8.6 Consider the wage equation

$$WAGE_i = \beta_1 + \beta_2 EDUC_i + \beta_3 EXPER_i + \beta_4 METRO_i + e_i \quad \text{(XR8.6a)}$$

where wage is measured in dollars per hour, education and experience are in years, and $METRO = 1$ if the person lives in a metropolitan area. We have $N = 1000$ observations from 2013.

a. We are curious whether holding education, experience, and *METRO* constant, there is the same amount of random variation in wages for males and females. Suppose $\text{var}(e_i|\mathbf{x}_i, FEMALE = 0) = \sigma_M^2$ and $\text{var}(e_i|\mathbf{x}_i, FEMALE = 1) = \sigma_F^2$. We specifically wish to test the null hypothesis $\sigma_M^2 = \sigma_F^2$ against $\sigma_M^2 \neq \sigma_F^2$. Using 577 observations on males, we obtain the sum of squared OLS residuals, $SSE_M = 97161.9174$. The regression using data on females yields $\hat{\sigma}_F = 12.024$. Test the null hypothesis at the 5% level of significance. Clearly state the value of the test statistic and the rejection region, along with your conclusion.

b. We hypothesize that married individuals, relying on spousal support, can seek wider employment types and hence holding all else equal should have more variable wages. Suppose $\text{var}(e_i|\mathbf{x}_i, MARRIED = 0) = \sigma_{SINGLE}^2$ and $\text{var}(e_i|\mathbf{x}_i, MARRIED = 1) = \sigma_{MARRIED}^2$. Specify the null hypothesis $\sigma_{SINGLE}^2 = \sigma_{MARRIED}^2$ versus the alternative hypothesis $\sigma_{MARRIED}^2 > \sigma_{SINGLE}^2$. We add *FEMALE* to the wage equation as an explanatory variable, so that

$$WAGE_i = \beta_1 + \beta_2 EDUC_i + \beta_3 EXPER_i + \beta_4 METRO_i + \beta_5 FEMALE + e_i \quad \text{(XR8.6b)}$$

Using $N = 400$ observations on single individuals, OLS estimation of (XR8.6b) yields a sum of squared residuals is 56231.0382. For the 600 married individuals, the sum of squared errors is 100,703.0471. Test the null hypothesis at the 5% level of significance. Clearly state the value of the test statistic and the rejection region, along with your conclusion.

c. Following the regression in part (b), we carry out the NR^2 test using the right-hand-side variables in (XR8.6b) as candidates related to the heteroskedasticity. The value of this statistic is 59.03. What do we conclude about heteroskedasticity, at the 5% level? Does this provide evidence about the issue discussed in part (b), whether the error variation is different for married and unmarried individuals? Explain.

d. Following the regression in part (b) we carry out the White test for heteroskedasticity. The value of the test statistic is 78.82. What are the degrees of freedom of the test statistic? What is the 5% critical value for the test? What do you conclude?

e. The OLS fitted model from part (b), with usual and robust standard errors, is

$\widehat{WAGE} =$	-17.77	$+\ 2.50EDUC$	$+\ 0.23EXPER$	$+\ 3.23METRO$	$-\ 4.20FEMALE$
(se)	(2.36)	(0.14)	(0.031)	(1.05)	(0.81)
(robse)	(2.50)	(0.16)	(0.029)	(0.84)	(0.80)

For which coefficients have interval estimates gotten narrower? For which coefficients have interval estimates gotten wider? Is there an inconsistency in the results?

f. If we add *MARRIED* to the model in part (b), we find that its t-value using a White heteroskedasticity robust standard error is about 1.0. Does this conflict with, or is it compatible with, the result in (b) concerning heteroskedasticity? Explain.

a.

