10.20

The CAPM [see Exercises 10.14 and 2.16] says that the risk premium on security j is related to the risk premium on the market portfolio. That is

$$r_j - r_f = \alpha_j + \beta_j(r_m - r_f)$$

where r_j and r_f are the returns to security j and the risk–free rate, respectively, r_m is the return on the market portfolio, and β_j is the jth security's "beta" value. We measure the market portfolio using the Standard & Poor's value weighted index, and the risk–free rate by the 30–day LIBOR monthly rate of return. As noted in Exercise 10.14, if the market return is measured with error, then we face an errors–in–variables, or measurement error, problem.

a. Use the observations on Microsoft in the data file $capm5$ to estimate the CAPM model using OLS. How would you classify the Microsoft stock over this period? Risky or relatively safe, relative to the market portfolio?

   Ans. Microsoft beta = 1.20，表示股票風險高於市場平均（高風險）。

```
Call:
lm(formula = excess_msft ~ excess_mkt, data = capm5)

Residuals:
     Min       1Q   Median       3Q      Max
-0.27424 -0.04744 -0.00820  0.03869  0.35801

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 0.003250   0.006036   0.538    0.591
excess_mkt  1.201840   0.122152   9.839   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.08083 on 178 degrees of freedom
Multiple R-squared:  0.3523,    Adjusted R-squared:  0.3486
F-statistic:  96.8 on 1 and 178 DF,  p-value: < 2.2e-16
```

b. It has been suggested that it is possible to construct an IV by ranking the values of the explanatory variable and using the rank as the IV, that is, we sort $(r_m - r_f)$ from smallest to largest, and assign the values $RANK = 1,2,...,180$. Does this variable potentially satisfy the conditions IV1–IV3? Create $RANK$ and obtain the first–stage regression results. Is the coefficient of $RANK$ very significant? What is the R^2 of the first–stage regression? Can $RANK$ be regarded as a strong IV?

   Ans.

   變數 $RANK$ 對 Microsoft's return 沒有直接相關，符合 IV1。

   $R^2 = 0.9126$，與 excess_mkt 有極強的相關性。

   t value = 43.1（非常高）。F 統計量為 1858，遠超過 10 的常用門檻。

   因此 $RANK$ 可以被視為強工具變數。

```
Call:
lm(formula = excess_mkt ~ RANK, data = capm5)

Residuals:
      Min        1Q    Median        3Q       Max
-0.110497 -0.006308  0.001497  0.009433  0.029513

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept) -7.903e-02  2.195e-03   -36.0   <2e-16 ***
RANK         9.067e-04  2.104e-05    43.1   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.01467 on 178 degrees of freedom
Multiple R-squared:  0.9126,    Adjusted R-squared:  0.9121
F-statistic:  1858 on 1 and 178 DF,  p-value: < 2.2e-16
```

```
Linear hypothesis test:
RANK = 0

Model 1: restricted model
Model 2: excess_mkt ~ RANK

  Res.Df     RSS Df Sum of Sq      F    Pr(>F)
1    179 0.43784
2    178 0.03829  1   0.39955 1857.6 < 2.2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

c. **Compute the first-stage residuals, $\hat{v}$, and add them to the CAPM model. Estimate the resulting augmented equation by OLS and test the significance of $\hat{v}$ at the 1% level of significance. Can we conclude that the market return is exogenous?**

Ans.

$\hat{v}$ 的 t 值為 −2.040、p 值為 0.0428 > 0.01（1%顯著水準）

因此，在 1% 顯著水準下，我們無法拒絕虛無假設 $\hat{v}$ 係數 = 0，可以得出結論：市場報酬率可以被視為外生的。

如果採用 5% 顯著水準，結論會不同——在 5% 水準下，$\hat{v}$ 係數是顯著的（p 值 0.0428 < 0.05），這表示市場報酬率可能存在內生性問題。

```
Call:
lm(formula = excess_msft ~ excess_mkt + v_hat, data = capm5)

Residuals:
     Min       1Q   Median       3Q      Max
-0.27140 -0.04213 -0.00911  0.03423  0.34887

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept)  0.003018   0.005984   0.504   0.6146
excess_mkt   1.278318   0.126749  10.085   <2e-16 ***
v_hat       -0.874599   0.428626  -2.040   0.0428 *
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.08012 on 177 degrees of freedom
Multiple R-squared:  0.3672,    Adjusted R-squared:   0.36
F-statistic: 51.34 on 2 and 177 DF,  p-value: < 2.2e-16
```

d. **Use $RANK$ as an IV and estimate the CAPM model by IV/2SLS. Compare this IV estimate to the OLS estimate in part (a). Does the IV estimate agree with your expectations?**

Ans.

IV 估計的 Beta 高於 OLS 估計（1.27 > 1.20），這一結果與預期相符，因為理論上，如果市場報酬率存在測量誤差，則 OLS 估計會低估 Beta 值，而 IV 方法可以糾正這種偏誤。

```
Call:
ivreg(formula = excess_msft ~ excess_mkt | RANK, data = capm5)

Residuals:
      Min        1Q    Median        3Q       Max
-0.271625 -0.049675 -0.009693  0.037683  0.355579

