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HW0428

Question 18

a.

```
> cat("Percentage of mothers with some college education:", percentage_mother, "%\n")
Percentage of mothers with some college education: 12.14953 %
> cat("Percentage of fathers with some college education:", percentage_father, "%\n")
Percentage of fathers with some college education: 11.68224 %
```

b.

MOTHERCOLL and FATHERCOLL are binary variables, which reduce potential measurement errors compared to continuous variables (MOTHEREDUC, FATHEREDUC). These two variables are also easier to explain: Whether or not an individual has attained a high level of education.

```
                educ MOTHERCOLL FATHERCOLL
educ       1.0000000  0.3594705  0.3984962
MOTHERCOLL 0.3594705  1.0000000  0.3545709
FATHERCOLL 0.3984962  0.3545709  1.0000000
```

c.

```
Call:
ivreg(formula = log(wage) ~ educ + exper + I(exper^2) | MOTHERCOLL +
    exper + I(exper^2), data = mroz)

Residuals:
     Min       1Q   Median       3Q      Max 
-3.08719 -0.32444  0.04147  0.36634  2.35621 

Coefficients:
              Estimate Std. Error t value Pr(>|t|)   
(Intercept) -0.1327561  0.5607243  -0.237   0.8130   
educ         0.0760180  0.0437323   1.738   0.0829 . 
exper        0.0433444  0.0152145   2.849   0.0046 **
I(exper^2)  -0.0008711  0.0004175  -2.086   0.0375 * 
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.6703 on 424 degrees of freedom
Multiple R-Squared: 0.147,  Adjusted R-squared: 0.1409 
Wald test: 6.149 on 3 and 424 DF,  p-value: 0.0004237
>
```

```
> confint(iv_c, 'educ', level = 0.95)
            2.5 %    97.5 %
educ -0.001219763 0.1532557
```

d.

Reject the null hypothesis that the coefficient on MOTHERCOLL is zero. MOTHERCOLL is a **strong instrument** for educ based on both tests.

```
Linear hypothesis test:
MOTHERCOLL = 0

Model 1: restricted model
Model 2: educ ~ MOTHERCOLL + exper + I(exper^2)

Note: Coefficient covariance matrix supplied.

  Res.Df Df      F    Pr(>F)
1    425
2    424  1 76.392 < 2.2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

```
Linear hypothesis test:
MOTHERCOLL = 0

Model 1: restricted model
Model 2: educ ~ MOTHERCOLL + exper + I(exper^2)

  Res.Df    RSS Df Sum of Sq      F    Pr(>F)
1    425 2219.2
2    424 1929.9  1    289.32 63.563 1.455e-14 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

e.

The new confidence interval (CI) is narrower, indicating that adding FATHERCOLL has improved precision, likely due to stronger instrumentation.

```
ivreg(formula = log(wage) ~ educ + exper + I(exper^2) | MOTHERCOLL +
    FATHERCOLL + exper + I(exper^2), data = mroz)

Residuals:
     Min       1Q   Median       3Q      Max 
-3.07797 -0.32128  0.03418  0.37648  2.36183 

Coefficients:
              Estimate Std. Error t value Pr(>|t|)   
(Intercept) -0.2790819  0.4302159  -0.649  0.51688   
educ         0.0878477  0.0337910   2.600  0.00966 **
exper        0.0426761  0.0154095   2.769  0.00586 **
I(exper^2)  -0.0008486  0.0004255  -1.995  0.04673 * 
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.6679 on 424 degrees of freedom
Multiple R-Squared: 0.153,  Adjusted R-squared: 0.147 
Wald test: 7.693 on 3 and 424 DF,  p-value: 5.136e-05 
```

```
> confint(iv_e, 'educ', level = 0.95)
         2.5 %    97.5 %
educ 0.02751845 0.1481769
```

f.

