1、

由共變異數矩陣可知 $var(b_1)=3$ $var(b_2)=4$ $var(b_3)=3$

a、

$H_0: \beta_2=0$ vs. $H_1: \beta_2 \neq 0$ $\varphi=\frac{b_2-0}{SE(b_2)} \sim t(63-3-1)$

$RR: \{\varphi \mid \varphi > t_{0.025}(59) \doteq z_{0.025}=1.96\}$

$\varphi^* = \frac{3-0}{\sqrt{4}} = 1.5 \notin RR$

無法拒絕虛無假設，無證據支持 $\beta_2=0$

b、

$H_0: \beta_1+2\beta_2=5$ vs. $H_1: \beta_1+2\beta_2 \neq 5$

$\varphi=\frac{b_1+2b_2-5}{SE(b_1+2b_2)} \sim t(63-3-1)$ $RR: \{\varphi \mid \varphi > t_{0.025}(59) \doteq z_{0.025}=1.96\}$

其中 $SE(b_1+2b_2)=\sqrt{var(b_1+2b_2)}=\sqrt{3+4\times4+2\times2\times(-2)}=\sqrt{11}$

$\varphi^* = \frac{2+3\times2-5}{\sqrt{11}} = 0.9045 \notin RR$

無法拒絕虛無假設，無證據支持 $\beta_1+2\beta_2 \neq 5$

C、

$H_0: \beta_1 - \beta_2 + \beta_3 = 4$ vs. $H_1: \beta_1 - \beta_2 + \beta_3 \neq 4$

$\psi = \dfrac{b_1 - b_2 + b_3 - 4}{SE(b_1 - b_2 + b_3)} \sim t(63-3-1)$ $RR: \{\psi \mid |\psi| > t_{0.025}(59) \approx z_{0.025} = 1.96\}$

其中 $SE(b_1 - b_2 + b_3) = \sqrt{var(b_1 - b_2 + b_3)} = \sqrt{8}$

$\psi^* = \dfrac{2 - 3 + (-1) - 4}{\sqrt{8}} = -2.1213 \in RR$

拒絕虛無假設，有證據支持 $\beta_1 - \beta_2 + \beta_3 \neq 4$

2-

a.

```
> model1 <- lm(time~depart+reds+trains, commute5)
> smodel1 <- summary(model1)
> smodel1

Call:
lm(formula = time ~ depart + reds + trains, data = commute5)

Residuals:
     Min       1Q   Median       3Q      Max 
-18.4389  -3.6774  -0.1188   4.5863  16.4986 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)  20.8701     1.6758  12.454  < 2e-16 ***
depart        0.3681     0.0351  10.487  < 2e-16 ***
reds          1.5219     0.1850   8.225 1.15e-14 ***
trains        3.0237     0.6340   4.769 3.18e-06 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 6.299 on 245 degrees of freedom
Multiple R-squared:  0.5346,	Adjusted R-squared:  0.5289 
F-statistic: 93.79 on 3 and 245 DF,  p-value: < 2.2e-16
```

令 $\hat{y}_i = b_0 + b_1 depart + b_2 reds + b_3 trains$

b_0：當Bill準時6:30出發，且路上沒遇到紅燈，且Bill不用等火車時，以平均來說，通勤時間為20.8701分鐘

b_1：當Bill每晚1分出發，控制其他條件不變下，以平均來說Bill的通勤時間會增加0.3681分鐘

b_2：當Bill每多遇到1個紅燈，控制其他條件不變下，以平均來說，Bill的通勤時間會增加1.5219分鐘

b_3：當Bill每多等一班火車，控制其他條件不變下，以平均來說，Bill的通勤時間會增加3.0237分鐘

b.

```
> confint(model1,  ,level = 0.95)
                2.5 %    97.5 %
(Intercept) 17.5694018 24.170871
depart       0.2989851  0.437265
reds         1.1574748  1.886411
trains       1.7748867  4.272505
> 
```

都有落在區間估計裡

c.

