11.28

Supply and demand curves as traditionally drawn in economics principles classes have price (P) on the vertical axis and quantity (Q) on the horizontal axis.

a. Rewrite the truffle demand and supply equations in (11.11) and (11.12) with price P on the left–hand side. What are the anticipated signs of the parameters in this rewritten system of equations?

Ans.

$$\text{Demand: } Q_i = \alpha_1 + \alpha_2 P_i + \alpha_3 PS_i + \alpha_4 DI_i + e_{di} \quad (11.11)$$

$$\text{Supply: } Q_i = \beta_1 + \beta_2 P_i + \beta_3 PF_i + e_{si} \quad (11.12)$$

Demand Equation: $\alpha_2 P_i = Q_i - (\alpha_1 + +\alpha_3 PS_i + \alpha_4 DI_i + e_{di})$

$$\longrightarrow \quad P_i = -\frac{1}{\alpha_2}\alpha_1 + \frac{1}{\alpha_2}Q_i - \frac{1}{\alpha_2}\alpha_3 PS_i - \frac{1}{\alpha_2}\alpha_4 DI_i - \frac{1}{\alpha_2}e_{di} = \delta_1 + \delta_2 Q_i + \delta_3 PS_i + \delta_4 DI_i + v_{di}$$

$\delta_2 < 0$：數量↑ → 價格↓，符合需求法則（law of demand）

$\delta_3 > 0$：替代品價格↑ → truffles 需求↑ → 價格↑

$\delta_4 > 0$：所得↑ → truffles 需求↑ → 價格↑（正常財）

Supply Equation: $\beta_2 P_i = Q_i - (\beta_1 + \beta_3 PF_i + e_{si})$

$$\longrightarrow \quad P_i = -\frac{1}{\beta_2}\beta_1 + \frac{1}{\beta_2}Q_i - \frac{1}{\beta_2}\beta_3 PF_i - \frac{1}{\beta_2}e_{si} = \gamma_1 + \gamma_2 Q_i + \gamma_3 PF_i + v_{si}$$

$\gamma_2 > 0$：數量↑ → 價格↑，供給曲線向右上傾斜

$\gamma_3 > 0$：投入成本↑ → 價格↑（廠商轉嫁成本）

b. Using the data in the file $truffles$, estimate the supply and demand equations that you have formulated in (a) using two–stage least squares. Are the signs correct? Are the estimated coefficients significantly different from zero?

Ans.

```
2SLS estimates for 'eq1' (equation 1)
Model Formula: p ~ q + ps + di
Instruments: ~ps + di + pf

             Estimate Std. Error  t value   Pr(>|t|)
(Intercept) -11.42841   13.59161 -0.84084  0.4081026
q            -2.67052    1.17495 -2.27287  0.0315350 *
ps            3.46108    1.11557  3.10252  0.0045822 **
di           13.38992    2.74671  4.87490 4.6752e-05 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 13.165551 on 26 degrees of freedom
Number of observations: 30 Degrees of Freedom: 26
SSR: 4506.625289 MSE: 173.331742 Root MSE: 13.165551
Multiple R-Squared: 0.556717 Adjusted R-Squared: 0.505569
```

```
2SLS estimates for 'eq2' (equation 2)
Model Formula: p ~ q + pf
Instruments: ~ps + di + pf

             Estimate Std. Error  t value   Pr(>|t|)
(Intercept) -58.798223   5.859161 -10.0353 1.3165e-10 ***
q             2.936711   0.215772  13.6103 1.3212e-13 ***
pf            2.958486   0.155964  18.9690 < 2.22e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 4.399078 on 27 degrees of freedom
Number of observations: 30 Degrees of Freedom: 27
SSR: 522.500877 MSE: 19.351884 Root MSE: 4.399078
Multiple R-Squared: 0.948605 Adjusted R-Squared: 0.944798
```

所有估計係數的符號皆符合預期。

除了 Demand Equation 的截距之外，所有估計係數均在 5% 顯著水準下顯著不為零。

c. Estimate the price elasticity of demand “at the means” using the results from (b).

Ans.

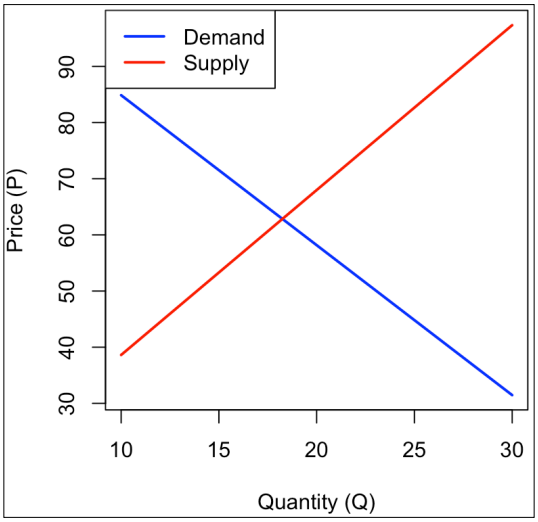
$$P_i = \delta_1 + \delta_2 Q_i + \delta_3 PS_i + \delta_4 DI_i + v_{di} \quad \longrightarrow \quad \frac{\partial Q}{\partial P} = \frac{1}{\delta_1}$$

$$\varepsilon = \frac{\partial Q}{\partial P} \cdot \frac{P}{Q} = \frac{1}{\delta_1} \cdot \frac{\bar{P}}{\bar{Q}}$$

```
> # 11.28(c)
> b_q <- coef(truff.sys$eq[[1]])["q"]
> p_bar <- mean(truffles$p)
> q_bar <- mean(truffles$q)
>
> elasticity <- (1 / b_q) * (p_bar / q_bar)
> elasticity
        q
-1.272464
```

d. Accurately sketch the supply and demand equations, with P on the vertical axis and Q on the horizontal axis, using the estimates from part (b). For these sketches set the values of the exogenous variables DI, PS, and PF to be $DI^* = 3.5$, $PF^* = 23$, and $PS^* = 22$.

