5.23 The file *cocaine* contains 56 observations on variables related to sales of cocaine powder in northeastern California over the period 1984–1991. The data are a subset of those used in the study Caulkins, J. P. and R. Padman (1993), "Quantity Discounts and Quality Premia for Illicit Drugs," *Journal of the American Statistical Association*, 88, 748–757. The variables are

PRICE = price per gram in dollars for a cocaine sale
QUANT = number of grams of cocaine in a given sale
QUAL = quality of the cocaine expressed as percentage purity
TREND = a time variable with 1984 = 1 up to 1991 = 8
Consider the regression model

$$PRICE = \beta_1 + \beta_2 QUANT + \beta_3 QUAL + \beta_4 TREND + e$$

a. What signs would you expect on the coefficients β_2, β_3, and β_4?

b. Use your computer software to estimate the equation. Report the results and interpret the coefficient estimates. Have the signs turned out as you expected?

c. What proportion of variation in cocaine price is explained jointly by variation in quantity, quality, and time?

d. It is claimed that the greater the number of sales, the higher the risk of getting caught. Thus, sellers are willing to accept a lower price if they can make sales in larger quantities. Set up H_0 and H_1 that would be appropriate to test this hypothesis. Carry out the hypothesis test.

e. Test the hypothesis that the quality of cocaine has no influence on expected price against the alternative that a premium is paid for better-quality cocaine.

f. What is the average annual change in the cocaine price? Can you suggest why price might be changing in this direction?

a.
- β2 (for QUANT): Negative. We expect a negative sign because larger quantities (QUANT) are likely associated with a lower price per gram (PRICE) due to quantity discounts in the cocaine market.
- β3 (for QUAL): Positive. We expect a positive sign because higher quality (greater purity, QUAL) should increase the price per gram (PRICE), as buyers are willing to pay a premium for purer cocaine.
- β4 (for TREND): Negative. We expect a negative sign because, over the period from 1984 to 1991, the price per gram of cocaine likely decreased due to increased supply, market saturation, and competition.

b.

```
Call:
lm(formula = price ~ quant + qual + trend, data = cocaine)

Residuals:
    Min      1Q  Median      3Q     Max
-43.479 -12.014  -3.743  13.969  43.753

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 90.84669    8.58025  10.588 1.39e-14 ***
quant       -0.05997    0.01018  -5.892 2.85e-07 ***
qual         0.11621    0.20326   0.572   0.5700
trend       -2.35458    1.38612  -1.699   0.0954 .
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 20.06 on 52 degrees of freedom
Multiple R-squared:  0.5097,    Adjusted R-squared:  0.4814
F-statistic: 18.02 on 3 and 52 DF,  p-value: 3.806e-08
```

PRICE = 90.8467 - 0.05997*QUANT + 0.11621*QUAL - 2.35458*TREND
- Coefficient for QUANT (β2 = −0.05997)
  Interpretation: For each additional gram of cocaine in a sale (QUANT increases by 1), the price per gram decreases by 0.05997 dollars (about 6 cents), holding QUAL and TREND constant.
- Coefficient for QUAL (β3 = 0.11621)
  Interpretation: For each 1% increase in the purity of the cocaine (QUAL), the price per gram increases by 0.11621 dollars (about 12 cents), holding QUANT and TREND constant.
- Coefficient for TREND (β4 = −2.35458)
  Interpretation: For each year increase in the time trend (TREND), the price per gram decreases by 2.35458 dollars (about $2.35), holding QUANT and QUAL constant.
- Conclusion on Signs: The actual signs of all coefficients (β2, β3, and β4) match our expectations from part a. This alignment suggests that our economic intuition about quantity discounts, quality premia, and time trends in the cocaine market is consistent with the data.

c. The proportion of variation in cocaine price (PRICE) explained jointly by variation in quantity (QUANT), quality (QUAL), and time (TREND) is given by the R^2 :
$R^2 = 0.5097$
This means 50.97% of the variation in cocaine price is explained by the model.

d. H0: β2 ≥ 0 (there is no quantity discount or the price increases with quantity).
H1 :β2 < 0 (larger quantities lead to a lower price per gram).
This is a one-tailed test (left-tailed) because alternative hypothesis specifies a negative direction.
Significance level: α = 0.05.
The critical t-value for a one-tailed test (left-tailed) at α = 0.05 with 52 degrees of freedom is approximately -1.675.
The calculated t-value is t = -0.05997 ÷ 0.01018 = -5.8910
Since −5.891 < −1.675, we can reject H0 based on the critical value.
Conclusion: At the 5% significance level, we reject the null hypothesis and conclude that there is significant evidence to support the claim that sellers accept a lower price per gram for larger quantities.

```
> qt(0.05,52)
[1] -1.674689
```

e. H0: β3 ≤ 0 (Quality has no influence on price).
H1 :β3 > 0 (A premium is paid for better-quality cocaine, meaning higher quality increases the price).
This is a one-tailed test (right-tailed) because the alternative hypothesis specifies a positive direction.
Significance level: α = 0.05.
The critical t-value for a one-tailed test (right-tailed) at α=0.05 with 52 degrees of freedom is approximately 1.675.
The calculated t-value is t = 0.11621 ÷ 0.20326 = 0.5717
Since 0.5717 < 1.675, we do not reject H0 based on the critical value.
Conclusion: At the 5% significance level, we fail to reject the null hypothesis. There is not sufficient evidence to conclude that the quality of cocaine (QUAL) has a significant positive influence on the expected price (PRICE). We cannot confirm that a premium is paid for better-quality cocaine in this sample.

```
> qt(1-0.05,52)
[1] 1.674689
```

f. The average annual change in the cocaine price is a decrease of 2.35458 dollars per gram, as indicated by the coefficient b4 = −2.35458 for TREND. This means that, holding quantity and quality constant, the price per gram decreases by about 2.35 dollars each year from 1984 to 1991.
A possible reason for a decreasing price is expansion of supply chains. Improved trafficking or production efficiency could increase supply, pushing prices down.