Q11、1

(a) 1、 $y_1 = a_1 y_2 + e_1$

2、 $y_2 = a_2 y_1 + \beta_1 x_1 + \beta_2 x_2 + e_2$

1、代入2 得 $y_2 = a_2(a_1 y_2 + e_1) + \beta_1 x_1 + \beta_2 x_2 + e_2$

$\Rightarrow y_2 = a_1 a_2 y_2 + a_2 e_1 + \beta_1 x_1 + \beta_2 x_2 + e_2$

$\Rightarrow y_2 - a_1 a_2 y_2 = a_2 e_1 + \beta_1 x_1 + \beta_2 x_2 + e_2$ 提出 $y_2(1-a_1 a_2)$

$\Rightarrow y_2 = \underbrace{\frac{\beta_1}{(1-a_1 a_2)}}_{\pi_1} x_1 + \underbrace{\frac{\beta_2}{(1-a_1 a_2)}}_{\pi_2} x_2 + \underbrace{\frac{a_2 e_1 + e_2}{(1-a_1 a_2)}}_{v_2}$

v_2 中含 e_1，所以 y_2 的誤差項和 e_1 有關

表示 y_2 是內生變數

$$Cov(y_2, e_1 | x) = E(y_2, e_1 | x) = \frac{E(e_1 e_2 | x) + a_2 E(e_1^2 | x)}{1 - a_1 a_2} = \frac{\sigma_1^2 a_2}{1 - a_1 a_2}$$

假設 2個方程式的誤差項不相關，$Cov(y_2, e_1 | x) \neq 0$ 除非 $a_2 = 0$

(b) 對於 $y_1 = a_1 y_2 + e_1$，由於 y_2 是內生的（和 e_1 相關），用OLS估計會有偏誤

對於 $y_2 = a_2 y_1 + \beta_1 x_1 + \beta_2 x_2 + e_2$ 同樣 y_1 是內生變數（也受 e_1 影響） OLS 估計 a_2 也不一致，只用 OLS 估 β_1、β_2 也受 y_1 干擾，不一定

沒任何參數在原始結構中用 OLS 一致估計

(c)

方程(1) 省略 2個外生變數（多於1個變數） 可識別

方程(2) 沒省略任何變數 不可識別

(d)

$$E(x_{i1} v_{i1} \mid x) = E(x_{i2} v_{i2} \mid x)$$

$$\Rightarrow E\left[x_{ik}\left(\frac{a_1 e_1 + e_2}{1 - a_1 a_2}\right) \mid x\right]$$

$$\Rightarrow E\left[\left(\frac{a_1 e_1}{1 - a_1 a_2}\right) x_{ik} \mid x\right] + E\left[\left(\frac{e_2}{1 - a_1 a_2}\right) x_{ik} \mid x\right] = 0$$

(e)

$$\frac{\partial S(\pi_1, \pi_2 \mid y, x)}{\partial \pi_1} = 2\Sigma(y_2 - \pi_1 x_1 - \pi_2 x_2)(-x_1) = 0$$

$$\frac{\partial S(\pi_1, \pi_2 \mid y, x)}{\partial \pi_2} = 2\Sigma(y_2 - \pi_1 x_1 - \pi_2 x_2)(-x_2) = 0$$

$$\times \frac{1}{2} \times N^{-1} = (d)$$

(f)

$$N^{-1}\sum x_{i1}(y_2-\pi_1 x_{i1}-\pi_2 x_{i2})=0$$

$$N^{-1}\sum x_{i2}(y_2-\pi_1 x_{i1}-\pi_2 x_{i2})=0$$

$$\Rightarrow \sum x_{i1}y_2-\pi_1(x_{i1})^2-\pi_2 x_{i1}x_{i2}=0$$

$$\sum x_{i2}y_2-\pi_1 x_{i1}x_{i2}-\pi_2(x_{i2})^2=0$$

代入 得

$$\Rightarrow 3-\hat{\pi}_1=0 \qquad \hat{\pi}_1=3 \qquad \hat{\pi}_2=4$$

$$4-\hat{\pi}_2=0$$

(g) $y_1=\alpha_1 y_2+e_1$ $\qquad \hat{y}_2=\hat{\pi}_1 x_1+\hat{\pi}_2 x_2$

$$\sum \hat{y}_{i2}(y_{i1}-\alpha_1 y_{i2})=0$$

$$\Rightarrow \sum \hat{y}_{i2}y_{i1}-\alpha\sum \hat{y}_{i2}y_{i2}=0$$

$$\Rightarrow \alpha=\frac{\sum \hat{y}_{i2}y_{i1}}{\sum \hat{y}_{i2}y_{i2}} \Rightarrow \hat{\alpha}=\frac{\sum(\hat{\pi}_1 x_1+\hat{\pi}_2 x_2)y_{i1}}{\sum(\hat{\pi}_1 x_1+\hat{\pi}_2 x_2)y_{i2}}=\frac{3\times 2+4\times 3}{3\times 3+4\times 4}=\frac{18}{25}$$

