Q3.1.a

$H_0: \beta_2 = 0$ against $H_1: \beta_2 > 0$

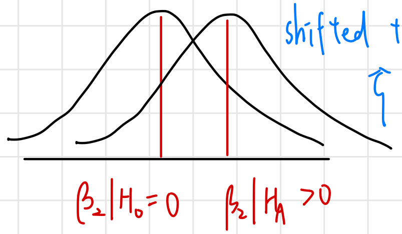
Q3.1.b

$$\varphi = \frac{b_2}{SE(b_2)} \sim t(62) \overset{A}{\sim} Z$$

$$\varphi^* = \frac{0.01309}{0.00215} = 6.088$$

Q3.1.c

$$b_2 \overset{A}{\sim} N(\beta_2, SE(b_2))$$




If $H_A$ is true, the distribution shifted to the right.

Q3.1.d

$RR = \{\varphi \mid \varphi > Z_{0.01} = 2.33\}$ The test statistic should be larger than about 2.33 to reject the null that $\beta_2 = 0$ at 1% confidence level.

Q3.1.e

$\varphi^* = 6.088 > Z_{0.01} = 2.33$, $\varphi^* \in RR$ (reject region)

We are 99% confident that GDP do positively affect the total number of medal won in one country.

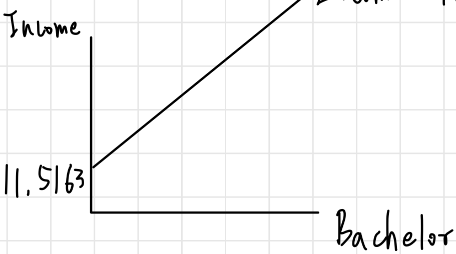
Q 3.7.a

$$\widehat{INCOME} = (a) + 1.029 BACHELOR$$
$$\text{se} \quad (2.672) \quad (c)$$
$$t \quad (4.31) \quad (10.75)$$

$$\frac{a}{2.672} = 4.31, \quad a = 11.5163$$

Q 3.7.b

$$\widehat{Income} = 11.5163 + 1.029 \text{ Bachelor}$$




$$\frac{\partial \widehat{Income}}{\partial \text{Bachelor}} = 1.029 \quad \text{increase}$$

$$\frac{\partial^2 \widehat{Income}}{\partial \text{Bachelor}^2} = 0 \quad \text{constant}$$

→ The fitted line increase constantly

Q 3.7.c

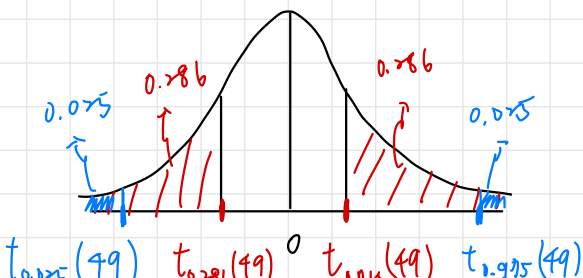
$$\frac{1.029}{c} = 10.75, \quad c = 0.0957$$

Q 3.7.d

$$\varphi = \frac{\alpha - A}{SE(\alpha)}, \quad \varphi^{v} = \frac{11.5163 - 10}{2.672} = 0.5675$$

Q 3.7.e

$$\varphi = \frac{\alpha - A}{SE(\alpha)} \sim t(49)$$




two-tailed p value = 0.572

Q 3.7.f

$$\varphi = \frac{b - \beta}{SE(\alpha)} \sim t(49)$$

$$P\left[ t_{0.005}(49) < \frac{1.029 - \beta}{0.0975} < t_{0.995}(49) \right] = 1 - 0.01$$

$$CI\ 1\% = [1.029 \pm 2.68 \times 0.0957] = [0.7725, 1.2855]$$

We are 99% confident that the slope parameter won't be outside the confident interval $[0.7725, 1.2855]$

Q 3.17.g

$H_0: \beta = 1$ against $H_1: \beta \neq 1$

$$\varphi = \frac{b - \beta}{SE(b)} \sim t(49)$$

$$RR = \{ \varphi \mid |\varphi^*| > t_{0.975}(49) = 2.01 \}$$

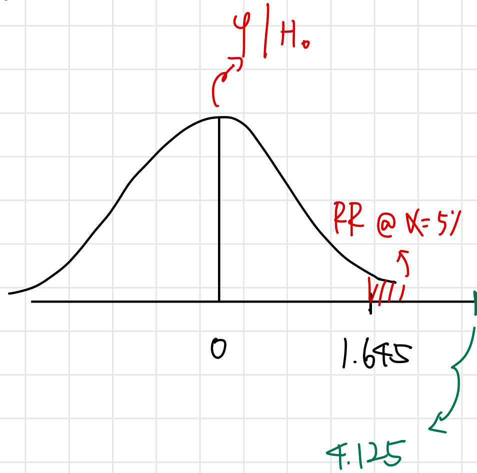
$$\varphi^* | H_0 = \frac{1.029 - 1}{0.0957} = 0.303 \notin RR$$

There's no significant evidence that shows every 1% increase in bachelor degree or more would bring 1 thousand dollar increase in income per capita.

