

5.6 Suppose that, from a sample of 63 observations, the least squares estimates and the corresponding estimated covariance matrix are given by

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix} \quad \widehat{\text{cov}}(b_1, b_2, b_3) = \begin{bmatrix} 3 & -2 & 1 \\ -2 & 4 & 0 \\ 1 & 0 & 3 \end{bmatrix}$$

Using a 5% significance level, and an alternative hypothesis that the equality does not hold, test each of the following null hypotheses:

- a. $\beta_2 = 0$
- b. $\beta_1 + 2\beta_2 = 5$
- c. $\beta_1 - \beta_2 + \beta_3 = 4$

$$t_{0.975, 60} = 2.000298$$

a.

$$\frac{3-0}{\sqrt{4}} = 1.5 < t_c \quad \text{fail to reject } H_0$$

b. $\text{Var}(\beta_1 + 2\beta_2) = \text{Var}(b_1) + 4\text{Var}(b_2) + 4\text{cov}(b_1, b_2)$

$$= 3 + 4 \times 4 + 4 \times -2 = 11$$

$$\frac{2+2 \times 3 - 5}{\sqrt{11}} = \frac{3}{\sqrt{11}} = 0.9035 < t_c \quad \text{fail to reject } H_0$$

c. $\text{Var}(\beta_1 - \beta_2 + \beta_3) = \text{Var}(b_1) - \text{Var}(b_2) + \text{Var}(b_3) - 2\text{cov}(b_1, b_2) + 2\text{cov}(b_1, b_3) - 2\text{cov}(b_2, b_3)$

$$= 3 - 4 + 3 + 4 + 2 = 8$$

$$\frac{2-3-1+4}{\sqrt{8}} = \frac{-6}{\sqrt{8}} = -2.1213 < -2.000298 \quad \text{reject } H_0$$

5.31 Each morning between 6:30 AM and 8:00 AM Bill leaves the Melbourne suburb of Carnegie to drive to work at the University of Melbourne. The time it takes Bill to drive to work (*TIME*), depends on the departure time (*DEPART*), the number of red lights that he encounters (*REDS*), and the number of trains that he has to wait for at the Murrumbeena level crossing (*TRAINS*). Observations on these

variables for the 249 working days in 2015 appear in the file *commute5*. *TIME* is measured in minutes.

DEPART is the number of minutes after 6:30 AM that Bill departs.

- a. Estimate the equation

$$TIME = \beta_1 + \beta_2 DEPART + \beta_3 REDS + \beta_4 TRAINS + e$$

Report the results and interpret each of the coefficient estimates, including the intercept β_1 .

- b. Find 95% interval estimates for each of the coefficients. Have you obtained precise estimates of each of the coefficients?
- c. Using a 5% significance level, test the null hypothesis that Bill's expected delay from each red light is 2 minutes or more against the alternative that it is less than 2 minutes.
- d. Using a 10% significance level, test the null hypothesis that the expected delay from each train is 3 minutes against the alternative that it is not 3 minutes.
- e. Using a 5% significance level, test the null hypothesis that Bill can expect a trip to be at least 10 minutes longer if he leaves at 7:30 AM instead of 7:00 AM, against the alternative that it will not be 10 minutes longer. (Assume other things are equal.)
- f. Using a 5% significance level, test the null hypothesis that the expected delay from a train is at least three times greater than the expected delay from a red light against the alternative that it is less than three times greater.
- g. Suppose that Bill encounters six red lights and one train. Using a 5% significance level, test the null hypothesis that leaving Carnegie at 7:00 AM is early enough to get him to the university on or before 7:45 AM against the alternative that it is not. [Carry out the test in terms of the expected time $E(TIME|X)$ where X represents the observations on all explanatory variables.]
- h. Suppose that, in part (g), it is imperative that Bill is not late for his 7:45 AM meeting. Have the null and alternative hypotheses been set up correctly? What happens if these hypotheses are reversed?

9.

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	20.8701	1.6758	12.454	< 2e-16 ***
depart	0.3681	0.0351	10.487	< 2e-16 ***
reds	1.5219	0.1850	8.225	1.15e-14 ***
trains	3.0237	0.6340	4.769	3.18e-06 ***

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 6.299 on 245 degrees of freedom

Multiple R-squared: 0.5346, Adjusted R-squared: 0.5289

F-statistic: 93.79 on 3 and 245 DF, p-value: < 2.2e-16

> confint(model, level = 0.95)

2.5 % 97.5 %

(Intercept)	17.5694018	24.170871
depart	0.2989851	0.437265
reds	1.1574748	1.886411
trains	1.7748867	4.272505

c. $H_0: \beta_3 \geq 2$

$$\frac{1.5219 - 2}{0.185} = -2.5843 < -1.65 \quad \text{reject } H_0$$

 $H_a: \beta_3 \leq 2$ d. $H_0: \beta_4 = 3$

$$\frac{3.0237 - 3}{0.634} \approx 0.0394 < 1.65 \quad \text{fail to reject } H_0$$

