(a)

$y_1 = a_1 y_2 + e_1$

$y_2 = a_2 y_1 + \beta_1 x_1 + \beta_2 x_2 + e_2$

$y_2 = a_2 (a_1 y_2 + e_1) + \beta_1 x_1 + \beta_2 x_2 + e_2$

$\Rightarrow y_2 = \frac{\beta_1}{(1-a_1 a_2)} x_1 + \frac{\beta_2}{(1-a_1 a_2)} x_2 + \frac{e_2 + a_2 e_1}{(1-a_1 a_2)}$

$= \pi_1 x_1 + \pi_2 x_2 + v_2$

$corr(y_2, e_1 | x) = E(y_2, e_1 | x)$

$= E\left[\frac{\beta_1}{(1-a_1a_2)} x_1 e_1 \middle| x\right] + E\left[\frac{\beta_2}{(1-a_1a_2)} x_2 \middle| x\right] + E\left[\left(\frac{e_2 + a_2 e_1}{(1-a_1 a_2)}\right) e_1 \middle| x\right]$

$= E\left[\left(\frac{e_2 e_1 + a_2 e_1^2}{1-a_1a_2}\right) \middle| x\right] = \frac{E(e_2 e_1 | x) + a_2 E(e_1^2 | x)}{1-a_1 a_2} = \frac{a_2}{1-a_1 a_2} \sigma_1^2$. if $a_2 = 0$, then simultaneity

(b)

(1) (2) cannot use OLS, because there are endogenous on the righ-hand side,

however we can use reduced form equation. it can be estimated consistently using OLS.

(c)

(1) is identify. (2) has 2 exogenous variable not identify.

(d) x are exogenous, mom uncorrelated. assumption $E(x_1 v_1 | x) = E(x_2 v_2 | x) = 0$.

(e) $S(\pi_1 \pi_2 | y, x) = \sum (y_2 - \pi_1 x_1 - \pi_2 x_2)^2$

$\frac{\partial S}{\partial \pi_1} = 2\sum (y_2 - \pi_1 x_1 - \pi_2 x_2) x_1 = 0 / 2N$

$\frac{\partial S}{\partial \pi_2} = 2\sum (y_2 - \pi_1 x_1 - \pi_2 x_2) x_2 = 0 / 2N$

(f)

$\sum x_{i1} y_{i2} - \pi_1 \sum x_{i1}^2 - \pi_2 \sum x_{i1} x_{i2} = 0$

$\sum x_{i2} y_{i2} - \pi_1 \sum x_{i1} x_{i2} - \pi_2 \sum x_{i2}^2 = 0$

$3 - \hat{\pi}_1 = 0 \qquad \hat{\pi}_1 = 3$

$4 - \hat{\pi}_2 = 0. \qquad \hat{\pi}_2 = 4$

(g)

first $y_1 = a_1 y_2 + e_1 \Rightarrow E[(\pi_1 x_1 + \pi_2 x_2) e_1 | x] = 0$

$\sum (\hat{\pi}_1 x_{i1} + \hat{\pi}_2 x_{i2})(y_{i1} - a_1 y_{i2}) = \sum \hat{y}_{i2} (y_{i1} - a_1 y_{i2}) = 0$

$\sum \hat{y}_{i2} y_{i1} - a_1 \sum \hat{y}_{i2} y_{i2} = 0 \Rightarrow \hat{a}_{1,IV} = \frac{\sum \hat{y}_{i2} y_{i1}}{\sum \hat{y}_{i2} y_{i2}}$

$\hat{\alpha}_{1IV} = \frac{\hat{\pi}_1 \sum x_{i1} y_{i1} + \hat{\pi}_2 \sum x_{i2} y_{i1}}{\hat{\pi}_1 \sum x_{i1} y_{i2} + \hat{\pi}_2 \sum x_{i2} y_{i2}} = \frac{18}{25}$

(h)

$\hat{\alpha}_{1,2SLS} = \frac{\sum \hat{y}_{i2} y_{i1}}{\sum \hat{y}_{i2}^2} \qquad \hat{y}_2 = y_2 - \hat{v}_2$

$\sum \hat{y}_{i2}^2 = \sum \hat{y}_{i2} (y_2 - \hat{v}_2) = \sum \hat{y}_{i2} y_2$

