

11.28 Supply and demand curves as traditionally drawn in economics principles classes have price (P) on the vertical axis and quantity (Q) on the horizontal axis.

- Rewrite the truffle demand and supply equations in (11.11) and (11.12) with price P on the left-hand side. What are the anticipated signs of the parameters in this rewritten system of equations?
- Using the data in the file *truffles*, estimate the supply and demand equations that you have formulated in (a) using two-stage least squares. Are the signs correct? Are the estimated coefficients significantly different from zero?
- Estimate the price elasticity of demand "at the means" using the results from (b).
- Accurately sketch the supply and demand equations, with P on the vertical axis and Q on the horizontal axis, using the estimates from part (b). For these sketches set the values of the exogenous variables DI , PS , and PF to be $DI^* = 3.5$, $PF^* = 23$, and $PS^* = 22$.
- What are the equilibrium values of P and Q obtained in part (d)? Calculate the predicted equilibrium values of P and Q using the estimated reduced-form equations from Table 11.2, using the same values of the exogenous variables. How well do they agree?
- Estimate the supply and demand equations that you have formulated in (a) using OLS. Are the signs correct? Are the estimated coefficients significantly different from zero? Compare the results to those in part (b).

b.

2SLS estimates for 'demand' (equation 1)

Model Formula: $p \sim q + ps + di$

Instruments: $\sim ps + di + pf$

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	-11.42841	13.59161	-0.84084	0.4081026
q	-2.67052	1.17495	-2.27287	0.0315350 *
ps	3.46108	1.11557	3.10252	0.0045822 **
di	13.38992	2.74671	4.87490	4.6752e-05 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 13.165551 on 26 degrees of freedom

Number of observations: 30 Degrees of Freedom: 26

SSR: 4506.625289 MSE: 173.331742 Root MSE: 13.165551

Multiple R-Squared: 0.556717 Adjusted R-Squared: 0.505569

c.

$$\text{Demand: } Q_i = \alpha_1 + \alpha_2 P_i + \alpha_3 PS_i + \alpha_4 DI_i + e_{di}$$

$$\Rightarrow P_i = -\frac{\alpha_1}{\alpha_2} + \frac{1}{\alpha_2} Q_i - \frac{\alpha_3}{\alpha_2} PS_i - \frac{\alpha_4}{\alpha_2} DI_i - \frac{1}{\alpha_2} e_{di}$$

$$\text{Supply: } Q_i = \beta_1 + \beta_2 P_i + \beta_3 PF_i + e_{pf}$$

$$\Rightarrow P_i = -\frac{\beta_1}{\beta_2} + \frac{1}{\beta_2} Q_i - \frac{\beta_3}{\beta_2} PF_i - \frac{1}{\beta_2} e_{pf}$$

2SLS estimates for 'supply' (equation 2)

Model Formula: $p \sim q + pf$

Instruments: $\sim ps + di + pf$

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	-58.798223	5.859161	-10.0353	1.3165e-10 ***
q	2.936711	0.215772	13.6103	1.3212e-13 ***
pf	2.958486	0.155964	18.9690	< 2.22e-16 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 4.399078 on 27 degrees of freedom

Number of observations: 30 Degrees of Freedom: 27

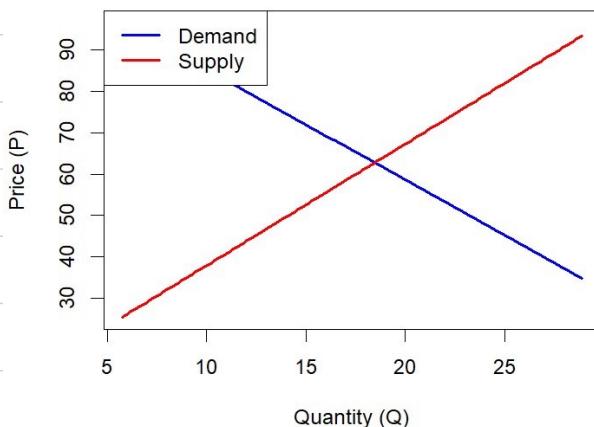
SSR: 522.500877 MSE: 19.351884 Root MSE: 4.399078

Multiple R-Squared: 0.948605 Adjusted R-Squared: 0.944798

All coefficient are exactly as predicts, except the intercept in demand equation.

d.

