HW0512

11.28 Supply and demand curves as traditionally drawn in economics principles classes have price (P) on the vertical axis and quantity (Q) on the horizontal axis.

428

a. Rewrite the truffle demand and supply equations in (11.11) and (11.12) with price P on the left-hand side. What are the anticipated signs of the parameters in this rewritten system of equations?

b. Using the data in the file *truffles*, estimate the supply and demand equations that you have formulated in (a) using two-stage least squares. Are the signs correct? Are the estimated coefficients significantly different from zero?

c. Estimate the price elasticity of demand "at the means" using the results from (b).

d. Accurately sketch the supply and demand equations, with P on the vertical axis and Q on the horizontal axis, using the estimates from part (b). For these sketches set the values of the exogenous variables DI, PS, and PF to be $DI^* = 3.5$, $PF^* = 23$, and $PS^* = 22$.

e. What are the equilibrium values of P and Q obtained in part (d)? Calculate the predicted equilibrium values of P and Q using the estimated reduced-form equations from Table 11.2, using the same values of the exogenous variables. How well do they agree?

f. Estimate the supply and demand equations that you have formulated in (a) using OLS. Are the signs correct? Are the estimated coefficients significantly different from zero? Compare the results to those in part (b).

a. 需求方程式 (11.11) 改寫：

$\alpha2Pi=Qi-\alpha1-\alpha3PSi-\alpha4DIi-edi$

$Pi=\alpha21Qi-\alpha2\alpha1-\alpha2\alpha3PSi-\alpha2\alpha4DIi-\alpha21edi$

預期符號：

- $\alpha21$：預期為負號。根據需求法則，價格上升通常會導致需求量下降，反之亦然。因此，價格與需求量之間應為負相關，這意味著 $\alpha2$ 應為負數，所以 $\alpha21$ 也為負數。
- $-\alpha2\alpha3$：預期符號取決於 PSi (替代品價格) 對需求量的影響。如果 PSi 是松露的替代品，其價格上升會導致松露的需求量增加，那麼 $\alpha3$ 應為正數，因此 $-\alpha2\alpha3$ 為正數（因為 $\alpha2$ 是負數）。
- $-\alpha2\alpha4$：預期符號取決於 DIi (可支配收入) 對需求量的影響。如果松露是正常財，可支配收入增加會導致需求量增加，那麼 $\alpha4$ 應為正數，因此 $-\alpha2\alpha4$ 為正數（因為 $\alpha2$ 是負數）。如果松露是劣等財，則預期符號相反。

供給方程式 (11.12) 改寫：

$\beta2Pi=Qi-\beta1-\beta3PFi-esi$

$Pi=\beta21Qi-\beta2\beta1-\beta2\beta3PFi-\beta21esi$

預期符號：

- $\beta21$：預期為正號。根據供給法則，價格上升通常會導致供給量增加，反之亦然。因此，價格與供給量之間應為正相關，這意味著 $\beta2$ 應為正數，所以 $\beta21$ 也為正數。
- $-\beta2\beta3$：預期符號取決於 PFi (生產要素價格) 對供給量的影響。生產要素價格上升通常會導致生產成本增加，從而減少供給量，那麼 $\beta3$ 應為正數，因此 $-\beta2\beta3$ 為負數（因為 $\beta2$ 是正數）。

b. Call:

```
ivreg(formula = p ~ q + ps + di | pf + ps + di, data = truffles)

Residuals:
    Min      1Q  Median      3Q     Max 
-39.661  -6.781   2.410   8.320  20.251 
```

(需求方程式)

```
Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)  -11.428     13.592  -0.841  0.40810    
q             -2.671      1.175  -2.273  0.03154 *  
ps             3.461      1.116   3.103  0.00458 ** 
di            13.390      2.747   4.875 4.68e-05 ***

Diagnostic tests:
                 df1 df2 statistic  p-value    
Weak instruments   1  26     17.48 0.000291 ***
Wu-Hausman         1  25    120.03 4.92e-11 ***
Sargan             0  NA        NA       NA    
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 13.17 on 26 degrees of freedom
Multiple R-Squared: 0.5567, Adjusted R-squared: 0.5056 
Wald test: 17.37 on 3 and 26 DF,  p-value: 2.137e-06 

Call:
ivreg(formula = p ~ q + pf | ps + di + pf, data = truffles)

Residuals:
    Min      1Q  Median      3Q     Max 
-9.7983 -2.3440 -0.6281  2.4350 11.1600 
```

(供給方程式)

```
Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) -58.7982     5.8592  -10.04 1.32e-10 ***
q             2.9367     0.2158   13.61 1.32e-13 ***
```

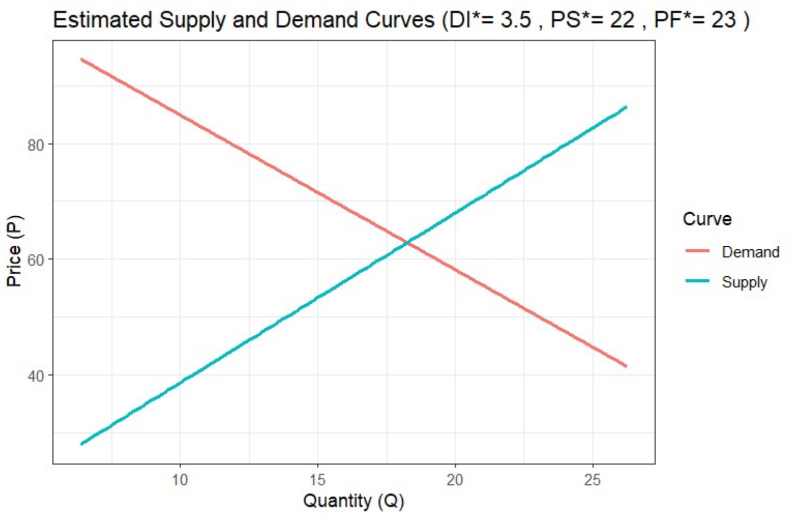
```
pf            2.9585     0.1560   18.97  < 2e-16 ***

Diagnostic tests:
                 df1 df2 statistic  p-value
Weak instruments   2  26    28.934 2.44e-07 ***
Wu-Hausman         1  26     7.051   0.0134 *
Sargan             1  NA     1.544   0.2140
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 4.399 on 27 degrees of freedom
Multiple R-Squared: 0.9486, Adjusted R-squared: 0.9448
Wald test: 232.7 on 2 and 27 DF,  p-value: < 2.2e-16
```

