HW0317_313707049俞懷蕥

4.4 The general manager of a large engineering firm wants to know whether the experience of technical artists influences their work quality. A random sample of 50 artists is selected. Using years of work experience (*EXPER*) and a performance rating (*RATING*, on a 100-point scale), two models are estimated by least squares. The estimates and standard errors are as follows:

Model 1:

$$\widehat{RATING} = 64.289 + 0.990EXPER \quad N = 50 \quad R^2 = 0.3793$$
$$(\text{se}) \qquad (2.422) \quad (0.183)$$

Model 2:

$$\widehat{RATING} = 39.464 + 15.312\ln(EXPER) \quad N = 46 \quad R^2 = 0.6414$$
$$(\text{se}) \qquad (4.198) \quad (1.727)$$

a.

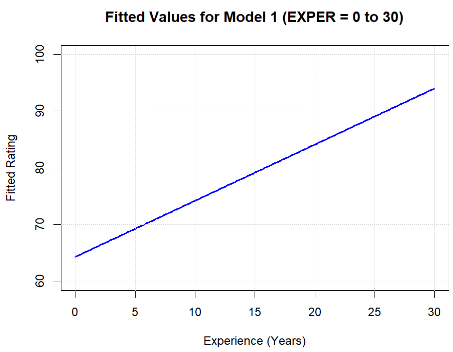
- For $\text{EXPER} = 0$:

$$\widehat{\text{RATING}} = 64.289 + 0.990 \times 0 = 64.289$$

- For $\text{EXPER} = 30$:

$$\widehat{\text{RATING}} = 64.289 + 0.990 \times 30 = 64.289 + 29.7 = 93.989$$

The relationship is linear, so the plot will be a straight line from (0, 64.289) to (30, 93.989).



b.

- For $\text{EXPER} = 1$:

$$\ln(1) = 0 \quad \Rightarrow \quad \widehat{\text{RATING}} = 39.464 + 15.312 \times 0 = 39.464$$

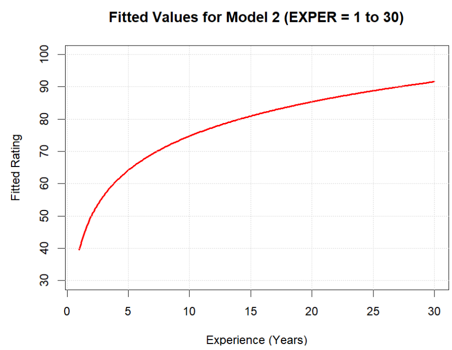
- For $\text{EXPER} = 10$:

$$\ln(10) \approx 2.3026 \quad \Rightarrow \quad \widehat{\text{RATING}} = 39.464 + 15.312 \times 2.3026 \approx 39.464 + 35.278 = 74.742$$

- For $\text{EXPER} = 30$:

$$\ln(30) \approx 3.4012 \quad \Rightarrow \quad \widehat{\text{RATING}} = 39.464 + 15.312 \times 3.4012 \approx 39.464 + 52.079 = 91.543$$

The relationship is logarithmic, so the plot will show a curve that increases rapidly at first and then levels off.



c.

Model 1 is linear: $\widehat{\text{RATING}} = 64.289 + 0.990 \times \text{EXPER}$.

The marginal effect of another year of experience is the derivative of $\widehat{\text{RATING}}$ with respect to EXPER:

$$\frac{d(\widehat{\text{RATING}})}{d(\text{EXPER})} = 0.990$$

Since Model 1 is linear, the marginal effect is constant and does not depend on the level of experience.

Artist with 10 and 20 years of experience: The marginal effect both= 0.990.

d.

Model 2 is logarithmic: $\widehat{\text{RATING}} = 39.464 + 15.312 \times \ln(\text{EXPER})$.

The marginal effect is the derivative of $\widehat{\text{RATING}}$ with respect to EXPER:

$$\frac{d(\widehat{\text{RATING}})}{d(\text{EXPER})} = \frac{d}{d(\text{EXPER})}(39.464 + 15.312 \times \ln(\text{EXPER})) = 15.312 \times \frac{d}{d(\text{EXPER})}(\ln(\text{EXPER})) = 15.312 \times \frac{1}{\text{EXPER}}$$

So, the marginal effect is: 1.5312/EXPER

- **(i) Artist with 10 years of experience:** Marginal effect=15.312/10=1.5312
- **(ii) Artist with 20 years of experience:** Marginal effect=15.312/20=0.7656

  In Model 2, the marginal effect decreases as experience increases, reflecting the logarithmic shape where gains in rating diminish with more experience.

e.

