15.6 Using the NLS panel data on $N = 716$ young women, we consider only years 1987 and 1988. We are interested in the relationship between ln(*WAGE*) and experience, its square, and indicator variables for living in the south and union membership. Some estimation results are in Table 15.10.

TABLE 15.10 **Estimation Results for Exercise 15.6**

	(1) OLS 1987	(2) OLS 1988	(3) FE	(4) FE Robust	(5) RE
C	0.9348	0.8993	1.5468	1.5468	1.1497
	(0.2010)	(0.2407)	(0.2522)	(0.2688)	(0.1597)
EXPER	0.1270	0.1265	0.0575	0.0575	0.0986
	(0.0295)	(0.0323)	(0.0330)	(0.0328)	(0.0220)
$EXPER^2$	−0.0033	−0.0031	−0.0012	−0.0012	−0.0023
	(0.0011)	(0.0011)	(0.0011)	(0.0011)	(0.0007)
SOUTH	−0.2128	−0.2384	−0.3261	−0.3261	−0.2326
	(0.0338)	(0.0344)	(0.1258)	(0.2495)	(0.0317)
UNION	0.1445	0.1102	0.0822	0.0822	0.1027
	(0.0382)	(0.0387)	(0.0312)	(0.0367)	(0.0245)
N	716	716	1432	1432	1432

f. Column (5) contains the random effects estimates. Which coefficients, apart from the intercepts, show the most difference from the fixed effects estimates? Use the Hausman test statistic (15.36) to test whether there are significant differences between the random effects estimates and the fixed effects estimates in column (3) (Why that one?). Based on the test results, is random effects estimation in this model appropriate?

(f)

變數 EXPER 和 EXPER² 的隨機效果估計係數幾乎是固定效果估計係數的兩倍，分別為 1.71 倍和 1.92 倍。SOUTH 的隨機效果估計係數約為固定效果的 0.71 倍，而 UNION 的隨機效果估計係數約為固定效果的 1.25 倍。

$$t = \frac{b_{FE,k} - b_{RE,k}}{\left[\widehat{\operatorname{var}}(b_{FE,k}) - \widehat{\operatorname{var}}(b_{RE,k})\right]^{1/2}} = \frac{b_{FE,k} - b_{RE,k}}{\left[\operatorname{se}(b_{FE,k})^2 - \operatorname{se}(b_{RE,k})^2\right]^{1/2}} \tag{15.36}$$

Hausman t 統計量：

	t-value
EXPER	-1.67
EXPER²	1.29
SOUTH	-0.77
UNION	-1.06

只有 **EXPER** 係數在 10%的顯著水準下顯示有顯著差異。因此內生性的證據較弱，無法根據這些結果強烈反對使用隨機效果估計量。故使用隨機效果模型是適當的。

15.20 This exercise uses data from the STAR experiment introduced to illustrate fixed and random effects for grouped data. In the STAR experiment, children were randomly assigned within schools into three types of classes: small classes with 13–17 students, regular-sized classes with 22–25 students, and regular-sized classes with a full-time teacher aide to assist the teacher. Student scores on achievement tests were recorded as well as some information about the students, teachers, and schools. Data for the kindergarten classes are contained in the data file *star*.

a. Estimate a regression equation (with no fixed or random effects) where *READSCORE* is related to *SMALL*, *AIDE*, *TCHEXPER*, *BOY*, *WHITE_ASIAN*, and *FREELUNCH*. Discuss the results. Do students perform better in reading when they are in small classes? Does a teacher's aide improve scores? Do the students of more experienced teachers score higher on reading tests? Does the student's sex or race make a difference?

b. Reestimate the model in part (a) with school fixed effects. Compare the results with those in part (a). Have any of your conclusions changed? [*Hint*: specify *SCHID* as the cross-section identifier and *ID* as the "time" identifier.]

c. Test for the significance of the school fixed effects. Under what conditions would we expect the inclusion of significant fixed effects to have little influence on the coefficient estimates of the remaining variables?

