3.7 We have 2008 data on *INCOME* = income per capita (in thousands of dollars) and *BACHELOR* = percentage of the population with a bachelor's degree or more for the 50 U.S. States plus the District of Columbia, a total of $N = 51$ observations. The results from a simple linear regression of *INCOME* on *BACHELOR* are

$$\begin{array}{rcc} \widehat{INCOME} = & (a) & +\ 1.029BACHELOR \\ \text{se} & (2.672) & (c) \\ t & (4.31) & (10.75) \end{array}$$

a. Using the information provided calculate the estimated intercept. Show your work.

b. Sketch the estimated relationship. Is it increasing or decreasing? Is it a positive or inverse relationship? Is it increasing or decreasing at a constant rate or is it increasing or decreasing at an increasing rate?

c. Using the information provided calculate the standard error of the slope coefficient. Show your work.

d. What is the value of the t-statistic for the null hypothesis that the intercept parameter equals 10?

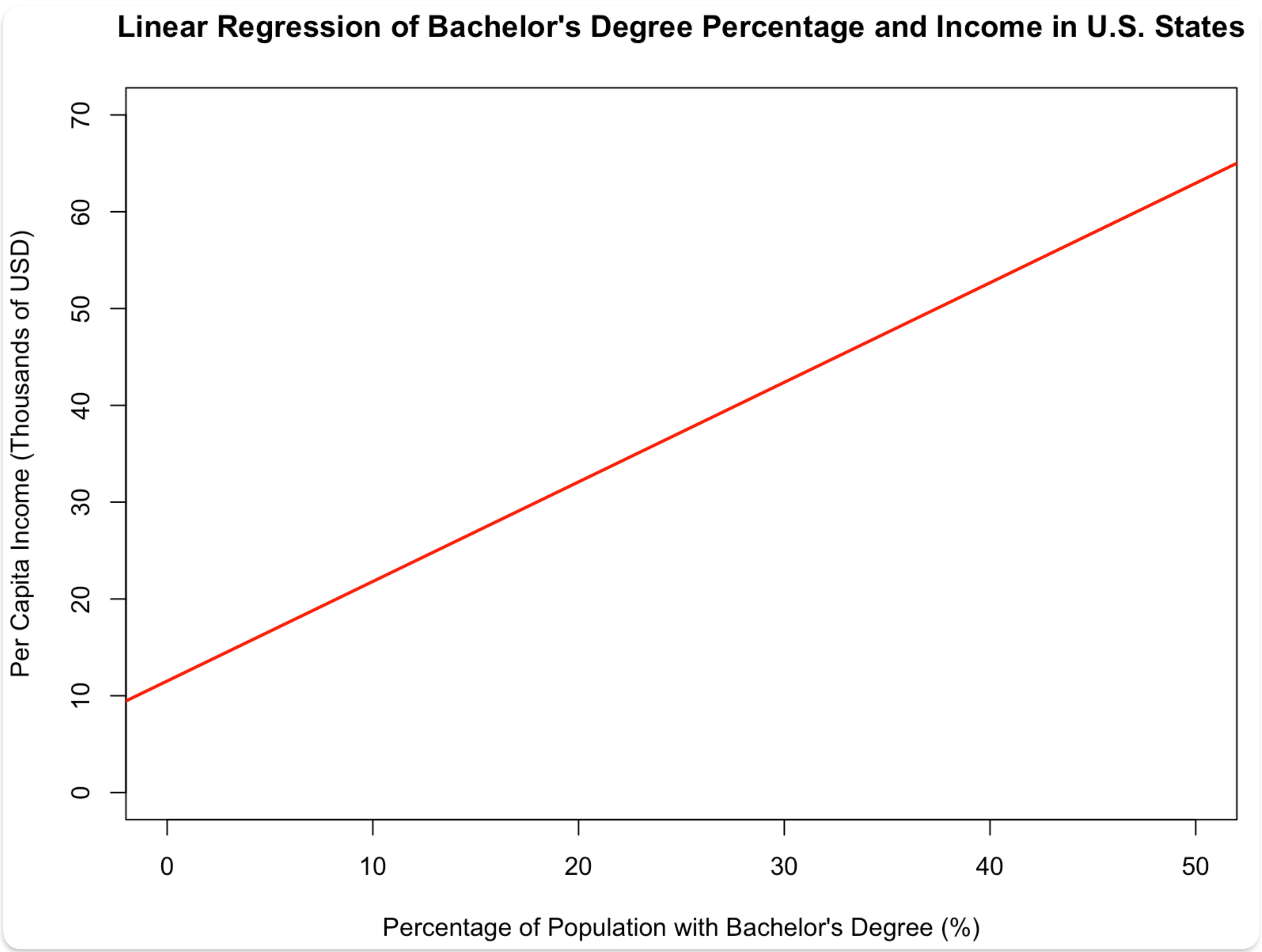
e. The p-value for a two-tail test that the intercept parameter equals 10, from part (d), is 0.572. Show the p-value in a sketch. On the sketch, show the rejection region if $\alpha = 0.05$.

f. Construct a 99% interval estimate of the slope. Interpret the interval estimate.

g. Test the null hypothesis that the slope coefficient is one against the alternative that it is not one at the 5% level of significance. State the economic result of the test, in the context of this problem.

a. $\because t = \dfrac{b_1}{se(b_1)}$ $\quad \therefore b_1 = t \times se(b_1) = 4.31 \times 2.672 = 11.51632$

b. 斜率為 正數 (1.029)，表示 INCOME 與 BACHELOR 之間為正向關係，即教育程度越高，收入越高。
由於斜率為常數，因此此關係是 線性且以恆定速率上升，不是加速或減速變化。