 $H_a: \beta_4 \neq 3$ e. $H_0: 3\beta_2 \geq 10 \quad \beta_2 \geq \frac{1}{3}$ $H_a: 3\beta_2 < 10$

$$\frac{0.3681 - \frac{1}{3}}{0.0351} \approx 0.9905 < 1.65 \quad \text{fail to reject } H_0$$

b.: 当 Bill 6:30 出发，没遇到红灯，没等火车，平均通勤时间为 20.87 min

b.: 当 Bill 每晚 1 min 出发，通勤时间多 0.3681 min

b.: 当 Bill 每天遇到 1 个红灯，平均多 1.5219 min

b.: 当 Bill 每天等一班车，平均多 3.0237 min

$$t_{0.95, 245} = 1.65$$

f. $H_0: \beta_4 \geq 3\beta_3 \quad \beta_4 - 3\beta_3 \geq 0$

$H_a: \beta_4 < 3\beta_3 \quad \beta_4 - 3\beta_3 < 0$

$$\frac{3.6239 - 3 \times 1.5219}{0.8444992} = -1.8249 < -1.65 \quad \text{reject } H_0$$

g:

$$H_0: \beta_1 + 3\beta_2 + 6\beta_3 + \beta_4 \leq 45$$

$$H_a: \beta_1 + 3\beta_2 + 6\beta_3 + \beta_4 > 45$$

$$\frac{44.0629 - 45}{0.5392689} = -1.726 < 1.65 \quad \text{fail to reject } H_0$$

h.

$$H_0: \beta_1 + 3\beta_2 + 6\beta_3 + \beta_4 > 45$$

$$H_a: \beta_1 + 3\beta_2 + 6\beta_3 + \beta_4 \leq 45$$

$$-1.726 < -1.65 \quad \text{reject } H_0 \quad \text{不会通过}$$

5.33 Use the observations in the data file *cps5_small* to estimate the following model:

$$\ln(WAGE) = \beta_1 + \beta_2 EDUC + \beta_3 EDUC^2 + \beta_4 EXPER + \beta_5 EXPER^2 + \beta_6 (EDUC \times EXPER) + e$$

- At what levels of significance are each of the coefficient estimates “significantly different from zero”?
- Obtain an expression for the marginal effect $\partial E[\ln(WAGE)|EDUC, EXPER]/\partial EDUC$. Comment on how the estimate of this marginal effect changes as *EDUC* and *EXPER* increase.
- Evaluate the marginal effect in part (b) for all observations in the sample and construct a histogram of these effects. What have you discovered? Find the median, 5th percentile, and 95th percentile of the marginal effects.
- Obtain an expression for the marginal effect $\partial E[\ln(WAGE)|EDUC, EXPER]/\partial EXPER$. Comment on how the estimate of this marginal effect changes as *EDUC* and *EXPER* increase.
- Evaluate the marginal effect in part (d) for all observations in the sample and construct a histogram of these effects. What have you discovered? Find the median, 5th percentile, and 95th percentile of the marginal effects.
- David has 17 years of education and 8 years of experience, while Svetlana has 16 years of education and 18 years of experience. Using a 5% significance level, test the null hypothesis that Svetlana’s expected log-wage is equal to or greater than David’s expected log-wage, against the alternative that David’s expected log-wage is greater. State the null and alternative hypotheses in terms of the model parameters.
- After eight years have passed, when David and Svetlana have had eight more years of experience, but no more education, will the test result in (f) be the same? Explain this outcome?
- Wendy has 12 years of education and 17 years of experience, while Jill has 16 years of education and 11 years of experience. Using a 5% significance level, test the null hypothesis that their marginal effects of extra experience are equal against the alternative that they are not. State the null and alternative hypotheses in terms of the model parameters.
- How much longer will it be before the marginal effect of experience for Jill becomes negative? Find a 95% interval estimate for this quantity.

b. $\frac{\partial E(\ln(WAGE))}{\partial EDUC} | EDUC, EXPER$ = $\beta_2 + 2\beta_3 EDUC + \beta_6 EXPER$

a.

Coefficients:

	Estimate	Std. Error	t value	Pr(> t)
(Intercept)	1.038e+00	2.757e-01	3.764	0.000175 ***
educ	8.954e-02	3.108e-02	2.881	0.004038 **
I(educ^2)	1.458e-03	9.242e-04	1.578	0.114855
exper	4.488e-02	7.297e-03	6.150	1.06e-09 ***
I(exper^2)	-4.680e-04	7.601e-05	-6.157	1.01e-09 ***
I(educ * exper)	-1.010e-03	3.791e-04	-2.665	0.007803 **

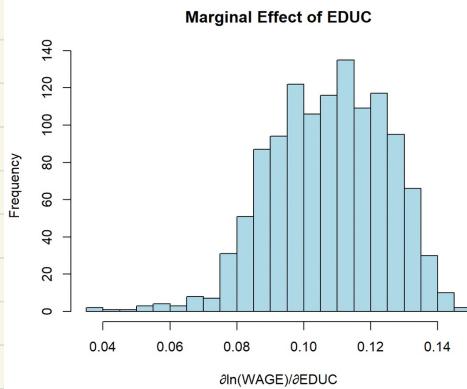
Signif. codes:	0 ***	0.001 **	0.01 *	0.05 .
Residual standard error:	0.4638	on 1194 degrees of freedom		
Multiple R-squared:	0.3227,	Adjusted R-squared:	0.3198	
F-statistic:	113.8	on 5 and 1194 DF,	p-value:	< 2.2e-16

All expect from $EDUC^2$

$$= 0.08954 + 0.002191 EDUC - 0.0010178 EXPER$$

marginal effect 會隨 EDUC 增加而增加，隨 EXPER 增加而下降

C.



