11.1

(a) ① $y_1 = \alpha_1 y_2 + e_1$

② $y_2 = \alpha_2 y_1 + \beta_1 x_1 + \beta_2 x_2 + e_2 \Rightarrow y_2 = \alpha_2(\alpha_1 y_2 + e_1) + \beta_1 x_1 + \beta_2 x_2 + e_2$
$= \alpha_1\alpha_2 y_2 + \alpha_2 e_1 + \beta_1 x_1 + \beta_2 x_2 + e_2 \Rightarrow y_2(1-\alpha_1\alpha_2) = \beta_1 x_1 + \beta_2 x_2 + e_2 + \alpha_2 e_1$

$$\Rightarrow y_2 = \frac{\beta_1}{1-\alpha_1\alpha_2}x_1 + \frac{\beta_2}{1-\alpha_1\alpha_2}x_2 + \frac{e_2+\alpha_2 e_1}{1-\alpha_1\alpha_2} = \pi_1 x_1 + \pi_2 x_2 + v_2 .$$

$$cov(y_2, e_1 | x) = E(y_2, e_1 | x)$$
$$= E\left[\left(\frac{\beta_1}{1-\alpha_1\alpha_2}x_1 + \frac{\beta_2}{1-\alpha_1\alpha_2}x_2 + \frac{\alpha_2 e_1 + e_2}{1-\alpha_1\alpha_2}\right) e_1 \Big| x\right]$$
$$= E\left[\left(\frac{\beta_1}{1-\alpha_1\alpha_2}x_1 e_1 | x\right] + E\left[\left(\frac{\beta_2}{1-\alpha_1\alpha_2}x_2 e_1 | x\right] + E\left[\left(\frac{\alpha_2 e_1 + e_2}{1-\alpha_1\alpha_2} e_1 | x\right]$$
$$= E\left[\left(\frac{\alpha_2 e_1 + e_2}{1-\alpha_1\alpha_2} e_1 | x\right] = \frac{\alpha_2 E(e_1^2|x) + E(e_1 e_2|x)}{1-\alpha_1\alpha_2}$$
$$= \frac{\alpha_2 \sigma_1^2}{1-\alpha_1\alpha_2}$$

(b) Equation ① and ② have an endogeneous variable

$\Rightarrow$ OLS is biased and consistent.

(c) $M=2 \Rightarrow M-1=1$. So it requires at least 1 omitted variable to be omitted from equation ① and ②

equation ① is identified (two engeneous variables)

equation ② is not identified (no variables)

(d) $E(x_{i1} v_{i1}|x) = E(x_{i2} v_{i2}|x) = 0 \Rightarrow E\left[x_{ik}\left(\frac{\alpha_2 e_1+e_2}{1-\alpha_1\alpha_2}|x\right)\right] = E\left[\frac{1}{1-\alpha_1\alpha_2}x_{ik}e_2|x\right] + E\left[\frac{\alpha_2}{1-\alpha_1\alpha_2}x_{ik}e_1|x\right] = 0$

(e) $\frac{\partial S(\pi_1,\pi_2|y\ x)}{\partial \pi_1} = 2\Sigma(y_2 - \pi_1 x_1 - \pi_2 x_2)x_1 = 0 \Rightarrow N^{-1}\Sigma x_{i1}(y_2 - \pi_1 x_{i1} - \pi_2 x_{i2}) = 0$

$\frac{\partial S(\pi_1,\pi_2|y\ x)}{\partial \pi_2} = 2\Sigma(y_2 - \pi_1 x_1 - \pi_2 x_2)x_2 = 0 \Rightarrow N^{-1}\Sigma x_{i2}(y_2 - \pi_1 x_{i1} - \pi_2 x_{i2}) = 0$

(f) $N^{-1}\Sigma x_{i1}(y_2 - \pi_1 x_{i1} - \pi_2 x_{i2}) = 0 \Rightarrow \Sigma x_{i1}y_{i2} - \pi_1\Sigma x_{i1}^2 - \pi_2\Sigma x_{i1}x_{i2} = 0 \Rightarrow 3 - \pi_1 = 0,\ \pi_1 = 3$

$N^{-1}\Sigma x_{i2}(y_2 - \pi_1 x_{i1} - \pi_2 x_{i2}) = 0 \Rightarrow \Sigma x_{i2}y_{i2} - \pi_2\Sigma x_{i2}^2 - \pi_1\Sigma x_{i1}x_{i2} = 0 \Rightarrow 4 - \pi_2 = 0,\ \pi_2 = 4$

