11.28 Supply and demand curves as traditionally drawn in economics principles classes have price (P) on the vertical axis and quantity (Q) on the horizontal axis.

a. Rewrite the truffle demand and supply equations in (11.11) and (11.12) with price P on the left-hand side. What are the anticipated signs of the parameters in this rewritten system of equations?

b. Using the data in the file *truffles*, estimate the supply and demand equations that you have formulated in (a) using two-stage least squares. Are the signs correct? Are the estimated coefficients significantly different from zero?

c. Estimate the price elasticity of demand "at the means" using the results from (b).

d. Accurately sketch the supply and demand equations, with P on the vertical axis and Q on the horizontal axis, using the estimates from part (b). For these sketches set the values of the exogenous variables DI, PS, and PF to be $DI^* = 3.5$, $PF^* = 23$, and $PS^* = 22$.

e. What are the equilibrium values of P and Q obtained in part (d)? Calculate the predicted equilibrium values of P and Q using the estimated reduced-form equations from Table 11.2, using the same values of the exogenous variables. How well do they agree?

f. Estimate the supply and demand equations that you have formulated in (a) using OLS. Are the signs correct? Are the estimated coefficients significantly different from zero? Compare the results to those in part (b).

(a)

(a). Demand equation: $Q_i = \alpha_1 + \alpha_2 P_i + \alpha_3 PS_i + \alpha_4 DI_i + e_{di}$

$$\Rightarrow P = \frac{1}{\alpha_2}(Q - \alpha_1 + \alpha_3 PS_i + \alpha_4 DI_i + e_{di})$$

$$= \underline{\delta_1 + \delta_2 Q + \delta_3 PS + \delta_4 DI + u_d}$$

預期 $\delta_2 < 0$ 需求和數量成反比（需求法則）

$\delta_3 > 0$ 若替代品價格上升，Truffles需求上升、其價格上升。

$\delta_4 > 0$ 可支配所得 $DI\uparrow$，若Truffles為正常財、需求上升、價格上升。

Supply equation: $Q_i = \beta_1 + \beta_2 P_i + \beta_3 PF_i + e_{si}$

$$\Rightarrow P = \frac{1}{\beta_2}(Q - \beta_1 + \beta_3 PF + e_{si})$$

$$= \underline{\phi_1 + \phi_2 Q + \phi_3 PF + u_s}$$

預期 $\phi_2 > 0$ 價格和數量成正比

$\phi_3 > 0$ 若生產成本上升、價格上升

(b)

```
> summary(demand_2sls, diagnostics = TRUE)

Call:
ivreg(formula = p ~ q + ps + di | ps + di + pf, data = truffles)

Residuals:
    Min      1Q  Median      3Q     Max
-39.661  -6.781   2.410   8.320  20.251

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  -11.428     13.592  -0.841  0.40810
q             -2.671      1.175  -2.273  0.03154 *
ps             3.461      1.116   3.103  0.00458 **
di            13.390      2.747   4.875 4.68e-05 ***

Diagnostic tests:
                 df1 df2 statistic  p-value
Weak instruments   1  26     17.48 0.000291 ***
Wu-Hausman         1  25    120.03 4.92e-11 ***
Sargan             0  NA        NA       NA
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 13.17 on 26 degrees of freedom
Multiple R-Squared: 0.5567,	Adjusted R-squared: 0.5056
Wald test: 17.37 on 3 and 26 DF,  p-value: 2.137e-06
```

