3.19 The owners of a motel discovered that a defective product was used during construction. It took 7 months to correct the defects during which approximately 14 rooms in the 100-unit motel were taken out of service for 1 month at a time. The data are in the file *motel*.

a. Plot *MOTEL_PCT* and *COMP_PCT* versus *TIME* on the same graph. What can you say about the occupancy rates over time? Do they tend to move together? Which seems to have the higher occupancy rates? Estimate the regression model $MOTEL\_PCT = \beta_1 + \beta_2 COMP\_PCT + e$. Construct a 95% interval estimate for the parameter β_2. Have we estimated the association between *MOTEL_PCT* and *COMP_PCT* relatively precisely, or not? Explain your reasoning.

b. Construct a 90% interval estimate of the expected occupancy rate of the motel in question, *MOTEL_PCT*, given that $COMP\_PCT = 70$.

c. In the linear regression model $MOTEL\_PCT = \beta_1 + \beta_2 COMP\_PCT + e$, test the null hypothesis $H_0: \beta_2 \leq 0$ against the alternative hypothesis $H_0: \beta_2 > 0$ at the $\alpha = 0.01$ level of significance. Discuss your conclusion. Clearly define the test statistic used and the rejection region.

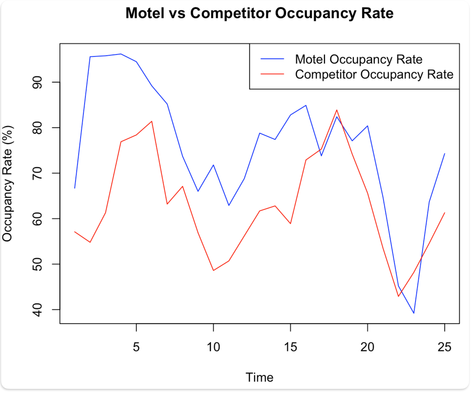
d. In the linear regression model $MOTEL\_PCT = \beta_1 + \beta_2 COMP\_PCT + e$, test the null hypothesis $H_0: \beta_2 = 1$ against the alternative hypothesis $H_0: \beta_2 \neq 1$ at the $\alpha = 0.01$ level of significance. If the null hypothesis were true, what would that imply about the motel's occupancy rate versus their competitor's occupancy rate? Discuss your conclusion. Clearly define the test statistic used and the rejection region.

e. Calculate the least squares residuals from the regression of *MOTEL_PCT* on *COMP_PCT* and plot them against *TIME*. Are there any unusual features to the plot? What is the predominant sign of the residuals during time periods 17–23 (July, 2004 to January, 2005)?

---

a.

入住率的趨勢分析
從圖表來看：
- motel_pct（藍色）和 comp_pct（紅色）大致上有類似的波動趨勢，代表兩者的入住率在時間上的變動具有某種程度的同步性。
- motel_pct 的值通常比 comp_pct 高，表示這間汽車旅館的入住率在大部分時間內優於競爭對手。
- 在某些時期（如時間值約 20 左右），兩者的入住率出現劇烈下降，可能與修繕期間的房間不可用有關。



motel_pct = 21.4 + 0.8646×comp_pct

beta_2 的 95% 信賴區間 = [0.4453, 1.2840]

```
> # 計算 β2 的 95% 置信區間
> confint(model, "comp_pct", level = 0.95)
              2.5 %   97.5 %
comp_pct 0.4452978 1.283981
>
> # 註解：評估 β2 的估計精確度
```

```
Call:
lm(formula = motel_pct ~ comp_pct, data = motel)

Residuals:
    Min      1Q  Median      3Q     Max
-23.876  -4.909  -1.193   5.312  26.818

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  21.4000    12.9069   1.658 0.110889
comp_pct      0.8646     0.2027   4.265 0.000291 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 11.02 on 23 degrees of freedom
Multiple R-squared:  0.4417,    Adjusted R-squared:  0.4174
F-statistic: 18.19 on 1 and 23 DF,  p-value: 0.0002906
```

Given that the standard error of β2 is relatively small compared to its estimate and the p-value is very low (0.000291), we can conclude that the estimate of the relationship between motel and competitor occupancy rates is reasonably precise.

b. C.I. = [77.3822, 87.4673]

```
> # 設定 COMP_PCT = 70
> new_data <- data.frame(comp_pct = 70)
>
> # 預測 MOTEL_PCT 的 90% 信賴區間
> predict(model, newdata = new_data, interval = "confidence", level = 0.90)
       fit      lwr      upr
1 81.92474 77.38223 86.46725
```

c. $H_0: \beta_2 \leq 0$, $H_1: \beta_2 > 0$

$\alpha = 0.01$, $df = n-2 = 25-2 = 23$

$$t = \frac{b_2 - 0}{se(b_2)} = \frac{0.8646 - 0}{0.2027} = 4.2654 > t_{0.99,23} = 2.5000$$

```
> qt(1-0.01, 23)
[1] 2.499867
```

因此，我們拒絕虛無假設 H0，接受對立假設 H1，即：β2 >0
這表示 競爭對手的入住率（COMP_PCT）對汽車旅館入住率（MOTEL_PCT）有顯著的正向影響。
The rejection region is |t| >2.5000. However, t = 4.2654. This value is in the rejection region.

d. $H_0: \beta_2 = 1$, $H_1: \beta_2 \neq 1$

$\alpha = 0.01$, $df = n-2 = 25-2 = 23$, $t_{0.995,23} = 2.8073$

```
> qt(1-0.01/2, 23)
[1] 2.807336
```

$$t = \frac{b_2 - 1}{se(b_2)} = \frac{0.8646 - 1}{0.2027} = -0.6680 \text{ , } -2.8073 < t < 2.8073$$

我們無法拒絕 H0，表示沒有足夠的證據表明 $\beta_2 \neq 1$。
這意味著，數據無法否定「競爭對手的入住率變動與汽車旅館的入住率變動同步（$\beta_2 = 1$）」的假設。
The rejection region is |t| >2.8073 .
Since t = −0.6680, which is in the non-rejection region, , we fail to reject H0 .
For every 1% increase in the competitor's occupancy rate, the motel's occupancy rate also increases by approximately 1%, indicating a proportional relationship.

e. 時間 17–23 的殘差僅有一年為正值，其餘皆為負值。
這段期間的入住率低於預測，與施工期間重疊，可能反映了施工的影響。