4.4

313707014 陳紀蓁

4.4 The general manager of a large engineering firm wants to know whether the experience of technical artists influences their work quality. A random sample of 50 artists is selected. Using years of work experience (*EXPER*) and a performance rating (*RATING*, on a 100-point scale), two models are estimated by least squares. The estimates and standard errors are as follows:

Model 1:

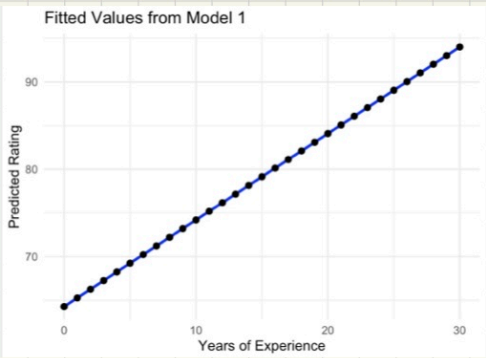
$$\widehat{RATING} = 64.289 + 0.990EXPER \quad N = 50 \quad R^2 = 0.3793$$
$$(\text{se}) \qquad (2.422) \quad (0.183)$$

Model 2:

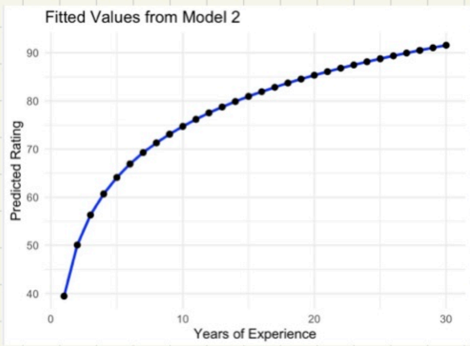
$$\widehat{RATING} = 39.464 + 15.312\ln(EXPER) \quad N = 46 \quad R^2 = 0.6414$$
$$(\text{se}) \qquad (4.198) \quad (1.727)$$

a. Sketch the fitted values from Model 1 for *EXPER* = 0 to 30 years.
b. Sketch the fitted values from Model 2 against *EXPER* = 1 to 30 years. Explain why the four artists with no experience are not used in the estimation of Model 2.
c. Using Model 1, compute the marginal effect on *RATING* of another year of experience for (i) an artist with 10 years of experience and (ii) an artist with 20 years of experience.
d. Using Model 2, compute the marginal effect on *RATING* of another year of experience for (i) an artist with 10 years of experience and (ii) an artist with 20 years of experience.
e. Which of the two models fits the data better? Estimation of Model 1 using just the technical artists with some experience yields $R^2 = 0.4858$.
f. Do you find Model 1 or Model 2 more reasonable, or plausible, based on economic reasoning? Explain.

a、



b、



c、 Model 1 : Margin effect: $\frac{d\ \widehat{RATING}}{d\ EXPER} = 0.99$

(i) 10年經驗，每增加1年，邊際效應增加 0.99
(ii) 20年經驗，每增加1年，邊際效應增加 0.99

→ 代表不管有幾年的經驗，邊際效應都是一樣的.

d. Model 2. Margin effect: $\frac{d\,\widehat{RATING}}{d\,EXPER} = 15.312 \cdot \frac{1}{EXPER}$

(i) 10年經驗 邊際效應: $15.312 \times \frac{1}{10} = 1.5312$

(ii) 20年經驗 邊際效應: $15.312 \times \frac{1}{20} = 0.7656$

⇒ 代表經驗帶來的效益是遞減的.

e. Model 1: $R^2 = 0.3793$

Model 1 (just technical artists): $R^2 = 0.4858$

Model 2: $R^2 = 0.6414$

R^2 越高代表模型擬合度好.

⇒ Model 2 的擬合度高.

f. Model 1 (線性模型): 隨著經驗上升，但邊際效應皆不變. 在現實中不合理.

Model 2 (對數模型): 隨著經驗上升、邊際效應遞減，符合現實，因為會在後期成長放緩.

4.28

A.

