

$$\text{Demand: } Q_i = \alpha_1 + \alpha_2 P_i + \alpha_3 PS_i + \alpha_4 DI_i + e_{di} \quad (11.11)$$

$$\text{Supply: } Q_i = \beta_1 + \beta_2 P_i + \beta_3 PF_i + e_{si} \quad (11.12)$$

(a) Demand Equation: $\alpha_2 P_i = Q_i - (\alpha_1 + \alpha_3 PS_i + \alpha_4 DI_i + e_{di})$
 $\Rightarrow P_i = -\frac{\alpha_1}{\alpha_2} + \frac{1}{\alpha_2} Q_i - \frac{\alpha_3}{\alpha_2} PS_i - \frac{\alpha_4}{\alpha_2} DI_i + \frac{1}{\alpha_2} e_{di} = \delta_1 + \delta_2 Q_i + \delta_3 PS_i + \delta_4 DI_i + u_i$
 $\delta_2 < 0$, law of demand
 $\delta_3 > 0$, substitute goods
 $\delta_4 > 0$, truffles are normal goods

Supply Equation: $\beta_2 P_i = Q_i - (\beta_1 + \beta_3 PF_i + e_{si})$
 $\Rightarrow P_i = -\frac{\beta_1}{\beta_2} + \frac{1}{\beta_2} Q_i - \frac{\beta_3}{\beta_2} PF_i + \frac{1}{\beta_2} e_{si} = \pi_1 + \pi_2 Q_i + \pi_3 PF_i + u_i$
 $\pi_2 > 0$, $q \uparrow \rightarrow s \uparrow$
 $\pi_3 > 0$, cost of a factor of production

(b)

All of the coefficients are exactly as theory predicts. Except for the intercept in the demand equation, all the other slope coefficients are statistically significant different from zero.

2SLS estimates for 'demand' (equation 1)
 Model Formula: $p \sim q + ps + di$
 Instruments: $\sim ps + di + pf$

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	-11.42841	13.59161	-0.84084	0.4081026
q	-2.67052	1.17495	-2.27287	0.0315350 *
ps	3.46108	1.11557	3.10252	0.0045822 **
di	13.38992	2.74671	4.87490	4.6752e-05 ***

 Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 13.165551 on 26 degrees of freedom
 Number of observations: 30 Degrees of Freedom: 26
 SSR: 4506.625289 MSE: 173.331742 Root MSE: 13.165551
 Multiple R-Squared: 0.556717 Adjusted R-Squared: 0.505569

2SLS estimates for 'supply' (equation 2)
 Model Formula: $p \sim q + pf$
 Instruments: $\sim ps + di + pf$

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	-58.798223	5.859161	-10.0353	1.3165e-10 ***
q	2.936711	0.215772	13.6103	1.3212e-13 ***
pf	2.958486	0.155964	18.9690	< 2.22e-16 ***

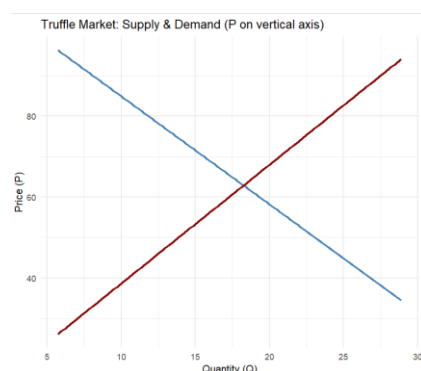
 Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 4.399078 on 27 degrees of freedom
 Number of observations: 30 Degrees of Freedom: 27
 SSR: 522.500877 MSE: 19.351884 Root MSE: 4.399078
 Multiple R-Squared: 0.948605 Adjusted R-Squared: 0.944798

(c)

q
 -1.272464

(d)



(e) The reduced-form predicts $\hat{Q} = 18.26$ and $\hat{P} = 62.815$, comparing the result, we think the two approaches are in very good agreement.

```
Q_eq. (Intercept) P_eq. (Intercept) Q_hat.1 P_hat.1
18.25021          62.84257 18.26040 62.81537
```

