$$K=2 \Rightarrow \beta = \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix} \qquad e = \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_n \end{bmatrix}$$

$$X = \begin{bmatrix} 1 & X_1 \\ 1 & X_2 \\ \vdots & \vdots \\ 1 & X_n \end{bmatrix}$$

$$Y = X\beta + e = \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix} \begin{bmatrix} 1 & X_1 \\ 1 & X_2 \\ \vdots & \vdots \\ 1 & X_n \end{bmatrix} + \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_n \end{bmatrix}$$

$$X'X = \begin{bmatrix} 1, & 1, & \cdots & 1 \\ X_1, & X_2, & \cdots, & X_n \end{bmatrix} \times \begin{bmatrix} 1 & X_1 \\ 1 & X_2 \\ \vdots & \vdots \\ 1 & X_n \end{bmatrix}$$

$$= \begin{bmatrix} n & \sum X_i \\ \sum X_i & \sum X_i^2 \end{bmatrix}$$

$$(X'X)^{-1} = \frac{1}{n\sum X_i^2 - [\sum X_i]^2} \times \begin{bmatrix} \sum X_i^2 & -\sum X_i \\ -\sum X_i & n \end{bmatrix}$$

$$X'Y = \begin{bmatrix} 1, & 1, & \cdots, & 1 \\ X_1, & X_2, & \cdots, & X_n \end{bmatrix} \begin{bmatrix} Y_1 \\ Y_2 \\ \vdots \\ Y_n \end{bmatrix}$$

$$= \begin{bmatrix} \sum Y_i \\ \sum X_i Y_i \end{bmatrix}$$

$$(X'X)^{-1} X'Y = \frac{1}{n\sum X_i^2 - (\sum X_i)^2} \times \begin{bmatrix} \sum X_i^2 & -\sum X_i \\ -\sum X_i & n \end{bmatrix} \begin{bmatrix} \sum Y_i \\ \sum X_i Y_i \end{bmatrix}$$

$$= \frac{1}{n\sum X_i^2 \, (\sum X_i)^2} \begin{bmatrix} \sum X_i^2 \sum Y_i \cdot \sum X_i \sum X_i Y_i \\ -\sum X_i \sum Y_i + n \sum X_i Y_i \end{bmatrix}$$

$$\Rightarrow \beta = \frac{n\sum X_i Y_i - \sum X_i \sum Y_i}{n\sum X_i^2 - (\sum X_i^2)}$$

$$= \frac{\sum X_i Y_i - n\bar{X}\bar{Y}}{\sum X_i^2 - n\bar{X}^2} = \frac{\sum (X_i - \bar{X})(Y_i - \bar{Y})}{\sum (X_i - \bar{X})^2} \quad \#$$

$$\frac{\sum X_i^2 \sum Y_i - \sum X_i \sum X_i Y_i}{n\sum X_i^2 - (\sum X_i)^2}$$

$$= \frac{\sum X_i^2 \bar{Y} - \bar{X}\sum X_i Y_i + n\bar{X}^2\bar{Y} - n\bar{X}^2\bar{Y}}{\sum X_i^2 - n\bar{X}^2}$$

$$= \frac{\bar{Y}\sum X_i^2}{\sum X_i^2 - n\bar{X}^2} - \frac{\bar{X}\sum X_i Y_i}{\sum X_i^2 - n\bar{X}^2}$$

$$= \bar{Y}\left(\frac{\sum X_i^2 - n\bar{X}^2}{\sum X_i^2 - n\bar{X}^2}\right) - \frac{\sum X_i Y_i}{\sum X_i^2 - n\bar{X}^2}\bar{X}$$

$$\frac{\sum X_i^2 \sum Y_i - \sum X_i \sum X_i Y_i}{n \sum X_i^2 - \left(\sum X_i\right)^2}$$

$$= \frac{\frac{1}{n}\sum Y_i \sum X_i^2 - \frac{1}{n}\sum X_i \sum X_i Y_i}{\sum X_i^2 - n \bar{X}^2}$$

$$= \frac{\bar{Y} \sum X_i^2}{\sum X_i^2 - n\bar{X}^2} - \frac{\bar{X} \sum X_i Y_i}{\sum X_i^2 - n\bar{X}^2}$$