回歸

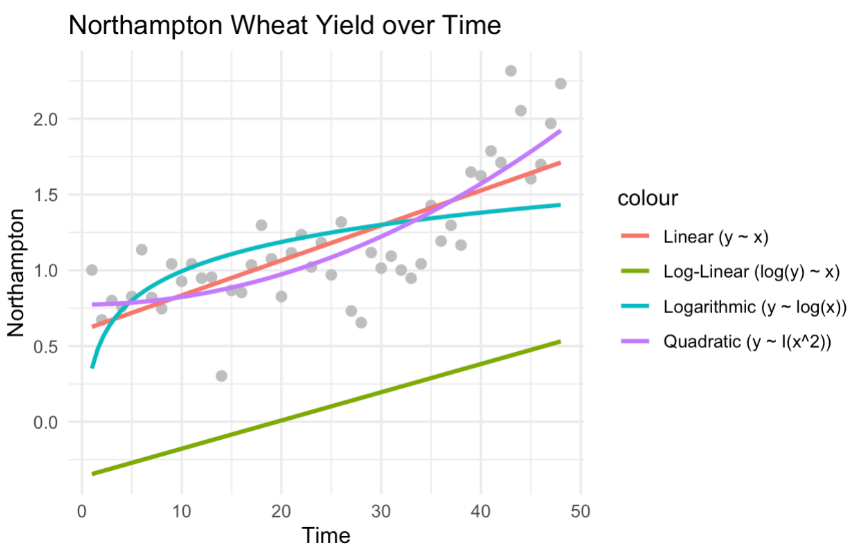
```
> summary(mod1)

Call:
lm(formula = northampton ~ time, data = wa_wheat)

Residuals:
     Min       1Q   Median       3Q      Max 
-0.62394 -0.17302  0.03342  0.12996  0.72050 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 0.603245   0.081858   7.369 2.55e-09 ***
time        0.023078   0.002908   7.935 3.69e-10 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.2791 on 46 degrees of freedom
Multiple R-squared:  0.5778,	Adjusted R-squared:  0.5687 
F-statistic: 62.96 on 1 and 46 DF,  p-value: 3.689e-10
```

```
> summary(mod2)

Call:
lm(formula = northampton ~ log(time), data = wa_wheat)

Residuals:
     Min       1Q   Median       3Q      Max 
-0.78488 -0.20711 -0.06382  0.15447  0.91573 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)   0.3510     0.1759   1.995    0.052 .  
log(time)     0.2790     0.0575   4.852 1.44e-05 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.3494 on 46 degrees of freedom
Multiple R-squared:  0.3386,	Adjusted R-squared:  0.3242 
F-statistic: 23.55 on 1 and 46 DF,  p-value: 1.44e-05
```

