

(a) $\beta_1 \Rightarrow$ zero or positive \Rightarrow hours can't be negative.

$\beta_2 \Rightarrow$ positive. It will be more attractive if wage is higher.

$\beta_3 \Rightarrow$ positive. high-level education will lead to more willing and ability in job.

$\beta_4 \Rightarrow$ not sure. getting older may lead to more experience. But maybe their productivity will be lower due to age.

$\beta_5 \Rightarrow$ negative. It may cause them focus on children rather than work.

$\beta_6 \Rightarrow$ negative. If other family member has the financial capability may cause women don't have the pressure to working.

(b) 因為 Wage 可能有內生性: hours, wage 之間存在 simultaneity bias or omitted variable bias. 例如工作能力同時影響工資、工作時數, 讓 OLS 估計有偏。

(c) Relevance: EXPER 與 WAGE 高度相關. EXPER 越多, Wage 越高

Exogeneity: 假設 EXPER, EXPER² 直接影響 HOURS 的唯一途徑是 WAGE, 並未直接影響 HOURS.

(d) Yes. Because the instruments \neq endogenous variable.
(EXPER, EXPER²) (WAGE)

(e) Step 1 = Regress that

$$WAGE = \gamma_1 + \gamma_2 EDUC + \gamma_3 AGE + \gamma_4 KIDSL6 + \gamma_5 NWIFEING + \gamma_6 EXPER + \gamma_7 EXPER^2 + e$$

Step 2: use the \widehat{WAGE} to regress HOURS and OLS estimate.

$$HOURS = \beta_1 + \beta_2 \widehat{WAGE} + \beta_3 EDUC + \beta_4 AGE + \beta_5 KIDSL6 + \beta_6 NWIFEING + e$$

10.3

$$(a) X = \gamma_1 + \theta_1 Z + V$$

$$\rightarrow E(X) = \gamma_1 + \theta_1 E(Z)$$

$$\theta_1 [Z - E(Z)] + V$$

$$[X - E(X)][Z - E(Z)] = \theta_1 [Z - E(Z)]^2 + [Z - E(Z)] \cdot V$$

$$E([X - E(X)][Z - E(Z)]) = \theta_1 E[Z - E(Z)]^2 + E([Z - E(Z)] \cdot V)$$

$$E([Z - E(Z)][X - E(X)]) = \theta_1 E([Z - E(Z)]^2) \Rightarrow \theta_1 = \frac{\text{cov}(Z, X)}{\text{var}(Z)}$$

$$(b) Y = \pi_0 + \pi_1 Z + u$$

$$\rightarrow E(Y) = \pi_0 + \pi_1 E(Z)$$

$$\pi_1 [Z - E(Z)] + u$$

$$[Z - E(Z)][Y - E(Y)] = \pi_1 [Z - E(Z)]^2 + [Z - E(Z)] \cdot u$$

$$E([Z - E(Z)][Y - E(Y)]) = \pi_1 E[Z - E(Z)]^2 + E([Z - E(Z)] \cdot u) \Rightarrow \pi_1 = \frac{\text{cov}(Z, Y)}{\text{var}(Z)}$$

$$(c) Y = \beta_1 + \beta_2 (\gamma_1 + \theta_1 Z + V) + e = \underbrace{(\beta_1 + \beta_2 \gamma_1)}_{\pi_0} + \underbrace{(\beta_2 \theta_1)}_{\pi_1} Z + \underbrace{(\beta_2 V + e)}_u$$

$$(d) \pi_1 = \beta_2 \theta_1 \Rightarrow \beta_2 = \frac{\pi_1}{\theta_1}$$

$$(e) \hat{\theta}_1 = \frac{\widehat{\text{cov}}(Z, X)}{\widehat{\text{var}}(Z)} = \frac{\sum (z_i - \bar{z})(x_i - \bar{x})}{\sum (z_i - \bar{z})^2}$$

$$\hat{\pi}_1 = \frac{\widehat{\text{cov}}(Z, Y)}{\widehat{\text{var}}(Z)} = \frac{\sum (z_i - \bar{z})(y_i - \bar{y})}{\sum (z_i - \bar{z})^2}$$

$$\hat{\beta}_2 = \frac{\hat{\pi}_1}{\hat{\theta}_1} = \frac{\widehat{\text{cov}}(Z, Y)}{\widehat{\text{cov}}(Z, X)} \xrightarrow{P} \frac{\text{cov}(Z, Y)}{\text{cov}(Z, X)} = \beta_2$$