(f) Except for the sign of demand estimated from OLS, all the other coefficient signs are correct. Except for the intercept and coefficients for Q of demand estimated from OLS, all the other coefficient are statistically significant different from zero.

```
# A tibble: 14 × 6
  model      term      estimate std.error statistic p.value
<chr>    <chr>    <dbl>    <dbl>    <dbl>    <dbl>
1 Demand 2SLS (Intercept) -11.4      13.6    -0.841 4.08e- 1
2 Demand OLS (Intercept) -13.6      9.09    -1.50 1.46e- 1
3 Supply 2SLS (Intercept) -58.8      5.86   -10.0 1.32e-10
4 Supply OLS (Intercept) -52.9      5.02   -10.5 4.68e-11
5 Demand 2SLS di      13.4      2.75     4.87 4.68e- 5
6 Demand OLS di      12.4      1.83     6.77 3.48e- 7
7 Supply 2SLS pf       2.96     0.156    19.0 3.88e-17
8 Supply OLS pf       2.92     0.148    19.7 1.47e-17
9 Demand 2SLS ps       3.46     1.12     3.10 4.58e- 3
10 Demand OLS ps      1.36     0.594     2.29 3.03e- 2
11 Demand 2SLS q      -2.67     1.17    -2.27 3.15e- 2
12 Demand OLS q       0.151    0.499     0.303 7.64e- 1
13 Supply 2SLS q       2.94     0.216    13.6 1.32e-13
14 Supply OLS q       2.66     0.171    15.5 5.42e-15
```

11.30 Example 11.3 introduces Klein's Model I. Use the data file *klein* to answer the following questions.

- Estimate the investment function in equation (11.18) by OLS. Comment on the signs and significance of the coefficients.
- Estimate the reduced-form equation for profits, P_t , using all eight exogenous and predetermined variables as explanatory variables. Test the joint significance of all the variables except lagged profits, P_{t-1} , and lagged capital stock, K_{t-1} . Save the residuals, \hat{v}_t and compute the fitted values, \hat{P}_t .
- The Hausman test for the presence of endogenous explanatory variables is discussed in Section 10.4.1. It is implemented by adding the reduced-form residuals to the structural equation and testing their significance, that is, using OLS estimate the model

$$I_t = \beta_1 + \beta_2 P_t + \beta_3 P_{t-1} + \beta_4 K_{t-1} + \delta \hat{v}_t + e_{2t}$$

Use a t -test for the null hypothesis $H_0: \delta = 0$ versus $H_1: \delta \neq 0$ at the 5% level of significance. By rejecting the null hypothesis, we conclude that P_t is endogenous. What do we conclude from the test? In the context of this simultaneous equations model what result should we find?

- Obtain the 2SLS estimates of the investment equation using all eight exogenous and predetermined variables as IVs and software designed for 2SLS. Compare the estimates to the OLS estimates in part (a). Do you find any important differences?
- Estimate the second-stage model $I_t = \beta_1 + \beta_2 \hat{P}_t + \beta_3 P_{t-1} + \beta_4 K_{t-1} + e_{2t}$ by OLS. Compare the estimates and standard errors from this estimation to those in part (d). What differences are there?
- Let the 2SLS residuals from part (e) be \hat{e}_{2t} . Regress these residuals on all the exogenous and predetermined variables. If these instruments are valid, then the R^2 from this regression should be low, and none of the variables are statistically significant. The Sargan test for instrument validity is discussed in Section 10.4.3. The test statistic TR^2 has a chi-square distribution with degrees of freedom equal to the number of "surplus" IVs if the surplus instruments are valid. The investment equation includes three exogenous and/or predetermined variables out of the total of eight possible. There are $L = 5$ external instruments and $B = 1$ right-hand side endogenous variables. Compare the value of the test statistic to the 95th percentile value from the $\chi^2_{(4)}$ distribution. What do we conclude about the validity of the surplus instruments in this case?