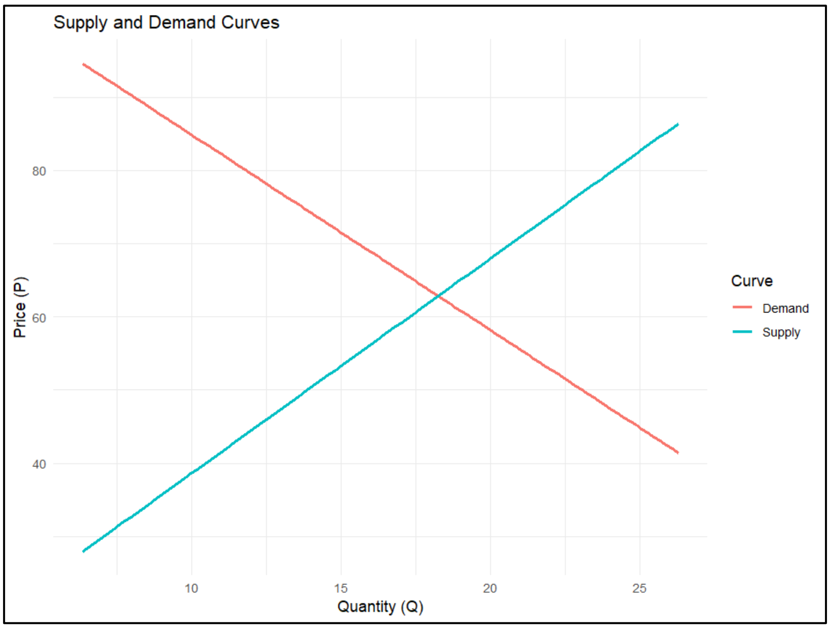
```
> summary(supply_2sls, diagnostics = TRUE)

Call:
ivreg(formula = p ~ q + pf | di + ps + pf, data = truffles)

Residuals:
    Min      1Q  Median      3Q     Max
-9.7983 -2.3440 -0.6281  2.4350 11.1600

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) -58.7982     5.8592  -10.04 1.32e-10 ***
q             2.9367     0.2158   13.61 1.32e-13 ***
pf            2.9585     0.1560   18.97  < 2e-16 ***

Diagnostic tests:
                 df1 df2 statistic  p-value
Weak instruments   2  26    28.934 2.44e-07 ***
Wu-Hausman         1  26     7.051   0.0134 *
Sargan             1  NA     1.544   0.2140
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 4.399 on 27 degrees of freedom
Multiple R-Squared: 0.9486,	Adjusted R-squared: 0.9448
Wald test: 232.7 on 2 and 27 DF,  p-value: < 2.2e-16
```

(c)

$$\varepsilon_D = \frac{\%\Delta Q}{\%\Delta P} = \frac{\Delta Q/\overline{Q}}{\Delta P/\overline{P}} = \frac{1}{\delta_2} \times \frac{\overline{P}}{\overline{Q}}$$

```
> cat("平均價格 P =", P_at_means, "\n")
平均價格 P = 62.724
> cat("平均數量 Q =", means["q"], "\n")
平均數量 Q = 18.45833
>
> # 計算需求價格彈性
> delta_2 <- coefs["q"]
> elasticity <- (1 / delta_2) * (P_at_means / means["q"])
>
> # 印出彈性結果
> cat("需求價格彈性 =", elasticity, "\n")
需求價格彈性 = -1.272464
```

(d)



(e)

TABLE 11.2a Reduced Form for Quantity of Truffles (*Q*)

Variable	Coefficient	Std. Error	*t*-Statistic	Prob.
C	7.8951	3.2434	2.4342	0.0221
PS	0.6564	0.1425	4.6051	0.0001
DI	2.1672	0.7005	3.0938	0.0047
PF	−0.5070	0.1213	−4.1809	0.0003

TABLE 11.2b Reduced Form for Price of Truffles (*P*)

Variable	Coefficient	Std. Error	*t*-Statistic	Prob.
C	−32.5124	7.9842	−4.0721	0.0004
PS	1.7081	0.3509	4.8682	0.0000
DI	7.6025	1.7243	4.4089	0.0002
PF	1.3539	0.2985	4.5356	0.0001