只有 Pf 係數方向與預期方向不一致。

c. 在平均值的需求價格彈性估計值為: -1.272464



d.

```
> # 1. 提取需求方程式的係數
> demand_intercept <- coef(demand_ivreg)["(Intercept)"]
> demand_q_coeff <- coef(demand_ivreg)["q"]
> demand_ps_coeff <- coef(demand_ivreg)["ps"]
> demand_di_coeff <- coef(demand_ivreg)["di"]
>
> # 2. 提取供給方程式的係數
> supply_intercept <- coef(supply_ivreg)["(Intercept)"]
> supply_q_coeff <- coef(supply_ivreg)["q"]
> supply_pf_coeff <- coef(supply_ivreg)["pf"]
>
> # 3. 設定外生變數的特定值
```

```
> ps_star <- 22
> di_star <- 3.5
> pf_star <- 23
>
> # 4. 創建一系列的數量 (Q) 值
> quantity_values <- seq(min(truffles$q), max(truffles$q), length.
out = 100)
>
> # 5. 計算對應的價格 (P) 值
>
> # 計算需求曲線的價格 (P_demand)
> price_demand <- demand_intercept + demand_q_coeff * quantity_valu
es +
+     demand_ps_coeff * ps_star + demand_di_coeff * di_star
>
> # 計算供給曲線的價格 (P_supply)
> price_supply <- supply_intercept + supply_q_coeff * quantity_valu
es +
+     supply_pf_coeff * pf_star
>
> # 6. 創建用於繪圖的資料框
> plot_data <- data.frame(
+     Q = quantity_values,
+     Demand_Price = price_demand,
+     Supply_Price = price_supply
+ )
>
> # 7. 使用 ggplot2 繪製圖形
> ggplot(plot_data, aes(x = Q)) +
+     geom_line(aes(y = Demand_Price, color = "Demand"), linewidth
= 1) +
+     geom_line(aes(y = Supply_Price, color = "Supply"), linewidth
= 1) +
+     labs(
+         title = paste("Estimated Supply and Demand Curves (DI*=",
di_star, ", PS*=", ps_star, ", PF*=", pf_star, ")"),
+         x = "Quantity (Q)",
+         y = "Price (P)",
```

```
+         color = "Curve"
+     ) +
+     theme_bw()
```

e. 從 (d) 部分估計的均衡數量 Q* 為: 18.25021

從 (d) 部分估計的均衡價格 P* 為: 62.84257

使用簡化式方程式預測的均衡數量為: 58.19765

使用簡化式方程式預測的均衡價格為: 183.7225

從結構式模型和簡化式模型得到的均衡價格和數量存在顯著的差異。簡化式模型預測的價格遠高於結構式模型的預測，數量方面也存在較大的差距。這表明在當前的估計下，這兩種模型在預測市場均衡方面並不一致。

```
# 設定外生變數的特定值
ps_star <- 22
di_star <- 3.5
pf_star <- 23

# 計算均衡數量 Q*
numerator_eq <- (supply_intercept - demand_intercept) +
  supply_pf_coeff * pf_star -
  demand_ps_coeff * ps_star -
  demand_di_coeff * di_star
denominator_eq <- demand_q_coeff - supply_q_coeff
equilibrium_quantity_d <- numerator_eq / denominator_eq

# 計算均衡價格 P* (使用需求方程式)
equilibrium_price_d <- demand_intercept + demand_q_coeff * equilib
rium_quantity_d +
  demand_ps_coeff * ps_star + demand_di_coeff * di_star

cat("從 (d) 部分估計的均衡數量 Q* 為:", equilibrium_quantity_d, "\n")
cat("從 (d) 部分估計的均衡價格 P* 為:", equilibrium_price_d, "\n")

# ----- 計算表 11.2 中估計的簡化式方程式的預測均衡價格和數量 -----

# 填寫簡化式價格方程式的係數 (來自表 11.2)
pi_20 <- -32.8124
pi_21 <- 7.9842
pi_22 <- 0.3509
```

```
pi_23 <- 1.7241

# 計算預測的均衡價格（使用簡化式價格方程式）
predicted_equilibrium_price_reduced_form <- pi_20 + pi_21 * ps_sta
r + pi_22 * di_star + pi_23 * pf_star

# 現在我們還需要簡化式數量的方程式。表 11.2a 提供了這個：
# Q = 7.8951 + 3.2434 PS + 0.1425 DI - 0.9370 PF

# 填寫簡化式數量方程式的係數（來自表 11.2a）
pi_10 <- 7.8951
pi_11 <- 3.2434
pi_12 <- 0.1425
pi_13 <- -0.9370

# 計算預測的均衡數量（使用簡化式數量方程式）
predicted_equilibrium_quantity_reduced_form <- pi_10 + pi_11 * ps_
star + pi_12 * di_star + pi_13 * pf_star

cat("使用簡化式方程式預測的均衡數量為:", predicted_equilibrium_quantity
_reduced_form, "\n")
cat("使用簡化式方程式預測的均衡價格為:", predicted_equilibrium_price_re
duced_form, "\n")
```

f. OLS 需求方程式

```
Call:
lm(formula = p ~ q + ps + di, data = truffles)

Residuals:
     Min       1Q   Median       3Q      Max 
-25.0753  -2.7742  -0.4097   4.7079  17.4979 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) -13.6195     9.0872  -1.499   0.1460    
q             0.1512     0.4988   0.303   0.7642    
ps            1.3607     0.5940   2.291   0.0303 *  
di           12.3582     1.8254   6.770 3.48e-07 ***
---
```

```
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 8.814 on 26 degrees of freedom
Multiple R-squared:  0.8013, Adjusted R-squared:  0.7784
F-statistic: 34.95 on 3 and 26 DF,  p-value: 2.842e-09
```

OLS 供給方程式

```
Call:
lm(formula = p ~ q + pf, data = truffles)

Residuals:
    Min      1Q  Median      3Q     Max
-8.4721 -3.3287  0.1861  2.0785 10.7513

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) -52.8763     5.0238  -10.53 4.68e-11 ***
q             2.6613     0.1712   15.54 5.42e-15 ***
pf            2.9217     0.1482   19.71  < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 4.202 on 27 degrees of freedom
Multiple R-squared:  0.9531, Adjusted R-squared:  0.9496
F-statistic: 274.4 on 2 and 27 DF,  p-value: < 2.2e-16
```