$$= \frac{\bar{Y}\cancel{\left(\sum X_i^2 - n\bar{X}^2\right)}}{\cancel{\left[\sum X_i^2 - n\bar{X}^2\right]}} - \frac{\bar{X}\left(\sum X_i Y_i - n\bar{X}\bar{Y}\right)}{\sum X_i^2 - n\bar{X}^2}$$

$$= \bar{Y} - \beta \bar{X}$$

$$Var(\beta) = Var\left(\begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix}\right) = \begin{bmatrix} Var(\beta_1) & Cov(\beta_1, \beta_2) \\ Cov(\beta_1, \beta_2) & Var(\beta_2) \end{bmatrix}$$

$$= \sigma^2 (X'X)^{-1}$$

$$= \sigma^2 \times \frac{1}{nS_{xx}} \begin{bmatrix} \sum X_i^2 & -\sum X_i \\ -\sum X_i & n \end{bmatrix}$$

對照位置可得

$$Var(\beta_1) = \sigma^2 \frac{\sum x_i^2}{n\sum(x_i-\bar{x})^2}$$

$$Var(\beta_2) = \sigma^2 \frac{n}{n\sum(x_i-\bar{x})^2}$$

$$= \frac{\sigma^2}{\sum(x_i-\bar{x})^2}$$

$$Cov(\beta_1, \beta_2) = \sigma^2 \times \frac{-\sum x_i}{n\sum(x_i-\bar{x})^2}$$

$$= -\frac{\sigma^2 \bar{x}}{\sum(x_i-\bar{x})^2}$$

5.3

$$WALC = \beta_1 + \beta_2 \ln TOTEXP + \beta_3 NK + \beta_4 AGE$$

$n = 1200$

Variable	Coefficient	Std. Err.	t. sta.	P
C	1.4515	2.2019	③	0.5099
ln TOTEXP	2.7648	②	5.7103	0.
NK	①	0.3695	-3.9376	0.0001

AGE	-0.1503	0.0235	-6.4019	0.

R^2 ④ $\overline{WALC} = 6.19434$

SSE ⑤ $SD(WALC) = 6.39547$

SSR 46221.62

a.

i. ③ : $\frac{1.4515}{2.2019} = 0.6592$

ii ② : $\frac{2.7648}{5.7103} = 0.4842$

iii ① : $-3.9376 \times 0.3695 = -1.4549$

iv. $S_y^2 = \frac{1}{n-1} \sum (Y_i - \bar{Y})^2 = \frac{SSTO}{1200-1}$

$SSTO = \sum (Y_i - \bar{Y})^2 = 1199 \times 6.39547^2 = 49041.54$

$SSE = SSTO - SSR = 2819.92$

$\Rightarrow R^2 = 1 - \frac{46221.62}{49041.54} = 0.0575 \doteq 5.75\%$

v. $\sqrt{\hat{\sigma}^2} = \sqrt{MSE} = \sqrt{\frac{46221.62}{1200-4}} \approx 6.2167$ #

b. 分別表示邊際變動量，但必須在其它條件不變下

c. $\hat{\beta}_4 \pm t_{0.025}(1200-4) \times SE(\hat{\beta}_4)$

$\Rightarrow -0.1503 \pm 1.96 \times 0.0235$

$\Rightarrow [-0.19636, -0.10424]$

表示 $\hat{\beta}_4$ 顯著 $\neq 0 \Rightarrow$ AGE這個變數對預測 WALL 具重要意義且呈現負相關

d. 截距項沒有，因 P-Value 遠大於 0.05

e.

① $H_0: \hat{\beta}_3 = -2$

$H_a: \hat{\beta}_3 \neq -2$

② $\alpha = 0.05$

③ test statistic : $t_0 = \dfrac{\hat{\beta}_3 - (-2)}{SE(\hat{\beta}_3)} \overset{H_0}{\sim} t_{0.025}(1200-4)$

④ $RR = \{ |t_0| \geq t_{0.025}(1200-4) = 1.96 \}$

⑤ $t_0 = \dfrac{-1.4549+2}{0.3695} = 1.4752 \notin RR$

⑥ Reject H_0: 在 5% 的顯著水準下，樣本並無足夠證據顯示 $\beta_3 = -2$