$\sum \hat{y}_{i2} \hat{v}_2 = \sum (\hat{\pi}_1 x_{i1} + \hat{\pi}_2 x_{i2}) \hat{v}_{i2} = \hat{\pi}_1 \underbrace{\sum x_{i1} \hat{v}_{i2}}_{0} + \hat{\pi}_2 \underbrace{\sum x_{i2} \hat{v}_{i2}}_{0} = 0.$

11.16 Consider the following supply and demand model

外生

$$\text{Demand: } Q_i = \alpha_1 + \alpha_2 P_i + e_{di}, \quad \text{Supply: } Q_i = \beta_1 + \beta_2 P_i + \beta_3 W_i + e_{si}$$

where Q is the quantity, P is the price, and W is the wage rate, which is assumed exogenous. Data on these variables are in Table 11.7.

TABLE 11.7 Data for Exercise 11.16

10

30

$\bar{Q} = 6$

Q		P	W
4	-2	2	2
6	0	4	3
9	3	3	1
3	-3	5	1
8	2	8	3

a. Derive the algebraic form of the reduced-form equations, $Q = \theta_1 + \theta_2 W + v_2$ and $P = \pi_1 + \pi_2 W + v_1$, expressing the reduced-form parameters in terms of the structural parameters.

b. Which structural parameters can you solve for from the results in part (a)? Which equation is "identified"?

c. The estimated reduced-form equations are $\hat{Q} = 5 + 0.5W$ and $\hat{P} = 2.4 + 1W$. Solve for the identified structural parameters. This is the method of **indirect least squares**.

d. Obtain the fitted values from the reduced-form equation for P, and apply 2SLS to obtain estimates of the demand equation.

(a)

$$\alpha_1 + \alpha_2 P_i + e_{di} = \beta_1 + \beta_2 P_i + \beta_3 W_i + e_{si}$$

$$P_i = \frac{\beta_1 - \alpha_1}{\alpha_2 - \beta_2} + \frac{\beta_3}{\alpha_2 - \beta_2} W_i + \frac{e_{si} - e_{di}}{\alpha_2 - \beta_2}$$

$$Q_i = \alpha_1 + \alpha_2 \left(\frac{\beta_1 - \alpha_1}{\alpha_2 - \beta_2} + \frac{\beta_3}{\alpha_2 - \beta_2} W_i + \frac{e_{si} - e_{di}}{\alpha_2 - \beta_2} \right) + e_{di}$$

$$= \alpha_1 + \frac{\beta_1 - \alpha_1}{\alpha_2 - \beta_2} \alpha_2 + \frac{\beta_3}{\alpha_2 - \beta_2} W_i \alpha_2 + \frac{e_{si} - e_{di}}{\alpha_2 - \beta_2} \alpha_2 + e_{di}$$

(b).

① yes, α_1 α_2 可

② no, $\beta_1, \beta_2, \beta_3$ 不可

(c)

$\hat{Q} = 5 + 0.5W$

$\hat{P} = 2.4 + 1W$

$$5 + 0.5W = \alpha_1 + \alpha_2 (2.4 + 1W)$$

$$= \alpha_1 + \alpha_2 \times 2.4 + W\alpha_2$$

$\alpha_2 = 0.5 \quad \alpha_1 = 3.8$

(d).

$\hat{P} = 2.4 + W$

W	$\hat{P}$	$\hat{P} - \bar{\hat{P}}$	$Q - \bar{Q}$
2	4.4	0	-2
3	5.4	1	0
1	3.4	-1	3
1	3.4	-1	-3
3	5.4	1	2

$\bar{\hat{P}} = 4.4$

$$Q = \alpha_1 + \alpha_2 \hat{P} + e_i$$

$$\hat{\alpha}_2 = \frac{\sum (\hat{P}_i - \bar{\hat{P}})(Q_i - \bar{Q})}{\sum (\hat{P}_i - \bar{\hat{P}})^2} = \frac{2}{4} = 0.5$$

$$\hat{\alpha}_1 = \bar{Q} - \alpha_2 \bar{\hat{P}} = 6 - 0.5 \times 4.4 = 3.8$$

$$\hat{Q} = 3.8 + 0.5P$$

11.17

The first equation is a consumption function, in which aggregate consumption in year t, CN_t is related to total wages earned by all workers, W_t. Total wages are divided into wages of workers earned in the private sector, W_{1t}, and wages of workers earned in the public sector, W_{2t}, so that total wages $W_t = W_{1t} + W_{2t}$. Private sector wages W_{1t} are endogenous and determined within the structure of the model, as we will see below. Public sector wages W_{2t} are exogenous. In addition, consumption expenditures are related to nonwage income (profits) in the current year, P_t, which are endogenous, and profits from the previous year, P_{t-1}. Thus, the consumption function is

$$CN_t = \alpha_1 + \alpha_2(W_{1t} + W_{2t}) + \alpha_3 P_t + \alpha_4 P_{t-1} + e_{1t} \quad (11.17)$$

