8.6 Consider the wage equation

8.6

$$WAGE_i = \beta_1 + \beta_2 EDUC_i + \beta_3 EXPER_i + \beta_4 METRO_i + e_i \qquad \text{(XR8.6a)}$$

where wage is measured in dollars per hour, education and experience are in years, and $METRO = 1$ if the person lives in a metropolitan area. We have $N = 1000$ observations from 2013.

a. We are curious whether holding education, experience, and *METRO* constant, there is the same amount of random variation in wages for males and females. Suppose $\text{var}(e_i|\mathbf{x}_i, FEMALE = 0) = \sigma_M^2$ and $\text{var}(e_i|\mathbf{x}_i, FEMALE = 1) = \sigma_F^2$. We specifically wish to test the null hypothesis $\sigma_M^2 = \sigma_F^2$ against $\sigma_M^2 \neq \sigma_F^2$. Using 577 observations on males, we obtain the sum of squared OLS residuals, $SSE_M = 97161.9174$. The regression using data on females yields $\hat{\sigma}_F = 12.024$. Test the null hypothesis at the 5% level of significance. Clearly state the value of the test statistic and the rejection region, along with your conclusion.

b. We hypothesize that married individuals, relying on spousal support, can seek wider employment types and hence holding all else equal should have more variable wages. Suppose $\text{var}(e_i|\mathbf{x}_i, MARRIED = 0) = \sigma_{SINGLE}^2$ and $\text{var}(e_i|\mathbf{x}_i, MARRIED = 1) = \sigma_{MARRIED}^2$. Specify the null hypothesis $\sigma_{SINGLE}^2 = \sigma_{MARRIED}^2$ versus the alternative hypothesis $\sigma_{MARRIED}^2 > \sigma_{SINGLE}^2$. We add *FEMALE* to the wage equation as an explanatory variable, so that

$$WAGE_i = \beta_1 + \beta_2 EDUC_i + \beta_3 EXPER_i + \beta_4 METRO_i + \beta_5 FEMALE + e_i \qquad \text{(XR8.6b)}$$

Using $N = 400$ observations on single individuals, OLS estimation of (XR8.6b) yields a sum of squared residuals is 56231.0382. For the 600 married individuals, the sum of squared errors is 100,703.0471. Test the null hypothesis at the 5% level of significance. Clearly state the value of the test statistic and the rejection region, along with your conclusion.

c. Following the regression in part (b), we carry out the NR^2 test using the right-hand-side variables in (XR8.6b) as candidates related to the heteroskedasticity. The value of this statistic is 59.03. What do we conclude about heteroskedasticity, at the 5% level? Does this provide evidence about the issue discussed in part (b), whether the error variation is different for married and unmarried individuals? Explain.

d. Following the regression in part (b) we carry out the White test for heteroskedasticity. The value of the test statistic is 78.82. What are the degrees of freedom of the test statistic? What is the 5% critical value for the test? What do you conclude?

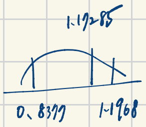
a-

Goldfeld-Quandt

$$GQ = \frac{\hat{\sigma}_M^2}{\hat{\sigma}_F^2} = \frac{169.56705}{144.59658} = 1.17285$$

$F(573,419,\ 0.025) = 0.8377$

$F(573,419,\ 0.975) = 1.1968$



不拒絕虛無假設，

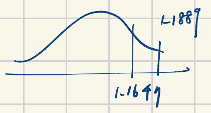
b.

$$\hat{\sigma}_{Mar}^2 = \frac{100703.0471}{600-5} = 169.24882$$

$$\hat{\sigma}_S^2 = \frac{56231.0382}{400-5} = 142.35706$$

$$GQ = \frac{\hat{\sigma}_{Mar}^2}{\hat{\sigma}_S^2} = \frac{169.24882}{142.35706} = 1.1889$$

$F(595, 395) = 1.1647$



我們拒絕虛無假設，即異質性變異存在

c、 卡方檢定

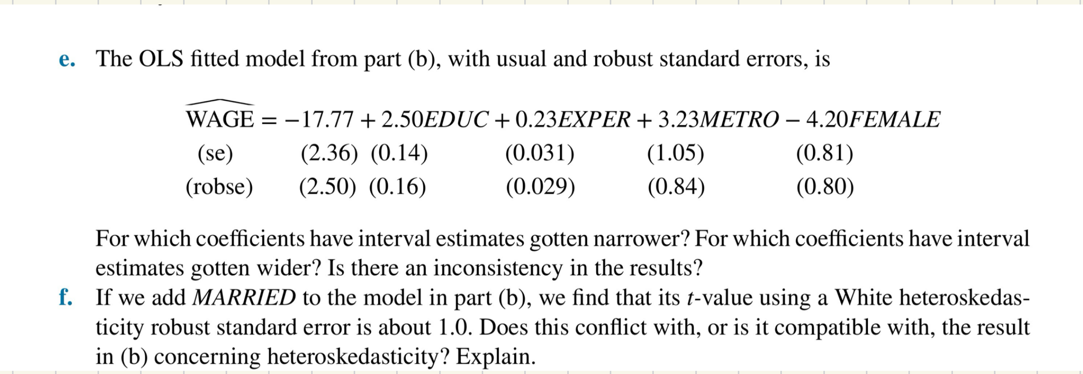
$\chi^2_{(95,4)} = 9.488$

$59.03 > 9.488$ ⇒ 拒絕虛無假設，即異質性變異存在

d. White 檢定.

$\chi^2_{(95,12)} = 21.026$

$78.82 > 21.026$，我們拒絕虛無假設，即異質性變異存在



e. The OLS fitted model from part (b), with usual and robust standard errors, is

$$\begin{array}{lllllll}
\widehat{\text{WAGE}} = & -17.77 & +\ 2.50EDUC & +\ 0.23EXPER & +\ 3.23METRO & -\ 4.20FEMALE \\
\text{(se)} & (2.36) & (0.14) & (0.031) & (1.05) & (0.81) \\
\text{(robse)} & (2.50) & (0.16) & (0.029) & (0.84) & (0.80)
\end{array}$$

For which coefficients have interval estimates gotten narrower? For which coefficients have interval estimates gotten wider? Is there an inconsistency in the results?

f. If we add *MARRIED* to the model in part (b), we find that its *t*-value using a White heteroskedasticity robust standard error is about 1.0. Does this conflict with, or is it compatible with, the result in (b) concerning heteroskedasticity? Explain.

e. 截距和 EDUC 的信賴區間變寬了

其他的變窄了.

∵ robust standard errors 可能會比不正確的 OLS 標準誤大，也可能更小

f. 控制了這些變數之後，已婚和未婚的預期工資差異並不顯著.

f小題是在看平均值是否不同

b小題是在看變異程度是否不同.