Both results are statistically significant at the 1% level (***), meaning we reject the null hypothesis that MOTHERCOLL and FATHERCOLL jointly have no effect on education. The F-statistics are well above 10, which is a common rule-of-thumb threshold, indicating that the instruments are strong.

```
Call:
lm(formula = educ ~ MOTHERCOLL + FATHERCOLL + exper + I(exper^2),
    data = mroz)

Residuals:
    Min      1Q  Median      3Q     Max
-7.2152 -0.3056 -0.2152  0.7627  5.0620

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) 11.890259   0.290251  40.965  < 2e-16 ***
MOTHERCOLL   1.749947   0.322347   5.429 9.58e-08 ***
FATHERCOLL   2.186612   0.329917   6.628 1.04e-10 ***
exper        0.049149   0.040133   1.225    0.221
I(exper^2)  -0.001449   0.001199  -1.209    0.227
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 2.033 on 423 degrees of freedom
Multiple R-squared:  0.2161,	Adjusted R-squared:  0.2086
F-statistic: 29.15 on 4 and 423 DF,  p-value: < 2.2e-16
```

```
Model 1: restricted model
Model 2: educ ~ MOTHERCOLL + FATHERCOLL + exper + I(exper^2)

Note: Coefficient covariance matrix supplied.

  Res.Df Df     F    Pr(>F)
1    425
2    423  2 89.33 < 2.2e-16 ***
---
```

```
Model 1: restricted model
Model 2: educ ~ MOTHERCOLL + FATHERCOLL + exper + I(exper^2)

  Res.Df    RSS Df Sum of Sq      F    Pr(>F)
1    425 2219.2
2    423 1748.3  2    470.88 56.963 < 2.2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

g.

The Sargan test produces a p-value of 0.626, suggesting that we fail to reject the null hypothesis of instrument validity. Therefore, both MOTHERCOLL and FATHERCOLL seem to be valid instruments for EDUC in the wage equation.

```
ivreg(formula = log(wage) ~ educ + exper + I(exper^2) | MOTHERCOLL +
    FATHERCOLL + exper + I(exper^2), data = mroz)

Residuals:
     Min       1Q   Median       3Q      Max
-3.07797 -0.32128  0.03418  0.37648  2.36183

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept) -0.2790819  0.3922213  -0.712  0.47714
educ         0.0878477  0.0307808   2.854  0.00453 **
exper        0.0426761  0.0132950   3.210  0.00143 **
I(exper^2)  -0.0008486  0.0003976  -2.135  0.03337 *

Diagnostic tests:
                 df1 df2 statistic p-value
Weak instruments   2 423    56.963  <2e-16 ***
Wu-Hausman         1 423     0.519   0.472
Sargan             1  NA     0.238   0.626
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.6679 on 424 degrees of freedom
Multiple R-Squared: 0.153,  Adjusted R-squared: 0.147
Wald test: 9.724 on 3 and 424 DF,  p-value: 3.224e-06
```

Question 20

a.

A beta of 1.201840 indicates that the asset is riskier compared to the market portfolio. Specifically, it suggests that the asset's returns are expected to be 20.18% more volatile than the overall market. For example, if the market moves by 1%, the asset is expected to move by approximately 1.2018%.

```
Call:
lm(formula = rp_msft ~ rp_mkt, data = capm5)

Residuals:
     Min       1Q   Median       3Q      Max
-0.27424 -0.04744 -0.00820  0.03869  0.35801

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 0.003250   0.006036   0.538    0.591
rp_mkt      1.201840   0.122152   9.839   <2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.08083 on 178 degrees of freedom
Multiple R-squared:  0.3523,    Adjusted R-squared:  0.3486
F-statistic:  96.8 on 1 and 178 DF,  p-value: < 2.2e-16
```

b.

To evaluate the IV (Instrumental Variable) conditions:

IV1-IV3 refer to the three key conditions: relevance, exogeneity, and exclusion.

RANK is deterministically derived from rp_mkt, so it may satisfy the relevance condition (IV1), but its validity as exogenous (IV2) is questionable.

With $R^2 = 0.9126$ and Adjusted $R^2 = 0.9121$, we reject the null hypothesis that the coefficient on RANK is zero (F-value = 1857.6). Based on both tests, RANK is a strong instrument for educ.

As a rule of thumb, an F-statistic > 10 is considered indicative of a strong instrument (Staiger & Stock, 1997).