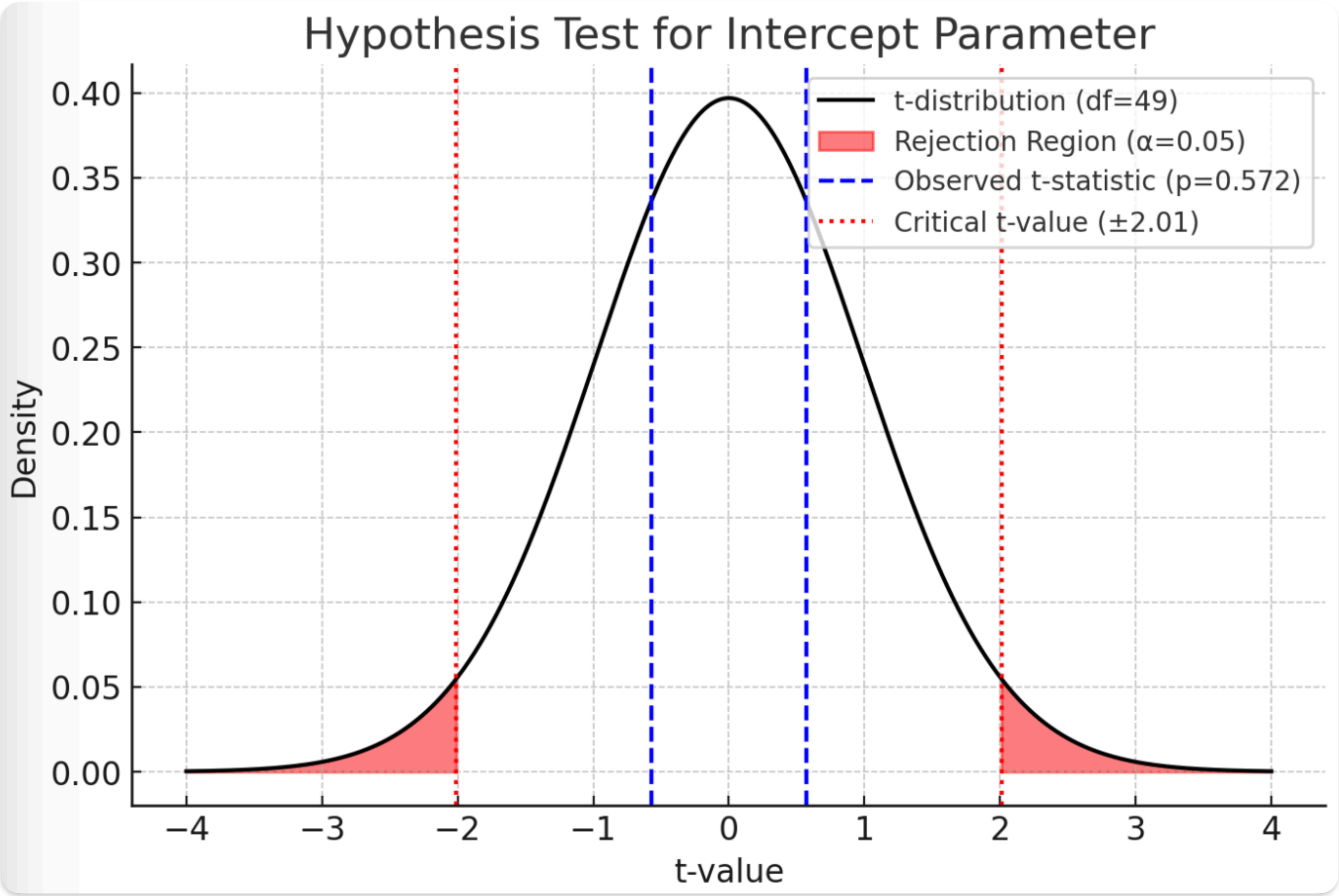
c. $SE(b_2) = \dfrac{1.029}{10.75} = 0.09572$

d. $H_0: \alpha = 10, \quad H_1: \alpha \neq 10$

$t = \dfrac{b_1 - 10}{se(b_1)} = \dfrac{11.51632 - 10}{2.672} = 0.5675$

e. $p = 0.572, \ \alpha = 0.05$

$\because p > \alpha$ $\quad \therefore$ We do not reject H_0 that the intercept $= 10$



f. $C.I = b_2 \pm t_{1-\frac{\alpha}{2}, df} \times SE(b_2)$

$b_2 = 1.029, \ \alpha = 0.01, \ df = n-2 = 51-2 = 49, \ SE(b_2) = 0.09572$

$CI = 1.029 \pm t_{0.995, 49} \times 0.09572 = 1.029 \pm 2.679952 \times 0.09572$

$= [0.7725, 1.2855]$

g. $H_0: \beta_2 = 1, \quad H_1: \beta_2 \neq 1 \qquad \alpha = 0.05$

$t = \dfrac{b_2 - 1}{SE(b_2)} = \dfrac{1.029 - 1}{0.09572} = 0.3030$

$t_{0.975, 49} = 2.0096$

```
> qt(1 - 0.01 / 2, 49)
[1] 2.679952
> qt(1 - 0.05 / 2, 49)
[1] 2.009575
```

$\because t = 0.3030 < 2.0096$

$\therefore$ we do not reject H_0 that the slope $= 1$