

CH5 Q3 a

$$(i) \quad t_{b_1} = \frac{1.4515}{2.2019} = 0.659$$

$$(ii) \quad se_{b_2} = \frac{2.7648}{5.7103} = 0.4842$$

$$(iii) \quad b_3 = 0.3695 \times 3.9376 = -1.4549$$

$$(iv) \quad 1199 \times 6.39547^2 = 49041.54 \quad R^2 = 1 - \frac{46221.62}{49041.54} = 0.0575$$

$$(v) \quad \hat{\sigma} = \sqrt{\frac{46221.62}{1196}} = 6.2167$$

Q3 (b)

b_2 : 總支出 (TOTEXP) 增加 1% 時, 家庭預算中酒精支出增加 2.76%

b_3 : 每增加 1 個孩子時, 家庭預算中酒精支出減少 1.4549

b_4 : 每增加 1 歲, 家庭預算中酒精支出減少 0.1503

Q3 (c)

$$95\% \Rightarrow -0.1503 \pm 1.96 \times 0.0235 = [-0.1964, -0.1042]$$

Dependent Variable: WALC				
Included observations: 1200				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	1.4515	2.2019	0.659	0.5099
ln(TOTEXP)	2.7648	0.4842	5.7103	0.0000
NK	-1.4549	0.3695	-3.9376	0.0001
AGE	-0.1503	0.0235	-6.4019	0.0000
R-squared	0.0575	Mean dependent var		6.19434
S.E. of regression	6.2167	S.D. dependent var		6.39547
Sum squared resid	46221.62			

Q3 d

沒有，因為 B_1 的 $p = 0.5099$ $0.5099 > 0.05$ 所以 B_1 不顯著

Q3 e

$$H_0: \beta_3 = -2$$

$$H_1: \beta_3 \neq -2$$

$$t = \frac{-1.4549 + 2}{0.3695} = 1.4752 \quad \text{vs} \quad t_{0.05, 1196} = 1.993$$

$$\because 1.4752 < 1.993 \quad \therefore \text{接受 } H_0 \quad \therefore \beta_3 = -2$$

CH5 Q23 a

B_2 : 銷售數量增加, 價格下降 (有折扣), 期望是負數

B_3 : 品質更好, 價格上升, 期望是正數

B_4 : 價格隨時間上升, 期望是正數

Q23 b

b_1 (數量): 每增加 1 單位, 價格減少 0.05997 單位 符合預期

b_2 (品質): 每增加 1 單位, 價格增加 0.11621 單位 符合預期

b_3 (時間): 每增加 1 年, 價格下降 2.35458 單位, 不符預期

$p\text{-value} = 0.0954 > 0.05$ $\therefore b_3$ 在 5% 顯著性水準下不顯著

Q23 c

$$R^2 = 0.5097$$

Q24 d

$H_0: B_2 \geq 0$ 沒影響

$H_1: B_2 < 0$ 有影響

$$t = \frac{-0.05997}{0.01018} = -5.8910 \quad \text{vs} \quad -t_{0.05, 52} = -1.675$$

$\therefore -5.8910 < -1.675$ 拒絕 H_0 $\therefore B_2 < 0$ 有影響

Q23 e

$H_0: \beta_3 \leq 0$ 沒影響

$H_1: \beta_3 > 0$ 有影響

$$t = \frac{0.11621}{0.20326} = 0.5717 \quad \text{vs} \quad t_{0.05, 52} = 1.675$$

$\because 0.5717 < 1.675 \quad \therefore \text{接受 } H_0 \quad \therefore \text{沒影響}$

Q23 f

年均變化：每克下降 2.35458 美元 ($b_4 = -2.35458$)

原因市場上的供給量提升，加上技術的進步造成供過於求，價格下降

Q1

$$Y = \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix} \quad B = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \quad X = \begin{bmatrix} 1 & x_1 \\ \vdots & \vdots \\ 1 & x_n \end{bmatrix} \quad \varepsilon = \begin{bmatrix} e_1 \\ \vdots \\ e_n \end{bmatrix}$$

$$(X^T X)^{-1} = \begin{bmatrix} 1 & 1 & \dots & 1 \\ x_1 & x_2 & \dots & x_n \end{bmatrix} \begin{bmatrix} 1 & x_1 \\ \vdots & \vdots \\ 1 & x_n \end{bmatrix} = \begin{bmatrix} n & x_1 + \dots + x_n \\ x_1 + \dots + x_n & \sum x_i^2 \end{bmatrix}^{-1} = \begin{bmatrix} n & n\bar{x} \\ n\bar{x} & \sum x_i^2 \end{bmatrix}^{-1}$$