虛無假設 H0:σM2=σF2

對立假設 H1:σM2≠σF2

$$\hat{\sigma}_M^2 = SSE_M/(n_M - k) = 97161.9174/(577 - 4) = 169.567$$

$$\text{test statistic: } F = \frac{\hat{\sigma}_M^2}{\hat{\sigma}_F^2} = \frac{169.567}{12.024^2} = 1.17285$$

$$RR = \{F^* \mid F^* > 1.19614 \text{ or } F^* < 0.83804\}$$

拒絕域：

RR={F*｜F*>1.19614 或 F*<0.83804}

我們未拒絕虛無假設，這表示「男性與女性之間的誤差變異沒有統計上顯著的差異」。

b.

虛無假設 $H_0 : \sigma^2_{MARRIED} = \sigma^2_{SINGLE}$
對立假設 $H_1 : \sigma^2_{MARRIED} > \sigma^2_{SINGLE}$

單身族群的誤差變異估計值：

$$\hat{\sigma}^2_{SINGLE} = \frac{SSE_{SINGLE}}{400-5} = \frac{56231.0382}{395} = 142.357$$

已婚族群的誤差變異估計值：

$$\hat{\sigma}^2_{MARRIED} = \frac{SSE_{MARRIED}}{600-5} = \frac{100703.0471}{595} = 169.2488$$

檢定統計量：

$$F = \frac{\hat{\sigma}^2_{MARRIED}}{\hat{\sigma}^2_{SINGLE}} = \frac{169.2488}{142.357} = 1.1889$$

拒絕域（Rejection Region）：

$$RR = \{F^* \mid F^* > 1.1647\}$$

我們拒絕虛無假設，這表示誤差項的變異數不是常數，並且與解釋變數存在系統性的關聯。

c.

虛無假設 H0: $\alpha 2=\alpha 3=\alpha 4=\alpha 5=0$

對立假設 H1:不是所有的 $\alpha i=0$, i = 2, 3, 4, 5

$$NR^2 = 59.03 > \chi^2_{0.95,4} = 9.488$$

因此我們拒絕池化迴歸中「同質變異性（homoskedasticity）」的虛無假設。

d.

原始變量：EDUC, EXPER, METRO, FEMALE

二次方變量：$EDUC^2$, $EXPER^2$

交互項：4×3/2=6

自由度為 12。

$\chi^2(0.95,12) = 21.026$，因此我們拒絕匯總回歸中同方差的虛無假設。

e.

截距項與 EDUC（教育）係數的信賴區間變寬了，而其他變數的信賴區間則變窄了。這個結果並不矛盾，因為在異質變異情況下所使用的穩健標準誤（heteroskedasticity-robust standard errors）可能比傳統的 OLS 標準誤大，也可能更小。當誤差項實際上具有異質變異時，假設同質變異所計算出的傳統 OLS 標準誤會產生偏誤，因此穩健標準誤提供了更準確的估計。

f.

在模型中納入虛擬變數 MARRIED，可以讓我們檢驗在控制了 EDUC（教育程度）、EXPER（工作經驗）、METRO（是否住在大都市）與 FEMALE（性別）之後，已婚與未婚者的預期薪資是否存在差異。結果顯示，兩者之間在預期薪資上並沒有統計上顯著的差異。

相對地，第 (b) 小題則著重於檢驗已婚與未婚者之間的薪資變異數是否不同。該分析確實發現兩組之間在變異程度上存在顯著差異。

這兩個問題探討的是模型中的不同面向：一個關注的是平均結果（即預期薪資），另一個則關注的是圍繞平均值的離散程度（即變異性）。

8.16 A sample of 200 Chicago households was taken to investigate how far American households tend to travel when they take a vacation. Consider the model

$$MILES = \beta_1 + \beta_2 INCOME + \beta_3 AGE + \beta_4 KIDS + e$$

MILES is miles driven per year, *INCOME* is measured in \$1000 units, *AGE* is the average age of the adult members of the household, and *KIDS* is the number of children.

a. Use the data file *vacation* to estimate the model by OLS. Construct a 95% interval estimate for the effect of one more child on miles traveled, holding the two other variables constant.

b. Plot the OLS residuals versus *INCOME* and *AGE*. Do you observe any patterns suggesting that heteroskedasticity is present?

c. Sort the data according to increasing magnitude of income. Estimate the model using the first 90 observations and again using the last 90 observations. Carry out the Goldfeld–Quandt test for heteroskedastic errors at the 5% level. State the null and alternative hypotheses.

