5.6 Suppose that, from a sample of 63 observations, the least squares estimates and the corresponding estimated covariance matrix are given by

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix} \qquad \widehat{\operatorname{cov}}(b_1, b_2, b_3) = \begin{bmatrix} \overset{V(b_1)}{3} & -2 & 1 \\ -2 & \overset{V(b_2)}{4} & 0 \\ 1 & 0 & \overset{V(b_3)}{3} \end{bmatrix}$$

Using a 5% significance level, and an alternative hypothesis that the equality does not hold, test each of the following null hypotheses:

a. $\beta_2 = 0$
b. $\beta_1 + 2\beta_2 = 5$
c. $\beta_1 - \beta_2 + \beta_3 = 4$

(a). $\begin{cases} H_0 : \beta_2 = 0 \\ H_1 : \beta_2 \neq 0 \end{cases}$ $\quad \alpha = 5\%$ $\quad df = 63 - 3 = 60$ $\quad t_{(0.025, 60)} = -2.0003$ $\quad t_{(0.975, 60)} = 2.0003$

$$t = \frac{b_2 - 0}{se(b_2)} = \frac{3 - 0}{\sqrt{4}} = 1.5$$

$\because -2.0003 < 1.5 < 2.0003$ $\quad \therefore$ Do not reject H_0 #

(b). $\begin{cases} H_0 : \beta_1 + 2\beta_2 = 5 \\ H_1 : \beta_1 + 2\beta_2 \neq 5 \end{cases}$

$$t = \frac{(b_1 + 2b_2) - 5}{\sqrt{var(b_1) + 2^2 var(b_2) + 2 \cdot 1 \cdot 2 \cdot cov(b_1, b_2)}} = \frac{(2 + 2 \times 3) - 5}{\sqrt{3 + 2^2 \cdot 4 + 4 \cdot (-2)}}$$

$$= \frac{3}{\sqrt{11}} = 0.9045$$

$\because -2 < 0.9045 < 2$ $\quad \therefore$ Do not reject H_0 #

(c). $\begin{cases} \beta_1 - \beta_2 + \beta_3 = 4 \\ \beta_1 - \beta_2 + \beta_3 \neq 4 \end{cases}$

$$t = \frac{(b_1 - b_2 + b_3) - 4}{\sqrt{var(b_1) + var(b_2) + var(b_3) - 2 \cdot cov(b_1, b_2) - 2 \cdot cov(b_2, b_3) + 2 \cdot cov(b_1, b_3)}}$$

$$= \frac{(2 - 3 + (-1)) - 4}{\sqrt{3 + 4 + 3 - 2 \cdot (-2) - 2 \cdot 0 + 2 \cdot 1}} = \frac{-6}{\sqrt{16}} = -1.5$$

$\because -2 < -1.5 < 2$ $\quad \therefore$ Do not reject H_0 #

5.31 Each morning between 6:30 AM and 8:00 AM Bill leaves the Melbourne suburb of Carnegie to drive to work at the University of Melbourne. The time it takes Bill to drive to work (*TIME*), depends on the departure time (*DEPART*), the number of red lights that he encounters (*REDS*), and the number of trains that he has to wait for at the Murrumbeena level crossing (*TRAINS*). Observations on these variables for the 249 working days in 2015 appear in the file *commute5*. *TIME* is measured in minutes. *DEPART* is the number of minutes after 6:30 AM that Bill departs.

a. Estimate the equation

$$TIME = \beta_1 + \beta_2 DEPART + \beta_3 REDS + \beta_4 TRAINS + e$$

Report the results and interpret each of the coefficient estimates, including the intercept β_1.

b. Find 95% interval estimates for each of the coefficients. Have you obtained precise estimates of each of the coefficients?

c. Using a 5% significance level, test the null hypothesis that Bill's expected delay from each red light is 2 minutes or more against the alternative that it is less than 2 minutes.

d. Using a 10% significance level, test the null hypothesis that the expected delay from each train is 3 minutes against the alternative that it is not 3 minutes.