Model 2's R square=0.6414 is higher than Model 1's R square=0.4858 , indicating that Model 2 explains a greater proportion of the variance in RATINGt and thus fits the data better.

f.

Model 1 (Linear): This model assumes that each additional year of experience increases the rating by a constant 0.990 points, regardless of the artist's current level of experience. Economically, this might not be realistic. Especially when an artist with 50 years of experience would have a rating of 64.289+0.990×50=113.789, which exceeds the 100-point scale, making the model implausible for high levels of experience.

Model 2 (Logarithmic): This model assumes that the effect of experience on rating follows a logarithmic pattern, where the marginal effect decreases as experience increases.

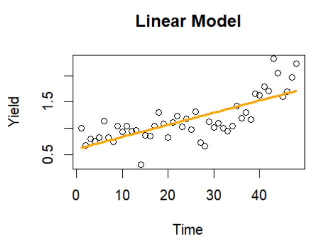
Conclusion: Model 2 is more reasonable. The logarithmic relationship better captures the diminishing returns to experience, which is a common phenomenon in skill-based professions like engineering. Model 1's linear assumption leads to unrealistic predictions at high levels of experience and doesn't reflect how learning and performance typically evolve over time.

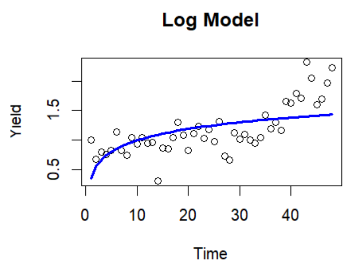
4.28 The file *wa-wheat.dat* contains observations on wheat yield in Western Australian shires. There are 48 annual observations for the years 1950–1997. For the Northampton shire, consider the following four equations:

$$YIELD_t = \beta_0 + \beta_1 TIME + e_t$$
$$YIELD_t = \alpha_0 + \alpha_1 \ln(TIME) + e_t$$
$$YIELD_t = \gamma_0 + \gamma_1 TIME^2 + e_t$$
$$\ln(YIELD_t) = \phi_0 + \phi_1 TIME + e_t$$

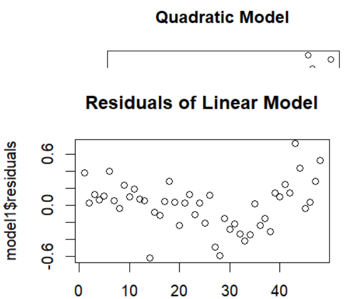
a.

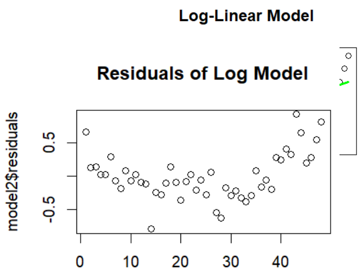
(i)





(ii)





R square

```
> summary(model1)$r.squared
[1] 0.5778369
> summary(model2)$r.squared
[1] 0.3385733
> summary(model3)$r.squared
[1] 0.6890101
> summary(model4)$r.squared
[1] 0.5073566
```

model 3的R square 最大，且殘差常態，最適合。

normality test

```
        Shapiro-Wilk normality test

data:  model1$residuals
W = 0.98236, p-value = 0.6792

> shapiro.test(model2$residuals)

        Shapiro-Wilk normality test

data:  model2$residuals
W = 0.96657, p-value = 0.1856

> shapiro.test(model3$residuals)

        Shapiro-Wilk normality test

data:  model3$residuals
W = 0.98589, p-value = 0.8266

> shapiro.test(model4$residuals)

        Shapiro-Wilk normality test

data:  model4$residuals
W = 0.86894, p-value = 7.205e-05
```

b. 隨時間推移，小麥的增長速度會逐漸加快。

```
> # 模型3 (二次模型)
> summary(model3)$coefficients
                Estimate   Std. Error  t value     Pr(>|t|)
(Intercept) 0.7736655220 5.221813e-02 14.81603 3.953882e-19
I(time^2)   0.0004986181 4.939119e-05 10.09528 3.007857e-13
```

