814 < 9.488$

Don't reject H_0, surplus IVs are valid.

11.30 R

```r
#11.30(a)
model_invest <- lm(i ~ p + plag+klag, data = klein)
summary(model_invest)

#(b)
# 建立資料子集：剔除有 NA 的觀測值
klein_clean <- na.omit(klein[, c("year","p", "w2", "tx", "g", "time", "elag", "plag", "klag")])
# 建立模型
model_reduced <- lm(p ~ w2 + tx + g + time + elag + plag + klag, data = klein_clean)
summary(model_reduced)
# 限制模型（只包含 plag 與 klag）
model_restricted <- lm(p ~ plag + klag, data = klein)
# F 檢定
anova(model_restricted, model_reduced)

# 儲存殘差與預測值
klein_clean$vresid <- resid(model_reduced)
klein_clean$pfitted <- fitted(model_reduced)
# 印出選定的結果欄位
print(klein_clean[, c("year", "p", "pfitted", "vresid")])


#(c)
klein_clean <- na.omit(klein[, c("year", "i", "p", "plag", "klag", "w2", "tx", "g", "time", "elag")])
# 預測 p，產生 vresid（你應該已經做過）
model_reduced <- lm(p ~ w2 + tx + g + time + elag + plag + klag, data = klein_clean)
klein_clean$vresid <- resid(model_reduced)

# Hausman 檢定模型
hausman_model <- lm(i ~ p + plag + klag + vresid, data = klein_clean)

# 檢視結果
summary(hausman_model)

#(d)
iv_model <- ivreg(i ~ p + plag + klag | w2 + tx + g + time + elag + plag + klag, data = klein_clean)

# 查看結果
summary(iv_model)


#(e)
klein_clean$pfitted <- fitted(model_reduced)
stage2_model <- lm(i ~ pfitted + plag + klag, data = klein_clean)
summary(stage2_model)

#(f)
model_2sls_stage2 <- lm(i ~ pfitted + plag + klag, data = klein_clean)
klein_clean$e2hat <- resid(model_2sls_stage2)
sargan_model <- lm(e2hat ~ w2 + tx + g + time + elag + plag + klag, data = klein_clean)
summary(sargan_model)
R2 <- summary(sargan_model)$r.squared
T <- nrow(klein_clean)
sargan_stat <- T * R2
qchisq(0.95, df = 4)
```