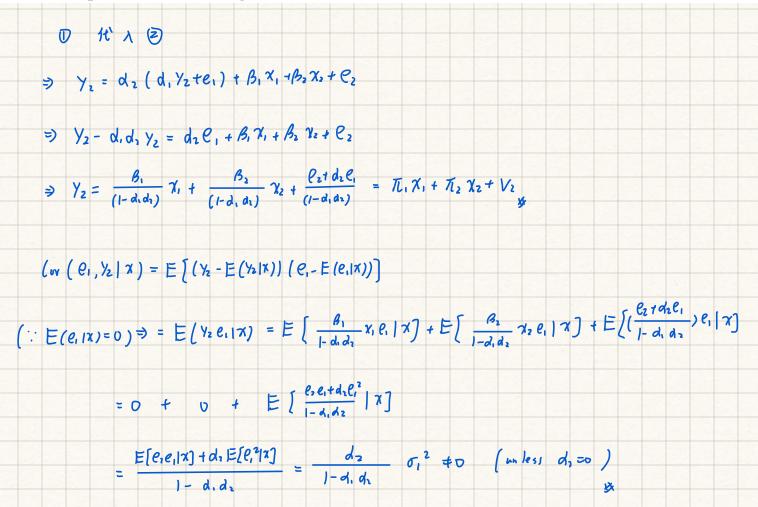
11.1 Our aim is to estimate the parameters of the simultaneous equations model

$$y_1 = \alpha_1 y_2 + e_1$$

$$y_2 = \alpha_2 y_1 + \beta_1 x_1 + \beta_2 x_2 + e_2$$

We assume that x_1 and x_2 are exogenous and uncorrelated with the error terms e_1 and e_2 .

a. Solve the two structural equations for the reduced-form equation for y_2 , that is, $y_2 = \pi_1 x_1 + \pi_2 x_2 + v_2$. Express the reduced-form parameters in terms of the **structural parameters** and the reduced-form error in terms of the structural parameters and e_1 and e_2 . Show that y_2 is correlated with e_1 .



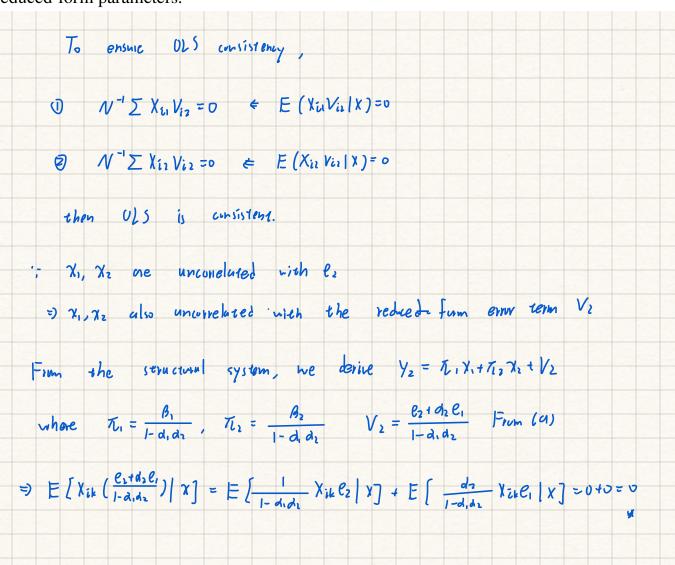
b. Which equation parameters are consistently estimated using OLS? Explain.

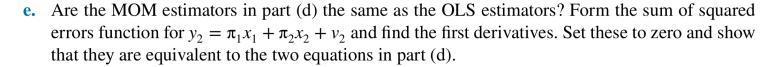
C. Which parameters are "identified," in the simultaneous equations sense? Explain your reasoning. M = 2

d. To estimate the parameters of the reduced-form equation for y_2 using the method of moments (MOM), which was introduced in Section 10.3, the two moment equations are

$$N^{-1} \sum_{i=1}^{n} x_{i1} (y_2 - \pi_1 x_{i1} - \pi_2 x_{i2}) = 0$$
$$N^{-1} \sum_{i=1}^{n} x_{i2} (y_2 - \pi_1 x_{i1} - \pi_2 x_{i2}) = 0$$

Explain why these two moment conditions are a valid basis for obtaining consistent estimators of the reduced-form parameters.





$$0LS - SS \not\models \Rightarrow S (\overline{\Lambda}_1, \overline{\Lambda}_2 \mid Y, Y) = \sum (Y_2 - \overline{\Lambda}_1 X_1 - \overline{\Lambda}_2 Y_2)^2$$

$$(\Rightarrow) \frac{\partial S}{\partial \overline{\Lambda}_1} = 2 \sum (Y_2 - \overline{\Lambda}_1 X_1 - \overline{\Lambda}_1 Y_2) Y_2 = 0$$

$$\Rightarrow \frac{\partial S}{\partial \overline{\Lambda}_2} = 2 \sum (Y_2 - \overline{\Lambda}_1 X_1 - \overline{\Lambda}_1 Y_2) Y_2 = 0$$

f. Using $\sum x_{i1}^2 = 1$, $\sum x_{i2}^2 = 1$, $\sum x_{i1}x_{i2} = 0$, $\sum x_{i1}y_{1i} = 2$, $\sum x_{i1}y_{2i} = 3$, $\sum x_{i2}y_{1i} = 3$, $\sum x_{i2}y_{2i} = 4$, and the two moment conditions in part (d) show that the MOM/OLS estimates of π_1 and π_2 are $\hat{\pi}_1 = 3$ and $\hat{\pi}_2 = 4$.

g. The fitted value $\hat{y}_2 = \hat{\pi}_1 x_1 + \hat{\pi}_2 x_2$. Explain why we can use the moment condition $\sum \hat{y}_{i2} (y_{i1} - \alpha_1 y_{i2}) = 0$ as a valid basis for consistently estimating α_1 . Obtain the IV estimate of α_1 .

