**11.1** Our aim is to estimate the parameters of the simultaneous equations model

$$y_1 = \alpha_1 y_2 + e_1$$

$$y_2 = \alpha_2 y_1 + \beta_1 x_1 + \beta_2 x_2 + e_2$$

We assume that $x_1$ and $x_2$ are exogenous and uncorrelated with the error terms $e_1$ and $e_2$.

**a.** Solve the two structural equations for the reduced-form equation for $y_2$, that is, $y_2 = \pi_1 x_1 + \pi_2 x_2 + v_2$. Express the reduced-form parameters in terms of the **structural parameters** and the reduced-form error in terms of the structural parameters and $e_1$ and $e_2$. Show that $y_2$ is correlated with $e_1$.

**b.** Which equation parameters are consistently estimated using OLS? Explain.

**c.** Which parameters are "identified," in the simultaneous equations sense? Explain your reasoning.

**d.** To estimate the parameters of the reduced-form equation for $y_2$ using the method of moments (MOM), which was introduced in Section 10.3, the two moment equations are

$$N^{-1}\sum x_{i1}(y_2 - \pi_1 x_{i1} - \pi_2 x_{i2}) = 0$$

$$N^{-1}\sum x_{i2}(y_2 - \pi_1 x_{i1} - \pi_2 x_{i2}) = 0$$

Explain why these two moment conditions are a valid basis for obtaining consistent estimators of the reduced-form parameters.

**e.** Are the MOM estimators in part (d) the same as the OLS estimators? Form the sum of squared errors function for $y_2 = \pi_1 x_1 + \pi_2 x_2 + v_2$ and find the first derivatives. Set these to zero and show that they are equivalent to the two equations in part (d).

**f.** Using $\sum x_{i1}^2 = 1$, $\sum x_{i2}^2 = 1$, $\sum x_{i1}x_{i2} = 0$, $\sum x_{i1}y_{1i} = 2$, $\sum x_{i1}y_{2i} = 3$, $\sum x_{i2}y_{1i} = 3$, $\sum x_{i2}y_{2i} = 4$, and the two moment conditions in part (d) show that the MOM/OLS estimates of $\pi_1$ and $\pi_2$ are $\hat{\pi}_1 = 3$ and $\hat{\pi}_2 = 4$.

**g.** The fitted value $\hat{y}_2 = \hat{\pi}_1 x_1 + \hat{\pi}_2 x_2$. Explain why we can use the moment condition $\sum \hat{y}_{i2}(y_{i1} - \alpha_1 y_{i2}) = 0$ as a valid basis for consistently estimating $\alpha_1$. Obtain the IV estimate of $\alpha_1$.

**h.** Find the 2SLS estimate of $\alpha_1$ by applying OLS to $y_1 = \alpha_1 \hat{y}_2 + e_1^*$. Compare your answer to that in part (g).

**11.16** Consider the following supply and demand model

$$\text{Demand: } Q_i = \alpha_1 + \alpha_2 P_i + e_{di}, \quad \text{Supply: } Q_i = \beta_1 + \beta_2 P_i + \beta_3 W_i + e_{si}$$

where $Q$ is the quantity, $P$ is the price, and $W$ is the wage rate, which is assumed exogenous. Data on these variables are in Table 11.7.

**TABLE 11.7** Data for Exercise 11.16

| $Q$ | $P$ | $W$ |
|---|---|---|
| 4 | 2 | 2 |
| 6 | 4 | 3 |
| 9 | 3 | 1 |
| 3 | 5 | 1 |
| 8 | 8 | 3 |

**a.** Derive the algebraic form of the reduced-form equations, $Q = \theta_1 + \theta_2 W + v_2$ and $P = \pi_1 + \pi_2 W + v_1$, expressing the reduced-form parameters in terms of the structural parameters.

**b.** Which structural parameters can you solve for from the results in part (a)? Which equation is "identified"?

**c.** The estimated reduced-form equations are $\hat{Q} = 5 + 0.5W$ and $\hat{P} = 2.4 + 1W$. Solve for the identified structural parameters. This is the method of **indirect least squares**.

**d.** Obtain the fitted values from the reduced-form equation for $P$, and apply 2SLS to obtain estimates of the demand equation.

of the demand equation.

**11.17** Example 11.3 introduces Klein's Model I.

- **a.** Do we have an adequate number of IVs to estimate each equation? Check the necessary condition for the identification of each equation. The necessary condition for identification is that in a system of $M$ equations at least $M - 1$ variables must be omitted from each equation.
- **b.** An equivalent identification condition is that the number of excluded exogenous variables from the equation must be at least as large as the number of included right-hand side endogenous variables. Check that this condition is satisfied for each equation.
- **c.** Write down in econometric notation the first-stage equation, the reduced form, for $W_{1t}$, wages of workers earned in the private sector. Call the parameters $\pi_1, \pi_2, \ldots$
- **d.** Describe the two regression steps of 2SLS estimation of the consumption function. This is not a question about a computer software command.
- **e.** Does following the steps in part (d) produce regression results that are identical to the 2SLS estimates provided by software specifically designed for 2SLS estimation? In particular, will the $t$-values be the same?