

4.4 The general manager of a large engineering firm wants to know whether the experience of technical artists influences their work quality. A random sample of 50 artists is selected. Using years of work experience (*EXPER*) and a performance rating (*RATING*, on a 100-point scale), two models are estimated by least squares. The estimates and standard errors are as follows:

Model 1:

$$\widehat{RATING} = 64.289 + 0.990EXPER \quad N = 50 \quad R^2 = 0.3793$$

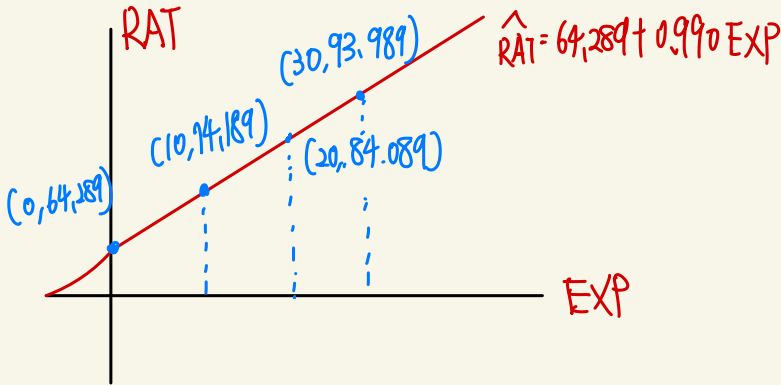
(se) (2.422) (0.183)

Model 2:

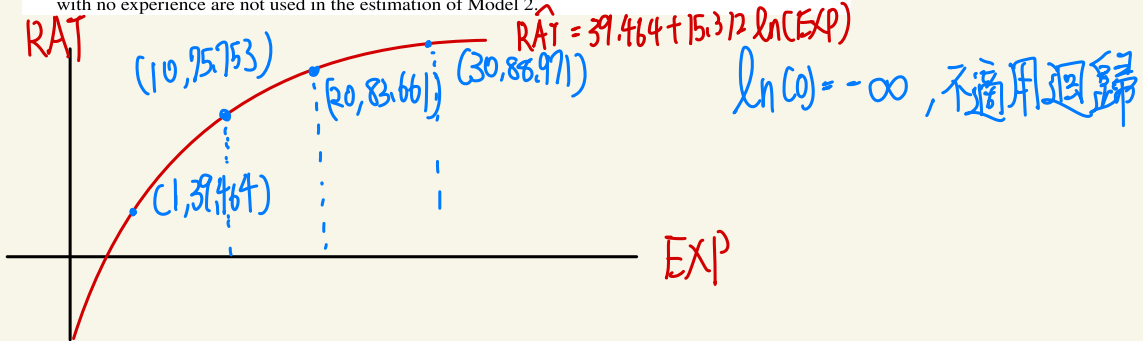
$$\widehat{RATING} = 39.464 + 15.312 \ln(EXPER) \quad N = 46 \quad R^2 = 0.6414$$

(se) (4.198) (1.727)

a. Sketch the fitted values from Model 1 for *EXPER* = 0 to 30 years.



b. Sketch the fitted values from Model 2 against *EXPER* = 1 to 30 years. Explain why the four artists with no experience are not used in the estimation of Model 2.



- c. Using Model 1, compute the marginal effect on *RATING* of another year of experience for (i) an artist with 10 years of experience and (ii) an artist with 20 years of experience.
- d. Using Model 2, compute the marginal effect on *RATING* of another year of experience for (i) an artist with 10 years of experience and (ii) an artist with 20 years of experience.

