HW0428 – Pinyo 312712017

10.18 Consider the data file *mroz* on working wives. Use the 428 observations on married women who participate in the labor force. In this exercise, we examine the effectiveness of a parent's college education as an instrumental variable.

a. Create two new variables. *MOTHERCOLL* is a dummy variable equaling one if *MOTHEREDUC* > 12, zero otherwise. Similarly, *FATHERCOLL* equals one if *FATHEREDUC* > 12 and zero otherwise. What percentage of parents have some college education in this sample?

```
Percentage of mothers with some college education: 10.09%
> cat(sprintf("Percentage of fathers with some college education: %.2f%
%\n", percent_fathercoll))
Percentage of fathers with some college education: 10.76%
```

b. Find the correlations between *EDUC*, *MOTHERCOLL*, and *FATHERCOLL*. Are the magnitudes of these correlations important? Can you make a logical argument why *MOTHERCOLL* and *FATHERCOLL* might be better instruments than *MOTHEREDUC* and *FATHEREDUC*?

```
             educ mothercoll fathercoll
educ        1.000      0.337      0.319
mothercoll  0.337      1.000      0.367
fathercoll  0.319      0.367      1.000
```

c. Estimate the wage equation in Example 10.5 using *MOTHERCOLL* as the instrumental variable. What is the 95% interval estimate for the coefficient of *EDUC*?

```
> cat(sprintf("95%% Confidence Interval for EDUC coefficient: [%.4f, %
4f]\n", lower_bound, upper_bound))
95% Confidence Interval for EDUC coefficient: [0.1885, 0.7440]
```

d. For the problem in part (c), estimate the first-stage equation. What is the value of the *F*-test statistic for the hypothesis that *MOTHERCOLL* has no effect on *EDUC*? Is *MOTHERCOLL* a strong instrument?

```
Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept)  8.7349066  0.2878732  30.343  < 2e-16 ***
mothercoll   0.9460059  0.2889619   3.274  0.00111 **
exper        0.0866125  0.0253869   3.412  0.00068 ***
exper2      -0.0019759  0.0008231  -2.401  0.01661 *
mothereduc   0.1367272  0.0298274   4.584 5.35e-06 ***
fathereduc   0.1842070  0.0243406   7.568 1.12e-13 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.951 on 747 degrees of freedom
Multiple R-squared:  0.2728,    Adjusted R-squared:  0.268
F-statistic: 56.05 on 5 and 747 DF,  p-value: < 2.2e-16
```

```
> cat(sprintf("F-statistic for mothercoll: %.2f\n", f_stat_mothercoll))
F-statistic for mothercoll: 116.68
```

Interpretation:

- If the F-statistic is greater than 10, mothercoll is generally considered a strong instrument158.
- If it is much lower, the instrument may be weak.

Conclusion, mothercoll is a strong instrument.

e. Estimate the wage equation in Example 10.5 using *MOTHERCOLL* and *FATHERCOLL* as the instrumental variables. What is the 95% interval estimate for the coefficient of *EDUC*? Is it narrower or wider than the one in part (c)?

```
> cat(sprintf("\n95%% Confidence Interval for EDUC coefficient (2 instr
uments): [%.4f, %.4f]\n",
+             lower_bound, upper_bound))

95% Confidence Interval for EDUC coefficient (2 instruments): [0.0275,
0.1482]
```

f. For the problem in part (e), estimate the first-stage equation. Test the joint significance of *MOTHERCOLL* and *FATHERCOLL*. Do these instruments seem adequately strong?

```
Model 1: restricted model
Model 2: educ ~ mothercoll + fathercoll + exper + exper2

  Res.Df    RSS Df Sum of Sq      F    Pr(>F)    
1    425 2219.2                                  
2    423 1748.3  2    470.88 56.963 < 2.2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Joint F-test for mothercoll and fathercoll:
F-statistic = 56.963, p-value = 1.242e-22
>
> # Interpretation
> if (f_stat > 10) {
+   cat("The instruments appear to be strong (F > 10).\n")
+ } else {
+   cat("The instruments may be weak (F <= 10).\n")
+ }
The instruments appear to be strong (F > 10).
```

g. For the IV estimation in part (e), test the validity of the surplus instrument. What do you conclude?