c.

```
> studentized_res
          1           2           3           4           5           6 
 0.97117127 -0.43892315  0.09154376 -0.09362102  0.17150978  1.48946942 
          7           8           9          10          11          12 
 0.09207526 -0.25121369  0.97031031  0.43928373  0.88308253  0.43133925 
         13          14          15          16          17          18 
 0.40663499 -2.56068246 -0.07921998 -0.20039139  0.49312413  1.55776314 
         19          20          21          22          23          24 
 0.51140018 -0.61544215  0.51116364  0.92505667 -0.06616263  0.50898647 
         25          26          27          28          29          30 
-0.48508765  0.87138263 -1.74863798 -2.24684727 -0.31870520 -0.87713750 
> leverage
         1          2          3          4          5          6 
0.04743473 0.04723338 0.04689948 0.04643560 0.04584531 0.04513318 
         7          8          9         10         11         12 
0.04430484 0.04336691 0.04232704 0.04119390 0.03997718 0.03868759 
        13         14         15         16         17         18 
0.03733687 0.03593775 0.03450401 0.03305043 0.03159284 0.03014805 
        19         20         21         22         23         24 
0.02873391 0.02736930 0.02607410 0.02486923 0.02377660 0.02281918 
        25         26         27         28         29         30 
0.02202093 0.02140683 0.02100290 0.02083617 0.02093468 0.02132750 
```

```
> dffits_values
          1           2           3           4           5           6 
 0.21671884 -0.09772816  0.02030689 -0.02065970  0.03759473  0.32382335 
          7           8           9          10          11          12 
 0.01982480 -0.05348720  0.20399098  0.09105340  0.18020490  0.08653117 
         13          14          15          16          17          18 
 0.08008229 -0.49440017 -0.01497595 -0.03704808  0.08906797  0.27464917 
         19          20          21          22          23          24 
 0.08796079 -0.10323931  0.08363762  0.14772978 -0.01032555  0.07778015 
         25          26          27          28          29          30 
-0.07279025  0.12887955 -0.25612316 -0.32775913 -0.04660328 -0.12948485 
         31          32          33          34          35          36 
-0.10055048 -0.18482622 -0.25204730 -0.21652582  0.03075755 -0.17405621 
         37          38          39          40          41          42 
-0.12830329 -0.28361722  0.10433872  0.04921896  0.17999300  0.06210342 
         43          44          45          46          47          48 
 0.78231995  0.39666614 -0.23822701 -0.18484656  0.14168569  0.50778020 
```

```
> dfbetas_values
    (Intercept)   I(Time^2)
1   0.216718802 -0.162293335
2  -0.097727849  0.073063005
3   0.020306565 -0.015139023
4  -0.020658634  0.015340433
5   0.037589949 -0.027768566
6   0.323736684 -0.237608499
7   0.019814787 -0.014429582
8  -0.053440095  0.038555438
9   0.203696093 -0.145364271
10  0.090847178 -0.064013954
11  0.179588800 -0.124702495
12  0.086097858 -0.058783799
13  0.079509348 -0.053242206
14 -0.489450177  0.320519995
15 -0.014769753  0.009426586
16 -0.036357010  0.022524814
17  0.086845864 -0.051978455
18  0.265587279 -0.152662876
19  0.084160844 -0.046123387
20 -0.097452600  0.050450814
21  0.077606439 -0.037496828
22  0.134136097 -0.059512311
23 -0.009122960  0.003632899
24  0.066410121 -0.022945195
25 -0.059553461  0.016903995
26  0.100005566 -0.021094711
27 -0.186175532  0.023013433
28 -0.219908713  0.003822742
```

29 -0.028358757 -0.003242530
30 -0.069984024 -0.019709984
31 -0.047073157 -0.023570795
32 -0.072666378 -0.058096993
33 -0.079964118 -0.098380081
34 -0.052431496 -0.099841197
35 0.005207561 0.016173251
36 -0.017387057 -0.101664741
37 -0.004450857 -0.081583039
38 0.007330872 -0.193256227
39 -0.008509056 0.075244047
40 -0.006520055 0.037193451
41 -0.032182354 0.141393550
42 -0.013713793 0.050388324
43 -0.202525494 0.652179762
44 -0.116349648 0.338316939
45 0.077302871 -0.207150911
46 0.065201030 -0.163400687
47 -0.053605470 0.127022369
48 -0.203926321 0.460766575

d.

```
> pred_i
       fit      lwr      upr
1 1.922482 1.412563 2.432401
```

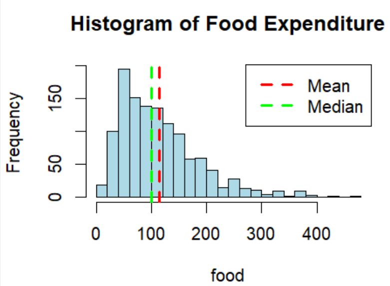
In repeat sampling, the 95% prediction interval for 1997 is [1.412563, 2.432401].

