3.1

$n = 64$

a.

$H_0: \beta = 0$

$H_a: \beta_2 > 0.$

b.

$t = \frac{\hat{\beta}_2}{SE(\hat{\beta}_2)} \sim t_{(64-2)}$

$t_0 = \frac{0.01309}{0.00215} = 6.088$

c.

If alternative hypothesis is true, then the distribution shifts to the right relative to the original one.

Due to that GDPB will affect # Medals, which means $\beta_2 > 0 \Rightarrow$ shift to the right.

d.

$RR \in \{ t_0 \geq t_{0.05}(62) \doteq 1.645 \}$

$\Rightarrow$ Fail to reject H_0 when $t_0 \leq 1.645$.

e.

$t_0 = 6.088 \in RR \Rightarrow$ Reject H_0.

3.7. $N = 51$

$$\widehat{INCOME} = \hat{\beta}_0 + 1.029\ BACHELOR$$

a.

$(a) = 4.31 \times 2.672 = 11.516$

b.

It's increasing and constant.

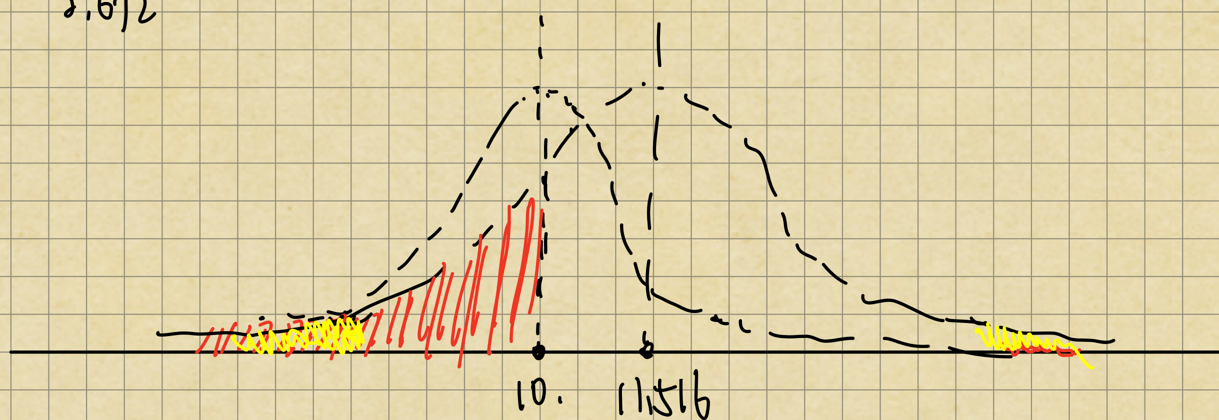
c.

$t_0 = \frac{\hat{\beta}_2}{SE(\hat{\beta}_2)} \Rightarrow 10.75 = \frac{1.029}{(c)}$, $(c) = 0.0957$

d.

$\frac{10}{2.672} = 3.7425$

e.



: p-value

: reject region

f.

slope of 99% confidence interval.

$\Rightarrow t_1 \pm t_{0.005}(51-2) \times SE(\hat{\beta}_2)$

$\Rightarrow 1.029 \pm 2.576 \times 0.0957$

$\Rightarrow [0.7824 , 1.2755]$

In repeated sampling, about $100(1-0.01)\%$ intervals constructed this way will contain the true value of the true parameter β_2

g.

$H_0: \beta_1 = 1$ don't reject H_0 according to CI in f.

$H_a: \beta_1 \neq 1$

3.17

a. ① $H_0: \beta_2^{URBAN} = 1.8$ ② $\alpha = 0.05$

$H_a: \beta_2^{URBAN} > 1.8$

③ $t_0 = \dfrac{\hat{\beta}_2^{URBAN} - 1.8}{SE(\hat{\beta}_2)} \sim t_{0.05}(986-2)$

④ $RR = \{ t_0 \geq t_{0.05}(984) = 1.645 \}$.

⑤ $t_0 = \frac{2.46}{0.16} = 4.125 > 1.645$

⑥ Reject H_0

b.

$\widehat{WAGE} \mid EDUC=16 = -4.88 + 1.8 \times 16 = 23.92$

$\widehat{WAGE} \pm t_{0.025}(214-2) \times SE(\widehat{WAGE})$

$\Rightarrow 23.92 \pm 1.96 \times 0.833$

$\Rightarrow [22.287, 25.553]$

c.

URBAN: $\widehat{WAGE} = -10.76 + 2.46\ EDUC$, $N = 986$

$EDUC = 16$. $Cov(\hat{\beta}_1, \hat{\beta}_2) = -0.345$

$\widehat{WAGE}$ 95% CI: $\widehat{WAGE} \pm t_{0.025}(984) \times SE(\widehat{WAGE})$

$(\widehat{WAGE} \mid EDUC=16) = -10.76 + 2.46 \times 16 = 28.6$

$t_{0.025}(984) = 1.96$

$Var(\widehat{WAGE}) = Var(\hat{\beta}_1) + EDUC^2\, Var(\hat{\beta}_2) + 2EDUC\, Cov(\hat{\beta}_1, \hat{\beta}_2)$

$SE(\widehat{WAGE}) = 0.8164$

$\widehat{WAGE}$ 95% CI : [27. 30.2]

The sample size of URBAN is larger the Rural which cause the SE is smaller

$\Rightarrow$ We can get a narrower CI.

d.

① $H_0: \beta_1 \geq 4$
$H_1: \beta_1 < 4$

② $\alpha = 0.01$

③ $t_0 = \dfrac{\hat{\beta}_1 - 4}{SE(\hat{\beta}_1)} \sim t_{0.01}(212)$ ④ RR $= \{ |t_0| > 2.326 \}$.

⑤ $|t_0| = \left| \dfrac{(-4.88) - 4}{3.29} \right| = 2.69 > 2.326$.

⑥ Reject H_0 : It is significant that $\beta_1 < 4$

3.19. c.

① $H_0: \beta_2 \le 0$

$H_a: \beta_2 > 0$

② $\alpha = 0.01$

③ $t_0 = \dfrac{\hat{\beta}_2 - 0}{SE(\hat{\beta}_2)} \sim t_{0.01}(25-2)$

④ $RR = \{ |t_0| \geq t_{0.01}(23) = 2.5 \}$.

⑤ $\dfrac{0.8646}{0.2027} = 4.265 \in RR$

⑥ Reject H_0

3.19 d.

① $H_0: \beta_2 = 1$

$H_a: \beta_2 \neq 1$

② $\alpha = 0.01$

③ test statistic: $t_0 = \dfrac{(\hat{\beta}_2 - 1)}{SE(\hat{\beta}_2)} \sim t_{0.01}(23)$

④ $RR = \{ |t_0| \geq t_{0.005}(23) = 2.807 \}$.

⑤ $\dfrac{0.8646 - 1}{0.2027} = -0.668 \notin RR$

⑥ don't Reject H_0.