10.2

a. WAGE, EDUC (+), AGE (+ or −), KIDSL6, NWIFEINC (−)

b. WAGE is edogenous. unobserved factors in e such as taste for work or career ambition influence both HOURS & WAGE, so $Cov(WAGE, e) \neq 0$.

$\therefore$ OLS produces biased & inconsistent esitimates of β_2 and any oefficients correlated with WAGE.

C. Relevence:

Past labour-market experience is a strong determinant of current hourly wage $\therefore Cov(EXPER, WAGE) \neq 0$.

Exogeneity:

Cumulative experience affects current HOURS primarily through its influence on the market wage, not through contemporaneous preference shocks in e. $\therefore Cov(EXPER, e) = 0$

$\because$ they satisfy both conditions, EXPER & $EXPER^2$ are valid instruments for WAGE.

d.

$\because$ endogenous regressor (WAGE) + excluded instruments (EXPER & $EXPER^2$) $\therefore$ the model is over-identified.

Provided the instruments are relevant, the structural parameters are identified & can be estimated consistently with IV / 2SLS.

e.

1. First stage:

$$\widehat{WAGE} = \pi_0 + \pi_1 EXPER + \pi_2 EXPER^2 + \dots$$

2. Second stage:

$$HOURS = \beta_1 + \beta_2 \widehat{WAGE} + \beta_3 EDUC + \beta_4 AGE + \beta_5 KIDSL6 + \beta_7 NWIFEINC + u$$

3. Inference & diagnostics:

- Use robust SE appropriate to 2SLS.
- Check first-stage F-statistic > 10
- Test over-identifying restrictions to assess instrument exogeneity.

10.3

a. $x = \gamma_1 + \theta_1 z + v \qquad E(x) = \gamma_1 + \theta_1 E(z)$

$x - E(x) = \theta_1 [z - E(z)] + v$ 同 − E(x)

$[x - E(x)][z - E(z)] = \theta_1 [z - E(z)]^2 + [z - E(z)]v$ 同 × [z − E(z)]

$E([x - E(x)][z - E(z)]) = \theta_1 E[z - E(z)]^2 + E([z - E(z)]v)$ 取 E( )

$\Rightarrow Cov(z, x) = \theta_1 Var(z) + Cov(z, v)$

$\Rightarrow \theta_1 = \dfrac{Cov(z, x) - Cov(z, v)}{Var(x_i)}$

$\because Cov(z, v) = 0$

$\therefore \theta_1 = \dfrac{Cov(z, x)}{Var(x_i)}$, this is OLS estimator of θ in $x = \gamma_1 + \theta_1 z + v$

b. $y = \pi_0 + \pi_1 z + u \qquad E(y) = \pi_0 + \pi_1 E(z)$

$y - E(y) = \pi_1 [z - E(z)] + u$ 同 − E(y)

$[y - E(y)][z - E(z)] = \pi_1 [z - E(z)]^2 + [z - E(z)]u$ 同 × z − E(z)

$E([y - E(y)][z - E(z)]) = \pi_1 E[z - E(z)]^2 + E([z - E(z)]u)$ 取 E( )

$\Rightarrow \pi_1 = \dfrac{E([y - E(y)][z - E(z)])}{E[z - E(z)]^2}$, this is the OLS estimator of π_1 in $y = \pi_0 + \pi_1 z + u$

c.

$$y = \beta_1 + \beta_2(\gamma_1 + \theta_1 z + v) + e = \underbrace{\beta_1 + \beta_2\gamma_1}_{=\pi_0} + \underbrace{\beta_2\theta_1}_{=\pi_1} z + \underbrace{\beta_2 v + e}_{=u}$$

d.

$\because \pi_1 = \beta_2\theta_1 \qquad \therefore \beta_2 = \dfrac{\pi_1}{\theta_1}$

e.

From a.

$$\hat{\theta}_1 = \frac{\widehat{Cov}(z, x)}{\widehat{Var}(z)} = \frac{\frac{\Sigma(z_i - \bar{z})(x_i - \bar{x})}{N}}{\frac{\Sigma(z_i - \bar{z})^2}{N}} = \frac{\Sigma(z_i - \bar{z})(x_i - \bar{x})}{\Sigma(z_i - \bar{z})^2}$$

, this estimator is consistent if z is uncorrelated with v.

$$\hat{\pi}_1 = \frac{\widehat{Cov}(z, y)}{\widehat{Var}(z)} = \frac{\frac{\Sigma(z_i - \bar{z})(y_i - \bar{y})}{N}}{\frac{\Sigma(z_i - \bar{z})^2}{N}} = \frac{\Sigma(z_i - \bar{z})(y_i - \bar{y})}{\Sigma(z_i - \bar{z})^2}$$

is a consistent estimator if z is uncorrelated with u.

then,

$$\hat{\beta}_2 = \frac{\hat{\pi}_1}{\hat{\theta}_1} = \frac{\frac{\Sigma(z_i - \bar{z})(y_i - \bar{y})}{\Sigma(z_i - \bar{z})^2}}{\frac{\Sigma(z_i - \bar{z})(x_i - \bar{x})}{\Sigma(z_i - \bar{z})^2}} = \frac{\Sigma(z_i - \bar{z})(y_i - \bar{y})}{\Sigma(z_i - \bar{z})(x_i - \bar{x})} = \frac{\frac{\Sigma(z_i - \bar{z})(y_i - \bar{y})}{N}}{\frac{\Sigma(z_i - \bar{z})(x_i - \bar{x})}{N}} = \frac{\widehat{Cov}(z, y)}{\widehat{Cov}(z, x)}$$

$$\hat{\beta}_2 = \frac{\hat{\pi}_1}{\hat{\theta}_1} = \frac{\widehat{Cov}(z, y)}{\widehat{Cov}(z, x)} \xrightarrow{p} \frac{Cov(z, y)}{Cov(z, x)} = \beta_2$$