11.28 Supply and demand curves as traditionally drawn in economics principles classes have price (P) on the vertical axis and quantity (Q) on the horizontal axis.

a. Rewrite the truffle demand and supply equations in (11.11) and (11.12) with price P on the left-hand side. What are the anticipated signs of the parameters in this rewritten system of equations?

b. Using the data in the file *truffles*, estimate the supply and demand equations that you have formulated in (a) using two-stage least squares. Are the signs correct? Are the estimated coefficients significantly different from zero?

```
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 20.03280    1.22311   16.38 1.50e-15 ***
p            0.33798    0.02492   13.56 1.43e-13 ***
pf          -1.00091    0.08253  -12.13 1.95e-12 ***

Diagnostic tests:
                 df1 df2 statistic  p-value
Weak instruments   2  26    41.487 8.12e-09 ***
Wu-Hausman         1  26     0.000    1.000
Sargan             1  NA     1.533    0.216
```

供給：$\hat{q}_i = 20.033 + 0.338\,p_i - 1.000\,pf_i$ 、需求 $\hat{q}_i = -4.2795 - 0.3745\,p_i + 5.0140\,di_i + 1.2960\,ps_i$

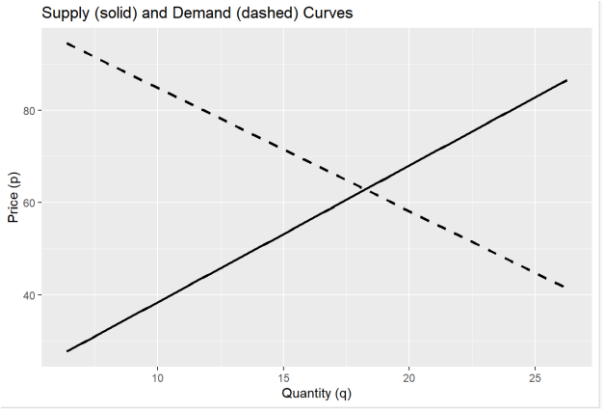
使用 2SLS 估計結果顯示，供給與需求方程式中價格變數的符號皆符合經濟理論，估計係數在統計上顯著，且工具變數檢定證實供給方無內生性問題、需求方存在明顯內生性，證明使用 2SLS 是適當的

c. Estimate the price elasticity of demand "at the means" using the results from (b)

```
Demand price elasticity at means = -1.272
```

需求在平均點的價格彈性為 −1.272，表示需求對價格變化具備彈性，價格每上升 1%，需求將減少約 1.27%

d. Accurately sketch the supply and demand equations, with P on the vertical axis and Q on the horizontal axis, using the estimates from part (b). For these sketches set the values of the exogenous variables DI, PS, and PF to be $DI^* = 3.5$, $PF^* = 23$, and $PS^* = 22$.



e. What are the equilibrium values of P and Q obtained in part (d)? Calculate the predicted equilibrium values of P and Q using the estimated reduced-form equations from Table 11.2, using the same values of the exogenous variables. How well do they agree?

```
Structural eq:  p* = 62.841 , q* = 18.251
> # Reduced-form: p ~ di + ps + pf; q ~ di
> red_p <- lm(p ~ di + ps + pf, data = truf
> red_q <- lm(q ~ di + ps + pf, data = truf
> p_eq_rf <- predict(red_p, newdata = data.
> q_eq_rf <- predict(red_q, newdata = data.
> cat("Reduced-form eq:  p* =", round(p_eq_
+     ", q* =", round(q_eq_rf,3), "\n")
Reduced-form eq:  p* = 62.815 , q* = 18.26
```

根據 d 的供給與需求曲線，解出結構型模型的均衡點為：p*=62.841、q*=18.251

同時使用簡化型模型對同樣的外生變數值（di=3.5, pf=23, ps=22）進行預測，得到：p*=62.815,q*=18.260

結構型與簡化型模型所估計的均衡價格與數量幾乎一致，差異極小，顯示模型設計一致性良好，且所選用的外生變數對供需系統具有穩定的預測能力

f. Estimate the supply and demand equations that you have formulated in (a) using OLS. Are the signs correct? Are the estimated coefficients significantly different from zero? Compare the results to those in part (b).

```
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 10.12579    5.46555   1.853 0.081374 .
p            0.47964    0.09711   4.939 0.000125 ***
plag         0.33304    0.10086   3.302 0.004212 **
klag        -0.11179    0.02673  -4.183 0.000624 ***
```

```
Coefficients:
             Estimate Std. Error    t value Pr(>|t|)
(Intercept)  9.234e-14  3.234e-14  2.855e+00  0.01146 *
cn          -7.069e-15  2.370e-15 -2.983e+00  0.00879 **
w1          -1.000e+00  3.874e-15 -2.581e+14  < 2e-16 ***
g           -5.477e-15  3.352e-15 -1.634e+00  0.12177
tx          -1.000e+00  3.169e-15 -3.155e+14  < 2e-16 ***
e            1.000e+00  1.672e-15  5.980e+14  < 2e-16 ***
```

