8.6 Consider the wage equation

$$WAGE_i = \beta_1 + \beta_2 EDUC_i + \beta_3 EXPER_i + \beta_4 METRO_i + e_i \quad \text{(XR8.6a)}$$

where wage is measured in dollars per hour, education and experience are in years, and $METRO = 1$ if the person lives in a metropolitan area. We have $N = 1000$ observations from 2013.

a. We are curious whether holding education, experience, and $METRO$ constant, there is the same amount of random variation in wages for males and females. Suppose $\text{var}(e_i|\mathbf{x}_i, FEMALE = 0) = \sigma_M^2$ and $\text{var}(e_i|\mathbf{x}_i, FEMALE = 1) = \sigma_F^2$. We specifically wish to test the null hypothesis $\sigma_M^2 = \sigma_F^2$ against $\sigma_M^2 \neq \sigma_F^2$. Using 577 observations on males, we obtain the sum of squared OLS residuals, $SSE_M = 97161.9174$. The regression using data on females yields $\hat{\sigma}_F = 12.024$. Test the null hypothesis at the 5% level of significance. Clearly state the value of the test statistic and the rejection region, along with your conclusion.

$$H_0: \sigma_m^2 = \sigma_F^2, H_1: \sigma_m^2 \neq \sigma_F^2$$

$$F = \frac{\sigma_m^2}{\sigma_F^2} = \frac{\frac{97161.9174}{573}}{12.024^2} = 1.173$$

Reject region: $F > 1.196 \; or \; F < 0.838$ ➔fail to reject H_0

b. We hypothesize that married individuals, relying on spousal support, can seek wider employment types and hence holding all else equal should have more variable wages. Suppose $\text{var}(e_i|\mathbf{x}_i, MARRIED = 0) = \sigma_{SINGLE}^2$ and $\text{var}(e_i|\mathbf{x}_i, MARRIED = 1) = \sigma_{MARRIED}^2$. Specify the null hypothesis $\sigma_{SINGLE}^2 = \sigma_{MARRIED}^2$ versus the alternative hypothesis $\sigma_{MARRIED}^2 > \sigma_{SINGLE}^2$. We add $FEMALE$ to the wage equation as an explanatory variable, so that

$$WAGE_i = \beta_1 + \beta_2 EDUC_i + \beta_3 EXPER_i + \beta_4 METRO_i + \beta_5 FEMALE + e_i \quad \text{(XR8.6b)}$$

Using $N = 400$ observations on single individuals, OLS estimation of (XR8.6b) yields a sum of squared residuals is 56231.0382. For the 600 married individuals, the sum of squared errors is 100,703.0471. Test the null hypothesis at the 5% level of significance. Clearly state the value of the test statistic and the rejection region, along with your conclusion.

$$H_0: \sigma_{SINGE}^2 = \sigma_{MARRIED}^2, H_1: \sigma_{MARRIED}^2 > \sigma_{SINGE}^2$$

$$F = \frac{\sigma_{MARRIED}^2}{\sigma_{SINGE}^2} = \frac{100703.0471}{595} / \frac{56231.0382}{395} = 1.189$$

Reject region: $F > 1.165$ ➔reject H_0

c. Following the regression in part (b), we carry out the NR^2 test using the right-hand-side variables in (XR8.6b) as candidates related to the heteroskedasticity. The value of this statistic is 59.03. What do we conclude about heteroskedasticity, at the 5% level? Does this provide evidence about the issue discussed in part (b), whether the error variation is different for married and unmarried individuals? Explain.

$$H_0: \alpha_2 = \alpha_3 = \alpha_4 = \alpha_5 = 0, H_1: not \; all \; \alpha \; in \; H_0 = 0$$

$$F = \frac{\sigma_{MARRIED}^2}{\sigma_{SINGE}^2} = \frac{100703.0471}{595} / \frac{56231.0382}{395} = 1.189$$

Reject region: $NR^2 > \chi^2_{0.95,4} = 9.49$➔59.03>9.49 ➔reject H_0

d. Following the regression in part (b) we carry out the White test for heteroskedasticity. The value of the test statistic is 78.82. What are the degrees of freedom of the test statistic? What is the 5% critical value for the test? What do you conclude?