e. Using a 5% significance level, test the null hypothesis that Bill can expect a trip to be at least 10 minutes longer if he leaves at 7:30 AM instead of 7:00 AM, against the alternative that it will not be 10 minutes longer. (Assume other things are equal.)

f. Using a 5% significance level, test the null hypothesis that the expected delay from a train is at least three times greater than the expected delay from a red light against the alternative that it is less than three times greater.

g. Suppose that Bill encounters six red lights and one train. Using a 5% significance level, test the null hypothesis that leaving Carnegie at 7:00 AM is early enough to get him to the university on or before 7:45 AM against the alternative that it is not. [Carry out the test in terms of the expected time $E(TIME|\mathbf{X})$ where $\mathbf{X}$ represents the observations on all explanatory variables.]

h. Suppose that, in part (g), it is imperative that Bill is not late for his 7:45 AM meeting. Have the null and alternative hypotheses been set up correctly? What happens if these hypotheses are reversed?

(a).

```
Call:
lm(formula = time ~ depart + reds + trains, data = commute5)

Residuals:
     Min       1Q   Median       3Q      Max 
-18.4389  -3.6774  -0.1188   4.5863  16.4986 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)  20.8701     1.6758  12.454  < 2e-16 ***
depart        0.3681     0.0351  10.487  < 2e-16 ***
reds          1.5219     0.1850   8.225 1.15e-14 ***
trains        3.0237     0.6340   4.769 3.18e-06 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 6.299 on 245 degrees of freedom
Multiple R-squared:  0.5346,    Adjusted R-squared:  0.5289 
F-statistic: 93.79 on 3 and 245 DF,  p-value: < 2.2e-16
```

$$\widehat{TIME} = 20.8701 + 0.3681\,DEPART + 1.5219\,REDS + 3.0237\,TRAINS$$
$$(se) \quad (1.6758) \quad (0.0351) \quad (0.1850) \quad (0.6340)$$

- β_1：若6:30 AM出門，則DEPART、REDS、TRAINS為0 通勤時間為20.8701分。
- β_2：其他條件不變下，每晚於6:30AM 10分鐘出門則通勤時間增加3.681分鐘。
- β_3：其他條件不變下，每遇到一個REDS則通勤時間增加1.5219分鐘。
- β_4：其他條件不變下，每遇到一個TRAINS則通勤時間增加3.0237分鐘。

(b)、

```
> # 計算95%的區間估計
> confint(model, level = 0.95)
                 2.5 %     97.5 %
(Intercept) 17.5694018 24.170871
depart       0.2989851  0.437265
reds         1.1574748  1.886411
trains       1.7748867  4.272505
```

β_1 β_2 β_3 β_4

區間相對狹窄，表示獲得了每個係數的精確估計。

(c)、

$$\begin{cases} H_0: \beta_3 \geq 2 \\ H_1: \beta_3 < 2 \end{cases} \quad \alpha = 5\% \quad df = 249-4 = 245$$

$$t = \frac{b_3 - 2}{se(b_3)} = \frac{1.5219 - 2}{0.1850} = -2.5843$$

$$t_{(0.05,245)} = -1.651 \quad \because -2.5843 < -1.651 \quad \therefore \text{拒絕} H_0$$

(d)、

$$\begin{cases} H_0: \beta_4 = 3 \\ H_1: \beta_4 \neq 3 \end{cases} \quad \alpha = 0.1 \quad df = 245$$

$$t = \frac{b_4 - 3}{se(b_4)} = \frac{3.0237 - 3}{0.6340} = 0.0374$$

$$t_{(1-\frac{0.1}{2},245)} = 1.651 \quad t_{(\frac{0.1}{2},245)} = -1.651$$

$$\because -1.651 < 0.0374 < 1.651 \quad \therefore \text{Do not reject } H_0 \#$$