(h) $\hat{\alpha}=\dfrac{\sum \hat{y}_{i2}y_{i1}}{\sum(\hat{y}_{i2})^2}$ $\qquad \hat{v}_2=y_2-\hat{y}_2 \qquad \hat{y}_2=y_2-\hat{v}_2$

$$\sum \hat{y}_{i2}^2=\sum(\hat{y}_{i2})(y_2-\hat{v}_2)=\sum \hat{y}_{i2}y_2-\underbrace{\sum \hat{y}_{i2}\hat{v}_2}_{=0}=\sum \hat{y}_{i2}y_2$$

Q11-16

(a)

$$\begin{cases} Q = \alpha_1 + \alpha_2 P_i + e_{di} \\ P = \beta_1 + \beta_2 P_i + \beta_3 w_i + e_{si} \end{cases}$$

$$\alpha_1 + \alpha_2 P_i + e_{di} = \beta_1 + \beta_2 P_i + \beta_3 w_i + e_{si}$$

$$\Rightarrow \alpha_2 P_i - \beta_2 P_i = \beta_1 - \alpha_1 + \beta_3 w_i + e_{si} - e_{di} \quad \text{提 } P_i$$

$$\Rightarrow P_i = \frac{\beta_1 - \alpha_1}{\alpha_2 - \beta_2} + \frac{\beta_3}{\alpha_2 - \beta_2} w_i + \frac{e_{si} - e_{di}}{\alpha_2 - \beta_2} \quad \text{代 ②}$$

$$Q = \alpha_1 + \alpha_2 \left(\frac{\beta_1 - \alpha_1}{\alpha_2 - \beta_2} + \frac{\beta_3}{\alpha_2 - \beta_2} w_i + \frac{e_{si} - e_{di}}{\alpha_2 - \beta_2} \right) + e_{di}$$

$$= \left(\frac{\alpha_2 \beta_1 - \alpha_1 \beta_2}{\alpha_2 - \beta_2} + \frac{\beta_3 \alpha_2}{\alpha_2 - \beta_2} w_i + \frac{\alpha_2 e_{si} - e_{di} \beta_2}{\alpha_2 - \beta_2} \right)$$

(b) $M=2$ 省略至少 1個 variable

方程(需求) 可以 識別

方程(供給) 不可識別

(c)

$\hat{Q} = 5 + 0.5w$

$\hat{P} = 2.4 + 1w$

$\Rightarrow$

$$5 + 0.5w = \alpha_1 + \alpha_2 (2.4 + 1w)$$

$$5 + 0.5w = \alpha_1 + 2.4\alpha_2 + \alpha_2 w$$

$\alpha_2 = 0.5$ $\quad 5 = \alpha_1 + 1.2 \quad \alpha_1 = 3.8$

(d)

$\hat{P} = 2.4 + w$

w	2	3	1	1	3	
$\hat{P}$	4.4	5.4	3.4	7.4	5.4	$u_{\hat{P}} = 4.4$
$Q =$	4	6	9	3	8	$u_Q = 6$
$\hat{P} - u_{\hat{P}}$	0	1	-1	-1	1	
$Q - u_Q$	-2	0	3	-3	2	
$(\hat{P} - u_{\hat{P}})^2$	0	1	1	1	1	$\Rightarrow 4$
$(Q - u_Q)^2$	4	0	9	9	4	$\Rightarrow 26$
$(P - u_P)(Q - u_Q)$	0	0	-3	3	2	$\Rightarrow 2$

$Q = a_1 + a_2 \hat{P} + e_{di}$

$a_2 = \frac{2}{4} = \frac{1}{2}$ $\quad 6 = a_1 + \frac{1}{2} \times 4.4$ $\quad a_1 = 3.8$

$Q = \frac{1}{2} P + 3.8$

Q11-17

(a) $M=8$ $M-1=7$ 個外生變數

消費方程式：用6個變數，排除10個

投資方程式：用6個變數，排除11個

工資方程式：用5個變數，排除11個

都符合，可識別

(b)

	RHS 內生	排除外生	
消費方程式：	2個	5個	
投資方程式：	1個	5個	都符合，可識別
工資方程式：	1個	5個	

(c)

$$W_{1t} = \pi_1 + \pi_2 G_t + \pi_3 W_{2t} + \pi_4 TX_t + \pi_5 TIME_t + \pi_6 P_{t-1} + \pi_7 K_{t-1} + \pi_8 E_{t-1} + V$$

(d) 取得內生預測

$$\hat{W}_{1t} = \pi_1 + \pi_2 G_t + \pi_3 W_{2t} + \pi_4 TX_t + \pi_5 TIME_t + \pi_6 P_{t-1} + \pi_7 K_{t-1} + \pi_8 E_{t-1}$$

$$\hat{P}_t = \hat{W}_{2t}$$

接著用 OLS 估計：

$$CN_t = \alpha_0 + \alpha_1(\hat{W}_{1t} + \hat{W}_{2t}) + \alpha_2 \hat{P}_t + \alpha_3 \hat{P}_t + u_t$$

(e)

估計值會相同，但 t 值不同，$\hat{W}_{1t}$、P_t 有估計誤差 (d) 的 OLS 未考慮