Supply and Demand for Truffles



f. Demand

lm(formula = p ~ q + ps + di, data = truffles)

Residuals:

Min	1Q	Median	3Q	Max
-25.0753	-2.7742	-0.4097	4.7079	17.4979

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	-13.6195	9.0872	-1.499	0.1460
q	0.1512	0.4988	0.303	0.7642
ps	1.3607	0.5940	2.291	0.0303 *
di	12.3582	1.8254	6.770	3.48e-07 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 8.814 on 26 degrees of freedom

Multiple R-squared: 0.8013, Adjusted R-squared: 0.7784

F-statistic: 34.95 on 3 and 26 DF, p-value: 2.842e-09

c.

q

-1.272464

```
> cat("Equilibrium Quantity (Q):", eq_Q, "\n")
Equilibrium Quantity (Q): 18.25021
> cat("Equilibrium Price (P):", eq_P, "\n")
Equilibrium Price (P): 62.84257
> cat("Reduced-form predicted Q:", Q_rf, "\n")
Reduced-form predicted Q: 18.45833
> cat("Reduced-form predicted P:", P_rf, "\n")
Reduced-form predicted P: 62.724
```

2 methods are similar, we can say they agree.

Supply

lm(formula = p ~ q + pf, data = truffles)

Residuals:

Min	1Q	Median	3Q	Max
-8.4721	-3.3287	0.1861	2.0785	10.7513

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	-52.8763	5.0238	-10.53	4.68e-11 ***
q	2.6613	0.1712	15.54	5.42e-15 ***
pf	2.9217	0.1482	19.71	< 2e-16 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 4.202 on 27 degrees of freedom

Multiple R-squared: 0.9531, Adjusted R-squared: 0.9496

F-statistic: 274.4 on 2 and 27 DF, p-value: < 2.2e-16

Except the intercept & q's coefficients, all the other coefficient are significant different from zero

- 11.30 Example 11.3 introduces Klein's Model I. Use the data file *klein* to answer the following questions.
- Estimate the investment function in equation (11.18) by OLS. Comment on the signs and significance of the coefficients.
 - Estimate the reduced-form equation for profits, P_t , using all eight exogenous and predetermined variables as explanatory variables. Test the joint significance of all the variables except lagged profits, P_{t-1} , and lagged capital stock, K_{t-1} . Save the residuals, \hat{v}_t , and compute the fitted values, \hat{P}_t .
 - The Hausman test for the presence of endogenous explanatory variables is discussed in Section 10.4.1. It is implemented by adding the reduced-form residuals to the structural equation and testing their significance, that is, using OLS estimate the model

$$I_t = \beta_1 + \beta_2 P_t + \beta_3 P_{t-1} + \beta_4 K_{t-1} + \delta \hat{v}_t + e_{2t}$$

Use a t -test for the null hypothesis $H_0: \delta = 0$ versus $H_1: \delta \neq 0$ at the 5% level of significance. By rejecting the null hypothesis, we conclude that P_t is endogenous. What do we conclude from the test? In the context of this simultaneous equations model what result should we find?

- Obtain the 2SLS estimates of the investment equation using all eight exogenous and predetermined variables as IVs and software designed for 2SLS. Compare the estimates to the OLS estimates in part (a). Do you find any important differences?
- Estimate the second-stage model $I_t = \beta_1 + \beta_2 \hat{P}_t + \beta_3 P_{t-1} + \beta_4 K_{t-1} + e_{2t}$ by OLS. Compare the estimates and standard errors from this estimation to those in part (d). What differences are there?
- Let the 2SLS residuals from part (e) be \hat{e}_{2t} . Regress these residuals on all the exogenous and predetermined variables. If these instruments are valid, then the R^2 from this regression should be low, and none of the variables are statistically significant. The Sargan test for instrument validity is discussed in Section 10.4.3. The test statistic TR^2 has a chi-square distribution with degrees of freedom equal to the number of "surplus" IVs if the surplus instruments are valid. The investment equation includes three exogenous and/or predetermined variables out of the total of eight possible. There are $L = 5$ external instruments and $B = 1$ right-hand side endogenous variables. Compare the value of the test statistic to the 95th percentile value from the $\chi^2_{(4)}$ distribution. What do we conclude about the validity of the surplus instruments in this case?

