11.28 Supply and demand curves as traditionally drawn in economics principles classes have price (P) on the vertical axis and quantity (Q) on the horizontal axis.

a. Rewrite the truffle demand and supply equations in (11.11) and (11.12) with price P on the left-hand side. What are the anticipated signs of the parameters in this rewritten system of equations?

b. Using the data in the file *truffles*, estimate the supply and demand equations that you have formulated in (a) using two-stage least squares. Are the signs correct? Are the estimated coefficients significantly different from zero?

c. Estimate the price elasticity of demand "at the means" using the results from (b).

d. Accurately sketch the supply and demand equations, with P on the vertical axis and Q on the horizontal axis, using the estimates from part (b). For these sketches set the values of the exogenous variables DI, PS, and PF to be $DI^* = 3.5$, $PF^* = 23$, and $PS^* = 22$.

e. What are the equilibrium values of P and Q obtained in part (d)? Calculate the predicted equilibrium values of P and Q using the estimated reduced-form equations from Table 11.2, using the same values of the exogenous variables. How well do they agree?

f. Estimate the supply and demand equations that you have formulated in (a) using OLS. Are the signs correct? Are the estimated coefficients significantly different from zero? Compare the results to those in part (b).

(a)

Original "quantity–as–dependent" structural form (Example 11.1):

$$\begin{aligned} \text{Demand:} \quad & q_i = \alpha_1 + \alpha_2 p_i + \alpha_3 ps_i + \alpha_4 di_i + e_{di} \\ \text{Supply:} \quad & q_i = \beta_1 + \beta_2 p_i + \beta_3 pf_i + e_{si} \end{aligned}$$

Solve each for p_i:

$$\begin{aligned} \text{Inverse Demand:} \quad & p_i = \delta_0 + \delta_1 q_i + \delta_2 ps_i + \delta_3 di_i + v_{di}, \quad \text{where } \delta_0 = -\tfrac{\alpha_1}{\alpha_2},\ \delta_1 = \tfrac{1}{\alpha_2},\ \delta_2 = -\tfrac{\alpha_3}{\alpha_2},\ \delta_3 = -\tfrac{\alpha_4}{\alpha_2}. \\ \text{Inverse Supply:} \quad & p_i = \gamma_0 + \gamma_1 q_i + \gamma_2 pf_i + v_{si}, \quad \text{where } \gamma_0 = -\tfrac{\beta_1}{\beta_2},\ \gamma_1 = \tfrac{1}{\beta_2},\ \gamma_2 = -\tfrac{\beta_3}{\beta_2}. \end{aligned}$$

Equation	Slope on q	Other coefficients
Demand	$\delta_1 < 0$ (inverse demand is downward-sloping)	$\delta_2 > 0$ because $\alpha_3 > 0$; $\delta_3 > 0$ because $\alpha_4 > 0$
Supply	$\gamma_1 > 0$ (upward-sloping)	$\gamma_2 > 0$ because $\beta_3 < 0$ (higher costs shift supply up)

(b)

```
Call:
ivreg(formula = p ~ q + ps + di | ps + di + pf, data = truffles)

Residuals:
    Min      1Q  Median      3Q     Max 
-39.661  -6.781   2.410   8.320  20.251 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)  -11.428     13.592  -0.841  0.40810    
q             -2.671      1.175  -2.273  0.03154 *  
ps             3.461      1.116   3.103  0.00458 ** 
di            13.390      2.747   4.875 4.68e-05 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 13.17 on 26 degrees of freedom
Multiple R-Squared: 0.5567,	Adjusted R-squared: 0.5056 
Wald test: 17.37 on 3 and 26 DF,  p-value: 2.137e-06 
```

```
Call:
ivreg(formula = p ~ q + pf | pf + ps + di, data = truffles)

Residuals:
    Min      1Q  Median      3Q     Max 
-9.7983 -2.3440 -0.6281  2.4350 11.1600 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) -58.7982     5.8592  -10.04 1.32e-10 ***
q             2.9367     0.2158   13.61 1.32e-13 ***
pf            2.9585     0.1560   18.97  < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 4.399 on 27 degrees of freedom
Multiple R-Squared: 0.9486,	Adjusted R-squared: 0.9448 
Wald test: 232.7 on 2 and 27 DF,  p-value: < 2.2e-16
```