$H_0: \beta_2 \geq 2$ vs. $H_1: \beta_2 < 2$ $\quad \psi = \dfrac{b_2 - 2}{SE(b_2)} \sim t(249-3-1)$

$RR: \{\psi \mid \psi > -t_{0.05}(245) \approx z_{0.05} = 1.645\}$

$\psi^* = \dfrac{1.5219-2}{0.1850} = -2.5843 \in RR$

拒絕虛無假設，有證據支持 $\beta_2 < 2$

d.

$H_0: \beta_3 = 3$ vs. $H_1: \beta_3 \neq 3$ $\quad \psi = \dfrac{b_3 - 3}{SE(b_3)} \sim t(249-3-1)$

$RR: \{\psi \mid \psi > t_{0.05}(245) \doteq z_{0.05} = 1.645 \text{ or } \psi < -t_{0.05}(245) \doteq -z_{0.05} = -1.645\}$

$\psi^* = \dfrac{3.0237-3}{0.634} = 0.0374 \notin RR$

無法拒絕虛無假設，無證據支持 $\beta_3 \neq 3$

e.

$H_0: 30\cdot\beta_1 \geq 10 \Rightarrow H_0: \beta_1 \geq \frac{1}{3}$ vs. $H_1: \beta_1 < \frac{1}{3}$

$\varphi = \frac{b_1 - \frac{1}{3}}{SE(b_1)} \sim t_{0.05}(249-3-1)$ RR: $\{\varphi \mid \varphi < -t_{0.05}(245) \fallingdotseq z_{0.05} = -1.645\}$

$\varphi^* = 0.9905 \notin RR$

無法拒絕虛無假設，無證據支持 $\beta_1 < \frac{1}{3}$

f.

$H_0: \beta_3 \geq 3\beta_2$ vs. $H_1: \beta_3 < 3\beta_2$

$\Rightarrow H_0: \beta_3 - 3\beta_2 \geq 0$ vs. $H_1: \beta_3 - 3\beta_2 < 0$

$\varphi = \frac{b_3 - 3b_2 - 0}{SE(b_3 - 3b_2)} \sim t(249-3-1)$ RR: $\{\varphi \mid \varphi < -t_{0.05}(245) \fallingdotseq -z_{0.05} = -1.645\}$

由共變異數矩陣可知 $var(b_2) = 0.0342$ $var(b_3) = 0.4020$

$cov(b_2, b_3) = -0.0006$

$SE(b_2 - 3b_2) = \sqrt{var(b_2 - 3b_2)} = 0.0342 + 9 \times 0.402 - 2 \times 3 \times (-0.0006)$

$= \sqrt{3.6558} = 1.912$

$\varphi^* = \frac{1.5219 - 3 \times 3.0237}{1.912} = -3.9483 \in RR$

拒絕虛無假設，有證據支持 $\beta_3 - 3\beta_2 < 0$

g、

$H_0: E(TIME|X) \le 45$ vs. $H_1: E(TIME|X) > 45$

令 $\widehat{TIME} = \hat{\theta} = \hat{b}_0 + 30\hat{b}_1 + 6\hat{b}_2 + 1\hat{b}_3 = 44.0682$

$$\varphi = \frac{\hat{\theta} - 45}{SE(\hat{\theta})} \sim t(249-3-1)$$

$RR: \{\varphi \mid \varphi > t_{0.05}(245) \doteqdot Z_{0.05} = 1.645\}$

$$SE(\hat{\theta}) = \sqrt{Var(\hat{\theta})} = \sqrt{Var(\hat{b}_0 + 30\hat{b}_1 + 6\hat{b}_2 + \hat{b}_3)}$$

$$= \sqrt{0.2784} = 0.5276$$

$$\varphi^* = \frac{44.0682 - 45}{0.5276} = -1.7661 \notin RR$$

無法拒絕虛無假設，無證據支持 $E(TIME|X) > 45$

f、

g小題的假設是合理的 ∵ $E(TIME|X) \le 45$ 代表不會遲到

通常會把要證明的放在虛無假設

反過來可能會犯 Type I error

3.