(e)、

6:30~7:30　6:30~7:00

$$\begin{cases} H_0: 60\beta_2 - 30\beta_2 \geq 10 \\ H_1: 60\beta_2 - 30\beta_2 < 10 \end{cases} \Rightarrow \begin{cases} H_0: \beta_2 \geq \frac{1}{3} \\ H_1: \beta_2 < \frac{1}{3} \end{cases} \quad \alpha = 5\%, df = 245$$

$$t = \frac{b_2 - \frac{1}{3}}{se(b_2)} = \frac{0.3681 - \frac{1}{3}}{0.0351} = 0.9905$$

$$t_{(0.05,245)} = -1.651 \quad \because 0.9905 > -1.651 \quad \therefore \text{Do not reject } H_0 \#$$

$\alpha = 5\%$

(f).
$$\begin{cases} H_0: \beta_4 \geq 3\beta_3 \\ H_1: \beta_4 < 3\beta_3 \end{cases} \Rightarrow \begin{cases} H_0: \beta_4 - 3\beta_3 \geq 0 \\ H_1: \beta_4 - 3\beta_3 < 0 \end{cases}$$

```
> print(cov_matrix)
             (Intercept)        depart          reds         trains
(Intercept)  2.808171830 -0.0260985055 -0.2690250770  0.0010777876
depart      -0.026098505  0.0012321419  0.0004557753 -0.0104185104
reds        -0.269025077  0.0004557753  0.0342390502 -0.0006481936
trains       0.001077788 -0.0104185104 -0.0006481936  0.4019709090
```

$$t = \frac{(b_4 - 3b_3) - 0}{\sqrt{var(b_4) + 3^2 \cdot var(b_3) + 2 \cdot 1 \cdot (-3) \cdot cov(b_3, b_4)}} = \frac{3.0237 - 3 \cdot 1.5219 - 0}{\sqrt{0.40197 + 9 \cdot 0.03424 + (-6) \cdot (-0.0006)}}$$

$$= \frac{-1.542}{0.8448} = -1.8253$$

$t_{(0.05, 45)} = -1.651 \quad \because -1.8253 < -1.651 \quad \therefore$ Reject H_0 #

(g). 6 REDS, 1 TRAINS, Leave at 7:00 $\quad \alpha = 5\%$

$$\begin{cases} H_0: \beta_1 + 30\beta_2 + 6\beta_3 + \beta_4 \leq 45 \\ H_1: \beta_1 + 30\beta_2 + 6\beta_3 + \beta_4 > 45 \end{cases}$$

$$t = \frac{(b_1 + 30b_2 + 6b_3 + b_4) - 45}{\sqrt{var(b_1) + 30^2 \cdot var(b_2) + 6^2 \cdot var(b_3) + var(b_4) + 2 \cdot 30 \cdot cov(b_1, b_2) + 2 \cdot 6 \cdot cov(b_1, b_3) + 2 \cdot cov(b_1, b_4) + 2 \cdot 30 \cdot 6 \cdot cov(b_2, b_3) + 2 \cdot 30 \cdot cov(b_2, b_4) + 2 \cdot 6 \cdot cov(b_3, b_4)}}$$

$$= \frac{20.8701 + 30 \cdot 0.3681 + 6 \cdot 1.5219 + 3.0237 - 45}{\sqrt{2.80817 + 900 \cdot 0.0012 + 36 \cdot 0.0342 + 0.402 + 60(-0.0261) + 12 \cdot (-0.269) + 2 \cdot 0.0011 + 360 \cdot 0.00045 + 60 \cdot (-0.0104) + 12 \cdot (-0.0006)}}$$

$$= \frac{-0.9318}{0.5392} = -1.726$$

$t_{(0.95, 45)} = 1.651 \quad \because -1.726 < 1.651 \quad \therefore$ Do not reject H_0 #

(h).

$$\begin{cases} H_0 : \beta_1 + 30\beta_2 + 6\beta_3 + \beta_4 \geq 45 \\ H_1 : \beta_1 + 30\beta_2 + 6\beta_3 + \beta_4 < 45 \end{cases} \qquad \alpha = 5\%$$

$t = -1.726$

$t_{(0.05, 245)} = -1.651$ $\quad \because -1.726 < -1.651$ Reject H_0 #

Next ?

