Q1:

$$X = \begin{bmatrix} 1 & x_1 \\ \vdots & \vdots \\ n & x_n \end{bmatrix}, \quad Y = \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix}$$

k=2:

$$X'X = \begin{bmatrix} 1 & 1 \\ x_1 & x_2 \end{bmatrix} \cdot \begin{bmatrix} 1 & x_1 \\ 1 & x_2 \end{bmatrix} = \begin{bmatrix} n & \Sigma x_i \\ \Sigma x_i & \Sigma x_i^2 \end{bmatrix}$$

$$X'Y = \begin{bmatrix} 1 & 1 \\ x_1 & x_2 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} \Sigma y_i \\ \Sigma x_i y_i \end{bmatrix}$$

from p.29.

$$b = (X'X)^{-1}(X'Y) \qquad (X'X)^{-1} = \frac{1}{n \cdot \Sigma x_i^2 - (\Sigma x_i)^2} \begin{bmatrix} \Sigma x_i^2 & -\Sigma x_i \\ -\Sigma x_i & n \end{bmatrix}$$

$$\Rightarrow \frac{1}{n \cdot \Sigma x_i^2 - (\Sigma x_i)^2} \begin{bmatrix} \Sigma x_i^2 & -\Sigma x_i \\ -\Sigma x_i & n \end{bmatrix} \cdot \begin{bmatrix} \Sigma y_i \\ \Sigma x_i y_i \end{bmatrix}$$

$$= \frac{1}{n \cdot \Sigma x_i^2 - (\Sigma x_i)^2} \begin{bmatrix} \Sigma x_i^2 y_i - \Sigma x_i \cdot \Sigma x_i y_i \\ -\Sigma x_i \cdot \Sigma y_i + n \cdot \Sigma x_i y_i \end{bmatrix}$$

$$\Rightarrow \hat{b}_2^{OLS} = \frac{n \cdot \Sigma x_i y_i - \Sigma x_i y_i}{n \cdot \Sigma x_i^2 - (\Sigma x_i)^2} = \frac{\Sigma x_i y_i - \frac{\Sigma x_i y_i}{n}}{\Sigma x_i^2 - \frac{\Sigma (x_i)^2}{n}}$$

$$= \frac{\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n} (x_i - \bar{x})^2}$$

#

$$\hat{b}_1^{OLS} = \frac{\sum x_i^2 y_i - \sum x_i \cdot \sum x_i y_i}{n \cdot \sum x_i^2 - (\sum x_i)^2}$$

$$= \frac{\bar{y} \cdot n \cdot \sum x_i^2 - n \cdot \bar{x} \cdot \sum x_i y_i}{n \cdot \sum x_i^2 - n \cdot \bar{x}^2}$$

$$= \frac{\bar{y} \cdot \sum x_i^2 - \bar{x} \cdot \sum x_i y_i}{\sum x_i^2 - \bar{x}^2}$$

$$= \frac{\sum x_i^2 \bar{y} - n \cdot \bar{x}^2 \cdot \bar{y}}{\sum (x_i - \bar{x})^2} - \bar{x} \, \frac{\sum x_i y_i - n \cdot \bar{x} \bar{y}}{\sum (x_i - \bar{x})^2}$$

$$= \frac{\bar{y}(\sum x_i - \bar{y}^2)}{\sum (x_i - \bar{x})^2} - \bar{x} \cdot \frac{\sum (x_i - \bar{x}) \sum (y_i - \bar{y})}{\sum (x_i - \bar{x})^2}$$

$$= \bar{y} - \bar{x} \cdot \hat{b}_2$$

#

Q2.

$K=2,$

$$Var(b) = \sigma^2 \cdot (X'X)^{-1}$$

$$= \sigma^2 \frac{1}{n \cdot \Sigma x_i^2 - (\Sigma x_i)^2} \begin{bmatrix} \Sigma x_i^2 & -\Sigma x_i \\ -\Sigma x_i & n \end{bmatrix}$$

$$Var(\hat{b}_1 | x) = \frac{\sigma^2 \cdot \Sigma x_i^2}{n \cdot \Sigma x_i^2 - (\Sigma x_i)^2} = \frac{\sigma^2 \cdot \Sigma x_i^2}{n \cdot \left(\Sigma x_i^2 - \frac{(\Sigma x_i)^2}{n}\right)} = \frac{\sigma^2 \cdot \Sigma x_i^2}{n \cdot \sum_{i=1}^{n} (x_i - \bar{x})^2} \#$$

$$Var(\hat{b}_2 | x) = \frac{\sigma^2 \cdot n}{n \cdot \Sigma x_i^2 - (\Sigma x_i)^2} = \frac{\sigma^2 \cdot n}{n \cdot \Sigma (x_i - \bar{x})^2} = \frac{\sigma^2}{\Sigma (x_i - \bar{x})^2} \#$$

$$cov(\hat{b}_1, \hat{b}_2 | x) = \frac{\sigma^2 \cdot -\Sigma x_i}{n \cdot \Sigma x_i^2 - (\Sigma x_i)^2} = \frac{-\sigma^2 \cdot n \cdot \bar{x}}{n \cdot \Sigma (x_i - \bar{x})^2} = \frac{-\sigma^2 \cdot \bar{x}}{\Sigma (x_i - \bar{x})^2}$$

$$= \frac{-\bar{x}}{\Sigma (x_i - \bar{x})^2} \cdot \sigma^2$$

5.3

a (i) t-statistic for b_1:

$$\frac{b_1}{se(\hat{b_1})} = \frac{1.4515}{2.2019} = 0.6592. \quad \#$$

(ii) The standard error for b_2:

