Q01

2.1 Consider the following five observations. You are to do all the parts of this exercise using only a calculator.

x	y	$x-\bar{x}$	$(x-\bar{x})^2$	$y-\bar{y}$	$(x-\bar{x})(y-\bar{y})$
3	4	2	4	2	4
2	2	1	1	0	0
1	3	0	0	1	0
–1	1	-2	4	-1	2
0	0	-1	1	-2	2
$\sum x_i =$	$\sum y_i =$	$\sum(x_i-\bar{x}) =$	$\sum(x_i-\bar{x})^2 =$	$\sum(y_i-\bar{y}) =$	$\sum(x_i-\bar{x})(y_i-\bar{y}) =$
5	10	0	10	0	8

a. Complete the entries in the table. Put the sums in the last row. What are the sample means $\bar{x}$ and $\bar{y}$?

$\bar{x}=1$, $\bar{y}=2$ #

b. Calculate b_1 and b_2 using (2.7) and (2.8) and state their interpretation.

$$b_2 = \frac{\sum(x_i-\bar{x})(y_i-\bar{y})}{\sum(x_i-\bar{x})^2} = 0.8$$

b_2代表當x每增加一單位，y會增加0.8單位 #

$$b_1 = \bar{y} - b_2\bar{x} = 1.2$$

b_1代表y的期望值，($x=0$時) #

c. Compute $\sum_{i=1}^{5} x_i^2$, $\sum_{i=1}^{5} x_i y_i$. Using these numerical values, show that $\sum(x_i-\bar{x})^2 = \sum x_i^2 - N\bar{x}^2$ and $\sum(x_i-\bar{x})(y_i-\bar{y}) = \sum x_i y_i - N\bar{x}\bar{y}$.

$\sum_{i=1}^{5} x_i^2 = 15$, $\sum_{i=1}^{5} x_i y_i = 18$, $N=5$

$\bar{x}=1$, $\bar{y}=2$

$10 = 15 - 5\times 1$, $\Rightarrow$ 成立 #

$8 = 18 - 5\times 1\times 2$, $\Rightarrow$ 成立 #

d. Use the least squares estimates from part (b) to compute the fitted values of y, and complete the remainder of the table below. Put the sums in the last row.
Calculate the sample variance of y, $s_y^2 = \sum_{i=1}^{N}(y_i - \bar{y})^2/(N-1)$, the sample variance of x, $s_x^2 = \sum_{i=1}^{N}(x_i - \bar{x})^2/(N-1)$, the sample covariance between x and y, $s_{xy} = \sum_{i=1}^{N}(y_i - \bar{y})(x_i - \bar{x})/(N-1)$, the sample correlation between x and y, $r_{xy} = s_{xy}/(s_x s_y)$ and the coefficient of variation of x, $CV_x = 100(s_x/\bar{x})$. What is the median, 50th percentile, of x?

x_i	y_i	$\hat{y}_i$	$\hat{e}_i$	$\hat{e}_i^2$	$x_i\hat{e}_i$
3	4	3.6	0.4	0.16	1.2
2	2	2.8	-0.8	0.64	-1.6
1	3	2	1	1	1
–1	1	0.4	0.6	0.36	0.6
0	0	1.2	-1.2	1.44	0
$\sum x_i =$	$\sum y_i =$	$\sum \hat{y}_i =$	$\sum \hat{e}_i =$	$\sum \hat{e}_i^2 =$	$\sum x_i\hat{e}_i =$
5	10	10	0	3.6	0

b_1, b_2 已在(b)估計出為 1.2 和 0.8，$\hat{y}_i = b_1 + b_2 x_i$

$\hat{e}_i = y_i - \hat{y}_i$

$s_y^2 = \frac{10}{4} = 2.5$ #

$s_x^2 = \frac{10}{4} = 2.5$ #

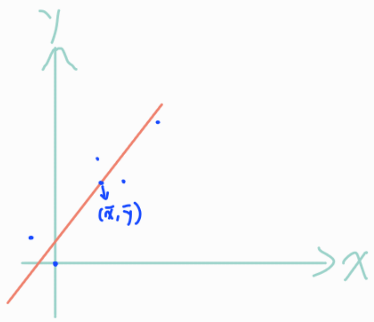
$s_{xy} = \frac{8}{4} = 2$ #

$r_{xy} = \frac{2}{\sqrt{2.5} \times \sqrt{2.5}} = 0.8$ #

$CV_x = 100 \times \left(\frac{\sqrt{2.5}}{1}\right) = 158.1\%$ #

median of $x = 1$ #

e. On graph paper, plot the data points and sketch the fitted regression line $\hat{y}_i = b_1 + b_2 x_i$.