如下圖計算，兩種方法計算出的均衡數量分別是 18.2503 和 18.2604，均衡價格分別是 62.8427 與 62.8154，差異非常小，幾乎相等。

1. 供需方程式交點求均衡

- 利用供給與需求方程式，並且將外生變數（$DI = 3.5, PF = 23, PS = 22$）代入後，將兩個方程式相等得到關於均衡數量 Q_{EQM} 的方程式：

$$111.5801 - 2.6705Q_{EQM} = 9.2470 + 2.9367Q_{EQM}$$

- 解出均衡數量：

$$Q_{EQM} = 18.2503$$

- 把此數量帶入需求方程式計算均衡價格：

$$P_{EQM} = 111.5801 - 2.6705 \times 18.2503 = 62.8427$$

2. 利用簡約式估計均衡

- 直接用簡約式的係數和固定外生變數計算均衡數量與價格：

$$Q_{EQM\_RF} = 7.8951 + 0.6564 \times 22 + 2.1672 \times 3.5 - 0.5070 \times 23 = 18.2604$$

$$P_{EQM\_RF} = -32.5124 + 1.7081 \times 22 + 7.6025 \times 3.5 + 1.3539 \times 23 = 62.8154$$

(f)

```
> summary(demand_ols)

Call:
lm(formula = p ~ q + ps + di, data = truffles)

Residuals:
     Min       1Q   Median       3Q      Max 
-25.0753  -2.7742  -0.4097   4.7079  17.4979 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) -13.6195     9.0872  -1.499   0.1460    
q             0.1512     0.4988   0.303   0.7642    
ps            1.3607     0.5940   2.291   0.0303 *  
di           12.3582     1.8254   6.770 3.48e-07 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 8.814 on 26 degrees of freedom
Multiple R-squared:  0.8013,	Adjusted R-squared:  0.7784 
F-statistic: 34.95 on 3 and 26 DF,  p-value: 2.842e-09
```

```
> summary(supply_ols)

Call:
lm(formula = p ~ q + pf, data = truffles)

Residuals:
    Min      1Q  Median      3Q     Max 
-8.4721 -3.3287  0.1861  2.0785 10.7513 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) -52.8763     5.0238  -10.53 4.68e-11 ***
q             2.6613     0.1712   15.54 5.42e-15 ***
pf            2.9217     0.1482   19.71  < 2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 4.202 on 27 degrees of freedom
Multiple R-squared:  0.9531,	Adjusted R-squared:  0.9496 
F-statistic: 274.4 on 2 and 27 DF,  p-value: < 2.2e-16
```

需求方程式中，使用 OLS 模型下 Q 的係數正，代表價格與需求量存在正向關係，違反經濟學理論。與 2SLS 的結果相比，其 Q 的係數為負，OLS 模型的截距項與替代品價格 PS 的係數值也更小。

供給方程式中，所有估計值皆顯著不同於零，兩種模型下係數正負號皆相同，且差異不大。

11.30 Example 11.3 introduces Klein's Model I. Use the data file *klein* to answer the following questions.

a. Estimate the investment function in equation (11.18) by OLS. Comment on the signs and significance of the coefficients.

b. Estimate the reduced-form equation for profits, P_t, using all eight exogenous and predetermined variables as explanatory variables. Test the joint significance of all the variables except lagged profits, P_{t-1}, and lagged capital stock, K_{t-1}. Save the residuals, $\hat{v}_t$ and compute the fitted values, $\hat{P}_t$.

c. The Hausman test for the presence of endogenous explanatory variables is discussed in Section 10.4.1. It is implemented by adding the reduced-form residuals to the structural equation and testing their significance, that is, using OLS estimate the model

$$I_t = \beta_1 + \beta_2 P_t + \beta_3 P_{t-1} + \beta_4 K_{t-1} + \delta \hat{v}_t + e_{2t}$$

Use a t-test for the null hypothesis $H_0: \delta = 0$ versus $H_1: \delta \neq 0$ at the 5% level of significance. By rejecting the null hypothesis, we conclude that P_t is endogenous. What do we conclude from the test? In the context of this simultaneous equations model what result should we find?

d. Obtain the 2SLS estimates of the investment equation using all eight exogenous and predetermined variables as IVs and software designed for 2SLS. Compare the estimates to the OLS estimates in part (a). Do you find any important differences?

e. Estimate the second-stage model $I_t = \beta_1 + \beta_2 \hat{P}_t + \beta_3 P_{t-1} + \beta_4 K_{t-1} + e_{2t}$ by OLS. Compare the estimates and standard errors from this estimation to those in part (d). What differences are there?