- (a) Both current profits (p) and lagged profits (plag) enter with positive coefficients: when firms earn higher profits and enjoy ample internal funds, they raise their investment spending, consistent with the expected positive profit–investment linkage. By contrast, the coefficient on lagged capital stock (k_{lag}) is negative: the larger the existing capital base, the lower the marginal need for additional capital, in line with the accelerator model. All three core slope coefficients are significantly different from zero at the 1–5 percent levels.

```
> summary(ols_inv)

Call:
lm(formula = i ~ p + plag + klag, data = klein)

Residuals:
    Min       1Q   Median       3Q      Max
-2.56562 -0.63169  0.03687  0.41542  1.49226

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 10.12579     5.46555   1.853 0.081374 .
p             0.47964     0.09711   4.939 0.000125 ***
plag         0.33304     0.10086   3.302 0.004212 **
klag        -0.11179     0.02673  -4.183 0.000624 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

- (b) The value of F-test=1.93<3.025, so we fail to reject the null hypothesis that all coefficients of these variables are zero.

```
> summary(rf_P)

Call:
lm(formula = p ~ g + w2 + tx + time + plag + klag + elag, data = klein)

Residuals:
    Min       1Q   Median       3Q      Max
-3.9067 -1.3050  0.3226  1.3613  2.8881

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 50.38442    31.63026   1.593  0.1352
g             0.43902     0.39114   1.122  0.2820
w2           -0.07961     2.53382  -0.031  0.9754
tx           -0.92310     0.43376  -2.128  0.0530 .
time          0.31941     0.77813   0.410  0.6881
plag          0.80250     0.51886   1.547  0.1459
klag         -0.21610     0.11911  -1.814  0.0928 .
elag          0.02200     0.28216   0.078  0.9390
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
Model 1: restricted model
Model 2: p ~ g + w2 + tx + time + plag + klag + elag

    Res.Df  RSS Df Sum of Sq    F Pr(>F)
1      18  108
2      13   62    5    46.1 1.93   0.16
```

```
> cat("Critical F(5,13;0.95) =", round(F_crit, 3), "\n")
Critical F(5,13;0.95) = 3.025
```

```
> df$phat
[1] 13.255556 16.577368 19.282347 20.960143 19.766509 18.238731 17.573065
[8] 19.541720 20.375101 17.180415 12.705026  8.999780  9.054102 12.671263
[15] 14.421338 14.711907 19.796405 19.206691 17.419605 20.305654 22.657273
```

- (c) Since \hat{v} are significantly at 0.001 levels, so we know P is endogenous. This is what we expected from the simultaneous equation model.

```
> summary(hausman)

Call:
lm(formula = i ~ p + plag + klag + vhat, data = df)

Residuals:
    Min       1Q   Median       3Q      Max
-1.04645 -0.56030  0.06189  0.25348  1.36700

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 20.27821    4.70179   4.313 0.000536 ***
p            0.15022    0.10798   1.391 0.183222
plag         0.61594    0.10147   6.070 1.62e-05 ***
klag        -0.15779    0.02252  -7.007 2.96e-06 ***
vhat         0.57451    0.14261   4.029 0.000972 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

(d) The strong OLS effect appears to be simultaneity bias: high-investment years are also high-profit years, inflating the naive slope. 2SLS sacrifices some precision—especially for p —because it relies on variation supplied by the instruments, not by the endogenous regressor itself.

```
> summary(iv_inv)

Call:
ivreg(formula = i ~ p + plag + klag | g + w2 + tx + time + plag +
      klag + elag, data = df)

Residuals:
    Min       1Q   Median       3Q      Max
-3.2909 -0.8069  0.1423  0.8601  1.7956

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 20.27821    8.38325   2.419 0.02707 *
p            0.15022    0.19253   0.780 0.44598
plag         0.61594    0.18093   3.404 0.00338 **
klag        -0.15779    0.04015  -3.930 0.00108 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
OLS vs 2SLS coefficients:
> print(compare_slopes, n = Inf)
# A tibble: 8 × 6
  model term      estimate std.error statistic p.value
  <chr> <chr>      <dbl>      <dbl>      <dbl>    <dbl>
1 OLS   (Intercept) 10.1        5.47       1.85  0.0814
2 OLS   p            0.480      0.0971     4.94  0.000125
3 OLS   plag         0.333      0.101     3.30  0.00421
4 OLS   klag        -0.112     0.0267    -4.18  0.000624
5 2SLS  (Intercept) 20.3        8.38       2.42  0.0271
6 2SLS  p            0.150     0.193     0.780 0.446
7 2SLS  plag         0.616     0.181     3.40  0.00338
8 2SLS  klag        -0.158     0.0402    -3.93  0.00108
```

