10.18 Consider the data file *mroz* on working wives. Use the 428 observations on married women who participate in the labor force. In this exercise, we examine the effectiveness of a parent's college education as an instrumental variable.

a. Create two new variables. *MOTHERCOLL* is a dummy variable equaling one if $MOTHEREDUC > 12$, zero otherwise. Similarly, *FATHERCOLL* equals one if $FATHEREDUC > 12$ and zero otherwise. What percentage of parents have some college education in this sample?

b. Find the correlations between *EDUC*, *MOTHERCOLL*, and *FATHERCOLL*. Are the magnitudes of these correlations important? Can you make a logical argument why *MOTHERCOLL* and *FATHERCOLL* might be better instruments than *MOTHEREDUC* and *FATHEREDUC*?

c. Estimate the wage equation in Example 10.5 using *MOTHERCOLL* as the instrumental variable. What is the 95% interval estimate for the coefficient of *EDUC*?

d. For the problem in part (c), estimate the first-stage equation. What is the value of the *F*-test statistic for the hypothesis that *MOTHERCOLL* has no effect on *EDUC*? Is *MOTHERCOLL* a strong instrument?

e. Estimate the wage equation in Example 10.5 using *MOTHERCOLL* and *FATHERCOLL* as the instrumental variables. What is the 95% interval estimate for the coefficient of *EDUC*? Is it narrower or wider than the one in part (c)?

f. For the problem in part (e), estimate the first-stage equation. Test the joint significance of *MOTHERCOLL* and *FATHERCOLL*. Do these instruments seem adequately strong?

g. For the IV estimation in part (e), test the validity of the surplus instrument. What do you conclude?

(a)

```
> print(paste("Percentage of mothers with some college education:", percentage_mothercoll, "%"))
[1] "Percentage of mothers with some college education: 12.1495327102804 %"
> print(paste("Percentage of fathers with some college education:", percentage_fathercoll, "%"))
[1] "Percentage of fathers with some college education: 11.6822429906542 %"
```

(b)

EDUC 和 MOTHERCOLL 的相關係數為 0.3595；EDUC 和 FATHERCOLL 的相關係數為 0.3985。

```
> print(correlation_matrix)
                educ mothercoll fathercoll
educ       1.0000000  0.3594705  0.3984962
mothercoll 0.3594705  1.0000000  0.3545709
fathercoll 0.3984962  0.3545709  1.0000000
```

(c) The 95% interval estimate for the coefficient of *EDUC* is [-0.0012, 0.1533].

```
> summary(wage_iv_model)

Call:
ivreg(formula = log(wage) ~ educ + exper + I(exper^2) | MOTHERCOLL +
    exper + I(exper^2), data = mroz1)

Residuals:
     Min       1Q   Median       3Q      Max
-3.08719 -0.32444  0.04147  0.36634  2.35621

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept) -0.1327561  0.4965325  -0.267  0.78932
educ         0.0760180  0.0394077   1.929  0.05440 .
exper        0.0433444  0.0134135   3.231  0.00133 **
I(exper^2)  -0.0008711  0.0004017  -2.169  0.03066 *
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.6703 on 424 degrees of freedom
Multiple R-Squared: 0.147,	Adjusted R-squared: 0.1409
Wald test:   8.2 on 3 and 424 DF,  p-value: 2.569e-05
```

```
> cat("The 95% interval estimate for the coefficient of EDUC is [",round(educ_95percent_interval,4),"]")
The 95% interval estimate for the coefficient of EDUC is [ -0.0012 0.1533 ]
```

(d)