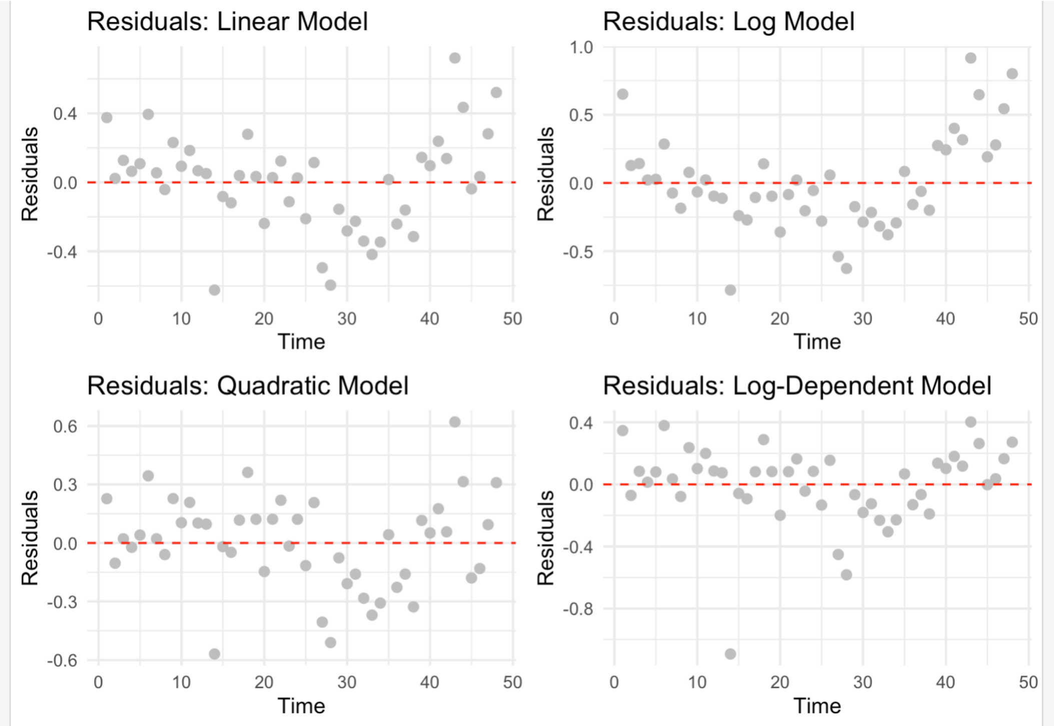
```
> summary(mod3)

Call:
lm(formula = northampton ~ I(time^2), data = wa_wheat)

Residuals:
     Min       1Q   Median       3Q      Max 
-0.56899 -0.14970  0.03119  0.12176  0.62049 

Coefficients:
             Estimate Std. Error t value Pr(>|t|)    
(Intercept) 7.737e-01  5.222e-02   14.82  < 2e-16 ***
I(time^2)   4.986e-04  4.939e-05   10.10 3.01e-13 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.2396 on 46 degrees of freedom
Multiple R-squared:  0.689,	Adjusted R-squared:  0.6822 
F-statistic: 101.9 on 1 and 46 DF,  p-value: 3.008e-13
```

```
> summary(mod4)

Call:
lm(formula = log(northampton) ~ time, data = wa_wheat)

Residuals:
     Min       1Q   Median       3Q      Max 
-1.09292 -0.10049  0.07125  0.14140  0.40263 

Coefficients:
             Estimate Std. Error t value Pr(>|t|)    
(Intercept) -0.363938   0.076192  -4.777 1.85e-05 ***
time         0.018632   0.002707   6.883 1.37e-08 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.2598 on 46 degrees of freedom
Multiple R-squared:  0.5074,	Adjusted R-squared:  0.4966 
F-statistic: 47.37 on 1 and 46 DF,  p-value: 1.366e-08
```

回歸圖



殘差圖



殘差常態分佈檢定

```
Shapiro-Wilk normality test

data:  residuals(mod1)
W = 0.98236, p-value = 0.6792
```

```
Shapiro-Wilk normality test

data:  residuals(mod2)
W = 0.96657, p-value = 0.1856
```

```
Shapiro-Wilk normality test

data:  residuals(mod3)
W = 0.98589, p-value = 0.8266
```

```
Shapiro-Wilk normality test

data:  residuals(mod4)
W = 0.86894, p-value = 7.205e-05
```

p–value:
Mod1 > 0.05，殘差呈現常態分佈
Mod2 > 0.05，殘差呈現常態分佈
Mod3 > 0.05，殘差呈現常態分佈
Mod4 < 0.05 ，殘差非常態分佈

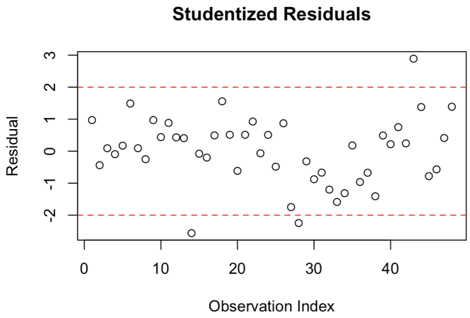
R^2 :
Mod1 R^2 : 0.5778
Mod2 R^2 : 0.3494
Mod3 R^2 : 0.689
Mod4 R^2 : 0.2598

