Q11.1

(a)

$$\begin{cases} ① \; y_1 = \alpha_1 y_2 + e_1 \\ ② \; y_2 = \alpha_2 y_1 + \beta_1 x_1 + \beta_2 x_2 + e_2 \end{cases}$$

①代入②

$$y_2 = \alpha_2 \cdot (\alpha_1 y_2 + e_1) + \beta_1 x_1 + \beta_2 x_2 + e_2$$

$$\Rightarrow y_2 = \alpha_1 \alpha_2 y_2 + \alpha_2 e_1 + \beta_1 x_1 + \beta_2 x_2 + e_2$$

$$\Rightarrow (1 - \alpha_1 \cdot \alpha_2) y_2 = \beta_1 x_1 + \beta_2 x_2 + e_2 + \alpha_2 e_1$$

$$\Rightarrow y_2 = \underbrace{\frac{\beta_1}{(1-\alpha_1\cdot\alpha_2)}}_{\pi_1} x_1 + \underbrace{\frac{\beta_2}{(1-\alpha_1\cdot\alpha_2)}}_{\pi_2} x_2 + \underbrace{\frac{\alpha_2 e_1 + e_2}{(1-\alpha_1\cdot\alpha_2)}}_{v_2} \;\#$$

v_2中包含 e_1，

除非 $\alpha_2 = 0$，否則 $cov(y_2, e_1) \neq 0$ #

(b) 兩個式子中，都有 endogenous variable，使用 OLS 會有 biased 和 inconsistent #

(c) $M=2$，$M-1$ 個外生變數不存在才可以

$\Rightarrow$ ①式 = identified，②式 = not identified #

(d) $E(x_{i1} v_{i1} \mid x) = E(x_{i2} v_{i2} \mid x) = 0$

等同 x_1, x_2 和 error 不相關

$\Rightarrow x_1, x_2$ 是外生變數

$$\Rightarrow E\left[x_{ik}\left(\frac{\alpha_2 e_1 + e_2}{(1-\alpha_1\cdot\alpha_2)}\right) \Big| x\right] = E\left[\frac{\alpha_2}{1-\alpha_1\cdot\alpha_2} e_1 x_{ik} \Big| x\right] + E\left[\frac{1}{1-\alpha_1\cdot\alpha_2} e_2 x_{ik} \Big| x\right] = 0 \;\#$$

(e) minimize $\sum (y_{2i} - \pi_1 x_{i1} - \pi_2 x_{i2})^2$ → 平方誤差和

對 π_1 微分：$\sum x_{i1}(y_{2i} - \pi_1 x_{i1} - \pi_2 x_{i2}) = 0$

對 π_2 微分：$\sum x_{i2}(y_{2i} - \pi_1 x_{i1} - \pi_2 x_{i2}) = 0$

$\Rightarrow$ 和 part (d) 相同 #

(f) $\sum x_{i1}(y_{2i} - \pi_1 x_{i1} - \pi_2 x_{i2}) = 0 \Rightarrow \sum x_{i1} y_2 - \pi_1 \sum x_{i1}^2 - \pi_2 \sum x_{i1} x_{i2} = 0$

$\sum x_{i2}(y_{2i} - \pi_1 x_{i1} - \pi_2 x_{i2}) = 0 \Rightarrow \sum x_{i2} y_2 - \pi_1 \sum x_{i1} x_{i2} - \pi_2 x_{i2}^2 = 0$

代入 $\Rightarrow$ $3 - \hat{\pi}_1 \times 1 - \hat{\pi}_2 \times 0 = 0$，$4 - \hat{\pi}_1 \times 0 - \hat{\pi}_2 \times 1 = 0$ $\Rightarrow$ $\hat{\pi}_1 = 3$，$\hat{\pi}_2 = 4$ #

(g) $\hat{y}_2 = \pi_1 x_1 + \pi_2 x_2$ ⟶ as IV

$= 3x_1 + 4x_2$

$$\sum \hat{y}_{i2}(y_{i1} - \alpha_1 y_{i2}) = 0 \Rightarrow \sum \hat{y}_2 y_1 = \alpha_1 \sum \hat{y}_2 y_2 \Rightarrow \alpha_1 = \frac{\sum \hat{y}_2 y_1}{\sum \hat{y}_2 \cdot y_2} = \frac{\sum(3x_1 + 4x_2)\cdot y_1}{\sum(3x_1+4x_2)\cdot y_2} = \frac{6+12}{9+16} = \frac{18}{25} = 0.72 \;\#$$

(h) 原本的 $\hat{\alpha}_1 = \dfrac{\sum \hat{y}_2 y_1}{\sum \hat{y}_2^2} = \dfrac{\sum \hat{y}_2 y_1}{\sum \hat{y}_2 \cdot y_2}$，$\sum \hat{y}_2 (y_2 - \hat{v}_2) = \sum \hat{y}_2 y_2 - \underbrace{\cancel{\sum \hat{y}_2 \hat{v}_2}}_{=0} = \sum \hat{y}_2 y_2$ #

$\hat{y}$和 error 無關

Q 11.16

(a) reduced-form, Demand = Supply, $\alpha_1 + \alpha_2 P_i + e_{di} = \beta_1 + \beta_2 P_i + \beta_3 W_i + e_{si}$

$(\alpha_2 - \beta_2) P_i = (\beta_1 - \alpha_1) + \beta_3 w_i + e_{si} - e_{di}$

