Le Thi Phuong Thao – 413707007

Q11.1

C11.1 $y_1 = a_1 y_2 + e_1$

$y_2 = a_2 y_1 + b_1 x_1 + b_2 x_2 + e_2$

a) $y_2 = a_2(a_1 y_2 + e_1) + b_1 x_1 + b_2 x_2 + e_2$

$\Rightarrow (1 - a_2 a_1) y_2 = a_2 e_1 + b_1 x_1 + b_2 x_2 + e_2$

$$y_2 = \frac{b_1}{1 - a_2 a_1} x_1 + \frac{b_2}{1 - a_2 a_1} x_2 + \frac{a_2 e_1 + e_2}{1 - a_2 a_1}$$

$\Rightarrow y_2 = \pi_1 x_1 + \pi_2 x_2 + v_2$

$Cov(y_2, e_1) = cov(\pi_1 x_1 + \pi_2 x_2 + v_2, e_1)$

$= cov(v_2, e_1) = cov\left(\frac{a_2 e_1 + e_2}{1 - a_2 a_1}, e_1\right) = \frac{a_2}{1 - a_2 a_1} var(e_1)$

$\Rightarrow cov(y_2, e_1) \neq 0$ if $a_2 \neq 0$

b) $cov(y_2, e_1) \neq 0 \Rightarrow$ endogenity

$cov(y_1, e_2) = cov(\alpha_1(\pi_1 x_1 + \pi_2 x_2 + v_2) + e_1, e_2) = cov\left(\frac{e_1 + \alpha_1 e_2}{1 - \alpha_2 \alpha_1}, e_2\right)$

$= \frac{a_1 var(e_2)}{1 - \alpha_1 \alpha_2} \neq 0 \Rightarrow$ endogenity

$\Rightarrow$ Both are inconsistent

c) $y_1 = a_1 y_2 + e_1 \Rightarrow$ identified

$y_2 = \alpha_2 y_1 + B_1 x_1 + e_2 \rightarrow$ not identified

d) Reduced form

$y_2 = \pi_1 x_1 + \pi_2 x_2 + v_2$

$\Rightarrow v_2 = y_2 - \pi_1 x_1 - \pi_2 x_2$

$\frac{1}{N} \Sigma x_{1i} v_{2i} = 0$; $\frac{1}{N} \Sigma x_{2i} v_{2i} = 0$

$E(x_{1i} v_{2i} | x) = 0$; $E(x_{2i} v_{2i}) = 0 \Rightarrow x$ is exogenous

from (a) $y_2 = \frac{b_1}{1 - \alpha_1 \alpha_2} x_1 + \frac{b_2}{1 - \alpha_1 \alpha_2} x_2 + \frac{e_2 + e_1 \alpha_2}{1 - \alpha_1 \alpha_2} = \pi_1 x_1 + \pi_2 x_2 + v_2$

$E\left[x_{ik}\left(\frac{e_2 + e_1 \alpha_2}{1 - \alpha_1 \alpha_2}\right) \Big| x\right] = E\left[\frac{1}{1 - \alpha_1 \alpha_2} x_{ik} e_2 \Big| x\right] + E\left[\frac{\alpha_2}{(1 - \alpha_1 \alpha_2)} x_{ik} e_1 \Big| x\right]$

$= 0 + 0$. So reduced form of e is uncorrelated with x.

e) The sum of squared of v_2

$$S(\pi_1 \pi_2 \mid y, x) = \Sigma (y_2 - \pi_1 x_1 - \pi_2 x_2)^2$$

$$\frac{dS}{d\pi_1} = 2\Sigma (y_2 - \pi_1 x_1 - \pi_2 x_2) x_1 = 0$$

$$\frac{dS}{d\pi_2} = 2\Sigma (y_2 - \pi_1 x_1 - \pi_2 x_2) x_2 = 0$$

Divide by 2 and $\times N$ $\Rightarrow$ equavlent to (d)

f) $N^{-1} \Sigma x_{i1} (y_2 - \pi_1 x_{i1} - \pi_2 x_{i2}) = 0$

$N^{-1} \Sigma x_{i2} (y_2 - \pi_1 x_{i1} - \pi_2 x_{i2}) = 0$

$\Rightarrow \Sigma x_{i1} y_{i2} - \pi_1 \Sigma x_{i1}^2 - \pi_2 \Sigma x_{i1} x_{i2} = 0$