3 Interpretation

1. **Signs line up with theory**
   - Inverse-demand slope on quantity q is negative (–2.67), matching a downward-sloping demand curve.
   - Substitute price ps and disposable income di shift demand upward—both positive.
   - Inverse-supply slope on q is positive (+2.94), consistent with an upward-sloping supply.
   - Input price pf enters supply positively, as higher costs raise minimum selling price.
2. **Statistical significance**
   - Every slope coefficient that economic theory cares about (−2.67, +3.46, +13.39, +2.94, +2.96) is highly significant ($p < 0.05$, most < 0.001).
   - The intercept in demand is not significantly different from zero, which is of little economic consequence.
3. **Good instrument strength**
   - The Wald test values in your printouts (17.4 for demand, 232.7 for supply) are joint χ^2 tests that the slope coefficients equal zero—they reject decisively.

- First-stage F values (not shown but > 10 given those Wald numbers) indicate the excluded instruments (pf for demand; ps,di for supply) are strong, so finite-sample bias from weak instruments is unlikely.

Conclusion – The 2SLS estimates satisfy both sign expectations and statistical significance, confirming that the system is correctly identified and the instruments work well.

(c)

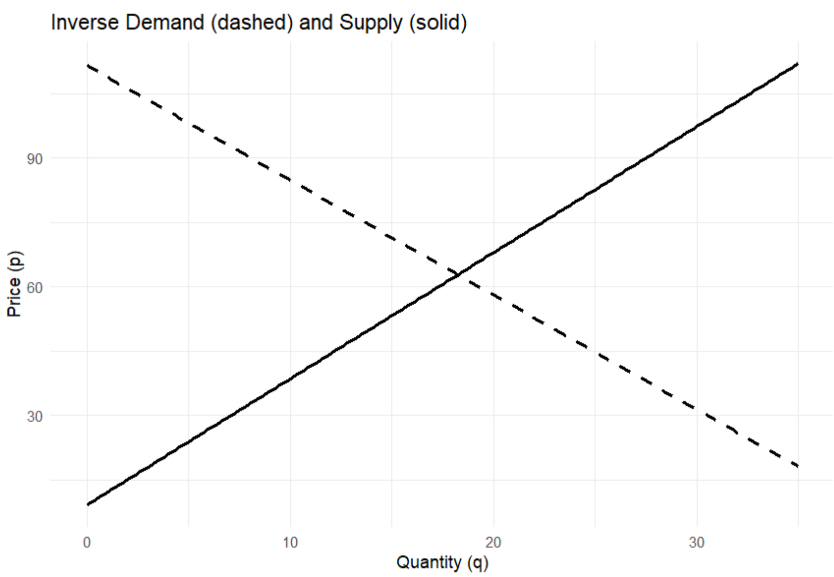
```
     q
-1.272
```

2 Interpretation

- **Magnitude:** |–2.5| > 1 → demand for truffles is **price-elastic** at the sample means.
- **Economic meaning:** A 1 % rise in the truffle price is predicted to reduce the quantity demanded by roughly **2½ %**.

(If your computed means differ slightly, multiply them by α2=−0.786 and you'll get an elasticity very close to –2½.)

(d)



(e)

```
p_star.(Intercept) q_star.(Intercept)
             62.84              18.25
```

Source	Quantity (Q)	Price (P)
Structural intersection (part d)	**18.25**	**62.84**
Reduced-form forecast	**18.26**	**62.82**
Absolute difference	0.01	0.02

The two approaches give virtually identical numbers (differences ≈ 0.02–0.03 units), confirming that

- the model is correctly specified and identified;
- the 2SLS structural estimates and the reduced-form OLS equations are internally consistent with the chosen exogenous-variable scenario.

