8.16 A sample of 200 Chicago households was taken to investigate how far American households tend to travel when they take a vacation. Consider the model

$$MILES = \beta_1 + \beta_2 INCOME + \beta_3 AGE + \beta_4 KIDS + e$$

MILES is miles driven per year, *INCOME* is measured in \$1000 units, *AGE* is the average age of the adult members of the household, and *KIDS* is the number of children.

a. Use the data file *vacation* to estimate the model by OLS. Construct a 95% interval estimate for the effect of one more child on miles traveled, holding the two other variables constant.

b. Plot the OLS residuals versus *INCOME* and *AGE*. Do you observe any patterns suggesting that heteroskedasticity is present?

c. Sort the data according to increasing magnitude of income. Estimate the model using the first 90 observations and again using the last 90 observations. Carry out the Goldfeld–Quandt test for heteroskedastic errors at the 5% level. State the null and alternative hypotheses.

d. Estimate the model by OLS using heteroskedasticity robust standard errors. Construct a 95% interval estimate for the effect of one more child on miles traveled, holding the two other variables constant. How does this interval estimate compare to the one in (a)?

e. Obtain GLS estimates assuming $\sigma_i^2 = \sigma^2 INCOME_i^2$. Using both conventional GLS and robust GLS standard errors, construct a 95% interval estimate for the effect of one more child on miles traveled, holding the two other variables constant. How do these interval estimates compare to the ones in (a) and (d)?

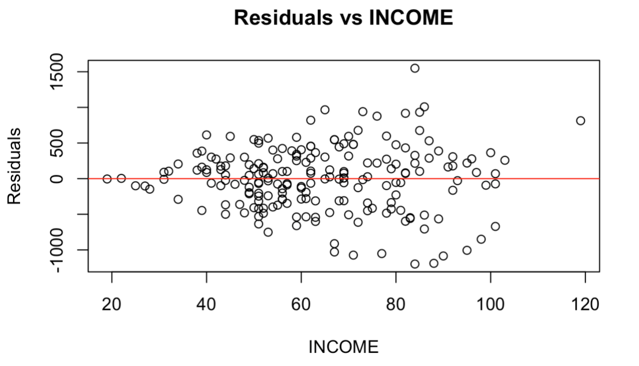
a.

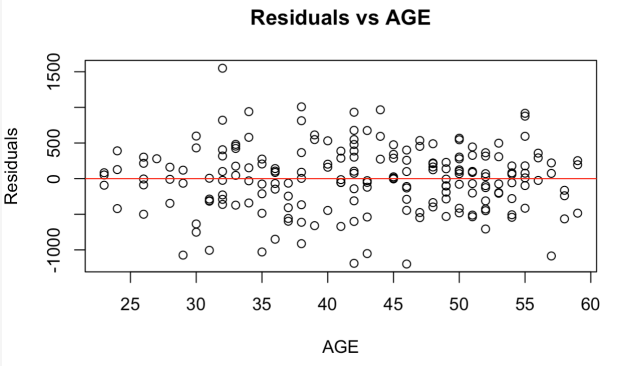
```
Table: The basic multiple regression model

|            | coefficient | Std. Error | t-value | p-value |
|:-----------|:-----------:|:----------:|:-------:|:-------:|
|(Intercept) |  -391.548   |  169.775   | -2.306  |  0.022  |
|income      |   14.201    |   1.800    |  7.889  |  0.000  |
|age         |   15.741    |   3.757    |  4.189  |  0.000  |
|kids        |   -81.826   |  27.130    | -3.016  |  0.003  |
```

```
Call:
lm(formula = miles ~ income + age + kids, data = vacation)

Residuals:
     Min       1Q   Median       3Q      Max 
-1198.14  -295.31    17.98   287.54  1549.41 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) -391.548    169.775  -2.306   0.0221 *  
income        14.201      1.800   7.889 2.10e-13 ***
age           15.741      3.757   4.189 4.23e-05 ***
kids         -81.826     27.130  -3.016   0.0029 ** 
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 452.3 on 196 degrees of freedom
Multiple R-squared:  0.3406,	Adjusted R-squared:  0.3305 
F-statistic: 33.75 on 3 and 196 DF,  p-value: < 2.2e-16
```

```
> #估計kids的信賴區間
> confint(mod1, "kids", level = 0.95)
          2.5 %    97.5 %
kids -135.3298 -28.32302
```

b.





