HW0505-2 王彩寧 313707006

11.28

(a)

Demand: $Q_i = \alpha_1 + \alpha_2 P_i + \alpha_3 PS_i + \alpha_4 DI_i + e_{di}$ (11.11)

Supply: $Q_i = \beta_1 + \beta_2 P_i + \beta_3 PF_i + e_{si}$ (11.12)

Demand: $\alpha_2 P_i = Q_i - \alpha_i - \alpha_3 PS_i + \alpha_4 DI_i + e_{di}$

$\Rightarrow P_i = -\frac{\alpha_i}{\alpha_2} + \frac{Q_i}{\alpha_2} - \frac{\alpha_3}{\alpha_2} PS_i + \frac{\alpha_4}{\alpha_2} DI_i + \frac{e_{di}}{\alpha_2} = \delta_1 + \delta_2 Q_i - \delta_3 PS_i + \delta_4 DI_i + u^d$

$\delta_2 < 0$: law of demand

$\delta_4 > 0$: normal goods

$\delta_3 > 0$: subsitute goods

Supply: $\beta_2 P_i = Q_i - \beta_1 - \beta_3 PF_i + e_{si}$

$\Rightarrow P_i = -\frac{\beta_1}{\beta_2} + \frac{Q_i}{\beta_2} - \frac{\beta_3}{\beta_2} PF_i + \frac{e_{si}}{\beta_2} = \pi_1 + \pi_2 Q_i + \pi_3 PF_i + \pi_4 e_{si}$

$\pi_2 > 0$: law of supply

$\pi_3 > 0$: cost of production factor $\uparrow \Rightarrow$ supply $\uparrow$

(b)

```
Call:
ivreg(formula = p ~ q + ps + di | ps + di + pf, data = truffles)

Residuals:
    Min      1Q  Median      3Q     Max 
-39.661  -6.781   2.410   8.320  20.251 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)  -11.428     13.592  -0.841  0.40810    
q             -2.671      1.175  -2.273  0.03154 *  
ps             3.461      1.116   3.103  0.00458 ** 
di            13.390      2.747   4.875 4.68e-05 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 13.17 on 26 degrees of freedom
Multiple R-Squared: 0.5567,	Adjusted R-squared: 0.5056 
Wald test: 17.37 on 3 and 26 DF,  p-value: 2.137e-06 
```

```
Call:
ivreg(formula = p ~ q + pf | ps + di + pf, data = truffles)

Residuals:
    Min      1Q  Median      3Q     Max 
-9.7983 -2.3440 -0.6281  2.4350 11.1600 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) -58.7982     5.8592  -10.04 1.32e-10 ***
q             2.9367     0.2158   13.61 1.32e-13 ***
pf            2.9585     0.1560   18.97  < 2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 4.399 on 27 degrees of freedom
Multiple R-Squared: 0.9486,	Adjusted R-squared: 0.9448 
Wald test: 232.7 on 2 and 27 DF,  p-value: < 2.2e-16 
```

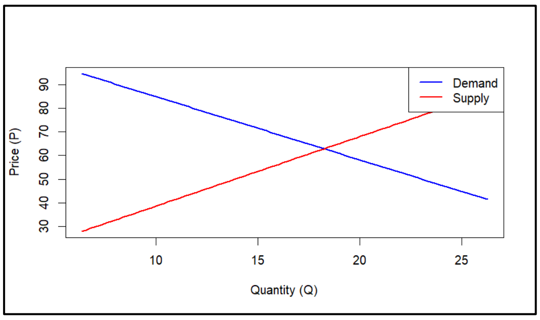
All signs are as expected.

Only the intercept is not significantly different from zero; the other coefficients are.

(c)

Elasticity:-1.272464

(d)



(e)

Equilibrium values of P: 62.84257 and Q: 18.25021

Reduced-form of P_hat:62.81537 and Q_hat:18.26040

There is only a slight difference between the results of the two methods.

