10.2 The labor supply of married women has been a subject of a great deal of economic research. Consider the following supply equation specification

$$HOURS = \beta_1 + \beta_2 WAGE + \beta_3 EDUC + \beta_4 AGE + \beta_5 KIDSL6 + \beta_6 NWIFEINC + e$$

where *HOURS* is the supply of labor, *WAGE* is hourly wage, *EDUC* is years of education, *KIDSL6* is the number of children in the household who are less than 6 years old, and *NWIFEINC* is household income from sources other than the wife's employment.

a. Discuss the signs you expect for each of the coefficients.

$\beta_2 > 0$, 工資高，工作越多回報增加多➔勞動供給增加

β_3不確定，教育程度高越能得到較佳的工作機會與薪資，但要考量具有較好薪資而有休閒與工作之間的 trade off，因此β_3不確定

β_4不確定，年齡增加(年輕時)在一開始可能增加勞動供給，而在老年時可能因為健康因素降低勞動供給

$\beta_5 < 0$,家中兒童多，為照顧兒童可能使工作時間減少➔降低勞動供給

$\beta_6 < 0$,其他收入增加，減少工作誘因➔降低勞動供給

b. Explain why this supply equation cannot be consistently estimated by OLS regression.

因為工資與 error 可能有內生問題，部分特質如工作能力並未加入，同時影響工時工資，且包含於 error 中

c. Suppose we consider the woman's labor market experience *EXPER* and its square, *EXPER*^{2}, to be instruments for *WAGE*. Explain how these variables satisfy the logic of instrumental variables.

$EXPER, EXPER^2$與工資具有相關性

$EXPER, EXPER^2$不直接影響$Hours$，只透過$Wage$影響

d. Is the supply equation identified? Explain.

Yes,有使用一個內生變數以及至少一個工具變數

e. Describe the steps [not a computer command] you would take to obtain IV/2SLS estimates.

1.先以$EXPER, EXPER^2$及其他外生變數對$Wage$做迴歸得 $\widehat{Wage}$

2.將原先迴歸中的$Wage$用$\widehat{Wage}$替代進行 OLS 迴歸

10.3 In the regression model $y = \beta_1 + \beta_2 x + e$, assume x is endogenous and that z is a valid instrument. In Section 10.3.5, we saw that $\beta_2 = \text{cov}(z, y)/\text{cov}(z, x)$.

a. Divide the denominator of $\beta_2 = \text{cov}(z, y)/\text{cov}(z, x)$ by $\text{var}(z)$. Show that $\text{cov}(z, x)/\text{var}(z)$ is the coefficient of the simple regression with dependent variable x and explanatory variable z, $x = \gamma_1 + \theta_1 z + v$. [*Hint*: See Section 10.2.1.] Note that this is the first-stage equation in two-stage least squares.

$$x = \gamma_1 + \theta_1 z + v$$
$$E(x) = \gamma_1 + \theta_1 E(z) \Rightarrow x - E(x) = \theta_1 (z - E(z)) + v$$
$$\Rightarrow E[(x - E(x))(z - E(z))] = \theta_1 E[(z - E(z))^2]$$
$$\Rightarrow \theta_1 = \frac{\text{cov}(z, x)}{\text{Var}(z)}$$

b. Divide the numerator of $\beta_2 = \text{cov}(z, y)/\text{cov}(z, x)$ by $\text{var}(z)$. Show that $\text{cov}(z, y)/\text{var}(z)$ is the coefficient of a simple regression with dependent variable y and explanatory variable z, $y = \pi_0 + \pi_1 z + u$. [*Hint*: See Section 10.2.1.]

$$y = \pi_0 + \pi_1 z + u$$
$$E(y) = \pi_0 + \pi_1 E(z) \Rightarrow y - E(y) = \pi_1 (z - E(z)) + u$$
$$\Rightarrow E[(y - E(y))(z - E(z))] = \pi_1 E[(z - E(z))^2]$$
$$\Rightarrow \pi_1 = \frac{\text{Cov}(z, y)}{\text{Var}(z)}$$

c. In the model $y = \beta_1 + \beta_2 x + e$, substitute for x using $x = \gamma_1 + \theta_1 z + v$ and simplify to obtain $y = \pi_0 + \pi_1 z + u$. What are π_0, π_1, and u in terms of the regression model parameters and error and the first-stage parameters and error? The regression you have obtained is a **reduced-form** equation.

$$y = \beta_1 + \beta_2(\gamma_1 + \theta_1 z + v) + e = \beta_1 + \beta_2\gamma_1 + \beta_2\theta_1 z + \beta_2 v + e$$
$$\Rightarrow \pi_0 = \beta_1 + \beta_2 \gamma_1$$
$$\pi_1 = \beta_2 \theta_1$$
$$u = \beta_2 v + e$$

d. Show that $\beta_2 = \pi_1/\theta_1$.

$$\pi_1 = \beta_2 \theta_1 \Rightarrow \beta_2 = \frac{\pi_1}{\theta_1}$$

e. If $\hat{\pi}_1$ and $\hat{\theta}_1$ are the OLS estimators of π_1 and θ_1, show that $\hat{\beta}_2 = \hat{\pi}_1/\hat{\theta}_1$ is a consistent estimator of $\beta_2 = \pi_1/\theta_1$. The estimator $\hat{\beta}_2 = \hat{\pi}_1/\hat{\theta}_1$ is an **indirect least squares** estimator.

$$\hat{\theta}_1 = \frac{\widehat{cov}(z,x)}{\widehat{var}(z)} = \frac{\sum(z_i-\bar{z})(x_i-\bar{x})}{\sum(z_i-\bar{z})^2} \xrightarrow{p} \theta$$

$$\hat{\pi}_1 = \frac{\widehat{cov}(z,y)}{\widehat{var}(z)} = \frac{\sum(z_i-\bar{z})(y_i-\bar{y})}{\sum(z_i-\bar{z})^2} \xrightarrow{p} \pi_1$$

$$\hat{\beta}_2 = \frac{\hat{\pi}_1}{\hat{\theta}_1} = \frac{\sum(z_i-\bar{z})(x_i-\bar{x})}{\sum(z_i-\bar{z})(y_i-\bar{y})} = \frac{\widehat{cov}(z,x)}{\widehat{cov}(z,y)} \xrightarrow{p} \beta_2$$

$$\hat{\beta}_2 \xrightarrow{p} \beta_2$$