c. 0.990, 邊際效果固定

d. (i) $15.312/10 = 1.5312$ (ii) $15.312/20 = 0.7656$

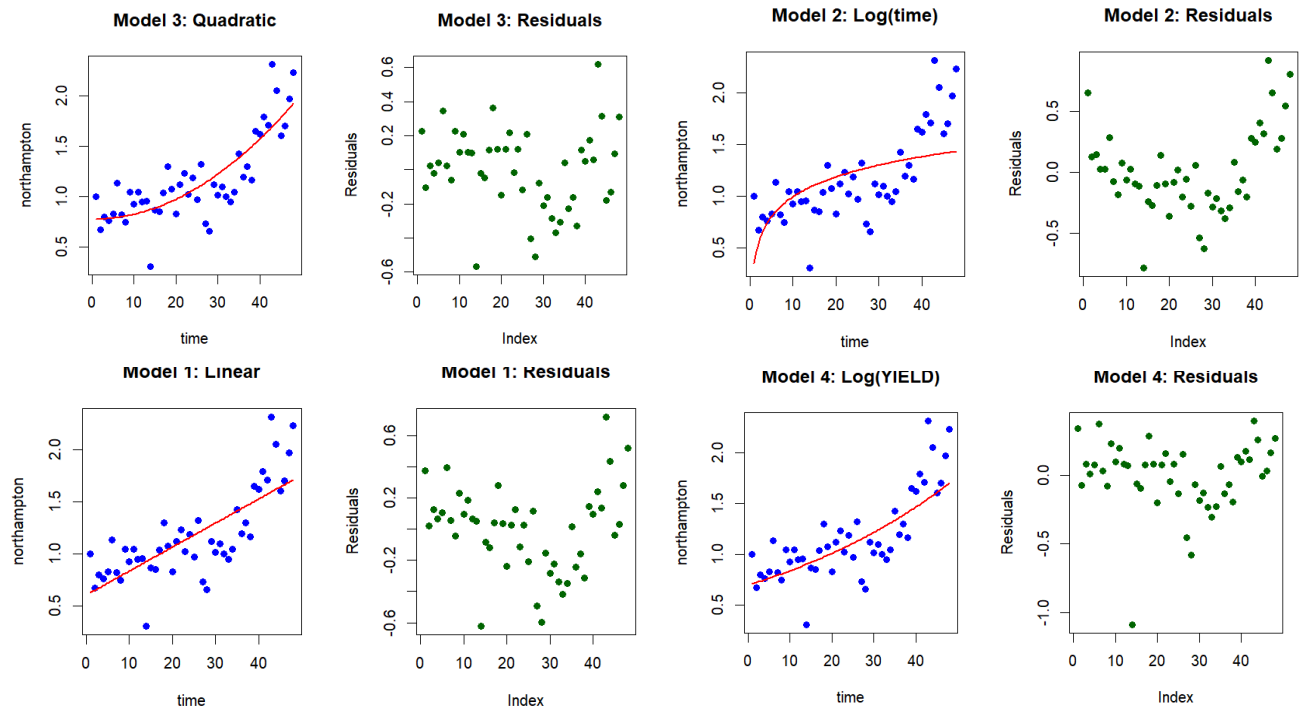
- e. Which of the two models fits the data better? Estimation of Model 1 using just the technical artists with some experience yields $R^2 = 0.4858$.
- f. Do you find Model 1 or Model 2 more reasonable, or plausible, based on economic reasoning? Explain.

e. Model 2 之 R^2 較高, Model 2 效果較好

f. Model 2 較合理, 在吸取一定經驗, 對 rating 的增量效果應逐漸降低

Q28

(a)應選擇model3(Quadratic Model), 因其殘差最近常態分配, 且擁有最高的R平方(解釋力強)



```
> shapiro.test(model3$residuals)
```

Shapiro-wilk normality test

data: model3\$residuals

W = 0.98589, p-value = 0.8266

```
> # R平方比較
```

```
> summary(model1)$r.squared
```

```
[1] 0.5778369
```

```
> summary(model2)$r.squared
```

```
[1] 0.3385733
```

```
> summary(model3)$r.squared
```

```
[1] 0.6890101
```

```
> summary(model4)$r.squared
```

```
[1] 0.5073566
```

(b)

Residuals:

	Min	1Q	Median	3Q	Max
	-0.56899	-0.14970	0.03119	0.12176	0.62049

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	7.737e-01	5.222e-02	14.82	< 2e-16 ***
I(time^2)	4.986e-04	4.939e-05	10.10	3.01e-13 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.2396 on 46 degrees of freedom