(f)

```
=== Demand Function Comparison ===
> print(compare_demand)
               Variable OLS_Estimate OLS_StdError OLS_pValue IV_Estimate IV_StdError IV_pValue
(Intercept) (Intercept)     -13.6195       9.0872     0.1460    -11.4284     13.5916    0.4081
q                     q       0.1512       0.4988     0.7642     -2.6705      1.1750    0.0315
ps                   ps       1.3607       0.5940     0.0303      3.4611      1.1156    0.0046
di                   di      12.3582       1.8254     0.0000     13.3899      2.7467    0.0000
>
> cat("\n=== Supply Function Comparison ===\n")

=== Supply Function Comparison ===
> print(compare_supply)
               Variable OLS_Estimate OLS_StdError OLS_pValue IV_Estimate IV_StdError IV_pValue
(Intercept) (Intercept)     -52.8763       5.0238          0    -58.7982      5.8592         0
q                     q       2.6613       0.1712          0      2.9367      0.2158         0
pf                   pf       2.9217       0.1482          0      2.9585      0.1560         0
```

In the demand equation, the OLS estimate for quantity is positive and insignificant, whereas the IV estimate is negative and statistically significant, aligning with economic theory. This suggests that OLS suffers from endogeneity bias. In contrast, the supply equation results are consistent across both methods, with similar coefficients and high significance, indicating that endogeneity is less of a concern in the supply specification.

11.30

(a)

```
Call:
lm(formula = i ~ p + plag + klag, data = klein)

Residuals:
     Min       1Q   Median       3Q      Max 
-2.56562 -0.63169  0.03687  0.41542  1.49226 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 10.12579    5.46555   1.853 0.081374 .  
p            0.47964    0.09711   4.939 0.000125 ***
plag         0.33304    0.10086   3.302 0.004212 ** 
klag        -0.11179    0.02673  -4.183 0.000624 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.009 on 17 degrees of freedom
  (因為不存在，1 個觀察量被刪除了)
Multiple R-squared:  0.9313,	Adjusted R-squared:  0.9192 
F-statistic: 76.88 on 3 and 17 DF,  p-value: 4.299e-10
```

The OLS estimation of the investment function yields statistically significant results for all variables. Current and lagged profits both have positive and significant effects on investment, consistent with economic theory. The lagged capital stock has a negative and significant coefficient, indicating that higher past capital stock may reduce current investment due to adjustment costs or capital saturation.

(b)

```
Call:
lm(formula = p ~ g + w2 + tx + time + plag + klag + elag, data =

Residuals:
    Min      1Q  Median      3Q     Max
-3.9067 -1.3050  0.3226  1.3613  2.8881

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 50.38442   31.63026   1.593   0.1352
g            0.43902    0.39114   1.122   0.2820
w2          -0.07961    2.53382  -0.031   0.9754
tx          -0.92310    0.43376  -2.128   0.0530 .
time         0.31941    0.77813   0.410   0.6881
plag         0.80250    0.51886   1.547   0.1459
klag        -0.21610    0.11911  -1.814   0.0928 .
elag         0.02200    0.28216   0.078   0.9390
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 2.183 on 13 degrees of freedom
  (因為不存在，1 個觀察量被刪除了)
Multiple R-squared:  0.8261,	Adjusted R-squared:  0.7324
F-statistic: 8.821 on 7 and 13 DF,  p-value: 0.0004481
```

```
Linear hypothesis test:
g = 0
w2 = 0
tx = 0
time = 0
elag = 0

Model 1: restricted model
Model 2: p ~ g + w2 + tx + time + plag + klag + elag

  Res.Df RSS Df Sum of Sq    F Pr(>F)
1     18 108
2     13  62  5      46.1 1.93   0.16
```

```
> cat("F statistic =", round(F_stat, 3), "\n")
F statistic = 1.934
> cat("Critical F(5,13;0.95) =", round(F_crit, 3), "\n")
Critical F(5,13;0.95) = 3.025
```

```
> df$phat
[1] 13.255556 16.577368 19.282347 20.960143 19.766509 18.238731 17.573065 19.541720 20.375101
[10] 17.180415 12.705026  8.999780  9.054102 12.671263 14.421338 14.711907 19.796405 19.206691
[19] 17.419605 20.305654 22.657273
```

A joint F-test for the coefficients of g, w2, tx, time, and elag yields a p-value of 0.16. We fail to reject the null hypothesis, suggesting that these variables are not jointly significant in explaining the dependent variable.

