Q10.2

Assignment
C10Q02(a)
C10Q02(b)
C10Q02(c)
C10Q02(d,e)
C10Q03(a)
C10Q03(b)
C10Q03(c)
C10Q03(d)
C10Q03(e)

10.2 The labor supply of married women has been a subject of a great deal of economic research. Consider the following supply equation specification

$$HOURS = \beta_1 + \beta_2 WAGE + \beta_3 EDUC + \beta_4 AGE + \beta_5 KIDSL6 + \beta_6 NWIFEINC + e$$

where *HOURS* is the supply of labor, *WAGE* is hourly wage, *EDUC* is years of education, *KIDSL6* is the number of children in the household who are less than 6 years old, and *NWIFEINC* is household income from sources other than the wife's employment.

a. Discuss the signs you expect for each of the coefficients.
b. Explain why this supply equation cannot be consistently estimated by OLS regression.
c. Suppose we consider the woman's labor market experience *EXPER* and its square, $EXPER^2$, to be instruments for *WAGE*. Explain how these variables satisfy the logic of instrumental variables.
d. Is the supply equation identified? Explain.
e. Describe the steps [not a computer command] you would take to obtain IV/2SLS estimates.

a.

Parameters	β_2	β_3	β_4	β_5	β_6
Prediction	+	−	−	−	−

b.
We can only observe the equalibrium of labor quantity and labor price in the market. The equalibrium is decided simultaneously by labor supply and demand, which thus leads to endogeniety for the OLS regression, making parameters inconsistent and biased.

$$\begin{cases} Q_s = a + b\, Wage_s + e \\ Q_d = c + d\, Wage_d + u \end{cases}$$

c. A good instrument variable should be correlated to wage but uncorrelated to e. Wage will increase as experience increase, but Experience doesn't bring excess explanation to quantity supplied other than wage do. Therefore, $Cov(Wage, Experience) \neq 0$

$$Cov(Experience, e) = 0$$

d. Wage is the only endogenous variable in the equation, and EXPER, $EXPER^2$ are 2 IV. The equation is thus overidentified.

e. 1. Regress wage on EXPER and $EXPER^2$

$$\hat{Wage} = \alpha_1 + \alpha_2 EXPER + \alpha_3 EXPER^2$$

and extract fitted Wage

2. Replace Wage with $\hat{Wage}$ and estimate original regression

$$HOURS = \beta_1 + \beta_2 \hat{Wage} + X'\beta + e$$

$\hat{\beta}_2$ IV is consistent and unbiased.

Q 10.3

10.3 In the regression model $y = \beta_1 + \beta_2 x + e$, assume x is endogenous and that z is a valid instrument. In Section 10.3.5, we saw that $\beta_2 = \text{cov}(z, y)/\text{cov}(z, x)$.

a. Divide the denominator of $\beta_2 = \text{cov}(z, y)/\text{cov}(z, x)$ by $\text{var}(z)$. Show that $\text{cov}(z, x)/\text{var}(z)$ is the coefficient of the simple regression with dependent variable x and explanatory variable z, $x = \gamma_1 + \theta_1 z + v$. [*Hint*: See Section 10.2.1.] Note that this is the first-stage equation in two-stage least squares.

b. Divide the numerator of $\beta_2 = \text{cov}(z, y)/\text{cov}(z, x)$ by $\text{var}(z)$. Show that $\text{cov}(z, y)/\text{var}(z)$ is the coefficient of a simple regression with dependent variable y and explanatory variable z, $y = \pi_0 + \pi_1 z + u$. [*Hint*: See Section 10.2.1.]

c. In the model $y = \beta_1 + \beta_2 x + e$, substitute for x using $x = \gamma_1 + \theta_1 z + v$ and simplify to obtain $y = \pi_0 + \pi_1 z + u$. What are π_0, π_1, and u in terms of the regression model parameters and error and the first-stage parameters and error? The regression you have obtained is a **reduced-form** equation.

d. Show that $\beta_2 = \pi_1/\theta_1$.

e. If $\hat{\pi}_1$ and $\hat{\theta}_1$ are the OLS estimators of π_1 and θ_1, show that $\hat{\beta}_2 = \hat{\pi}_1/\hat{\theta}_1$ is a consistent estimator of $\beta_2 = \pi_1/\theta_1$. The estimator $\hat{\beta}_2 = \hat{\pi}_1/\hat{\theta}_1$ is an **indirect least squares** estimator.

a.

$$X = \gamma_1 + \theta_1 Z + V$$

$$E(X) = \gamma_1 + \theta_1 E(Z)$$

$$X - E(X) = \gamma_1 + \theta_1 Z + V - \gamma_1 - \theta E(Z)$$

$$= \theta_1 \left[Z - E(Z) \right] + V$$

$$[X - E(X)][Z - E(Z)] = \theta_1 [Z - E(Z)]^2 + V[Z - E(Z)]$$

$Cov(V, Z) = 0$

$$E[X - E(X)][Z - E(Z)] = \theta_1 E[Z - E(Z)]^2 + \underline{E\{V[Z - E(Z)]\}}$$

$$Cov(X, Z) = \theta_1 \, Var(Z), \qquad \theta_1 = \frac{Cov(X, Z)}{Var(Z)}$$

b.

$$y = \pi_0 + \pi_1 Z + u$$

$$E(y) = \pi_0 + \pi_1 E(Z)$$

$$y - E(y) = \pi_1 [Z - E(Z)] + u$$

$Cov(u, Z) = 0$

$$E[y - E(y)][Z - E(Z)] = \pi_1 E[Z - E(Z)]^2 + \underline{E\{u[Z - E(Z)]\}}$$

$$Cov(y, Z) = \pi_1 \, Var(Z)$$

$$\pi_1 = \frac{Cov(y, Z)}{Var(Z)}$$

c. $y = \beta_1 + \beta_2 X + e$

$X = \gamma_1 + \theta_1 Z + V$ $\rightarrow$ $y = \beta_1 + \beta_2(\gamma_1 + \theta_1 Z + V) + e$

$y = \pi_0 + \pi_1 Z + u$

$$= \underbrace{\beta_1 + \beta_2\gamma_1}_{=\pi_0} + \underbrace{\beta_2\theta_1}_{=\pi_1} Z + \underbrace{\beta_2 V + e}_{=u}$$

d.

$$\beta_2 = \frac{\pi_1}{\theta_1} = \frac{Cov(y,z)/Var(z)}{Cov(X,z)/Var(z)} = \frac{Cov(y,z)}{Cov(X,z)}$$

e.

$$\hat{\pi}_1 = \frac{\Sigma(y_i - \bar{y})(z_i - \bar{z})}{\Sigma(z_i - \bar{z})^2}$$

$$\hat{\theta}_1 = \frac{\Sigma(X_i - \bar{X})(z_i - \bar{z})}{\Sigma(z_i - \bar{z})^2}$$

$$\hat{\beta}_2 = \frac{\hat{\pi}_1}{\hat{\theta}_1} = \frac{\Sigma(y_i - \bar{y})(z_i - \bar{z})}{\Sigma(X_i - \bar{X})(z_i - \bar{z})}$$

$$\rightarrow \frac{Cov(y,z)}{Cov(X,z)} = \beta_2$$