$$\det = n \sum x_i^2 - (n\bar{x})^2 \Rightarrow \frac{1}{n \sum x_i^2 - (n\bar{x})^2} \begin{bmatrix} \sum x_i^2 & -n\bar{x} \\ -n\bar{x} & n \end{bmatrix}$$

$$b = \frac{1}{n \sum x_i^2 - (n\bar{x})^2} \begin{bmatrix} \sum x_i^2 & -n\bar{x} \\ -n\bar{x} & n \end{bmatrix} \begin{bmatrix} n\bar{y} \\ \sum xy \end{bmatrix} = \frac{1}{n \sum x_i^2 - (n\bar{x})^2} \begin{bmatrix} n\bar{y} \sum x_i^2 - n\bar{x} \sum xy \\ -n^2 \bar{x} \bar{y} + n \sum xy \end{bmatrix}$$

$$b_2 = \frac{-n^2 \bar{x} \bar{y} + n \sum xy}{n \sum x_i^2 - (n\bar{x})^2} = \frac{\sum xy - n\bar{x} \bar{y}}{\sum x_i^2 - n\bar{x}^2} \quad \sum x \bar{y} = \sum x$$

$$b_2 = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (x_i - \bar{x})^2} = \frac{\sum xy - \bar{x} \sum y - \bar{y} \sum x + \bar{x} \bar{y} \sum 1}{\sum x_i^2 - 2\bar{x} \sum x + \bar{x}^2 \sum 1} = \frac{\sum xy - \sum \bar{x} y - \sum x \bar{y} + \sum \bar{x} \bar{y}}{\sum x_i^2 - 2\sum x_i \bar{x} + \sum \bar{x}^2}$$

$$= \frac{\sum xy - n\bar{x} \bar{y}}{\sum x_i^2 - n\bar{x}^2}$$

$$b_1 = \frac{(\sum x_i^2)(n\bar{y}) - n\bar{x}(\sum xy)}{n \sum x_i^2 - (n\bar{x})^2} = \frac{\bar{y} \sum x_i^2 - \bar{x} \sum xy}{\sum x_i^2 - n\bar{x}^2}$$

$$b_1 = \bar{y} - b_2 \bar{x} = \frac{\bar{y} \sum x_i^2 - \bar{y} n \bar{x}^2 - \bar{x} \sum xy + n \bar{x}^2 \bar{y}}{\sum x_i^2 - n\bar{x}^2} = \frac{\bar{y} \sum x_i^2 - \bar{x} \sum xy}{\sum x_i^2 - n\bar{x}^2}$$

Q2

$$\text{Var}(b) = \sigma^2 (X^T X)^{-1} = \sigma^2 \frac{1}{n \sum x_i^2 - n \bar{x}^2} \begin{bmatrix} \sum x_i^2 & -n\bar{x} \\ -n\bar{x} & n \end{bmatrix}$$

$$\text{Var}(b_1) = \sigma^2 \times \frac{\sum x_i^2}{n \sum x_i^2 - (n\bar{x})^2}$$

$$\text{vs } \text{Var}(b_1) = \frac{\sigma^2 \sum x_i^2}{n \sum (x_i - \bar{x})^2} = \frac{\sigma^2 \sum x_i^2}{n \sum x_i^2 - (n\bar{x})^2}$$

$$\text{Var}(b_2) = \sigma^2 \times \frac{1}{\sum x_i^2 - \frac{(n\bar{x})^2}{n}} = \frac{\sigma^2}{\sum x_i^2 - n\bar{x}^2}$$

$$\text{vs } \text{Var}(b_2) = \frac{\sigma^2}{\sum (x_i - \bar{x})^2} = \frac{\sigma^2}{\sum x_i^2 - n\bar{x}^2}$$

$$\text{Cov}(b_1, b_2) = \sigma^2 \frac{-\sum x_i}{n \sum x_i^2 - (n\bar{x})^2} \Rightarrow \frac{-\sigma^2 (n\bar{x})}{n \sum x_i^2 - (n\bar{x})^2} \Rightarrow \frac{-\sigma^2 \bar{x}}{\sum x_i^2 - n\bar{x}^2}$$

$$\text{Cov}(b_1, b_2) = \frac{-\sigma^2 \bar{x}}{\sum (x_i - \bar{x})^2} \Rightarrow \frac{-\sigma^2 \bar{x}}{\sum x_i^2 - \underbrace{2 \sum x_i \bar{x}}_{n\bar{x}^2} + \sum \bar{x}^2} \Rightarrow \frac{-\sigma^2 \bar{x}}{\sum x_i^2 - n\bar{x}^2}$$