df: 一般項 4+平方項 4+交乘項 6=14

$78.82 > \chi^2_{0.95,14} = 23.685$ ➔reject H_0 ➔heteroskedasticity

e. The OLS fitted model from part (b), with usual and robust standard errors, is

$$\begin{array}{llllll} \widehat{WAGE} = & -17.77 & + 2.50EDUC & + 0.23EXPER & + 3.23METRO & - 4.20FEMALE \\ \text{(se)} & (2.36) & (0.14) & (0.031) & (1.05) & (0.81) \\ \text{(robse)} & (2.50) & (0.16) & (0.029) & (0.84) & (0.80) \end{array}$$

CI for EDUC 變更寬, CI for EXPER, MRTRO, FEMALR 變窄

➔受 heteroskedasticity 影響

f. If we add *MARRIED* to the model in part (b), we find that its *t*-value using a White heteroskedasticity robust standard error is about 1.0. Does this conflict with, or is it compatible with, the result in (b) concerning heteroskedasticity? Explain.

b 小題是討論 married vs single 的 error variation 是否相同，但在 f 小題則是加入 married 當解釋變數，不顯著表示 married 對薪資影響不顯著，因此兩題結果不衝突

8.16 A sample of 200 Chicago households was taken to investigate how far American households tend to travel when they take a vacation. Consider the model

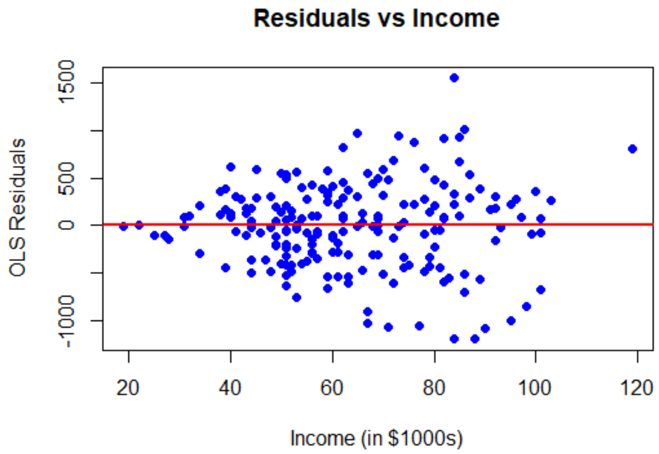
$$MILES = \beta_1 + \beta_2 INCOME + \beta_3 AGE + \beta_4 KIDS + e$$

MILES is miles driven per year, *INCOME* is measured in $1000 units, *AGE* is the average age of the adult members of the household, and *KIDS* is the number of children.

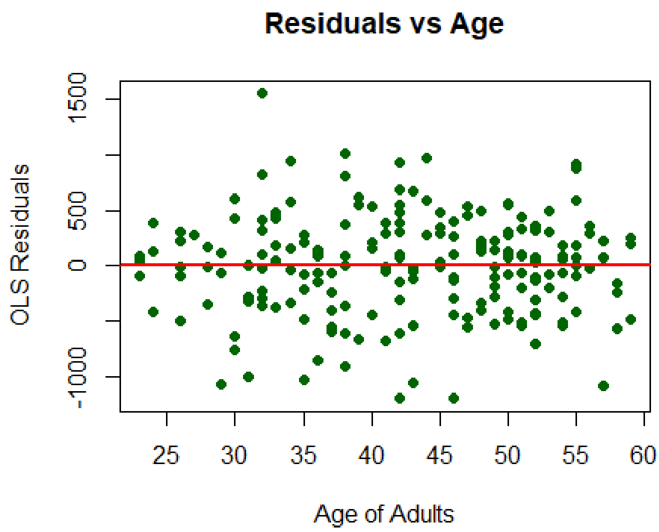
a. Use the data file *vacation* to estimate the model by OLS. Construct a 95% interval estimate for the effect of one more child on miles traveled, holding the two other variables constant.

```
> summary(model)

Call:
lm(formula = miles ~ income + age + kids, data = vacation)

Residuals:
     Min       1Q   Median       3Q      Max
-1198.14  -295.31    17.98   287.54  1549.41

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) -391.548    169.775  -2.306   0.0221 *
income        14.201      1.800   7.889 2.10e-13 ***
age           15.741      3.757   4.189 4.23e-05 ***
kids         -81.826     27.130  -3.016   0.0029 **
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 452.3 on 196 degrees of freedom
Multiple R-squared:  0.3406,    Adjusted R-squared:  0.3305
F-statistic: 33.75 on 3 and 196 DF,  p-value: < 2.2e-16

> confint(model, "kids", level = 0.95)
         2.5 %    97.5 %
kids -135.3298 -28.32302
```

b. Plot the OLS residuals versus *INCOME* and *AGE*. Do you observe any patterns suggesting that heteroskedasticity is present?