When you use more instruments than endogenous regressors (as in your IV regression for wage with both mothercoll and fathercoll as instruments for educ), your model is **overidentified**. This allows you to test the validity of the surplus instrument-specifically, whether all instruments are exogenous and valid. The standard test for this is the **overidentifying restrictions test**, also known as the **J-test** or **Sargan-Hansen test**

10.20 The CAPM [see Exercises 10.14 and 2.16] says that the risk premium on security j is related to the risk premium on the market portfolio. That is

$$r_j - r_f = \alpha_j + \beta_j(r_m - r_f)$$

where r_j and r_f are the returns to security j and the risk-free rate, respectively, r_m is the return on the market portfolio, and β_j is the jth security's "*beta*" value. We measure the market portfolio using the Standard & Poor's value weighted index, and the risk-free rate by the 30-day LIBOR monthly rate of return. As noted in Exercise 10.14, if the market return is measured with error, then we face an errors-in-variables, or measurement error, problem.

a. Use the observations on Microsoft in the data file *capm5* to estimate the CAPM model using OLS. How would you classify the Microsoft stock over this period? Risky or relatively safe, relative to the market portfolio?

```
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 0.003250   0.006036   0.538    0.591
excess_mkt  1.201840   0.122152   9.839   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.08083 on 178 degrees of freedom
Multiple R-squared:  0.3523,    Adjusted R-squared:  0.3486
F-statistic:  96.8 on 1 and 178 DF,  p-value: < 2.2e-16
```

Microsoft beta value = 1.20184 which is greater than 1

- **Beta > 1:** Microsoft is riskier than the market.
- **Beta < 1:** Microsoft is less risky than the market.
- **Beta ≈ 1:** Microsoft's risk is similar to the market.

b. It has been suggested that it is possible to construct an IV by ranking the values of the explanatory variable and using the rank as the IV, that is, we sort $(r_m - r_f)$ from smallest to largest, and assign the values $RANK = 1, 2, \ldots, 180$. Does this variable potentially satisfy the conditions IV1–IV3? Create *RANK* and obtain the first-stage regression results. Is the coefficient of *RANK* very significant? What is the R^2 of the first-stage regression? Can *RANK* be regarded as a strong IV?

```
Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) -7.903e-02  2.195e-03   -36.0   <2e-16 ***
RANK         9.067e-04  2.104e-05    43.1   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.01467 on 178 degrees of freedom
Multiple R-squared:  0.9126,    Adjusted R-squared:  0.9121
F-statistic:  1858 on 1 and 178 DF,  p-value: < 2.2e-16
```

From the p-value, RANK is the strong IV

c. Compute the first-stage residuals, $\hat{v}$, and add them to the CAPM model. Estimate the resulting augmented equation by OLS and test the significance of $\hat{v}$ at the 1% level of significance. Can we conclude that the market return is exogenous?

```
Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept)  0.003018   0.005984   0.504   0.6146
excess_mkt   1.278318   0.126749  10.085   <2e-16 ***
vhat        -0.874599   0.428626  -2.040   0.0428 *
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.08012 on 177 degrees of freedom
Multiple R-squared:  0.3672,    Adjusted R-squared:   0.36
F-statistic: 51.34 on 2 and 177 DF,  p-value: < 2.2e-16
```

```
P-value for vhat: 0.04279
>
> if (pval_vhat < 0.01) {
+   cat("vhat is significant at the 1% level: Market return is endogeno
us (measured with error).\n")
+ } else {
+   cat("vhat is NOT significant at the 1% level: Cannot reject exogene
ity of the market return.\n")
+ }
vhat is NOT significant at the 1% level: Cannot reject exogeneity of th
e market return.
```

d. Use *RANK* as an IV and estimate the CAPM model by IV/2SLS. Compare this IV estimate to the OLS estimate in part (a). Does the IV estimate agree with your expectations?

