HW3

C03Q01

(a)

H0: $\beta_2 = 0$ H1: $\beta_2 > 0$

Reject H0:

there is a positive relationship between the number of medals won and GDP

Fail to reject H0:

there is no relationship between the number of medals won and GDP

(b)

test statistic: $t = \frac{b_2 - 0}{SE(b_2)}$

distribution: Student's t-distribution, $df = 64 - 2 = 62$

C03Q01

(c)

$E[b_2] > 0$ => the distribution shift to the right

(d)

$\alpha = 0.01,\ df = 62$

critical value $= qt(1 - \alpha, df) \approx 2.388$

Reject H0: $t > 2.388$

Fail to reject H0: $t \leq 2.388$

C03Q01

(e)

$t = \frac{b_2 - 0}{SE(b_2)} \approx 6.0884 > 2.388$

=> Reject H0, it suggests that there is a statistically significant positive relationship between the number of medals won and GDP.

$\alpha = 0.01$

=> The probability of type I error occurring (H0 is true but we reject it) < 1%

C03Q07

(a)

$$t = \frac{b_1}{SE(b_1)} \quad \Rightarrow \quad b_1 = t \times SE(b_1) = 11.51632$$

(b)

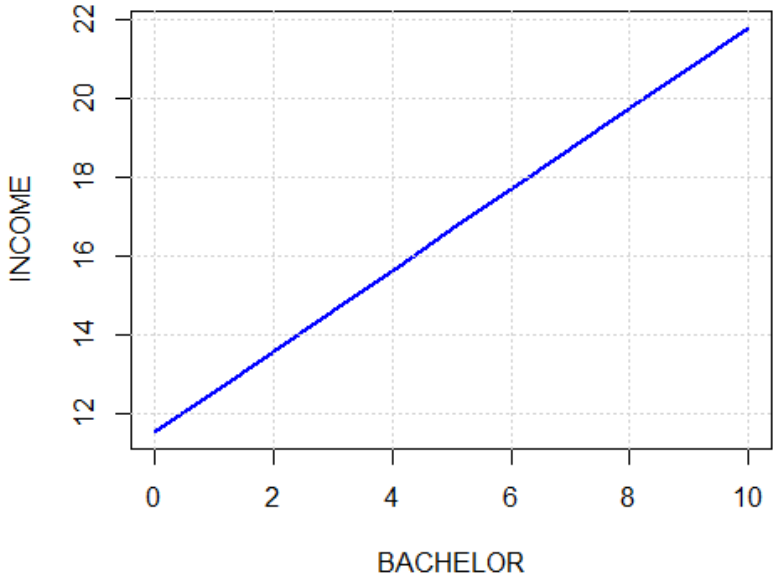
$$\widehat{INCOME} = 11.51632 + 1.029BACHELOR$$

Relationship is increasing ($b_1 > 0$),

so is a positive relationship.

Since the is a simple linear regression,

it is increasing at a constant rate ($b_1 = 1.029$).



C03Q07

(c)

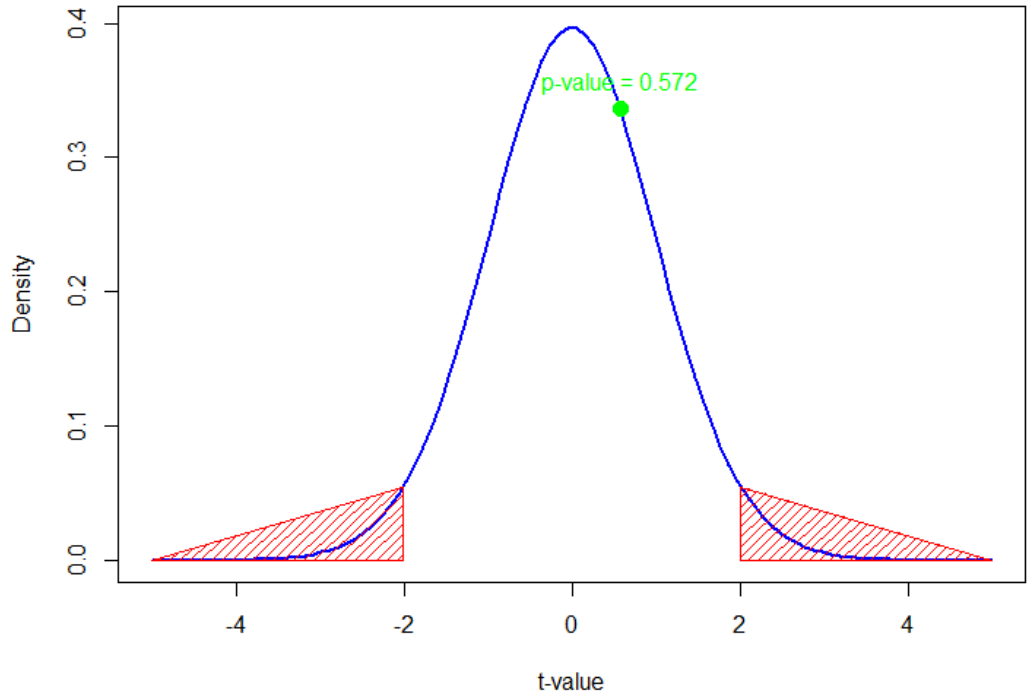
$$t = \frac{b_2}{SE(b_2)} \Rightarrow SE(b_2) = \frac{b_2}{t} \approx 0.0957$$