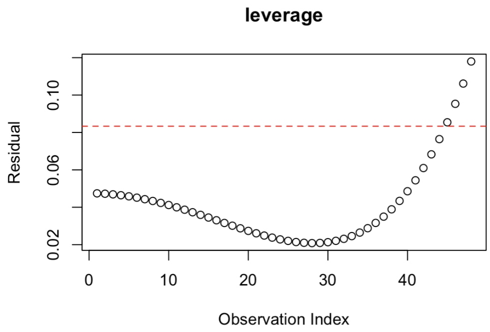
因此我覺得模型 3 的表現較好，因為他的 R^2 較接近 1

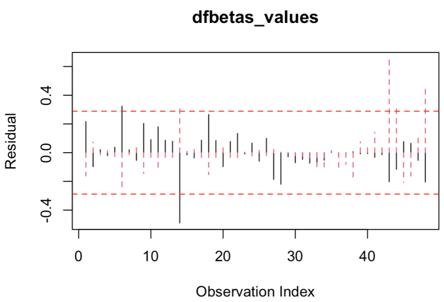
B. 係數解釋

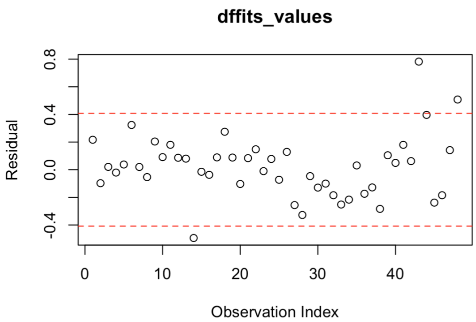
當 time = 0 時，值為 0.7737
The coefficient of $time^2$ is 0.0004986，代表時間越大，產出會隨之增加，也就是每增加 1 單位的 $time^2$，yield 會增加約 0.0004986 單位

C.









Studentized residuals :
大部分觀測值的殘差皆落在 ±2 範圍內，顯示模型預測結果整體穩定。然而，仍有少數觀測值超出此範圍，可能為潛在的離群點或異常觀測值

Leverage:
大部分觀測值的槓桿值均低於參考線，顯示其對模型的影響有限。然而，在資料序列後段，有數個觀測值的槓桿值明顯偏高，可能為潛在的高影響力資料點

dfbetas_values:
這張 DFBETAs 圖顯示大多數觀測值的 DFBETA 值均落在 ±0.3 的門檻範圍內，顯示這些資料對模型係數的影響有限。然而，在觀測值編號接近 43 至 47 的區間中，有數筆資料的 DFBETA 超出紅線，表示這些觀測值可能對某一或多個迴歸係數產生顯著影響，可能為潛在的高影響觀測點

dffits_values:
這張 DFFITS 值圖用於檢查每筆觀測值對模型預測的影響程度。結果顯示，大多數觀測值的 DFFITS 值均落在 ±0.4 的警戒範圍內，顯示其對模型預測的影響有限。然而，在觀測值編號約 45 至 47 處，DFFITS 值超出警戒門檻，顯示這些資料對模型的預測結果影響較大，，可能為潛在的高影響觀測點