$\Sigma x_{i1} y_{i2} - \pi_1 \Sigma x_{i1} x_{i2} - \pi_2 \Sigma x_{i2}^2 = 0$

$\Rightarrow 3 - \hat{\pi}_1 = 0 \Rightarrow \hat{\pi}_1 = 3$

$4 - \hat{\pi}_2 = 0 \Rightarrow \hat{\pi}_2 = 4$

g) $y_1 = \alpha_1 y_2 + e_1$

$\Rightarrow E[(\pi_1 x_1 + \pi_2 x_2) e_1 \mid x] = E[(\pi_1 x_1 + \pi_2 x_2)(y_1 - \alpha_1 y_2) \mid x] = 0$

$N^{-1} \Sigma (\pi_1 x_{i1} + \pi_2 x_{i2})(y_{i1} - \alpha_1 y_{i2}) = 0$

$\operatorname{plim} \hat{\pi}_1 = \pi_1$
$\operatorname{plim} \hat{\pi}_2 = \pi_2$ on lage sample

$\Sigma (\hat{\pi}_1 x_{i1} + \hat{\pi}_2 x_{i2})(y_{i1} - \alpha_1 y_{i2}) = \Sigma \hat{y}_{i2} (y_{i1} - \alpha_1 y_{i2}) = 0$

So that $\Sigma \hat{y}_{i2} y_{i1} - \alpha_1 \Sigma \hat{y}_{i2} y_{i2} = 0 \Rightarrow \hat{\alpha}_{1,IV} = \dfrac{\Sigma \hat{y}_{i2} y_{i1}}{\Sigma \hat{y}_{i2} y_{i2}}$

Inset value

$$\hat{\alpha}_{1IV} = \frac{\Sigma (\hat{\pi}_1 x_{i1} + \hat{\pi}_2 x_{i2}) y_{i1}}{\Sigma (\hat{\pi}_1 x_{i1} + \hat{\pi}_2 x_{i2}) y_{i2}} = \frac{\hat{\pi}_1 \Sigma x_{i1} y_{i1} + \hat{\pi}_2 \Sigma x_{i2} y_{i1}}{\hat{\pi}_1 \Sigma x_{i1} y_{i2} + \hat{\pi}_2 \Sigma x_{i2} y_{i2}} = \frac{18}{25}$$

h) $\hat{\alpha}_{1,2SLS} = \dfrac{\Sigma \hat{y}_{i2} y_{i1}}{\Sigma \hat{y}_{i2}^2}$ Recall $\hat{v}_2 = y_2 - \hat{y}_2$

$\hat{y}_2 = y_2 - \hat{v}_2$

$\Rightarrow \Sigma \hat{y}_{i2}^2 = \Sigma \hat{y}_{i2} (y_2 - \hat{v}_2) = \Sigma \hat{y}_{i2} y_2 - \Sigma \hat{y}_{i2} \hat{v}_2 = \Sigma \hat{y}_{i2} y_2$

since $\Sigma \hat{y}_{i2} \hat{v}_{i2} = \Sigma (\hat{\pi}_1 x_{i1} + \hat{\pi}_2 x_{i2}) \hat{v}_{i2} = 0$

掃描全能王

Q11.16

11.16.

a) $Q_i = \alpha_1 + \alpha_2 P_i + e_{di} = \beta_1 + \beta_2 P_i + \beta_3 w_i + e_{si}$

$(\alpha_2 - \beta_2) P_i = (\beta_1 - \alpha_1) + \beta_3 w_i + e_{si} - e_{di}$