a.

```
lm(formula = i ~ p + plag + klag, data = klein)

Residuals:
    Min      1Q  Median      3Q     Max 
-2.56562 -0.63169  0.03687  0.41542  1.49226 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 10.12579   5.46555  1.853 0.081374 .  
p           0.47964   0.09711  4.939 0.000125 *** 
plag        0.33304   0.10086  3.302 0.004212 ** 
klag        -0.11179   0.02673 -4.183 0.000624 *** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.009 on 17 degrees of freedom
Multiple R-squared:  0.9313, Adjusted R-squared:  0.9192 
F-statistic: 76.88 on 3 and 17 DF  p-value: 4.299e-10
```

The signs are consistent with expectations, and high statistical significance of the key var. indicates a well-specified model.

b.

```
lm(formula = p ~ g + w2 + tx + plag + klag + time + elag, data = klein)
```

Residuals:

Min	1Q	Median	3Q	Max
-3.9067	-1.3050	0.3226	1.3613	2.8881

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	50.38442	31.63026	1.593	0.1352
g	0.43902	0.39114	1.122	0.2820
w2	-0.07961	2.53382	-0.031	0.9754
tx	-0.92310	0.43376	-2.128	0.0530 .
plag	0.80250	0.51886	1.547	0.1459
klag	-0.21610	0.11911	-1.814	0.0928 .
time	0.31941	0.77813	0.410	0.6881
elag	0.02200	0.28216	0.078	0.9390

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 2.183 on 13 degrees of freedom
Multiple R-squared: 0.8261, Adjusted R-squared: 0.7324
F-statistic: 8.821 on 7 and 13 DF, p-value: 0.0004481

Analysis of Variance Table

Model 1: p ~ plag + klag		Model 2: p ~ g + w2 + tx + plag + klag + time + elag			
Res.Df	RSS	Df	Sum of Sq	F	Pr(>F)
1	18	108.04			
2	13	61.95	5	46.093	1.9345 0.1566

c.

```
lm(formula = i ~ p + plag + klag + uhat, data = klein)
```

Residuals:

Min	1Q	Median	3Q	Max
-1.04645	-0.56030	0.06189	0.25348	1.36700

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	20.27821	4.70179	4.313	0.000536 ***
p	0.15022	0.10798	1.391	0.183222
plag	0.61594	0.10147	6.070	1.62e-05 ***
klag	-0.15779	0.02252	-7.007	2.96e-06 ***
uhat	0.57451	0.14261	4.029	0.000972 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.7331 on 16 degrees of freedom
Multiple R-squared: 0.9659, Adjusted R-squared: 0.9574
F-statistic: 113.4 on 4 and 16 DF, p-value: 1.588e-11

Conclusion:

The result ($F = 1.93$, $p = 0.1566$) indicates that the additional variables are not jointly statistically significant at the 5% level. Thus, conditional on lagged profit and capital stock, these exogenous variables do not significantly improve the explanatory power of the model.

	p	Phat	uhat
2	12.4	13.255556	-0.85555556
3	16.9	16.577368	0.3226319
4	18.4	19.282347	-0.8823465
5	19.4	20.960143	-1.5601433
6	20.1	19.766509	0.3334910
7	19.6	18.238731	1.3612688
8	19.8	17.573065	2.2269354
9	21.1	19.541720	1.5582796
10	21.7	20.375101	1.3248995
11	15.6	17.180415	-1.5804148
12	11.4	12.705026	-1.3050261
13	7.0	8.999780	-1.9997802
14	11.2	9.054102	2.1458976
15	12.3	12.671263	-0.3712632
16	14.0	14.421338	-0.4213385
17	17.6	14.711907	2.8880932
18	17.3	19.796405	-2.4964049
19	15.3	19.206691	-3.9066913
20	19.0	17.419605	1.5803947
21	21.1	20.305654	0.7943462
22	23.5	22.657273	0.8427268