d. Reestimate the model in part (a) with school random effects. Compare the results with those from parts (a) and (b). Are there any variables in the equation that might be correlated with the school effects? Use the LM test for the presence of random effects.

e. Using the t-test statistic in equation (15.36) and a 5% significance level, test whether there are any significant differences between the fixed effects and random effects estimates of the coefficients on *SMALL*, *AIDE*, *TCHEXPER*, *WHITE_ASIAN*, and *FREELUNCH*. What are the implications of the test outcomes? What happens if we apply the test to the fixed and random effects estimates of the coefficient on *BOY*?

f. Create school-averages of the variables and carry out the Mundlak test for correlation between them and the unobserved heterogeneity.

(d)

使用隨機效果模型。如下頁圖表。

OLS 模型假設所有個體是同質的，忽略了個體之間可能存在的異質性或不可觀察因素。而相較之下，**隨機效果模型**允許存在個體間不可觀察的隨機異質性，且假設這些異質性與自變數不相關。

(i) 在 STAR 計畫中，學生是在同一學校內隨機分配的，但不是跨學校分配。部分學校位於富裕的學區，部分學校位於較貧困的學區。家庭收入會影響家庭的學習環境和資源，這在一定程度上由「免費午餐（FREELUNCH）」指標所反映。此外，富裕的學區可能支付更高的薪水，且擁有能力更強和經驗更豐富的教師。

(ii) LM 統計量為 2247.96，遠大於標準常態分布的臨界值 1.96。LM 平方統計量為 6677.42，也大於卡方分布自由度 1 下的臨界值 3.84。因此，有證據顯示學校間存在未觀察的異質性。

```
> summary(re_model)
Oneway (individual) effect Random Effect Model
   (Swamy-Arora's transformation)

Call:
plm(formula = readscore ~ small + aide + tchexper + boy + white_asian +
    freelunch, data = pdata, model = "random")

Unbalanced Panel: n = 79, T = 34-137, N = 5766

Effects:
                 var std.dev share
idiosyncratic 751.43   27.41 0.829
individual    155.31   12.46 0.171
theta:
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
 0.6470  0.7225  0.7523  0.7541  0.7831  0.8153

Residuals:
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
-97.483 -17.236  -3.282   0.037  12.803 192.346

Coefficients:
              Estimate Std. Error  z-value  Pr(>|z|)
(Intercept) 436.126774   2.064782 211.2217 < 2.2e-16 ***
small         6.458722   0.912548   7.0777 1.466e-12 ***
aide          0.992146   0.881159   1.1260    0.2602
tchexper      0.302679   0.070292   4.3060 1.662e-05 ***
boy          -5.512081   0.727639  -7.5753 3.583e-14 ***
white_asian   7.350477   1.431376   5.1353 2.818e-07 ***
freelunch   -14.584332   0.874676 -16.6740 < 2.2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Total Sum of Squares:    6158000
Residual Sum of Squares: 4332100
R-Squared:      0.29655
Adj. R-Squared: 0.29582
Chisq: 493.205 on 6 DF, p-value: < 2.22e-16
```

```
> plmtest(fe_model, type = "bp")

        Lagrange Multiplier Test - (Breusch-Pagan)

data:  readscore ~ small + aide + tchexper + boy + white_asian + freelunch
chisq = 6677.4, df = 1, p-value < 2.2e-16
alternative hypothesis: significant effects
```

(e)

$$t = \frac{b_{FE,k} - b_{RE,k}}{\sqrt{\mathrm{var}(b_{FE,k}) - \mathrm{var}(b_{RE,k})}} = \frac{b_{FE,k} - b_{RE,k}}{\sqrt{se(b_{FE,k})^2 - se(b_{RE,k})^2}}$$

- $b_{FE,k}$：第 k 個變數的固定效果模型係數
- $b_{RE,k}$：第 k 個變數的隨機效果模型係數
- $\mathrm{var}(b)$ 與 $se(b)$ 分別為該係數的變異數與標準誤
- 用此統計量檢定兩個估計值是否有系統性差異（檢定統計量符合標準常態分布）

