v、

10.2 The labor supply of married women has been a subject of a great deal of economic research. Consider the following supply equation specification

$$HOURS = \beta_1 + \beta_2 WAGE + \beta_3 EDUC + \beta_4 AGE + \beta_5 KIDSL6 + \beta_6 NWIFEINC + e$$

where *HOURS* is the supply of labor, *WAGE* is hourly wage, *EDUC* is years of education, *KIDSL6* is the number of children in the household who are less than 6 years old, and *NWIFEINC* is household income from sources other than the wife's employment.

a. Discuss the signs you expect for each of the coefficients.
b. Explain why this supply equation cannot be consistently estimated by OLS regression.
c. Suppose we consider the woman's labor market experience *EXPER* and its square, $EXPER^2$, to be instruments for *WAGE*. Explain how these variables satisfy the logic of instrumental variables.
d. Is the supply equation identified? Explain.
e. Describe the steps [not a computer command] you would take to obtain IV/2SLS estimates.

a、 β_2 是正向的，WAGE 越高 HOURS↑

β_3 是正向的，EDUC 越高，HOURS↑

β_4 是不確定、AGE ↑ 可能越專業，HOURS↑.
也可能投入家庭，HOURS↓.

β_5 負向的，kids 越多，HOURS↓

β_6 負向的，NWIFEINC 越多，HOURS↓

b、 為何不符合 OLS

∵ 有內生性問題

c、 為何 EXPER 可以做為工具變數

∵ EXPER、$EXPER^2$ 和 wage 有關

EXPER、$EXPER^2$ 和 HOURS 無關

d、 是的，因為我們用工具變數，所以模型是可識別的

e. 先估計出 $\widehat{wage}$，$wage = r_1 + r_2 EDUC + r_3 AGE + r_4 KIDSL6 + r_5 KIDS618 + r_6 NWIFEINC + \theta_1 EXPER + \theta_2 EXPER^2 + u$.

然在原本的模型中用 $\widehat{wage}$ 取代，再用 OLS 去估計.

3.

10.3 In the regression model $y = \beta_1 + \beta_2 x + e$, assume x is endogenous and that z is a valid instrument. In Section 10.3.5, we saw that $\beta_2 = \text{cov}(z, y)/\text{cov}(z, x)$.

a. Divide the denominator of $\beta_2 = \text{cov}(z, y)/\text{cov}(z, x)$ by $\text{var}(z)$. Show that $\text{cov}(z, x)/\text{var}(z)$ is the coefficient of the simple regression with dependent variable x and explanatory variable z, $x = \gamma_1 + \theta_1 z + v$. [*Hint*: See Section 10.2.1.] Note that this is the first-stage equation in two-stage least squares.

b. Divide the numerator of $\beta_2 = \text{cov}(z, y)/\text{cov}(z, x)$ by $\text{var}(z)$. Show that $\text{cov}(z, y)/\text{var}(z)$ is the coefficient of a simple regression with dependent variable y and explanatory variable z, $y = \pi_0 + \pi_1 z + u$. [*Hint*: See Section 10.2.1.]

c. In the model $y = \beta_1 + \beta_2 x + e$, substitute for x using $x = \gamma_1 + \theta_1 z + v$ and simplify to obtain $y = \pi_0 + \pi_1 z + u$. What are π_0, π_1, and u in terms of the regression model parameters and error and the first-stage parameters and error? The regression you have obtained is a **reduced-form** equation.

d. Show that $\beta_2 = \pi_1/\theta_1$.

e. If $\hat{\pi}_1$ and $\hat{\theta}_1$ are the OLS estimators of π_1 and θ_1, show that $\hat{\beta}_2 = \hat{\pi}_1/\hat{\theta}_1$ is a consistent estimator of $\beta_2 = \pi_1/\theta_1$. The estimator $\hat{\beta}_2 = \hat{\pi}_1/\hat{\theta}_1$ is an **indirect least squares** estimator.

a.

$$x = \gamma_1 + \theta_1 z + v$$

$$E(x) = \gamma_1 + \theta_1 E(z)$$

$$x - E(x) = \theta_1 (z - E(z)) + v$$

$$(z - E(x))(x - E(x)) = \theta_1 (z - E(z))^2 + (z - E(z))v$$

$$E[(z - E(x))(x - E(x))] = \theta_1 E[(z - E(z))^2] + v E[z - E(z)]$$

$$= \theta_1 E[(z - E(z))^2].$$

$$\theta_1 = \frac{E[(z - E(x))(x - E(x))]}{E[(z - E(z))^2]} = \frac{\text{cov}(z, x)}{\text{var}(z)}, \quad x = \gamma_1 + \theta_1 z + v$$

b.

$$y = \pi_0 + \pi_1 z + u$$

$$E(y) = \pi_0 + \pi_1 E(z)$$

$$y - E(y) = \pi_1 (z - E(z)) + u$$

$$(z - E(z))(y - E(y)) = \pi_1 (z - E(z))^2 + (z - E(z))u.$$

$$E[(z - E(z))(y - E(y))] = \pi_1 E[(z - E(z))^2]$$

$$\pi_1 = \frac{E[(z - E(z))(y - E(y))]}{E[(z - E(z))^2]} = \frac{\text{cov}(z, y)}{\text{Var}(z)}, \quad y = \pi_0 + \pi_1 z + u.$$

c.

$$y = \beta_1 + \beta_2 X + e$$

$$= \beta_1 + \beta_2(\gamma_1 + \theta_1 z + v) + e$$

$$= (\beta_1 + \beta_2\gamma_1) + \beta_2\theta_1 z + \beta_2 v + e$$

$$= \pi_0 + \pi_1 z + u.$$

$\pi_0 = \beta_1 + \beta_2\gamma_1$, $\pi_1 = \beta_2\theta_1$. $u = (\beta_2 v + e)$

d. $\pi_1 = \beta_2\theta_1$, $\beta_2 = \dfrac{\pi_1}{\theta_1}$

e.

$$\hat{\theta}_1 = \frac{\widehat{cov}(z, x)}{\widehat{var}(z)} = \frac{\sum(z_i - \bar{z})(x_i - \bar{x})/N}{\sum(z_i - \bar{z})^2/N} = \frac{\sum(z_i - \bar{z})(x_i - \bar{x})}{\sum(z_i - \bar{z})^2}$$

$$\hat{\pi}_1 = \frac{\widehat{cov}(z, y)}{\widehat{var}(z)} = \frac{\sum(z_i - \bar{z})(y_i - \bar{y})/N}{\sum(z_i - \bar{z})^2/N} = \frac{\sum(z_i - \bar{z})(y_i - \bar{y})}{\sum(z_i - \bar{z})^2}$$

$$\hat{\beta}_2 = \frac{\hat{\pi}_1}{\hat{\theta}_1} = \frac{\sum(z_i - \bar{z})(y_i - \bar{y})}{\sum(z_i - \bar{z})(x_i - \bar{x})} = \frac{\widehat{cov}(z, y)}{\widehat{cov}(z, x)}$$

$$\widehat{cov}(z, y) \xrightarrow{p} cov(z, y)$$

$$\widehat{cov}(z, x) \xrightarrow{p} cov(z, x)$$

$$\hat{\beta} = \frac{\hat{\pi}_1}{\hat{\theta}_1} = \frac{\widehat{cov}(z, y)}{\widehat{cov}(z, x)} \xrightarrow{p} \frac{cov(x, y)}{cov(z, x)} = \beta_2$$