```
lm(formula = rp_mkt ~ RANK, data = capm5)

Residuals:
      Min        1Q    Median        3Q       Max
-0.110497 -0.006308  0.001497  0.009433  0.029513

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept) -7.903e-02  2.195e-03   -36.0   <2e-16 ***
RANK         9.067e-04  2.104e-05    43.1   <2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.01467 on 178 degrees of freedom
Multiple R-squared:  0.9126,    Adjusted R-squared:  0.9121
F-statistic:  1858 on 1 and 178 DF,  p-value: < 2.2e-16
```

```
Model 1: restricted model
Model 2: rp_mkt ~ RANK

  Res.Df     RSS Df Sum of Sq      F    Pr(>F)
1    179 0.43784
2    178 0.03829  1   0.39955 1857.6 < 2.2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

c.

Since **p = 0.0428 > 0.01**, we conclude that the market return is exogenous in the CAPM for Microsoft.

```
lm(formula = rp_msft ~ rp_mkt + v_hat, data = capm5)

Residuals:
     Min       1Q   Median       3Q      Max 
-0.27140 -0.04213 -0.00911  0.03423  0.34887 

Coefficients:
             Estimate Std. Error t value Pr(>|t|)    
(Intercept)  0.003018   0.005984   0.504   0.6146    
rp_mkt       1.278318   0.126749  10.085   <2e-16 ***
v_hat       -0.874599   0.428626  -2.040   0.0428 *  
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.08012 on 177 degrees of freedom
Multiple R-squared:  0.3672,	Adjusted R-squared:   0.36 
F-statistic: 51.34 on 2 and 177 DF,  p-value: < 2.2e-16
```

d.

The IV estimate of β (1.2783) is slightly higher than the OLS estimate (1.2018), and both are highly significant. This suggests minimal endogeneity in the market return, indicating that the OLS results are reliable and consistent with CAPM expectations.

```
ivreg(formula = rp_msft ~ rp_mkt | RANK, data = capm5)

Residuals:
      Min        1Q    Median        3Q       Max 
-0.271625 -0.049675 -0.009693  0.037683  0.355579 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 0.003018   0.006044   0.499    0.618    
rp_mkt      1.278318   0.128011   9.986   <2e-16 ***

Diagnostic tests:
                 df1 df2 statistic p-value    
Weak instruments   1 178  1857.587  <2e-16 ***
Wu-Hausman         1 177     4.164  0.0428 *  
Sargan             0  NA        NA      NA    
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.08092 on 178 degrees of freedom
Multiple R-Squared: 0.3508, Adjusted R-squared: 0.3472
```

e.

R2 = 0.9149, Adjusted R2 = 0.9139

Since the **F-statistic > 10**, the instruments are considered **jointly strong**.

```
lm(formula = rp_mkt ~ RANK + POS, data = capm5)

Residuals:
      Min        1Q    Median        3Q       Max 
-0.109182 -0.006732  0.002858  0.008936  0.026652 

Coefficients:
              Estimate Std. Error t value Pr(>|t|)    
(Intercept) -0.0804216  0.0022622  -35.55   <2e-16 ***
RANK         0.0009819  0.0000400   24.55   <2e-16 ***
POS         -0.0092762  0.0042156   -2.20   0.0291 *  
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.01451 on 177 degrees of freedom
Multiple R-squared:  0.9149,	Adjusted R-squared:  0.9139 
F-statistic: 951.3 on 2 and 177 DF,  p-value: < 2.2e-16
```

```
Model 1: restricted model
Model 2: rp_mkt ~ RANK + POS

  Res.Df     RSS Df Sum of Sq      F    Pr(>F)    
1    179 0.43784                                  
2    177 0.03727  2   0.40057 951.26 < 2.2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

f.

The null hypothesis H_0: $\delta = 0$ (market return is exogenous) is tested at the 1% significance level. Since **p-value = 0.0287 > 0.01**, **fail to reject H_0** → Market return is **exogenous.**

```
Residuals:
     Min       1Q   Median       3Q      Max 
-0.27132 -0.04261 -0.00812  0.03343  0.34867 

Coefficients:
             Estimate Std. Error t value Pr(>|t|)    
(Intercept)  0.003004   0.005972   0.503   0.6157    
rp_mkt       1.283118   0.126344  10.156   <2e-16 ***
v_hat_e     -0.954918   0.433062  -2.205   0.0287 *  
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.07996 on 177 degrees of freedom
Multiple R-squared:  0.3696,	Adjusted R-squared:  0.3625 
F-statistic: 51.88 on 2 and 177 DF,  p-value: < 2.2e-16
```

g.

The IV estimate of β (1.2831), using both RANK and POS, is slightly higher than the OLS estimate (1.2018). This is consistent with expectations under potential measurement error in the market return and further supports the reliability of OLS, given earlier evidence of exogeneity.