H_0 : P_t is exogenous. ($\delta = 0$)

$\therefore P\text{-value} = 0.000972 < 0.05$

H_1 : P_t is endogenous. ($\delta \neq 0$)

\therefore We reject H_0 , means that P_t is endogenous.

d.

```
ivreg(formula = i ~ p + plag + klag | g + w2 + tx + elag + time +
      plag + klag, data = klein)

Residuals:
    Min     1Q Median     3Q    Max 
-3.2909 -0.8069  0.1423  0.8601  1.7956 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 20.27821   8.38325  2.419  0.02707 *  
p           0.15022   0.19253  0.780  0.44598    
plag        0.61594   0.18093  3.404  0.00338 **  
klag        -0.15779   0.04015 -3.930  0.00108 **  
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.307 on 17 degrees of freedom
Multiple R-Squared:  0.8849,    Adjusted R-squared:  0.8646 
Wald test:  41.2 on 3 and 17 DF,  p-value: 5.148e-08
```

OLS estimation is 0.48 & significant; 2SLS estimation is 0.15 & not significant.

It shows that OLS estimate was likely biased due to endogeneity.

e.

```
lm(formula = i ~ Phat + plag + klag, data = klein)

Residuals:
    Min     1Q Median     3Q    Max 
-3.8778 -1.0029  0.3058  0.7275  2.1831 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 20.27821   9.97663  2.033  0.05802 .  
Phat        0.15022   0.22913  0.656  0.52084    
plag        0.61594   0.21531  2.861  0.01083 *  
klag        -0.15779   0.04778 -3.302  0.00421 **  
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.556 on 17 degrees of freedom
Multiple R-squared:  0.837,    Adjusted R-squared:  0.8082 
F-statistic: 29.09 on 3 and 17 DF,  p-value: 6.393e-07
```

All slopes keep their signs & magnitudes, only the

SE of 2SLS are larger which leading to higher p-values.

f.

```
> cat("Sargan test statistic: ", Sargan_stat, "\n")
Sargan test statistic: 1.281519
>
> crit_val <- qchisq(0.95, df = 4)
> cat("Critical value (chi^2(4), 95%): ", crit_val, "\n")
Critical value (chi^2(4), 95%): 9.487729
```

$\therefore 1.2815 < 9.4877 \Rightarrow$ We fail to reject H_0 , which means the surplus instruments appear valid.

15.6 Using the NLS panel data on $N = 716$ young women, we consider only years 1987 and 1988. We are interested in the relationship between $\ln(WAGE)$ and experience, its square, and indicator variables for living in the south and union membership. Some estimation results are in Table 15.10.

	(1) OLS 1987	(2) OLS 1988	(3) FE	(4) FE Robust	(5) RE
C	0.9348 (0.2010)	0.8993 (0.2407)	1.5468 (0.2522)	1.5468 (0.2688)	1.1497 (0.1597)
EXPER	0.1270 (0.0295)	0.1265 (0.0323)	0.0575 (0.0330)	0.0575 (0.0328)	0.0986 (0.0220)
EXPER ²	-0.0033 (0.0011)	-0.0031 (0.0011)	-0.0012 (0.0011)	-0.0012 (0.0011)	-0.0023 (0.0007)
SOUTH	-0.2128 (0.0338)	-0.2384 (0.0344)	-0.3261 (0.1258)	-0.3261 (0.2495)	-0.2326 (0.0317)
UNION	0.1445 (0.0382)	0.1102 (0.0387)	0.0822 (0.0312)	0.0822 (0.0367)	0.1027 (0.0245)
N	716	716	1432	1432	1432

(standard errors in parentheses)