MOTHERCOLL 係數的 t 值是 7.97， F-值是 63.21。F-值遠大於常用的臨界值 10，因此拒絕 MOTHERCOLL 對 EDUC 沒有影響的虛無假設，表示 MOTHERCOLL 是一個強的工具變數。另外，Stock-Yogo 臨界值是一個用於檢測工具變數強度的標準。如果願意接受 10%的拒絕率來進行 5%顯著性水平的檢定，則臨界值為 16.38。MOTHERCOLL 的 F-統計量 63.21 遠大於 16.38，進一步證明它是強的工具變數。

```
> # 設置第一階段回歸模型
> first_stage_model <- lm(educ ~ MOTHERCOLL + exper + I(exper^2), data = mroz1)
>
> # 顯示回歸結果
> summary(first_stage_model)

Call:
lm(formula = educ ~ MOTHERCOLL + exper + I(exper^2), data = mroz1)

Residuals:
    Min      1Q  Median      3Q     Max 
-7.4267 -0.4826 -0.3731  1.0000  4.9353 

Coefficients:
             Estimate Std. Error t value Pr(>|t|)    
(Intercept) 12.079094   0.303118  39.849  < 2e-16 ***
MOTHERCOLL   2.517068   0.315713   7.973 1.46e-14 ***
exper        0.056230   0.042101   1.336    0.182    
I(exper^2)  -0.001956   0.001256  -1.557    0.120    
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 2.133 on 424 degrees of freedom
Multiple R-squared:  0.1347,	Adjusted R-squared:  0.1285 
F-statistic: 21.99 on 3 and 424 DF,  p-value: 2.965e-13

>
> # 計算 F-統計量
> f_statistic <- summary(first_stage_model)$fstatistic
> f_statistic_value <- pf(f_statistic[1], f_statistic[2], f_statistic[3], lower.tail = FALSE)
> cat("F-test statistic for the hypothesis that MOTHERCOLL has no effect on EDUC:", f_test, "\n")
F-test statistic for the hypothesis that MOTHERCOLL has no effect on EDUC: 63.21602
```

(e)

The 95% interval estimate for the coefficient of *EDUC* is [0.0275, 0.1478], which is slightly narrower than (c).

```
> summary(wage_iv_model_ef)

Call:
ivreg(formula = log(wage) ~ educ + exper + I(exper^2) | MOTHERCOLL + 
    FATHERCOLL + exper + I(exper^2), data = mroz1)

Residuals:
     Min       1Q   Median       3Q      Max 
-3.07797 -0.32128  0.03418  0.37648  2.36183 

Coefficients:
              Estimate Std. Error t value Pr(>|t|)   
(Intercept) -0.2790819  0.3922213  -0.712  0.47714   
educ         0.0878477  0.0307808   2.854  0.00453 **
exper        0.0426761  0.0132950   3.210  0.00143 **
I(exper^2)  -0.0008486  0.0003976  -2.135  0.03337 * 
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.6679 on 424 degrees of freedom
Multiple R-Squared: 0.153,	Adjusted R-squared: 0.147 
Wald test: 9.724 on 3 and 424 DF,  p-value: 3.224e-06 

>
> # 提取 95% 置信區間
> educ_95percent_interval_ef <- confint(wage_iv_model_ef, level = 0.95)["educ",]
>
> # 顯示結果
> cat("The 95% interval estimate for the coefficient of EDUC using both MOTHERCOLL and FATHERCOLL is [", round(educ_95percent_interval_ef, 4), "]\n")
The 95% interval estimate for the coefficient of EDUC using both MOTHERCOLL and FATHERCOLL is [ 0.0275 0.1482 ]
```

(f)

