5.6 Suppose that, from a sample of 63 observations, the least squares estimates and the corresponding estimated covariance matrix are given by

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix} \qquad \widehat{\text{cov}}(b_1, b_2, b_3) = \begin{bmatrix} 3 & -2 & 1 \\ -2 & 4 & 0 \\ 1 & 0 & 3 \end{bmatrix}$$

Using a 5% significance level, and an alternative hypothesis that the equality does not hold, test each of the following null hypotheses:

a. $\beta_2 = 0$
b. $\beta_1 + 2\beta_2 = 5$
c. $\beta_1 - \beta_2 + \beta_3 = 4$

(a) $H_0: \beta_2 = 0 \quad H_1: \beta_2 \neq 0 \quad \alpha = 0.05$

Test statistic: $T = \dfrac{b_2}{se(b_2)} \sim t_{(0.975, 60)}$

$T_0 = \dfrac{3}{\sqrt{4}} = 1.5$

$RR = \{|T| \geq t_{(0.975, 60)} = 2\}$

$\because T_0 \notin RR \therefore$ don't reject H_0, there is no evidence to say $\beta_2 \neq 0$

(b) $H_0: \beta_1 + 2\beta_2 = 5 \quad H_1: \beta_1 + 2\beta_2 \neq 5$

Test statistic: $T = \dfrac{b_1 + 2b_2 - 5}{se(b_1 + 2b_2)} \sim t_{(0.975, 60)}$

$T_0 = \dfrac{2 + 2\times 3 - 5}{\sqrt{Var(b_1) + 4Var(b_2) + 4cov(b_1, b_2)}} = \dfrac{3}{\sqrt{11}} \approx 0.9045$

$RR = \{|T| \geq 2\}$

$\because T_0 \notin RR$ $\therefore$ don't reject H_0, there is no evidence to say $\beta_1 + 2\beta_2 \neq 5$

(c) $H_0: \beta_1 - \beta_2 + \beta_3 = 4$ $H_1: \beta_1 - \beta_2 + \beta_3 \neq 4$

Test statistic: $T = \dfrac{b_1 - b_2 + b_3 - 4}{se(b_1 - b_2 + b_3)} \sim t_{(0.975, 60)}$

$$T_0 = \frac{2-3-1-4}{\sqrt{Var(b_1) + Var(b_2) + Var(b_3) - 2cov(b_1,b_2) + 2cov(b_1,b_3) - 2cov(b_2,b_3)}}$$

$$= \frac{-6}{\sqrt{16}} = -1.5$$

$RR = \{|T| \geq 2\}$

$\because T_0 \notin RR$ $\therefore$ don't reject H_0, there is no evidence to say $\beta_1 - \beta_2 + \beta_3 \neq 4$

5.31 Each morning between 6:30 AM and 8:00 AM Bill leaves the Melbourne suburb of Carnegie to drive to work at the University of Melbourne. The time it takes Bill to drive to work (*TIME*), depends on the departure time (*DEPART*), the number of red lights that he encounters (*REDS*), and the number of trains that he has to wait for at the Murrumbeena level crossing (*TRAINS*). Observations on these variables for the 249 working days in 2015 appear in the file *commute5*. *TIME* is measured in minutes. *DEPART* is the number of minutes after 6:30 AM that Bill departs.

a. Estimate the equation

$$TIME = \beta_1 + \beta_2 DEPART + \beta_3 REDS + \beta_4 TRAINS + e$$

Report the results and interpret each of the coefficient estimates, including the intercept β_1.

(a)

```
Call:
lm(formula = time ~ depart + reds + trains, data = commute5)

Residuals:
     Min       1Q   Median       3Q      Max
-18.4389  -3.6774  -0.1188   4.5863  16.4986

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  20.8701     1.6758  12.454  < 2e-16 ***
depart        0.3681     0.0351  10.487  < 2e-16 ***
reds          1.5219     0.1850   8.225 1.15e-14 ***
trains        3.0237     0.6340   4.769 3.18e-06 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' '

Residual standard error: 6.299 on 245 degrees of freedom
Multiple R-squared:  0.5346,	Adjusted R-squared:  0.5289
F-statistic: 93.79 on 3 and 245 DF,  p-value: < 2.2e-16
```

$\beta_1 = 20.8701$, means Bill's expected commute time when leave at 6:30 AM and encounters no red lights and no trains will be 20.8701 min

$\beta_2 = 0.3681$, means the other conditions hold, if Bill leaves later one minute after 6:30 AM, the expected commute time will increase 0.3681 min.