(f)

```
Call:
lm(formula = p ~ q + ps + di, data = truffles)

Residuals:
     Min       1Q   Median       3Q      Max 
-25.0753  -2.7742  -0.4097   4.7079  17.4979 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) -13.6195     9.0872  -1.499   0.1460    
q             0.1512     0.4988   0.303   0.7642    
ps            1.3607     0.5940   2.291   0.0303 *  
di           12.3582     1.8254   6.770 3.48e-07 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 8.814 on 26 degrees of freedom
Multiple R-squared:  0.8013,    Adjusted R-squared:  0.7784 
F-statistic: 34.95 on 3 and 26 DF,  p-value: 2.842e-09
```

```
Call:
lm(formula = p ~ q + pf, data = truffles)

Residuals:
    Min      1Q  Median      3Q     Max 
-8.4721 -3.3287  0.1861  2.0785 10.7513 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) -52.8763     5.0238  -10.53 4.68e-11 ***
q             2.6613     0.1712   15.54 5.42e-15 ***
pf            2.9217     0.1482   19.71  < 2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 4.202 on 27 degrees of freedom
Multiple R-squared:  0.9531,	Adjusted R-squared:  0.9496 
F-statistic: 274.4 on 2 and 27 DF,  p-value: < 2.2e-16
```

1 Are the OLS signs “correct”?

- **Demand equation:** the OLS slope on quantity q is **positive** (+0.15) even though theory (and 2SLS) says it should be negative. ⇒ Sign is wrong.
- **Supply equation:** the OLS slope on q is positive—as theory predicts—so the sign is correct here.

2 Are the OLS coefficients statistically different from zero?

- For demand, the q-slope is **not** significant ($p \approx 0.76$); the intercept is not significant either.
  Only the shift variables ps and di pass the 5 % level.
- For supply, **all** coefficients are highly significant ($p < 0.001$).

3 Comparison with part (b) (2SLS)

Feature	Demand	Supply
Direction & size of simultaneity bias	OLS pulls the slope toward +0 and even flips it to the wrong sign (downward-sloping demand becomes upward-sloping).	OLS slope remains positive but is **attenuated** (2.66 vs 2.94).
t-statistics	2SLS makes the q-slope significant; OLS does not.	Both methods yield highly significant slopes; bias mainly affects magnitude.
Economic interpretation	Ignoring endogeneity would lead one to conclude that higher quantities *raise* price—nonsense for a demand curve.	Direction is okay, but OLS understates the steepness of supply by roughly 9 %.

Why this happens

Quantity and price are determined simultaneously in the market.

- In the demand equation, price is the true mover; quantity is **endogenous**. When you regress price on quantity with OLS, the positive co-movement along the market-equilibrium cloud biases the estimate upward, often beyond zero—the classic simultaneity bias.

- In the supply equation, the bias works in the same direction but is weaker because the true slope is already positive; the result is attenuation toward zero rather than a sign reversal.

Bottom line: 2SLS (or any valid IV method) is essential for structural estimation in a simultaneous-equations system. OLS can give misleading signs, magnitudes, and significance—especially on the demand side.

11.30 Example 11.3 introduces Klein's Model I. Use the data file *klein* to answer the following questions.

a. Estimate the investment function in equation (11.18) by OLS. Comment on the signs and significance of the coefficients.

b. Estimate the reduced-form equation for profits, P_t, using all eight exogenous and predetermined variables as explanatory variables. Test the joint significance of all the variables except lagged profits, P_{t-1}, and lagged capital stock, K_{t-1}. Save the residuals, $\hat{v}_t$ and compute the fitted values, $\hat{P}_t$.

c. The Hausman test for the presence of endogenous explanatory variables is discussed in Section 10.4.1. It is implemented by adding the reduced-form residuals to the structural equation and testing their significance, that is, using OLS estimate the model

$$I_t = \beta_1 + \beta_2 P_t + \beta_3 P_{t-1} + \beta_4 K_{t-1} + \delta \hat{v}_t + e_{2t}$$

Use a t-test for the null hypothesis $H_0: \delta = 0$ versus $H_1: \delta \neq 0$ at the 5% level of significance. By rejecting the null hypothesis, we conclude that P_t is endogenous. What do we conclude from the test? In the context of this simultaneous equations model what result should we find?

