

15.20

This exercise uses data from the STAR experiment introduced to illustrate fixed and random effects for grouped data. In the STAR experiment, children were randomly assigned within schools into three types of classes: small classes with 13–17 students, regular-sized classes with 22–25 students, and regular-sized classes with a full-time teacher aide to assist the teacher. Student scores on achievement tests were recorded as well as some information about the students, teachers, and schools. Data for the kindergarten classes are contained in the data file *star*.

- a. Estimate a regression equation (with no fixed or random effects) where *READSCORE* is related to *SMALL*, *AIDE*, *TCHEXPER*, *BOY*, *WHITE_ASIAN*, and *FREELUNCH*. Discuss the results. Do students perform better in reading when they are in small classes? Does a teacher's aide improve scores? Do the students of more experienced teachers score higher on reading tests? Does the student's sex or race make a difference?

Ans.

```
Call:
lm(formula = readscore ~ small + aide + tchexper + boy + white_asian +
    freelunch, data = star)

Residuals:
    Min       1Q   Median       3Q      Max
-107.220  -20.214   -3.935   14.339  185.956

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept)  437.76425    1.34622  325.180 < 2e-16 ***
small         5.82282     0.98933    5.886 4.19e-09 ***
aide          0.81784     0.95299    0.858  0.391
tchexper      0.49247     0.06956    7.080 1.61e-12 ***
boy          -6.15642     0.79613   -7.733 1.23e-14 ***
white_asian   3.90581     0.95361    4.096 4.26e-05 ***
freelunch    -14.77134     0.89025  -16.592 < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 30.19 on 5759 degrees of freedom
(20 observations deleted due to missingness)
Multiple R-squared:  0.09685,    Adjusted R-squared:  0.09591
F-statistic: 102.9 on 6 and 5759 DF,  p-value: < 2.2e-16
```

Do students perform better in reading when they are in small classes?

小班制 *SMALL* 對閱讀成績有顯著正向影響，估計值為 +5.82 分，*p* 值極小，顯示統計上非常顯著。

Does a teacher's aide improve scores?

沒有明顯提升，*AIDE* 的係數為 +0.82，但 *p* 值為 0.391，不顯著。

Do the students of more experienced teachers score higher on reading tests?

會。*TCHEXPER* 的係數為 +0.49，*p* 值 < 0.001，表示教師每多一年經驗，閱讀成績平均提高約 0.49 分，顯著正向。

Does the student's sex or race make a difference?

性別有差異：男生 *BOY* 的閱讀分數平均顯著較低，估計為 -6.16，*p* < 0.001。

種族有差異：白人或亞洲學生 *WHITE_ASIAN* 分數顯著較高，估計為 +3.91，*p* < 0.001。

預估領取免費午餐 *FREELUNCH* 的學生，其平均閱讀成績比未領取者低 14.8 分。

除 *AIDE* 外，所有迴歸係數在個別檢定下皆具有顯著性。

- b. Reestimate the model in part (a) with school fixed effects. Compare the results with those in part (a). Have any of your conclusions changed? [Hint: specify *SCHID* as the cross-section identifier and *ID* as the “time” identifier.]

Ans.

SMALL：固定效果估計結果為 +6.49 分，比 OLS 的 +5.82 分更高。表示在同一學校內部比較時，小班的效益更明確，排除學校選擇偏誤後效果變大。

AIDE：在兩個模型中皆為不顯著，表示是否有教學助理對閱讀成績沒有明顯影響。

TCHEXPER：OLS 預估為每年 +0.49，FE 模型下降為 +0.29。教師經驗仍顯著，但效果下降。顯示學校間選擇偏誤（如好學校配資深老師）可能存在。

BOY：男生閱讀成績較低，估計值略為下降（-6.16 → -5.46），但仍顯著。表示性別差異穩定存在，不受學校影響太大。

WHITE_ASIAN：種族差異從 OLS 的 +3.91 上升到 +8.03。效果大幅上升。表示控除學校後，族群差異更明顯。白人與亞洲學生的成績優勢，原本被學校特性（如優質學校學生偏多）低估。

FREELUNCH：貧窮 proxy 的負面影響一致，代表低社經背景學生成績普遍較差，即使在同一學校內。

```
Oneway (individual) effect Within Model

Call:
plm(formula = readscore ~ small + aide + tchexper + boy + white_asian +
      freelunch, data = pdata, effect = "individual", model = "within")

Unbalanced Panel: n = 79, T = 34-137, N = 5766

Residuals:
    Min.   1st Qu.   Median   3rd Qu.    Max.
-102.6381 -16.7834  -2.8473  12.7591  198.4169

Coefficients:
              Estimate Std. Error t-value Pr(>|t|)
small          6.490231   0.912962   7.1090 1.313e-12 ***
aide            0.996087   0.881693   1.1297  0.2586
tchexper        0.285567   0.070845   4.0309 5.629e-05 ***
boy            -5.455941   0.727589  -7.4987 7.440e-14 ***
white_asian     8.028019   1.535656   5.2277 1.777e-07 ***
freelunch     -14.593572   0.880006 -16.5835 < 2.2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Total Sum of Squares:  4628000
Residual Sum of Squares: 4268900
R-Squared: 0.077592
Adj. R-Squared: 0.063954
F-statistic: 79.6471 on 6 and 5681 DF, p-value: < 2.22e-16
```

加入學校固定效果後，大多數結論未改變，但部分變數的影響力更清楚。

固定效果模型能排除學校間異質性的干擾，更準確估計學生層級變數的真實效果。

小班效益與種族差異在控制學校後更加明顯，教師年資影響被部分吸收。

- c. Test for the significance of the school fixed effects. Under what conditions would we expect the inclusion of significant fixed effects to have little influence on the coefficient estimates of the remaining variables?

Ans.

```
F test for individual effects

data: readscore ~ small + aide + tchexper + boy + white_asian + freelunch
F = 16.698, df1 = 78, df2 = 5681, p-value < 2.2e-16
alternative hypothesis: significant effects

> qf(0.95, df1 = 78, df2 = 5681)
[1] 1.27979
```

H_0 ：所有學校固定效果都等於零

H_1 ：至少一個學校固定效果不為零

$\alpha = 0.05$

$df_1 = n - 1 = 79 - 1 = 78$

$df_2 = N - n - k = 5766 - 79 - 6 = 5681$

F 統計量 = 16.698

> 臨界值 = 1.2798

結論：拒絕虛無假設，表示學校固定效果顯著存在，應使用 fixed effects 而非 pooled OLS

當以下情況成立時，加入固定效果不會明顯改變其他變數的係數估計：

解釋變數（如 *BOY*, *FREELUNCH* 等）主要在學校內部變動，而非學校間變動。如果變數的變化主要是「同校學生之間」的差異，而不是「學校之間」的差異，則固定效果的納入不會太影響這些變數的估計。