MOTHERCOLL 和 FATHERCOLL 兩個工具變數的聯合顯著性 F-檢驗統計量為 56.96，這遠大於臨界值 10。這個值也大大超過了 Stock-Yogo 臨界值，該臨界值是根據樣本大小標準來確定的。如果我們願意接受 10% 的拒絕率來進行 5% 顯著性水平的檢定，臨界值為 19.93，拒絕工具變數是弱的虛無假設。

```
> summary(first_stage_model_ef)

Call:
lm(formula = educ ~ MOTHERCOLL + FATHERCOLL + exper + I(exper^2),
    data = mroz1)

Residuals:
    Min      1Q  Median      3Q     Max
-7.2152 -0.3056 -0.2152  0.7627  5.0620

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) 11.890259   0.290251  40.965  < 2e-16 ***
MOTHERCOLL   1.749947   0.322347   5.429 9.58e-08 ***
FATHERCOLL   2.186612   0.329917   6.628 1.04e-10 ***
exper        0.049149   0.040133   1.225    0.221
I(exper^2)  -0.001449   0.001199  -1.209    0.227
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 2.033 on 423 degrees of freedom
Multiple R-squared:  0.2161,	Adjusted R-squared:  0.2086
F-statistic: 29.15 on 4 and 423 DF,  p-value: < 2.2e-16
```

```
> # 顯示 F-檢驗結果
> print(f_test_result)

Linear hypothesis test:
MOTHERCOLL = 0
FATHERCOLL = 0

Model 1: restricted model
Model 2: educ ~ MOTHERCOLL + FATHERCOLL + exper + I(exper^2)

  Res.Df    RSS Df Sum of Sq      F    Pr(>F)
1    425 2219.2
2    423 1748.3  2    470.88 56.963 < 2.2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

(g)

使用(e)的估計結果獲得了殘差$\widehat{e_{IV}}$。用所有外生變數和工具變數對殘差$\widehat{e_{IV}}$回歸。Sargan 測試的結果 NR^2 為 0.2375，該統計量在虛無假設下服從 $\chi^2_{(1)}$分佈，虛無假設是工具變數有效。5%的臨界值為 3.841＞0.2375，故無法拒絕虛無假設。

```
> summary(first_stage_model_sargan)

Call:
lm(formula = residuals_iv ~ MOTHERCOLL + FATHERCOLL + exper +
    I(exper^2), data = mroz1)

Residuals:
     Min       1Q   Median       3Q      Max
-3.07856 -0.31888  0.03683  0.37513  2.36217

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept) -1.259e-04  9.545e-02  -0.001    0.999
MOTHERCOLL  -4.424e-02  1.060e-01  -0.417    0.677
FATHERCOLL   4.122e-02  1.085e-01   0.380    0.704
exper       -1.304e-04  1.320e-02  -0.010    0.992
I(exper^2)   1.016e-05  3.944e-04   0.026    0.979

Residual standard error: 0.6685 on 423 degrees of freedom
Multiple R-squared:  0.0005551, Adjusted R-squared:  -0.008896
F-statistic: 0.05874 on 4 and 423 DF,  p-value: 0.9936
```