D.

```
> pred
       fit      lwr      upr
1 1.881111 1.372403 2.389819
> origin
[1] 2.2318
```

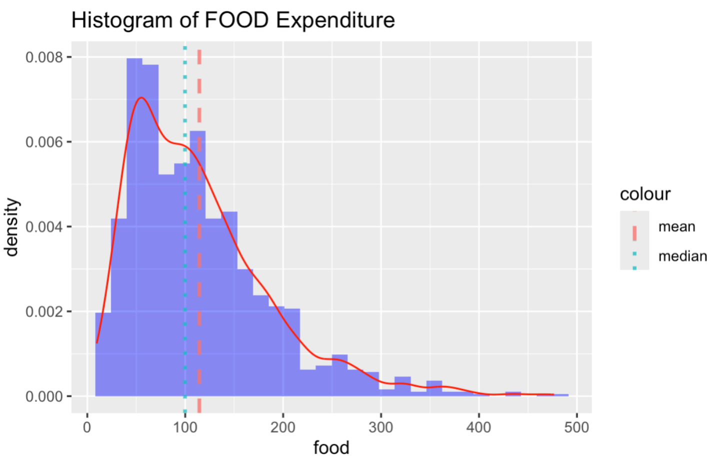
2.2318有落在區間之中

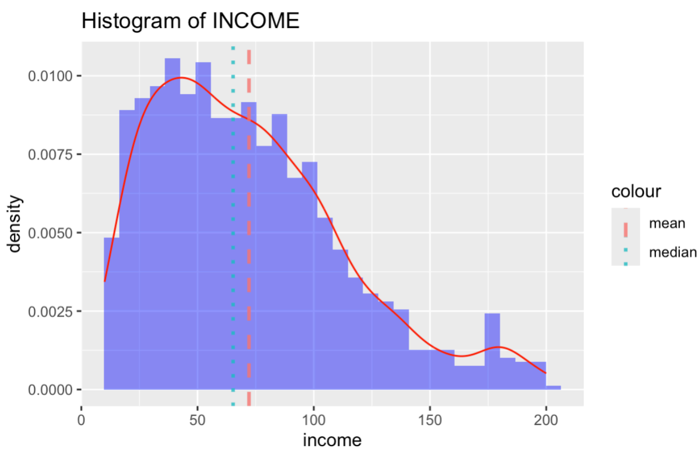
4.29

A.

```
> describe(cex5_small$food)
   vars    n   mean    sd median trimmed   mad  min    max  range skew kurtosis  se
X1    1 1200 114.44 72.66   99.8  105.03 66.18 9.63 476.67 467.04 1.35     2.36 2.1
> describe(cex5_small$income)
   vars    n  mean    sd median trimmed   mad min max range skew kurtosis  se
X1    1 1200 72.14 41.65  65.29   67.94 41.65  10 200   190 0.84     0.32 1.2
```

	mean	median	Minimum	maximum	Standard Deviation
food	114.44	99.8	9.63	476.67	72.66
income	72.14	65.29	10	200	41.65





The histogram of food expenditure are not symmetrical and bell-shaped curve.
food_mean > food_median： right-skewed.

The histogram of income are not symmetrical and bell-shaped curve.
income_mean > income_median： right-skewed

```
> print(jb_food)

        Jarque-Bera Normality Test

data:  cex5_small$food
JB = 648.65, p-value < 2.2e-16
alternative hypothesis: greater
```

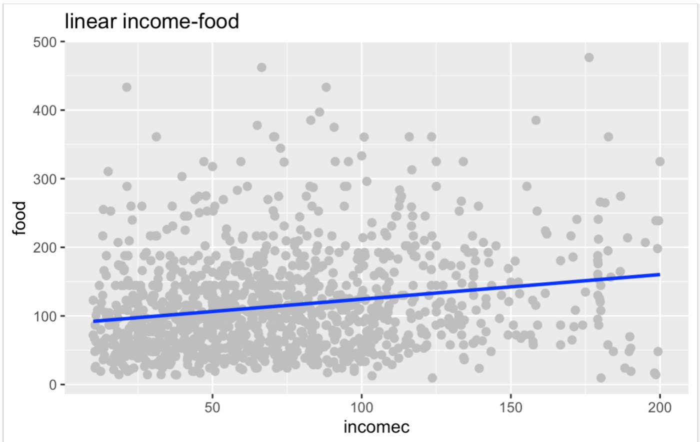
```
> print(jb_income)

        Jarque-Bera Normality Test

data:  cex5_small$income
JB = 148.21, p-value < 2.2e-16
alternative hypothesis: greater
```

Food of JB = 648.65
P–value < 0.05, 所以拒絕虛無假設
也就是 food 並非常態分佈

Income of JB = 648.65
P–value < 0.05, 所以拒絕虛無假設
也就是 income 並非常態分佈

B.

```
Call:
lm(formula = food ~ income, data = cex5_small)

Residuals:
    Min      1Q  Median      3Q     Max
-145.37  -51.48  -13.52   35.50  349.81

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 88.56650    4.10819  21.559  < 2e-16 ***
income       0.35869    0.04932   7.272 6.36e-13 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 71.13 on 1198 degrees of freedom
Multiple R-squared:  0.04228,	Adjusted R-squared:  0.04148
F-statistic: 52.89 on 1 and 1198 DF,  p-value: 6.357e-13
```




```
> confint(mod1, level = 0.95)
                 2.5 %    97.5 %
(Intercept) 80.5064570 96.626543
income       0.2619215  0.455452
>
```

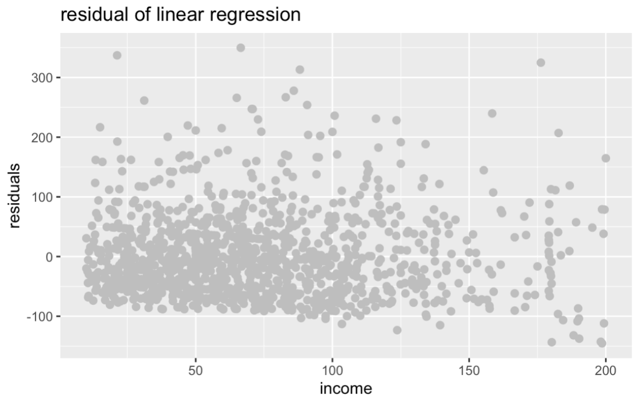
Estimate linear regression:
Food = 88.5665 + 0.3587*Income