(c)

```
Call:
lm(formula = i ~ p + plag + klag + vhat, data = df)

Residuals:
     Min       1Q   Median       3Q      Max
-1.04645 -0.56030  0.06189  0.25348  1.36700

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 20.27821    4.70179   4.313 0.000536 ***
p            0.15022    0.10798   1.391 0.183222
plag         0.61594    0.10147   6.070 1.62e-05 ***
klag        -0.15779    0.02252  -7.007 2.96e-06 ***
vhat         0.57451    0.14261   4.029 0.000972 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.7331 on 16 degrees of freedom
Multiple R-squared:  0.9659,	Adjusted R-squared:  0.9574
F-statistic: 113.4 on 4 and 16 DF,  p-value: 1.588e-11
```

Since the p-value is less than 0.05, we reject the null hypothesis that v hat equals zero and conclude that P is endogenous.

(d)

```
Call:
ivreg(formula = i ~ p + plag + klag | g + w2 + tx + time + elag +
    plag + klag, data = df)

Residuals:
    Min      1Q  Median      3Q     Max
-3.2909 -0.8069  0.1423  0.8601  1.7956

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 20.27821    8.38325   2.419  0.02707 *
p            0.15022    0.19253   0.780  0.44598
plag         0.61594    0.18093   3.404  0.00338 **
klag        -0.15779    0.04015  -3.930  0.00108 **
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 1.307 on 17 degrees of freedom
Multiple R-Squared: 0.8849,	Adjusted R-squared: 0.8646
Wald test:  41.2 on 3 and 17 DF,  p-value: 5.148e-08
```

```
> print(compare, row.names = FALSE)
 OLS.estimate  OLS.p.value 2SLS.estimate 2SLS.p.value
   10.1257885 0.0813741769    20.2782089  0.027070529
    0.4796356 0.0001245554     0.1502218  0.445979836
    0.3330387 0.0042117328     0.6159436  0.003375496
   -0.1117947 0.0006244484    -0.1577876  0.001079721
```

The discrepancy between the OLS and 2SLS estimates suggests that the OLS results suffer from simultaneity bias. In periods of high investment, firms also tend to report high profits, which inflates the estimated effect of profits on investment. In contrast, the 2SLS approach addresses this bias by using external instruments, though it leads to a less precise estimate for profits due to relying on variation from the instruments rather than from the endogenous variable itself. This trade-off between bias and variance is typical in simultaneous equations modeling.

(e)

```
Call:
lm(formula = i ~ phat + plag + klag, data = df)

Residuals:
    Min      1Q  Median      3Q     Max
-3.8778 -1.0029  0.3058  0.7275  2.1831

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 20.27821    9.97663   2.033  0.05802 .
phat         0.15022    0.22913   0.656  0.52084
plag         0.61594    0.21531   2.861  0.01083 *
klag        -0.15779    0.04778  -3.302  0.00421 **
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 1.556 on 17 degrees of freedom
Multiple R-squared:  0.837,	Adjusted R-squared:  0.8082
F-statistic: 29.09 on 3 and 17 DF,  p-value: 6.393e-07
```

The second-stage OLS regression using P^t\hat{P}_tP^t produces coefficient estimates that are nearly identical to those obtained from the 2SLS estimation in part (d), which is expected since both methods follow the same two-stage procedure. However, the standard errors from this manual estimation are smaller, particularly for P^t\hat{P}_tP^t, because the model does not adjust for the uncertainty introduced by the first-stage estimation. As a result, the coefficient on profits remains statistically insignificant, consistent with the ivreg() output, but the confidence in that insignificance may be overstated due to underestimated standard errors.

(f)

```
> cat("   TR^2   =", round(TR2, 3), "\n")
   TR^2   = 1.282
> cat("   χ²_0.95(df=4) =", round(crit95, 3), "\n")
   χ²_0.95(df=4) = 9.488
> if (TR2 < crit95) {
+   cat("   → Fail to reject H0: surplus instruments appear valid.\n")
+ } else {
+   cat("   → Reject H0: at least one surplus instrument may be invalid.\n")
+ }
   → Fail to reject H0: surplus instruments appear valid.
```