(e) All slopes keep their signs and magnitudes; only their standard errors change.

```
> summary(stage2)

Call:
lm(formula = i ~ phat + plag + klag, data = df)

Residuals:
    Min       1Q   Median       3Q      Max
-3.8778 -1.0029  0.3058  0.7275  2.1831

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 20.27821    9.97663   2.033 0.05802 .
phat         0.15022    0.22913   0.656 0.52084
plag         0.61594    0.21531   2.861 0.01083 *
klag        -0.15779    0.04778  -3.302 0.00421 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

(f)

```
(f) Sargan test:
> cat("  TR^2 = ", round(TR2, 3), "\n")
  TR^2 = 1.815
> cat("  X^2_0.95(df=4) = ", round(crit95, 3), "\n")
  X^2_0.95(df=4) = 9.488
> if (TR2 < crit95) {
+   cat("  - Fail to reject H0 : surplus instruments appear valid.\n")
+ } else {
+   cat("  - Reject H0 : at least one surplus instrument may be invalid.\n")
+ }
- Fail to reject H0 : surplus instruments appear valid.
```

15.6 Using the NLS panel data on $N = 716$ young women, we consider only years 1987 and 1988. We are interested in the relationship between $\ln(WAGE)$ and experience, its square, and indicator variables for living in the south and union membership. Some estimation results are in Table 15.10.

- a. The OLS estimates of the $\ln(WAGE)$ model for each of the years 1987 and 1988 are reported in columns (1) and (2). How do the results compare? For these individual year estimations, what are you assuming about the regression parameter values across individuals (heterogeneity)?

(a) OLS 在 1987, 1988 的估計差異不大 → 個體間無異質性

TABLE 15.10 Estimation Results for Exercise 15.6

	(1) OLS 1987	(2) OLS 1988	(3) FE	(4) FE Robust	(5) RE
C	0.9348 (0.2010)	0.8993 (0.2407)	1.5468 (0.2522)	1.5468 (0.2688)	1.1497 (0.1597)
EXPER	0.1270 (0.0295)	0.1265 (0.0323)	0.0575 (0.0330)	0.0575 (0.0328)	0.0986 (0.0220)
EXPER ²	-0.0033 (0.0011)	-0.0031 (0.0011)	-0.0012 (0.0011)	-0.0012 (0.0011)	-0.0023 (0.0007)
SOUTH	-0.2128 (0.0338)	-0.2384 (0.0344)	-0.3261 (0.1258)	-0.3261 (0.2495)	-0.2326 (0.0317)
UNION	0.1445 (0.0382)	0.1102 (0.0387)	0.0822 (0.0312)	0.0822 (0.0367)	0.1027 (0.0245)
N	716	716	1432	1432	1432

(standard errors in parentheses)

- b. The $\ln(WAGE)$ equation specified as a panel data regression model is

$$\ln(WAGE_{it}) = \beta_1 + \beta_2 EXPER_{it} + \beta_3 EXPER_{it}^2 + \beta_4 SOUTH_{it} + \beta_5 UNION_{it} + (u_i + e_{it}) \quad (XR15.6)$$

Explain any differences in assumptions between this model and the models in part (a).

加了時間和個體如下標，假設每個個體會自己隨時間變動的误差 e_{it}
允許 和解釋變數相關但不隨時間改變 u_i

- c. Column (3) contains the estimated fixed effects model specified in part (b). Compare these estimates with the OLS estimates. Which coefficients, apart from the intercepts, show the most difference?

截距 0.9 → 1.5 變化最大
EXPER, EXPER² 的效果剩約 1/4
SOUTH, UNION 係數變小