檢驗固定效果（FE）與隨機效果（RE）模型中各個變數係數是否有顯著差異。在 5% significance level 下，t 值都沒有大於臨界值 1.96，都不顯著，代表固定效果（FE）與隨機效果（RE）模型估計的係數沒有統計上顯著差異。

vars	t-value
SMALL	1.38
AIDE	0.15
TCHEXPER	-2.08
WHITE_ASIAN	1.23
FREELUNCH	-0.10

(f)

```
Linear hypothesis test:
small_avg = 0
aide_avg = 0
tchexper_avg = 0
boy_avg = 0
white_asian_avg = 0
freelunch_avg = 0

Model 1: restricted model
Model 2: readscore ~ small + aide + tchexper + boy + white_asian + freelunch +
    small_avg + aide_avg + tchexper_avg + boy_avg + white_asian_avg +
    freelunch_avg

Note: Coefficient covariance matrix supplied.

  Res.Df Df      F Pr(>F)
1   5759
2   5753  6 2.2413 0.0366 *
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

虛無假設（H0）：所有學校平均變數（small_avg、aide_avg、tchexper_avg、boy_avg、white_asian_avg、freelunch_avg）的係數同時等於 0。

對立假設（H1）：至少有一個學校平均變數的係數不為 0。

p-value = 0.0366 < 0.05，**拒絕虛無假設**。表示學校平均變數的係數**不全是 0**，即這些學校平均特徵對解釋變數（讀寫分數）有顯著影響。Mundlak 測試結果顯示學校間平均差異是顯著的，代表個體特有的未觀察異質性和解釋變數之間有相關性，所以不適合使用隨機效果模型，要使用固定效果模型來控制這些未觀察到的異質性。

15.17 The data file *liquor* contains observations on annual expenditure on liquor (*LIQUOR*) and annual income (*INCOME*) (both in thousands of dollars) for 40 randomly selected households for three consecutive years.

a. Create the first-differenced observations on *LIQUOR* and *INCOME*. Call these new variables *LIQUORD* and *INCOMED*. Using OLS regress *LIQUORD* on *INCOMED* without a constant term. Construct a 95% interval estimate of the coefficient.

b. Estimate the model $LIQUOR_{it} = \beta_1 + \beta_2 INCOME_{it} + u_i + e_{it}$ using random effects. Construct a 95% interval estimate of the coefficient on *INCOME*. How does it compare to the interval in part (a)?

c. Test for the presence of random effects using the LM statistic in equation (15.35). Use the 5% level of significance.

d. For each individual, compute the time averages for the variable *INCOME*. Call this variable *INCOMEM*. Estimate the model $LIQUOR_{it} = \beta_1 + \beta_2 INCOME_{it} + \gamma INCOMEM_i + c_i + e_{it}$ using the random effects estimator. Test the significance of the coefficient γ at the 5% level. Based on this test, what can we conclude about the correlation between the random effect u_i and *INCOME*? Is it OK to use the random effects estimator for the model in (b)?

(b)

$$\widehat{LIQUORD} = 0.9690 + 0.026576 \cdot \mathrm{INCOMED}$$

以 95%的信心水準估計，每增加 1000 美元的收入，家庭在酒類上的支出將增加介於 12.83 美元至 40.32 美元之間。隨機效果估計的係數略小於差分估計的係數，但隨機效果估計的標準誤約 first-differenced 標準誤的 25%，因此具有統計上的顯著性。

```
> summary(re_model)
Oneway (individual) effect Random Effect Model
   (Swamy-Arora's transformation)

Call:
plm(formula = liquor ~ income, data = pdata, model = "random")

Balanced Panel: n = 40, T = 3, N = 120

Effects:
                 var std.dev share
idiosyncratic 0.9640  0.9819 0.571
individual    0.7251  0.8515 0.429
theta: 0.4459

