# Chapter 10

2.

a. $\beta_2 \rightarrow$ positive, it has more labor supply as the wage increases.

$\beta_3 \rightarrow$ not sure, as the education level increases, it has more qualified labor can supply.
However, since labors are more efficient, labor needs decreases and so as labor supply.

$\beta_4 \rightarrow$ not sure, as women become older, it has more experienced labors to offer labor supply but also their health worsen and thus labor supply decreases.

$\beta_5 \rightarrow$ negative, when the kids become more, women have less time on work.

$\beta_6 \rightarrow$ negative, when women can earn more on other sources, women supply less labor

b. WAGE is Endogeneous.
This is because other factors can influence both HOURS and WAGE. Thus, $cov(WAGE, e) \neq 0$.

c. It becomes an instrumental variable because
1) Relevance: EXPER and $EXPER^2$ must be correlated with WAGE.
2) Exogeneity: EXPER and $EXPER^2$ must be uncorrelated with the error term e.

d. Yes. since it has at least as many valid instruments as endogeneous regressors.

e. step 1: Regress $WAGE = \gamma_1 + \gamma_2 EDUC + \gamma_3 AGE + \gamma_4 KIDSL6 + \gamma_5 NWIFEINC + \gamma_6 EXPER + \gamma_7 EXPER^2 + e$
step 2: Get $\hat{WAGE}$ and use OLS estimate.

3.

a. $X = \gamma_1 + \theta_1 Z + V$

$\underline{E(X) = \gamma_1 + \theta_1 E(Z)}$

$(X - E(X)) = \theta_1 (Z - E(Z)) + V$

$\rightarrow (Z - E(Z))(X - E(X)) = \theta_1 (Z - E(Z))^2 + V(Z - E(Z))$

$E((Z - E(Z))(X - E(X))) = \theta_1 E((Z - E(Z))^2)$

$\theta_1 = \frac{cov(Z, X)}{var(Z)}$

b. $y = \pi_0 + \pi_1 Z + u$

$\underline{E(y) = \pi_0 + \pi_1 E(Z)}$

$y - E(y) = \pi_1 (Z - E(Z)) + u$

$\rightarrow (Z - E(Z))(y - E(y)) = \pi_1 (Z - E(Z))^2 + u(Z - E(Z))$

$E((Z - E(Z))(y - E(y))) = \theta_1 E((Z - E(Z))^2)$

$\theta_1 = \frac{cov(Z, y)}{var(Z)}$

c. $y = \beta_1 + \beta_2(\gamma_1 + \theta_1 z + v) + e$

$= (\beta_1 + \beta_2\gamma_1) + \beta_2\theta_1 z + (\beta_2 v + e)$

$\longrightarrow \pi_0 = \beta_1 + \beta_2\gamma_1$

$\pi_1 = \beta_2\theta_1$

$u = \beta_2 v + e$

d. $\pi_1 = \beta_2\theta_1$ , $\beta_2 = \dfrac{\pi_1}{\theta_1}$

e. $\hat{\theta}_1 = \dfrac{\widehat{\text{cov}}(z,x)}{\widehat{\text{var}}(z)} = \dfrac{\sum(z_i-\bar{z})(x_i-\bar{x})}{\sum(z_i-\bar{z})^2}$

$\hat{\pi}_1 = \dfrac{\widehat{\text{cov}}(z,y)}{\widehat{\text{var}}(z)} = \dfrac{\sum(z_i-\bar{z})(y_i-\bar{y})}{\sum(z_i-\bar{z})^2}$

$\hat{\beta}_2 = \dfrac{\hat{\pi}_1}{\hat{\theta}_1} = \dfrac{\sum(z_i-\bar{z})(y_i-\bar{y})}{\sum(z_i-\bar{z})(x_i-\bar{x})} = \dfrac{\widehat{\text{cov}}(z,y)}{\widehat{\text{cov}}(z,x)}$

$\longrightarrow$ It is IV estimator.

$\because \widehat{\text{cov}}(z,y) \xrightarrow{p} \text{cov}(z,y)$

$\widehat{\text{cov}}(z,x) \xrightarrow{p} \text{cov}(z,x)$

$\therefore \hat{\beta}_2 = \dfrac{\widehat{\text{cov}}(z,y)}{\widehat{\text{cov}}(z,x)} \xrightarrow{p} = \dfrac{\text{cov}(z,y)}{\text{cov}(z,x)} = \beta_2$