HW5

6.

a. $H_0: \beta_2 = 0$, $H_1: \beta_2 \neq 0$

$t = \frac{b_2 - \beta_2}{se(b_2)} = \frac{3-0}{\sqrt{4}} = 1.5 < t_c \approx 2.00298 \longrightarrow$ Fail to reject H_0

b. $H_0: \beta_1 + 2\beta_2 = 5$, $H_1: \beta_1 + 2\beta_2 \neq 5$

$t = \frac{(b_1 + 2b_2) - 5}{se(\beta_1 + 2\beta_2)} = \frac{2 + 2\times3 - 5}{\sqrt{3 + 2^2\times4 - 2\times2\times2}} = \frac{3}{\sqrt{11}} \approx 0.9035 < t_c \approx 2.00298 \longrightarrow$ Fail to reject H_0.

c. $H_0: \beta_1 - \beta_2 + \beta_3 = 4$, $H_1: \beta_1 - \beta_2 + \beta_3 \neq 4$

$t = \frac{(b_1 - b_2 + b_3) - 4}{se(\beta_1 - \beta_2 + \beta_3)} = \frac{2 - 3 + (-1) - 4}{\sqrt{3 + 4 + 3 + 2(-1)(-2) + 2\times1}} = \frac{-6}{4} = -1.5 > -t_c \approx -2.00298 \longrightarrow$ Fail to reject H_0.

31.

a.

```
Residuals:
     Min       1Q   Median       3Q      Max 
-18.4389  -3.6774  -0.1188   4.5863  16.4986 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)  20.8701     1.6758  12.454  < 2e-16 ***
depart        0.3681     0.0351  10.487  < 2e-16 ***
reds          1.5219     0.1850   8.225 1.15e-14 ***
trains        3.0237     0.6340   4.769 3.18e-06 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 6.299 on 245 degrees of freedom
Multiple R-squared:  0.5346,	Adjusted R-squared:  0.5289 
F-statistic: 93.79 on 3 and 245 DF,  p-value: < 2.2e-16
```

$\longrightarrow \widehat{TIME} = 20.8701 + 0.3681\, DEPART + 1.5219\, REDS + 3.0237\, TRAINS$

$\longrightarrow$ β_1 Intercept (20.8701):
If Bill leaves at exactly 6:30 AM (depart = 0), and encounters 0 red lights and 0 trains, the expected travel time is approximately 20.87 minutes.

β_2 depart (0.3681):
For each additional minute after 6:30 AM that Bill departs, the expected travel time increases by 0.368 minutes (~22 seconds), holding reds and trains constant.
This effect is highly significant ($p < 2e-16$).

β_3 reds (1.5219):
Each red light Bill encounters adds 1.52 minutes to his travel time on average.
Also highly significant ($p < 0.001$).

β_4 trains (3.0237):
Each train adds about 3.02 minutes of delay.
Also statistically significant ($p < 0.001$).

b.

```
                 2.5 %    97.5 %
(Intercept) 17.5694018 24.170871
depart       0.2989851  0.437265
reds         1.1574748  1.886411
trains       1.7748867  4.272505
```

c. $H_0: \beta_3 \geq 2$, $H_1: \beta_3 < 2$

$t = \frac{b_3 - \beta_3}{se(\beta_3)} = \frac{1.5219 - 2}{0.185} \approx -2.5843 < t_c \approx -1.645 \longrightarrow$ reject H_0, means the expected delay due to a red light is less than 2 min.

d. $H_0: \beta_4 = 3$, $H_1: \beta_4 \neq 3$

$t = \frac{b_4 - \beta_4}{se(\beta_4)} = \frac{3.0237 - 3}{0.634} \approx 0.0374 < t_c \approx 1.6511 \longrightarrow$ fail to reject H_0, means the expected delay due to a train is 3 min.

e. $H_0: 30\beta_2 \geq 10$, $H_1: 30\beta_2 < 10$

$t = \frac{b_2 - \beta_2}{se(\beta_2)} = \frac{0.3681 - \frac{1}{3}}{0.0351} \approx 0.9905 < t_c \approx 1.645 \longrightarrow$ fail to reject H_0, means the expected delay due to 30 min. more of departure time is at least 10 mins.

f. $H_0: \beta_4 \geq 3\beta_3$, $H_1: \beta_4 < 3\beta_3$

$t = \frac{3.0237 - 3\times1.5219 - 0}{\sqrt{0.634^2 + 3^2\times0.185^2}} \approx -1.83 < t_c = -1.645 \longrightarrow$ reject H_0, means the expected delay of a train is less than 3 times of red light.

g. $H_0: \beta_1 + 30\beta_2 + 6\beta_3 + \beta_4 \le 45$ $\quad H_1: \beta_1 + 30\beta_2 + 6\beta_3 + \beta_4 > 45$

$t \approx \frac{44.07 - 45}{1.5} = 20.62 < t_c = -1.645$ $\longrightarrow$ fail to reject H_0, cannot conclude that Bill will arrive after 7.45 AM.

h. $H_0: \beta_1 + 30\beta_2 + 6\beta_3 + \beta_4 \ge 45$ $\quad H_1: \beta_1 + 30\beta_2 + 6\beta_3 + \beta_4 < 45$

$t \approx -1.726 < t_c = -1.6511$ $\longrightarrow$ reject H_0, means Bill will on time expectedly.

