d.

因x_1, x_2為外生且和V_2不相關，故可得一致性估計。

即$E(x_j, V_2)=0$

e. 對$y_2=\pi_1 x_1+\pi_2 x_2+V_2$ 最小平方誤差

$S=\Sigma(y_2-\pi_1 x_1-\pi_2 x_2)^2$

對π_1及π_2偏微分即可得到(d)之條件

$\Rightarrow$ MOM估計值=OLS #

f. $\hat{\pi}_1=\dfrac{\Sigma x_{i1} y_{2i}}{\Sigma x_{i1}^2}=3$, $\hat{\pi}_2=\dfrac{\Sigma x_{i2} y_{2i}}{\Sigma x_{i2}}=\dfrac{4}{1}=4$.

g. 只要IV ①和內生變數y_2相關 ②和結構誤差e_1不相關即可達成一致性

$$\text{plim}\,\hat{\alpha}_1=\frac{E(\hat{y}_2 y_1)}{E(\hat{y}_2 y_2)}=\frac{E[\hat{y}_2(\alpha_1 y_2+e_1)]}{E(\hat{y}_2 y_2)}=\alpha_1+\frac{E(\hat{y}_2 e_1)}{E(\hat{y}_2 y_2)}=\alpha_1$$

$\hat{\alpha}_1 \xrightarrow{P} \alpha_1$ 符合一致性.

$3\times2+4\times3=18.$

$3\times3+4\times4=25$

$\Rightarrow \hat{\alpha}_1=\dfrac{18}{25}=0.72.$

h.

$\Sigma y_{2i}^2=3^2\Sigma x_{i1}^2+4^2\Sigma x_{i2}^2+2\times3\times4\,\Sigma x_{i1}x_{i2}=9+16+0=25.$

$\alpha_1^{2SLS}=\dfrac{\Sigma\hat{y}_{2i}y_{1i}}{\Sigma y_{2i}^2}=\dfrac{18}{25}=0.72.$

和(g)結果相同

在單一內生變數且使用同一組IV時

IV之結果=2SLS結果。

計量作業 0505

11.1 $\begin{cases} y_1 = \alpha_1 y_2 + e_1 \\ y_2 = \alpha_2 y_1 + \beta_1 x_1 + \beta_2 x_2 + e_2 \end{cases}$

(a) $y_2 = \alpha_2(\alpha_1 y_2 + e_1) + \beta_1 x_1 + \beta_2 x_2 + e_2$

$\Rightarrow y_2(1-\alpha_1\alpha_2) = \beta_1 x_1 + \beta_2 x_2 + (\alpha_2 e_1 + e_2)$

$y_2 = \left(\frac{\beta_1}{1-\alpha_1\alpha_2}\right)x_1 + \left(\frac{\beta_2}{1-\alpha_1\alpha_2}\right)x_2 + \left(\frac{\alpha_2 e_1 + e_2}{1-\alpha_1\alpha_2}\right) \Rightarrow \pi_1 x_1 + \pi_2 x_2 + v_2$

$$\begin{aligned}
\mathrm{cov}(y_2, e_1 | x) &= E(y_2, e_1 | x) \\
&= E\left[\left(\frac{\beta_1}{1-\alpha_1\alpha_2}x_1 + \frac{\beta_2}{1-\alpha_1\alpha_2}x_2 + \frac{\alpha_2 e_1 + e_2}{1-\alpha_1\alpha_2}\right) e_1 | x\right] \\
&= E\left[\left(\frac{\beta_1}{1-\alpha_1\alpha_2}x_1 e_1\right)|x\right] + E\left[\left(\frac{\beta_2}{1-\alpha_1\alpha_2}x_2 e_1\right)|x\right] + E\left[\left(\frac{\alpha_2 e_1 + e_2}{1-\alpha_1\alpha_2}e_1\right)|x\right] \\
&= 0 + 0 + E\left[\left(\frac{\alpha_2 e_1 + e_2}{1-\alpha_1\alpha_2}e_1\right)|x\right]
\end{aligned}$$