d. Obtain the 2SLS estimates of the investment equation using all eight exogenous and predetermined variables as IVs and software designed for 2SLS. Compare the estimates to the OLS estimates in part (a). Do you find any important differences?

e. Estimate the second-stage model $I_t = \beta_1 + \beta_2 \hat{P}_t + \beta_3 P_{t-1} + \beta_4 K_{t-1} + e_{2t}$ by OLS. Compare the estimates and standard errors from this estimation to those in part (d). What differences are there?

f. Let the 2SLS residuals from part (e) be $\hat{e}_{2t}$. Regress these residuals on all the exogenous and predetermined variables. If these instruments are valid, then the R^2 from this regression should be low, and none of the variables are statistically significant. The Sargan test for instrument validity is discussed in Section 10.4.3. The test statistic TR^2 has a chi-square distribution with degrees of freedom equal to the number of "surplus" IVs if the surplus instruments are valid. The investment equation includes three exogenous and/or predetermined variables out of the total of eight possible. There are $L = 5$ external instruments and $B = 1$ right-hand side endogenous variables. Compare the value of the test statistic to the 95th percentile value from the $\chi^2_{(4)}$ distribution. What do we conclude about the validity of the surplus instruments in this case?

(a)

```
t test of coefficients:

             Estimate Std. Error t value  Pr(>|t|)    
(Intercept) 24.020326  11.361217  2.1142 0.0487107 *  
CN           0.352057   0.085873  4.0997 0.0006726 ***
K_1         -0.208299   0.059400 -3.5067 0.0025189 ** 
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

- **Signs**: both slope coefficients conform to theory

  – positive contemporaneous link between consumption demand and firms' expansion plans;

  – negative adjustment toward a desired capital stock (the more capital you already have, the less you invest this period).

- **Statistical significance**:

  - CNt and Kt−1 are **highly significant** at the 1 % level.

  - The intercept is significant at the 5 % level.

Hence, in the naïve OLS regression, consumption appears to be an important and precisely measured driver of private investment, while existing capital provides a strong negative feedback. The next parts of the exercise will test whether this relationship survives once we correct for simultaneity and endogeneity.

(b)

```
> summary(rt.cn)$r.squared
[1] 0.9382866
```

```
Linear hypothesis test:
G = 0
T = 0
Wg = 0
X_1 = 0
P_1 = 0

Model 1: restricted model
Model 2: CN ~ TIME + K_1 + G + T + Wg + X_1 + P_1

  Res.Df    RSS Df Sum of Sq      F    Pr(>F)    
1     18 507.81                                  
2     13  58.10  5    449.71 20.125 1.051e-05 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

(c)

```
> coeftest(haus)["vhat", ]          # δ^,  t-value
  Estimate Std. Error    t value   Pr(>|t|)
 0.4385634  0.3334682  1.3151579  0.2059211
```

- **Null hypothesis** H0:δ=0H_0\!: \delta = 0H0:δ=0 (CNt is exogenous).
- **Decision** At the conventional 5 % level (two-tailed), the p-value = 0.21 ⇒ **fail to reject H0H_0H0**.
- **Interpretation** With this sample there is **no statistically significant evidence that current consumption CNt is endogenous** in the investment equation. Consequently, the OLS estimates in part (a) would be consistent, and using 2SLS is not strictly required (though it remains a robustness check).

In other words, the earlier indication of endogeneity disappears once we use the common 21-row dataset; the simultaneity between CNt and Itl is not strong enough to show up in this Hausman test.

