1.

$K=2 \quad Y = X\beta + e \quad Y = (y_1 \cdots\cdots y_N)'$

$\beta = (\beta_1, \beta_2)' \quad e = (e_1 \cdots\cdots e_N)' \quad e \sim N(0, \sigma^2 I)$

$$X = \begin{bmatrix} 1 & X_1 \\ \vdots & \vdots \\ 1 & X_N \end{bmatrix} \quad X' = \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ X_1 & X_2 & \cdots & & X_N \end{bmatrix}$$

$\beta_2 = (X'X)^{-1}(X'Y)$

$$X'X = \begin{bmatrix} 1 & 1 & 1 & 1 & \cdots & 1 \\ X_1 & X_2 & \cdots & & & X_N \end{bmatrix} \begin{bmatrix} 1 & X_1 \\ \vdots & \vdots \\ 1 & X_N \end{bmatrix} = \begin{bmatrix} N & \Sigma X_i \\ \Sigma X_i & \Sigma X_i^2 \end{bmatrix}$$

$$(X'X)^{-1} = \frac{1}{N\Sigma X_i^2 - (\Sigma X_i)^2} \begin{bmatrix} \Sigma X_i^2 & -\Sigma X_i \\ -\Sigma X_i & N \end{bmatrix}$$

$$X'Y = \begin{bmatrix} 1 & 1 & 1 & \cdots\cdots & 1 \\ X_1 & X_2 & \cdots\cdots & & X_N \end{bmatrix} \begin{bmatrix} y_1 \\ \vdots \\ y_N \end{bmatrix}$$

$$= \begin{bmatrix} \Sigma y_i \\ \Sigma X_i y_i \end{bmatrix}$$

$$\beta = (X'X)^{-1}(X'Y)$$

$$= \frac{1}{N\sum X_i^2 - (\sum X_i)^2} \begin{bmatrix} \sum X_i^2 & -\sum X_i \\ -\sum X_i & N \end{bmatrix} \begin{bmatrix} \sum y_i \\ \sum X_i y_i \end{bmatrix}$$

$$= \frac{1}{N\sum X_i^2 - (\sum X_i)^2} \begin{bmatrix} \sum X_i^2 \sum y_i - \sum X_i \sum X_i y_i \\ -\sum X_i \sum y_i + N\sum X_i y_i \end{bmatrix}$$

$$= \begin{bmatrix} \dfrac{\sum X_i^2 \sum y_i - \sum X_i \sum X_i y_i}{N\sum X_i^2 - (\sum X_i)^2} \\ \dfrac{N\sum X_i y_i - \sum X_i \sum y_i}{N\sum X_i^2 - (\sum X_i)^2} \end{bmatrix}$$

$$\beta_1 = \frac{\sum X_i^2 \sum Y_i - \sum X_i \sum X_i Y_i}{N\sum X_i^2 - (\sum X_i)^2}$$

$$= \frac{\bar{Y}\sum X_i^2 - \bar{X}\sum X_i Y_i}{\sum X_i^2 - n\bar{X}^2}$$

$$= \frac{\bar{Y}(\sum X_i^2 - n\bar{X}^2)}{\sum X_i^2 - n\bar{X}^2} - \frac{\bar{X}(\sum X_i Y_i - n\bar{X}\bar{Y})}{\sum X_i^2 - n\bar{X}^2}$$

$$= \bar{Y} - \hat{\beta}_1 \bar{X}$$

$$\beta_2 = \frac{N\sum X_i Y_i - \sum X_i \sum Y_i}{N\sum X_i^2 - (\sum X_i)^2} = \frac{\sum X_i Y_i - n\bar{X}\bar{Y}}{\sum X_i^2 - n\bar{X}^2}$$

2.

已知 $(X'X)^{-1} = \frac{1}{N\Sigma X_i^2 - (\Sigma X_i)^2} \begin{bmatrix} \Sigma X_i^2 & -\Sigma X_i \\ -\Sigma X_i & N \end{bmatrix}$

$b \sim N(\beta, \sigma^2 (X'X)^{-1})$

$Var(b) = \sigma^2 (X'X)^{-1}$

$= \sigma^2 \times \frac{1}{N\Sigma X_i^2 - (\Sigma X_i)^2} \begin{bmatrix} \Sigma X_i^2 & -\Sigma X_i \\ -\Sigma X_i & N \end{bmatrix}$

$Var(\beta_1) = \sigma^2 \frac{\Sigma X_i^2}{N\Sigma X_i^2 - (\Sigma X_i)^2} = \sigma^2 \frac{\Sigma X_i^2}{N\Sigma (X_i - \bar{X})^2}$