5%	50%	95%
0.08008187	0.10843125	0.13361880

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- d. Obtain an expression for the marginal effect $\partial E[\ln(WAGE)|EDUC, EXPER]/\partial EXPER$. Comment on how the estimate of this marginal effect changes as EDUC and EXPER increase.
- e. Evaluate the marginal effect in part (d) for all observations in the sample and construct a histogram of these effects. What have you discovered? Find the median, 5th percentile, and 95th percentile of the marginal effects.
- f. David has 17 years of education and 8 years of experience, while Svetlana has 16 years of education and 18 years of experience. Using a 5% significance level, test the null hypothesis that Svetlana's expected log-wage is equal to or greater than David's expected log-wage, against the alternative that David's expected log-wage is greater. State the null and alternative hypotheses in terms of the model parameters.

d.

$$\partial E[\ln(WAGE)|EDUC, EXPER]$$

$\partial EXPER$

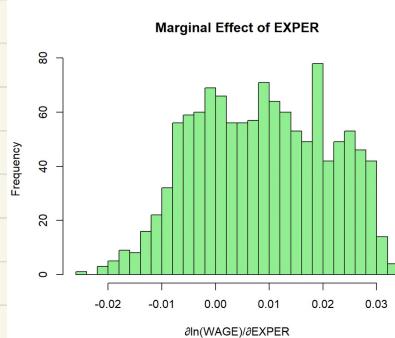
$$= \beta_4 + 2\beta_5 EXPER + \beta_6 EDUC$$

$$= 0.04488 + 2 \times (-0.00468) \times EXPER + (-0.0011) \times EDUC$$

marginal effect decreases as EDUC or EXPER increases

e.

	5%	50%	95%
-0.010376212	0.008418878	0.027931151	



most marginal effects are positive, only a small portion may experience negative.

$$H_0: \beta_1 + 17\beta_2 + 17^2\beta_3 + 8\beta_4 + 8^2\beta_5 + 8 \times 17\beta_6 \leq \beta_1 + 16\beta_2 + 16^2\beta_3 + 18\beta_4 + 18^2\beta_5 + 18 \times 16\beta_6$$

$$H_1: \beta_1 + 17\beta_2 + 17^2\beta_3 + 8\beta_4 + 8^2\beta_5 + 8 \times 17\beta_6 > \beta_1 + 16\beta_2 + 16^2\beta_3 + 18\beta_4 + 18^2\beta_5 + 18 \times 16\beta_6$$

$$H_0: \theta_2 + 33\beta_3 - 10\beta_4 - 260\beta_5 - 152\beta_6 \leq 0$$

$$t_{0.95, 114} = 1.646131$$

$$H_1: \theta_2 + 33\beta_3 - 10\beta_4 - 260\beta_5 - 152\beta_6 > 0$$

$$t = \frac{-0.03388 - 0}{0.021489} = -1.6697 < 1.646 \quad \text{fail to reject } H_0$$

$$g. H_0: \beta_2 + 3\beta_3 - 10\beta_4 - 4\beta_5 - 144\beta_6 \leq 0$$

$$H_1: \beta_2 + 3\beta_3 - 10\beta_4 - 4\beta_5 - 144\beta_6 > 0$$

$$t = \frac{0.03092 - 0}{0.01499} = 2.0627 > 1.646 \quad \text{reject } H_0$$

$$h. \text{ marginal effect} = \beta_4 + 2\beta_5 \text{ GDPER} + \beta_6 \text{ EDUC}$$

$$H_0: \beta_4 + 2\beta_5 + 12\beta_6 = \beta_4 + 2\beta_5 + 16\beta_6$$

$$H_0: 12\beta_5 - 4\beta_6 = 0$$

$$H_1: \beta_4 + 2\beta_5 + 12\beta_6 \neq \beta_4 + 2\beta_5 + 16\beta_6 \Rightarrow H_1: 12\beta_5 - 4\beta_6 \neq 0$$

$$t = \frac{-0.001575}{0.0015334} = -1.02713 > -1.962 \quad \text{fail to reject } H_0$$

$$P_4 + 2 P_6 \times (11 + \chi) + 16 P_6 = 0$$

$$\chi = \frac{-P_4 - 16 P_6}{2 P_6} = -11 = 19.677 \cdot b$$

$$Se = 1.8959$$

$$19.6771 \pm t \times 1.8959 = [15.958, 22.396]$$