f. Let the 2SLS residuals from part (e) be $\hat{e}_{2t}$. Regress these residuals on all the exogenous and predetermined variables. If these instruments are valid, then the R^2 from this regression should be low, and none of the variables are statistically significant. The Sargan test for instrument validity is discussed in Section 10.4.3. The test statistic TR^2 has a chi-square distribution with degrees of freedom equal to the number of "surplus" IVs if the surplus instruments are valid. The investment equation includes three exogenous and/or predetermined variables out of the total of eight possible. There are $L = 5$ external instruments and $B = 1$ right-hand side endogenous variables. Compare the value of the test statistic to the 95th percentile value from the $\chi^2_{(4)}$ distribution. What do we conclude about the validity of the surplus instruments in this case?

(a)

```
> summary(investment_ols)

Call:
lm(formula = i ~ p + plag + klag, data = klein)

Residuals:
     Min       1Q   Median       3Q      Max 
-2.56562 -0.63169  0.03687  0.41542  1.49226 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 10.12579    5.46555   1.853 0.081374 .  
p            0.47964    0.09711   4.939 0.000125 ***
plag         0.33304    0.10086   3.302 0.004212 ** 
klag        -0.11179    0.02673  -4.183 0.000624 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.009 on 17 degrees of freedom
  (因為不存在，1 個觀察量被刪除了)
Multiple R-squared:  0.9313,	Adjusted R-squared:  0.9192 
F-statistic: 76.88 on 3 and 17 DF,  p-value: 4.299e-10
```

投資函數中，利潤（當年 p 及前一年 plag）對投資有正向顯著影響，符合經濟理論預期。前一年資本存量 klag 的係數為負且顯著，顯示資本存量越大，新增投資越少，可能反映投資替代效應或資本折舊影響。

(b)

```
> summary(rf_P)

Call:
lm(formula = p ~ g + w2 + tx + time + plag + klag + elag, data = df)

Residuals:
    Min      1Q  Median      3Q     Max 
-3.9067 -1.3050  0.3226  1.3613  2.8881 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)  
(Intercept) 50.38442   31.63026   1.593   0.1352  
g            0.43902    0.39114   1.122   0.2820  
w2          -0.07961    2.53382  -0.031   0.9754  
tx          -0.92310    0.43376  -2.128   0.0530 .
time         0.31941    0.77813   0.410   0.6881  
plag         0.80250    0.51886   1.547   0.1459  
klag        -0.21610    0.11911  -1.814   0.0928 .
elag         0.02200    0.28216   0.078   0.9390  
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 2.183 on 13 degrees of freedom
Multiple R-squared:  0.8261,	Adjusted R-squared:  0.7324 
F-statistic: 8.821 on 7 and 13 DF,  p-value: 0.0004481
```

```
> # 聯合假設檢定 (除plag, klag外的五個變數係數是否全為0)
> joint_test <- linearHypothesis(
+     rf_P,
+     c("g = 0", "w2 = 0", "tx = 0", "time = 0", "elag = 0")
+ )
>
> cat("\nJoint F-test for (g, w2, tx, time, elag):\n")

Joint F-test for (g, w2, tx, time, elag):
> print(joint_test)

Linear hypothesis test:
g = 0
w2 = 0
tx = 0
time = 0
elag = 0

Model 1: restricted model
Model 2: p ~ g + w2 + tx + time + plag + klag + elag

  Res.Df    RSS Df Sum of Sq      F Pr(>F)
1     18 108.04                           
2     13  61.95  5    46.093 1.9345 0.1566
```

```
> # 設定自由度與信心水準
> df1 <- 5       # 分子自由度：5個限制
> df2 <- 13      # 分母自由度：殘差自由度13
> alpha <- 0.05 # 95%信心水準
>
> # 計算F分布的臨界值
> F_critical <- qf(1 - alpha, df1, df2)
> cat("F檢定的臨界值 (α=0.05):", round(F_critical, 3), "\n")
F檢定的臨界值 (α=0.05): 3.025
```