$$5.7103 = \frac{2.7648}{se(\hat{b_2})}, \quad se(\hat{b_2}) = 0.4842$$

(iii) The estimated b_3:

$$\hat{b_3} = t\text{-statistic}(\hat{b_3}) \cdot se(\hat{b_3})$$

$$= -3.9376 \cdot 0.3695$$

$$= -1.4549 \quad \#$$

(iv) $$R^2 = 1 - \frac{SSR}{SST}, \quad SST = (n-1) \cdot \hat{s}^2_y$$

$$= 1 - \frac{46221.62}{6.39547^2 \times (1200-1)} = 0.0575 \quad \#$$

(v) $$\hat{\sigma}^2 = \frac{SSR}{n-K} = \frac{46221.62}{1196}, \quad \hat{\sigma} = 6.2167 \quad \#$$

b. $b_2 = 2.7648$.

when total expenditure increase by 1%

the WALC will increase by 0.027648%

holding other variable constant.

$b_3 = -1.4549$.

when number of children increase 1 unit,

the WALC will decrease by 1.4549%

holding other variable constant.

$b_4 = -0.1503$.

when household age increase 1 unit.

the WALC will decrease 0.1503%

holding other variable constant.

c. 95% interval estimate for β_4

$\Rightarrow [\beta_4 - t_{0.025}(n-k) \cdot se(\hat{b_4}),\ \beta_4 + t_{0.025}(n-k)\, se \cdot \hat{b_4}]$

$\Rightarrow [-0.1503 - 1.96 \cdot 0.0235,\ -0.1503 + 1.96 \cdot 0.0235]$

$\Rightarrow [-0.19636,\ -0.10424]$

當 household head AGE increase 1 unit, then the WALC have 95% confidence will change between -0.19636% ~ -0.10424% #

(d) ln(TOTEXP)、NK、AGE are significant at 5% level, because their p-value are less than 0.05, but the intercept is not significant at 5% level #

(e) $H_0: b_3 = -2$

$H_a: b_3 \neq -2$

$\alpha = 0.05$

test-statistic: $\dfrac{\hat{b}_3 - 2}{se(\hat{b}_3)} \overset{H_0}{\sim} t(n-k)$

$RR = \{ |T| \geq t_{0.025}(1196) \}$

$= |T| \geq 1.96.$

$T_0 = \dfrac{-1.4549-(-2)}{0.3695} = 1.4752$

$T_0 \notin RR$, do not reject H_0

there is no evidence to suggest that having an extra child leads to a decline in the alcohol budget share that is different from 2%. #

5.23)

a. the sign of b_2 is expected to be negative, because as the number of grams in a giver sale increase, the price per gram should decrease, implying a discount for larger sales.

the sign of b_3 is expected to be positive, because with the purer the purity, the higher the price.

the sign of b_4 is expected to be uncertain, because it depend on how demand and supply are changing over time.

b.

```
Residuals:
    Min      1Q  Median      3Q 
-43.479 -12.014  -3.743  13.969 
    Max 
 43.753 

Coefficients:
            Estimate Std. Error
(Intercept) 90.84669    8.58025
quant       -0.05997    0.01018
qual         0.11621    0.20326
trend       -2.35458    1.38612
            t value Pr(>|t|)    
(Intercept)  10.588 1.39e-14 ***
quant        -5.892 2.85e-07 ***
qual          0.572   0.5700    
trend        -1.699   0.0954 .  
---
Signif. codes:  
  0 '***' 0.001 '**' 0.01 '*'
  0.05 '.' 0.1 ' ' 1

Residual standard error: 20.06 on 52 degrees of freedom
Multiple R-squared:  0.5097,	Adjusted R-squared:  0.4814 
F-statistic: 18.02 on 3 and 52 DF,  p-value: 3.806e-08
```

$\widehat{price} = 90.8467 - 0.05997\, QUANT + 0.11621\, QUAL - 2.35458\, TREND$

the estimated value for $\beta_2, \beta_3, \beta_4$ are -0.05997, 0.11621, -2.35458

β_2 implys a quantity increase by 1 unit, the price will decrease by 0.05997 (per gram), holding other variable constant.

β_3 implys a quality increase by 1%, the price will increase by 0.11621 (per gram), holding othe variable constant.

β_4 implys supply has been increasing fuster than demand. so the sign of β_4 is negative.

c. multiple R- squared is 0.5097 #

d

$H_0: \beta_2 \geq 0$

$H_a: \beta_2 < 0$

$\alpha = 0.05$

test statistic $= \dfrac{\hat{\beta_2}}{se(\hat{\beta_2})} \overset{H_0}{\sim} t(n-k)$

RR: $T < t_{0.05}(52) = -1.675$

Test statistic $= -5.892$, $\in$ RR. reject H_0.

Sellers are willing to accept a lower price if they can make sales in larger quantity.

e. $H_0: \beta_3 \leq 0$

$H_a: \beta_3 > 0$

$\alpha = 0.05$

test statistic $= \dfrac{\hat{\beta}_3}{se(\hat{\beta}_3)} \overset{H_0}{\sim} t(n-k)$

RR: $T > t_{0.05}(52) = 1.675$

Test statistic $= 0.5717$

$\notin$ RR, do not reject Ho,

we can't conclude that a premium is paid for better quality cocaine.

(f) the average annual change in the cocaine price is $b_4 = -2.3546$. A possible reason for a decreasing price is the supplier can produce more cocaine at the same cost, so the sign of β_4 is negative.