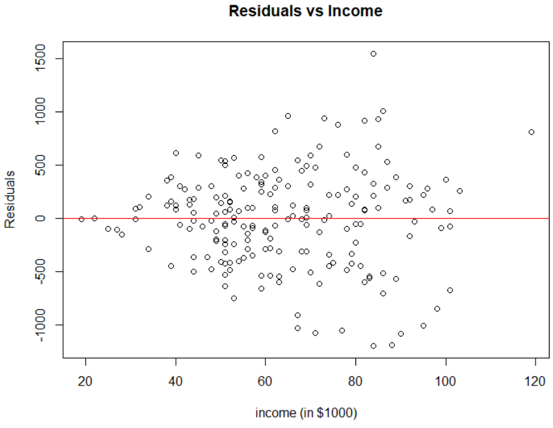
d. Estimate the model by OLS using heteroskedasticity robust standard errors. Construct a 95% interval estimate for the effect of one more child on miles traveled, holding the two other variables constant. How does this interval estimate compare to the one in (a)?

e. Obtain GLS estimates assuming $\sigma_i^2 = \sigma^2 INCOME_i^2$. Using both conventional GLS and robust GLS standard errors, construct a 95% interval estimate for the effect of one more child on miles traveled, holding the two other variables constant. How do these interval estimates compare to the ones in (a) and (d)?

a.

kids 的 95% 信心區間為： -135.3298 到 -28.32302

b.



Income 的殘差隨著 income 增加而增加，表明存在異方差性。

c.

```
> # 輸出結果
> cat("Goldfeld-Quandt 檢驗統計量：", gq_stat, "\n")
Goldfeld-Quandt 檢驗統計量： 3.104061
> cat("F 臨界值 (5% 水平)：", f_critical, "\n")
F 臨界值 (5% 水平)： 1.530373
>
> # 判斷是否拒絕虛無假設
> if (gq_stat > f_critical) {
+   cat("拒絕虛無假設，存在異方差性。\n")
+ } else {
+   cat("無法拒絕虛無假設，無顯著異方差性。\n")
+ }
拒絕虛無假設，存在異方差性。
```

d.

```
> # 輸出穩健信心區間
> cat("kids 的 95% 穩健信心區間為：", ci_lower_robust, "到", ci_upper_robust, "\n")
kids 的 95% 穩健信心區間為： -139.323 到 -24.32986
>
> # 問題 (a) 的普通 OLS 信心區間（參考之前的計算）
> summary_model <- summary(model)
> beta_kids_ols <- coef(summary_model)["kids", "Estimate"]
> se_kids_ols <- coef(summary_model)["kids", "Std. Error"]
> ci_lower_ols <- beta_kids_ols - t_value * se_kids_ols
> ci_upper_ols <- beta_kids_ols + t_value * se_kids_ols
>
> # 輸出普通 OLS 信心區間
> cat("問題 (a) 的 OLS 信心區間為：", ci_lower_ols, "到", ci_upper_ols, "\n")
問題 (a) 的 OLS 信心區間為： -135.3298 到 -28.32302
```

兩者的 95% 信心區間非常接近，寬度幾乎相同，穩健信心區間略微向左偏移。在這個數據集中，異方差性對標準誤的影響較小，因此穩健標準誤和普通 OLS 的信心區間差異不大。穩健方法更可靠，但這裡的調整效果有限。

e.

```
> # 輸出 GLS 信心區間
> cat("kids 的 95% GLS 信心區間為：", ci_lower_gls, "到", ci_upper_gls, "\n")
kids 的 95% GLS 信心區間為： -127.1453 到 -29.5814

> # 輸出穩健 GLS 信心區間
> cat("kids 的 95% 穩健 GLS 信心區間為：", ci_lower_robust_gls, "到", ci_upper_robust_gls, "\n")
kids 的 95% 穩健 GLS 信心區間為： -127.7173 到 -29.00934
```