```
Residuals:
     Min       1Q   Median       3Q      Max 
-0.27168 -0.04960 -0.00983  0.03762  0.35543 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 0.003004   0.006044   0.497     0.62    
rp_mkt      1.283118   0.127866  10.035   <2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.08093 on 178 degrees of freedom
Multiple R-Squared: 0.3507,	Adjusted R-squared: 0.347 
Wald test: 100.7 on 1 and 178 DF,  p-value: < 2.2e-16 
```

h.

To perform the Sargan test for instrument validity, we first compute the residuals from the IV/2SLS model in part (g), then regress these residuals on all instruments (RANK and POS). The $\mathbf{NR^2}$ statistic from this regression is used for the test.

The Sargan test yields a statistic of **0.5585** with a **p-value of 0.4549**, meaning we fail to reject the null hypothesis at the 5% significance level. This indicates that the surplus instrument **POS** is valid and that the instruments, as a whole, are exogenous.

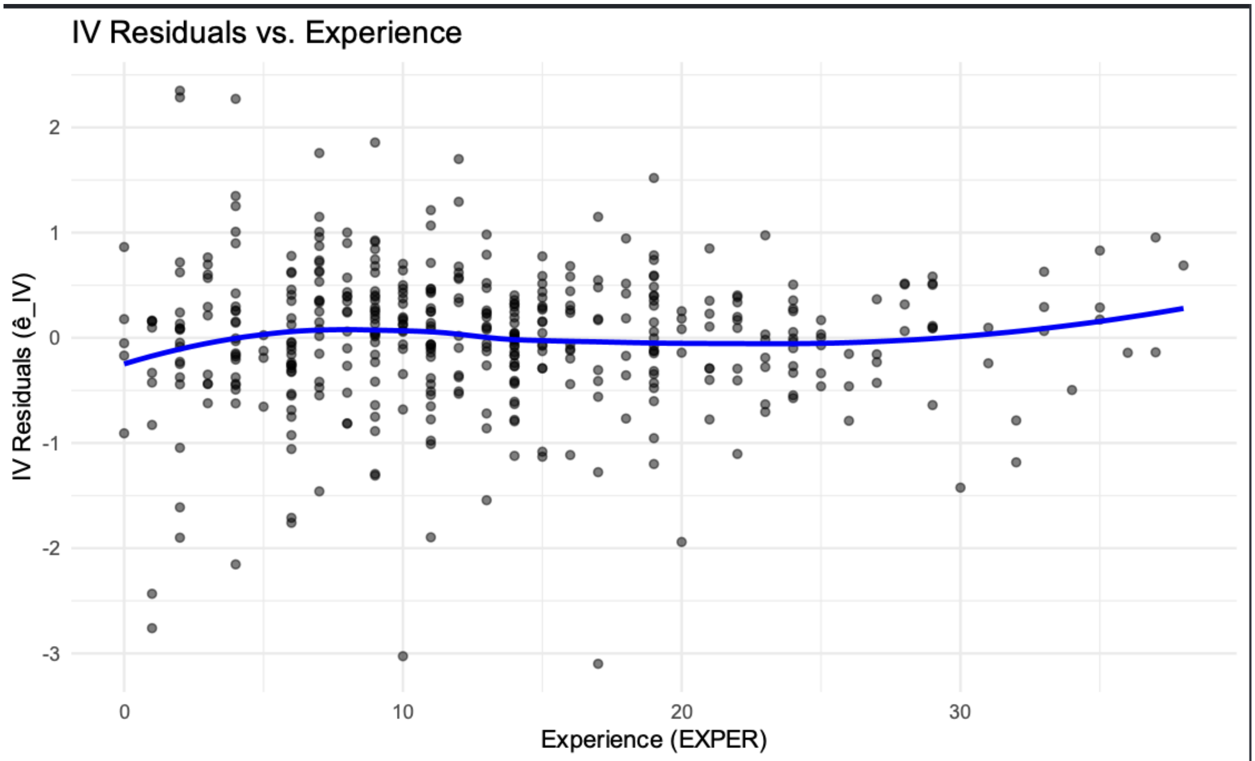
```
Residuals:
     Min       1Q   Median       3Q      Max
-0.27168 -0.04960 -0.00983  0.03762  0.35543