Multiple R-squared: 0.689, Adjusted R-squared: 0.6822

F-statistic: 101.9 on 1 and 46 DF, p-value: 3.008e-13

(C) 打星號為可能影響迴歸線的資料

	dfb.1_	dfb.I..2	dffit	cov.r	cook.d	hat	inf							
1	0.21672	-0.16229	0.2167	1.052	2.35e-02	0.0474		25	-0.05955	0.01690	-0.0728	1.057	2.69e-03	0.0220
2	-0.09773	0.07306	-0.0977	1.087	4.86e-03	0.0472		26	0.10001	-0.02109	0.1289	1.033	8.35e-03	0.0214
3	0.02031	-0.01514	0.0203	1.096	2.11e-04	0.0469		27	-0.18618	0.02301	-0.2561	0.936	3.14e-02	0.0210
4	-0.02066	0.01534	-0.0207	1.095	2.18e-04	0.0464		28	-0.21991	0.00382	-0.3278	0.863	4.94e-02	0.0208
5	0.03759	-0.02777	0.0376	1.094	7.22e-04	0.0458		29	-0.02836	-0.00324	-0.0466	1.062	1.11e-03	0.0209
6	0.32374	-0.23761	0.3238	0.994	5.11e-02	0.0451		30	-0.06998	-0.01971	-0.1295	1.032	8.43e-03	0.0213
7	0.01981	-0.01443	0.0198	1.093	2.01e-04	0.0443		31	-0.04707	-0.02357	-0.1006	1.048	5.12e-03	0.0220
8	-0.05344	0.03856	-0.0535	1.089	1.46e-03	0.0434		32	-0.07267	-0.05810	-0.1848	1.004	1.69e-02	0.0231
9	0.20370	-0.14536	0.2040	1.047	2.08e-02	0.0423		33	-0.07996	-0.09838	-0.2520	0.961	3.07e-02	0.0246
10	0.09085	-0.06401	0.0911	1.081	4.22e-03	0.0412		34	-0.05243	-0.09984	-0.2165	0.996	2.31e-02	0.0265
11	0.17959	-0.12470	0.1802	1.052	1.63e-02	0.0400		35	0.00521	0.01617	0.0308	1.074	4.83e-04	0.0288
12	0.08610	-0.05878	0.0865	1.078	3.81e-03	0.0387		36	-0.01739	-0.10166	-0.1741	1.036	1.52e-02	0.0316
13	0.07951	-0.05324	0.0801	1.078	3.27e-03	0.0373		37	-0.00445	-0.08158	-0.1283	1.061	8.33e-03	0.0350
14	-0.48945	0.32052	-0.4944	0.826	1.09e-01	0.0359	*	38	0.00733	-0.19326	-0.2836	0.997	3.94e-02	0.0389
15	-0.01477	0.00943	-0.0150	1.082	1.15e-04	0.0345		39	-0.00851	0.07524	0.1043	1.081	5.53e-03	0.0434
16	-0.03636	0.02252	-0.0370	1.079	7.01e-04	0.0331		40	-0.00652	0.03719	0.0492	1.096	1.24e-03	0.0486
17	0.08685	-0.05198	0.0891	1.067	4.03e-03	0.0316		41	-0.03218	0.14139	0.1800	1.078	1.64e-02	0.0544
18	0.26559	-0.15266	0.2746	0.970	3.66e-02	0.0301		42	-0.01371	0.05039	0.0621	1.110	1.97e-03	0.0610
19	0.08416	-0.04612	0.0880	1.063	3.93e-03	0.0287		43	-0.20253	0.65218	0.7823	0.798	2.64e-01	0.0683
20	-0.09745	0.05045	-0.1032	1.056	5.40e-03	0.0274		44	-0.11635	0.33832	0.3967	1.042	7.72e-02	0.0764
21	0.07761	-0.03750	0.0836	1.061	3.55e-03	0.0261		45	0.07730	-0.20715	-0.2382	1.112	2.86e-02	0.0854
22	0.13414	-0.05951	0.1477	1.032	1.09e-02	0.0249		46	0.06520	-0.16340	-0.1848	1.139	1.73e-02	0.0953
23	-0.00912	0.00363	-0.0103	1.070	5.45e-05	0.0238		47	-0.05361	0.12702	0.1417	1.160	1.02e-02	0.1061
24	0.06641	-0.02295	0.0778	1.057	3.07e-03	0.0228		48	-0.20393	0.46077	0.5078	1.089	1.26e-01	0.1180