$\beta_3 = 1.5219$, means the other conditions hold, Bill encounters one more red light will increase the expected commute time 1.5219 min.

$\beta_4 = 3.0237$, means the other conditions hold, Bill waits for one more train will increase the expected commute time 3.0237 min.

b. Find 95% interval estimates for each of the coefficients. Have you obtained precise estimates of each of the coefficients?

```
> confint(model, level = 0.95)
                 2.5 %     97.5 %
(Intercept) 17.5694018 24.170871
depart       0.2989851  0.437265
reds         1.1574748  1.886411
trains       1.7748867  4.272505
```

se = 1.6158
se = 0.0351
se = 0.1850
se = 0.6340

$\because$ se is small $\therefore$ the interval is narrow and p-value of each coefficients < 0.05, so we obtain precise estimates

c. Using a 5% significance level, test the null hypothesis that Bill's expected delay from each red light is 2 minutes or more against the alternative that it is less than 2 minutes.

$H_0: \beta_3 \geq 2 \quad H_1: \beta_3 < 2 \quad \alpha = 0.05$

test statistic: $T = \dfrac{b_3 - 2}{se(b_3)} \sim t_{(0.05, 249-4)}$

$T_0 = \dfrac{1.52 - 2}{0.185} = -2.59$

$RR = \{ T < t_{(0.05, 245)} = -1.65 \}$

$\because T_0 \in RR \therefore$ reject H_0, we conclude that expected delay from each red light is less than 2 mins

d. Using a 10% significance level, test the null hypothesis that the expected delay from each train is 3 minutes against the alternative that it is not 3 minutes.

$H_0: \beta_4 = 3 \quad H_1: \beta_4 \neq 3 \quad \alpha = 0.10$

test statistic: $T = \frac{b_4 - 3}{se(b_4)} \sim t_{(0.95, 249-4)}$

$T_0 = \frac{3.0237 - 3}{0.634} = 0.037$

$RR = \{ |T| \geq t_{(0.95, 245)} = 1.645 \}$

$\because T_0 \notin RR \therefore$ don't reject H_0. there is no evidence to say the expected delay from each train is not 3 mins.

e. Using a 5% significance level, test the null hypothesis that Bill can expect a trip to be at least 10 minutes longer if he leaves at 7:30 AM instead of 7:00 AM, against the alternative that it will not be 10 minutes longer. (Assume other things are equal.)

$H_0: 30\beta_2 \geq 10 \quad H_1: 30\beta_2 < 10 \quad \alpha = 0.05$

test statistic: $T = \frac{b_2 - \frac{1}{3}}{se(b_2)} \sim t_{(0.05, 249-4)}$

$T_0 = \frac{0.3681 - \frac{1}{3}}{0.0351} = 0.991$

$RR = \{ T < t_{(0.05, 245)} = -1.645 \}$

$\because T_0 \notin RR \therefore$ don't reject H_0. there is no evidence to say that if he leaves at 7:30 AM, trip will not be 10 minutes longer

f. Using a 5% significance level, test the null hypothesis that the expected delay from a train is at least three times greater than the expected delay from a red light against the alternative that it is less than three times greater.

$H_0: \beta_4 \geq 3\beta_3 \quad H_1: \beta_4 < 3\beta_3 \quad \alpha = 0.05$

test statistic: $T = \dfrac{b_4 - 3b_3}{se(b_4 - 3b_3)} \sim t_{(0.05, 245)}$

$T_0 = \dfrac{3.02337 - 3(1.52191)}{\sqrt{Var(b_4) + 9Var(b_3) - 6cov(b_3, b_4)}} = -1.831$

$RR = \{T < -1.645\}$

$\because T_0 \in RR$, $\therefore$ reject H_0, we conclude that the expected delay from a train is less than three times the expected delay from red light

g. Suppose that Bill encounters six red lights and one train. Using a 5% significance level, test the null hypothesis that leaving Carnegie at 7:00 AM is early enough to get him to the university on or before 7:45 AM against the alternative that it is not. [Carry out the test in terms of the expected time $E(TIME|\mathbf{X})$ where $\mathbf{X}$ represents the observations on all explanatory variables.]