f. On the sketch in part (e), locate the point of the means $(\bar{x}, \bar{y})$. Does your fitted line pass through that point? If not, go back to the drawing board, literally.

Yes, pass through the $(\bar{x}, \bar{y})$ point #

g. Show that for these numerical values $\bar{y} = b_1 + b_2\bar{x}$.

h. Show that for these numerical values $\bar{\hat{y}} = \bar{y}$, where $\bar{\hat{y}} = \sum \hat{y}_i / N$.

i. Compute $\hat{\sigma}^2$.

j. Compute $\widehat{\text{var}}(b_2|\mathbf{x})$ and $\text{se}(b_2)$.

(g) $y = 1.2 + 0.8x$, 代入 $\bar{x}$, $\bar{y}$

$\Rightarrow 2 = 1.2 + 0.8 \times 1$ #

(h) $\bar{\hat{y}} = 2$, $\dfrac{\sum \hat{y}_i}{N} = \dfrac{10}{5} = 2$ #

(i) $\hat{\sigma}^2 = \dfrac{\sum \hat{e}_i^2}{N-2} = \dfrac{3.6}{3} = 1.2$ #

(j) $\widehat{\text{var}}(b_2|x) = \dfrac{\hat{\sigma}^2}{\sum(x_i - \bar{x})^2} = \dfrac{1.2}{10} = 0.12$ #

$se(b_2) = \sqrt{\text{var}} = \sqrt{0.12}$ #

Q 14

2.14 Consider the regression model $WAGE = \beta_1 + \beta_2 EDUC + e$, where $WAGE$ is hourly wage rate in U.S. 2013 dollars and $EDUC$ is years of education, or schooling. The regression model is estimated twice using the least squares estimator, once using individuals from an urban area, and again for individuals in a rural area.

$$\text{Urban} \quad \begin{array}{ll} \widehat{WAGE} = -10.76 + 2.46\, EDUC, & N = 986 \\ \text{(se)} \qquad\quad (2.27) \quad (0.16) & \end{array}$$

$$\text{Rural} \quad \begin{array}{ll} \widehat{WAGE} = -4.88 + 1.80\, EDUC, & N = 214 \\ \text{(se)} \qquad\quad (3.29) \quad (0.24) & \end{array}$$

a. Using the estimated rural regression, compute the elasticity of wages with respect to education at the "point of the means." The sample mean of $WAGE$ is \$19.74.

$$\overline{EDUC} = (19.74 + 4.88)/1.8 \doteq 13.68$$

$$\text{elasticity} = \frac{\partial WAGE}{\partial EDUC} \times \frac{\overline{EDUC}}{\overline{WAGE}} = 1.8 \times \frac{13.68}{19.74} \doteq 1.25 \ \#$$

b. The sample mean of $EDUC$ in the urban area is 13.68 years. Using the estimated urban regression, compute the standard error of the elasticity of wages with respect to education at the "point of the means." Assume that the mean values are "givens" and not random.

c. What is the predicted wage for an individual with 12 years of education in each area? With 16 years of education?