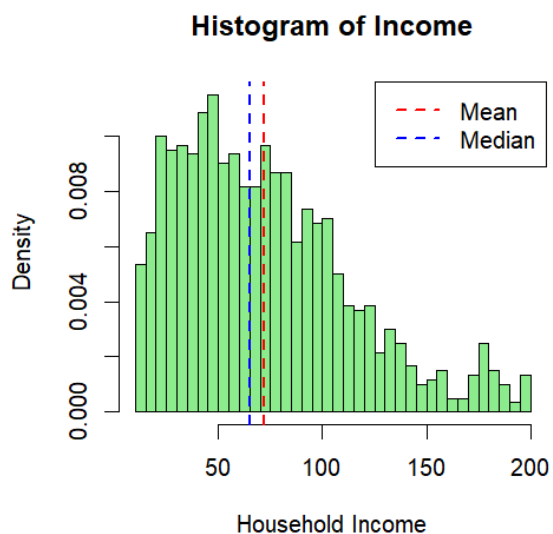
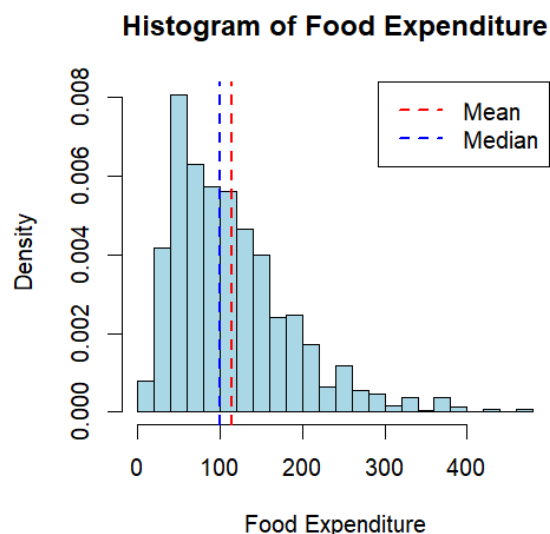
(d) 實際值為2.2318, 落在95%預測區間內

	fit	lwr	upr
1	1.922482	1.412563	2.432401

Q29.

(a) food、income之敘述統計量與直方圖，兩者都不是bell-shaped，且平均數都大於中位數，進行Jarque-bera Test後，皆無法拒絕分布非常態分配，皆可能為正偏。

	food	income
Mean	114.4431	72.14264
Median	99.8000	65.29000
Min	9.6300	10.00000
Max	476.6700	200.00000
SD	72.6575	41.65228



```
data: cex5_small$food
JB = 648.65, p-value < 2.2e-16
alternative hypothesis: greater
```

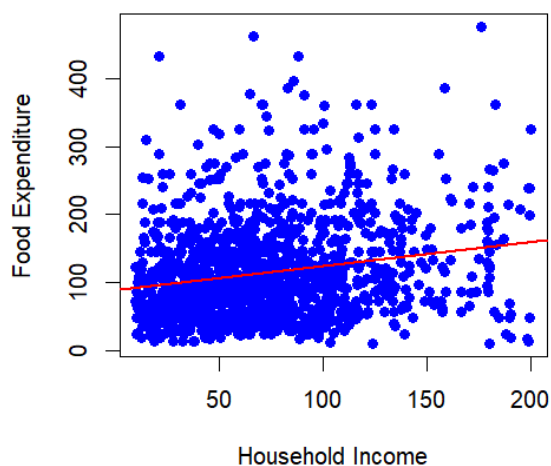
```
>
> jb_income <- jarque.test(cex5_small$income)
> print(jb_income)
```

Jarque-Bera Normality Test

```
data: cex5_small$income
JB = 148.21, p-value < 2.2e-16
alternative hypothesis: greater
```

(b)