```
> # 計算 Sargan 測試統計量（N*R^2）
> sargan_statistic <- nrow(mroz1) * summary(first_stage_model_sargan)$r.squared
>
> # 顯示 Sargan 測試統計量
> cat("Sargan test statistic (NR^2) is", round(sargan_statistic, 6), "\n")
Sargan test statistic (NR^2) is 0.237585
>
> # 計算臨界值（假設5%的顯著性水平，χ^2 分佈，1自由度）
> critical_value <- qchisq(0.95, df = 1)
>
> # 顯示 Sargan 測試結果
> cat("The 5% critical value for the Sargan test is", round(critical_value, 3), "\n")
The 5% critical value for the Sargan test is 3.841
>
> # 根據 Sargan 測試統計量與臨界值進行判斷
> if (sargan_statistic > critical_value) {
+     cat("We reject the null hypothesis that the surplus IV is valid.\n")
+ } else {
+     cat("We fail to reject the null hypothesis that the surplus IV is valid.\n")
+ }
We fail to reject the null hypothesis that the surplus IV is valid.
```

10.20 The CAPM [see Exercises 10.14 and 2.16] says that the risk premium on security j is related to the risk premium on the market portfolio. That is

$$r_j - r_f = \alpha_j + \beta_j(r_m - r_f)$$

where r_j and r_f are the returns to security j and the risk-free rate, respectively, r_m is the return on the market portfolio, and β_j is the jth security's "*beta*" value. We measure the market portfolio using the Standard & Poor's value weighted index, and the risk-free rate by the 30-day LIBOR monthly rate of return. As noted in Exercise 10.14, if the market return is measured with error, then we face an errors-in-variables, or measurement error, problem.

a. Use the observations on Microsoft in the data file *capm5* to estimate the CAPM model using OLS. How would you classify the Microsoft stock over this period? Risky or relatively safe, relative to the market portfolio?

b. It has been suggested that it is possible to construct an IV by ranking the values of the explanatory variable and using the rank as the IV, that is, we sort $(r_m - r_f)$ from smallest to largest, and assign the values $RANK = 1, 2, \ldots, 180$. Does this variable potentially satisfy the conditions IV1–IV3? Create *RANK* and obtain the first-stage regression results. Is the coefficient of *RANK* very significant? What is the R^2 of the first-stage regression? Can *RANK* be regarded as a strong IV?

c. Compute the first-stage residuals, $\hat{v}$, and add them to the CAPM model. Estimate the resulting augmented equation by OLS and test the significance of $\hat{v}$ at the 1% level of significance. Can we conclude that the market return is exogenous?

d. Use *RANK* as an IV and estimate the CAPM model by IV/2SLS. Compare this IV estimate to the OLS estimate in part (a). Does the IV estimate agree with your expectations?

e. Create a new variable $POS = 1$ if the market return $(r_m - r_f)$ is positive, and zero otherwise. Obtain the first-stage regression results using both *RANK* and *POS* as instrumental variables. Test the joint significance of the IV. Can we conclude that we have adequately strong IV? What is the R^2 of the first-stage regression?

f. Carry out the Hausman test for endogeneity using the residuals from the first-stage equation in (e). Can we conclude that the market return is exogenous at the 1% level of significance?

g. Obtain the IV/2SLS estimates of the CAPM model using *RANK* and *POS* as instrumental variables. Compare this IV estimate to the OLS estimate in part (a). Does the IV estimate agree with your expectations?

h. Obtain the IV/2SLS residuals from part (g) and use them (not an automatic command) to carry out a Sargan test for the validity of the surplus IV at the 5% level of significance.

(a)

Microsoft 的 beta 值(即 rmrf) 估計為 1.2018，95%信賴水準下區間估計為 [0.9607882, 1.442891]。因為大於 1，Microsoft 是屬於風險高於市場投組的標的。

```
> summary(msftrf)

Call:
lm(formula = msftrf ~ rmrf, data = capm5)

Residuals:
     Min       1Q   Median       3Q      Max 
-0.27424 -0.04744 -0.00820  0.03869  0.35801 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 0.003250   0.006036   0.538    0.591    
rmrf        1.201840   0.122152   9.839   <2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.08083 on 178 degrees of freedom
Multiple R-squared:  0.3523,	Adjusted R-squared:  0.3486 
F-statistic:  96.8 on 1 and 178 DF,  p-value: < 2.2e-16

> 
> # 計算 beta (rmrf) 的 95% 區間估計
> beta_95ci <- confint(msftrf, level = 0.95)["rmrf", ]
> 
> # 顯示 beta 的 95% 區間估計
> cat("The 95% confidence interval for beta is [", round(beta_95ci[1], 4), ", ", round(beta_95ci[2], 4), "]\n")
The 95% confidence interval for beta is [ 0.9608 ,  1.4429 ]
```

(b)

$R^2 = 0.9126$，RANK 的 t 值為 43.10，對應的 F 值是 1857.61。因此 RANK 是一個很強的工具變數。

```
> capm5$rank = rank(capm5$rmrf)
> rmrf.ols=lm(rmrf~rank , data=capm5)
> summary(rmrf.ols)

Call:
lm(formula = rmrf ~ rank, data = capm5)

Residuals:
      Min        1Q    Median        3Q       Max 
-0.110497 -0.006308  0.001497  0.009433  0.029513 

Coefficients:
              Estimate Std. Error t value Pr(>|t|)    
(Intercept) -7.903e-02  2.195e-03   -36.0   <2e-16 ***
rank         9.067e-04  2.104e-05    43.1   <2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.01467 on 178 degrees of freedom
Multiple R-squared:  0.9126,	Adjusted R-squared:  0.9121 
F-statistic:  1858 on 1 and 178 DF,  p-value: < 2.2e-16
```

(c)

使用對內生性的 Hausman 測試。v_hat 的 t-統計量為-2.04，對應的 p 值為 0.043。在 5%顯著水準（p 值 <0.05）下，v_hat 顯著。在 1% 顯著水準（p 值 <0.01）下，v_hat 不顯著，因為 p 值大於 0.01。在 1%顯著水準下，我們無法拒絕市場回報外生性的虛無假設。