a.

```
> model1 <- lm(log(wage)~educ+I(educ^2)+exper+I(exper^2)+I(educ*exper), cps5_smal
l)
> smodel1 <- summary(model1)
> smodel1

Call:
lm(formula = log(wage) ~ educ + I(educ^2) + exper + I(exper^2) + 
    I(educ * exper), data = cps5_small)

Residuals:
    Min      1Q  Median      3Q     Max 
-1.6628 -0.3138 -0.0276  0.3140  2.1394 

Coefficients:
                  Estimate Std. Error t value Pr(>|t|)    
(Intercept)      1.038e+00  2.757e-01   3.764 0.000175 ***
educ             8.954e-02  3.108e-02   2.881 0.004038 ** 
I(educ^2)        1.458e-03  9.242e-04   1.578 0.114855    
exper            4.488e-02  7.297e-03   6.150 1.06e-09 ***
I(exper^2)      -4.680e-04  7.601e-05  -6.157 1.01e-09 ***
I(educ * exper) -1.010e-03  3.791e-04  -2.665 0.007803 ** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.4638 on 1194 degrees of freedom
Multiple R-squared:  0.3227,	Adjusted R-squared:  0.3198 
F-statistic: 113.8 on 5 and 1194 DF,  p-value: < 2.2e-16
```

*** 表示 0.1% 1% 5% 都顯著

** 表示 1% 5% 顯著

無星號表示不顯著

b.

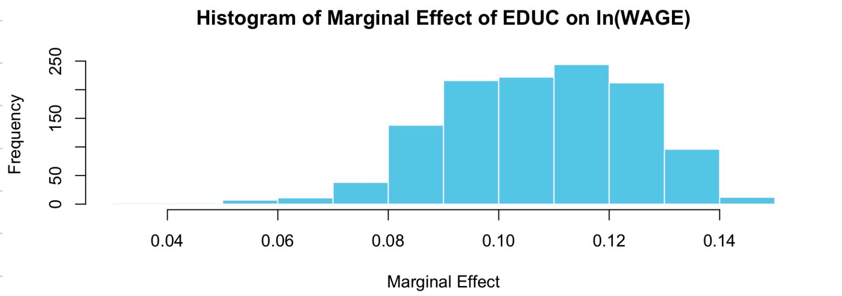
$$\ln(wage) = \beta_1 + \beta_2 educ + \beta_3 educ^2 + \beta_4 exper + \beta_5 exper^2 + \beta_6 \times (educ \times exper) + e$$

$$\frac{\partial \ln(wage)}{\partial educ} = \beta_2 + 2\beta_3 educ + \beta_6 exper$$

當 educ、exper 都增加，educ 的邊際效果會因為交乘項(β_6)而減少

c.

```
> cat("median marginal effect of educ:", round(median_effect, 4))
median marginal effect of educ: 0.1084
> cat("5th percentile:", round(p5_effect, 4))
5th percentile: 0.0801
> cat("95th percentile:", round(p95_effect, 4))
95th percentile: 0.1336
```



大部份人的邊際效果落在8.01%~13.36%(log(wage)可近似成%)
直方圖成右偏分配，代表少部份人有特別高的影響

d.

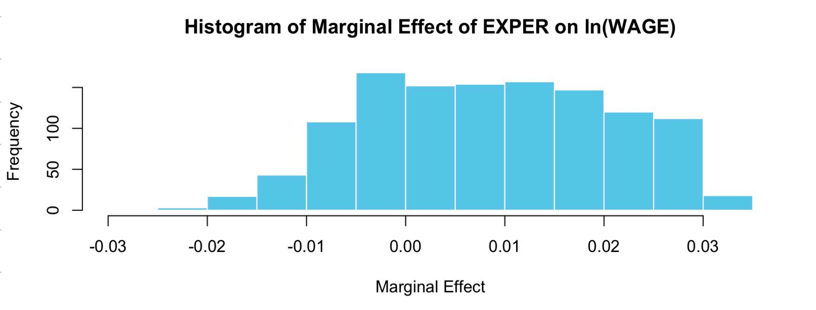
$$\ln(wage)=\beta_1+\beta_2 educ+\beta_3 educ^2+\beta_4 exper+\beta_5 exper^2+\beta_6 \times(educ\times exper)+e$$

$$\frac{\partial \ln(wage)}{\partial exper}=\beta_4+2\beta_5 exper+\beta_6 \times educ$$