$H_0: \beta_1 + 30\beta_2 + 6\beta_3 + \beta_4 \leq 45 \qquad H_0: E(TIME|X) \leq 45$

$H_1: \beta_1 + 30\beta_2 + 6\beta_3 + \beta_4 > 45 \Rightarrow H_1: E(TIME|X) > 45$

test statistic: $T = \dfrac{b_1 + 30b_2 + 6b_3 + b_4 - 45}{se(b_1 + 30b_2 + 6b_3 + b_4)} \sim t_{(0.05, 245)}$

$$Time = 20.8701 + 30\cdot(0.3681) + 6(1.5219) + 3.0237 = 44.0682$$

```
> cat("標準誤 (SE) =", SE_hat_TIME, "\n")
標準誤 (SE) = 0.5392687
```

$$T_0 = \frac{44.0682 - 45}{0.54} = -1.726$$

$$RR = \{T > 1.645\}$$

$T_0 \notin RR$ $\therefore$ don't reject H_0, there is no evidence to say it is early enough

h. Suppose that, in part (g), it is imperative that Bill is not late for his 7:45 AM meeting. Have the null and alternative hypotheses been set up correctly? What happens if these hypotheses are reversed?

reversed hypothesis

$H_0: E(TIME|X) > 45$

$H_1: E(TIME|X) \le 45$

$RR = \{T < -1.645\}$

$\therefore T_0 = -1.726 \in RR$ $\therefore$ reject H_0

$\therefore$ the answer is same as in part (g)

5.31R

```
model <- lm(time ~ depart + reds + trains, data = commute5)
summary(model)
confint(model, level = 0.95)
qt(0.95, df = 245)
(g)
beta_hat <- coef(model)  # 取得回歸係數
vcov_matrix <- vcov(model)  # 取得變異數-共變數矩陣

X_new <- c(1, 30, 6, 1)  # 包含截距項 (1)

# 6. 計算預測值的標準誤
SE_hat_TIME <- sqrt(t(X_new) %*% vcov_matrix %*% X_new)

# 7. 顯示結果
cat("標準誤 (SE) =", SE_hat_TIME, "\n")
```

5.33 Use the observations in the data file *cps5_small* to estimate the following model:

$$\ln(WAGE) = \beta_1 + \beta_2 EDUC + \beta_3 EDUC^2 + \beta_4 EXPER + \beta_5 EXPER^2 + \beta_6 (EDUC \times EXPER) + e$$

a. At what levels of significance are each of the coefficient estimates "significantly different from zero"?

```
Call:
lm(formula = log(wage) ~ educ + I(educ^2) + exper + I(exper^2) +
    I(educ * exper), data = cps5_small)

Residuals:
    Min      1Q  Median      3Q     Max
-1.6628 -0.3138 -0.0276  0.3140  2.1394

Coefficients:
                  Estimate Std. Error t value Pr(>|t|)
(Intercept)      1.038e+00  2.757e-01   3.764 0.000175 ***
educ             8.954e-02  3.108e-02   2.881 0.004038 **
I(educ^2)        1.458e-03  9.242e-04   1.578 0.114855
exper            4.488e-02  7.297e-03   6.150 1.06e-09 ***
I(exper^2)      -4.680e-04  7.601e-05  -6.157 1.01e-09 ***
I(educ * exper) -1.010e-03  3.791e-04  -2.665 0.007803 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.4638 on 1194 degrees of freedom
Multiple R-squared:  0.3227,	Adjusted R-squared:  0.3198
F-statistic: 113.8 on 5 and 1194 DF,  p-value: < 2.2e-16
```

from p-value, except for $EDUC^2$. other coefficient estimates are significantly different from zeros at 1% level, $EDUC^2$ are different from zeros at 12% significant level

b. Obtain an expression for the marginal effect $\partial E[\ln(WAGE)|EDUC, EXPER]/\partial EDUC$. Comment on how the estimate of this marginal effect changes as *EDUC* and *EXPER* increase.