33.

a.

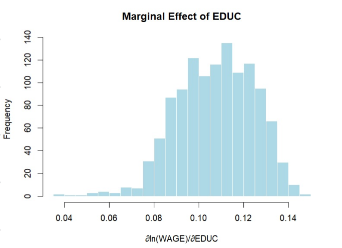
```
Residuals:
    Min      1Q  Median      3Q     Max 
-1.6628 -0.3138 -0.0276  0.3140  2.1394 

Coefficients:
                  Estimate Std. Error t value Pr(>|t|)    
(Intercept)      1.038e+00  2.757e-01   3.764 0.000175 ***
educ             8.954e-02  3.108e-02   2.881 0.004038 ** 
I(educ^2)        1.458e-03  9.242e-04   1.578 0.114855    
exper            4.488e-02  7.297e-03   6.150 1.06e-09 ***
I(exper^2)      -4.680e-04  7.601e-05  -6.157 1.01e-09 ***
I(educ * exper) -1.010e-03  3.791e-04  -2.665 0.007803 ** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.4638 on 1194 degrees of freedom
Multiple R-squared:  0.3227,	Adjusted R-squared:  0.3198 
F-statistic: 113.8 on 5 and 1194 DF,  p-value: < 2.2e-16
```

$\longrightarrow$ Except $EDUC^2$ is at 12% significant level, others are at 1% significant level.

b. $\frac{\partial(\ln(WAGE))}{\partial(EDUC)} = \beta_2 + 2\beta_3 EDUC + \beta_6 EXPER = 0.08954 + 0.002916 EDUC - 0.00101 EXPER$

$\longrightarrow$ The marginal effect increases as EDUC increases but decrease at EXPER increases.

c.

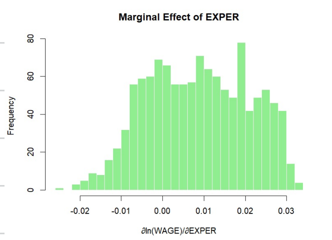



```
        5%        50%        95% 
0.08008187 0.10843125 0.13361880 
```

d. $\frac{\partial \ln(WAGE)}{\partial EXPER} = \beta_4 + 2\beta_5 EXPER + \beta_6 EDUC = 0.04488 - 0.000936 EXPER - 0.00101 EDUC$

$\longrightarrow$ The marginal effect decreases as EDUC or EXPER increases.

e.




```
          5%          50%          95% 
-0.010376212  0.008418878  0.027931151 
```

f. $H_0: -\beta_2 - 33\beta_3 + 10\beta_4 + 260\beta_5 + 152\beta_6 \ge 0$ $\quad H_1: -\beta_2 - 33\beta_3 + 10\beta_4 + 260\beta_5 + 152\beta_6 < 0$

$t = -1.6699 > t_c = -1.6463$ $\longrightarrow$ fail to reject H_0, cannot conclude David's $\ln(WAGE)$ is greater.

5.33 Use the observations in the data file *cps5_small* to estimate the following model:

$$\ln(WAGE) = \beta_1 + \beta_2 EDUC + \beta_3 EDUC^2 + \beta_4 EXPER + \beta_5 EXPER^2 + \beta_6 (EDUC \times EXPER) + e$$

g. After eight years have passed, when David and Svetlana have had eight more years of experience, but no more education, will the test result in (f) be the same? Explain this outcome?

h. Wendy has 12 years of education and 17 years of experience, while Jill has 16 years of education and 11 years of experience. Using a 5% significance level, test the null hypothesis that their marginal effects of extra experience are equal against the alternative that they are not. State the null and alternative hypotheses in terms of the model parameters.

i. How much longer will it be before the marginal effect of experience for Jill becomes negative? Find a 95% interval estimate for this quantity.

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g. $H_0: -\beta_2 - 33\beta_3 + 10\beta_4 + 420\beta_5 + 144\beta_6 \geq 0$ $\quad H_1: -\beta_2 - 33\beta_3 + 10\beta_4 + 420\beta_5 + 144\beta_6 < 0$

$t = \frac{-0.03092 - 0}{0.01499} = -2.0627 < t_c = -1.6463 \longrightarrow$ reject H_0, can conclude David's $\ln(WAGE)$ is greater, not same with problem (f).

h. $H_0: 12\beta_5 - 4\beta_6 = 0$ $\quad H_1: 12\beta_5 - 4\beta_6 \neq 0$.

$t = \frac{-0.001575}{0.0015334} \approx -1.027 > t_c = -1.962 \longrightarrow$ fail to reject H_0, marginal effect of extra experience is different for Jill and Wendy.

i. $X = \frac{-\beta_4 - 22\beta_5 - 16\beta_6}{2\beta_5} \approx 19.6771$

C.I. $= X \pm t_c \times se = 19.6771 \pm 1.96 \times 1.8957 \Rightarrow (15.958, 23.396)$