FE 95% confidence interval

→ EXPER (-0.0065, 0.1035) SOUTH (-0.5777, -0.0745)
EXPER² (-0.0034, 0.001) UNION (0.0198, 0.1146)

EXPER 用 OLS 的估計值不在區間裡
所以可知 OLS 估計值和 FE 估計值有顯著差異

- d. The F -statistic for the null hypothesis that there are no individual differences, equation (15.20), is 11.68. What are the degrees of freedom of the F -distribution if the null hypothesis (15.19) is true? What is the 1% level of significance critical value for the test? What do you conclude about the null hypothesis.

$$F = 11.68 > F_{0.01, 2, 111} = F_{0.01, 2, 111}$$

→ FE 拒絕虛無假設 (個體無差異)，支持採用固定效果

- e. Column (4) contains the fixed effects estimates with cluster-robust standard errors. In the context of this sample, explain the different assumptions you are making when you estimate with and without cluster-robust standard errors. Compare the standard errors with those in column (3). Which ones are substantially different? Are the robust ones larger or smaller?

within transformation $\tilde{e}_{it} = e_{it} - \bar{e}_{it}$

問題 (4) 標準誤普通偏大

- f. Column (5) contains the random effects estimates. Which coefficients, apart from the intercepts, show the most difference from the fixed effects estimates? Use the Hausman test statistic (15.36) to test whether there are significant differences between the random effects estimates and the fixed effects estimates in column (3) (Why that one?). Based on the test results, is random effects estimation in this model appropriate?

$$0.0003 / 0.0002 = 1.7278$$

$$Hausman \text{ test } \tau_j = \frac{\hat{\beta}_{FE} - \hat{\beta}_{RE}}{\sqrt{\text{var}(\hat{\beta}_{FE}) - \text{var}(\hat{\beta}_{RE})}}$$

$$\tau_{EXPER} = \frac{0.0575 - 0.0986}{\sqrt{0.0033 - 0.022}} = -1.67$$

$$\tau_{EXPER^2} = \frac{-0.0012 - (-0.0023)}{\sqrt{0.0011 - 0.0007}} = -1.296$$

$$\tau_{SOUTH} = \frac{-0.3261 - (-0.2326)}{\sqrt{0.1258^2 - 0.0317^2}} = -0.97$$

$$\tau_{UNION} = \frac{0.0822 - 0.1027}{\sqrt{0.0312^2 - 0.0245^2}} = -1.06$$

只有 EXPER 的係數在 10% 顯著水準上有顯著差異

⇒ random effects estimation is appropriate

15.17 The data file *liquor* contains observations on annual expenditure on liquor (*LIQUOR*) and annual income (*INCOME*) (both in thousands of dollars) for 40 randomly selected households for three consecutive years.

- a. Create the first-differenced observations on *LIQUOR* and *INCOME*. Call these new variables *LIQUORD* and *INCOMED*. Using OLS regress *LIQUORD* on *INCOMED* without a constant term. Construct a 95% interval estimate of the coefficient.

(a.)

$$\widehat{LIQUORD}_{it} = 0.02975 INCOMED_{it}$$

```
> summary(ols_a)
```

```
Call:
lm(formula = readscore ~ small + aide + tchexper + boy + white_asian +
    freelunch, data = star)
```

```
Residuals:
    Min       1Q   Median       3Q      Max
-107.220  -20.214   -3.935   14.339  185.956
```