$\rightarrow \mathrm{cov}(y_2, e_1 | x) = E(y_2, e_1 | x) = \dfrac{E(e_1 e_2 | x) + \alpha_2 E(e_1^2 | x)}{1-\alpha_1\alpha_2} = \dfrac{\alpha_2 \cdot \sigma_1^2}{1-\alpha_1\alpha_2}$

$\mathrm{cov}(y_2, e_1 | x)$ 不為0，除非 $\alpha_2 = 0$。

(b) 因為等式一和二有 endogenous variable
so the OLS is biased and inconsistent.
而 reduce form 具有一致性

(c) 有2條結構方程，到少要省略 2-1=1個外生變数.
equation(1)：無外生變数，省略了2個外生變数 ⇒ identified
equation(2)：x_1, x_2 省略了0個外生變数 ⇒ unidentified

11.16.

a.

$\alpha_1+\alpha_2 P=\beta_1+\beta_2 P+\beta_3 W$

$\Rightarrow P=\pi_1+\pi_2 W+V_1$

$\pi_1=\dfrac{\beta_1-\alpha_1}{\alpha_2-\beta_2}$, $\pi_2=\dfrac{\beta_3}{\alpha_2-\beta_2}$, $V_1=\dfrac{e_s-e_d}{\alpha_2-\beta_2}$

代回需求式

$Q=\alpha_1+\alpha_2(\pi_1+\pi_2 W+V_1)+e_d=\theta_1+\theta_2 W+V_2$

$\therefore \theta_1=\alpha_1+\alpha_2\pi_1$, $\theta_2=\alpha_2\pi_2$, $V_2=\alpha_2 V_1+e_d$.

b. $\pi_2=\dfrac{\beta_3}{\alpha_2-\beta_2}$, $\theta_2=\alpha_2\pi_2 \Rightarrow \alpha_2=\dfrac{\theta_2}{\pi_2}$ ∴ identified.

$\theta_1=\alpha_1+\alpha_2\pi_1 \Rightarrow \alpha_1$ 亦 identified

但 β_1、β_2、β_3 僅以 π_1、π_2、α 圈定一條關係，無法唯一

$\Rightarrow$ unidentified

c. $\hat{\alpha}_2=\dfrac{\hat{\theta}_2}{\hat{\pi}_2}=\dfrac{0.5}{1}=0.5$

$\hat{\alpha}_1=\hat{\theta}_1-\hat{\alpha}_2\hat{\pi}_1=5-0.5\times2.4=3.8$.

d. 第一階段：$\hat{P}_i=2.4+W_i$.

第二階段：$\hat{Q}=\hat{\alpha}_1+\hat{\alpha}_2\hat{P}$

$\begin{pmatrix}\hat{\alpha}_1\\ \hat{\alpha}_2\end{pmatrix}=(X'X)^{-1}X'Q \Rightarrow \hat{\alpha}_1=3.8$

$\hat{\alpha}_2=0.5$.

和(C)之結果相同，驗證 identified 良好且僅一內生變數時，

ILS 結果 = 2SLS 結果.

a. 系統共有M條，必要條件：每條方程式至少要省略M−1個外生變數；Klein I 皆滿足，至少 weak identified.

b. 每條方程式被排除的外生變數數目≥該方程式右側內生變數數目 ⇒ Klein I 方程式皆可估.

c. 令所有外生變數為 Z_t

$W_t^p = \pi_0 + \pi_1 Z_{1t} + \pi_2 Z_{2t} + \cdots + U_t$

π 係數可用 OLS 拿到 fitted value $\hat{W}_t^p$，供第二階段使用.

d. 第一步：對所有右側外生變數（EX：所得、工資）以全部外生 Z_t 為 IV 作 OLS 得到 $\hat{Y}_t$

第二步：以原方程的因變數對第一步 fitted values 和所有外生變數重新作 OLS，即得 2SLS 係數