(g) $\hat{y}_2 = \hat\pi_1 x_1 + \hat\pi_2 x_2$, $(\hat y_2, e_1) = \Sigma\hat y_2(y_1 - \alpha_1 y_2) = 0 \Rightarrow \Sigma\hat y_2 y_1 - \alpha_1\Sigma\hat y_2 y_2 = 0 \Rightarrow \alpha_1 = \frac{\Sigma\hat y_2 y_1}{\Sigma\hat y_2 y_2}$

$$\Rightarrow \hat\alpha_1 = \frac{\Sigma(\hat\pi_1 x_1 + \hat\pi_2 x_2)y_1}{\Sigma(\hat\pi_1 x_1 + \hat\pi_2 x_2)y_2} = \frac{3\cdot2+4\cdot3}{3\cdot3+4\cdot4} = \frac{18}{25}$$

(h) $\hat v_2 = y_2 - \hat y_2 \Rightarrow \hat y_2 = y_2 - \hat v_2$

$\Sigma\hat y_2^2 = \Sigma\hat y_2(y_2 - \hat v_2) = \Sigma\hat y_2 y_2 - \Sigma\hat y_2\hat v_2 = \Sigma\hat y_2 y_2$

$\boxed{11.16}$

(a) $\alpha_1+\alpha_2 P_i + e_{di} = \beta_1+\beta_2 P_i + \beta_3 W_i + e_{si}$

$\Rightarrow (\alpha_2-\beta_2)P_i = (\beta_1-\alpha_1)+\beta_3 W_i + (e_{si}-e_{di})$

$\Rightarrow P_i = \frac{\beta_1-\alpha_1}{\alpha_2-\beta_2} + \frac{\beta_3}{\alpha_2-\beta_2}W_i + \frac{e_{si}-e_{di}}{\alpha_2-\beta_2}$

$Q_i = \alpha_1+\alpha_2\left(\frac{\beta_1-\alpha_1}{\alpha_2-\beta_2} + \frac{\beta_3}{\alpha_2-\beta_2}W_i + \frac{e_{si}-e_{di}}{\alpha_2-\beta_2}\right)+e_{di}$

$= \alpha_1 + \frac{\beta_1-\alpha_1}{\alpha_2-\beta_2}\alpha_2 + \frac{\beta_3}{\alpha_2-\beta_2}W_i\alpha_2 + \frac{e_{si}-e_{di}}{\alpha_2-\beta_2}\alpha_2 + e_{di}$

(b) $M-1=1 \Rightarrow$ at least 1 variable omitted. equation ① is identified (two variables)

equation ② isn't identified (no variables)

(c) $\begin{cases}\hat{Q} = 5+0.5W \\ \hat{P} = 2.4+W\end{cases}$

$5+0.5W = \alpha_1+\alpha_2(2.4+W) = \alpha_1+2.4\alpha_2+\alpha_2 W$

$\Rightarrow \alpha_2 = 0.5,\ \alpha_1 = 3.8$

(d) $\hat{P} = 2.4+W$

	$\hat{P}$	$\hat{P}-\bar{P}$	$Q-\bar{Q}$
W= 2	4.4	0	-2
W= 3	5.4	1	0
W= 1	3.4	-1	3
W= 1	3.4	-1	-3
W= 3	5.4	1	2

$\Rightarrow \bar{P} = 4.4,\ \bar{Q} = 6$

$Q = \alpha_1+\alpha_2\hat{P}+e_i$

$\hat{\alpha}_2 = \frac{\Sigma(\hat{P}_i-\bar{P})(Q_i-\bar{Q})}{\Sigma(\hat{P}_i-\bar{P})^2} = \frac{-3+3+2}{4} = \frac{1}{2}$

$\hat{\alpha}_1 = 6-0.5\times 4.4 = 3.8 \Rightarrow \hat{Q} = 3.8+0.5P$

$\boxed{11.17}$

(a) $M-1=7$, at least 7 omitted variables

Consumption : 6 vairables and omit 10 variables

Investment : 5 variables and omit 11 variables ⇒ all fuctions are identified.

Wage : 5 variables and omit 11 variables

(b) Consumption : 2RHS endogenous vairables and excludes 5 exgenous variables

Investment : 1RHS endogenous variables and excludes 5 exgenous variables ⇒ all fuctions are satisfied.

Wage : 1 RHS endogenous variables and excludes 5 exgenous variables

(c) $W_{1t} = \pi_1 + \pi_2 G_t + \pi_3 W_{2t} + \pi_4 TX_t + \pi_5 TIME_t + \pi_6 P_{t-1} + \pi_7 K_{t-1} + \pi_8 E_{t-1} + V$

(d) We get $\hat{W}_{1t}$ from (c) and use the same way $\hat{P}_t$, then create $W_t^* = \hat{W}_{1t} + W_{2t}$ ⇒ Regress CN_t by OLS.

(e) The two coefficient estimates will be the same, but t-value will be different.