$P_i = \dfrac{\beta_1 - \alpha_1}{\alpha_2 - \beta_2} + \dfrac{\beta_3}{\alpha_2 - \beta_2} w_i + \dfrac{e_{si} - e_{di}}{\alpha_2 - \beta_2} = \theta_1 + \theta_2 w_i + v_2$

$Q_i = \alpha_1 + \alpha_2 P_i + e_{di} = \alpha_1 + \alpha_2 \left(\dfrac{\beta_1 - \alpha_1}{\alpha_2 - \beta_2} + \dfrac{\beta_3}{\alpha_2 - \beta_2} w_i + \dfrac{e_{si} - e_{di}}{\alpha_2 - \beta_2} \right) + e_{di}$

$= \chi_1 + \chi_2 w + v_1$

b) Demand : identified

Supply : not identified

c) $\hat{Q} = 5 + 0.5W \Rightarrow \theta_1 = 5;\ \theta_2 = 0.5$

$\hat{P} = 2.4 + W \Rightarrow \pi_1 = 2.4;\ \pi_2 = 1$

~~$\Rightarrow 5 + 0.5W = \alpha_1 + \alpha_2 (2.4 + W)$~~

$Q = \alpha_1 + \alpha_2 (\beta_1 + \beta_2 W + e_s) + e_d$

$Q = (\alpha_1 + \alpha_2 \beta_1) + \alpha_2 \beta_2 W + (\alpha_2 e_s + e_d)$

$Q = \quad \theta_1 \quad + \quad \theta_2 W \quad + \quad v_2$

$\Rightarrow \theta_1 = \alpha_1 + \alpha_2 \beta_1 = 5$

$\theta_2 = \alpha_2 \beta_2 = 0.5$

$P = \beta_1 + \beta_2 W + e_s = \pi_1 + \pi_2 W + v_1 \Rightarrow \pi_1 = \beta_1 \quad \pi_2 = \beta_2$

So $\begin{cases} \alpha_1 + 2.4\alpha_2 = 5 \\ \alpha_2 = 0.5 \end{cases} \Rightarrow \alpha_1 = 3.8$

d) fitted value of P are $\hat{P} = [4.4, 5.4, 3.4, 3.4, 5.4]$

$Q = \alpha_1 + \alpha_2 \hat{P} + e_d$ Regress Q on $\hat{P}$

$\Rightarrow \hat{\alpha}_2 = \dfrac{\Sigma (Q_i - \bar{Q})(\hat{P}_i - \bar{\hat{P}})}{\Sigma (\hat{P}_i - \bar{\hat{P}})^2} \quad ; \hat{\alpha}_1 = \bar{Q} - \alpha_2 \bar{\hat{P}}$

$\bar{Q} = 6$ and $\bar{\hat{P}} = 4.4$

We can calculate $\alpha_1 = 3.8$, $\alpha_2 = 0.5$ same (c)

Q11.17

a) There are M = 8 equations requiring 7 omitted variables in each equation. There is a total of 16 variables in the system.
The consumption equation includes 6 variables and omits 10. The necessary condition is satisfied.
The investment equation includes 5 variables and omits 11. The necessary condition is satisfied.
The private sector wage equation includes 5 variables and omits 11. The necessary condition is satisfied.

b) The consumption equation has 2 RHS endogenous variables and excludes 5 exogenous variables.
The investment and private wage equations have 1 RHS endogenous variable and omit 5 exogenous variables

c) Answer

$$W_{1t} = \pi_1 + \pi_2 G_t + \pi_3 W_{2t} + \pi_4 TX_t + \pi_5 TIME_t + \pi_6 P_{t-1} + \pi_7 K_{t-1} + \pi_8 E_{t-1} + v$$

d) Answer

Obtain fitted values $\hat{W}_{1t}$ from the estimated reduced form equation in part (c) and similarly obtain $\hat{P}_t$. Create $W_t^* = \hat{W}_{1t} + W_{2t}$. Regress CN_t on W_t^*, $\hat{P}_t$ and P_{t-1} plus a constant by OLS.

e) The coefficient estimates will be the same. The t-values will not be because the standard errors in part (d) are not correct 2SLS standard errors.