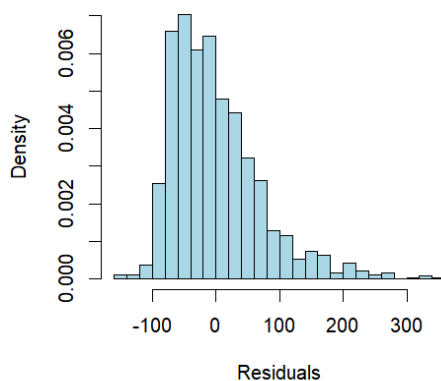
Scatterplot of Food vs. Income



```
> confint(lm_model, level = 0.95)
                2.5 %    97.5 %
(Intercept) 80.5064570 96.626543
income       0.2619215  0.455452
> |
```

(c) 無法拒絕殘差不是常態分配(可能為正偏)

Histogram of Residuals



```
> jarque.test(residuals_lm)
```

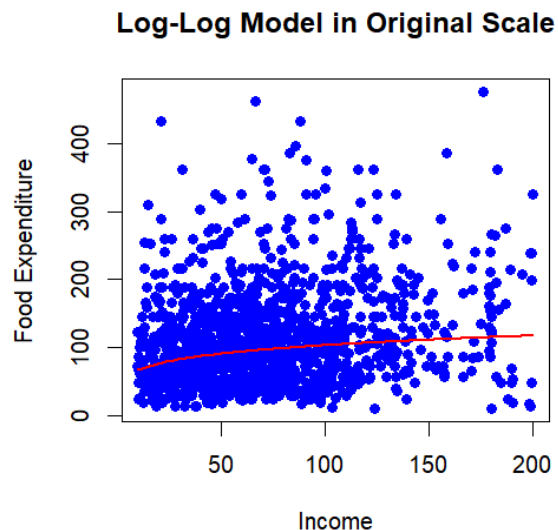
Jarque-Bera Normality Test

```
data: residuals_lm
JB = 624.19, p-value < 2.2e-16
alternative hypothesis: greater
```

(d)彈性隨收入增加而單調遞增, 且信賴區間不重疊。收入增加代表家庭可能花更多錢在食物上(更為精緻的飲食, 而非單純滿足生理需求)。

```
> print(result)
  income_values  food_hat elasticity lower_bound upper_bound
1             19   95.38155  0.07145038  0.05219387  0.09070689
2             65  111.88114  0.20838756  0.15222527  0.26454986
3            160  145.95638  0.39319883  0.28722827  0.49916940
```

(e)log-log Model的表現無較佳

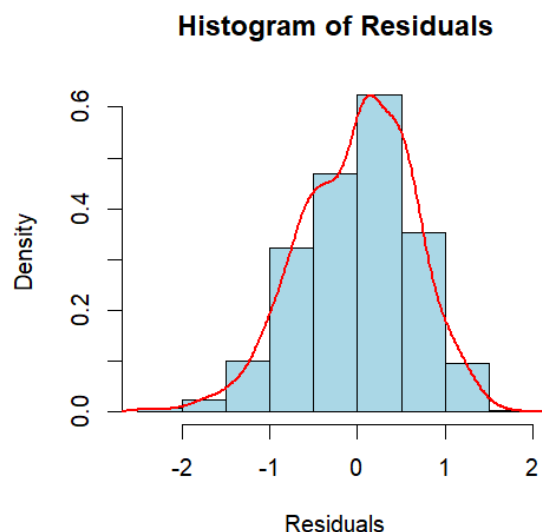


```
> cat("Linear Model R²: ", r2_linear, "\n")
Linear Model R²:  0.0422812
> cat("Log-Log Model R²: ", r2_log_log, "\n")
Log-Log Model R²:  0.03322915
> cat("Generalized R² for Log-Log Model: ", ge
Generalized R² for Log-Log Model:  0.0332564
```