Q.3.17.a

$H_0: \beta_{2,urban} = 1.8$ against $H_1: \beta_{2,urban} > 1.8$

$$\varphi = \frac{b_{2,urban} - \beta_{2,urban}}{SE(b_{2,urban})} \overset{A}{\sim} Z$$




$$RR = \{ \varphi \mid |\varphi^d| > Z_{0.95} = 1.645 \}$$

$$\varphi^d = \frac{2.46 - 1.8}{0.16} = 4.125 \in RR$$

We have strong evidence $\beta_2 > 1.8$

Q3.17.b

$$CI @ \alpha = 5\% = [\widehat{WAGE} | Educ = 16 \pm t_{0.025}(212) \cdot SE(\widehat{WAGE})]$$

$$= [-4.88 + 1.8 \times 16 \pm 1.971 \times 0.833]$$

$$= [22.2782, 25.5618]$$

Q3.17.c

$$SE(\widehat{WAGE}) = \sqrt{2.27^2 + 16^2 \times 0.16^2 + 2 \times 16 \times (-0.345)} = 0.8164$$

$$CI @ \alpha = 5\% = [\widehat{WAGE} \pm Z_{0.05} \times SE(\widehat{WAGE})]$$

$$= [-10.17 + 2.46 \times 16 \pm 1.96 \times 0.8164]$$

$$= [27.5899, 30.79]$$

for urban, $30.79 - 27.5899 = 3.2$ $\quad t_{0.05}(984) \times 0.8164$

for rural, $25.5618 - 22.2782 = 3.24$ $\quad t_{0.05}(212) \times 0.8133$

as $N \uparrow$, $SE(\widehat{WAGE}) \downarrow$ and also, as $N \to \infty$, $\varphi \xrightarrow{A} Z$,

and the tail will become thinner, and the critical value $\downarrow$

It is resonable that CI for urban regression than it for the rural one.

Q.3.17.d

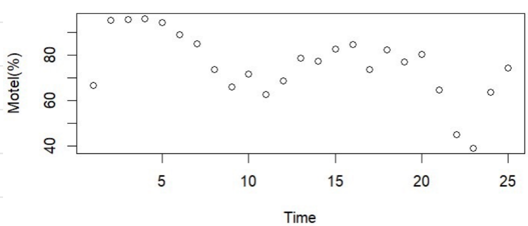
$H_0: \beta_{1,rural} = 4$ against $\beta_{1,rural} < 4$

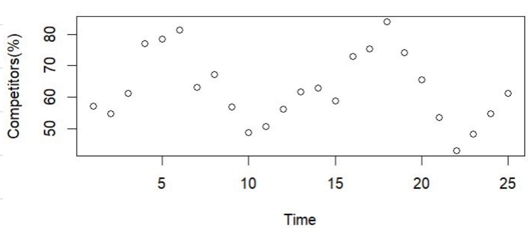
$$\varphi = \frac{b_{1,rural} - \beta_{1,rural}}{SE(b_{2,rural})} \sim t(212)$$

$$RR = \{\varphi \mid \varphi^* < t_{0.01}(212) = -2.344\}$$

$$\varphi^* = \frac{-4.88 - 4}{3.29} = -2.699 \in RR$$

Q3.19.a




```
> mean(data$motel_pct)
[1] 75.644
> mean(data$comp_pct)
[1] 62.736
> cor(data$motel_pct, data$comp_pct , use = "everything",
+      method = c("pearson"))
[1] 0.6645726
```

| | 2.5 % | 97.5 % |
|---|---|---|
| (Intercept) | -5.2998960 | 48.099873 |
| data$comp_pct | 0.4452978 | 1.283981 |

The mean of motel_pct > comp_pct and the correlation coefficient is positive.

CI @ 5% for $\beta_2$: $[0.4453, 1.2840]$

## Q3.19.b

```
predict(mod1,
        interval = "confidence",
        level = 0.90,
        newdata = data.frame(comp_pct = 70))
     fit      lwr      upr
81.92474 77.38223 86.46725
```

CI @ $\alpha = 0.1$ for $\widehat{\text{MODEL\_PCT}}$

: $[77.3822, 86.4673]$

## Q3.19.c.

$H_0: \beta_2 \le 0$ against $\beta_2 > 0$

$$\varphi = \frac{b_2}{SE(b_2)} \sim t(23)$$

$$RR = \{\varphi \mid \varphi > t_{0.99}(23)\}$$

```
> df <- df.residual(mod1)
>
> # Critical t value at alpha = 0.01, one-tailed
> t_critical <- qt(0.99, df)
>
> # t-statistic for comp_pct
> t_stat <- summary(mod1)$coefficients["comp_pct", "t
value"]
>
> # Conclusion
> if (t_stat > t_critical) {
+   print("Reject H0: There is strong evidence that b
eta2 > 0.")
+ } else {
+   print("Fail to reject H0: No strong evidence that
beta2 > 0.")
+ }
[1] "Reject H0: There is strong evidence that beta2 >
0."
```

## Q3.19.d.

$H_0: \beta_2 = 1$ against $\beta_2 \neq 1$

$$\varphi = \frac{b_2 - 1}{SE(b_2)} \sim t(23)$$

$$RR = \{\varphi \mid |\varphi| > t_{0.995}(23)\}$$

```
> if (abs(t_stat) > t_critical) {
+   print("Reject H0: beta2 is significantly differen
t from 1.")
+ } else {
+   print("Fail to reject H0: No significant differen
ce from 1.")
+ }
[1] "Fail to reject H0: No significant difference fro
m 1."
```

## Q3.19.e

There seems to be a problem of serial correlation and at t= 17-23, the sign is also negative