➔heteroskedasticity



➔homoskedasticity

c. Sort the data according to increasing magnitude of income. Estimate the model using the first 90 observations and again using the last 90 observations. Carry out the Goldfeld–Quandt test for heteroskedastic errors at the 5% level. State the null and alternative hypotheses.

H0: homoskedasticity

H1: heteroskedasticity

```
> # 顯示結果
> cat("F statistic =", F_stat, "\np-value =", p_value, "\n")
F statistic = 3.104061
p-value = 1.64001e-07
```

3.104>1.4286, 拒絕虛無假設，顯示模型的誤差項存在異質變異

d. Estimate the model by OLS using heteroskedasticity robust standard errors. Construct a 95% interval estimate for the effect of one more child on miles traveled, holding the two other variables constant. How does this interval estimate compare to the one in (a)?

a 小題
```
> confint(model, "kids", level = 0.95)
         2.5 %    97.5 %
kids -135.3298 -28.32302
```

d 小題
```
> confint_robust["kids", ]
      2.5 %      97.5 %
-139.32297   -24.32986
```
➔比 a 寬

```
t test of coefficients:

             Estimate Std. Error t value  Pr(>|t|)
(Intercept) -391.5480   142.6548 -2.7447 0.0066190 **
income        14.2013     1.9389  7.3246 6.083e-12 ***
age           15.7409     3.9657  3.9692 0.0001011 ***
kids         -81.8264    29.1544 -2.8067 0.0055112 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

e. Obtain GLS estimates assuming $\sigma_i^2 = \sigma^2 INCOME_i^2$. Using both conventional GLS and robust GLS standard errors, construct a 95% interval estimate for the effect of one more child on miles traveled, holding the two other variables constant. How do these interval estimates compare to the ones in (a) and (d)?

```
> cat("傳統 GLS 信賴區間(kids): \n")
傳統 GLS 信賴區間(kids):
> print(confint_traditional)
    2.5 %     97.5 %
-127.1453  -29.5814
> cat("\nRobust GLS 信賴區間(kids): \n")

Robust GLS 信賴區間(kids):
> print(confint_robust)
     2.5 %      97.5 %
-127.71734   -29.00934
```

➔GLS 點估計較小，區間左移

8.18 Consider the wage equation,

$$\ln(WAGE_i) = \beta_1 + \beta_2 EDUC_i + \beta_3 EXPER_i + \beta_4 EXPER_i^2 + \beta_5 FEMALE_i + \beta_6 BLACK + \beta_7 METRO_i + \beta_8 SOUTH_i + \beta_9 MIDWEST_i + \beta_{10} WEST + e_i$$

where *WAGE* is measured in dollars per hour, education and experience are in years, and $METRO = 1$ if the person lives in a metropolitan area. Use the data file *cps5* for the exercise.

a. We are curious whether holding education, experience, and *METRO* equal, there is the same amount of random variation in wages for males and females. Suppose $\text{var}(e_i|\mathbf{x}_i, FEMALE = 0) = \sigma_M^2$ and $\text{var}(e_i|\mathbf{x}_i, FEMALE = 1) = \sigma_F^2$. We specifically wish to test the null hypothesis $\sigma_M^2 = \sigma_F^2$ against $\sigma_M^2 \neq \sigma_F^2$. Carry out a Goldfeld–Quandt test of the null hypothesis at the 5% level of significance. Clearly state the value of the test statistic and the rejection region, along with your conclusion.

```
> # 印出結果
> cat("F 統計量:", f_statistic, "\n")
F 統計量: 1.058757
> cat("臨界值:", critical_value, "\n")
臨界值: 1.048488
```

```
> # 檢定結果
> if (f_statistic > critical_value | f_statistic < 1 / critical_v
alue) {
+   cat("拒絕虛無假設，認為男性與女性的工資變異數不同。\n")
+ } else {
+   cat("未拒絕虛無假設，認為男性與女性的工資變異數相同。\n")
+ }
拒絕虛無假設，認為男性與女性的工資變異數不同。
>
```

b. Estimate the model by OLS. Carry out the NR^2 test using the right-hand-side variables *METRO*, *FEMALE*, *BLACK* as candidates related to the heteroskedasticity. What do we conclude about heteroskedasticity, at the 1% level? Do these results support your conclusions in (a)? Repeat the test using all model explanatory variables as candidates related to the heteroskedasticity.