當educ、exper都增加，exper的邊際效果會因為交乘項(β_6)而減少

e.

```
> median_effect_exper <- median(marginal_exper, TRUE)
> p5_effect_exper <- quantile(marginal_exper, 0.05, TRUE)
> p95_effect_exper <- quantile(marginal_exper, 0.95, TRUE)
> cat("median marginal effect of exper:", round(median_effect_exper, 4))
median marginal effect of exper: 0.0084
> cat("5th percentile of exper:", round(p5_effect_exper, 4))
5th percentile of exper: -0.0104
> cat("95th percentile of exper:", round(p95_effect_exper, 4))
95th percentile of exper: 0.0279
```



邊際效果偏左代表有部份人額外增加經驗會導致邊際報酬為負

5th percentile 為負表示經驗愈多，薪資成長可能下降

f

```
> # 建立模型向量 (intercept, EDUC, EDUC^2, EXPER, EXPER^2, EDUC*EXPER)
> x_david <- c(1, 17, 17^2, 8, 8^2, 17*8)
> x_svet  <- c(1, 16, 16^2, 18, 18^2, 16*18)
> # 差值向量 (x_d - x_s)
> x_diff <- x_david - x_svet
> # 提取模型係數與共變異數矩陣
> b <- coef(model1)
> vcov_mat <- vcov(model1)
> # 計算期望 log-wage 差異 (估計值)
> delta_hat <- sum(x_diff * b)
> # 計算標準誤
> se_delta <- sqrt(t(x_diff) %*% vcov_mat %*% x_diff)
> # 計算 t 統計量
> t_stat <- delta_hat / se_delta
> # 印出檢定統計量與結論
> cat("Estimated difference (David - Svetlana):", round(delta_hat, 4), "\n")
Estimated difference (David - Svetlana): -0.0359
> cat("Standard error of difference:", round(se_delta, 4), "\n")
Standard error of difference: 0.0215
> cat("t-statistic:", round(t_stat, 4), "\n")
t-statistic: -1.6699
> # 臨界值 (one-tailed test, 5%)
> crit_value <- qt(0.95, df = model1$df.residual)
> cat("Critical value (5% level):", round(crit_value, 4), "\n")
Critical value (5% level): 1.6461
> if (t_stat > crit_value) {
+   cat("Conclusion: Reject H0. David is expected to earn more.\n")
+ } else {
+   cat("Conclusion: Fail to reject H0. Not enough evidence that David earns mor
e.\n")
+ }
Conclusion: Fail to reject H0. Not enough evidence that David earns more.
```

f.

不一定一樣；模型是非線性的且增加的經驗不一定會提升相同幅度

```
> # g.
> x_david <- c(1, 17, 17^2, 16, 16^2, 17*16)
> x_svet  <- c(1, 16, 16^2, 26, 26^2, 16*26)
> # 差值向量 (x_d - x_s)
> x_diff <- x_david - x_svet
> # 提取模型係數與共變異數矩陣
> b <- coef(model1)
> vcov_mat <- vcov(model1)
> # 計算期望 log-wage 差異 (估計值)
> delta_hat <- sum(x_diff * b)
> # 計算標準誤
> se_delta <- sqrt(t(x_diff) %*% vcov_mat %*% x_diff)
> # 計算 t 統計量
> t_stat <- delta_hat / se_delta
> # 印出檢定統計量與結論
> cat("Estimated difference (David - Svetlana):", round(delta_hat, 4), "\n")
Estimated difference (David - Svetlana): 0.0309
> cat("Standard error of difference:", round(se_delta, 4), "\n")
Standard error of difference: 0.015
> cat("t-statistic:", round(t_stat, 4), "\n")
t-statistic: 2.0624
> # 臨界值 (one-tailed test, 5%)
> crit_value <- qt(0.95, df = model1$df.residual)
> cat("Critical value (5% level):", round(crit_value, 4), "\n")
Critical value (5% level): 1.6461
> if (t_stat > crit_value) {
+   cat("Conclusion: Reject H0. David is expected to earn more.\n")
+ } else {
+   cat("Conclusion: Fail to reject H0. Not enough evidence that David earns mor
e.\n")
+ }
Conclusion: Reject H0. David is expected to earn more.
```