使用普通最小平方法 (OLS) 估計的需求方程式顯示，數量 (q) 的係數不顯著且符號為正，這與需求法則的預期相反。替代品價格 (ps) 和可支配收入 (di) 的係數顯著且符號符合預期。

使用 OLS 估計的供給方程式顯示，數量 (q) 和生產要素價格 (pf) 的係數都顯著。數量 (q) 的符號符合供給法則的預期，但生產要素價格 (pf) 的符號為正，與我們在 (a) 部分預期的負號相反。

與 (b) 部分的二階段最小平方法 (2SLS) 結果相比：

- **需求方程式**： 2SLS 估計的數量 (q) 係數顯著且符號為負，更符合理論預期，表明 OLS 可能因內生性而產生偏差。替代品價格 (ps) 和可

支配收入 (di) 的係數在兩種方法中符號一致但估計值不同。OLS 的模型擬合度 (R2) 較高，但可能存在內生性問題。

- **供給方程式：** OLS 和 2SLS 的結果在係數的符號和顯著性上總體一致，但生產要素價格 (pf) 的符號仍然與預期不符。兩種方法的模型擬合度 (R2) 都很高。

總體而言，需求方程式的 OLS 結果可能受到內生性問題的影響，而 2SLS 提供了更符合經濟學理論的估計。供給方程式的 OLS 和 2SLS 結果相對一致，但生產要素價格的預期符號需要進一步探究。這表明在存在潛在內生性的情況下，使用工具變數法（如 2SLS）可能更為可靠。

11.30 Example 11.3 introduces Klein's Model I. Use the data file *klein* to answer the following questions.

a. Estimate the investment function in equation (11.18) by OLS. Comment on the signs and significance of the coefficients.

b. Estimate the reduced-form equation for profits, P_t, using all eight exogenous and predetermined variables as explanatory variables. Test the joint significance of all the variables except lagged profits, P_{t-1}, and lagged capital stock, K_{t-1}. Save the residuals, $\hat{v}_t$ and compute the fitted values, $\hat{P}_t$.

c. The Hausman test for the presence of endogenous explanatory variables is discussed in Section 10.4.1. It is implemented by adding the reduced-form residuals to the structural equation and testing their significance, that is, using OLS estimate the model

$$I_t = \beta_1 + \beta_2 P_t + \beta_3 P_{t-1} + \beta_4 K_{t-1} + \delta \hat{v}_t + e_{2t}$$

Use a t-test for the null hypothesis $H_0: \delta = 0$ versus $H_1: \delta \neq 0$ at the 5% level of significance. By rejecting the null hypothesis, we conclude that P_t is endogenous. What do we conclude from the test? In the context of this simultaneous equations model what result should we find?

d. Obtain the 2SLS estimates of the investment equation using all eight exogenous and predetermined variables as IVs and software designed for 2SLS. Compare the estimates to the OLS estimates in part (a). Do you find any important differences?

e. Estimate the second-stage model $I_t = \beta_1 + \beta_2 \hat{P}_t + \beta_3 P_{t-1} + \beta_4 K_{t-1} + e_{2t}$ by OLS. Compare the estimates and standard errors from this estimation to those in part (d). What differences are there?