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 0.003018   0.006044   0.499    0.618
excess_mkt  1.278318   0.128011   9.986   <2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.08092 on 178 degrees of freedom
Multiple R-Squared: 0.3508,     Adjusted R-squared: 0.3472
Wald test: 99.72 on 1 and 178 DF,  p-value: < 2.2e-16
```

e. **Create a new variable POS = 1 if the market return $(r_m - r_f)$ is positive, and zero otherwise. Obtain the first-stage regression results using both $RANK$ and POS as instrumental variables. Test the joint significance of the IV. Can we conclude that we have adequately strong IV? What is the R^2 of the first-stage regression?**

Ans.

$R^2 = 0.9149$（非常高）。F 統計量為 951.26，遠高於 10 的臨界值，表明兩個工具變數聯合起來高度顯著。因此 $RANK$、POS 可以被視為強工具變數。

```
Call:
lm(formula = excess_mkt ~ RANK + POS, data = capm5)

Residuals:
      Min        1Q    Median        3Q       Max
-0.109182 -0.006732  0.002858  0.008936  0.026652

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) -0.0804216  0.0022622  -35.55   <2e-16 ***
RANK         0.0009819  0.0000400   24.55   <2e-16 ***
POS         -0.0092762  0.0042156   -2.20   0.0291 *
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.01451 on 177 degrees of freedom
Multiple R-squared:  0.9149,    Adjusted R-squared:  0.9139
F-statistic: 951.3 on 2 and 177 DF,  p-value: < 2.2e-16
```

```
Linear hypothesis test:
RANK = 0
POS = 0

Model 1: restricted model
Model 2: excess_mkt ~ RANK + POS

  Res.Df     RSS Df Sum of Sq      F    Pr(>F)
1    179 0.43784
2    177 0.03727  2   0.40057 951.26 < 2.2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

f. **Carry out the Hausman test for endogeneity using the residuals from the first-stage equation in (e). Can we conclude that the market return is exogenous at the 1% level of significance?**

Ans.

$\hat{v}$ 的 p 值為 0.0287 > 大於 0.01（1%顯著水準）

因此，在1%顯著水準下，我們無法拒絕虛無假設 $\hat{v}$ 係數 = 0，可以得出結論：市場報酬率可以被視為外生的。

```
Call:
lm(formula = excess_msft ~ excess_mkt + v_hat_both, data = capm5)

Residuals:
     Min       1Q   Median       3Q      Max
-0.27132 -0.04261 -0.00812  0.03343  0.34867

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept)  0.003004   0.005972   0.503   0.6157
excess_mkt   1.283118   0.126344  10.156   <2e-16 ***
v_hat_both  -0.954918   0.433062  -2.205   0.0287 *
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.07996 on 177 degrees of freedom
Multiple R-squared:  0.3696,    Adjusted R-squared:  0.3625
F-statistic: 51.88 on 2 and 177 DF,  p-value: < 2.2e-16
```

g. **Obtain the IV/2SLS estimates of the CAPM model using $RANK$ and POS as instrumental variables. Compare this IV estimate to the OLS estimate in part (a). Does the IV estimate agree with your expectations?**

Ans.

IV/2SLS 估計 Beta 高於 OLS 估計（1.283 > 1.20），這一結果與預期相符，因為理論上，如果市場報酬率存在測量誤差，則 OLS 估計會低估 Beta 值，而 IV/2SLS 方法可以糾正這種偏誤。

```
Call:
ivreg(formula = excess_msft ~ excess_mkt | RANK + POS, data = capm5)

Residuals:
     Min       1Q   Median       3Q      Max
-0.27168 -0.04960 -0.00983  0.03762  0.35543

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 0.003004   0.006044   0.497     0.62
excess_mkt  1.283118   0.127866  10.035   <2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.08093 on 178 degrees of freedom
Multiple R-Squared: 0.3507,     Adjusted R-squared: 0.347
Wald test: 100.7 on 1 and 178 DF,  p-value: < 2.2e-16
```

h. **Obtain the IV/2SLS residuals from part (g) and use them (not an automatic command) to carry out a Sargan test for the validity of the surplus IV at the 5% level of significance.**

Ans.

Sargan 檢定 p值 = 0.4549 > 0.05：無法拒絕”所有工具變數都有效"的虛無假設。表明沒有證據顯示工具變數無效。支持工具變數的有效性。

```
Call:
ivreg(formula = excess_msft ~ excess_mkt | RANK + POS, data = capm5)

Residuals:
     Min       1Q   Median       3Q      Max
-0.27168 -0.04960 -0.00983  0.03762  0.35543

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 0.003004   0.006044   0.497     0.62
excess_mkt  1.283118   0.127866  10.035   <2e-16 ***

Diagnostic tests:
                 df1 df2 statistic p-value
Weak instruments   2 177   951.262  <2e-16 ***
Wu-Hausman         1 177     4.862  0.0287 *
Sargan             1  NA     0.558  0.4549
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.08093 on 178 degrees of freedom
Multiple R-Squared: 0.3507,     Adjusted R-squared: 0.347
Wald test: 100.7 on 1 and 178 DF,  p-value: < 2.2e-16
```