```
> cat("使用 METRO, FEMALE, BLACK 變數進行 NR2 測試：\n")
使用 METRO, FEMALE, BLACK 變數進行 NR2 測試：
> cat("R^2:", r_squared1, "\n")
R^2: 0.002404002
> cat("NR2 統計量:", nr2_statistic1, "\n")
NR2 統計量: 23.55681
> cat("p 值:", p_value1, "\n")
p 值: 3.441691e-15
>
> # 根據 p 值來得出結論
> if (p_value1 < 0.01) {
+   cat("在 1% 顯著性水準下，拒絕虛無假設，表明存在異質變異性。\n")
+ } else {
+   cat("在 1% 顯著性水準下，未能拒絕虛無假設，表明不存在異質變異性。\n")
+ }
在 1% 顯著性水準下，拒絕虛無假設，表明存在異質變異性。
```

➔23.56>$\chi^2_{0.99,3} = 11.34$➔reject H_0

```
使用所有解釋變數進行 NR2 測試：
> cat("R^2:", r_squared2, "\n")
R^2: 0.01116688
> cat("NR2 統計量:", nr2_statistic2, "\n")
NR2 統計量: 109.4243
> cat("p 值:", p_value2, "\n")
p 值: 0
>
> # 根據 p 值來得出結論
> if (p_value2 < 0.01) {
+   cat("在 1% 顯著性水準下，拒絕虛無假設，表明存在異質變異性。\n")
+ } else {
+   cat("在 1% 顯著性水準下，未能拒絕虛無假設，表明不存在異質變異性。\n")
+ }
在 1% 顯著性水準下，拒絕虛無假設，表明存在異質變異性。
```

➔109.4243>$\chi^2_{0.99,9} = 21.67$➔reject H_0

c. Carry out the White test for heteroskedasticity. What is the 5% critical value for the test? What do you conclude?

```
> # 印出結果
> cat("White 檢定統計量:", white_statistic, "\n")
White 檢定統計量: 109.4243
> cat("臨界值:", critical_value_white, "\n")
臨界值: 16.91898
>
> # 檢定結論
> if (white_statistic > critical_value_white) {
+   cat("拒絕虛無假設，表明存在異質變異性。\n")
+ } else {
+   cat("未拒絕虛無假設，表明不存在異質變異性。\n")
+ }
拒絕虛無假設，表明存在異質變異性。
```

d. Estimate the model by OLS with White heteroskedasticity robust standard errors. Compared to OLS with conventional standard errors, for which coefficients have interval estimates gotten narrower? For which coefficients have interval estimates gotten wider? Is there an inconsistency in the results?

```
> # 計算 White 標準誤
> white_se_results <- coeftest(model_ols, vcov = white_se)
> print(white_se_results)

t test of coefficients:

               Estimate  Std. Error  t value  Pr(>|t|)
(Intercept)  1.2014e+00  3.2831e-02  36.5932 < 2.2e-16 ***
educ         1.0123e-01  1.9082e-03  53.0492 < 2.2e-16 ***
exper        2.9622e-02  1.3171e-03  22.4899 < 2.2e-16 ***
I(exper^2)  -4.4578e-04  2.7654e-05 -16.1199 < 2.2e-16 ***
metro        1.1902e-01  1.1591e-02  10.2683 < 2.2e-16 ***
south       -4.5755e-02  1.3910e-02  -3.2895  0.001007 **
midwest     -6.3943e-02  1.3732e-02  -4.6566 3.258e-06 ***
west        -6.5891e-03  1.4565e-02  -0.4524  0.650989
female      -1.6550e-01  9.4932e-03 -17.4337 < 2.2e-16 ***
black       -1.1153e-01  1.6114e-02  -6.9211 4.762e-12 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

```
> # 95% 置信區間
> confint_ols <- confint(model_ols)
> confint_white <- confint(model_ols, vcov = white_se)
>
> # 輸出置信區間
> cat("傳統OLS標準誤的95%置信區間：\n")
傳統OLS標準誤的95%置信區間：
> print(confint_ols)
                    2.5 %        97.5 %
(Intercept)  1.1384302204  1.2643338265
educ         0.0977830603  0.1046761665
exper        0.0270727569  0.0321706349
I(exper^2)  -0.0004974407 -0.0003941203
metro        0.0948966363  0.1431441846
south       -0.0723384657 -0.0191724010
midwest     -0.0915893895 -0.0362971859
west        -0.0348207138  0.0216425095
female      -0.1841810529 -0.1468229075
black       -0.1447358548 -0.0783146449
> cat("\n使用White穩健標準誤的95%置信區間：\n")

使用White穩健標準誤的95%置信區間：
> print(confint_white)
                    2.5 %        97.5 %
(Intercept)  1.1384302204  1.2643338265
educ         0.0977830603  0.1046761665
exper        0.0270727569  0.0321706349
I(exper^2)  -0.0004974407 -0.0003941203
metro        0.0948966363  0.1431441846
south       -0.0723384657 -0.0191724010
midwest     -0.0915893895 -0.0362971859
west        -0.0348207138  0.0216425095
female      -0.1841810529 -0.1468229075
black       -0.1447358548 -0.0783146449
```

```
> cat("\n傳統OLS置信區間寬度：\n")

傳統OLS置信區間寬度：
> print(interval_width_ols)
 (Intercept)         educ        exper   I(exper^2)
0.1259036061 0.0068931063 0.0050978780 0.0001033205
       metro        south      midwest         west
0.0482475482 0.0531660647 0.0552922035 0.0564632233
      female        black
0.0373581454 0.0664212100
> cat("\n使用White穩健標準誤置信區間寬度：\n")

使用White穩健標準誤置信區間寬度：
> print(interval_width_white)
 (Intercept)         educ        exper   I(exper^2)
0.1259036061 0.0068931063 0.0050978780 0.0001033205
       metro        south      midwest         west
0.0482475482 0.0531660647 0.0552922035 0.0564632233
      female        black
0.0373581454 0.0664212100
```

e. Obtain FGLS estimates using candidate variables *METRO* and *EXPER*. How do the interval estimates compare to OLS with robust standard errors, from part (d)?

```
FGLS回歸的置信區間（使用White穩健標準誤）：
> print(confint_fgls_white)
                 2.5 %      97.5 %
(Intercept) 2.68215178 2.742283284
metro       0.17025703 0.221728599
exper       0.00433356 0.006022831
>
```

```
FGLS置信區間寬度：
> print(interval_width_fgls)
(Intercept)       metro       exper
0.060131502 0.051471564 0.001689271
> cat("\nOLS置信區間寬度：\n")

OLS置信區間寬度：
> print(interval_width_ols)
 (Intercept)         educ        exper   I(exper^2)
0.1259036061 0.0068931063 0.0050978780 0.0001033205
       metro        south      midwest         west
0.0482475482 0.0531660647 0.0552922035 0.0564632233
      female        black
0.0373581454 0.0664212100
>
```

```
FGLS置信區間變窄的係數：
> print(names(narrower_fgls))
[1] "(Intercept)" "exper"       "south"       "west"
[5] "female"      "black"
>
> cat("\nFGLS置信區間變寬的係數：\n")

FGLS置信區間變寬的係數：
> print(names(wider_fgls))
[1] "educ"       "I(exper^2)" "metro"      "midwest"
>
```

f. Obtain FGLS estimates with robust standard errors using candidate variables *METRO* and *EXPER*. How do the interval estimates compare to those in part (e) and OLS with robust standard errors, from part (d)?

```
> # 輸出結果
> cat("FGLS（使用 robust 標準誤）95% 信賴區間:\n")
FGLS（使用 robust 標準誤）95% 信賴區間:
> print(confint_fgls_robust)
                    2.5 %        97.5 %
(Intercept)  1.127694057  1.2515350381
educ         0.098351366  0.1052682659
exper        0.027590905  0.0326693606
I(exper^2)  -0.000509177 -0.0004041652
female      -0.184317568 -0.1471399412
black       -0.144166923 -0.0776164205
metro        0.094808099  0.1401225846
south       -0.071252312 -0.0182311336
midwest     -0.090708494 -0.0358393299
west        -0.033747215  0.0226111169
```

West 寬度最窄

g. If reporting the results of this model in a research paper which one set of estimates would you present? Explain your choice.

我會選 FGLS robust，因其考量了變異誤差並確保區間估計的穩健性