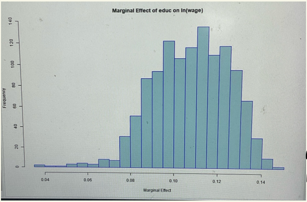
$$\text{Marginal effect} = \frac{\partial \ln(WAGE)}{\partial EDUC}$$

$$= \beta_2 + 2\beta_3 EDUC + \beta_6 EXPER$$

$$\text{estimate } \widehat{ME} = 0.08954 + 0.002916\, EDUC - 0.00101\, EXPER$$

the EDUC ↑ , ME ↑
EXPER ↑ , ME ↓

c. Evaluate the marginal effect in part (b) for all observations in the sample and construct a histogram of these effects. What have you discovered? Find the median, 5th percentile, and 95th percentile of the marginal effects.



Min.	1st Qu.	Median	Mean	3rd Qu.	Max.
0.03565	0.09513	0.10843	0.10735	0.12050	0.14787

5%	95%
0.08008187	0.13361880

the histogram is negative skewness, and most of observations are concentrated between 0.08 and 0.13

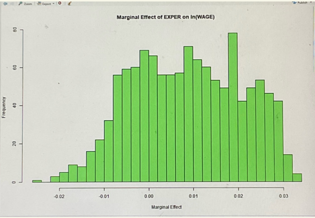
d. Obtain an expression for the marginal effect $\partial E\left[\ln(WAGE)|EDUC, EXPER\right]/\partial EXPER$. Comment on how the estimate of this marginal effect changes as $EDUC$ and $EXPER$ increase.

$$\text{Marginal effect} = \frac{\partial \ln(WAGE)}{\partial EXPER}$$
$$= \beta_4 + 2\beta_5 EXPER + \beta_6 EDUC$$

estimate $\widehat{ME} = 0.04488 - 0.000936\, EXPER - 0.00101\, EDUC$

the EXPER ↑, ME ↓ , EDUC ↑ , ME ↓

e. Evaluate the marginal effect in part (d) for all observations in the sample and construct a histogram of these effects. What have you discovered? Find the median, 5th percentile, and 95th percentile of the marginal effects.




```
        5%        95%
-0.01037621  0.02793115
> summary(marginal_effect)
     Min.    1st Qu.    Median     Mean   3rd Qu.     Max.
-0.025279 -0.001034  0.008419  0.008652  0.018586  0.033989
>
```

there is a large proportion that are negative

f. David has 17 years of education and 8 years of experience, while Svetlana has 16 years of education and 18 years of experience. Using a 5% significance level, test the null hypothesis that Svetlana's expected log-wage is equal to or greater than David's expected log-wage, against the alternative that David's expected log-wage is greater. State the null and alternative hypotheses in terms of the model parameters.

$$\beta_1 + 17\beta_2 + 17^2\beta_3 + 8\beta_4 + 64\beta_5 + (17\times 8)\beta_6$$

$$-)\ \beta_1 + 16\beta_2 + 16^2\beta_3 + 18\beta_4 + 18^2\beta_5 + (16\times 18)\beta_6$$

$$\beta_2 + 33\beta_3 - 10\beta_4 - 260\beta_5 - 152\beta_6$$

$$H_0: \beta_2 + 33\beta_3 - 10\beta_4 - 260\beta_5 - 152\beta_6 \leq 0$$

$$H_1: \beta_2 + 33\beta_3 - 10\beta_4 - 260\beta_5 - 152\beta_6 > 0$$

```
            [,1]
[1,] -0.03588456
> (sef2 <- sqrt(t(c_vec2) %*% vcov_matrix2 %*% c_vec2))
            [,1]
[1,] 0.02148902
```

```
> t<-betaf/sef2
> print(t)
          [,1]
[1,] -1.669902
> qt(0.95, 1194)
[1] 1.646131
```

$$T_0 = \frac{-0.03588}{0.021489} = -1.6699 < t_{(0.95, 1194)} = 1.646$$

$\therefore$ don't reject H_0, there is no evidence to say David's expected log-wage is greater

g. After eight years have passed, when David and Svetlana have had eight more years of experience, but no more education, will the test result in (f) be the same? Explain this outcome?

$$\beta_1 + 17\beta_2 + 17^2\beta_3 + 16\beta_4 + 256\beta_5 + (17\times16)\beta_6$$
$$-)\ \beta_1 + 16\beta_2 + 16^2\beta_3 + 26\beta_4 + 26^2\beta_5 + (16\times26)\beta_6$$
$$\beta_2 + 33\beta_3 - 10\beta_4 - 420\beta_5 - 144\beta_6$$