```
> summary(capm5_model)

Call:
lm(formula = msftrf ~ rmrf + v_hat, data = capm5)

Residuals:
     Min       1Q   Median       3Q      Max 
-0.27140 -0.04213 -0.00911  0.03423  0.34887 

Coefficients:
             Estimate Std. Error t value Pr(>|t|)    
(Intercept)  0.003018   0.005984   0.504   0.6146    
rmrf         1.278318   0.126749  10.085   <2e-16 ***
v_hat       -0.874599   0.428626  -2.040   0.0428 *  
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.08012 on 177 degrees of freedom
Multiple R-squared:  0.3672,	Adjusted R-squared:   0.36 
F-statistic: 51.34 on 2 and 177 DF,  p-value: < 2.2e-16
```

(d)

95%的信賴區間是 [1.0274, 1.5292]。所有數值都大於 1，因此我們拒絕微軟的 beta 等於 1 的假設。

```
> summary(iv_model)

Call:
ivreg(formula = msftrf ~ rmrf | rank, data = capm5)

Residuals:
      Min        1Q    Median        3Q       Max 
-0.271625 -0.049675 -0.009693  0.037683  0.355579 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 0.003018   0.006044   0.499    0.618    
rmrf        1.278318   0.128011   9.986   <2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.08092 on 178 degrees of freedom
Multiple R-Squared: 0.3508,	Adjusted R-squared: 0.3472 
Wald test: 99.72 on 1 and 178 DF,  p-value: < 2.2e-16 

> 
> # 計算 beta 的 95% 信賴區間
> beta_95CI = confint(iv_model, level = 0.95)["rmrf", ]
> cat("The 95% confidence interval for beta is: [", round(beta_95CI, 4), "]")
The 95% confidence interval for beta is: [ 1.0274 1.5292 ]>
```

(e)

F 統計量為 951.3 大於 10。RANK 仍然顯著，而 POS 則不顯著。p 值<0.01，拒絕誤差項與解釋變數之間不存在相關性的虛無假設。R^2 = 0.9149，這些工具變數並不弱。

```
> summary(first_stage_rank_pos)

Call:
lm(formula = RP ~ RANK + POS, data = capm5)

Residuals:
      Min        1Q    Median        3Q       Max 
-0.109182 -0.006732  0.002858  0.008936  0.026652 

Coefficients:
              Estimate Std. Error t value Pr(>|t|)    
(Intercept) -0.0804216  0.0022622  -35.55   <2e-16 ***
RANK         0.0009819  0.0000400   24.55   <2e-16 ***
POS         -0.0092762  0.0042156   -2.20   0.0291 *  
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.01451 on 177 degrees of freedom
Multiple R-squared:  0.9149,	Adjusted R-squared:  0.9139 
F-statistic: 951.3 on 2 and 177 DF,  p-value: < 2.2e-16
```

(f)

v_hat2(residuals_first_stage)的 p 值大於 0.01，無法拒絕 market return 為外生性的假設。

```
> summary(capm5_model)

Call:
lm(formula = msftrf ~ rmrf + residuals_first_stage, data = capm5)

Residuals:
     Min       1Q   Median       3Q      Max 
-0.27132 -0.04261 -0.00812  0.03343  0.34867 

Coefficients:
                       Estimate Std. Error t value Pr(>|t|)    
(Intercept)            0.003004   0.005972   0.503   0.6157    
rmrf                   1.283118   0.126344  10.156   <2e-16 ***
residuals_first_stage -0.954918   0.433062  -2.205   0.0287 *  
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.07996 on 177 degrees of freedom
Multiple R-squared:  0.3696,	Adjusted R-squared:  0.3625 
F-statistic: 51.88 on 2 and 177 DF,  p-value: < 2.2e-16
```

(g)

beta 係數在 2SLS 模型中的估計值 1.283118 大於 OLS 模型中的估計值 1.20184。如果存在測量誤差，則 OLS 估計量會受到衰減偏誤（偏向下估計）。故此 2SLS 模型中估計值略大是我們所預期的。其 95%的區間估計為[1.032505 , 1.533731]。範圍都大於 1，因此拒絕 Microsoft 的 beta 等於 1 的虛無假設。