5.33 Use the observations in the data file *cps5_small* to estimate the following model:

$$\ln(WAGE) = \beta_1 + \beta_2 EDUC + \beta_3 EDUC^2 + \beta_4 EXPER + \beta_5 EXPER^2 + \beta_6 (EDUC \times EXPER) + e$$

a. At what levels of significance are each of the coefficient estimates "significantly different from zero"?

b. Obtain an expression for the marginal effect $\partial E[\ln(WAGE)|EDUC, EXPER]/\partial EDUC$. Comment on how the estimate of this marginal effect changes as *EDUC* and *EXPER* increase.

c. Evaluate the marginal effect in part (b) for all observations in the sample and construct a histogram of these effects. What have you discovered? Find the median, 5th percentile, and 95th percentile of the marginal effects.

d. Obtain an expression for the marginal effect $\partial E[\ln(WAGE)|EDUC, EXPER]/\partial EXPER$. Comment on how the estimate of this marginal effect changes as *EDUC* and *EXPER* increase.

e. Evaluate the marginal effect in part (d) for all observations in the sample and construct a histogram of these effects. What have you discovered? Find the median, 5th percentile, and 95th percentile of the marginal effects.

f. David has 17 years of education and 8 years of experience, while Svetlana has 16 years of education and 18 years of experience. Using a 5% significance level, test the null hypothesis that Svetlana's expected log-wage is equal to or greater than David's expected log-wage, against the alternative that David's expected log-wage is greater. State the null and alternative hypotheses in terms of the model parameters.

g. After eight years have passed, when David and Svetlana have had eight more years of experience, but no more education, will the test result in (f) be the same? Explain this outcome?

h. Wendy has 12 years of education and 17 years of experience, while Jill has 16 years of education and 11 years of experience. Using a 5% significance level, test the null hypothesis that their marginal effects of extra experience are equal against the alternative that they are not. State the null and alternative hypotheses in terms of the model parameters.

i. How much longer will it be before the marginal effect of experience for Jill becomes negative? Find a 95% interval estimate for this quantity.

(a).

```
Call:
lm(formula = log(wage) ~ educ + I(educ^2) + exper + I(exper^2) +
    educ * exper, data = cps5_small)

Residuals:
    Min      1Q  Median      3Q     Max
-1.6628 -0.3138 -0.0276  0.3140  2.1394

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  1.038e+00  2.757e-01   3.764 0.000175 ***
educ         8.954e-02  3.108e-02   2.881 0.004038 **
I(educ^2)    1.458e-03  9.242e-04   1.578 0.114855
exper        4.488e-02  7.297e-03   6.150 1.06e-09 ***
I(exper^2)  -4.680e-04  7.601e-05  -6.157 1.01e-09 ***
educ:exper  -1.010e-03  3.791e-04  -2.665 0.007803 **
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.4638 on 1194 degrees of freedom
Multiple R-squared:  0.3227,    Adjusted R-squared:  0.3198
F-statistic: 113.8 on 5 and 1194 DF,  p-value: < 2.2e-16
```

$$\ln(WAGE) = 1.038 + 0.08954\,EDUC + 0.001458\,EDUC^2 + 0.04488\,EXPER - 0.000468\,EXPER^2 - 0.00101(EDUC \times EXPER)$$

所有係數在 1% 顯著水準下皆異於0 除了 $EDUC^2$ 是在 12% 顯著水準下

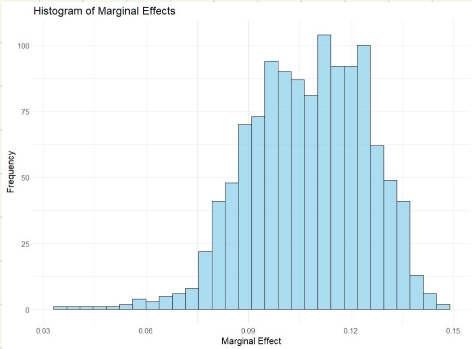
(b). $$\frac{\partial E[\ln(WAGE)|EDUC, EXPER]}{\partial EDUC} = \beta_2 + 2\beta_3 EDUC + EXPER\,\beta_6$$