虛無假設：g=w2=tx=time=elag=0，因為 f=1.9345<3.025，故無法拒絕虛無假設。代表這五個變數對利潤的影響不顯著。

(c) Hausman 檢定

對 Pt 建立簡約式模型（reduced form），用外生與預定變數去解釋 Pt，取出殘差 $\hat{v}_t$，將這個殘差加入原本的結構方程式中做迴歸。檢驗該殘差的係數 δ 是否顯著（用 t 檢定），故虛無假設為 δ=0，代表 Pt 是外生的；替代假設 δ 不等於 0，代表 Pt 是內生的。由下圖可發現$\hat{v}_t$的係數為 0.5745，t 值為 4.029 非常顯著，故拒絕虛無假設，Pt 為內生的。

```
> summary(hausman_test)

Call:
lm(formula = i ~ p + plag + klag + vhat, data = df)

Residuals:
     Min       1Q   Median       3Q      Max 
-1.04645 -0.56030  0.06189  0.25348  1.36700 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 20.27821    4.70179   4.313 0.000536 ***
p            0.15022    0.10798   1.391 0.183222    
plag         0.61594    0.10147   6.070 1.62e-05 ***
klag        -0.15779    0.02252  -7.007 2.96e-06 ***
vhat         0.57451    0.14261   4.029 0.000972 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.7331 on 16 degrees of freedom
Multiple R-squared:  0.9659,	Adjusted R-squared:  0.9574 
F-statistic: 113.4 on 4 and 16 DF,  p-value: 1.588e-11
```

(d)

```
> summary(iv_model)

Call:
ivreg(formula = i ~ p + plag + klag | g + w2 + tx + time + elag + 
    plag + klag, data = df)

Residuals:
    Min      1Q  Median      3Q     Max 
-3.2909 -0.8069  0.1423  0.8601  1.7956 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)   
(Intercept) 20.27821    8.38325   2.419  0.02707 * 
p            0.15022    0.19253   0.780  0.44598   
plag         0.61594    0.18093   3.404  0.00338 **
klag        -0.15779    0.04015  -3.930  0.00108 **
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 1.307 on 17 degrees of freedom
Multiple R-Squared: 0.8849,	Adjusted R-squared: 0.8646 
Wald test:  41.2 on 3 and 17 DF,  p-value: 5.148e-08
```

(e)

在第二階段 OLS 中，我們用第一階段所得到的 p_hat 替代原始的 p，再次估計投資方程式。估計出來的係數與(d)題中 ivreg()的結果非常接近，但標準誤在此模型中未經調整故有所不同。

```
> summary(second_stage)

Call:
lm(formula = i ~ phat + plag + klag, data = df)

Residuals:
    Min      1Q  Median      3Q     Max 
-3.8778 -1.0029  0.3058  0.7275  2.1831 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)   
(Intercept) 20.27821    9.97663   2.033  0.05802 . 
phat         0.15022    0.22913   0.656  0.52084   
plag         0.61594    0.21531   2.861  0.01083 * 
klag        -0.15779    0.04778  -3.302  0.00421 **
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 1.556 on 17 degrees of freedom
Multiple R-squared:   0.837,	Adjusted R-squared:  0.8082 
F-statistic: 29.09 on 3 and 17 DF,  p-value: 6.393e-07
```

(f)

L=5 個外部工具變數

B=1 個右側內生變數（即 p）

有 L−B=4 個剩餘工具變數，作為檢定自由度。

以 2SLS 的殘差作為應變數，對所有的外生與預定變數進行回歸。如果這些工具變數是有效的，那麼這個回歸的 R^2 應該要很低。

$TR^2 < X^2_{0.95,4}$，故工具變數與誤差項無關，有效。

```
> cat("Sargan 檢定統計量 TR^2 =", round(TR2, 3), "\n")
Sargan 檢定統計量 TR^2 = 1.815
> cat("95% 卡方臨界值 χ²(4) =", round(crit_val, 3), "\n")
95% 卡方臨界值 χ²(4) = 9.488
>
> if(TR2 > crit_val){
+     cat("結論：拒絕虛無假設，工具變數可能無效。\n")
+ } else {
+     cat("結論：無法拒絕虛無假設，工具變數有效。\n")
+ }
結論：無法拒絕虛無假設，工具變數有效。
```