```
> summary(capm_iv_model)

Call:
ivreg(formula = msftrf ~ rmrf | rank + POS, data = capm5)

Residuals:
     Min       1Q   Median       3Q      Max 
-0.27168 -0.04960 -0.00983  0.03762  0.35543 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 0.003004   0.006044   0.497     0.62    
rmrf        1.283118   0.127866  10.035   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.08093 on 178 degrees of freedom
Multiple R-Squared: 0.3507,	Adjusted R-squared: 0.347 
Wald test: 100.7 on 1 and 178 DF,  p-value: < 2.2e-16
```

```
> summary(ols_model)

Call:
lm(formula = msftrf ~ rmrf, data = capm5)

Residuals:
     Min       1Q   Median       3Q      Max 
-0.27424 -0.04744 -0.00820  0.03869  0.35801 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 0.003250   0.006036   0.538    0.591    
rmrf        1.201840   0.122152   9.839   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.08083 on 178 degrees of freedom
Multiple R-squared:  0.3523,	Adjusted R-squared:  0.3486 
F-statistic:  96.8 on 1 and 178 DF,  p-value: < 2.2e-16
```

(h)

5%顯著水準下，p-value 大於 0.05，故不拒絕工具變數有效的虛無假設。

```
> summary(OLS_eIV)

Call:
lm(formula = residuals_iv ~ rank + POS, data = capm5)

Residuals:
     Min       1Q   Median       3Q      Max 
-0.26914 -0.04702 -0.00801  0.03771  0.35674 

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept) -0.0022220  0.0126326  -0.176    0.861
rank         0.0001370  0.0002234   0.613    0.540
POS         -0.0174499  0.0235409  -0.741    0.460

Residual standard error: 0.08103 on 177 degrees of freedom
Multiple R-squared:  0.003103,  Adjusted R-squared:  -0.008162 
F-statistic: 0.2754 on 2 and 177 DF,  p-value: 0.7596
```