(b)

$$\overline{WAGE} = -10.76 + 2.46 \times 13.68 = 22.89$$

$$\text{elasticity (SE)} = se(\beta_1) \times \frac{\overline{EDUC}}{\overline{WAGE}} = 0.16 \times \frac{13.68}{22.89} = 0.096 \ \#$$

(c)

	12 years	16 years
Urban.	$wage = -10.76 + 2.46 \times 12 = 18.76$ #	$wage = -10.76 + 2.46 \times 16 = 28.6$ #
Rural.	$wage = -4.88 + 1.8 \times 12 = 16.72$ #	$wage = -4.88 + 1.8 \times 16 = 23.92$ #

Q 16

(a)

$$\underline{r_j - r_f} = \underline{\alpha_j} + \underline{\beta_j} \, \underline{(r_m - r_f)} + \underline{e_j}$$

$$\Downarrow \qquad \Downarrow \qquad \Downarrow \qquad \Downarrow \qquad \Downarrow$$

$$y = \beta_0 + \beta_1 \cdot X + e$$

數學形式相同

(b)

```
[1] "(b)"
  ge.MKT_rp  ibm.MKT_rp ford.MKT_rp msft.MKT_rp  dis.MKT_rp  xom.MKT_rp
  1.2400877   1.0690254   1.7541663   1.2939753   1.1036563   0.5486564
most aggressive: ford.MKT_rp
most defensive xom.MKT_rp
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(c)

```
[1] "(c)"
                    Stock       Alpha      P_Value Significant
ge.(Intercept)         ge -0.06010112 1.251570e-22         Yes
ibm.(Intercept)       ibm -0.05308990 3.499514e-17         Yes
ford.(Intercept)     ford -0.05536354 9.423409e-07         Yes
msft.(Intercept)     msft -0.05589285 5.316667e-15         Yes
dis.(Intercept)       dis -0.05809553 1.295343e-18         Yes
xom.(Intercept)       xom -0.05385892 2.152167e-23         Yes
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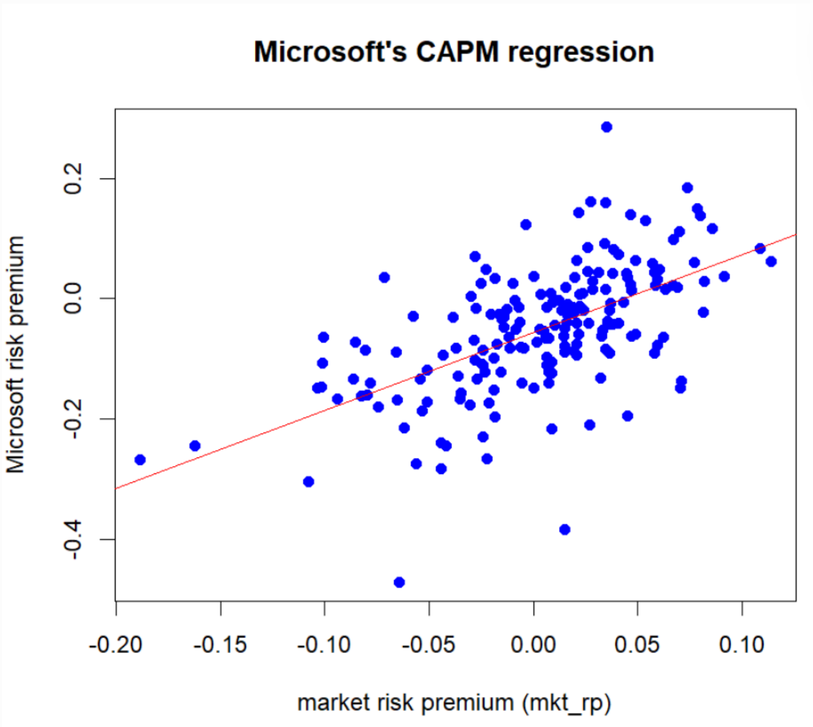
not correct

In Finance Theory, $\alpha_j = 0$

but in the result $\alpha_j \neq 0$.

表示有其他因素影響了结果



(d)

差異極小

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[1] "(d)"
            Stock Beta_with_intercept Beta_no_intercept Difference
ge.MKT_rp      ge           1.2400877          1.165559 0.07452914
ibm.MKT_rp    ibm           1.0690254          1.003191 0.06583479
ford.MKT_rp  ford           1.7541663          1.685512 0.06865424
msft.MKT_rp  msft           1.2939753          1.224665 0.06931062
dis.MKT_rp    dis           1.1036563          1.031614 0.07204208
xom.MKT_rp    xom           0.5486564          0.481868 0.06678842
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