$$\widehat{ME}_{EDUC} = 0.08954 + 2 \cdot 0.001458\,EDUC - 0.00101\,EXPER$$

$$= 0.08954 + 0.002916\,EDUC - 0.00101\,EXPER$$

當 EDUC 增加 ME_{EDUC} 增加，但 EXPER 增加會使 $\widehat{ME}_{EDUC}$ 減少。

(c)、




```
> print(marginal_effects_summary)
       5%       50%       95%
0.0800863 0.1084290 0.1336117
```

$$\widehat{ME}_{(EDUC,\,0.05)} = 0.08$$

$$\widehat{ME}_{(EDUC,\,0.5)} = 0.108$$

$$\widehat{ME}_{(EDUC,\,0.95)} = 0.134$$

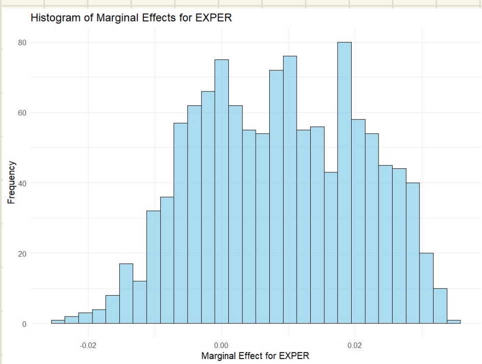
(d)、

$$\frac{\partial E[\ln(WAGE) \mid EDUC, EXPER]}{\partial EXPER} = \beta_4 + 2\beta_5 EXPER + \beta_6 EDUC$$

$$\widehat{ME}_{EXPER} = 0.04488 - 0.000936\, EXPER - 0.00101\, EDUC$$

當EXPER增加、EDUC增加會使 $\widehat{ME}_{EXPER}$ 減少。

(e)、




```
> print(marginal_effects_exper_summary)
        5%        50%        95%
-0.0103700  0.0084240  0.0279357
```

$$\widehat{ME}_{(EXPER,\,0.05)} = -0.01037$$

$$\widehat{ME}_{(EXPER,\,0.5)} = 0.008424$$

$$\widehat{ME}_{(EXPER,\,0.95)} = 0.0279357$$

(f).
$$\begin{cases} H_0: \cancel{\beta_1} + 17\beta_2 + 17^2\beta_3 + 8\beta_4 + 8^2\beta_5 + \beta_6(17\times 8) \le \cancel{\beta_1} + 16\beta_2 + 16^2\beta_3 + 18\beta_4 + 18^2\beta_5 + (18\times 16)\beta_6 \\ H_1: \beta_1 + 17\beta_2 + 17^2\beta_3 + 8\beta_4 + 8^2\beta_5 + \beta_6(17\times 8) > \beta_1 + 16\beta_2 + 16^2\beta_3 + 18\beta_4 + 18^2\beta_5 + (18\times 16)\beta_6 \end{cases}$$

$$\Rightarrow \begin{cases} H_0: \beta_2 + 33\beta_3 - 10\beta_4 - 260\beta_5 - 152\beta_6 \le 0 \\ H_1: \beta_2 + 33\beta_3 - 10\beta_4 - 260\beta_5 - 152\beta_6 > 0 \end{cases}$$

```
> cat("Variance:", var_combo, "\n")
Variance: 0.0004617778
> cat("Standard Error:", se_combo, "\n")
Standard Error: 0.02148902
```

$$t = \frac{b_2 + 33b_3 - 10b_4 - 260b_5 - 152b_6 - 0}{se(b_2 + 33b_3 - 10b_4 - 260b_5 - 152b_6)} = \frac{-0.035946}{0.021489} = -1.6727$$