```
> # 輸出 Sargan 統計量與臨界值
> cat("Sargan test statistic (NR^2) is", round(Rsqard*N, 6), "\n")
Sargan test statistic (NR^2) is 0.558463 
> cat("The 5% critical value for the Sargan test is", round(chi_2_critical, 3), "\n")
The 5% critical value for the Sargan test is 3.841 
> 
> # 根據 Sargan 統計量進行假設檢驗
> if (Rsqard*N > chi_2_critical) {
+     cat("We reject the null hypothesis that the surplus IV is valid.\n")
+ } else {
+     cat("We fail to reject the null hypothesis that the surplus IV is valid.\n")
+ }
We fail to reject the null hypothesis that the surplus IV is valid.
```

10.24 Consider the data file *mroz* on working wives. Use the 428 observations on married women who participate in the labor force. In this exercise, we examine the effectiveness of alternative standard errors for the IV estimator. Estimate the model in Example 10.5 using IV/2SLS using both *MOTHEREDUC* and *FATHEREDUC* as IV. These will serve as our baseline results.

a. Calculate the IV/2SLS residuals, $\hat{e}_{IV}$. Plot them versus *EXPER*. Do the residuals exhibit a pattern consistent with homoskedasticity?

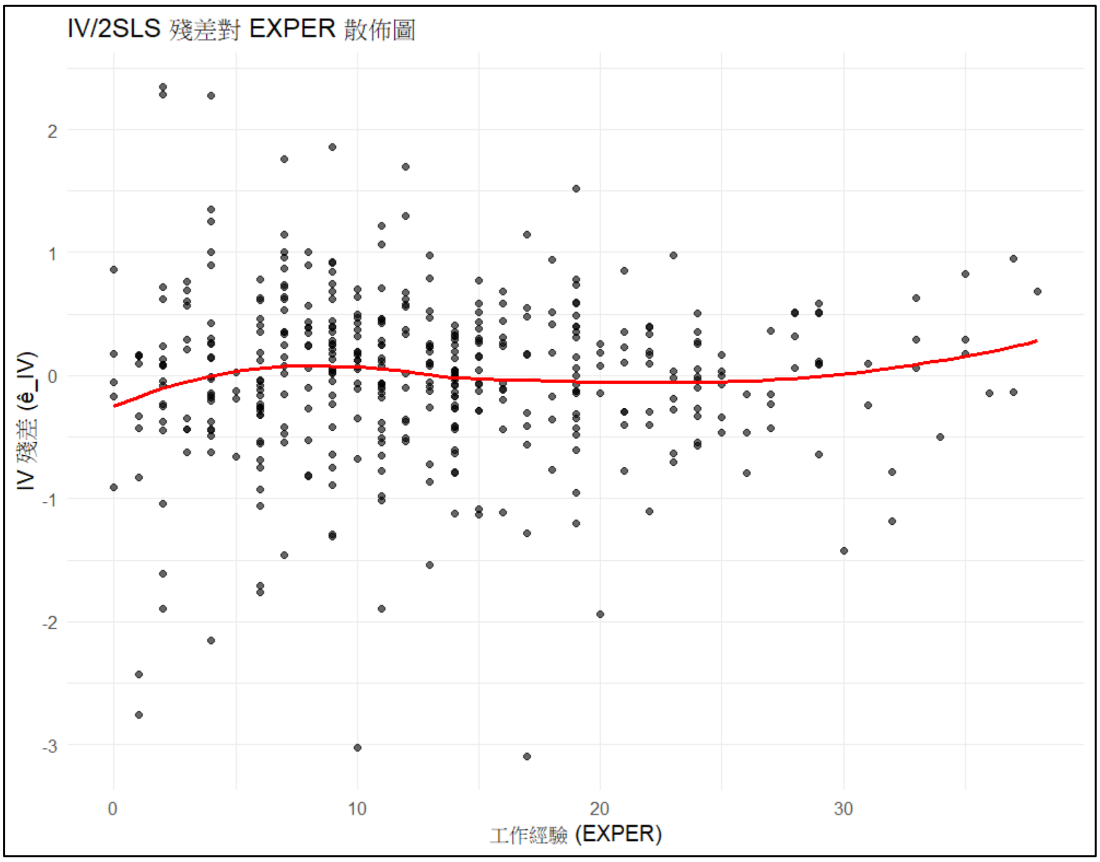
b. Regress $\hat{e}_{IV}^2$ against a constant and *EXPER*. Apply the NR^2 test from Chapter 8 to test for the presence of heteroskedasticity.

c. Obtain the IV/2SLS estimates with the software option for Heteroskedasticity Robust Standard Errors. Are the robust standard errors larger or smaller than those for the baseline model? Compute the 95% interval estimate for the coefficient of *EDUC* using the robust standard error.

d. Obtain the IV/2SLS estimates with the software option for Bootstrap standard errors, using $B = 200$ bootstrap replications. Are the bootstrap standard errors larger or smaller than those for the baseline model? How do they compare to the heteroskedasticity robust standard errors in (c)? Compute the 95% interval estimate for the coefficient of *EDUC* using the bootstrap standard error.

(a)

紅色的擬合曲線顯示殘差隨著工作經驗 (EXPER) 的增加而呈現輕微上升的趨勢。隨著 EXPER 的增長，散佈圖的點似乎變得較為分散，這表示殘差的分布可能會隨著工作經驗變數的增長而變得更加異方差(Heteroskedasticity)。



(b) P-value <0.05, NR^2 is larger than critical value, thus we reject the null hypothesis of homoskedasticity.

