5.6 Suppose that, from a sample of 63 observations, the least squares estimates and the corresponding estimated covariance matrix are given by

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix} \qquad \widehat{\operatorname{cov}}(b_1, b_2, b_3) = \begin{array}{c|c} & \begin{array}{ccc} b_1 & b_2 & b_3 \end{array} \\ \hline \begin{array}{c} \\ b_2 \\ b_3 \end{array} & \begin{bmatrix} 3 & -2 & 1 \\ -2 & 4 & 0 \\ 1 & 0 & 3 \end{bmatrix} \end{array}$$

Using a 5% significance level, and an alternative hypothesis that the equality does not hold, test each of the following null hypotheses:

a. $\beta_2 = 0$
b. $\beta_1 + 2\beta_2 = 5$
c. $\beta_1 - \beta_2 + \beta_3 = 4$

(a.)

1. $H_0: \beta_2 = 0$ against $H_1: \beta_2 \neq 0$.

2. $\varphi = \dfrac{3-0}{\sqrt{4}} = 1.5 \sim t_{0.025}(60)$

3. $RR = \{ \varphi \geq 1.96 \text{ or } \varphi \leq -1.96 \}$.

$3+16-8$

4. $\varphi \notin RR$, not reject H_0, can't conclude that $\beta_2 \neq 0$.

(b.) $SE(b_1 + 2b_2) = \sqrt{Var(b_1) + 4Var(b_2) + 4\operatorname{cov}(b_1, b_2)} = \sqrt{3 + 4 \times 4 + 4 \times -2} = 3.3166$.

1. $H_0: \beta_1 + 2\beta_2 = 5$ against $H_1: \beta_1 + 2\beta_2 \neq 5$

2. $\varphi = \dfrac{\hat{\beta}_1 + 2\hat{\beta}_2 - 5}{3.3166} = \dfrac{2+6-5}{3.3166} = 0.9045 \sim t_{0.025}(60)$

3. $RR: \{ \varphi \geq 1.96 \text{ or } \varphi \leq -1.96 \}$.

$10+4+2$

4. $\varphi \notin RR$, not reject H_0, can't conclude that $\beta_1 + 2\beta_2 \neq 5$

(c.) $SE(b_1 - b_2 + b_3) = \sqrt{Var(b_1) + Var(b_2) + Var(b_3) - 2\operatorname{cov}(b_1, b_2) + 2\operatorname{cov}(b_1, b_3) - 2\operatorname{cov}(b_2, b_3)}$

$= \sqrt{3 + 4 + 3 - 2 \times -2 + 2 \times 1 - 2 \times 0} = 4$

1. $H_0: \beta_1 - \beta_2 + \beta_3 = 4$ against $H_1: \beta_1 - \beta_2 + \beta_3 \neq 4$

3. $RR: \{ \varphi \geq 1.96 \text{ or } \varphi \leq -1.96 \}$.

2. $\varphi = \dfrac{2-3-1-4}{4} = \dfrac{-6}{4} = -1.5 \sim t_{0.025}(60)$

4. $\varphi \notin RR$, can't conclude that $\beta_1 - \beta_2 + \beta_3 \neq 4$.