Ans.



e. What are the equilibrium values of P and Q obtained in part (d)? Calculate the predicted equilibrium values of P and Q using the estimated reduced-form equations from Table 11.2, using the same values of the exogenous variables. How well do they agree?

Ans.

1. Equilibrium from Structural Supply and Demand Equations

```
> cat(q_star, p_star)
18.25021 62.84257
```

Demand: $P = \delta_0^* + \delta_1 Q$ Supply: $P = \gamma_0^* + \gamma_1 Q$

equilibrium: $\delta_0^* + \delta_1 Q = \gamma_0^* + \gamma_1 Q \longrightarrow Q^* = \dfrac{\gamma_0^* - \delta_0^*}{\delta_1 - \gamma_1}$; $P^* = \delta_0^* + \delta_1 Q^*$

the equilibrium values of $P^* = 62.84257$ and the equilibrium values of $Q^* = 18.25021$

2. Predicted Equilibrium from Reduced-Form Equations

From Tables 11.2a and 11.2b, the reduced-form equations are:

```
> cat(q_rform, p_rform)
18.2601 62.81425
```

$$Q = 7.8951 + 0.6564 \cdot PS + 2.1672 \cdot DI - 0.5070 \cdot PF$$

$$P = -32.5124 + 1.7081 \cdot PS + 7.6025 \cdot DI + 1.3539 \cdot PF$$

the predicted equilibrium values of $P = 62.81425$ and $Q = 18.2601$

兩種方法所得到的均衡價格與數量相當接近，表示透過結構模型估計出的供需線條與 reduced-form 模型是一致的。這代表所使用的工具變數（IV）合理，識別條件成立，模型穩定。

f. Estimate the supply and demand equations that you have formulated in (a) using OLS. Are the signs correct? Are the estimated coefficients significantly different from zero? Compare the results to those in part (b).

Ans.

```
Call:
lm(formula = p ~ q + ps + di, data = truffles)

Residuals:
     Min       1Q   Median       3Q      Max
-25.0753  -2.7742  -0.4097   4.7079  17.4979

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) -13.6195     9.0872  -1.499   0.1460
q             0.1512     0.4988   0.303   0.7642
ps            1.3607     0.5940   2.291   0.0303 *
di           12.3582     1.8254   6.770 3.48e-07 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 8.814 on 26 degrees of freedom
Multiple R-squared:  0.8013,    Adjusted R-squared:  0.7784
F-statistic: 34.95 on 3 and 26 DF,  p-value: 2.842e-09
```

```
Call:
lm(formula = p ~ q + pf, data = truffles)

Residuals:
    Min      1Q  Median      3Q     Max
-8.4721 -3.3287  0.1861  2.0785 10.7513

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) -52.8763     5.0238  -10.53 4.68e-11 ***
q             2.6613     0.1712   15.54 5.42e-15 ***
pf            2.9217     0.1482   19.71  < 2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 4.202 on 27 degrees of freedom
Multiple R-squared:  0.9531,    Adjusted R-squared:  0.9496
F-statistic: 274.4 on 2 and 27 DF,  p-value: < 2.2e-16
```

需求方程 (P ~ q + ps + di)

```
> print(demand_compare, right = FALSE)
            OLS_Estimate OLS_SE OLS_t   OLS_p    2SLS_Estimate 2SLS_SE 2SLS_t  2SLS_p
(Intercept) -13.6195     9.0872 -1.4987 1.46e-01 -11.4284      13.5916 -0.8408 4.08e-01
q             0.1512     0.4988  0.3032 7.64e-01  -2.6705       1.1750 -2.2729 3.15e-02
ps            1.3607     0.5940  2.2909 3.03e-02   3.4611       1.1156  3.1025 4.58e-03
di           12.3582     1.8254  6.7701 3.48e-07  13.3899       2.7467  4.8749 4.68e-05
```

變數 q 在 OLS 中為非顯著且方向錯誤，顯示 OLS 模型受到內生性干擾

2SLS 結果顯示 q 為顯著且負值，符合經濟理論

其他變數在兩種模型下皆為顯著、且符號一致，支持 2SLS 模型的有效性

供給方程 (P ~ q + pf)

```
> print(supply_compare, right = FALSE)
            OLS_Estimate OLS_SE OLS_t    OLS_p    2SLS_Estimate 2SLS_SE 2SLS_t   2SLS_p
(Intercept) -52.8763     5.0238 -10.5252 4.68e-11 -58.7982      5.8592  -10.0353 1.32e-10
q             2.6613     0.1712  15.5436 5.42e-15   2.9367      0.2158   13.6103 1.32e-13
pf            2.9217     0.1482  19.7150 1.47e-17   2.9585      0.1560   18.9691 0.00e+00
```

OLS 與 2SLS 結果差距非常小，因為供給式中的 q 為內生性問題較小的變數

可見在供給方程中使用 OLS 並未導致明顯偏誤，但使用 2SLS 更能保險排除潛在內生性問題