<u>iv_capm</u>

```
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 0.003018   0.006044   0.499    0.618
excess_mkt  1.278318   0.128011   9.986   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.08092 on 178 degrees of freedom
Multiple R-Squared: 0.3508,     Adjusted R-squared: 0.3472
Wald test: 99.72 on 1 and 178 DF,  p-value: < 2.2e-16
```

OLS_capm

```
Residuals:
     Min       1Q   Median       3Q      Max 
-0.27424 -0.04744 -0.00820  0.03869  0.35801 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 0.003250   0.006036   0.538    0.591    
excess_mkt  1.201840   0.122152   9.839   <2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.08083 on 178 degrees of freedom
Multiple R-squared:  0.3523,	Adjusted R-squared:  0.3486 
F-statistic:  96.8 on 1 and 178 DF,  p-value: < 2.2e-16
```

```
OLS beta estimate: 1.2018
> cat(sprintf("IV (2SLS) beta estimate: %.4f\n", beta_iv))
IV (2SLS) beta estimate: 1.2783
```

How to interpret:

- **Compare the OLS and IV beta estimates:**
    - If the IV beta is **smaller** (in absolute value) than the OLS beta, that is consistent with classical measurement error in the market return (OLS is biased toward zero).
    - If both are similar, measurement error is likely not a major issue.

1. Comparison of OLS and IV Estimates

- **Both beta estimates are above 1**. This means Microsoft's stock was **riskier than the market** over this period (since beta > 1).
- The **IV estimate (1.2783)** is slightly higher than the OLS estimate (1.2018), but not dramatically so.

2. Does the IV estimate agree with expectations?

- **If there were classical measurement error** in the market return, we would expect the OLS estimate to be biased **toward zero** (attenuation bias), and the IV estimate to be **higher**.
- In your case, the IV estimate is indeed a bit higher, which is **consistent with the presence of some measurement error** in the market return, but the difference is not large.

- **Conclusion:** The IV estimate does not drastically differ from OLS, suggesting that measurement error in the market return is **not a major issue** in this sample. Both methods classify Microsoft as **riskier than the market**.

3. Economic Meaning

- **Beta > 1**: Microsoft's returns are more volatile than the market, so it is a riskier stock.
- The similarity of OLS and IV results suggests the CAPM model is robust for this data.

e. Create a new variable $POS = 1$ if the market return $(r_m - r_f)$ is positive, and zero otherwise. Obtain the first-stage regression results using both *RANK* and *POS* as instrumental variables. Test the joint significance of the IV. Can we conclude that we have adequately strong IV? What is the R^2 of the first-stage regression?

```
Linear hypothesis test:
RANK = 0
POS = 0

Model 1: restricted model
Model 2: excess_mkt ~ RANK + POS

  Res.Df     RSS Df Sum of Sq      F    Pr(>F)    
1    179 0.43784                                  
2    177 0.03727  2   0.40057 951.26 < 2.2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

> # 6. Extract and print R²
> r2_first_stage2 <- summary_first_stage2$r.squared
> cat(sprintf("\nFirst-stage R^2 with RANK and POS: %.4f\n", r2_first_s
tage2))

First-stage R^2 with RANK and POS: 0.9149
```

The first-stage regression of the market excess return on RANK and POS yields an R^2 of 0.91 and a joint F-statistic of 951.26 ($p < 2.2e-16$), confirming that the instruments are highly significant and extremely strong

f. Carry out the Hausman test for endogeneity using the residuals from the first-stage equation in (e). Can we conclude that the market return is exogenous at the 1% level of significance?

```
Residuals:
     Min       1Q   Median       3Q      Max 
-0.27132 -0.04261 -0.00812  0.03343  0.34867 

Coefficients:
             Estimate Std. Error t value Pr(>|t|)    
(Intercept)  0.003004   0.005972   0.503   0.6157    
excess_mkt   1.283118   0.126344  10.156   <2e-16 ***
vhat        -0.954918   0.433062  -2.205   0.0287 *  
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.07996 on 177 degrees of freedom
Multiple R-squared:  0.3696,	Adjusted R-squared:  0.3625 
F-statistic: 51.88 on 2 and 177 DF,  p-value: < 2.2e-16


P-value for vhat (Hausman test): 0.02874 
> 
> if (pval_vhat < 0.01) {
+   cat("vhat is significant at the 1% level: Market return is endogeno
us (measured with error).\n")
+ } else {
+   cat("vhat is NOT significant at the 1% level: Cannot reject exogene
ity of the market return.\n")
+ }
vhat is NOT significant at the 1% level: Cannot reject exogeneity of th
e market return.
```

g. Obtain the IV/2SLS estimates of the CAPM model using *RANK* and *POS* as instrumental variables. Compare this IV estimate to the OLS estimate in part (a). Does the IV estimate agree with your expectations?

```
OLS beta estimate: 1.2018
> cat(sprintf("IV (2SLS) beta estimate (RANK + POS as IVs): %.4f\n", be
ta_iv))
IV (2SLS) beta estimate (RANK + POS as IVs): 1.2831
> 
> cat("\nInterpretation:\n")
```