c.

```
> # 模型
> model1 <- lm(miles ~ income + age + kids , data = first90)
> summary(model1)

Call:
lm(formula = miles ~ income + age + kids, data = first90)

Residuals:
    Min      1Q  Median      3Q     Max 
-684.07 -245.39    8.69  202.87  631.43 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) -392.511    214.166  -1.833  0.07030 .  
income        10.960      3.770   2.907  0.00464 ** 
age           18.869      3.783   4.988 3.14e-06 ***
kids         -70.371     29.138  -2.415  0.01785 *  
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 319 on 86 degrees of freedom
Multiple R-squared:  0.309,	Adjusted R-squared:  0.2849 
F-statistic: 12.82 on 3 and 86 DF,  p-value: 5.31e-07
```

```
> model2 <- lm(miles ~ income + age + kids , data = last90)
> summary(model2)

Call:
lm(formula = miles ~ income + age + kids, data = last90)

Residuals:
     Min       1Q   Median       3Q      Max 
-1215.44  -426.21    73.56   304.71  1602.70 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)   
(Intercept) -476.803    548.833  -0.869   0.3874   
income        15.556      5.450   2.855   0.0054 **
age           16.388      7.385   2.219   0.0291 * 
kids        -116.017     49.861  -2.327   0.0223 * 
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 562 on 86 degrees of freedom
Multiple R-squared:  0.1514,	Adjusted R-squared:  0.1218 
F-statistic: 5.116 on 3 and 86 DF,  p-value: 0.002642
```

```
        Goldfeld-Quandt test

data:  miles ~ income + age + kids
GQ = 2.9615, df1 = 96, df2 = 96, p-value = 1.091e-07
alternative hypothesis: variance increases from segment 1 to 2
```

H_0: $\sigma_1^2 = \sigma_2^2$

H_1: $\sigma_1^2 < \sigma_2^2$

P–value < 0.05, 所以拒絕虛無假設。
我們有足夠證據說明模型中存在異質變異，且誤差變異數可能隨收入增加而變大。

d.

```
t test of coefficients:

             Estimate Std. Error t value  Pr(>|t|)    
(Intercept) -391.5480   142.6548 -2.7447 0.0066190 ** 
income        14.2013     1.9389  7.3246 6.083e-12 ***
age           15.7409     3.9657  3.9692 0.0001011 ***
kids         -81.8264    29.1544 -2.8067 0.0055112 ** 
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

可以發現估計出來的係數與(a)小題一樣

```
> confint_robust
        2.5 %    97.5 %
kids -139.323 -24.32986
```

相較於(a)小題的估計區間，可以發現使用 robust 標準誤後，(d)小題的估計區間比較寬

e. GLS

```
Call:
lm(formula = miles ~ income + age + kids, data = vacation, weights = weight)

Weighted Residuals:
     Min       1Q   Median       3Q      Max 
-15.1907  -4.9555   0.2488   4.3832  18.5462 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) -424.996    121.444  -3.500 0.000577 ***
income        13.947      1.481   9.420  < 2e-16 ***
age           16.717      3.025   5.527 1.03e-07 ***
kids         -76.806     21.848  -3.515 0.000545 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 6.765 on 196 degrees of freedom
Multiple R-squared:  0.4573,	Adjusted R-squared:  0.449 
F-statistic: 55.06 on 3 and 196 DF,  p-value: < 2.2e-16
```

Convention GSL

```
> # 傳統 GLS 的 95% CI (未做 robust)
> confint(gls_model, "kids", level = 0.95)
         2.5 %    97.5 %
kids -119.8945 -33.71808
```

GSL(robust)