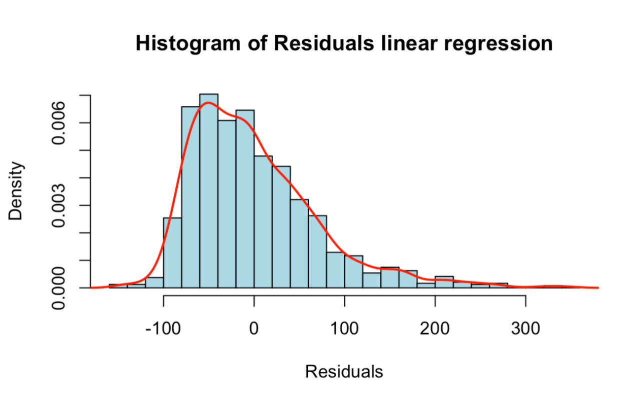
95% interval estimate for β_2 is
[0.2619, 0.4555]

從係數的角度來看，t值為7.272 是具有顯著性的，且p值極小且顯著，再加上95%信賴區間為 [0.2619, 0.4555]，相對窄，因此可以說收入對於食物支出的影響是統計上精確的。

但是從 R^2 = 0.04228 的角度來看，此模型的解釋力很低，因此可能還有其他因素會影像食物支出。

C.





從散布圖中來看並沒有明顯的線性趨勢，但是可以發現隨著收入增加殘差的變異增大
從直方圖來看可以發現圖形有右偏，且非完全對稱

```
> jarque.bera.test(residuals(mod1))  #測試殘差的常態性

	Jarque Bera Test

data:  residuals(mod1)
X-squared = 624.19, df = 2, p-value < 2.2e-16
```

從 Jarque Bera Test 來看

虛無假設 H_0：殘差為常態分配
替代假設 H_1：殘差非常態分配

p–value < 0.05, 所以拒絕虛無假設，接受替代假設，也就是殘差非常態分佈

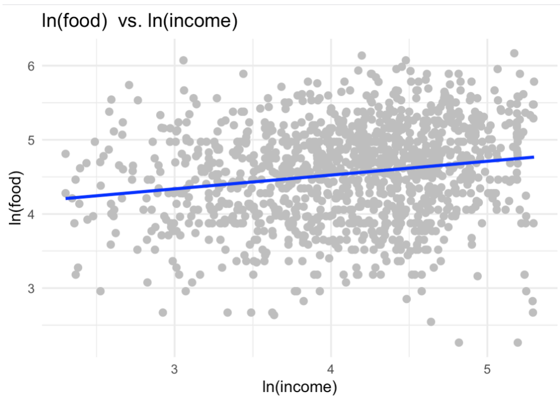
殘差為常態分佈比較重要，因為殘差為常態分佈，才可以確保後續的統計推論是正確的

D.

```
income = 19 expected food expenditure 95.38155 , elasticity = 0.07145038 ,se =  0.00982475 ,elasticity_interval = 0.05217475 0.09072601
income = 65 expected food expenditure 111.8811 , elasticity = 0.2083876 ,se =  0.02865423 ,elasticity_interval = 0.1521695 0.2646056
income = 160 expected food expenditure 145.9564 , elasticity = 0.3931988 ,se =  0.05406661 ,elasticity_interval = 0.287123 0.4992746
>
```

彈性隨著收入增加而增加，代表收入增加時會增加食物支出
信賴區間並沒有重疊，代表在統計上有顯著差異
根據經濟學原則，隨著收入上升，食物支出的彈性會降低，但在此模型中看到食物彈性隨著收入增加而增加，代表在模型設定的收入範圍內，食物仍是具有收入敏感性的商品

E.




```
Call:
lm(formula = ln_food ~ ln_income, data = df_log)

Residuals:
     Min       1Q   Median       3Q      Max 
-2.48175 -0.45497  0.06151  0.46063  1.72315 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)  3.77893    0.12035  31.400   <2e-16 ***
ln_income    0.18631    0.02903   6.417    2e-10 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.6418 on 1198 degrees of freedom
Multiple R-squared:  0.03323,	Adjusted R-squared:  0.03242 
F-statistic: 41.18 on 1 and 1198 DF,  p-value: 1.999e-10
```

log–log model 的 R^2 為 0.0332 ， linear model 的 R^2 為 0.0423
因此 log–log model 的模型適配度較差，也就是 linear model seems fit data better