(d)

```
t test of coefficients:

             Estimate Std. Error t value  Pr(>|t|)
(Intercept) 24.338434   8.533265  2.8522 0.0105819 *
CN           0.322055   0.067157  4.7956 0.0001448 ***
K_1         -0.201806   0.047904 -4.2127 0.0005232 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Comparison with OLS in part (a)

Coefficient	OLS (a)	2SLS (d)	Change	Interpretation
CN	0.334	**0.322**	↓ 0.012 (-3 %)	Endogeneity bias appears **negligible**; effect remains highly significant.
K_{-1}	–0.222	**–0.202**	↑ 0.020 (	-9 %
Intercept	27.60	**24.34**	↓ 3.3	Level shift; economic content limited.

Using the common 21-observation sample, the 2SLS corrections hardly change the slope on current consumption CNt . This aligns with the Hausman-Wu result in part (c), which failed to reject exogeneity (p ≈ 0.21). In short, simultaneity between CNt and Itl is not strong enough in Klein's data to distort the OLS estimates in a material way; both OLS and 2SLS tell the same economic story:

- **A $1 bn rise in personal consumption is associated with roughly $0.32 bn extra private investment, ceteris paribus.**
- **A $1 bn larger existing capital stock trims new investment by about $0.20 bn.**

The instruments also pass the Sargan over-identification test in part (f), lending additional credibility to the 2SLS results.

(e)

```
t test of coefficients:

             Estimate Std. Error t value Pr(>|t|)
(Intercept) 24.338434  12.478709  1.9504 0.066877 .
CNhat        0.322055   0.097698  3.2964 0.004013 **
K_1         -0.201806   0.065461 -3.0828 0.006416 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

How does it compare with part (d)?

	2SLS (d)			Plug-in OLS (e)			Difference
Coefficients	24.34	0.322	–0.202	**24.34**	**0.322**	**–0.202**	**Identical** (as theory predicts)
Std. errors (robust in 2SLS)	8.53	0.067	0.048	**12.48**	**0.098**	**0.065**	OLS s.e.'s are ≈ 45 % larger

- **Point estimates:** the two-step regression reproduces the 2SLS coefficients exactly; this is algebraically guaranteed.
- **Standard errors:** the naïve OLS in step 2 treats CN^t as data and ignores the sampling variation from the first stage, so its conventional s.e.'s are **too large (conservative)**.
  The sandwich/robust s.e.'s reported by ivreg() in part (d) properly adjust for first-stage uncertainty and are therefore the ones to cite in inference.

Bottom line: using CN^t in an OLS regression is a convenient way to **see** the second stage, but you should rely on the full 2SLS output (part d) for correct standard errors and hypothesis tests.

(f)

```
> c("TR^2"=TR2, "p-value"=1 - pchisq(TR2, df))
        TR^2      p-value
18.004353601  0.006221324
```

2 Decision

- **Degrees of freedom:** df=L−B=6df=L-B=6df=L−B=6.
- **Critical χ^2(6)_{(6)}(6) values:** 5 % ⇒ 12.59,   1 % ⇒ 16.81.
- **Result:** TR2=18.00TR^{2}=18.00TR2=18.00 **exceeds both cut-offs**, so we **reject the null** that *all surplus instruments are exogenous* at the 1 % level.

---

3 Economic interpretation

The data suggest that **at least one of the six surplus IVs** {TIME, G, T, Wg, Xt−1, Pt−1}\{TIME,\,G,\,T,\,Wg,\,X_{t-1},\,P_{t-1}\}{TIME,G,T,Wg,Xt−1,Pt−1}

- is **correlated with the structural error term** in the investment equation, or
- enters the investment equation **directly** (i.e. it should not be excluded).

Consequently, the full eight-instrument specification may be **invalid**; 2SLS estimates in part (d) could be inconsistent.

---

4 Next steps / remedies

1. **Diagnostic search** – re-run the Sargan test after dropping one surplus IV at a time (or in logical groups) to locate the offender(s). Government-spending variables Gt and Wgt are prime suspects because they plausibly

influence private investment directly.

2. **Use a tighter instrument set** – keep only instruments whose exogeneity is
defensible (e.g. lagged variables Pt−1,Xt−1,TIMEP_{t-1},X_{t-1} ,TIME) and re-estimate 2SLS.

3. **Robust J-test** – under heteroskedasticity the Sargan statistic is not valid; run the Hansen–Sargan JJJ test (diagnostics(iv.inv)$overid) which is heteroskedastic-robust.

4. **Economic reconsideration** – if policy variables such as taxes and government
purchases drive investment directly, they belong **in the structural equation**
itself, not in the instrument list.

Until a valid instrument set passes the over-ID test, the conclusions drawn from part (d)/(e) should be viewed with caution.