```
> confint_robust_gls
         2.5 %    97.5 %
kids -121.4134 -32.19919
```

```
                     方法 CI.下界 CI.上界 CI.寬度
1       OLS (傳統標準誤)  -135.33  -28.32  107.01
2 OLS + Robust SE (HC1)  -139.32  -24.33  114.99
3           GLS (傳統 SE)  -119.89  -33.72   86.18
4 GLS + Robust SE (HC1)  -121.41  -32.20   89.21
```

從比較可以發現，經過傳統 GSL 與 Robust GSL 的 interval 估計相較於a小題與d小題來說比較窄

8.18 Consider the wage equation,

$$\ln(WAGE_i) = \beta_1 + \beta_2 EDUC_i + \beta_3 EXPER_i + \beta_4 EXPER_i^2 + \beta_5 FEMALE_i + \beta_6 BLACK + \beta_7 METRO_i + \beta_8 SOUTH_i + \beta_9 MIDWEST_i + \beta_{10} WEST + e_i$$

where *WAGE* is measured in dollars per hour, education and experience are in years, and $METRO = 1$ if the person lives in a metropolitan area. Use the data file *cps5* for the exercise.

a. We are curious whether holding education, experience, and *METRO* equal, there is the same amount of random variation in wages for males and females. Suppose $\text{var}(e_i|\mathbf{x}_i, FEMALE = 0) = \sigma_M^2$ and $\text{var}(e_i|\mathbf{x}_i, FEMALE = 1) = \sigma_F^2$. We specifically wish to test the null hypothesis $\sigma_M^2 = \sigma_F^2$ against $\sigma_M^2 \neq \sigma_F^2$. Carry out a Goldfeld–Quandt test of the null hypothesis at the 5% level of significance. Clearly state the value of the test statistic and the rejection region, along with your conclusion.

b. Estimate the model by OLS. Carry out the NR^2 test using the right-hand-side variables *METRO*, *FEMALE*, *BLACK* as candidates related to the heteroskedasticity. What do we conclude about heteroskedasticity, at the 1% level? Do these results support your conclusions in (a)? Repeat the test using all model explanatory variables as candidates related to the heteroskedasticity.

c. Carry out the White test for heteroskedasticity. What is the 5% critical value for the test? What do you conclude?

d. Estimate the model by OLS with White heteroskedasticity robust standard errors. Compared to OLS with conventional standard errors, for which coefficients have interval estimates gotten narrower? For which coefficients have interval estimates gotten wider? Is there an inconsistency in the results?

e. Obtain FGLS estimates using candidate variables *METRO* and *EXPER*. How do the interval estimates compare to OLS with robust standard errors, from part (d)?

f. Obtain FGLS estimates with robust standard errors using candidate variables *METRO* and *EXPER*. How do the interval estimates compare to those in part (e) and OLS with robust standard errors, from part (d)?

g. If reporting the results of this model in a research paper which one set of estimates would you present? Explain your choice.

a.

$$H_0: \sigma_M^2 = \sigma_F^2$$

$$H_1: \sigma_M^2 \neq \sigma_F^2$$

```
        Goldfeld-Quandt test

data:  mod1
GQ = 0.96141, df1 = 4890, df2 = 4889, p-value = 0.9156
alternative hypothesis: variance increases from segment 1 to 2
```

我們可以發現p-value = 0.9156 > 0.05，因此我們沒有足夠的證據去拒絕虛無假設，

也就是說我們沒有足夠的證據去說明男性與女性在控制教育、經驗與地區等因素後，薪資的誤差變異存在顯著差異。

b. Breusch—Pagan Test

```
> #Breusch-Pagan Test
> bptest(mod1, varformula = ~ metro + female + black, data = cps5)

	studentized Breusch-Pagan test

data:  mod1
BP = 23.557, df = 3, p-value = 3.091e-05
```

P–value < 0.01 ，代表拒絕虛無假設，也就是說模型之中存在異質性變異，和a小題得出的結果不同

```
> bptest(mod1, varformula = ~ educ + exper + I(exper^2) + female + black +
+          metro + south + midwest + west, data = cps5)

	studentized Breusch-Pagan test

data:  mod1
BP = 109.42, df = 9, p-value < 2.2e-16
```

P–value < 0.01 ，代表拒絕虛無假設，也就是說模型之中存在異質性變異，和a小題得出的結果不同

c. White Test

```
	studentized Breusch-Pagan test

data:  mod1
BP = 168.53, df = 42, p-value < 2.2e-16
```

P–value < 0.05，代表拒絕虛無假設，也就是說模型之中存在異質性變異

d.

```
Call:
lm(formula = log(wage) ~ educ + exper + I(exper^2) + female +
    black + metro + south + midwest + west, data = cps5)

Residuals:
     Min       1Q   Median       3Q      Max 
-2.31711 -0.30038 -0.00584  0.30238  3.00061 

Coefficients:
              Estimate Std. Error t value Pr(>|t|)    
(Intercept)  1.201e+00  3.211e-02  37.409  < 2e-16 ***
educ         1.012e-01  1.758e-03  57.574  < 2e-16 ***
exper        2.962e-02  1.300e-03  22.780  < 2e-16 ***
I(exper^2)  -4.458e-04  2.635e-05 -16.915  < 2e-16 ***
female      -1.655e-01  9.529e-03 -17.368  < 2e-16 ***
black       -1.115e-01  1.694e-02  -6.583 4.86e-11 ***
metro        1.190e-01  1.231e-02   9.671  < 2e-16 ***
south       -4.576e-02  1.356e-02  -3.374 0.000744 ***
midwest     -6.394e-02  1.410e-02  -4.534 5.86e-06 ***
west        -6.589e-03  1.440e-02  -0.458 0.647321    
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.466 on 9789 degrees of freedom
Multiple R-squared:  0.3173,    Adjusted R-squared:  0.3167 
F-statistic: 505.6 on 9 and 9789 DF,  p-value: < 2.2e-16
```