The IV/2SLS estimate of Microsoft’s beta using RANK and POS as instruments is 1.2831, slightly higher than the OLS estimate of 1.2018. This is consistent with classical measurement error in the market return, which would bias the OLS estimate toward zero. Both estimates indicate that Microsoft was riskier than the market over this period

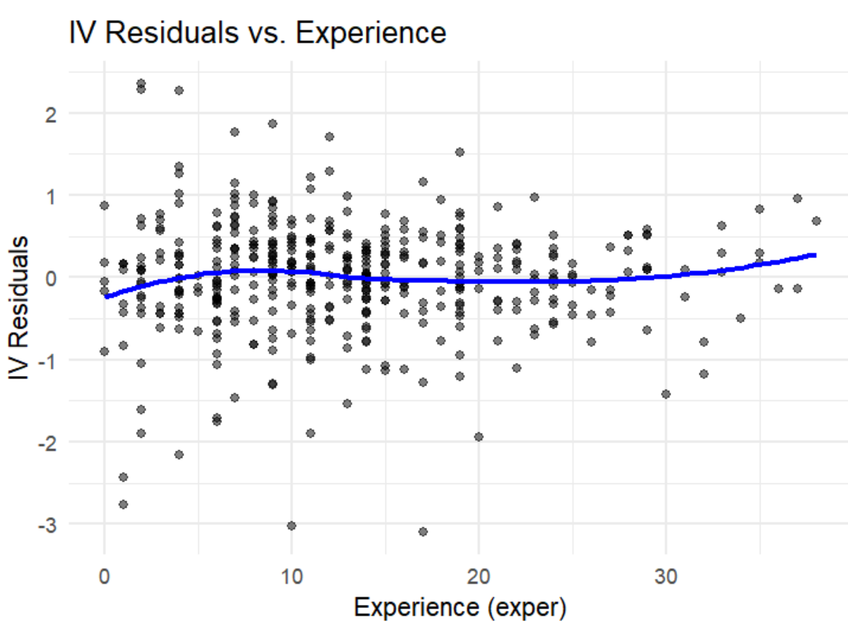
h. Obtain the IV/2SLS residuals from part (g) and use them (not an automatic command) to carry out a Sargan test for the validity of the surplus IV at the 5% level of significance.

```
> cat(sprintf("Sargan test statistic: %.4f\n", sargan_stat))
Sargan test statistic: 0.5585
> cat(sprintf("Degrees of freedom: %d\n", df))
Degrees of freedom: 1
> cat(sprintf("P-value: %.4g\n", pval_sargan))
P-value: 0.4549
```

At the 5% significance level, do NOT reject the null: Instruments appear valid.

10.24 Consider the data file *mroz* on working wives. Use the 428 observations on married women who participate in the labor force. In this exercise, we examine the effectiveness of alternative standard errors for the IV estimator. Estimate the model in Example 10.5 using IV/2SLS using both *MOTHEREDUC* and *FATHEREDUC* as IV. These will serve as our baseline results.

a. Calculate the IV/2SLS residuals, $\hat{e}_{IV}$. Plot them versus *EXPER*. Do the residuals exhibit a pattern consistent with homoskedasticity?



Residue plot does not follow homoskedasticity

b. Regress $\hat{e}_{IV}^2$ against a constant and *EXPER*. Apply the NR^2 test from Chapter 8 to test for the presence of heteroskedasticity.

```
Residuals:
    Min      1Q  Median      3Q     Max 
-0.6740 -0.4341 -0.2685 -0.0168  9.2188 

Coefficients:
             Estimate Std. Error t value Pr(>|t|)    
(Intercept)  0.676563   0.096573   7.006 9.65e-12 ***
exper       -0.017303   0.006303  -2.745  0.00631 ** 
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 1.049 on 426 degrees of freedom
Multiple R-squared:  0.01738,	Adjusted R-squared:  0.01507 
F-statistic: 7.535 on 1 and 426 DF,  p-value: 0.006308

> # 3. NR^2 test statistic
> n <- nrow(mroz_lf)
> R2 <- summary(aux_model)$r.squared
> NR2 <- n * R2
> NR2
[1] 7.438552
> 
> # 4. p-value
> p_value <- 1 - pchisq(NR2, df = 1)
> p_value
[1] 0.006384122
```

