Q1

$$Y = X\beta + e \quad , \quad \hat{\beta} = (X^T X)^{-1} X^T Y$$

$$X = \begin{bmatrix} 1 & X_1 \\ 1 & X_2 \\ \vdots & \vdots \\ 1 & X_n \end{bmatrix} , \quad X^T = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ X_1 & X_2 & \cdots & X_n \end{bmatrix} , \quad Y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} , \quad \beta = \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix}$$

$$X^T X = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ X_1 & X_2 & \cdots & X_n \end{bmatrix} \begin{bmatrix} 1 & X_1 \\ 1 & X_2 \\ \vdots & \vdots \\ 1 & X_n \end{bmatrix} = \begin{bmatrix} n & \Sigma X_i \\ \Sigma X_i & \Sigma X_i^2 \end{bmatrix}$$

$$(X^T X)^{-1} = \frac{1}{n\Sigma X_i^2 - (\Sigma X_i)^2} \begin{bmatrix} \Sigma X_i^2 & -\Sigma X_i \\ -\Sigma X_i & n \end{bmatrix}$$

$$X^T Y = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ X_1 & X_2 & \cdots & X_n \end{bmatrix} \begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_n \end{bmatrix} = \begin{bmatrix} \Sigma Y_i \\ \Sigma X_i Y_i \end{bmatrix}$$

$$\hat{\beta} = (X^T X)^{-1} X^T Y = \frac{1}{n\Sigma X_i^2 - (\Sigma X_i)^2} \begin{bmatrix} \Sigma X_i^2 & -\Sigma X_i \\ -\Sigma X_i & n \end{bmatrix} \begin{bmatrix} \Sigma Y_i \\ \Sigma X_i Y_i \end{bmatrix}$$

$$= \frac{1}{n\Sigma X_i^2 - (\Sigma X_i)^2} \begin{bmatrix} \Sigma X_i^2 \Sigma Y_i - \Sigma X_i \Sigma X_i Y_i \\ n\Sigma X_i Y_i - \Sigma X_i \Sigma Y_i \end{bmatrix}$$

$$\hat{\beta}_2 = \frac{n\Sigma X_i Y_i - \Sigma X_i \Sigma Y_i}{n\Sigma X_i^2 - (\Sigma X_i)^2} = \frac{\Sigma X_i Y_i - \frac{\Sigma X_i \Sigma Y_i}{n}}{\Sigma X_i^2 - \frac{(\Sigma X_i)^2}{n}} = \frac{\Sigma (X_i - \bar{X})(Y_i - \bar{Y})}{\Sigma (X_i - \bar{X})^2}$$

$$\hat{\beta}_1 = \frac{(\Sigma X_i^2)\Sigma Y_i - \Sigma X_i \Sigma X_i Y_i}{n\Sigma X_i^2 - (\Sigma X_i)^2} = \frac{\bar{Y}\Sigma X_i^2 - \bar{X}\Sigma XY}{\Sigma X_i^2 - \frac{(\Sigma X_i)^2}{n}}$$

$$\hat{\beta}_1 = \bar{Y} - \hat{\beta}_2 \bar{X} = \frac{\bar{Y}\Sigma X_i^2 - \cancel{\bar{Y}\frac{(\Sigma X_i)^2}{n}} - \bar{X}\Sigma XY + \cancel{n\bar{X}^2\bar{Y}}}{\Sigma X_i^2 - \frac{(\Sigma X_i)^2}{n}} = \frac{\bar{Y}\Sigma X_i^2 - \bar{X}\Sigma XY}{\Sigma X_i^2 - \frac{(\Sigma X_i)^2}{n}}$$

Q2

$$Var(\hat{\beta}) = \sigma^2 (X^T X)^{-1} = \frac{\sigma^2}{n\Sigma X_i^2 - (\Sigma X_i)^2} \begin{bmatrix} \Sigma X_i^2 & -\Sigma X_i \\ -\Sigma X_i & n \end{bmatrix}$$

$$Var(\hat{\beta}_2) = \frac{\sigma^2}{\Sigma (X_i - \bar{X})^2}$$ ，與多元迴歸 $K=2$ 相同

$$Var(\hat{\beta}_1) = Var(\bar{Y} - \hat{\beta}_2 \bar{X}) = Var(\bar{Y}) + \bar{X}^2 Var(\hat{\beta}_2) - 2\bar{X} Cov(\bar{Y}, \hat{\beta}_1)$$

