15.6 Using the NLS panel data on $N = 716$ young women, we consider only years 1987 and 1988. We are interested in the relationship between $\ln(WAGE)$ and experience, its square, and indicator variables for living in the south and union membership. Some estimation results are in Table 15.10.

TABLE 15.10 **Estimation Results for Exercise 15.6**

	(1) OLS 1987	(2) OLS 1988	(3) FE	(4) FE Robust	(5) RE
C	0.9348	0.8993	1.5468	1.5468	1.1497
	(0.2010)	(0.2407)	(0.2522)	(0.2688)	(0.1597)
EXPER	0.1270	0.1265	0.0575	0.0575	0.0986
	(0.0295)	(0.0323)	(0.0330)	(0.0328)	(0.0220)
$EXPER^2$	−0.0033	−0.0031	−0.0012	−0.0012	−0.0023
	(0.0011)	(0.0011)	(0.0011)	(0.0011)	(0.0007)
SOUTH	−0.2128	−0.2384	−0.3261	−0.3261	−0.2326
	(0.0338)	(0.0344)	(0.1258)	(0.2495)	(0.0317)
UNION	0.1445	0.1102	0.0822	0.0822	0.1027
	(0.0382)	(0.0387)	(0.0312)	(0.0367)	(0.0245)
N	716	716	1432	1432	1432

(standard errors in parentheses)

a. The OLS estimates of the $\ln(WAGE)$ model for each of the years 1987 and 1988 are reported in columns (1) and (2). How do the results compare? For these individual year estimations, what are you assuming about the regression parameter values across individuals (heterogeneity)?

OLS 1987 與 1988 係數有些微差異，可能有 heterogeneity

b. The $\ln(WAGE)$ equation specified as a panel data regression model is

$$\ln(WAGE_{it}) = \beta_1 + \beta_2 EXPER_{it} + \beta_3 EXPER_{it}^2 + \beta_4 SOUTH_{it} + \beta_5 UNION_{it} + (u_i + e_{it}) \quad \text{(XR15.6)}$$

Explain any differences in assumptions between this model and the models in part (a).

假設每個隨時間變動的殘差為 u

c. Column (3) contains the estimated fixed effects model specified in part (b). Compare these estimates with the OLS estimates. Which coefficients, apart from the intercepts, show the most difference?

改變最多的係數：

SOUTH（從 -0.2128/-0.2384 到 -0.0361）變化最多，顯示 FE 模型認為 south 很大部分是個體特徵引起的。

UNION（從 0.1445/0.1102 到 0.0127）次之，顯示 union 在 FE 中被大幅削弱。

d. The F-statistic for the null hypothesis that there are no individual differences, equation (15.20), is 11.68. What are the degrees of freedom of the F-distribution if the null hypothesis (15.19) is true? What is the 1% level of significance critical value for the test? What do you conclude about the null hypothesis.

OLS (1) 和 (2)：樣本數 716，每年 5 個參數（常數項 + 4 個變數），自由度 = 716 - 5 = 711。

FE (3)：考慮個體效應，每個個體一個虛擬變數，自由度減少為 716 - (5 + 715) = -4（實際上 plm 會調整）。

在 1% 顯著性水平下，假設自由度約為 (715, 711)，F 臨界值約為 2.64

拒絕虛無假設，個體無差異

e. Column (4) contains the fixed effects estimates with cluster-robust standard errors. In the context of this sample, explain the different assumptions you are making when you estimate with and without cluster-robust standard errors. Compare the standard errors with those in column (3). Which ones are substantially different? Are the robust ones larger or smaller?

標準誤皆變大

f. Column (5) contains the random effects estimates. Which coefficients, apart from the intercepts, show the most difference from the fixed effects estimates? Use the Hausman test statistic (15.36) to test whether there are significant differences between the random effects estimates and the fixed effects estimates in column (3) (Why that one?). Based on the test results, is random effects estimation in this model appropriate?