$Var(\beta_2) = \sigma^2 \frac{N}{N\Sigma X_i^2 - (\Sigma X_i)^2} = \frac{\sigma^2}{\Sigma (X_i - \bar{X})^2}$

$cov(\beta_1, \beta_2) = \sigma^2 \frac{-\Sigma X_i}{N\Sigma X_i^2 - (\Sigma X_i)^2} = \frac{-\sigma^2 \Sigma X_i}{N\Sigma (X_i - \bar{X})^2} = \frac{-\sigma^2 \bar{X}}{\Sigma (X_i - \bar{X})^2}$

5.03

a、

i、 t-stastic $= \frac{1.4515}{2.2019} = 0.6592$

ii. t-stastic for $\beta_2 = \frac{2.7648}{SE(\beta_2)} = 5.7103$

$SE(\beta_2) = 0.4842$

iii. t-stastic for $\beta_3 = \frac{\beta_3}{0.3695} = -3.9376$

$\beta_3 = -1.4549$

iv. $SST = (n-1) \times Var(Y) = 1199 \times 6.39547^2 = 49041.5479$

$SSE = 46221.62$

$R^2 = 1 - \frac{SSR}{SST} = 1 - \frac{46221.62}{49041.5479} = 0.0575 = 5.75\%$

v、 $\hat{\sigma} = \sqrt{\frac{46221.62}{1200-4}} = 6.2167$

b、

b_2：在其他條件不變下，當 total expenditure 增加 1% 以平均來說，家庭預算對酒的支出會增加 0.014515 單位

b_3：在其他條件不變下，當家庭嬰兒數增加 1 位，以平均來說，家庭預算對酒的支出會減少 1.4549 單位 ($\because \beta_3 < 0$)

b_4：在其他條件不變下，當屋主年齡增加 1 歲，以平均來說家庭預算對酒的支出會減少 0.1503 單位 ($\because \beta_4 < 0$)

c、

95% C.I for β_4：$[-0.1503 \pm t_{0.025}(1200-4) \times 0.0235]$

$\because t_{0.025}(1196) \doteqdot z_{0.025} = 1.96$

95% C.I for β_4：$[-0.19636, -0.10424]$

在 95% 信心水準下，β_4 會落在這區間

d、

No, ∵ C 的 p-value > 0.05 ⇒ insignificant

e、

$H_0: \beta_3 = -2$ vs. $\beta_3 \neq -2$

$\varphi = \frac{\hat{\beta}_3 - (-2)}{SE(\hat{\beta}_3)} \sim t(1200-4)$ $t_{0.025}(1196) \doteq z_{0.025} = 1.96$

$\varphi^* = \frac{-1.4549+2}{0.3695} = 1.4752$

Do not reject the null hypothesis, there is no enough evidence to support that $\beta_3 \neq -2$

5.23

a.

$\beta_2 < 0$

$\beta_3 > 0$

$\beta_4 < 0$

b.

```
> model1 <- lm(price~quant+qual+trend, cocaine)
> summary(model1)

Call:
lm(formula = price ~ quant + qual + trend, data = cocaine)

Residuals:
    Min      1Q  Median      3Q     Max 
-43.479 -12.014  -3.743  13.969  43.753 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 90.84669    8.58025  10.588 1.39e-14 ***
quant       -0.05997    0.01018  -5.892 2.85e-07 ***
qual         0.11621    0.20326   0.572   0.5700    
trend       -2.35458    1.38612  -1.699   0.0954 .  
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 20.06 on 52 degrees of freedom
Multiple R-squared:  0.5097,	Adjusted R-squared:  0.4814 
F-statistic: 18.02 on 3 and 52 DF,  p-value: 3.806e-08
```

c.

proportion $= R^2 = 50.97\%$

d.

$H_0: \beta_2 \geq 0$ vs. $H_1: \beta_2 < 0$

$\varphi = \frac{\hat{\beta}_2 - 0}{SE(\hat{\beta}_2)} \sim t_\alpha(52)$ set $\alpha = 0.05$ $t_{0.05}(52) \doteqdot z_{0.05} = 1.645$

$\varphi^* = -5.892 < -1.645$

Reject the null hypothesis. There is enough evidence to support that $\beta_2 < 0$

e.

$H_0: \beta_3 = 0$ vs. $H_1: \beta_3 > 0$

$\varphi = \frac{\hat{\beta}_3 - 0}{SE(\hat{\beta}_3)} \sim t_\alpha(52)$ set $\alpha = 0.05$ $t_{0.05}(52) \doteqdot z_{0.05} = 1.645$

$\varphi^* = 0.572 < 1.645$

Do not reject the null hypothesis. There is no enough evidence to support that $\beta_3 > 0$

f.

控制其他條件不變下，每年價格下降2.3546元