F.

```
> cat("the point of the elasticity", b2_log)
the point of the elasticity 0.1863054
> cat ("95% interval estimate of the elasticity",interval_log)
95% interval estimate of the elasticity 0.1293432 0.2432675
```

```
income = 19  ,elasticity_interval = 0.05217475 0.09072601  elasticity = 0.07145038
income = 65 ,elasticity_interval = 0.1521695 0.2646056    elasticity = 0.2083876
income = 160 ,elasticity_interval = 0.287123 0.4992746     elasticity = 0.3931988
>
```

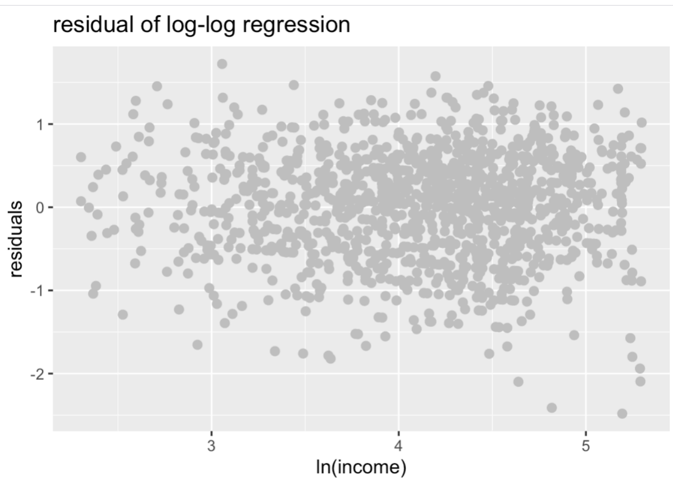
Linear–linear 資料

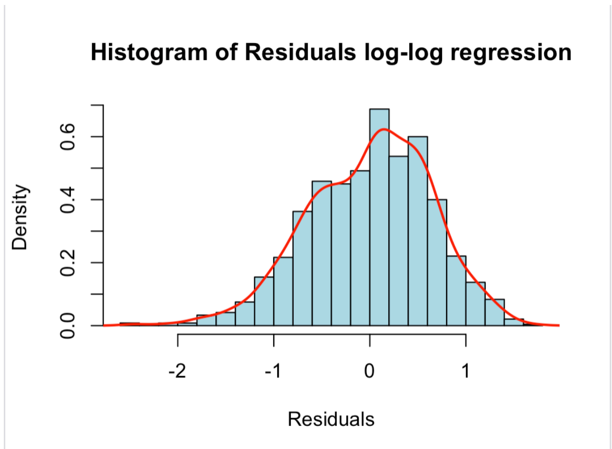
log–log model 的彈性區間 和 linear–linear model 在 Income = 19 與 160 時的彈性區間並沒有重疊，因此在統計上有顯著差異

log–log model 的彈性區間和 linear–linear model 在 Income = 65 時的彈性區間有重疊，因此在統計上沒有顯著差異

因此可以說 log–log model 所估計的彈性與 linear–linear model 在 income = 65 時的彈性相近，但與低收入與高收入時的彈性明顯不同。

G.

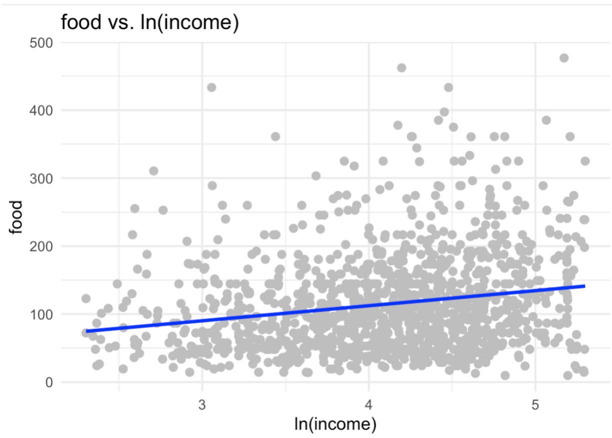





```
	Jarque Bera Test

data:  residuals(mod2)
X-squared = 25.85, df = 2, p-value = 2.436e-06
```