$EXPER^2$, Hausman test $t_j = \dfrac{\hat{\beta}_{FE} - \hat{\beta}_{RE}}{\sqrt{Var(\hat{\beta}_{FE}) - Var(\hat{\beta}_{RE})}}$

$\underline{t_{EXPER} = -1.67}$, $t_{EXPER^2} = 1.096$

$t_{SOUTH} = -0.77$, $t_{UNION} = -1.06$ → random effects estimation appropriate

15.20 This exercise uses data from the STAR experiment introduced to illustrate fixed and random effects for grouped data. In the STAR experiment, children were randomly assigned within schools into three types of classes: small classes with 13–17 students, regular-sized classes with 22–25 students, and regular-sized classes with a full-time teacher aide to assist the teacher. Student scores on achievement tests were recorded as well as some information about the students, teachers, and schools. Data for the kindergarten classes are contained in the data file *star*.

a. Estimate a regression equation (with no fixed or random effects) where *READSCORE* is related to *SMALL*, *AIDE*, *TCHEXPER*, *BOY*, *WHITE_ASIAN*, and *FREELUNCH*. Discuss the results. Do students perform better in reading when they are in small classes? Does a teacher's aide improve scores? Do the students of more experienced teachers score higher on reading tests? Does the student's sex or race make a difference?

```
Residuals:
    Min      1Q  Median      3Q     Max 
-110.05  -20.27   -4.02   14.45  189.12 

Coefficients:
             Estimate Std. Error t value Pr(>|t|)    
(Intercept) 434.52072    1.28572 337.958  < 2e-16 ***
small         5.81416    0.99437   5.847 5.28e-09 ***
aide          0.79682    0.95784   0.832    0.406    
tchexper      0.51286    0.06986   7.341 2.41e-13 ***
white_asian   3.74427    0.95823   3.907 9.43e-05 ***
freelunch   -14.75206    0.89478 -16.487  < 2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 30.34 on 5760 degrees of freedom
  (因為不存在，20 個觀察量被刪除了)
Multiple R-squared:  0.08748,	Adjusted R-squared:  0.08668 
F-statistic: 110.4 on 5 and 5760 DF,  p-value: < 2.2e-16
```

b. Reestimate the model in part (a) with school fixed effects. Compare the results with those in part (a). Have any of your conclusions changed? [*Hint*: specify *SCHID* as the cross-section identifier and *ID* as the “time” identifier.]

```
> # 比較兩個模型的係數
> summary(model_a)$coefficients
               Estimate Std. Error     t value     Pr(>|t|)
(Intercept) 434.5207225 1.28572307 337.9582536 0.000000e+00
small         5.8141611 0.99436939   5.8470837 5.277351e-09
aide          0.7968162 0.95784135   0.8318875 4.055069e-01
tchexper      0.5128553 0.06985898   7.3412937 2.408842e-13
white_asian   3.7442696 0.95823207   3.9074768 9.433391e-05
freelunch   -14.7520583 0.89477693 -16.4868560 1.045669e-59
> summary(model_b)$coefficients
                       Estimate Std. Error     t value     Pr(>|t|)
(Intercept)         407.4698192  4.1186586  98.9326520 0.000000e+00
small                 6.4814512  0.9173873   7.0651197 1.797342e-12
aide                  0.9869604  0.8859669   1.1139924 2.653296e-01
tchexper              0.3000867  0.0711614   4.2169874 2.514345e-05
white_asian           7.9348573  1.5430510   5.1423169 2.804640e-07
```

c. Test for the significance of the school fixed effects. Under what conditions would we expect the inclusion of significant fixed effects to have little influence on the coefficient estimates of the remaining variables?

```
> anova(model_a, model_b)
Analysis of Variance Table

Model 1: readscore ~ small + aide + tchexper + white_asian + freelunch
Model 2: readscore ~ small + aide + tchexper + white_asian + freelunch + 
    factor(schid)
  Res.Df     RSS Df Sum of Sq      F    Pr(>F)    
1   5760 5302072                                  
2   5682 4311147 78    990925 16.744 < 2.2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

d. Reestimate the model in part (a) with school random effects. Compare the results with those from parts (a) and (b). Are there any variables in the equation that might be correlated with the school effects? Use the LM test for the presence of random effects.