(f)與linear-Model不同, 因log-log彈性為常數

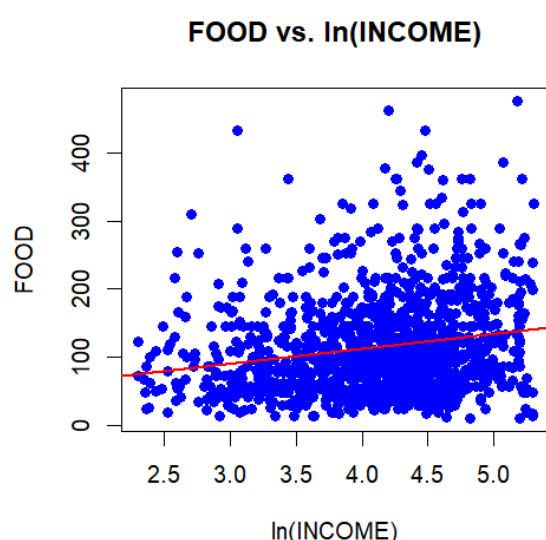
```
> cat("Point Estimate of Elasticity: ", beta2_hat, "\n")
Point Estimate of Elasticity:  0.1863054
> cat("95% Confidence Interval: (", CI_lower, ",", CI_upper, ")\n")
95% Confidence Interval: ( 0.1293997 , 0.243211 )
```

(g)log-log model之殘差並非常態分配



```
Jarque-Bera Test Statistic:  25.84998
> cat("p-value: ", jb_test$p.value, "\n")
p-value:  2.436404e-06
>
> if (jb_test$p.value < 0.05) {
+   cat("Conclusion: Residuals are NOT normally dist
+ } else {
+   cat("Conclusion: Residuals are normally distribu
+ }
Conclusion: Residuals are NOT normally distributed.
```

(h) linear-log Model的表現大同小異

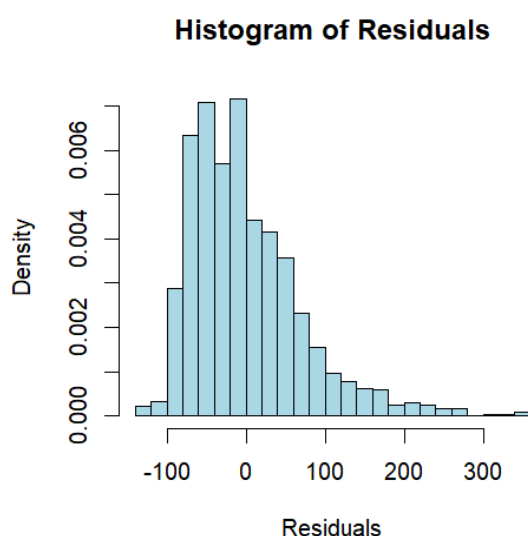
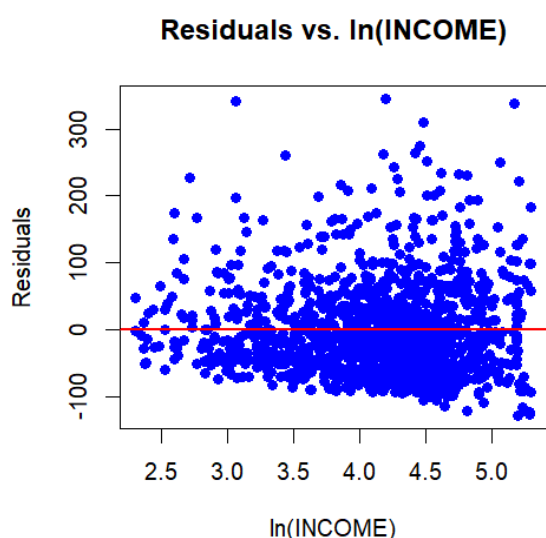


```
cat("R-squared for Linear Model: ", r2_linear)
R-squared for Linear Model: 0.0422812
cat("R-squared for Log-Log Model: ", r2_loglog)
R-squared for Log-Log Model: 0.03322915
cat("R-squared for Linear-Log Model: ", r2_linear_log)
R-squared for Linear-Log Model: 0.03799984
```

(i) linear-log之彈性與上述兩個模型完全不同, 因其主要取決於 $1/\ln(\text{Income})$ 。

Income	Elasticity_Estimate	Lower_95_CI	Upper_95_CI
19	1.1677571	0.83503867	1.5004756
65	0.3413444	0.24408823	0.4386005
160	0.1386712	0.09916084	0.1781815

(j) linear-log之殘差檢定=>可能也不是常態分配



Jarque-Bera Normality Test

```
data: residuals_linear_log
JB = 628.07, p-value < 2.2e-16
alternative hypothesis: greater
```

(h)在R平方差不多(0.03~0.04)的情況下, linear log Model的殘差更接近常態分配, 可能是較好的估計模型。