- (a) b2: + ,b3: + ,b4: - ,b5: - ,b6: -
- (b) 因為這個迴歸式存在內生性問題,WAGE應該是內生性變數,HOURS和WAGE很可能互項影響,有simultaneous equations bias
- (c)
 EXPER 不會直接影響HOURS,但EXPER 和EXPER² 和WAGE 應該有顯著相關
- (d) 如果WAGE是內生的,則式子是not identified

(e)

step1:

內生性變數(WAGE)當被解釋變數, instruments(EXPER,EXPER²)以 及外生性變數(EDUC,...)當解釋變數,進行迴歸得到WAGE的估計 WAGE

step2:

把原式中的WAGE用它的估計WAGE取代,進行迴歸,得到WAGE的係數即 IV/2SLS estimate

(a)
$$x = \gamma_1 + \theta_1 z + v \stackrel{?}{=} OLS, z \stackrel{?}{=} valid instrument \Rightarrow cov(z, v) = 0$$
 z的係數 $\theta_1 = \frac{cov(z, x)}{var(z)} - \frac{cov(z, v)}{var(z)} = \frac{cov(z, x)}{var(z)}$ (b)
$$y = \pi_0 + \pi_1 z + u \stackrel{?}{=} OLS, z \stackrel{?}{=} valid instrument \Rightarrow cov(z, u) = 0$$
 z的係數 $\pi_1 = \frac{cov(z, y)}{var(z)} - \frac{cov(z, u)}{var(z)} = \frac{cov(z, y)}{var(z)}$ (c)
$$y = \beta_1 + \beta_2 x + e$$

$$= \beta_1 + \beta_2 (\gamma_1 + \theta_1 z + v) + e$$

$$= \beta_1 + \beta_2 (\gamma_1 + \theta_1 z + v) + e$$

$$= \beta_1 + \beta_2 \gamma_1 + \beta_2 \theta_1 z + \beta_2 v + e \Rightarrow \pi_1 = \beta_2 \theta_1$$

$$u = \beta_2 v + e$$

$$\pi_1 = \beta_2 \theta_1$$

$$\Rightarrow \beta_2 = \frac{\pi_1}{\theta_1}$$

(e)

 $\hat{\pi}_1$, $\hat{\theta}_1$ 是 OLS estimator => consistent estimator

$$\hat{\pi}_1 \xrightarrow{p} \pi_1$$

$$\hat{\theta}_1 \xrightarrow{p} \theta_1$$

因此 $\hat{\beta}_2 = \frac{\hat{\pi}_1}{\hat{\theta}_1} \xrightarrow{p} \frac{\pi_1}{\theta_1} = \beta_2$ 也是 consistent estimator