Now refer back to equation (5.44) in Section 5.7.3. There we introduced the term **contemporaneously uncorrelated** to describe the situation in which an explanatory variable observed at time t, x_{tk} is uncorrelated with the random error at time t, e_t. In the terminology of Chapter 10, the variable x_{tk} is **exogenous** if it is contemporaneously uncorrelated with the random error e_t. And the variable x_{tk} is **endogenous** if it is contemporaneously correlated with the random error e_t. In the consumption equation, W_{1t} and P_t are endogenous and contemporaneously correlated with the random error e_t. On the other hand, wages in the public sector, W_{2t}, are set by public authority and are assumed exogenous and uncorrelated with the current period random error e_{1t}. What about profits in the previous year, P_{t-1}? They are **not** correlated with the random error occurring one year later. Lagged endogenous variables are called **predetermined variables** and are treated just like exogenous variables.

The second equation in the model is the investment equation. Net investment, I_t, is specified to be a function of current and lagged profits, P_t and P_{t-1}, as well as the capital stock at the end of the previous year, K_{t-1}. This lagged variable is predetermined and treated as exogenous. The investment equation is

$$I_t = \beta_1 + \beta_2 P_t + \beta_3 P_{t-1} + \beta_4 K_{t-1} + e_{2t} \quad (11.18)$$

Finally, there is an equation for wages in the private sector, W_{1t}. Let $E_t = CN_t + I_t + (G_t - W_{2t})$, where G_t is government spending. Consumption and investment are endogenous and government spending and public sector wages are exogenous. The sum, E_t, total national product minus public sector wages, is endogenous. Wages are taken to be related to E_t and the predetermined variable E_{t-1}, plus a time trend variable, $TIME_t = YEAR_t - 1931$, which is exogenous. The wage equation is

$$W_{1t} = \gamma_1 + \gamma_2 E_t + \gamma_3 E_{t-1} + \gamma_4 TIME_t + e_{3t} \quad (11.19)$$

Because there are eight endogenous variables in the entire system there must also be eight equations. Any system of M endogenous variables must have M equations to be complete. In addition to the three equations (11.17)–(11.19), which contain five endogenous variables, there are five other definitional equations to complete the system that introduce three further endogenous variables. In total, there are eight exogenous and predetermined variables, which can be used as IVs. The exogenous variables are government spending, G_t, public sector wages, W_{2t}, taxes, TX_t, and the time trend variable, $TIME_t$. Another exogenous variable is the constant term, the "intercept" variable in each equation, $X_{1t} \equiv 1$. The predetermined variables are lagged profits, P_{t-1}, the lagged capital stock, K_{t-1}, and the lagged total national product minus public sector wages, E_{t-1}.

(a)

内生: $CN_t, I_t, W_t, E_t, P_t, G_t, W_{2t}, TX_t$

外生: W_{2t} $P_{t-1}, E_{t-1}, K_{t-1}, TIME_t, G_t, TX_t, \alpha\beta\gamma$

consumption: V: 6 omit: 10 → ok

investment: V: 5 omit: 11 → ok

wage: V: 5 omit: 11 → ok.

b.

consumption: 2 en. 5 excluded ex

investment: 1 en. 5 excluded ex

wage: 1 en, 5 excluded ex

c. $W_{1t} = \pi_1 + \pi_2 G_t + \pi_3 W_{2t} + \pi_4 TX_t + \pi_5 TIME_t + \pi_6 P_{t-1} + \pi_7 K_{t-1} + \pi_8 E_{t-1} + v$

d. fitted value $\hat{W}_{1t}$ from (c), obtain $\hat{P}_t$ create $W_t^* = \hat{W}_{1t} + W_{2t}$ regress $CN_t, W_t^*, \hat{P}_t, P_{t-1}$ + α, OLS

e. coefficient yes, t-value no