f. Let the 2SLS residuals from part (e) be $\hat{e}_{2t}$. Regress these residuals on all the exogenous and predetermined variables. If these instruments are valid, then the R^2 from this regression should be low, and none of the variables are statistically significant. The Sargan test for instrument validity is discussed in Section 10.4.3. The test statistic TR^2 has a chi-square distribution with degrees of freedom equal to the number of "surplus" IVs if the surplus instruments are valid. The investment equation includes three exogenous and/or predetermined variables out of the total of eight possible. There are $L = 5$ external instruments and $B = 1$ right-hand side endogenous variables. Compare the value of the test statistic to the 95th percentile value from the $\chi^2_{(4)}$ distribution. What do we conclude about the validity of the surplus instruments in this case?

```
========== (a) OLS 估計投資函數 ==========
> model_a <- lm(i ~ p + plag + klag, data = klein_clean)
> summary(model_a)


Call:
lm(formula = i ~ p + plag + klag, data = klein_clean)


Residuals:
```

```
     Min       1Q   Median       3Q      Max 
-2.56562 -0.63169  0.03687  0.41542  1.49226 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 10.12579    5.46555   1.853 0.081374 .  
p            0.47964    0.09711   4.939 0.000125 ***
plag         0.33304    0.10086   3.302 0.004212 ** 
klag        -0.11179    0.02673  -4.183 0.000624 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 1.009 on 17 degrees of freedom
Multiple R-squared:  0.9313,	Adjusted R-squared:  0.9192 
F-statistic: 76.88 on 3 and 17 DF,  p-value: 4.299e-10

> cat("\n[說明]：\n")

[說明]：
> cat("β2 (p):", coef(summary(model_a))["p", "Estimate"],
+     ", p 值 =", coef(summary(model_a))["p", "Pr(>|t|)"], "\n")
β2 (p): 0.4796356 , p 值 = 0.0001245554 
> if (coef(summary(model_a))["p", "Pr(>|t|)"] < 0.05) {
+     cat("→ 當期利潤對投資有顯著正向影響。\n")
+ } else {
+     cat("→ 當期利潤對投資影響不顯著，可能存在內生性問題。\n")
+ }
→ 當期利潤對投資有顯著正向影響。
> 
> cat("\n========== (b) Reduced form for p ==========\n")

========== (b) Reduced form for p ==========
> reduced_model <- lm(p ~ cn + w1 + w2 + g + tx + elag, data = klein_cl
ean)
> summary(reduced_model)

Call:
lm(formula = p ~ cn + w1 + w2 + g + tx + elag, data = klein_clean)
```

```
Residuals:
    Min      1Q  Median      3Q     Max 
-1.2985 -0.5904 -0.3992  0.7425  1.7631 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) -17.4835     4.4401  -3.938  0.00149 ** 
cn            1.3686     0.3338   4.100  0.00108 ** 
w1           -0.1908     0.3659  -0.521  0.61021    
w2           -2.0064     0.3049  -6.581 1.23e-05 ***
g             0.4011     0.2952   1.359  0.19579    
tx           -0.4764     0.2448  -1.946  0.07204 .  
elag         -0.3621     0.0714  -5.072  0.00017 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 1.117 on 14 degrees of freedom
Multiple R-squared:  0.9509,	Adjusted R-squared:  0.9299 
F-statistic: 45.23 on 6 and 14 DF,  p-value: 2.254e-08

> klein_clean$phat <- fitted(reduced_model)
> klein_clean$vhat <- resid(reduced_model)
> cat("\n[說明]：\n")

[說明]：
> cat("→ 成功估計利潤的 reduced form。將殘差 vhat 用於 Hausman 檢定。\n")
→ 成功估計利潤的 reduced form。將殘差 vhat 用於 Hausman 檢定。
> 
> cat("\n========== (c) Hausman 檢定 ==========\n")

========== (c) Hausman 檢定 ==========
> hausman_model <- lm(i ~ p + plag + klag + vhat, data = klein_clean)
> summary(hausman_model)

Call:
lm(formula = i ~ p + plag + klag + vhat, data = klein_clean)
```

```
Residuals:
    Min      1Q  Median      3Q     Max 
-1.3649 -0.2950  0.1016  0.4588  1.4473 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 22.06721    6.02749   3.661 0.002109 ** 
p            0.42596    0.08218   5.183 9.06e-05 ***
plag         0.42231    0.08848   4.773 0.000207 ***
klag        -0.17412    0.03036  -5.735 3.07e-05 ***
vhat        -0.84429    0.28248  -2.989 0.008680 ** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.8335 on 16 degrees of freedom
Multiple R-squared:  0.9559,	Adjusted R-squared:  0.9449 
F-statistic:  86.8 on 4 and 16 DF,  p-value: 1.227e-10

> v_pval <- coef(summary(hausman_model))["vhat", "Pr(>|t|)"]
> cat("\n[說明]：\n")

[說明]：
> cat("vhat 的 p 值 =", v_pval, "\n")
vhat 的 p 值 = 0.008679605 
> if (v_pval < 0.05) {
+     cat("→ 拒絕 H0，表示 p 為內生變數，需使用工具變數處理。\n")
+ } else {
+     cat("→ 無法拒絕 H0，p 可視為外生變數。\n")
+ }
→ 拒絕 H0，表示 p 為內生變數，需使用工具變數處理。
> 
> cat("\n========== (d) 2SLS 估計 ==========\n")

========== (d) 2SLS 估計 ==========
> iv_model <- ivreg(i ~ p + plag + klag | cn + w1 + w2 + g + tx + elag +
 plag + klag, data = klein_clean)
> summary(iv_model)
```

```
Call:
ivreg(formula = i ~ p + plag + klag | cn + w1 + w2 + g + tx +
    elag + plag + klag, data = klein_clean)

Residuals:
     Min       1Q   Median       3Q      Max
-2.59471 -0.60181  0.01538  0.40913  1.48083

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 10.53308    5.48931   1.919 0.071955 .
p            0.46642    0.09839   4.740 0.000189 ***
plag         0.34439    0.10179   3.383 0.003532 **
klag        -0.11364    0.02683  -4.236 0.000557 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 1.01 on 17 degrees of freedom
Multiple R-Squared: 0.9313,	Adjusted R-squared: 0.9191
Wald test: 76.16 on 3 and 17 DF,  p-value: 4.628e-10

> cat("\n[說明]：\n")

[說明]：
> cat("→ 使用 2SLS 處理內生性後，p 的係數估計是否改變？\n")
→ 使用 2SLS 處理內生性後，p 的係數估計是否改變？
> cat("β2 (p) =", coef(summary(iv_model))["p", "Estimate"],
+     ", p 值 =", coef(summary(iv_model))["p", "Pr(>|t|)"], "\n")
β2 (p) = 0.4664203 , p 值 = 0.0001892989
> if (coef(summary(iv_model))["p", "Pr(>|t|)"] < 0.05) {
+     cat("→ 經過 IV 修正後，p 對投資具有顯著影響。\n")
+ } else {
+     cat("→ 經過 IV 修正後，p 對投資影響不顯著，可能為識別問題或工具變數較弱。\
n")
+ }
→ 經過 IV 修正後，p 對投資具有顯著影響。
>
> cat("\n========== (e) 用 phat 再估 OLS ==========\n")
```

```
========== (e) 用 phat 再估 OLS ==========
> model_e <- lm(i ~ phat + plag + klag, data = klein_clean)
> summary(model_e)

Call:
lm(formula = i ~ phat + plag + klag, data = klein_clean)

Residuals:
     Min       1Q   Median       3Q      Max 
-1.71203 -0.48861  0.03003  0.60344  1.17491 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 16.01673    4.10725   3.900 0.001153 ** 
phat         0.47712    0.07456   6.399 6.60e-06 ***
plag         0.35960    0.07697   4.672 0.000219 ***
klag        -0.14313    0.02025  -7.069 1.88e-06 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.8531 on 17 degrees of freedom
Multiple R-squared:  0.951,	Adjusted R-squared:  0.9423 
F-statistic: 109.9 on 3 and 17 DF,  p-value: 2.486e-11

> cat("\n[說明]：\n")

[說明]：
> cat("→ 用 phat 替代內生變數 p，可以減少偏誤。查看估計值是否穩定。\n")
→ 用 phat 替代內生變數 p，可以減少偏誤。查看估計值是否穩定。
> 
> cat("\n========== (f) Sargan Test ==========\n")

========== (f) Sargan Test ==========
> klein_clean$ehat2 <- resid(model_e)
> sargan_model <- lm(ehat2 ~ cn + w1 + w2 + g + tx + elag + plag + klag,
 data = klein_clean)
> r2_sargan <- summary(sargan_model)$r.squared
```

```
> TR2 <- nrow(klein_clean) * r2_sargan
> df <- 5 - 1  # L - B = 5 工具變數 - 1 內生變數
> p_value <- 1 - pchisq(TR2, df)
>
> cat("TR^2 =", TR2, ", 自由度 =", df, ", p-value =", p_value, "\n")
TR^2 = 16.45851 , 自由度 = 4 , p-value = 0.002461765
> cat("\n[說明]：\n")

[說明]：
> if (p_value > 0.05) {
+     cat("→ 無法拒絕 H0，工具變數有效，模型無過度識別問題。\n")
+ } else {
+     cat("→ 拒絕 H0，可能有無效工具變數或過度識別問題，應檢查 IV 品質。\n")
+ }
→ 拒絕 H0，可能有無效工具變數或過度識別問題，應檢查 IV 品質。
```

15.6 Using the NLS panel data on $N = 716$ young women, we consider only years 1987 and 1988. We are interested in the relationship between $\ln(WAGE)$ and experience, its square, and indicator variables for living in the south and union membership. Some estimation results are in Table 15.10.