```
Model a (OLS) Coefficients:
> print(summary(model_a)$coefficients)
               Estimate Std. Error     t value     Pr(>|t|)
(Intercept) 437.7642527  1.3462212 325.1800198 0.000000e+00
small         5.8228158  0.9893333   5.8855960 4.189826e-09
aide          0.8178369  0.9529935   0.8581768 3.908306e-01
tchexper      0.4924687  0.0695551   7.0802669 1.610506e-12
boy          -6.1564214  0.7961282  -7.7329526 1.232255e-14
white_asian   3.9058095  0.9536072   4.0958264 4.264330e-05
freelunch   -14.7713371  0.8902481 -16.5923825 1.965023e-60
> cat("\nModel b (Fixed Effects) Coefficients:\n")

Model b (Fixed Effects) Coefficients:
> print(summary(model_b)$coefficients)
                       Estimate Std. Error     t value     Pr(>|t|)
(Intercept)         411.1626296 4.12826475  99.5969624 0.000000e+00
small                 6.4902305 0.91296175   7.1089841 1.312946e-12
aide                  0.9960875 0.88169306   1.1297441 2.586318e-01
tchexper              0.2855668 0.07084451   4.0308950 5.629160e-05
boy                  -5.4559412 0.72758937  -7.4986543 7.439670e-14
white_asian           8.0280192 1.53565617   5.2277452 1.777245e-07
freelunch           -14.5935724 0.88000649 -16.5834828 2.362112e-60
factor(schid)123056  12.2362490 5.44228332   2.2483668 2.459099e-02
```

```
Model d (Random Effects) Coefficients:
> print(coef(model_d))
(Intercept)       small        aide    tchexper         boy white_asian   freelunch
436.1267737   6.4587216   0.9921460   0.3026787  -5.5120812   7.3504772 -14.5843317
>
> # 執行 Breusch-Pagan (LM) 檢驗以測試隨機效應
> plmtest(model_d, type = "bp")

	Lagrange Multiplier Test - (Breusch-Pagan)

data:  readscore ~ small + aide + tchexper + boy + white_asian + freelunch
chisq = 6677.4, df = 1, p-value < 2.2e-16
alternative hypothesis: significant effects
```

e. Using the t-test statistic in equation (15.36) and a 5% significance level, test whether there are any significant differences between the fixed effects and random effects estimates of the coefficients on *SMALL*, *AIDE*, *TCHEXPER*, *WHITE_ASIAN*, and *FREELUNCH*. What are the implications of the test outcomes? What happens if we apply the test to the fixed and random effects estimates of the coefficient on *BOY*?

```
T-tests for differences between fixed and random effects coefficients:
> for (i in 1:length(variables)) {
+   cat(sprintf("%s: t-statistic = %.3f, p-value = %.3f\n",
+               variables[i], t_results[[i]]$t_stat, t_results[[i]]$p_value))
+ }
small: t-statistic = 0.024, p-value = 0.981
aide: t-statistic = 0.003, p-value = 0.997
tchexper: t-statistic = -0.171, p-value = 0.864
white_asian: t-statistic = 0.323, p-value = 0.747
freelunch: t-statistic = -0.007, p-value = 0.994
boy: t-statistic = 0.055, p-value = 0.956
> 
```

f. Create school-averages of the variables and carry out the Mundlak test for correlation between them and the unobserved heterogeneity.

```
Linear hypothesis test:
mean_small = 0
mean_aide = 0
mean_tchexper = 0
mean_white_asian = 0
mean_freelunch = 0

Model 1: restricted model
Model 2: readscore ~ small + aide + tchexper + white_asian + freelunch +
    mean_small + mean_aide + mean_tchexper + mean_white_asian +
    mean_freelunch

  Res.Df     RSS Df Sum of Sq      F    Pr(>F)
1   5760 5302072
2   5755 5232672  5     69401 15.266 6.194e-15 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
>
```

15.17 The data file *liquor* contains observations on annual expenditure on liquor (*LIQUOR*) and annual income (*INCOME*) (both in thousands of dollars) for 40 randomly selected households for three consecutive years.

a. Create the first-differenced observations on *LIQUOR* and *INCOME*. Call these new variables *LIQUORD* and *INCOMED*. Using OLS regress *LIQUORD* on *INCOMED* without a constant term. Construct a 95% interval estimate of the coefficient.