h.

```
> # Wendy: 12 educ, 17 exper → 0*intercept + 0*educ + 0*educ2 + 1*exper + 2*17*exp
er2 + 12*educ*exper
> c_wendy <- c(0, 0, 0, 1, 2*17, 12)
> # Jill: 16 educ, 1 exper → 0*intercept + 0*educ + 0*educ2 + 1*exper + 2*1*exper2
+ 16*educ*exper
> c_jill  <- c(0, 0, 0, 1, 2*1, 16)
> # 差值 (H0: 差 = 0)
> c_diff <- c_wendy - c_jill
> # 模型係數與共變異數矩陣
> b <- coef(model1)
> vcov_mat <- vcov(model1)
> # 差異估計值
> delta_hat <- sum(c_diff * b)
> # 標準誤
> se_delta <- sqrt(t(c_diff) %*% vcov_mat %*% c_diff)
> # t 統計量
> t_stat <- delta_hat / se_delta
> # 臨界值
> crit_val <- qt(0.975, df = model1$df.residual)  # 兩尾檢定
> # 輸出
> cat("Estimated difference in marginal effects:", round(delta_hat, 4), "\n")
Estimated difference in marginal effects: -0.0109
> cat("Standard error:", round(se_delta, 4), "\n")
Standard error: 0.0025
> cat("t-statistic:", round(t_stat, 4), "\n")
t-statistic: -4.4146
> cat("Critical value (5% level): ±", round(crit_val, 4), "\n")
Critical value (5% level): ± 1.962
> if (abs(t_stat) > crit_val) {
+   cat("Conclusion: Reject H0. Marginal effects are significantly different.\n")
+ } else {
+   cat("Conclusion: Fail to reject H0. No significant difference in marginal effe
cts.\n")
+ }
Conclusion: Reject H0. Marginal effects are significantly different.
```

i.

```
> b4 <- coef(model1)[4]
> b5 <- coef(model1)[5]
> b6 <- coef(model1)[6]
> # 計算臨界年數 (使 Jill 的 marginal effect = 0)
> x_crit <- - (b4 + b6 * 16) / (2 * b5)
> cat("Point estimate of experience when marginal effect becomes negative:", round
(x_crit, 2), "years\n")
Point estimate of experience when marginal effect becomes negative: 30.68 years
> d_b4 <- -1 / (2 * b5)
> d_b5 <- (b4 + 16 * b6) / (2 * b5^2)
> d_b6 <- -16 / (2 * b5)
> # 將這三個導數組成梯度向量
> grad <- c(d_b4, d_b5, d_b6)
> # 取出這三個參數的變異數-共變異數矩陣
> vcov_sub <- vcov(model1)[c("exper", "I(exper^2)", "I(educ * exper)"),c("exper",
"I(exper^2)", "I(educ * exper)") ]
> vcov_sub
                        exper    I(exper^2) I(educ * exper)
exper            5.325294e-05 -3.903252e-07   -2.396544e-06
I(exper^2)      -3.903252e-07  5.777667e-09    8.121608e-09
I(educ * exper) -2.396544e-06  8.121608e-09    1.436988e-07
> # delta method 計算標準誤
> se_xcrit <- sqrt(t(grad) %*% vcov_sub %*% grad)
> # 95% 信賴區間
> crit_val <- qt(0.975, df = model1$df.residual)
> lower_bound <- x_crit - crit_val * se_xcrit
> upper_bound <- x_crit + crit_val * se_xcrit
> cat("95% confidence interval for turning point: [", round(lower_bound, 2), ",",
round(upper_bound, 2), "] years\n")
95% confidence interval for turning point: [ 26.96 , 34.4 ] years
```