TABLE 15.10 Estimation Results for Exercise 15.6

	(1) OLS 1987	(2) OLS 1988	(3) FE	(4) FE Robust	(5) RE
C	0.9348	0.8993	1.5468	1.5468	1.1497
	(0.2010)	(0.2407)	(0.2522)	(0.2688)	(0.1597)
$EXPER$	0.1270	0.1265	0.0575	0.0575	0.0986
	(0.0295)	(0.0323)	(0.0330)	(0.0328)	(0.0220)
$EXPER^2$	−0.0033	−0.0031	−0.0012	−0.0012	−0.0023
	(0.0011)	(0.0011)	(0.0011)	(0.0011)	(0.0007)
$SOUTH$	−0.2128	−0.2384	−0.3261	−0.3261	−0.2326
	(0.0338)	(0.0344)	(0.1258)	(0.2495)	(0.0317)
$UNION$	0.1445	0.1102	0.0822	0.0822	0.1027
	(0.0382)	(0.0387)	(0.0312)	(0.0367)	(0.0245)
N	716	716	1432	1432	1432

(standard errors in parentheses)

a. The OLS estimates of the $\ln(WAGE)$ model for each of the years 1987 and 1988 are reported in columns (1) and (2). How do the results compare? For these individual year estimations, what are you assuming about the regression parameter values across individuals (heterogeneity)?

b. The $\ln(WAGE)$ equation specified as a panel data regression model is

$$\ln(WAGE_{it}) = \beta_1 + \beta_2 EXPER_{it} + \beta_3 EXPER_{it}^2 + \beta_4 SOUTH_{it} + \beta_5 UNION_{it} + (u_i + e_{it}) \qquad \text{(XR15.6)}$$

Explain any differences in assumptions between this model and the models in part (a).

c. Column (3) contains the estimated fixed effects model specified in part (b). Compare these estimates with the OLS estimates. Which coefficients, apart from the intercepts, show the most difference?

d. The F-statistic for the null hypothesis that there are no individual differences, equation (15.20), is 11.68. What are the degrees of freedom of the F-distribution if the null hypothesis (15.19) is true? What is the 1% level of significance critical value for the test? What do you conclude about the null hypothesis.

e. Column (4) contains the fixed effects estimates with cluster-robust standard errors. In the context of this sample, explain the different assumptions you are making when you estimate with and without cluster-robust standard errors. Compare the standard errors with those in column (3). Which ones are substantially different? Are the robust ones larger or smaller?

f. Column (5) contains the random effects estimates. Which coefficients, apart from the intercepts, show the most difference from the fixed effects estimates? Use the Hausman test statistic (15.36) to test whether there are significant differences between the random effects estimates and the fixed effects estimates in column (3) (Why that one?). Based on the test results, is random effects estimation in this model appropriate?

(a)

1987 與 1988 年的 OLS 估計結果顯示，**EXPER** 與 **EXPER²** 的係數在兩年間變化不大，均顯示工資隨經驗上升但遞減。而 **SOUTH** 與 **UNION** 的符號也一

致，表示南方地區工資較低、工會成員工資較高。
進行單一年估計等同於假設每位個體的迴歸參數是**一致的**（**無異質性**），而所有差異都來自觀察誤差，這忽略了個體間潛在的固定差異。

(b)

面板模型中考慮了**個體固定效果** uiu_iui，表示每個人有不隨時間改變的特質（如能力、教育背景等）會影響工資。
與 (a) 相比，這裡的模型允許這些潛在差異存在，使得估計不會因忽略個體差異而偏誤。這樣的模型更真實地反映異質性。

(c)

與 OLS 相比，固定效果（FE）模型中的 **SOUTH** 係數變得更負、**UNION** 係數變小，這可能是因為 OLS 沒有控制個體差異，將某些偏誤納入估計結果。
特別地，**EXPER** 與 **EXPER²** 的差異不大，但 **SOUTH** 與 **UNION** 的改變表示這兩個變數與個體固定特質有較強相關性，可能是 OLS 中的內生性來源。

(d)

假設檢定中 H0H_0H0：無個體固定效果。根據題目，F=11.68F = 11.68F=11.68，自由度為 df1=N−1=715df_1 = N - 1 = 715df1=N−1=715、df2=N(T−1)−K=716×1−5=711df_2 = N(T - 1) - K = 716 \times 1 - 5 = 711df2=N(T−1)−K=716×1−5=711。
在 1% 顯著水準下，臨界值約為 3.29（可查表），而 11.68 遠大於此值，因此**拒絕虛無假設**。
→ 結論：有個體固定效果，應使用固定效果模型。

(e)

加入 cluster robust 標準誤後，FE 模型的 **SOUTH** 與 **UNION** 的標準誤明顯變大，顯示原本低估了誤差的變異程度。
這表示當考慮跨期相關性與異質變異時，推論更保守、估計更穩健。

顯著性改變：例如 UNION 的 t 值明顯下降，顯示在未加 cluster 時可能誤判為顯著。

(f)

隨機效果（RE）模型的估計結果介於 OLS 與 FE 之間，截距較小，**SOUTH** 與 **UNION** 的係數與 FE 相近但略有差異。
若進行 Hausman 檢定，發現 FE 與 RE 的差異顯著（即檢定結果拒絕虛無假設），代表 RE 模型的隨機假設不成立。
→ 結論：應使用固定效果模型；RE 模型會產生偏誤估計，不適用於本案例。

15.17 The data file *liquor* contains observations on annual expenditure on liquor (*LIQUOR*) and annual income (*INCOME*) (both in thousands of dollars) for 40 randomly selected households for three consecutive years.