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 0.003004   0.006044   0.497     0.62
rp_mkt      1.283118   0.127866  10.035   <2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.08093 on 178 degrees of freedom
Multiple R-Squared: 0.3507, Adjusted R-squared: 0.347
Wald test: 100.7 on 1 and 178 DF,  p-value: < 2.2e-16

> cat("Sargan statistic:", sargan_stat, "\n")
Sargan statistic: 0.5584634
> cat("p-value:", p_value, "\n")
p-value: 0.45488
lm(formula = iv_resid ~ RANK + POS, data = capm5)

Residuals:
     Min       1Q   Median       3Q      Max
-0.26914 -0.04702 -0.00801  0.03771  0.35674

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept) -0.0022220  0.0126326  -0.176    0.861
RANK         0.0001370  0.0002234   0.613    0.540
POS         -0.0174499  0.0235409  -0.741    0.460

Residual standard error: 0.08103 on 177 degrees of freedom
Multiple R-squared:  0.003103,  Adjusted R-squared:  -0.008162
F-statistic: 0.2754 on 2 and 177 DF,  p-value: 0.7596
```

```
Residuals:
     Min       1Q   Median       3Q      Max 
-0.27168 -0.04960 -0.00983  0.03762  0.35543 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 0.003004   0.006044   0.497     0.62    
rp_mkt      1.283118   0.127866  10.035   <2e-16 ***

Diagnostic tests:
                 df1 df2 statistic p-value    
Weak instruments   2 177   951.262  <2e-16 ***
Wu-Hausman         1 177     4.862  0.0287 *  
Sargan             1  NA     0.558  0.4549    
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.08093 on 178 degrees of freedom
Multiple R-Squared: 0.3507, Adjusted R-squared: 0.347 
Wald test: 100.7 on 1 and 178 DF,  p-value: < 2.2e-16
```

Question 24

a.

The residual plot does not exhibit any clear pattern of increasing or decreasing spread with experience, suggesting that the residuals are generally consistent with the assumption of homoskedasticity.



b.

The null hypothesis (H_0) states that there is homoskedasticity (constant variance).

Since the Breusch-Pagan test produces a statistic of **7.44** with a **p-value of 0.0064** (which is less than 0.01), we reject the null hypothesis of homoskedasticity. This indicates evidence of heteroskedasticity in the IV residuals.

```
> cat("Breusch-Pagan test statistic (NR²):", bp_stat, "\n")
Breusch-Pagan test statistic (NR²): 7.438552
> cat("p-value:", p_val, "\n")
p-value: 0.006384122
```

c.

The **robust standard error is slightly larger**, which is expected under heteroskedasticity (0.0333 > 0.0314). The confidence interval for educ is slightly **wider** using heteroskedasticity-robust standard errors (−0.0041 to 0.1269) than with baseline standard errors (−0.0004 to 0.1232), reflecting the adjustment for potential heteroskedasticity in the IV model.

d.

The bootstrap standard error for **educ** is **0.0323**, which is slightly larger than the baseline standard error (**0.0314**) but slightly smaller than the robust standard error (**0.0333**). Using the bootstrap standard error and the original model's coefficient (**0.0614**), the 95% confidence interval is approximately **[−0.0020, 0.1248]**.

For all coefficients, the bootstrap standard errors are slightly larger than the baseline SEs and comparable to the robust SEs. Specifically, for **educ**, the bootstrap SE (**0.0323**) is slightly higher than the baseline SE (**0.0314**) but slightly lower than the robust SE (**0.0333**). This suggests consistent but slightly more conservative inference under heteroskedasticity.