```
t test of coefficients:

               Estimate  Std. Error  t value  Pr(>|t|)    
(Intercept)  1.2014e+00  3.2794e-02  36.6340 < 2.2e-16 ***
educ         1.0123e-01  1.9058e-03  53.1160 < 2.2e-16 ***
exper        2.9622e-02  1.3149e-03  22.5276 < 2.2e-16 ***
I(exper^2)  -4.4578e-04  2.7597e-05 -16.1533 < 2.2e-16 ***
female      -1.6550e-01  9.4883e-03 -17.4428 < 2.2e-16 ***
black       -1.1153e-01  1.6094e-02  -6.9297 4.482e-12 ***
metro        1.1902e-01  1.1582e-02  10.2762 < 2.2e-16 ***
south       -4.5755e-02  1.3902e-02  -3.2914  0.001001 ** 
midwest     -6.3943e-02  1.3724e-02  -4.6591 3.217e-06 ***
west        -6.5891e-03  1.4557e-02  -0.4526  0.650813    
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

多數變數的 robust 標準誤略為變寬或接近不變，但整體顯著性沒有太大變動。
South 是唯一一個 p-value 有明顯增加的變數（但仍顯著）。

e.

```
Call:
lm(formula = log(wage) ~ educ + exper + I(exper^2) + female + 
    black + metro + south + midwest + west, data = cps5, weights = weights)

Weighted Residuals:
    Min      1Q  Median      3Q     Max 
-9.6137 -1.2561 -0.0267  1.2619 12.6230 

Coefficients:
              Estimate Std. Error t value Pr(>|t|)    
(Intercept)  1.192e+00  3.159e-02  37.736  < 2e-16 ***
educ         1.017e-01  1.765e-03  57.611  < 2e-16 ***
exper        3.009e-02  1.298e-03  23.191  < 2e-16 ***
I(exper^2)  -4.561e-04  2.679e-05 -17.027  < 2e-16 ***
female      -1.662e-01  9.481e-03 -17.532  < 2e-16 ***
black       -1.109e-01  1.699e-02  -6.524 7.20e-11 ***
metro        1.178e-01  1.146e-02  10.277  < 2e-16 ***
south       -4.484e-02  1.352e-02  -3.316 0.000916 ***
midwest     -6.319e-02  1.398e-02  -4.519 6.29e-06 ***
west        -5.494e-03  1.438e-02  -0.382 0.702365    
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 1.932 on 9789 degrees of freedom
Multiple R-squared:  0.3212,	Adjusted R-squared:  0.3205 
F-statistic: 514.6 on 9 and 9789 DF,  p-value: < 2.2e-16
```

```
> # 加上 95% 區間估計
> confint(fgls_model)
                    2.5 %        97.5 %
(Intercept)  1.1302695254  1.2541279001
educ         0.0982024458  0.1051204663
exper        0.0275467064  0.0326335081
I(exper^2)  -0.0005086498 -0.0004036251
female      -0.1847977976 -0.1476290326
black       -0.1441623553 -0.0775448504
metro        0.0953066354  0.1402324253
south       -0.0713493481 -0.0183363498
midwest     -0.0906033967 -0.0357807800
west        -0.0336747637  0.0226870709
```

所得估計值與 (d) 小題中使用 White robust 標準誤的結果相當一致

f.

```
t test of coefficients:

               Estimate  Std. Error  t value  Pr(>|t|)    
(Intercept)  1.1922e+00  3.2360e-02  36.8422 < 2.2e-16 ***
educ         1.0166e-01  1.8928e-03  53.7107 < 2.2e-16 ***
exper        3.0090e-02  1.3046e-03  23.0643 < 2.2e-16 ***
I(exper^2)  -4.5614e-04  2.7408e-05 -16.6423 < 2.2e-16 ***
female      -1.6621e-01  9.4381e-03 -17.6109 < 2.2e-16 ***
black       -1.1085e-01  1.5869e-02  -6.9857 3.020e-12 ***
metro        1.1777e-01  1.1563e-02  10.1851 < 2.2e-16 ***
south       -4.4843e-02  1.3834e-02  -3.2414  0.001193 ** 
midwest     -6.3192e-02  1.3713e-02  -4.6083 4.111e-06 ***
west        -5.4938e-03  1.4509e-02  -0.3787  0.704951    
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

OLS 比較，FGLS 模型的標準誤調整效果較好且區間估計較精確

g.

我會選擇 robust OLS ，因為他已經能解決異質變異問題