```
> summary(model_hetero)

Call:
lm(formula = e_iv_squared ~ exper, data = mroz_working)

Residuals:
    Min      1Q  Median      3Q     Max 
-0.6740 -0.4341 -0.2685 -0.0168  9.2188 

Coefficients:
             Estimate Std. Error t value Pr(>|t|)    
(Intercept)  0.676563   0.096573   7.006 9.65e-12 ***
exper       -0.017303   0.006303  -2.745  0.00631 ** 
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 1.049 on 426 degrees of freedom
Multiple R-squared:  0.01738,	Adjusted R-squared:  0.01507 
F-statistic: 7.535 on 1 and 426 DF,  p-value: 0.006308
```

```
> # 顯示 NR^2 測試結果
> cat("The NR^2 test statistic is", NR_squared_test_stat, "\n")
The NR^2 test statistic is 7.438552 
> 
> # 計算卡方臨界值
> critical_value <- qchisq(0.95, df = 1)  # 5% 顯著性水平，1 自由度
> 
> cat("The 5% critical value for the NR^2 test is", critical_value, "\n")
The 5% critical value for the NR^2 test is 3.841459 
```

```
> cat("R^2 from the regression is:", R_squared, "\n")
R^2 from the regression is: 0.0173798 
> cat("The p-value for EXPER is:", p_value, "\n")
The p-value for EXPER is: 0.006308017 
```

(c)

這題要求使用穩健標準誤來進行調整，以提高結果的可靠性。這會使估算結果對異方差性更具抵抗力。通常穩健標準誤會比基線模型的標準誤略大，因為它對異方差進行了修正。

95% 信賴水準下 EDUC 的區間估計為[-0.00457648, 0.1273697]。左圖呈現工具變數模型中 EDUC 的標準誤為 0.0314367；右圖為穩健模型中 EDUC 標準誤為 0.03366，略大於前者。

```
> summary(iv_model)

Call:
ivreg(formula = log(wage) ~ educ + exper + exper2 | mothereduc + 
    fathereduc + exper + exper2, data = mroz_working)

Residuals:
    Min      1Q  Median      3Q     Max 
-3.0986 -0.3196  0.0551  0.3689  2.3493 

Coefficients:
              Estimate Std. Error t value Pr(>|t|)   
(Intercept)  0.0481003  0.4003281   0.120  0.90442   
educ         0.0613966  0.0314367   1.953  0.05147 . 
exper        0.0441704  0.0134325   3.288  0.00109 **
exper2      -0.0008990  0.0004017  -2.238  0.02574 * 
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.6747 on 424 degrees of freedom
Multiple R-Squared: 0.1357,	Adjusted R-squared: 0.1296 
Wald test: 8.141 on 3 and 424 DF,  p-value: 2.787e-05 
```

```
> print(robust_se)

t test of coefficients:

               Estimate  Std. Error t value Pr(>|t|)   
(Intercept)  0.04810030  0.43377952  0.1109 0.911759   
educ         0.06139663  0.03365975  1.8240 0.068850 . 
exper        0.04417039  0.01576605  2.8016 0.005318 **
exper2      -0.00089897  0.00043908 -2.0474 0.041233 * 
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

> 
> # 計算EDUC係數的95%信賴區間
> educ_coef <- robust_se[2, 1]
> educ_se <- robust_se[2, 2]
> lower_bound <- educ_coef - 1.96 * educ_se
> upper_bound <- educ_coef + 1.96 * educ_se
> 
> # 顯示EDUC係數的95%信賴區間
> cat("95% confidence interval for EDUC: [", lower_bound, ", ", upper_bound, "]", sep = "")
95% confidence interval for EDUC: [-0.00457648, 0.1273697]>
```

(d)

EDUC 的標準誤 0.4379215，其信賴區間為[-0.8248245, 0.8504725]。工具變數模型中 EDUC 的標準誤為 0.0314367，穩健模型中 EDUC 標準誤為 0.03366，和兩者相比 Bootstrap 的標準誤差較大。

```
> cat("Bootstrap 標準誤差:", bootstrap_se, "\n")
Bootstrap 標準誤差: 0.4379215
> cat("95% 信賴區間: [", educ_lower_bound, ", ", educ_upper_bound, "]\n")
95% 信賴區間: [ -0.8248245 ,  0.8504725 ]
```