GLS 和穩健 GLS 的信心區間比 OLS 和穩健 OLS 窄得多（97.56 和 98.71 比 163.65），表明 GLS 提高了估計效率。

GLS 和穩健 GLS 的係數估計更負（中點約 -78.36 比 -53.50 和 -57.50），且區間完全在負數範圍，顯示 kids 的負向影響更確定。

8.18 Consider the wage equation,

$$\ln(WAGE_i) = \beta_1 + \beta_2 EDUC_i + \beta_3 EXPER_i + \beta_4 EXPER_i^2 + \beta_5 FEMALE_i + \beta_6 BLACK$$
$$+ \beta_7 METRO_i + \beta_8 SOUTH_i + \beta_9 MIDWEST_i + \beta_{10} WEST + e_i$$

where *WAGE* is measured in dollars per hour, education and experience are in years, and $METRO = 1$ if the person lives in a metropolitan area. Use the data file *cps5* for the exercise.

a. We are curious whether holding education, experience, and *METRO* equal, there is the same amount of random variation in wages for males and females. Suppose $\text{var}(e_i|\mathbf{x}_i, FEMALE = 0) = \sigma_M^2$ and $\text{var}(e_i|\mathbf{x}_i, FEMALE = 1) = \sigma_F^2$. We specifically wish to test the null hypothesis $\sigma_M^2 = \sigma_F^2$ against $\sigma_M^2 \neq \sigma_F^2$. Carry out a Goldfeld–Quandt test of the null hypothesis at the 5% level of significance. Clearly state the value of the test statistic and the rejection region, along with your conclusion.

b. Estimate the model by OLS. Carry out the NR^2 test using the right-hand-side variables *METRO*, *FEMALE*, *BLACK* as candidates related to the heteroskedasticity. What do we conclude about heteroskedasticity, at the 1% level? Do these results support your conclusions in (a)? Repeat the test using all model explanatory variables as candidates related to the heteroskedasticity.

c. Carry out the White test for heteroskedasticity. What is the 5% critical value for the test? What do you conclude?

d. Estimate the model by OLS with White heteroskedasticity robust standard errors. Compared to OLS with conventional standard errors, for which coefficients have interval estimates gotten narrower? For which coefficients have interval estimates gotten wider? Is there an inconsistency in the results?

e. Obtain FGLS estimates using candidate variables *METRO* and *EXPER*. How do the interval estimates compare to OLS with robust standard errors, from part (d)?

f. Obtain FGLS estimates with robust standard errors using candidate variables *METRO* and *EXPER*. How do the interval estimates compare to those in part (e) and OLS with robust standard errors, from part (d)?

g. If reporting the results of this model in a research paper which one set of estimates would you present? Explain your choice.

a.

```
> # 輸出結果
> cat("Goldfeld-Quandt Test Statistic:", GQ_stat, "\n")
Goldfeld-Quandt Test Statistic: 0.9683995
> cat("Critical Value (5% level):", critical_value, "\n")
Critical Value (5% level): 1.057889
> cat("Reject Null Hypothesis:", reject_null, "\n")
Reject Null Hypothesis: FALSE
```

b.

```
> # 輸出結果
> cat("NR² 檢定 (metro, female, black):\n")
NR² 檢定 (metro, female, black):
> cat("統計量:", NR2_stat1, "，臨界值:", critical_value1, "，是否拒絕虛無假設:", reject_null1, "\n
\n")
統計量: 23.55681 ，臨界值: 11.34487 ，是否拒絕虛無假設: TRUE

> cat("NR² 檢定 (所有變數):\n")
NR² 檢定 (所有變數):
> cat("統計量:", NR2_stat2, "，臨界值:", critical_value2, "，是否拒絕虛無假設:", reject_null2, "\n")
統計量: 109.4243 ，臨界值: 21.66599 ，是否拒絕虛無假設: TRUE
> 
```

