

This exercise uses data from the STAR experiment introduced to illustrate fixed and random effects for grouped data. In the STAR experiment, children were randomly assigned within schools into three types of classes: small classes with 13–17 students, regular-sized classes with 22–25 students, and regular-sized classes with a full-time teacher aide to assist the teacher. Student scores on achievement tests were recorded as well as some information about the students, teachers, and schools. Data for the kindergarten classes are contained in the data file *star*.

- d. Reestimate the model in part (a) with school random effects. Compare the results with those from parts (a) and (b). Are there any variables in the equation that might be correlated with the school effects? Use the LM test for the presence of random effects.

Ans.

整體來看，random effects 的估計看起來與 OLS 的估計非常接近，也與 fixed effects 的估計十分類似。

所有變數變化不大，跟 school effects 相關的可能性小。

LM test

$H_0: \sigma_u^2 = 0$ (No random effects)

$H_1: \sigma_u^2 > 0$ (Random effects exist)

LM 統計量 (Chi-squared) 為 6677.4 > 臨界值

3.84，且 p 值遠小於 0.05 顯著水準。

因此，拒絕虛無假設 $H_0: \sigma_u^2 = 0$ ，表示模型中存在顯著的 random effects，存在 school-level unobserved heterogeneity。

```
> qchisq(p = 0.95, df = 1)
[1] 3.841459
```

```
Lagrange Multiplier Test - (Breusch-Pagan)
data: readscore ~ small + aide + tchexper + boy + white_asian + freelunch
chisq = 6677.4, df = 1, p-value < 2.2e-16
alternative hypothesis: significant effects
```

```
Oneway (individual) effect Random Effect Model
(Swamy-Arora's transformation)

Call:
plm(formula = readscore ~ small + aide + tchexper + boy + white_asian +
      freelunch, data = pdata, effect = "individual", model = "random")

Unbalanced Panel: n = 79, T = 34-137, N = 5766

Effects:
              var std.dev share
idiosyncratic 751.43   27.41 0.829
individual    155.31   12.46 0.171
theta:
      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
0.6470  0.7225   0.7523   0.7541  0.7831   0.8153

Residuals:
      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
-97.483 -17.236   -3.282    0.037  12.803  192.346

Coefficients:
              Estimate Std. Error z-value Pr(>|z|)
(Intercept)  436.126774   2.064782  211.2217 < 2.2e-16 ***
small         6.458722    0.912548   7.0777 1.466e-12 ***
aide         0.992146    0.881159   1.1260 0.2602
tchexper     0.302679    0.070292   4.3060 1.662e-05 ***
boy        -5.512081    0.727639  -7.5753 3.583e-14 ***
white_asian  7.350477    1.431376   5.1353 2.818e-07 ***
freelunch   -14.584332   0.874676 -16.6740 < 2.2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Total Sum of Squares:    6158000
Residual Sum of Squares: 4332100
R-Squared:               0.29655
Adj. R-Squared:          0.29582
Chisq: 493.205 on 6 DF, p-value: < 2.22e-16
```

- e. Using the t -test statistic in equation (15.36) and a 5% significance level, test whether there are any significant differences between the fixed effects and random effects estimates of the coefficients on *SMALL*, *AIDE*, *TCHEXPER*, *BOY*, *WHITE_ASIAN*, and *FREELUNCH*. What are the implications of the test outcomes? What happens if we apply the test to the fixed and random effects estimates of the coefficient on *BOY*?

Ans.

Hausman test

$H_0: \beta_{FE,k} = \beta_{RE,k}$ (No endogeneity)

$H_1: \beta_{FE,k} \neq \beta_{RE,k}$ (Endogeneity)

因為 13.809 > 臨界值 = 12.59 → 拒絕虛無假設

應拒絕使用隨機效果模型 (RE)，因為其估計結果可能有偏誤。

推薦使用固定效果模型 (FE) 作為較穩健的選擇。

```
> qchisq(p = 0.95, df = 6)
[1] 12.59159
```

```
Hausman Test

data: readscore ~ small + aide + tchexper + boy + white_asian + freelunch
chisq = 13.809, df = 6, p-value = 0.03184
alternative hypothesis: one model is inconsistent
```

$$t = \frac{b_{FE,k} - b_{RE,k}}{[se(b_{FE,k})^2 - se(b_{RE,k})^2]^{1/2}}$$

個別變數的檢定結果 (t 檢定) 如右圖

係數 *BOY* 因 $se_{FE}^2 < se_{RE}^2$ ，會產生無意義的負數平方根

→ NaN，不適用 t 統計檢定

small	: t-value = 1.1460, p-value = 0.2518
aide	: t-value = 0.1284, p-value = 0.8978
tchexper	: t-value = -1.9377, p-value = 0.0527
white_asian	: t-value = 1.2181, p-value = 0.2232
freelunch	: t-value = -0.0956, p-value = 0.9239
boy	: t-value = NaN, p-value = NaN

- f. Create school-averages of the variables and carry out the Mundlak test for correlation between them and the unobserved heterogeneity.

Ans.

根據 Mundlak 檢定，在所有學校平均變數中，僅有學生性別的學校平均值 *boy_m* 在 5% 顯著水準下達到統計顯著 ($p = 0.0344$)，與學校特定效果 (individual effect) 具有顯著相關。表示變數 *boy* 違反 Random Effects 模型的外生性假設，其估計結果可能有偏。變數 *boy* 應改採 Fixed Effects 模型處理。其他變數如 *small*, *aide*, *tchexper*, *freelunch* 及其學校平均值皆不具顯著性，代表它們與未觀察異質性無明顯相關，仍適用 RE 模型估計。

```
Oneway (individual) effect Random Effect Model
(Swamy-Arora's transformation)

Call:
plm(formula = readscore ~ small + aide + tchexper + boy + white_asian +
      freelunch + small_m + aide_m + tchexper_m + boy_m + white_asian_m +
      freelunch_m, data = pdata_clean, model = "random")

Unbalanced Panel: n = 78, T = 34-136, N = 5681

Effects:
              var std.dev share
idiosyncratic 756.11  27.50 0.817
individual    169.40  13.02 0.183
theta:
      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
0.6593  0.7327  0.7615  0.7630  0.7892  0.8217

Residuals:
      Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
-98.886 -17.051  -3.166   0.039  12.846  193.321

Coefficients:
              Estimate Std. Error z-value Pr(>|z|)
(Intercept)  459.462989  20.529888  22.3802 < 2.2e-16 ***
small         6.637460   0.922068   7.1985 6.090e-13 ***
aide          1.157620   0.889542   1.3014  0.1931
tchexper      0.289286   0.071754   4.0316 5.539e-05 ***
boy          -5.386109   0.735063  -7.3274 2.346e-13 ***
white_asian    8.081423   1.550155   5.2133 1.855e-07 ***
freelunch     -14.699025   0.892109 -16.4767 < 2.2e-16 ***
small_m       -18.410060  22.273923  -0.8265  0.4085
aide_m        16.811358  20.793685   0.8085  0.4188
tchexper_m     1.006007   0.625690   1.6078  0.1079
boy_m        -53.353521  25.221654  -2.1154  0.0344 *
white_asian_m  -6.648191   6.320012  -1.0519  0.2928
freelunch_m   -3.318853   8.779553  -0.3780  0.7054
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Total Sum of Squares:    6007200
Residual Sum of Squares: 4281300
R-Squared:              0.28737
Adj. R-Squared:         0.28586
Chisq: 500.306 on 12 DF, p-value: < 2.22e-16
```