```
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  437.76425    1.34622  325.180   < 2e-16 ***
small         5.82282    0.98933   5.886 4.19e-09 ***
aide          0.81784    0.95299   0.858   0.391
tchexper      0.49247    0.06956   7.080 1.61e-12 ***
boy          -6.15642    0.79613  -7.733 1.23e-14 ***
white_asian   3.90581    0.95361   4.096 4.26e-05 ***
freelunch    -14.77134    0.89025 -16.592 < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
> confint(fd_mod, level = 0.95)
                2.5 %      97.5 %
INCOMED -0.02841457 0.08790818
```

Q.15.20.

(a.) Small-class effect (SMALL) significant.

Coefficient $\approx +5.82$. Small-class instruction has a statistically significant positive impact on reading performance.

Teacher's aide effect (AIDE) not statistically significant.

Coefficient $\approx +0.82$. There is no evidence that having a teacher's aide significantly improves reading scores.

Teacher experience (TCHEXPER) significant

Coefficient $\approx +0.49$. More experienced teachers are associated with better student reading outcomes.

Gender difference (BOY) significant

Coefficient ≈ -6.16 . Girls outperform boys in this reading assessment.

Race/ethnicity (WHITE ASIAN) significant

Coefficient $\approx +3.91$. White and Asian students achieve higher average reading scores.

Economic disadvantage (FREELUNCH) significant

Coefficient ≈ -14.77 . Economic disadvantage is strongly associated with lower reading performance.

```
> summary(ols_a)

Call:
lm(formula = readscore ~ small + aide + tchexper + boy + white_asian +
    freelunch, data = star)

Residuals:
    Min       1Q   Median       3Q      Max
-107.220  -20.214   -3.935   14.339   185.956

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  437.76425    1.34622  325.180 < 2e-16 ***
small         5.82282     0.98933   5.886 4.19e-09 ***
aide          0.81784     0.95299   0.858  0.391
tchexper      0.49247     0.06956   7.080 1.61e-12 ***
boy          -6.15642     0.79613  -7.733 1.23e-14 ***
white_asian   3.90581     0.95361   4.096 4.26e-05 ***
freelunch    -14.77134     0.89025 -16.592 < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

(b)

The small-class advantage actually grows once we control for school-specific factors, and remains highly significant The experience effect falls in magnitude—suggesting some of the OLS effect was driven by differences across schools—but remains positive and highly significant. Boys continue to score about 5–6 points below girls, a highly significant gap that persists within schools. Controlling for school heterogeneity roughly doubles the race/ethnicity premium, indicating that within-school differences are even larger than the pooled estimate suggested.

```
> summary(fe_b)
Oneway (individual) effect Within Model

Call:
plm(formula = readscore ~ small + aide + tchexper + boy + white_asian +
    freelunch, data = pdata, model = "within")

Unbalanced Panel: n = 79, T = 34-137, N = 5766

Residuals:
    Min.    1st Qu.    Median    3rd Qu.    Max.
-102.6381  -16.7834   -2.8473   12.7591   198.4169

Coefficients:
              Estimate Std. Error t-value Pr(>|t|)
small         6.490231    0.912962   7.1090 1.313e-12 ***
aide          0.996087    0.881693   1.1297  0.2586
tchexper      0.285567    0.070845   4.0309 5.629e-05 ***
boy          -5.455941    0.727589  -7.4987 7.440e-14 ***
white_asian   8.028019    1.535656   5.2277 1.777e-07 ***
freelunch    -14.593572    0.880006 -16.5835 < 2.2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

(c)

The F-test statistic is 16.70 larger than 1.2798, so we reject the null hypothesis that there are no significant differences between schools. If the between-school heterogeneity is negligible, then absorbing school-specific intercepts merely shifts the overall level (intercept) without altering the slope estimates. Similarly, if most of the variation in the key regressors comes from within-school differences rather than across-school differences, the inclusion of fixed effects has little impact on the estimated slopes.

F test for individual effects

```
data:  readscore ~ small + aide + tchexper + boy + white_asian + freelunch
F = 16.698, df1 = 78, df2 = 5681, p-value < 2.2e-16
alternative hypothesis: significant effects
```