(d)

$$t = \frac{b_1 - 10}{SE(b_1)} = \frac{11.51632 - 10}{2.672} \approx 0.5675$$

(e)

red area: reject region



C03Q07

(f)

99%CI: $[b_2 \pm t \times SE(b_2)]$

$\approx [0.7725,\ 1.2855]$

(g)

H0: $\beta_2 = 1$ H1: $\beta_2 \neq 0$

$t = \frac{b_2 - 1}{SE(b_2)} = \frac{1.029 - 1}{0.0957} \approx 0.3030$

critical value $= \pm qt(1 - \alpha, df) \approx \pm 2.0096$

=> Fail to reject H0, it suggests that for every 1% increase in the proportion of bachelor's degree holders, income per capita should increase by 1000.

C03Q17

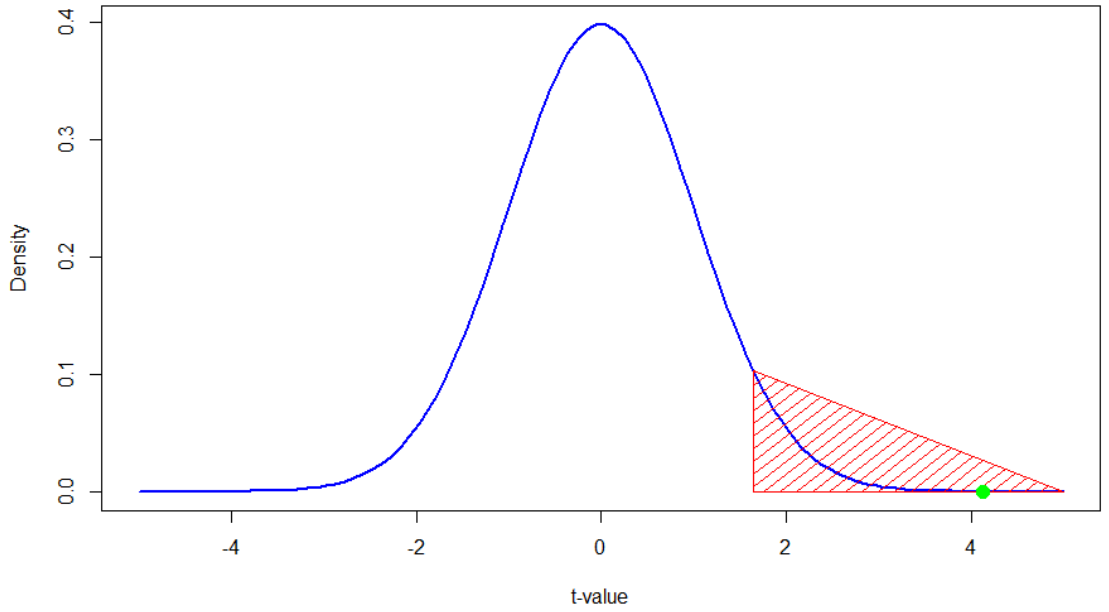
(a)

H0: $\beta_2 = 1.80$ H1: $\beta_2 > 0$

$t = \frac{b_2 - 1}{SE(b_2)} \approx 4.125$

critical value $= qt(1 - \alpha, df)$

$= 1.6464$



=> Reject H0, there is sufficient evidence to suggest that the slope should be 1.80 .

C03Q17

(b)

$$SE\left(\widehat{WAGE}\right) = \sqrt{SE(b_1)^2 + EDUC \times SE(b_2)^2 + 2 \times EDUC \times COV(b_1, b_2)}$$

$$\approx 1.1035$$

$$\widehat{WAGE} = b_1 + EDUC \times b_2 = 23.92$$

$$t = qt\left(1 - \frac{alpha}{2}, df\right) = 1.9712$$

95% CI: $\left[\widehat{WAGE} \pm t \times SE\left(\widehat{WAGE}\right)\right]$

$$\approx [21.7448,\ 26.0952]$$

C03Q17

(c)

$$SE\left(\widehat{WAGE}\right) = \sqrt{SE(b_1)^2 + EDUC \times SE(b_2)^2 + 2 \times EDUC \times COV(b_1, b_2)}$$

$$\approx 0.8164$$

$$\widehat{WAGE} = b_1 + EDUC \times b_2 = 28.6$$

$$t = qt\left(1 - \frac{alpha}{2}, df\right) = 1.9624$$

95% CI: $\left[\widehat{WAGE} \pm t \times SE\left(\widehat{WAGE}\right)\right]$

$$\approx [26.9979,\ 30.2021]$$

The interval is narrower than which for the rural.