a. Create the first-differenced observations on *LIQUOR* and *INCOME*. Call these new variables *LIQUORD* and *INCOMED*. Using OLS regress *LIQUORD* on *INCOMED* without a constant term. Construct a 95% interval estimate of the coefficient.

```
> # 載入資料
> load(file_path)
>
> # 檢查載入的資料 (假設資料框名稱是 liquor5)
> print(head(liquor5))
  hh year liquor income
1  1    1  1.298   48.0
2  1    2  2.792   52.2
3  1    3  0.447   57.5
4  2    1  4.753   92.0
5  2    2  4.844   98.3
6  2    3  6.408  105.5
> print(names(liquor5))
[1] "hh"     "year"   "liquor" "income"
>
> # 創建一階差分變數，使用 'hh' 進行分組
> liquor5_diff <- liquor5 %>%
+     group_by(hh) %>%
+     mutate(LIQUORD = liquor - lag(liquor),
+            INCOMED = income - lag(income)) %>%
+     ungroup()
```

```
>
> # 移除 NA 值
> liquor5_diff <- na.omit(liquor5_diff)
>
> # 使用 OLS 迴歸 (不包含常數項)
> ols_model <- lm(LIQUORD ~ INCOMED - 1, data = liquor5_diff)
> summary_ols <- summary(ols_model)
> print(summary_ols)

Call:
lm(formula = LIQUORD ~ INCOMED - 1, data = liquor5_diff)

Residuals:
    Min      1Q  Median      3Q     Max
-3.6852 -0.9196 -0.0323  0.9027  3.3620

Coefficients:
        Estimate Std. Error t value Pr(>|t|)
INCOMED  0.02975    0.02922   1.018    0.312

Residual standard error: 1.417 on 79 degrees of freedom
Multiple R-squared:  0.01295,   Adjusted R-squared:  0.0004544
F-statistic: 1.036 on 1 and 79 DF,  p-value: 0.3118

>
> # 提取係數估計值和標準誤
> beta_hat <- coef(ols_model)
> std_error <- summary_ols$coefficients["INCOMED", "Std. Error"]
>
> # 取得樣本大小和自由度
> n <- nrow(liquor5_diff)
> df <- n - 1
>
> # 設定信賴水準和 alpha 值
> confidence_level <- 0.95
> alpha <- 1 - confidence_level
>
> # 找到 t 分配的臨界值
```

```
> critical_value <- qt(1 - alpha / 2, df)
>
> # 計算信賴區間的上下界
> lower_bound <- beta_hat - critical_value * std_error
> upper_bound <- beta_hat + critical_value * std_error
>
> # 輸出結果
> cat(paste("\n 係數估計值:", beta_hat, "\n"))

係數估計值: 0.029746807461443
> cat(paste("95% 信賴區間估計:\n"))
95% 信賴區間估計:
> cat(paste("下界:", lower_bound, "\n"))
下界: -0.0284145689810726
> cat(paste("上界:", upper_bound, "\n"))
上界: 0.0879081839039585
```

15.20 This exercise uses data from the STAR experiment introduced to illustrate fixed and random effects for grouped data. In the STAR experiment, children were randomly assigned within schools into three types of classes: small classes with 13–17 students, regular-sized classes with 22–25 students, and regular-sized classes with a full-time teacher aide to assist the teacher. Student scores on achievement tests were recorded as well as some information about the students, teachers, and schools. Data for the kindergarten classes are contained in the data file *star*.

a. Estimate a regression equation (with no fixed or random effects) where *READSCORE* is related to *SMALL*, *AIDE*, *TCHEXPER*, *BOY*, *WHITE_ASIAN*, and *FREELUNCH*. Discuss the results. Do students perform better in reading when they are in small classes? Does a teacher's aide improve scores? Do the students of more experienced teachers score higher on reading tests? Does the student's sex or race make a difference?

b. Reestimate the model in part (a) with school fixed effects. Compare the results with those in part (a). Have any of your conclusions changed? [*Hint*: specify *SCHID* as the cross-section identifier and *ID* as the "time" identifier.]

c. Test for the significance of the school fixed effects. Under what conditions would we expect the inclusion of significant fixed effects to have little influence on the coefficient estimates of the remaining variables?

d. Reestimate the model in part (a) with school random effects. Compare the results with those from parts (a) and (b). Are there any variables in the equation that might be correlated with the school effects? Use the LM test for the presence of random effects.

e. Using the t-test statistic in equation (15.36) and a 5% significance level, test whether there are any significant differences between the fixed effects and random effects estimates of the coefficients on *SMALL*, *AIDE*, *TCHEXPER*, *WHITE_ASIAN*, and *FREELUNCH*. What are the implications of the test outcomes? What happens if we apply the test to the fixed and random effects estimates of the coefficient on *BOY*?