$\Rightarrow$ ① $P_i = \dfrac{(\beta_1 - \alpha_1)}{(\alpha_2 - \beta_2)} + \dfrac{\beta_3}{(\alpha_2 - \beta_2)} \cdot w_i + \dfrac{e_{si} - e_{di}}{(\alpha_2 - \beta_2)}$ ※

$Q_i = \alpha_1 + \alpha_2 \left(\dfrac{(\beta_1 - \alpha_1)}{(\alpha_2 - \beta_2)} + \dfrac{\beta_3}{(\alpha_2 - \beta_2)} \cdot w_i + \dfrac{e_{si} - e_{di}}{(\alpha_2 - \beta_2)} \right) + e_{di}$

$\Rightarrow$ ② $Q_i = \alpha_1 + \dfrac{(\beta_1 - \alpha_1)}{(\alpha_2 - \beta_2)} \cdot \alpha_2 + \dfrac{\beta_3}{(\alpha_2 - \beta_2)} \cdot w_i \cdot \alpha_2 + \dfrac{e_{si} - e_{di}}{(\alpha_2 - \beta_2)} \cdot \alpha_2 + e_{di}$ ※

(b) $M=2$，至少忽略 $M-1$ 個 exogenous variable

Demand equation $\Rightarrow$ identified，可以得出 α_1, α_2.

Supply equation $\Rightarrow$ not identified，無法估出 $\beta_1, \beta_2, \beta_3$

(c) $5 + 0.5w = \alpha_1 + \alpha_2(2.4 + w)$

$\Rightarrow 5 + 0.5w = \alpha_1 + 2.4\alpha_2 + w \cdot \alpha_2$

$\alpha_2 = 0.5$ ※. $(\alpha_1 + 2.4 \times 0.5) = 5$

$\alpha_1 = 3.8$ ※

(d) $\hat{P} = 2.4 + w$, $Q = \alpha_1 + \alpha_2 \hat{P} + e_i$

TABLE 11.7 Data for Exercise 11.16

Q	P	W
4	2	2
6	4	3
9	3	1
3	5	1
8	8	3

$\bar{P} = 4.4$

$\bar{Q} = 6$

$P_i - \bar{P}$	$Q_i - \bar{Q}$
0	-2
1	0
-1	3
-1	-3
1	2

$\hat{\alpha}_2 = \dfrac{\Sigma (P_i - \bar{P})(Q_i - \bar{Q})}{\Sigma (P_i - \bar{P})^2} = \dfrac{-3+3+2}{4} = \dfrac{1}{2}$

$\hat{\alpha}_1 = \bar{Q} - \alpha_2 \bar{P} = 6 - 0.5 \times 4.4 = 3.8$

$\hat{Q} = 3.8 + 0.5P$ ※

Q11.17

(a)

EXAMPLE 11.3 | Klein's Model I

One of the most widely used econometric examples in the past 50 years is the small, three equation, macroeconomic model of the U.S. economy proposed by Lawrence Klein, the 1980 Nobel Prize winner in Economics.[6] The model has three equations, which are estimated, and then a number of macroeconomic identities, or definitions, to complete the model. In all, there are eight endogenous variables and eight exogenous variables.

M = 8, M−1 = 7, at least 7 variables needs to be ommited

$$CN_t = \alpha_1 + \alpha_2(W_{1t} + W_{2t}) + \alpha_3 P_t + \alpha_4 P_{t-1} + e_{1t} \quad (11.17)$$ ⇒ ommit 10 variables

$$I_t = \beta_1 + \beta_2 P_t + \beta_3 P_{t-1} + \beta_4 K_{t-1} + e_{2t} \quad (11.18)$$ ⇒ ommit 11

$$W_{1t} = \gamma_1 + \gamma_2 E_t + \gamma_3 E_{t-1} + \gamma_4 TIME_t + e_{3t} \quad (11.19)$$ ⇒ ommit 11

#

(b)

$$CN_t = \alpha_1 + \alpha_2(W_{1t} + W_{2t}) + \alpha_3 P_t + \alpha_4 P_{t-1} + e_{1t} \quad (11.17)$$ 2 endogenous: W_{1t}, P_t, exclude 5 exogenous

$$I_t = \beta_1 + \beta_2 P_t + \beta_3 P_{t-1} + \beta_4 K_{t-1} + e_{2t} \quad (11.18)$$ 1 endogenous P_t, exclude 5 exogenous

$$W_{1t} = \gamma_1 + \gamma_2 E_t + \gamma_3 E_{t-1} + \gamma_4 TIME_t + e_{3t} \quad (11.19)$$ 1 endogenous E_t, exclude 5 exogenous

皆滿足 #

(c) $W_{1t} = \pi_1 + \pi_2 G_t + \pi_3 W_{2t} + \pi_4 TX_t + \pi_5 TIME_t + \pi_6 P_{t-1} + \pi_7 K_{t-1} + \pi_8 E_{t-1} + v$ #

(d) obtain $\hat{W}_{1t}$ from (c), and similarly obtain $\hat{P}_t$, create $W^*_t = \hat{W}_{1t} + W_{2t}$, 用 W^*_t, $\hat{P}_t$, P_{t-1} 對 CN_t 跑 OLS #

(e) coefficient 估計會相同，但 t-value 不一定，因為 (d) 的 SE 不是正確的 2SLS 的 SE #