HW 0421

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(a) $E(x) = \gamma_1 + \theta_1 E(z)$ from $x = \gamma_1 + \theta_1 z + v$ to obtain $x - E(x) = \theta_1 [z - E(z)] + v$

Multiply by $[z - E(z)]$

$\Rightarrow [z - E(z)][x - E(x)] = \theta_1 [z - E(z)]^2 + [z - E(z)] v$

$\Rightarrow E[(z - E(z))(x - E(x))] = \theta_1 E[(z - E(z))^2] + E(z - E(z))$

Assuming : $E(z - E(z)) v = 0$

$$\Rightarrow \theta_1 = \frac{E[(z - E(z))(x - E(x))]}{E[(z - E(z))^2]} = \frac{Cov(z, x)}{Var(z)}$$

$\Rightarrow x = \gamma_1 + \theta_1 z + v.$

b. Subtract $E(y) = \pi_0 + \pi_1 E(z)$ from $y = \pi_0 + \pi_1 z + u$

$\Rightarrow Y - E(Y) = \pi_1 (Z - E(Z)) + u.$

$\Leftrightarrow (z - E(z))(y - E(y)) = \pi_1 (z - E(z))^2 + (z - E(z)) u$

Assuming $E(z - E(z)) u = 0$,

$\Leftrightarrow E[(z - E(z))(y - E(y))] = \pi_1 E[(z - E(z))^2]$

Solving for π_1 we have : $\pi_1 = \dfrac{E[(z - E(z))(y - E(y))]}{E[(z - E(z))^2]} = \dfrac{Cov(z, y)}{Var(z)}$

c. $y = \beta_1 + \beta_2 + e = \beta_1 + \beta_2(\gamma_1 + \theta_{12} + v) + e$

$\rightarrow (\beta_1 + \beta_2\gamma_1) + \beta_2\theta_{12} + \beta_2 v + e)$

$\pi_0 = \beta_1 + \beta_2\gamma_1$, $\pi_1 = \beta_2\theta_1$, $u = \beta_2 v + e$

d. (c) $\pi_1 = \beta_2\theta_1$

$\rightarrow$ solving $\pi_1 = \beta_2\theta_1$ for β_2, we have $\beta_2 = \dfrac{\pi_1}{\theta_1}$

(e) from (a), $\hat{\theta}_1 = \dfrac{\widehat{Cov(z,x)}}{\widehat{Var(z)}} = \dfrac{\sum_i (z_i - \bar{z})(x_i - \bar{x})/N}{\sum_i (z_i - \bar{z})^2/N}$

$$\hat{\pi}_1 = \frac{\widehat{Cov(z,y)}}{\widehat{Var(z)}} = \frac{\sum_i (z_i - \bar{z})(y_i - \bar{y})/N}{\sum_i (z_i - \bar{z})^2/N}$$

A consistent estimator if z is uncorrelated with u

$$\rightarrow \hat{\beta}_2 = \hat{\pi}_1/\hat{\theta}_1 = \frac{\sum_i (z_i - \bar{z})(y_i - \bar{y})}{\sum_i (z_i - \bar{z})(x_i - \bar{x})} = \frac{\widehat{Cov(z,y)}}{\widehat{Cov(z,x)}}$$

$$\widehat{Cov(z,y)} \xrightarrow{P} Cov(z,y) \text{ \& } \widehat{Cov(z,x)} \xrightarrow{P} Cov(z,x)$$

$$\rightarrow \hat{\beta}_2 = \hat{\pi}_1/\hat{\theta}_1 = \frac{\widehat{Cov(z,y)}}{\widehat{Cov(z,x)}} \xrightarrow{P} \frac{Cov(z,y)}{Cov(z,x)} = \beta_2$$

C10Q02

β_2 (WAGE) (+) : $\nearrow$ wage $\rightarrow$ incentive for women to supply more labour hours)

β_3 (EDUC) (+) $\nearrow$ opportunities $\rightarrow$ more labour supply.

β_4 (AGE) : Uncertain

β_5 (KIDSL6) (−) taking care childrend $\rightarrow$ less time)

β_6 (NWIFEINC) (-) other sources → ↓ supply labor.

b. Explain: Endogeneity issues arising from

* Simultaneity: WAGE may be influence labour suply.

→ Violate OLS assumption.

* Omitted variable.

* measurement error.

c). EXPER & $EXPER^2$ can be valid instruments.

1. Relevance: both are expected to be positively corr with WAGE.

2. Exogeneity. Neither EXPER nor $EXPER^2$ should have a effect on hours beyond their influence on WAGE

3. Independence.

d., yes, the supply equation is identified

e) - First-stage $WAGE = \gamma_0 + \gamma_1 EXPER + \gamma_2 EXPEX^2 + \ldots$

- Obtain fitted values
- Second stage regression.

$$HOURS = \beta_0 + \beta_1 \widehat{WAGE} + \beta_2 EDU + \ldots$$

- Esess model fit
- Interprest results.