

3.1 There were 64 countries in 1992 that competed in the Olympics and won at least one medal. Let *MEDALS* be the total number of medals won, and let *GDPB* be GDP (billions of 1995 dollars). A linear regression model explaining the number of medals won is $MEDALS = \beta_1 + \beta_2 GDPB + e$. The estimated relationship is

$$\widehat{MEDALS} = b_1 + b_2 GDPB = 7.61733 + 0.01309GDPB$$

(se) (2.38994) (0.00215)

(XR3.1)

- We wish to test the hypothesis that there is no relationship between the number of medals won and *GDP* against the alternative there is a positive relationship. State the null and alternative hypotheses in terms of the model parameters.
- What is the test statistic for part (a) and what is its distribution if the null hypothesis is true?
- What happens to the distribution of the test statistic for part (a) if the alternative hypothesis is true? Is the distribution shifted to the left or right, relative to the usual *t*-distribution? [*Hint*: What is the expected value of b_2 if the null hypothesis is true, and what is it if the alternative is true?]
- For a test at the 1% level of significance, for what values of the *t*-statistic will we reject the null hypothesis in part (a)? For what values will we fail to reject the null hypothesis?
- Carry out the *t*-test for the null hypothesis in part (a) at the 1% level of significance. What is your economic conclusion? What does 1% level of significance mean in this example?

a. $H_0: \beta_2 = 0$, $H_1: \beta_2 > 0$

$$b_1 \quad t = \frac{b_2}{se(b_1)} \sim t_{(62)} \quad df = N - 2 = 64 - 2 = 62$$

If the null hypothesis is true, then $t \sim t_{(b_2)}$

c. If the alternative hypothesis is true, then t distribution is shifted to the right

d. $\alpha = 0.01$, $t_{0.99, 62} \approx 2.3880$

If $t \geq 2.3880$, we reject H_0 .

If $t < 2.3880$, we do not reject H_0 .

$$e. t = \frac{b_2}{se(b_2)} = \frac{0.01309}{0.00215} = 6.0884$$

Because $t > 2.3880$, we reject H_0 that there is no relationship between the number of medals won and GDP.

We accept H_1 , that there is a positive relationship between the number of medals won and GDP.

1% level of significance means that if H_0 is true, the probability of incorrectly rejecting H_0 (Type I error) is only 1%.

In other words, we are 99% confident in the conclusion that GDP has an impact on the number of models won.

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> qt(1 - 0.01, 62)
[1] 2.388011
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