$t_{(0.95,1194)} = 1.6461 > -1.6727$ $\therefore$ Do not reject H_0 #

(g)
$$\begin{cases} H_0: \cancel{\beta_1} + 17\beta_2 + 17^2\beta_3 + 16\beta_4 + 16^2\beta_5 + \beta_6(17\times 16) \le \cancel{\beta_1} + 16\beta_2 + 16^2\beta_3 + 26\beta_4 + 26^2\beta_5 + (16\times 26)\beta_6 \\ H_1: \beta_1 + 17\beta_2 + 17^2\beta_2 + 16\beta_4 + 16^2\beta_5 + \beta_6(17\times 16) > \beta_1 + 16\beta_2 + 16^2\beta_3 + 26\beta_4 + 26^2\beta_5 + (16\times 26)\beta_6 \end{cases}$$

$$\Rightarrow \begin{cases} H_0: \beta_2 + 33\beta_3 - 10\beta_4 - 420\beta_5 - 144\beta_6 \le 0 \\ H_1: \beta_2 + 33\beta_3 - 10\beta_4 - 420\beta_5 - 144\beta_6 > 0 \end{cases}$$

```
> cat("Variance:", var_combo, "\n")
Variance: 0.0002247335
> cat("Standard Error:", se_combo, "\n")
Standard Error: 0.01499112
```

$$t = \frac{\beta_2 + 33\beta_3 - 10\beta_4 - 420\beta_5 - 144\beta_6 - 0}{se(\beta_2 + 33\beta_3 - 10\beta_4 - 420\beta_5 - 144\beta_6)} = \frac{0.030854}{0.01499} = 2.0583 > 1.64$$

$\therefore$ Reject H_0 #

(h) $ME_{EXPER} = \beta_4 + 2\beta_5 EXPER + \beta_6 EDUC$

$$\begin{cases} H_0: \cancel{\beta_4} + 2\beta_5 \cdot 17 + \beta_6 \cdot 12 = \cancel{\beta_4} + 2\cdot\beta_5\, 11 + \beta_6 \cdot 16 \\ H_1: \beta_4 + 2\beta_5 \cdot 17 + \beta_6 \cdot 12 \ne \beta_4 + 2\cdot\beta_5\, 11 + \beta_6 \cdot 16 \end{cases}$$

$\alpha = 5\%$

$$\Rightarrow \begin{cases} H_0: 12\beta_5 - 4\beta_6 = 0 \\ H_1: 12\beta_5 - 4\beta_6 \ne 0 \end{cases}$$

```
> print(vcov_matrix)
```

	(Intercept)	educ	I(educ^2)	exper	I(exper^2)	I(educ * exper)
(Intercept)	7.603310e-02	-8.202345e-03	2.101167e-04	-1.463848e-03	5.245466e-06	8.306534e-05
educ	-8.202345e-03	9.660882e-04	-2.718150e-05	1.247850e-04	-2.585468e-07	-7.830715e-06
I(educ^2)	2.101167e-04	-2.718150e-05	8.541432e-07	-2.352932e-06	3.541430e-09	1.509737e-07
exper	-1.463848e-03	1.247850e-04	-2.352932e-06	5.325294e-05	-3.903252e-07	-2.396544e-06
I(exper^2)	5.245466e-06	-2.585468e-07	3.541430e-09	-3.903252e-07	5.777667e-09	8.121608e-09
I(educ * exper)	8.306534e-05	-7.830715e-06	1.509737e-07	-2.396544e-06	8.121608e-09	1.436988e-07

$$t = \frac{(12b_5 - 4b_6) - 0}{\sqrt{12^2 \cdot var(b_5) + (-4)^2 var(b_6) - 2 \cdot 12 \cdot (-4)\, cov(b_5, b_6)}}$$

$$= \frac{12 \cdot (-0.000468) + 4 \cdot 0.00101 - 0}{0.001533457} = -1.02774$$

$t_{(0.025,1194)} = -1.9619 < -1.02774 < 1.9619$ Do not reject H_0 #

(i) Jill: 16 EDUC, 11 EXPER $\Rightarrow ME_{EXPER} = \beta_4 + 2\beta_5 EXPER + \beta_6 EDUC$