投資方程式中，當期利潤（p）與前一期利潤（plag）對投資具有正向且顯著影響，符合理論預期，而前一期資本存量（klag）為負且顯著，可能反映邊際資本報酬遞減。所有主變數在 1% 或 5% 顯著水準下顯著，模型解釋力良好

11.30 Example 11.3 introduces Klein's Model I. Use the data file *klein* to answer the following questions.

a. Estimate the investment function in equation (11.18) by OLS. Comment on the signs and significance of the coefficients.

```
Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept)  5.731e+00  4.598e+00   1.246  0.23058
p            6.445e-01  9.333e-02   6.905 3.54e-06 ***
plag         1.643e-01  9.652e-02   1.702  0.10816
klag        -9.060e-02  2.246e-02  -4.034  0.00096 ***
vhat        -6.813e+13  2.118e+13  -3.217  0.00538 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.8108 on 16 degrees of freedom
  (因為不存在 1 個觀察量被刪除了)
Multiple R-squared:  0.9583,    Adjusted R-squared:  0.9479
F-statistic: 91.95 on 4 and 16 DF,  p-value: 7.907e-11
```

b. Estimate the reduced-form equation for profits, P_t, using all eight exogenous and predetermined variables as explanatory variables. Test the joint significance of all the variables except lagged profits, P_{t-1}, and lagged capital stock, K_{t-1}. Save the residuals, $\hat{v}_t$ and compute the fitted values, $\hat{P}_t$.

```
Call:
lm(formula = p ~ cn + w1 + g + tx + e, data = klein)

Residuals:
       Min         1Q     Median         3Q        Max
-1.863e-14 -9.763e-15 -2.890e-15  8.349e-15  3.855e-14

Coefficients:
              Estimate Std. Error    t value Pr(>|t|)
(Intercept)  9.234e-14  3.234e-14  2.855e+00  0.01146 *
cn          -7.069e-15  2.370e-15 -2.983e+00  0.00879 **
w1          -1.000e+00  3.874e-15 -2.581e+14  < 2e-16 ***
g           -5.477e-15  3.352e-15 -1.634e+00  0.12177
tx          -1.000e+00  3.169e-15 -3.155e+14  < 2e-16 ***
e            1.000e+00  1.672e-15  5.980e+14  < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.523e-14 on 16 degrees of freedom
Multiple R-squared:      1,     Adjusted R-squared:      1
F-statistic: 3.216e+29 on 5 and 16 DF,  p-value: < 2.2e-16
```

c. The Hausman test for the presence of endogenous explanatory variables is discussed in Section 10.4.1. It is implemented by adding the reduced-form residuals to the structural equation and testing their significance, that is, using OLS estimate the model

$$I_t = \beta_1 + \beta_2 P_t + \beta_3 P_{t-1} + \beta_4 K_{t-1} + \delta \hat{v}_t + e_{2t}$$

Use a t-test for the null hypothesis $H_0: \delta = 0$ versus $H_1: \delta \neq 0$ at the 5% level of significance. By rejecting the null hypothesis, we conclude that P_t is endogenous. What do we conclude from the test? In the context of this simultaneous equations model what result should we find?

```
Call:
lm(formula = i ~ p + plag + klag + vhat, data = klein)

Residuals:
    Min      1Q  Median      3Q     Max
-1.3537 -0.2834  0.1446  0.2808  1.4881

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept)  5.731e+00  4.598e+00   1.246  0.23058
p            6.445e-01  9.333e-02   6.905 3.54e-06 ***
plag         1.643e-01  9.652e-02   1.702  0.10816
klag        -9.060e-02  2.246e-02  -4.034  0.00096 ***
vhat        -6.813e+13  2.118e+13  -3.217  0.00538 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.8108 on 16 degrees of freedom
  (因為不存在 1 個觀察量被刪除了)
Multiple R-squared:  0.9583,    Adjusted R-squared:  0.9479
F-statistic: 91.95 on 4 and 16 DF,  p-value: 7.907e-11
```

H_0:δ=0、H_1:δ≠0，vhat 的係數 $= -6.813 \times 10^{13}$，t = –3.217，p = 0.0054，因 p-值 < 0.01，拒絕 H0，表示殘差與投資方程式顯著相關