c. Obtain the IV/2SLS estimates with the software option for Heteroskedasticity Robust Standard Errors. Are the robust standard errors larger or smaller than those for the baseline model? Compute the 95% interval estimate for the coefficient of *EDUC* using the robust standard error.

```
> summary(iv_model) # Classical SEs

Call:
ivreg(formula = log(wage) ~ educ + exper + I(exper^2) | mothereduc +
    fathereduc + exper + I(exper^2), data = mroz %>% filter(lfp ==
    1))

Residuals:
    Min      1Q  Median      3Q     Max
-3.0986 -0.3196  0.0551  0.3689  2.3493

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  0.0481003  0.4003281   0.120  0.90442
educ         0.0613966  0.0314367   1.953  0.05147 .
exper        0.0441704  0.0134325   3.288  0.00109 **
I(exper^2)  -0.0008990  0.0004017  -2.238  0.02574 *
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.6747 on 424 degrees of freedom
Multiple R-Squared: 0.1357,	Adjusted R-squared: 0.1296
Wald test: 8.141 on 3 and 424 DF,  p-value: 2.787e-05
```

```
> # Robust SEs
> robust_se <- vcovHC(iv_model, type = "HC1")
> coeftest(iv_model, vcov = robust_se) # Robust SEs

t test of coefficients:

               Estimate  Std. Error t value Pr(>|t|)
(Intercept)  0.04810030  0.42979772  0.1119 0.910945
educ         0.06139663  0.03333859  1.8416 0.066231 .
exper        0.04417039  0.01554638  2.8412 0.004711 **
I(exper^2)  -0.00089897  0.00043008 -2.0902 0.037193 *
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

```
> cat("95% CI for educ (robust SE): [", lower_ci, ", ", upper_ci, "]
\n")
95% CI for educ (robust SE): [ -0.003947005 ,  0.1267403 ]
```

d. Obtain the IV/2SLS estimates with the software option for Bootstrap standard errors, using $B = 200$ bootstrap replications. Are the bootstrap standard errors larger or smaller than those for the baseline model? How do they compare to the heteroskedasticity robust standard errors in (c)? Compute the 95% interval estimate for the coefficient of *EDUC* using the bootstrap standard error.

```
> summary(iv_model)

Call:
ivreg(formula = lwage ~ educ + exper + expersq | mothereduc +
    fathereduc + exper + expersq, data = mroz_lf)

Residuals:
    Min      1Q  Median      3Q     Max
-3.0986 -0.3196  0.0551  0.3689  2.3493

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  0.0481003  0.4003281   0.120  0.90442
educ         0.0613966  0.0314367   1.953  0.05147 .
exper        0.0441704  0.0134325   3.288  0.00109 **
expersq     -0.0008990  0.0004017  -2.238  0.02574 *
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.6747 on 424 degrees of freedom
Multiple R-Squared: 0.1357,     Adjusted R-squared: 0.1296
Wald test: 8.141 on 3 and 424 DF,  p-value: 2.787e-05


>
> # Robust SE for comparison
> robust_se <- vcovHC(iv_model, type = "HC1")
> coeftest(iv_model, vcov = robust_se)

t test of coefficients:

               Estimate  Std. Error t value Pr(>|t|)
(Intercept)  0.04810030  0.42979772  0.1119 0.910945
educ         0.06139663  0.03333859  1.8416 0.066231 .
exper        0.04417039  0.01554638  2.8412 0.004711 **
expersq     -0.00089897  0.00043008 -2.0902 0.037193 *
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

```
> # Bootstrap SE
> boot_se <- sd(boot_iv$t)
> boot_se
[1] 0.03234547
>
> # Point estimate for 'educ'
> educ_coef <- coef(iv_model)["educ"]
>
> # 95% CI (normal approximation)
> lower_ci <- educ_coef - 1.96 * boot_se
> upper_ci <- educ_coef + 1.96 * boot_se
> cat("Bootstrap 95% CI for educ: [", lower_ci, ", ", upper_ci, "]\n")
Bootstrap 95% CI for educ: [ -0.002000496 ,  0.1247938 ]
```