Residuals:
     Min.   1st Qu.    Median   3rd Qu.      Max.
-2.263634 -0.697383  0.078697  0.552680  2.225798

Coefficients:
             Estimate Std. Error z-value  Pr(>|z|)
(Intercept) 0.9690324  0.5210052  1.8599 0.0628957 .
income      0.0265755  0.0070126  3.7897 0.0001508 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Total Sum of Squares:    126.61
Residual Sum of Squares: 112.88
R-Squared:      0.1085
Adj. R-Squared: 0.10095
Chisq: 14.3618 on 1 DF, p-value: 0.00015083
```

```
> # 建立95%信賴區間
> confint(re_model, level = 0.95)
                  2.5 %     97.5 %
(Intercept) -0.05211904 1.99018381
income       0.01283111 0.04031983
```

(c)

$$\mathrm{LM} = \sqrt{\frac{NT}{2(T-1)}}\left\{\frac{\sum_{i=1}^{N}\left(\sum_{t=1}^{T}\hat{e}_{it}\right)^2}{\sum_{i=1}^{N}\sum_{t=1}^{T}\hat{e}_{it}^2} - 1\right\}$$

```
> cat("LM Statistic:", lm_test_result$statistic, "\n")
LM Statistic: 20.67973
> cat("p-value:", lm_test_result$p.value, "\n")
p-value: 5.428756e-06
> cat("df:", lm_test_result$parameter, "\n")
df: 1
```

LM 統計量的意義是檢查所有殘差的總和相對於殘差平方和的比例，判斷是否存在系統性的個體異質性（隨機效果）。虛無假設為無 random effects（pooled OLS 模型適用），$\sigma^2_u=0$；對立假設存在 random effects（pooled OLS 不適合），$\sigma^2_u>0$。因為 p 值小於 0.05，故拒絕虛無假設。

(d)

$$LIQUOR_{it} = \beta_1 + \beta_2 INCOME_{it} + \gamma INCOMEM_i + c_i + e_{it}$$

迴歸結果：

$$\widehat{LIQUOR}_{it} = 0.91633 + 0.02074 INCOME_{it} + 0.00658 INCOMEM_i$$
$$(\text{se}) \qquad\qquad\qquad\qquad\qquad\qquad (0.02220)$$

虛無假設 γ=0，表示隨機效果與 INCOME 無相關性，RE 模型估計是合理且有效的；若拒絕虛無假設 γ=0，表示隨機效果 ci 與 INCOME 存在相關，隨機效果模型假設不成立，應使用固定效果模型。

由以下結果可以發現，INCOMEM 的 p 值很大，且 INCOMEM 係數的 t 值為 0.30(=0.006579/0.0222058)，這在統計上是不顯著的，無法拒絕 γ=0 的虛無假設，表示隨機效果 ci 與 INCOME 沒有顯著相關性。

```
> summary(re_model)
Oneway (individual) effect Random Effect Model
   (Swamy-Arora's transformation)

Call:
plm(formula = liquor ~ income + INCOMEM, data = pdata, model = "random")

Balanced Panel: n = 40, T = 3, N = 120

Effects:
                 var std.dev share
idiosyncratic 0.9640  0.9819 0.571
individual    0.7251  0.8515 0.429
theta: 0.4459

Residuals:
     Min.   1st Qu.    Median   3rd Qu.      Max.
-2.300955 -0.703840  0.054992  0.560255  2.257325

Coefficients:
             Estimate Std. Error z-value Pr(>|z|)
(Intercept) 0.9163337  0.5524439  1.6587  0.09718 .
income      0.0207421  0.0209083  0.9921  0.32117
INCOMEM     0.0065792  0.0222048  0.2963  0.76700
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Total Sum of Squares:    126.61
Residual Sum of Squares: 112.79
R-Squared:      0.10917
Adj. R-Squared: 0.093945
Chisq: 14.3386 on 2 DF, p-value: 0.00076987
```