第一部分：用 metro、female、black 作為候選變數

- 虛無假設 (H0)：殘差變異與 metro、female、black 無關，即 Var(ϵi)=σ^2，是一個常數。
- 對立假設 (H1)：殘差變異與 metro、female、black 中的至少一個變數

相關，即 Var(ϵi)隨這些變數變化。

第二部分：用所有解釋變數作為候選變數

- 虛無假設 (H0)：殘差變異與所有解釋變數（educ、exper、exper^2、metro、female、black、south、midwest、west）無關，即 Var(ϵi)=σ^2
- 對立假設 (H1)：殘差變異與至少一個解釋變數相關，即 Var(ϵi)隨這些變數變化。

c.

```
> # 輸出結果
> cat("White 檢定統計量:", white_stat, "\n")
White 檢定統計量: 182.6723
> cat("自由度:", df, "\n")
自由度: 46
> cat("5% 顯著水準臨界值:", critical_value, "\n")
5% 顯著水準臨界值: 62.82962
> cat("是否拒絕虛無假設 (存在異質變異):", reject_null, "\n")
是否拒絕虛無假設 (存在異質變異): TRUE
```

d.

```
[1] "標準誤比較："
> print(comparison)
                   係數    OLS標準誤   穩健標準誤          差異
(Intercept) (Intercept) 3.211489e-02 3.277743e-02  6.625467e-04
educ               educ 1.758260e-03 1.904848e-03  1.465880e-04
exper             exper 1.300342e-03 1.314237e-03  1.389480e-05
exper2           exper2 2.635448e-05 2.758278e-05  1.228299e-06
female           female 9.529136e-03 9.483417e-03 -4.571842e-05
black             black 1.694240e-02 1.608548e-02 -8.569226e-04
metro             metro 1.230675e-02 1.157624e-02 -7.305096e-04
south             south 1.356134e-02 1.389454e-02  3.331966e-04
midwest         midwest 1.410367e-02 1.371725e-02 -3.864173e-04
west               west 1.440237e-02 1.454941e-02  1.470465e-04
[1] "置信區間寬度比較："
> print(width_comparison)
                   係數 OLS置信區間寬度 穩健置信區間寬度 更寬
(Intercept) (Intercept)    0.1259036061    0.1284875351 穩健
educ               educ    0.0068931063    0.0074670055 穩健
exper             exper    0.0050978780    0.0051518089 穩健
exper2           exper2    0.0001033205    0.0001081245 穩健
female           female    0.0373581454    0.0371749964  OLS
black             black    0.0664212100    0.0630550813  OLS
metro             metro    0.0482475482    0.0453788716  OLS
south             south    0.0531660647    0.0544665986 穩健
midwest         midwest    0.0552922035    0.0537716271  OLS
west               west    0.0564632233    0.0570337016 穩健
```

```
                   係數 OLS顯著性 穩健顯著性
(Intercept) (Intercept)      TRUE       TRUE
educ               educ      TRUE       TRUE
exper             exper      TRUE       TRUE
exper2           exper2      TRUE       TRUE
female           female      TRUE       TRUE
black             black      TRUE       TRUE
metro             metro      TRUE       TRUE
south             south      TRUE       TRUE
midwest         midwest      TRUE       TRUE
west               west     FALSE      FALSE
```

傳統 OLS 標準誤假設誤差方差恆定（同方差）。若此假設不成立（例如工資方差隨 metro 或 exper 變化），則傳統標準誤會有偏差，通常被低估。

White 穩健標準誤通過允許誤差方差變化來修正異方差問題，因此當存在異方差時，穩健標準誤通常更大，置信區間更寬。

更寬的置信區間表明傳統標準誤過於樂觀（置信區間過窄），可能導致對結果的過分自信。

e.