- a. The OLS estimates of the $\ln(WAGE)$ model for each of the years 1987 and 1988 are reported in columns (1) and (2). How do the results compare? For these individual year estimations, what are you assuming about the regression parameter values across individuals (heterogeneity)?
- b. The $\ln(WAGE)$ equation specified as a panel data regression model is

$$\ln(WAGE_{it}) = \beta_1 + \beta_2 \text{EXPER}_{it} + \beta_3 \text{EXPER}_{it}^2 + \beta_4 \text{SOUTH}_{it} + \beta_5 \text{UNION}_{it} + (u_i + e_{it}) \quad (\text{XR15.6})$$

- Explain any differences in assumptions between this model and the models in part (a).
- c. Column (3) contains the estimated fixed effects model specified in part (b). Compare these estimates with the OLS estimates. Which coefficients, apart from the intercepts, show the most difference?
- d. The F-statistic for the null hypothesis that there are no individual differences, equation (15.20), is 11.68. What are the degrees of freedom of the F-distribution if the null hypothesis (15.19) is true? What is the 1% level of significance critical value for the test? What do you conclude about the null hypothesis?
- e. Column (4) contains the fixed effects estimates with cluster-robust standard errors. In the context of this sample, explain the different assumptions you are making when you estimate with and without cluster-robust standard errors. Compare the standard errors with those in column (3). Which ones are substantially different? Are the robust ones larger or smaller?

e. Non-robust SE: 假設 e_{it} ~ i.i.d. & 無序列相關 & 無跨了件相關

i. Robust SE: 假設了件內誤差相關 (同人不同年)

SOUTH : 0.1258 → 0.2495 几乎翻倍, 表示 RE 的估計更保守

15.17 The data file *liquor* contains observations on annual expenditure on liquor (*LIQUOR*) and annual income (*INCOME*) (both in thousands of dollars) for 40 randomly selected households for three consecutive years.

- a. Create the first-differenced observations on *LIQUOR* and *INCOME*. Call these new variables *LIQORD* and *INCOMED*. Using OLS regress *LIQORD* on *INCOMED* without a constant term. Construct a 95% interval estimate of the coefficient.

$$\hat{\text{LIQORD}}_{it} = 0.02975 \text{ INCOMED}_{it}$$

Call:
`lm(formula = liquord ~ incomed - 1, data = liquor5)`

Residuals:

Min	1Q	Median	3Q	Max
-3.6852	-0.9196	-0.0323	0.9027	3.3620

2.5 % 97.5 %
`incomed` -0.02841457 0.08790818

Coefficients:

Estimate	Std. Error	t value	Pr(> t)	
incomed	0.02975	0.02922	1.018	0.312

Residual standard error: 1.417 on 79 degrees of freedom
Multiple R-squared: 0.01295, Adjusted R-squared: 0.0004544
F-statistic: 1.036 on 1 and 79 DF, p-value: 0.3118

a.

`lm(formula = readscore ~ small + aide + tchexper + boy + white_asian + freelunch, data = star)`

Residuals:

Min	1Q	Median	3Q	Max
-107.220	-20.214	-3.935	14.339	185.956

Coefficients:

(Intercept)	Estimate	Std. Error	t value	Pr(> t)
437.76425	1.34622	325.180	< 2e-16	***
small	5.82282	0.98933	5.886	4.19e-09 ***
aide	0.81784	0.95299	0.858	0.391
tchexper	0.49247	0.06956	7.080	1.61e-12 ***
boy	-6.15642	0.79613	-7.733	1.23e-14 ***
white_asian	3.90581	0.95361	4.096	4.26e-05 ***
freelunch	-14.77134	0.89025	-16.592	< 2e-16 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 30.19 on 5759 degrees of freedom

(因為不存在 20 個觀察量被刪除了)

Multiple R-squared: 0.09685, Adjusted R-squared: 0.09591

F-statistic: 102.9 on 6 and 5759 DF, p-value: < 2.2e-16

15.20 This exercise uses data from the STAR experiment introduced to illustrate fixed and random effects for grouped data. In the STAR experiment, children were randomly assigned within schools into three types of classes: small classes with 13–17 students, regular-sized classes with 22–25 students, and regular-sized classes with a full-time teacher aide to assist the teacher. Student scores on achievement tests were recorded as well as some information about the students, teachers, and schools. Data for the kindergarten classes are contained in the data file *star*.