$H_0: \beta_2 + 33\beta_3 - 10\beta_4 - 420\beta_5 - 144\beta_6 \le 0$

$H_1: \beta_2 + 33\beta_3 - 10\beta_4 - 420\beta_5 - 144\beta_6 > 0$

```
> (betag <- c_vec3 %*% coef_column_vector)
          [,1]
[1,] 0.03091716
> (seg <- sqrt(t(c_vec3) %*% vcov_matrix2 %*% c_vec3))
          [,1]
[1,] 0.01499112
```

```
> t<-betag/seg
> print(t)
         [,1]
[1,] 2.062365
```

$T_0 = 2.062 > 1.646$

$\therefore$ reject H_0, David's expected log-wage is greater after 8 years

h. Wendy has 12 years of education and 17 years of experience, while Jill has 16 years of education and 11 years of experience. Using a 5% significance level, test the null hypothesis that their marginal effects of extra experience are equal against the alternative that they are not. State the null and alternative hypotheses in terms of the model parameters.

from (d), marginal effect $= \beta_4 + 2\beta_5 EXPER + \beta_6 EDUC$

$H_0: \beta_4 + 2\times17\beta_5 + 12\beta_6 = \beta_4 + 2\times11\beta_5 + 16\beta_6$

$H_1: \beta_4 + 2\times17\beta_5 + 12\beta_6 \ne \beta_4 + 2\times11\beta_5 + 16\beta_6$

$$\Rightarrow \begin{cases} H_0: 12\beta_5 - 4\beta_6 = 0 \\ H_1: 12\beta_5 - 4\beta_6 \neq 0 \end{cases}$$

```
> vcov_matrix2 <- vcov(model)
> c_vec4 <- c(0, 0, 0, 0, 12, -4)
> (betah <- c_vec4 %*% coef_column_vector)
          [,1]
[1,] -0.001575327
> (seh <- sqrt(t(c_vec4) %*% vcov_matrix2 %*% c_vec4))
          [,1]
[1,] 0.001533457
> t<-betah/seh
> print(t)
          [,1]
[1,] -1.027304
```

$T_0 = -1.027$, $RR = \{|T| \geq t_{(0.995, 1194)} = 1.97\}$

$\because T_0 \notin RR$. $\therefore$ don't reject H_0, there is no evidence to say the marginal effects from extra experience are different.

i. How much longer will it be before the marginal effect of experience for Jill becomes negative? Find a 95% interval estimate for this quantity.

$$\beta_4 + 2\beta_5 EXPER + \beta_6 EDUC = 0$$

$$EXPER = \frac{-\beta_4 - \beta_6 EDUC}{2\beta_5}$$

from (h). $EDUC = 16$, $EXPER = 11 + x$

$$11 + x = \frac{-\beta_4 - \beta_6 \cdot (16)}{2\beta_5}$$

$$x = \frac{-\beta_4 - \beta_6 \cdot (16)}{2\beta_5} - 11 = 19.667 \text{ (臨界值)}$$

By delta method: $SE(EXPER) = \sqrt{\nabla' V \nabla}$

$$\nabla' = \left[\frac{\partial EXPER}{\partial \beta_4}, \frac{\partial EXPER}{\partial \beta_5}, \frac{\partial EXPER}{\partial \beta_6}\right]$$

$$\frac{\partial EXPER}{\partial \beta_4} = \frac{-1}{2\beta_5}, \quad \frac{\partial EXPER}{\partial \beta_5} = \frac{\beta_4 + \beta_6 \cdot 16}{2\beta_5^2}$$

$$\frac{\partial EXPER}{\partial \beta_6} = \frac{-16}{2\beta_5}$$

```
> print(cov_matrix_subset)  # 變異數-共變數矩陣
                         exper   I(exper^2) I(educ * exper)
exper             5.325294e-05 -3.903252e-07   -2.396544e-06
I(exper^2)       -3.903252e-07  5.777667e-09    8.121608e-09
I(educ * exper)  -2.396544e-06  8.121608e-09    1.436988e-07
> |
```

$$se^2 = \underbrace{\left[\frac{-1}{2\beta_5}, \frac{\beta_4+\beta_6 \cdot 16}{2\beta_5^2}, \frac{-16}{2\beta_5}\right]}_{1\times 3} \underbrace{V}_{3\times 3} \underbrace{\begin{bmatrix} \frac{-1}{2\beta_5} \\ \frac{\beta_4+\beta_6 \cdot 16}{2\beta_5^2} \\ \frac{-16}{2\beta_5} \end{bmatrix}}_{3\times 1}$$