$$= \frac{\sigma^2}{n} + \bar{X}^2 \frac{\sigma^2}{\Sigma(X_i - \bar{X})^2}$$

$$= \sigma^2 \left(\frac{1}{n} + \frac{\bar{X}^2}{\Sigma(X_i - \bar{X})^2} \right)$$

$$= \sigma^2 \frac{\Sigma(X_i - \bar{X})^2 + n\bar{X}^2}{n\Sigma(X_i - \bar{X})^2}$$

$$= \sigma^2 \frac{\Sigma X_i^2}{n\Sigma(X_i - \bar{X})^2}$$ ，與多元迴歸 $K=2$ 相同

CH5 Q3

(a)

i. $t_1 = \dfrac{b_1}{SE(b_1)} = \dfrac{1.4515}{2.2019} = 0.6592$

ii. $SE(b_2) = \dfrac{b_2}{t_2} = \dfrac{2.7648}{5.7103} = 0.4842$

iii. $b_3 = t_3 \times SE(b_3) = -1.4549$

iv. $\text{Variance of WALC} = (6.39547)^2 = 40.902$

$SST = (n-1) \times \text{Variance} = 1199 \times 40.902 \approx 49041.5$

$R^2 = 1 - \dfrac{SSE}{SST} = 1 - \dfrac{46221.62}{49041.5} = 0.0575$

v. $\hat{\sigma} = \sqrt{\dfrac{46221.62}{1200-4}} = 6.2167$

(b)

b_2: 總支出增加 1%，酒類支出增加 2.7648%

(因 WALC 為百分比)

b_3: 每增加1個小孩，酒類支出減少 1.4549%

b_4: 家主的年紀每增加1歲，酒類支出減少 0.1503%

(c) 95% CI of b4

$= -0.1503 \pm 1.96 \times 0.0235$

$= [-0.1964, -0.1042]$

(d)

No，因為 b_1 的 p-value $= 0.5099 > 0.05$，結果不顯著

(e)

$H_0 : \beta_3 = -2$

$H_1 : \beta_3 \neq -2$

$\alpha = 0.05$, $t\text{-stat} = \dfrac{\beta_3 + 2}{SE(\beta_3)} \sim N(0,1)$

$RR = \{ |T| \geq 1.96 \}$

$$t_3 = \frac{-1.4549 + 2}{0.3695} = 1.4751$$

$t_3 \notin RR$, do not reject H_0

CH5 Q23

(a) β_2: 應為負，因為大量批發，單位價格應下降

β_3: 應為正，品質越好，單價越高

β_4: 應為正，考慮通貨膨脹，單價上升

(b)

```
Residuals:
    Min      1Q  Median      3Q     Max
-43.479 -12.014  -3.743  13.969  43.753

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 90.84669    8.58025  10.588 1.39e-14 ***
quant       -0.05997    0.01018  -5.892 2.85e-07 ***
qual         0.11621    0.20326   0.572   0.5700
trend       -2.35458    1.38612  -1.699   0.0954 .
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 20.06 on 52 degrees of freedom
Multiple R-squared:  0.5097,	Adjusted R-squared:  0.4814
F-statistic: 18.02 on 3 and 52 DF,  p-value: 3.806e-08
```

β_2, β_3 與預期相同， β_4 與預期相反

(c)

$R^2 = 0.5097$

Adjusted $R^2 = 0.4814$

(d)

$H_0: \beta_2 = 0$

$H_1: \beta_2 < 0$ (數量越大，單價越低)

$\alpha = 0.05$

$RR = \{T \le -1.645\}$

$t_2 = -5.892$ ， $t_2 \in RR$

Reject H_0 ， 數量对價格有顯著負影響

(e)

$H_0: \beta_3 = 0$

$H_1: \beta_3 > 0$ (純度越高，單價越貴)

$\alpha = 0.05$

$RR = \{T \ge 1.645\}$

$t_3 = 0.5717$ ， $t_3 \notin RR$

Do not reject H_0 ， 純度影響不顯著

(f)

$\beta_4 = -2.3546$

單價隨年份而降低，可能是法律变嚴格，因此 cocaine 市場緊縮，導致價格降低