從殘差圖來看，並沒有明顯的趨勢，從直方圖來看，大致是鍾狀，但有偏態與峰態異常。
從JB檢定來看，p–value小於 0.05拒絕虛無假設，代表殘差非常態分佈

H.




```
Call:
lm(formula = food ~ ln_income, data = df_log)

Residuals:
    Min      1Q  Median      3Q     Max 
-129.18  -51.47  -13.98   35.05  345.54 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)   23.568     13.370   1.763   0.0782 .  
ln_income     22.187      3.225   6.879 9.68e-12 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 71.29 on 1198 degrees of freedom
Multiple R-squared:  0.038,	Adjusted R-squared:  0.0372 
F-statistic: 47.32 on 1 and 1198 DF,  p-value: 9.681e-12
```

Linear–log model 的 R^2 為 0.038
log–log model 的 R^2 為 0.0332 ， linear–linear model 的 R^2 為 0.0423
可以得出 linear–linear model R^2 最大，模型配適度最高

I.

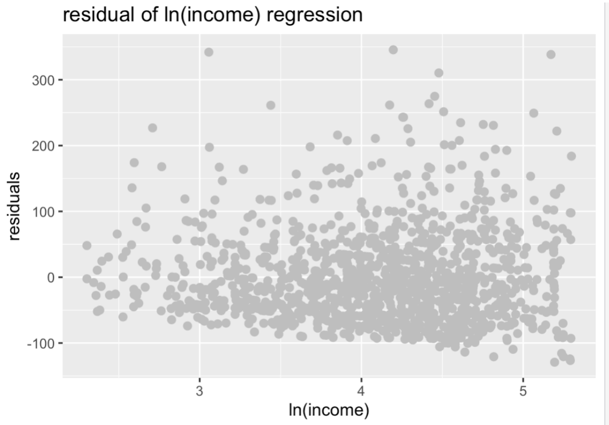
```
income = 19 expected food expenditure 88.89788 , elasticity = 0.2495828 ,se =  0.03628131 ,elasticity_interval = 0.1784009 0.3207648
income = 65 expected food expenditure 116.1872 , elasticity = 0.1909624 ,se =  0.02775978 ,elasticity_interval = 0.1364992 0.2454256
income = 160 expected food expenditure 136.1733 , elasticity = 0.1629349 ,se =  0.02368549 ,elasticity_interval = 0.1164652 0.2094046
>
```

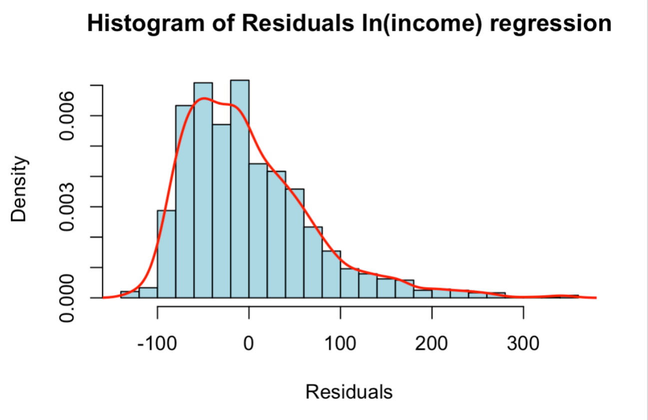
Linear–log model vs. log–log model：在不同的收入水準之下，彈性區間幾乎重疊，代表沒有顯著差異

Linear–log model vs. log–log model ：
linear–log model 的彈性區間 和 linear–linear model 在 Income = 19 與 160 時的彈性區間並沒有重疊，因此在統計上有顯著差異

linear–log model 的彈性區間和 linear–linear model 在 Income = 65 時的彈性區間有重疊，因此在統計上沒有顯著差異

J.






```
        Jarque Bera Test

data:  residuals(mod3)
X-squared = 628.07, df = 2, p-value < 2.2e-16
```

從散布圖來看是呈現隨機分佈，但是隨著收入越大有殘差有變大的趨勢，從直方圖可以看出有左偏的趨勢，從JB檢定來看，p–value 小於 0.05 ，殘差非常態分佈

K. 我會選擇log–log模型，因為他在三個模型之中殘差是最接近常態分佈的