It is plausible because the standard errors of $\widehat{WAGE}$ in urban areas is smaller than those in rural areas, so the CI cloud be more precise

C03Q17

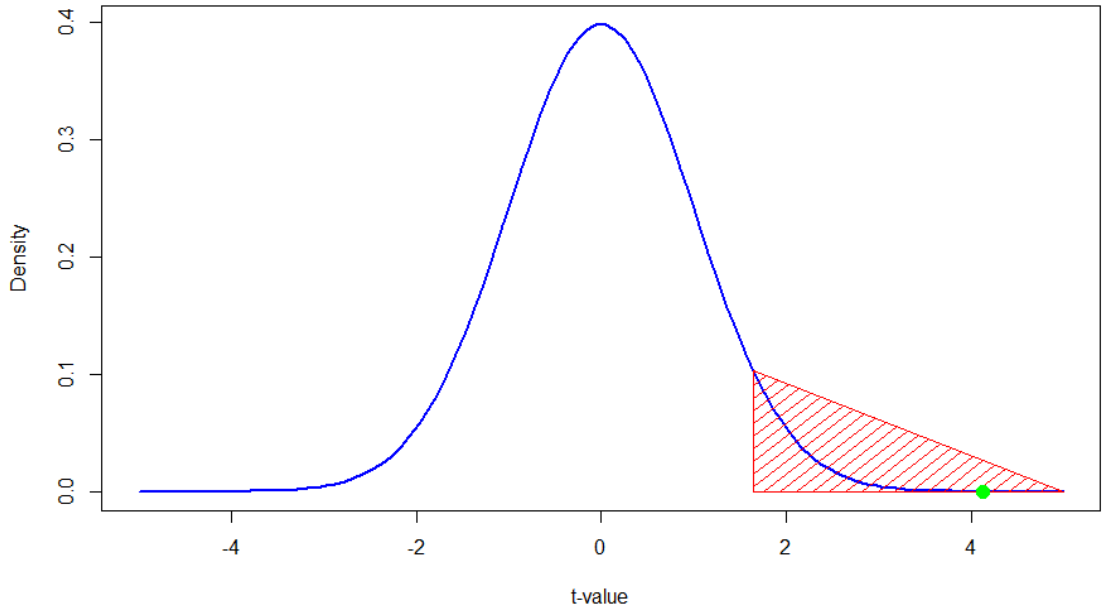
(d)

H0: $\beta_1 = 4$ H1: $\beta_1 < 4$

$t = \frac{b_2 - 4}{SE(b_2)} \approx -2.6991$

critical value $= qt(\alpha, df)$

$= -2.3441$



=> Reject H0, there is sufficient evidence to suggest that the intercept should be smaller than 4 .

C03Q19

(d)

Q1:

It seems to go up and down periodically

Q2:

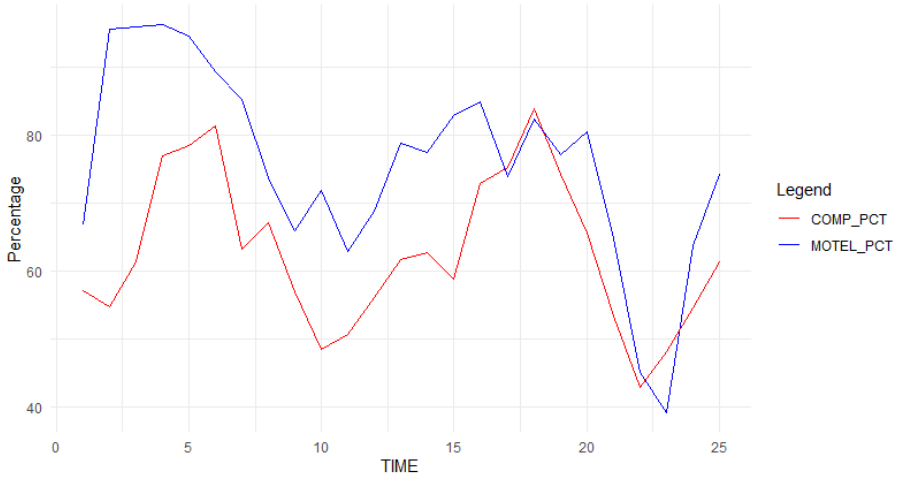
Yes, they do.

Q3:

MOTEL_PCT

Q4:

Yes, the p-value of b2 is significant.

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Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept)  21.4000    12.9069   1.658 0.110889
comp_pct      0.8646     0.2027   4.265 0.000291 ***

                2.5 %    97.5 %
(Intercept) -5.2998960 48.099873
comp_pct     0.4452978  1.283981
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C03Q19

(b)

	fit	lwr	upr
1	81.92474	77.38223	86.46725

(c)

$t \approx 4.2654$

critical value ≈ 2.4999

=> Reject H0, it suggests that COMP_PCT has a statistically significant positive effect on MOTEL_PCT.

C03Q19

(d)

$t \approx -0.6678$

critical value ≈ 2.4999

=> Fail to reject H0, it suggests that beta2 should be 1.

(e)

Q1:

The residuals in the earlier are mostly positive,

while those in the later are mostly negative.

Q2:

Negative.

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        17         18         19         20         21         22         23
-12.707328 -11.543226  -8.456225   2.279673  -2.958191 -13.293015 -23.875603
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