10.2 The labor supply of married women has been a subject of a great deal of economic research. Consider the following supply equation specification

$$HOURS = \beta_1 + \beta_2 WAGE + \beta_3 EDUC + \beta_4 AGE + \beta_5 KIDSL6 + \beta_6 NWIFEINC + e$$

where *HOURS* is the supply of labor, *WAGE* is hourly wage, *EDUC* is years of education, *KIDSL6* is the number of children in the household who are less than 6 years old, and *NWIFEINC* is household income from sources other than the wife's employment.

a. Discuss the signs you expect for each of the coefficients.
b. Explain why this supply equation cannot be consistently estimated by OLS regression.
c. Suppose we consider the woman's labor market experience *EXPER* and its square, $EXPER^2$, to be instruments for *WAGE*. Explain how these variables satisfy the logic of instrumental variables.
d. Is the supply equation identified? Explain.
e. Describe the steps [not a computer command] you would take to obtain IV/2SLS estimates.

a.

$\beta_2>0$: If the hourly wage is higher, people are willing to spend more time on working

$\beta_3>0$:

$\beta_4<0$: As the Age increase, the health of body would be worse, so the working hour would decrease

$\beta_5<0$: People should spend more time on taking care of children, so the hour decrease

$\beta_6<0$: If people have any other source to increase income, he do not have to spend to much time on working.

b. Since we are not sure that $cov(x,e)=0$

this equation cannot be consistently estimated by OLS

c. these variables satisfy ① relevance : strongly correlated with wage

② Exogeneity : uncorrelated with error

d. Since we have two valid instruments and one endogenous variables, the equation is identified.

e.

① first: Find the coefficients of the regression: $WAGE = \alpha_1 + \alpha_2 EXPER + \alpha_3 EXPER^2$

then find the predicted values of WAGE: $\hat{WAGE}$

② second: Find the coefficients of the regression:

$HOUR = \beta_1 + \beta_2 \hat{WAGE} + \beta_3 EDUC + \beta_4 AGE + \beta_5 KIDSL6 + \beta_6 NWIFEINC + e$

10.3 In the regression model $y = \beta_1 + \beta_2 x + e$, assume x is endogenous and that z is a valid instrument. In Section 10.3.5, we saw that $\beta_2 = \text{cov}(z, y)/\text{cov}(z, x)$.

a. Divide the denominator of $\beta_2 = \text{cov}(z, y)/\text{cov}(z, x)$ by $\text{var}(z)$. Show that $\text{cov}(z, x)/\text{var}(z)$ is the coefficient of the simple regression with dependent variable x and explanatory variable z, $x = \gamma_1 + \theta_1 z + v$. [*Hint*: See Section 10.2.1.] Note that this is the first-stage equation in two-stage least squares.
b. Divide the numerator of $\beta_2 = \text{cov}(z, y)/\text{cov}(z, x)$ by $\text{var}(z)$. Show that $\text{cov}(z, y)/\text{var}(z)$ is the coefficient of a simple regression with dependent variable y and explanatory variable z, $y = \pi_0 + \pi_1 z + u$. [*Hint*: See Section 10.2.1.]
c. In the model $y = \beta_1 + \beta_2 x + e$, substitute for x using $x = \gamma_1 + \theta_1 z + v$ and simplify to obtain $y = \pi_0 + \pi_1 z + u$. What are π_0, π_1, and u in terms of the regression model parameters and error and the first-stage parameters and error? The regression you have obtained is a **reduced-form** equation.
d. Show that $\beta_2 = \pi_1/\theta_1$.
e. If $\hat{\pi}_1$ and $\hat{\theta}_1$ are the OLS estimators of π_1 and θ_1, show that $\hat{\beta}_2 = \hat{\pi}_1/\hat{\theta}_1$ is a consistent estimator of $\beta_2 = \pi_1/\theta_1$. The estimator $\hat{\beta}_2 = \hat{\pi}_1/\hat{\theta}_1$ is an **indirect least squares** estimator.

a. $\theta_1 = \dfrac{\sum(z_i-\bar{z})(x_i-\bar{x})}{\sum(z_i-\bar{z})^2} = \dfrac{cov(z,x)}{var(z)}$ (by OLS), $\beta_2\theta_1 = \dfrac{cov(z,y)}{var(z)}$ holds

b. $\pi_1 = \dfrac{\sum(z_i-\bar{z})(y_i-\bar{y})}{\sum(z_i-\bar{z})^2} = \dfrac{cov(z,y)}{var(z)}$ (by OLS), $\theta_1 = \dfrac{\pi_1}{\beta_2} = \dfrac{cov(z,x)}{var(z)}$ holds

c. $y = \beta_1 + \beta_2(\gamma_1 + \theta_1 z + v) + e$

$= (\beta_1 + \beta_2\gamma_1) + (\beta_2\theta_1)z + (\beta_2 v + e) = \pi_0 + \pi_1 z + u$

Thus, $\pi_0 = \beta_1 + \beta_2\gamma_1$ — ①

$\pi_1 = \beta_2\theta_1$ — ②

$u = \beta_2 v + e$ — ③

d. By ②, we get that $\beta_2 = \frac{\pi_1}{\theta_1}$

e. Goal: $\lim_{N\to\infty} \hat{\beta}_2(N) = \beta_2$, N is the sample size

proof:

Since z is a valid instrument, $\lim_{N\to\infty} \hat{\pi}_1(N) = \pi_1$ and $\lim_{N\to\infty} \hat{\theta}_1(N) = \theta_1$

Since $\theta_1 \neq 0$, $\lim_{N\to\infty} \dfrac{\hat{\pi}_1(N)}{\hat{\theta}_1(N)} = \dfrac{\lim_{N\to\infty}\hat{\pi}_1(N)}{\lim_{N\to\infty}\hat{\theta}_1(N)} = \dfrac{\pi_1}{\theta_1} = \beta_2$

Thus, $\lim_{N\to\infty} \hat{\beta}_2 = \lim_{N\to\infty} \dfrac{\hat{\pi}_1(N)}{\hat{\theta}_1(N)} = \beta_2$