⇒ 利潤 p 是內生變數，必須使用 2SLS 而非 OLS 來估計投資方程式

d. Obtain the 2SLS estimates of the investment equation using all eight exogenous and predetermined variables as IVs and software designed for 2SLS. Compare the estimates to the OLS estimates in part (a). Do you find any important differences?

```
Call:
ivreg(formula = i ~ p + plag + klag | cn + w1 + g + tx + e +
    plag + klag, data = klein)

Residuals:
     Min       1Q   Median       3Q      Max
-2.56562 -0.63169  0.03687  0.41542  1.49226

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 10.12579    5.46555   1.853 0.081374 .
p            0.47964    0.09711   4.939 0.000125 ***
plag         0.33304    0.10086   3.302 0.004212 **
klag        -0.11179    0.02673  -4.183 0.000624 ***

Diagnostic tests:
                 df1 df2 statistic  p-value
Weak instruments   5  13  5.99e+29  < 2e-16 ***
Wu-Hausman         0  17       NaN      NaN
Sargan             4  NA  2.10e+01 0.000317 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 1.009 on 17 degrees of freedom
Multiple R-Squared: 0.9313,     Adjusted R-squared: 0.9192
Wald test: 76.88 on 3 and 17 DF,  p-value: 4.299e-10
```

2SLS：$\hat{i}_t = 10.1258 + 0.4796\,p_t + 0.3330\,plag_t - 0.1118\,klag_t$

在 2SLS 估計中，投資方程式中當期利潤 ptp_tpt 的係數為 +0.4796，與 OLS 的 +0.4796 幾乎完全相同，且在 1% 水準高度顯著；同時，落後一期利潤 plagplagplag 為正向、落後一期資本存量 klagklagklag 為負向，二者符號與 OLS 結果一致並在 5% 或 1% 水準下顯著，說明在此模型中 OLS 的內生性偏誤不大。弱工具檢定顯示 $\chi^2(5, 13)=5.99\times10^{29}$（$p < 2e^{-16}$），工具變數極為強勁；然而，Sargan 過度識別檢定 $\chi^2(4)=21.0$（$p = 0.0003$）拒絕了所有過剩工具變數有效的假設，提示至少有一個工具變數可能不完全有效，值得進一步檢視

e. Estimate the second-stage model $I_t = \beta_1 + \beta_2 \hat{P}_t + \beta_3 P_{t-1} + \beta_4 K_{t-1} + e_{2t}$ by OLS. Compare the estimates and standard errors from this estimation to those in part (d). What differences are there?

```
Call:
lm(formula = i ~ phat + plag + klag, data = klein)

Residuals:
     Min       1Q   Median       3Q      Max
-2.56562 -0.63169  0.03687  0.41542  1.49226

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 10.12579    5.46555   1.853 0.081374 .
phat         0.47964    0.09711   4.939 0.000125 ***
plag         0.33304    0.10086   3.302 0.004212 **
klag        -0.11179    0.02673  -4.183 0.000624 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 1.009 on 17 degrees of freedom
  (因為不存在 1 個觀察量被刪除了)
Multiple R-squared:  0.9313,    Adjusted R-squared:  0.9192
F-statistic: 76.88 on 3 and 17 DF,  p-value: 4.299e-10
```

手動第二階段的 OLS 結果與 ivreg() 直接產出的 2SLS 結果在係數估計值及標準誤上完全一致，證實我們的 2SLS 實作正確

f. Let the 2SLS residuals from part (e) be $\hat{e}_{2t}$. Regress these residuals on all the exogenous and predetermined variables. If these instruments are valid, then the R^2 from this regression should be low, and none of the variables are statistically significant. The Sargan test for instrument validity is discussed in Section 10.4.3. The test statistic TR^2 has a chi-square distribution with degrees of freedom equal to the number of "surplus" IVs if the surplus instruments are valid. The investment equation includes three exogenous and/or predetermined variables out of the total of eight possible. There are $L = 5$ external instruments and $B = 1$ right-hand side endogenous variables. Compare the value of the test statistic to the 95th percentile value from the $\chi^2_{(4)}$ distribution. What do we conclude about the validity of the surplus instruments in this case?

```
Sargan test TR^2 = 10.708
自由度 = 4
P 值 = 0.0301
```

由於 $p < 0.05$，我們在 5% 水準下拒絕虛無假設（「所有過剩的工具變數都是有效的」），說明這組工具變數不完全通過過度識別檢定，存在至少一個「過剩」工具變數可能無效。