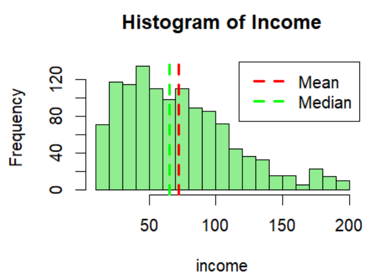
The actual Northampton yield was 2.2318, which is inside the interval.

4.29 Consider a model for household expenditure as a function of household income using the 2013 data from the Consumer Expenditure Survey, *cex5_small*. The data file *cex5* contains more observations. Our attention is restricted to three-person households, consisting of a husband, a wife, plus one other. In this exercise, we examine expenditures on a staple item, food. In this extended example, you are asked to compare the linear, log-log, and linear-log specifications.

a.

```
> #a.
> # 計算摘要統計量
> cat("Summary Statistics for food:\n")
Summary Statistics for food:
> print(c(Mean = mean(cex5_small$food, na.rm = TRUE),
+         Median = median(cex5_small$food, na.rm = TRUE),
+         Minimum = min(cex5_small$food, na.rm = TRUE),
+         Maximum = max(cex5_small$food, na.rm = TRUE),
+         Std_Dev = sd(cex5_small$food, na.rm = TRUE)))
    Mean   Median  Minimum  Maximum  Std_Dev
114.4431  99.8000   9.6300 476.6700  72.6575
> cat("\nSummary Statistics for income:\n")

Summary Statistics for income:
> print(c(Mean = mean(cex5_small$income, na.rm = TRUE),
+         Median = median(cex5_small$income, na.rm = TRUE)
+         Minimum = min(cex5_small$income, na.rm = TRUE),
+         Maximum = max(cex5_small$income, na.rm = TRUE),
+         Std_Dev = sd(cex5_small$income, na.rm = TRUE)))
     Mean    Median   Minimum   Maximum   Std_Dev
 72.14264  65.29000  10.00000 200.00000  41.65228
```

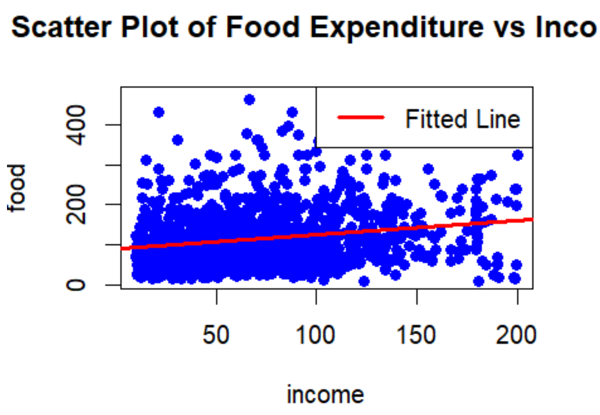




兩結果p-value皆小於0.05，皆拒絕正態假設。

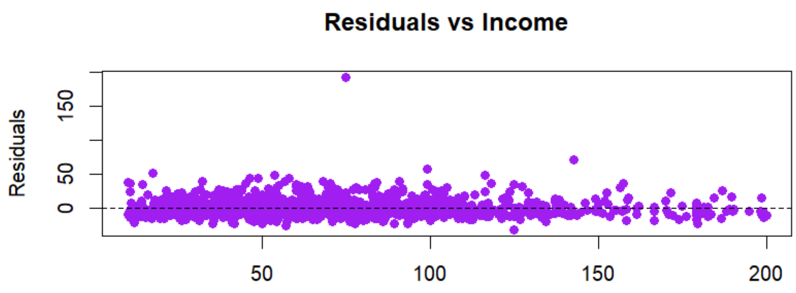
```
> jarque.bera.test(cex5_small$food)

        Jarque Bera Test

data:  cex5_small$food
X-squared = 648.65, df = 2, p-value < 2.2e-16

> jarque.bera.test(cex5_small$income)

        Jarque Bera Test

data:  cex5_small$income
X-squared = 148.21, df = 2, p-value < 2.2e-16
```

b.




```
> # 計算 95% 置信區間
> confint(model_linear, "income", level = 0.95)
          2.5 %   97.5 %
income 0.2619215 0.455452
```