```
Call:
lm(formula = LIQUORD ~ INCOMED, data = liquir)

Residuals:
    Min      1Q  Median      3Q     Max
-3.5012 -0.8399  0.0298  1.0077  3.5049

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) -0.40287    0.40600  -0.992    0.324
INCOMED      0.09815    0.07487   1.311    0.194

Residual standard error: 1.417 on 78 degrees of freedom
Multiple R-squared:  0.02156,	Adjusted R-squared:  0.009012
F-statistic: 1.718 on 1 and 78 DF,  p-value: 0.1937
```

```
                2.5 %    97.5 %
INCOMED -0.05090933 0.2472087
```

b. Estimate the model $LIQUOR_{it} = \beta_1 + \beta_2 INCOME_{it} + u_i + e_{it}$ using random effects. Construct a 95% interval estimate of the coefficient on *INCOME*. How does it compare to the interval in part (a)?

```
Effects:
                  var std.dev share
idiosyncratic 0.9640  0.9819 0.571
individual    0.7251  0.8515 0.429
theta: 0.4459

Residuals:
     Min.   1st Qu.    Median   3rd Qu.      Max.
-2.263634 -0.697383 0.078697  0.552680  2.225798

Coefficients:
             Estimate Std. Error z-value  Pr(>|z|)
(Intercept) 0.9690324  0.5210052  1.8599 0.0628957 .
income      0.0265755  0.0070126  3.7897 0.0001508 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Total Sum of Squares:    126.61
Residual Sum of Squares: 112.88
R-Squared:      0.1085
Adj. R-Squared: 0.10095
Chisq: 14.3618 on 1 DF, p-value: 0.00015083
>
> # 計算 INCOME 係數的 95% 信賴區間
> conf_interval_b <- confint(model_b, "income", level = 0.95)
> cat("\n95% Confidence Interval for INCOME (Random Effects):\n")

95% Confidence Interval for INCOME (Random Effects):
> print(conf_interval_b)
           2.5 %     97.5 %
income 0.01283111 0.04031983
```

c. Test for the presence of random effects using the LM statistic in equation (15.35). Use the 5% level of significance.

```
Breusch-Pagan LM Test for Random Effects:
> print(lm_test)

        Lagrange Multiplier Test - (Breusch-Pagan)

data:  liquor ~ income
chisq = 20.68, df = 1, p-value = 5.429e-06
alternative hypothesis: significant effects
```

d. For each individual, compute the time averages for the variable *INCOME*. Call this variable *INCOMEM*. Estimate the model $LIQUOR_{it} = \beta_1 + \beta_2 INCOME_{it} + \gamma INCOMEM_i + c_i + e_{it}$ using the random effects estimator. Test the significance of the coefficient γ at the 5% level. Based on this test, what can we conclude about the correlation between the random effect u_i and *INCOME*? Is it OK to use the random effects estimator for the model in (b)?

```
Call:
plm(formula = liquor ~ income + INCOMEM, data = liquor5, model = "random",
    index = c("hh", "year"))

Balanced Panel: n = 40, T = 3, N = 120

Effects:
                 var std.dev share
idiosyncratic 0.9640  0.9819 0.571
individual    0.7251  0.8515 0.429
theta: 0.4459

Residuals:
     Min.   1st Qu.    Median   3rd Qu.      Max.
-2.300955 -0.703840  0.054992  0.560255  2.257325

Coefficients:
             Estimate Std. Error z-value Pr(>|z|)
(Intercept) 0.9163337  0.5524439  1.6587  0.09718 .
income      0.0207421  0.0209083  0.9921  0.32117
INCOMEM     0.0065792  0.0222048  0.2963  0.76700
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Total Sum of Squares:    126.61
Residual Sum of Squares: 112.79
R-Squared:      0.10917
Adj. R-Squared: 0.093945
Chisq: 14.3386 on 2 DF, p-value: 0.00076987
>
> # 檢驗 INCOMEM 係數 (γ) 的顯著性
> cat("\nTest for significance of INCOMEM (γ):\n")

Test for significance of INCOMEM (γ):
> gamma_coef <- coef(summary(model_d))["INCOMEM", ]
> cat(sprintf("Coefficient of INCOMEM: %.3f, p-value: %.3f\n",
+             gamma_coef["Estimate"], gamma_coef["Pr(>|t|)"]))
Coefficient of INCOMEM: 0.007, p-value: NA
```