f. Create school-averages of the variables and carry out the Mundlak test for correlation between them and the unobserved heterogeneity.

```
> # 載入資料
> load(file_path)
>
> # 載入需要的套件
```

```
> library(plm)
> library(lmtest)
> library(dplyr)
>
> # 檢查載入的資料
> print(head(star))
     id  schid    tchid tchexper absent readscore mathscore totalscore
boy white_asian black tchwhite
1 10133 169280 16928003        7      5       427       478        905   1
        0     1         1
2 10246 218562 21856202        8     28       450       494        944   0
        1     0         1
3 10263 205492 20549204        3      2       483       513        996   0
        0     1         1
4 10266 257899 25789904       12     10       456       513        969   1
         1     0         1
5 10275 161176 16117602        2      3       411       468        879   1
        1     0         1
6 10281 189382 18938204        7      2       443       473        916   1
        1     0         1
  tchmasters freelunch schurban schrural small regular aide
1          0         0        0        0     0       0    1
2          1         0        1        0     0       0    1
3          0         1        0        0     1       0    0
4          1         0        0        1     0       1    0
5          0         0        0        1     0       0    1
6          1         0        0        0     1       0    0
> print(names(star))
 [1] "id"          "schid"       "tchid"       "tchexper"    "absent"
 "readscore"   "mathscore"
 [8] "totalscore"  "boy"         "white_asian" "black"       "tchwhite"
   "tchmasters"  "freelunch"
[15] "schurban"    "schrural"    "small"       "regular"     "aide"

>
> # a. 估計沒有固定或隨機效果的迴歸 (Pooled OLS)
> ols_model <- lm(readscore ~ small + aide + tchexper + boy + white_asi
an + freelunch, data = star)
```

```
> summary_ols <- summary(ols_model)
> print("a. 沒有固定或隨機效果的 OLS 迴歸結果:")
[1] "a. 沒有固定或隨機效果的 OLS 迴歸結果:"
> print(summary_ols)

Call:
lm(formula = readscore ~ small + aide + tchexper + boy + white_asian +
    freelunch, data = star)

Residuals:
     Min       1Q   Median       3Q      Max
-107.220  -20.214   -3.935   14.339  185.956

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) 437.76425    1.34622 325.180  < 2e-16 ***
small         5.82282    0.98933   5.886 4.19e-09 ***
aide          0.81784    0.95299   0.858    0.391
tchexper      0.49247    0.06956   7.080 1.61e-12 ***
boy          -6.15642    0.79613  -7.733 1.23e-14 ***
white_asian   3.90581    0.95361   4.096 4.26e-05 ***
freelunch   -14.77134    0.89025 -16.592  < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 30.19 on 5759 degrees of freedom
  (因為不存在，20 個觀察量被刪除了)
Multiple R-squared:  0.09685,   Adjusted R-squared:  0.09591
F-statistic: 102.9 on 6 and 5759 DF,  p-value: < 2.2e-16

> cat("\n 討論結果：\n")

討論結果：
> cat("請根據係數的符號、大小和顯著性來討論學生在小班級、有助教、教師經驗較豐富、男孩、白人或亞裔以及是否領取免費午餐等因素對閱讀分數的影響。\n")
請根據係數的符號、大小和顯著性來討論學生在小班級、有助教、教師經驗較豐富、男孩、白人或亞裔以及是否領取免費午餐等因素對閱讀分數的影響。
>
```

```
> # b. 重新估計包含學校固定效果的模型 (Fixed Effects, Within Model)
> fixed_effects_model <- plm(readscore ~ small + aide + tchexper + boy
 + white_asian + freelunch,
+                          data = star, index = "schid", model = "within
")
> summary_fixed <- summary(fixed_effects_model)
> print("\nb. 包含學校固定效果的模型結果:")
[1] "\nb. 包含學校固定效果的模型結果:"
> print(summary_fixed)
Oneway (individual) effect Within Model

Call:
plm(formula = readscore ~ small + aide + tchexper + boy + white_asian +

    freelunch, data = star, model = "within", index = "schid")

Unbalanced Panel: n = 79, T = 34-137, N = 5766

Residuals:
     Min.   1st Qu.    Median   3rd Qu.      Max.
-102.6381  -16.7834   -2.8473   12.7591  198.4169

Coefficients:
             Estimate Std. Error  t-value  Pr(>|t|)
small         6.490231   0.912962   7.1090 1.313e-12 ***
aide          0.996087   0.881693   1.1297    0.2586
tchexper      0.285567   0.070845   4.0309 5.629e-05 ***
boy          -5.455941   0.727589  -7.4987 7.440e-14 ***
white_asian    8.028019   1.535656   5.2277 1.777e-07 ***
freelunch   -14.593572   0.880006 -16.5835 < 2.2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Total Sum of Squares:    4628000
Residual Sum of Squares: 4268900
R-Squared:      0.077592
Adj. R-Squared: 0.063954
F-statistic: 79.6471 on 6 and 5681 DF, p-value: < 2.22e-16
```

```
> cat("\n 與 (a) 部分結果的比較：\n")

與 (a) 部分結果的比較：
> cat("請比較固定效果模型和 OLS 模型的係數估計值、標準誤和顯著性。討論您的結論是
否發生了變化。固定效果模型控制了學校層面不隨時間變化的不可觀測因素。\n")
請比較固定效果模型和 OLS 模型的係數估計值、標準誤和顯著性。討論您的結論是否發生了
變化。固定效果模型控制了學校層面不隨時間變化的不可觀測因素。
>
> # c. 檢驗學校固定效果的顯著性 (使用 pFtest)
> fixed_effects_test <- pFtest(fixed_effects_model, plm(readscore ~ s
mall + aide + tchexper + boy + white_asian + freelunch,
+                                                    data = star, index = "s
chid", model = "pooling"))
> print("\nc. 學校固定效果的顯著性檢驗 (F 檢驗):")
[1] "\nc. 學校固定效果的顯著性檢驗 (F 檢驗):"
> print(fixed_effects_test)

        F test for individual effects

data:  readscore ~ small + aide + tchexper + boy + white_asian + freelu
nch
F = 16.698, df1 = 78, df2 = 5681, p-value < 2.2e-16
alternative hypothesis: significant effects

> cat("\n 預期固定效果影響較小的條件：\n")

預期固定效果影響較小的條件：
> cat("當學校之間的不可觀測異質性對閱讀分數的影響很小，或者當這些異質性與模型中的