- a. Estimate a regression equation (with no fixed or random effects) where *READSCORE* is related to *SMALL*, *AIDE*, *TCHEXPER*, *BOY*, *WHITE_ASIAN*, and *FREELUNCH*. Discuss the results. Do students perform better in reading when they are in small classes? Does a teacher's aide improve scores? Do the students of more experienced teachers score higher on reading tests? Does the student's sex or race make a difference?
- b. Reestimate the model in part (a) with school fixed effects. Compare the results with those in part (a). Have any of your conclusions changed? [Hint: specify *SCHID* as the cross-section identifier and *ID* as the "time" identifier.]
- c. Test for the significance of the school fixed effects. Under what conditions would we expect the inclusion of significant fixed effects to have little influence on the coefficient estimates of the remaining variables?

SMALL is significant: small class has positive impact on reading performance.

AIDE is not significant: there's no evidence that having teacher's aide improves reading scores.

TCHEXPER is significant: more experienced teacher are associated with better student reading outcomes.

BOY is significant: Girls outperform boys in this reading assessment.

WHITE_ASIAN is significant: white and Asian students achieve higher average reading scores.

FREELUNCH is significant: economic disadvantage is strongly associated with lower reading performance.

a.

方向也一致

21 年份的 OLS estimate 結果相近 → 假設所有个体有相同的

回歸係數, 參數不隨个体變動 → Homogeneity

b.

random error

个体固定效應

↑ unobserved heterogeneity

XR15.6: 誤差項 = $u_{it} + e_{it}$, u_{it} 描述不隨時間變化的特徵

c.

EF OLS

EXPER: $0.0575 < 0.127$, OLS 可能高估經驗對工資的影響

EXPER: $-0.0012 > -0.0031$, 也顯著變化

SOUTH & UNION: 變動不大, 表示變數可能沒有太大變異

d.

$H_0: u_i = 0$ (Homogeneity) $df = (N-1, NT-N-K)$

= (716-1, 1432-716-5)

$H_1: u_i \neq 0$ (Heterogeneity) $= (715, 711)$

$F_{(715, 711)} = 1.12 < 11.68 \Rightarrow \text{reject } H_0$, means that it exists heterogeneity.
we use FE model.

```

plm(formula = readscore ~ small + aide + tchexper + boy + white_asian +
    freelunch, data = nswdata, model = "within")
Unbalanced Panel: n = 79, T = 34-137, N = 5766
Residuals:
    Min. 1st Qu. Median 3rd Qu. Max.
-102.6381 -16.7834 -2.8473 12.7591 198.4169
Coefficients:
            Estimate Std. Error t-value Pr(>|t|)
small      6.490231  0.912962  7.1090 1.313e-12 ***
aide       0.996087  0.881693  1.1297  0.2586
tchexper   0.000157  0.000157  0.4000  5.913e-05 ***
boy        -5.455941  0.727589 -7.4987 7.440e-14 ***
white_asian 8.028019  1.535656  5.2277 1.777e-07 ***
freelunch -14.593572  0.880006 -16.5835 < 2.2e-16 ***
---
Signif. codes: 0 '****' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Total Sum of Squares: 4628000
Residual Sum of Squares: 4268900
R-Squared: 0.077592
Adj. R-Squared: 0.06954
F-statistic: 79.6471 on 6 and 5681 DF, p-value: < 2.22e-16

```

The conclusion hasn't changed.

C. Reject H_0 , means that school fixed effects are significant.

F test for individual effects

```

data: readscore ~ small + aide + tchexper + boy + white_asian + freelunch
F = 16.698, df1 = 78, df2 = 5681, p-value < 2.2e-16
alternative hypothesis: significant effects

```