To estimate the seructural equation
$$Y_1 = d_1Y_2 + e_1$$
, we use $25US$. Since \hat{Y}_2 is generated from exagenous varietables X_1 and X_2 , it is uncontellated with the structural error term e_1 . This suitisfies the arthogonality condition regard for constitent wondition.

Use the moment condition $\sum \hat{Y}_{2i} (\hat{Y}_{ii} - d_1 \hat{Y}_{2i}) = 0 \Rightarrow d_1 = \frac{\sum \hat{Y}_{2i} \hat{Y}_{ii}}{\sum \hat{X}_{ii}^2}$

Substituting $\hat{Y}_{2i} = \hat{T}_1 \hat{Y}_{2i} + \hat{T}_1 \hat{X}_{12}$, $d_1 = \frac{\hat{T}_2 \hat{Y}_{2i} \hat{Y}_{2i}}{\hat{T}_2 \hat{X}_{12} \hat{Y}_{2i}} = \frac{3 \times 2 + 4 \times 3}{3 \times 5 + 4 \times 4} = \frac{18}{25}$

h. Find the 2SLS estimate of α_1 by applying OLS to $y_1 = \alpha_1 \hat{y}_2 + e_1^*$. Compare your answer to that in part (g).

$$\frac{\partial}{\partial x}, \quad 2525 = \frac{\sum_{i=1}^{N_{i}} y_{ii}}{\sum_{i=1}^{N_{i}} y_{ii}}$$

$$\frac{\sum_{i=1}^{N_{i}} y_{ii}}{\sum_{i=1}^{N_{i}} y_{ii}} = \sum_{i=1}^{N_{i}} y_{ii} = \sum_{i=1}^{N_{i}} y_{ii}$$

$$\frac{\sum_{i=1}^{N_{i}} y_{ii}}{\sum_{i=1}^{N_{i}} y_{ii}} = \sum_{i=1}^{N_{i}} y_{ii} = \sum_{i=1}^{N_{i}} y_{ii}$$

$$\frac{\sum_{i=1}^{N_{i}} y_{ii}}{\sum_{i=1}^{N_{i}} y_{ii}} = \sum_{i=1}^{N_{i}} y_{ii}$$

$$\frac{\sum_{i=1}^{N_{i}} y_{ii$$

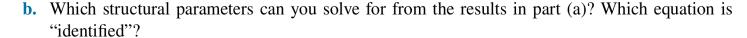
11.16 Consider the following supply and demand model

Demand:
$$Q_i = \alpha_1 + \alpha_2 P_i + e_{di}$$
, Supply: $Q_i = \beta_1 + \beta_2 P_i + \beta_3 W_i + e_{si}$

where Q is the quantity, P is the price, and W is the wage rate, which is assumed exogenous. Data on these variables are in Table 11.7.

TABLE 11.7		Data for Exercise 11.16
Q	P	W
4	2	2
6	4	3
9	3	1
3	5	1
8	8	3

a. Derive the algebraic form of the reduced-form equations, $Q = \theta_1 + \theta_2 W + v_2$ and $P = \pi_1 + \pi_2 W + v_1$, expressing the reduced-form parameters in terms of the structural parameters.



①
$$d_1, d_2$$
② $M=2$, $M-1=1$

[Demand =) identified (1)

Supply =) not identified (v)

c. The estimated reduced-form equations are $\hat{Q} = 5 + 0.5W$ and $\hat{P} = 2.4 + 1W$. Solve for the identified structural parameters. This is the method of **indirect least squares**.

$$\hat{\beta} = 2.4 + 10 = \pi_1 + \pi_2 n \qquad \hat{\pi}_1 = 2.4, \quad \hat{\pi}_2 = 1$$

$$\hat{Q} = 54 \text{ U.5W} = Q_1 + Q_2 \text{ W} \qquad \hat{Q}_1 = 5 \qquad \hat{Q}_2 = \text{U.5}$$

$$\Rightarrow 5 + \text{U.5W} = d_1 + d_2 (2.4 + \text{W}) = d_1 + 2.4 d_2 \text{ W}$$

$$\Rightarrow (3) = d_1 + 2.4 d_2$$

$$(4) = 3.8, \quad d_2 = \text{U.5}$$

d. Obtain the fitted values from the reduced-form equation for *P*, and apply 2SLS to obtain estimates of the demand equation.

$$\hat{p}_{i} = 2.44 \text{ Wi}$$

$$Q_{i} = d_{i} + d_{2} \hat{p}_{i} + e_{i}$$

$$d_{2} = \frac{\sum (\hat{p}_{i} - \bar{p})^{2}}{\sum (\hat{p}_{i} - \bar{p})^{2}} = \frac{2}{4} = 0.5$$

$$\hat{Z}_{i} = d_{i} + u.5 \hat{p}_{i}, \quad d_{i} = 6 - 0.5 \times 0.44 = 3.8$$

$$\Rightarrow \hat{Q}_{i} = 3.8 + 0.5 \hat{p}_{i}$$