$ME_{EXPER}: \beta_4 + 2\beta_5 EXPER + 16\beta_6 = 0$

使用 delta method 令 $g := g(\beta_4, \beta_5, \beta_6) = \frac{-16\beta_6 - \beta_4}{2\beta_5}$

點估計 $\hat{g} = \frac{-16 \cdot (-0.00101) - 0.04488}{2 \cdot (-0.000468)} = 30.68$

$30.68 - 11 = 19.6837$ 年 $\Rightarrow$ 再過 19.6837, ME 將 < 0

$$g_4 = \frac{\partial g}{\partial \beta_4} = \frac{(-1) \cdot \cancel{4\beta_5} - 0}{2\cancel{4}\beta_5^{\cancel{2}}} = \frac{-1}{2\beta_5}$$

$$g_5 = \frac{\partial g}{\partial \beta_5} = \frac{0 - (-16\beta_6 - \beta_4) \cdot \cancel{2}}{2\cancel{4}\beta_5^2} = \frac{16\beta_6 + \beta_4}{2\beta_5^2}$$

$$g_6 = \frac{\partial g}{\partial \beta_6} = \frac{\overset{8}{\cancel{-16}} \cdot \cancel{2\beta_5} - 0}{\cancel{2}\cancel{4}\beta_5^{\cancel{2}}} = \frac{-8}{\beta_5}$$

$$var(g(b_4, b_5, b_6))$$

$$\approx g_4^2 var(b_4) + g_5^2 var(b_5) + g_6^2 var(b_6) + 2 \cdot g_4 \cdot g_5 \, cov(b_4, b_5) + 2 \cdot g_5 g_6 \, cov(b_5, b_6) + 2 \cdot g_4 g_6 \, cov(b_4, b_6)$$

$$= \left(\frac{-1}{2\beta_5}\right)^2 var(b_4) + \left(\frac{16\beta_6 + \beta_4}{2\beta_5^2}\right)^2 var(b_5) + \left(\frac{-8}{\beta_5}\right)^2 var(b_6) + 2 \cdot \left(\frac{-1}{2\beta_5}\right)\left(\frac{16\beta_6 + \beta_4}{2\beta_5^2}\right) cov(b_4, b_5)$$

$$+ 2 \cdot \left(\frac{16\beta_6 + \beta_4}{(2\beta_5)^2}\right)\left(\frac{-8}{\beta_5}\right) cov(b_5, b_6) + 2 \cdot \left(\frac{-1}{2\beta_5}\right)\left(\frac{-8}{2\beta_5}\right) \cdot cov(b_4, b_6)$$

$$= 3.596955$$

```
> # 係數估計值
> b4 <- 4.488e-02  # β4 (exper)
> b5 <- -4.680e-04 # β5 (exper^2)
> b6 <- -1.010e-03 # β6 (educ * exper)
>
> # 計算偏微分（修正後，分母為2β5）
> partial_b4 <- -1 / (2 * b5)
> partial_b5 <- (16 * b6 + b4) / (2 * b5^2)
> partial_b6 <- -16 / (2 * b5)
>
> # 偏微分向量（對應於β4, β5, β6）
> grad_g <- c(partial_b4, partial_b5, partial_b6)
>
> # 共變異數矩陣
> V <- vcov(model)
>
> # 計算變異數（修正後使用行 4, 5, 6）
> var_g <- t(grad_g) %*% V[4:6, 4:6] %*% grad_g
>
> # 計算標準差（平方根）
> sd_g <- sqrt(var_g)
>
> # 顯示變異數和標準差
> cat("Variance of g(β4, β5, β6):", var_g, "\n")
Variance of g(β4, β5, β6): 3.596955
> cat("Standard Deviation of g(β4, β5, β6):", sd_g, "\n")
Standard Deviation of g(β4, β5, β6): 1.896564
```

$$se(g(b_4, b_5, b_6)) = \sqrt{3.596955} = 1.8965$$

$\alpha = 0.05$

區間估計：$19.6837 \pm t_{(0.975, 1196)} \times 1.8965$

$$= 19.6837 \pm 1.96195 \times 1.8965 = (15.9628, 23.4045)$$

#