	係數	FGLS置信區間寬度	OLS穩健置信區間寬度	更寬
(Intercept)	(Intercept)	0.1238583747	0.1284875351	OLS穩健
educ	educ	0.0069180205	0.0074670055	OLS穩健
exper	exper	0.0050868017	0.0051518089	OLS穩健
exper2	exper2	0.0001050246	0.0001081245	OLS穩健
female	female	0.0371687649	0.0371749964	OLS穩健
black	black	0.0666175049	0.0630550813	FGLS
metro	metro	0.0449257899	0.0453788716	OLS穩健
south	south	0.0530129983	0.0544665986	OLS穩健
midwest	midwest	0.0548226168	0.0537716271	FGLS
west	west	0.0563618347	0.0570337016	OLS穩健

FGLS 與 OLS（White 穩健）的置信區間比較：

FGLS 通過顯式建模異方差（用 metro 和 exper 估計方差結構），試圖提高估計效率。若異方差模型指定正確，FGLS 的標準誤通常比 OLS（White 穩健）更小，置信區間更窄。

但若異方差模型指定錯誤（例如漏掉其他影響方差的變量），FGLS 的標準誤可能不準確，置信區間可能比 OLS（White 穩健）更寬，但結果可能不可靠。

從 width_comparison 表中，若大多數係數的 FGLS 置信區間寬度 小於 OLS 穩健置信區間寬度，則表明 FGLS 在效率上可能優於 OLS（White 穩健）。但需要注意 FGLS 對異方差模型的依賴性。

OLS（White 穩健）不假設誤差方差結構，直接調整標準誤以應對異方差，結果更穩健但可能效率較低（置信區間較寬）。

FGLS 假設誤差方差可以通過 metro 和 exper 建模，若假設正確，FGLS 會更有效率（置信區間更窄）；若假設錯誤，FGLS 可能表現不佳。

f.

```
> # 顯示結果
> print("置信區間寬度比較（FGLS vs FGLS穩健 vs OLS穩健）：")
[1] "置信區間寬度比較（FGLS vs FGLS穩健 vs OLS穩健）："
> print(width_comparison)
```

	係數	FGLS置信區間寬度	FGLS穩健置信區間寬度	OLS穩健置信區間寬度
(Intercept)	(Intercept)	0.1238583747	0.1267849388	0.1284875351
educ	educ	0.0069180205	0.0074158315	0.0074670055
exper	exper	0.0050868017	0.0051114864	0.0051518089
exper2	exper2	0.0001050246	0.0001073856	0.0001081245
female	female	0.0371687649	0.0369783728	0.0371749964
black	black	0.0666175049	0.0621737099	0.0630550813
metro	metro	0.0449257899	0.0453033498	0.0453788716
south	south	0.0530129983	0.0542033173	0.0544665986
midwest	midwest	0.0548226168	0.0537263583	0.0537716271
west	west	0.0563618347	0.0568452827	0.0570337016

第(d)小題的 OLS（White 穩健）不假設任何方差結構，直接調整標準誤以應對異方差，置信區間通常較寬但更可靠
FGLS（穩健標準誤）可能比 OLS（White 穩健）更窄（如果異方差模型部分正確)，但比第(e)小題的 FGLS 更寬（因為穩健標準誤考慮了模型誤差)。

g.

選擇 FGLS（White 穩健）
FGLS（White 穩健）的標準誤比 OLS（White 穩健）略小（例如 educ 的標準誤：0.0074135815 vs 0.0074670055)，置信區間更窄，表明其效率略高，充分利用了 metro 和 exper 的異方差結構。
同時，使用 White 穩健標準誤確保即使異方差模型（metro 和 exper）指定錯誤，標準誤和置信區間仍然可靠，避免了 FGLS（傳統標準誤）可能低估不確定性的風險。