c.殘差分佈圖沒有明顯的系統趨勢，直方圖右偏。Jarque-Bera 統計量為 624.186，遠大於 5% 的臨界值 5.99，故殘差並不符合常態分佈。



Histogram of Residuals

```
Call:
lm(formula = food ~ income, data = cex5_small)

Residuals:
    Min      1Q  Median      3Q     Max 
-145.37  -51.48  -13.52   35.50  349.81 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 88.56650    4.10819  21.559  < 2e-16 ***
income       0.35869    0.04932   7.272 6.36e-13 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 71.13 on 1198 degrees of freedom
Multiple R-squared:  0.04228,	Adjusted R-squared:  0.04148 
F-statistic: 52.89 on 1 and 1198 DF,  p-value: 6.357e-13
```

```
  income predicted_food elasticity   lower_ci   upper_ci
1     19       95.38155 0.07145038 0.05219387 0.09070689
2     65      111.88114 0.20838756 0.15222527 0.26454986
3    160      145.95638 0.39319883 0.28722827 0.49916940
```

```
X-squared = 624.19, df = 2, p-value < 2.2e-16
```

d.

e. log-log model R square= 0.033，略小於linear model R square= 0.042。但是直接比較 R square可能不公平，因為兩模型因變量不同，故要計算原始尺度的R square。

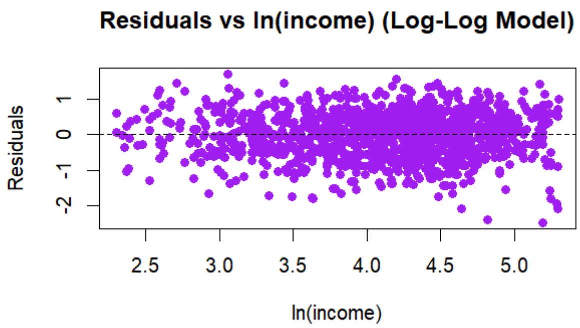
```
> cat("Generalized R^2 for Linear Model:", generalized_r2_linear, "\n")
Generalized R^2 for Linear Model: 0.0422812
> cat("Generalized R^2 for Log-Log Model:", generalized_r2_loglog, "\n")
Generalized R^2 for Log-Log Model: 0.03965161
```

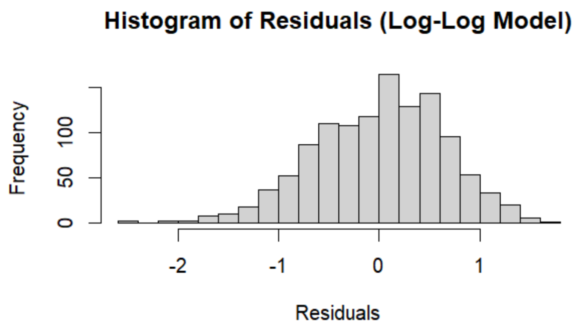
linear R square較大，故較佳。

f.

```
> cat("Point Estimate of Elasticity (log-log model): ", elasticity_point_estimate
"\n")
Point Estimate of Elasticity (log-log model):  0.1863054
> cat("95% Confidence Interval for Elasticity: ", conf_intervals_gamma2, "\n")
95% Confidence Interval for Elasticity:  0.1293432 0.2432675
```

g. 殘差非常態分布





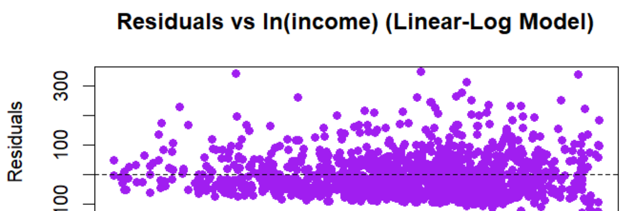

```
> library(tseries)
> jarque.bera.test(log_log_residuals)

        Jarque Bera Test

data:  log_log_residuals
X-squared = 25.85, df = 2, p-value = 2.436e-06
```

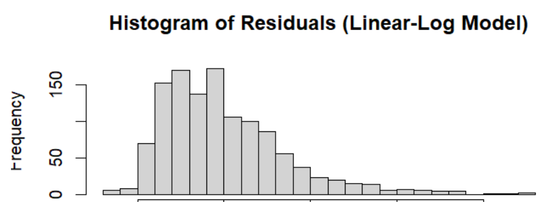
h.

Jarque-Bera 統計量為 628.07，遠大於臨界值 5.99，拒絕 linear-log model 的殘差為常態分佈的假設。

	INCOME	Predicted_FOOD	Elasticity	Lower_CI	Upper_CI
1	19	88.89788	0.2495828	0.1784009	0.3207648
2	65	116.18722	0.1909624	0.1364992	0.2454256
3	160	136.17332	0.1629349	0.1164652	0.2094046

j.





k.

linear-log model較合理，因為彈性隨收入減少，符合恩格爾定律，反映其為必需品。此外，其誤差呈隨機分布，滿足線性和同質變異數假設。