

10.2

a) $\beta_1 > 0$

$\beta_2, \beta_3 > 0$

β_4 can be positive or negative

$\beta_5, \beta_6 < 0$

b) Wage can be endogenous since it can be influenced by unobserved factors such as motivation, ability, job experience

e may capture these unobserved factors thus $E(e|wages) = 0$ does not hold anymore. \Rightarrow the estimators are biased and inconsistent

c) Expert and Expert² satisfy both requirements of an IV

d) IV: expert and expert² } The equation is overidentified
Endogenous: Wages \Rightarrow need the test for IV validity

e) 1st stage Wages = $\alpha_1 + \alpha_2 \text{expert} + \alpha_3 \text{expert}^2 + \alpha_4 \text{educ} + \alpha_5 \text{age} + \alpha_6 \text{kids} + \dots$

2nd stage Hours = $\beta_1 + \beta_2 \text{wages} + \beta_3 \text{educ} + \beta_4 \text{kids} + \beta_5 \text{whiteinc} + \epsilon$

(compare to OLS results)

10.3.

a) $x = \gamma_1 + \theta_1 z + v$

$$\theta_1 = \frac{\text{Cov}(z, x)}{\text{Var}(z)} = \frac{\text{Cov}(z, u)}{\text{Var}(z)}$$

Since $\text{Cov}(v, z) = 0$ so

$$\theta_1 = \frac{\text{Cov}(z, x)}{\text{Var}(z)}$$

b) $y = \beta_1 + \beta_2 x + e = \beta_1 + \beta_2 (\gamma_1 + \theta_1 z + v) + e$
 $= (\beta_1 + \beta_2 \gamma_1) + \beta_2 \theta_1 z + (\beta_2 v + e)$
 $= \pi_0 + \pi_1 z + u$

d) $\pi_1 = \beta_2 \theta_1 \Rightarrow \beta_2 = \pi_1 / \theta_1$

e) $\theta_1 = \frac{\sum (x_i - \bar{x})(z_i - \bar{z})}{\sum (z_i - \bar{z})^2} \Rightarrow \frac{\text{Cov}(z, x)}{\text{Var}(z)} = \phi_1$

$$\pi_1 = \frac{\sum (y_i - \bar{y})(z_i - \bar{z})}{\sum (z_i - \bar{z})^2} \Rightarrow \frac{\text{Cov}(y, z)}{\text{Var}(z)} = \pi_1$$

$$\hat{\beta}_2 = \frac{\hat{\pi}_1}{\hat{\theta}_1} \Rightarrow \beta_2 = \frac{\phi_1}{\phi_1}$$