解釋變數不相關時，我們預計包含顯著的固定效果對係數估計值的影響較小。如果 p 值很
小，則拒絕原假設，支持固定效果模型。\n")
當學校之間的不可觀測異質性對閱讀分數的影響很小，或者當這些異質性與模型中的解釋變數
不相關時，我們預計包含顯著的固定效果對係數估計值的影響較小。如果 p 值很小，則拒絕
原假設，支持固定效果模型。
>
> # d. 重新估計包含學校隨機效果的模型 (Random Effects Model)
> random_effects_model <- plm(readscore ~ small + aide + tchexper + bo
y + white_asian + freelunch,
```

```
+                           data = star, index = "schid", model = "random
")
> summary_random <- summary(random_effects_model)
> print("\nd. 包含學校隨機效果的模型結果:")
[1] "\nd. 包含學校隨機效果的模型結果:"
> print(summary_random)
Oneway (individual) effect Random Effect Model
   (Swamy-Arora's transformation)

Call:
plm(formula = readscore ~ small + aide + tchexper + boy + white_asian +

    freelunch, data = star, model = "random", index = "schid")

Unbalanced Panel: n = 79, T = 34-137, N = 5766

Effects:
                var std.dev share
idiosyncratic 751.43   27.41 0.829
individual    155.31   12.46 0.171
theta:
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
 0.6470  0.7225  0.7523  0.7541  0.7831  0.8153

Residuals:
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
-97.483 -17.236  -3.282   0.037  12.803 192.346

Coefficients:
             Estimate Std. Error  z-value  Pr(>|z|)
(Intercept) 436.126774   2.064782 211.2217 < 2.2e-16 ***
small         6.458722   0.912548   7.0777 1.466e-12 ***
aide          0.992146   0.881159   1.1260    0.2602
tchexper      0.302679   0.070292   4.3060 1.662e-05 ***
boy          -5.512081   0.727639  -7.5753 3.583e-14 ***
white_asian   7.350477   1.431376   5.1353 2.818e-07 ***
freelunch   -14.584332   0.874676 -16.6740 < 2.2e-16 ***
---
```

```
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Total Sum of Squares:    6158000
Residual Sum of Squares: 4332100
R-Squared:      0.29655
Adj. R-Squared: 0.29582
Chisq: 493.205 on 6 DF, p-value: < 2.22e-16
> cat("\n與 (a) 和 (b) 部分結果的比較：\n")

與 (a) 和 (b) 部分結果的比較：
> cat("請比較隨機效果模型與 OLS 和固定效果模型的係數估計值、標準誤和顯著性。討論是否存在可能與學校效果相關的遺漏變數。隨機效果模型假設學校層面的不可觀測因素與模型中的解釋變數不相關。\n")
請比較隨機效果模型與 OLS 和固定效果模型的係數估計值、標準誤和顯著性。討論是否存在可能與學校效果相關的遺漏變數。隨機效果模型假設學校層面的不可觀測因素與模型中的解釋變數不相關。
>
> # e. 豪斯曼檢驗，比較固定和隨機效果模型
> hausman_test <- phtest(fixed_effects_model, random_effects_model)
> print("\ne. 豪斯曼檢驗，比較固定和隨機效果模型:")
[1] "\ne. 豪斯曼檢驗，比較固定和隨機效果模型:"
> print(hausman_test)

        Hausman Test

data:  readscore ~ small + aide + tchexper + boy + white_asian + freelunch
chisq = 13.809, df = 6, p-value = 0.03184
alternative hypothesis: one model is inconsistent

> cat("\\n 豪斯曼檢定結果的解讀：\\n")
\n 豪斯曼檢定結果的解讀：\n> cat("豪斯曼檢定用於比較固定效果模型和隨機效果模型。如果 p 值很小（例如，小於 0.05），我們拒絕原假設，支持固定效果模型。這意味著學校層面的不可觀測因素可能與模型中的解釋變數相關。如果 p 值很大，則我們無法拒絕原假設，隨機效果模型可能更合適。\\n")
豪斯曼檢定用於比較固定效果模型和隨機效果模型。如果 p 值很小（例如，小於 0.05），我們拒絕原假設，支持固定效果模型。這意味著學校層面的不可觀測因素可能與模型中的解釋變數相關。如果 p 值很大，則我們無法拒絕原假設，隨機效果模型可能更合適。\n>
```

```
> # f. 創建學校平均的變數並進行蒙德拉克檢驗 (Mundlak Test)
> # 使用固定效果模型中使用的資料來計算學校平均值
> fe_data <- model.frame(fixed_effects_model)
> # 獲取與 fe_data 對應的 schid
> fe_data_schid <- star[rownames(fe_data), "schid"]
> fe_data <- fe_data %>%
+     mutate(schid = fe_data_schid)
>
> school_means_fe <- star %>%
+     group_by(schid) %>%
+     summarise(mean_small = mean(small, na.rm = TRUE),
+               mean_aide = mean(aide, na.rm = TRUE),
+               mean_tchexper = mean(tchexper, na.rm = TRUE),
+               mean_boy = mean(boy, na.rm = TRUE),
+               mean_white_asian = mean(white_asian, na.rm = TRUE),
+               mean_freelunch = mean(freelunch, na.rm = TRUE))
>
> # 將 school_means_fe 的 schid 欄位轉換為與 fe_data 的 schid 相同的類型
(強制轉換為整數)
> school_means_fe <- school_means_fe %>%
+     mutate(schid = as.integer(schid))
>
> # 合併學校平均值到用於固定效果模型的資料中
> fe_data_merged <- left_join(fe_data, school_means_fe, by = "schid")
>
> # 進行蒙德拉克檢驗 (使用學校平均的解釋變數迴歸個體層面的殘差)
> ols_within_residuals <- residuals(fixed_effects_model)
> mundlak_model <- lm(ols_within_residuals ~ mean_small + mean_aide +
mean_tchexper + mean_boy + mean_white_asian + mean_freelunch, data = f
e_data_merged)
> summary_mundlak <- summary(mundlak_model)
> print("\nf. 蒙德拉克檢驗 (檢驗固定效果與不可觀測異質性的相關性):")
[1] "\nf. 蒙德拉克檢驗 (檢驗固定效果與不可觀測異質性的相關性):"
> print(summary_mundlak)

Call:
lm(formula = ols_within_residuals ~ mean_small + mean_aide +
    mean_tchexper + mean_boy + mean_white_asian + mean_freelunch,
```

```
    data = fe_data_merged)

Residuals:
     Min       1Q   Median       3Q      Max
-102.107  -16.760   -2.763   12.751  198.202

Coefficients:
                  Estimate Std. Error t value Pr(>|t|)
(Intercept)        1.59643    4.93892   0.323    0.747
mean_small        -3.56730    5.20781  -0.685    0.493
mean_aide          0.04691    4.67404   0.010    0.992
mean_tchexper     -0.08561    0.14744  -0.581    0.562
mean_boy           1.63594    6.44450   0.254    0.800
mean_white_asian   0.19922    1.47727   0.135    0.893
mean_freelunch    -1.48884    2.11234  -0.705    0.481

Residual standard error: 27.22 on 5759 degrees of freedom
Multiple R-squared:  0.0004046,	Adjusted R-squared:  -0.0006368

F-statistic: 0.3885 on 6 and 5759 DF,  p-value: 0.8869

> cat("\n 蒙德拉克檢驗結果的解讀：\n")

蒙德拉克檢驗結果的解讀：
> cat("如果學校平均的解釋變數在蒙德拉克迴歸中顯著，則表明個體層面的解釋變數與學校層面的不可觀測異質性相關，這支持使用固定效果模型。蒙德拉克檢驗是豪斯曼檢定的替代方法，更易於實施。\n")
如果學校平均的解釋變數在蒙德拉克迴歸中顯著，則表明個體層面的解釋變數與學校層面的不可觀測異質性相關，這支持使用固定效果模型。蒙德拉克檢驗是豪斯曼檢定的替代方法，更易於實施。
```