$$= \left(\frac{-1}{2\beta_5}\right)^2 Var(\beta_4) + \left(\frac{\beta_4+\beta_6 \cdot 16}{2\beta_5^2}\right)^2 Var(\beta_5) + \left(\frac{-16}{2\beta_5}\right)^2 Var(\beta_6)$$

$$+ 2\left(\frac{-1}{2\beta_5}\right)\left(\frac{\beta_4+\beta_6 \cdot 16}{2\beta_5^2}\right) cov(\beta_4, \beta_5) + 2 \cdot \left(\frac{-1}{2\beta_5}\right)\left(\frac{-16}{2\beta_5}\right) cov(b_4, b_6)$$

$$+ 2\left(\frac{\beta_4+\beta_6 \cdot 16}{2\beta_5^2}\right)\left(\frac{-16}{2\beta_5}\right) cov(\beta_5, \beta_6) = 3.5937$$

$$se = 1.8957$$

$$\swarrow t_{(0.975, 1194)}$$

$$95\% \; CI = [19.667 \pm 1.97 \cdot (1.8957)]$$

$$= [15.93, 23.40]$$

5.33R

```r
model <- lm(log(wage) ~ educ + I(educ^2) + exper + I(exper^2) + I(educ * exper), data = cps5_small)
summary(model)
#(C)
b <- coef(model)
b2 <- b["educ"]
b3 <- b["I(educ^2)"]
b6 <- b["I(educ * exper)"]
marginal_effect <- b2 + 2 * b3 * cps5_small$educ + b6 * cps5_small$exper
summary(marginal_effect)
hist(marginal_effect, breaks = 30, main = "Marginal Effect of educ on ln(wage)", xlab = "Marginal Effect", col = "lightblue")
quantile(marginal_effect, c(0.05, 0.95))

#(e)
b <- coef(model)
b4 <- b["exper"]
b5 <- b["I(exper^2)"]
b6 <- b["I(educ * exper)"]
marginal_effect <- b4 + 2 * b5 * cps5_small$exper + b6 * cps5_small$educ
hist(marginal_effect, breaks = 30, main = "Marginal Effect of EXPER on ln(WAGE)", xlab = "Marginal Effect", col = "lightgreen")
quantile(marginal_effect, c(0.05, 0.95))
summary(marginal_effect)

#(f)
vcov_matrix2 <- vcov(model)
c_vec2 <- c(0, 1, 33, -10, -260, -152)
coef_column_vector <- matrix(coef(model), ncol = 1)
(betaf <- c_vec2 %*% coef_column_vector)
(sef2 <- sqrt(t(c_vec2) %*% vcov_matrix2 %*% c_vec2))
t<-betaf/sef2
print(t)
qt(0.95, 1194)

#(g)
vcov_matrix2 <- vcov(model)
c_vec3 <- c(0, 1, 33, -10, -420, -144)
(betag <- c_vec3 %*% coef_column_vector)
(seg <- sqrt(t(c_vec3) %*% vcov_matrix2 %*% c_vec3))
t<-betag/seg
print(t)
```

```r
#(h)
vcov_matrix2 <- vcov(model)
c_vec4 <- c(0, 0, 0, 0, 12, -4)
(betah <- c_vec4 %*% coef_column_vector)
(seh <- sqrt(t(c_vec4) %*% vcov_matrix2 %*% c_vec4))
t<-betah/seh
print(t)
qt(0.975, 1194)

(i)
# 取得 β4, β5, β6 的變異數-共變數矩陣
cov_matrix_subset<- vcov(model)[c("exper", "I(exper^2)", "I(educ * exper)"),
                               c("exper", "I(exper^2)", "I(educ * exper)")]

print(cov_matrix_subset)  # 變異數-共變數矩陣
```

```r
b6 = coef(model)[6]
b5 = coef(model)[5]
b4 = coef(model)[4]
g4 = -1/(2*b5)
g5 = (b4+16*b6)/(2*b5^2)
g6 = -8/b5
g = c(g4, g5, g6)
varr = t(g)%*%cov_matrix_subset%*%g
sd = varr^0.5
```