5.33 Use the observations in the data file *cps5_small* to estimate the following model:

$$\ln(WAGE) = \beta_1 + \beta_2 EDUC + \beta_3 EDUC^2 + \beta_4 EXPER + \beta_5 EXPER^2 + \beta_6 (EDUC \times EXPER) + e$$

a. At what levels of significance are each of the coefficient estimates "significantly different from zero"?

b. Obtain an expression for the marginal effect $\partial E\left[\ln(WAGE) | EDUC, EXPER\right] / \partial EDUC$. Comment on how the estimate of this marginal effect changes as *EDUC* and *EXPER* increase.

c. Evaluate the marginal effect in part (b) for all observations in the sample and construct a histogram of these effects. What have you discovered? Find the median, 5th percentile, and 95th percentile of the marginal effects.

d. Obtain an expression for the marginal effect $\partial E\left[\ln(WAGE) | EDUC, EXPER\right] / \partial EXPER$. Comment on how the estimate of this marginal effect changes as *EDUC* and *EXPER* increase.

e. Evaluate the marginal effect in part (d) for all observations in the sample and construct a histogram of these effects. What have you discovered? Find the median, 5th percentile, and 95th percentile of the marginal effects.

f. David has 17 years of education and 8 years of experience, while Svetlana has 16 years of education and 18 years of experience. Using a 5% significance level, test the null hypothesis that Svetlana's expected log-wage is equal to or greater than David's expected log-wage, against the alternative that David's expected log-wage is greater. State the null and alternative hypotheses in terms of the model parameters.

g. After eight years have passed, when David and Svetlana have had eight more years of experience, but no more education, will the test result in (f) be the same? Explain this outcome?

h. Wendy has 12 years of education and 17 years of experience, while Jill has 16 years of education and 11 years of experience. Using a 5% significance level, test the null hypothesis that their marginal effects of extra experience are equal against the alternative that they are not. State the null and alternative hypotheses in terms of the model parameters.

i. How much longer will it be before the marginal effect of experience for Jill becomes negative? Find a 95% interval estimate for this quantity.

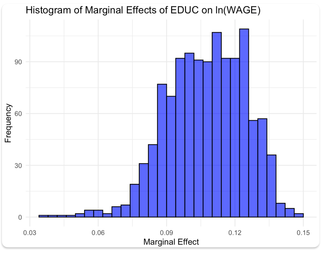
---

a. 由右圖 p_value 可知：
EDUC、EXPER、EXPER²、EDUC×EXPER 在 1% 顯著性水準下顯著。
EDUC² 在 12% 顯著性水準下顯著。

```
Call:
lm(formula = ln_wage ~ educ + educ2 + exper + exper2 + educ_exper, 
    data = data)

Residuals:
     Min       1Q   Median       3Q      Max 
-1.6628 -0.3138 -0.0276  0.3140  2.1394 

Coefficients:
              Estimate Std. Error t value Pr(>|t|)    
(Intercept)  1.038e+00  2.757e-01   3.764 0.000175 ***
educ         8.954e-02  3.108e-02   2.881 0.004038 ** 
educ2        1.458e-03  9.242e-04   1.578 0.114855    
exper        4.488e-02  7.297e-03   6.150 1.06e-09 ***
exper2      -4.680e-04  7.601e-05  -6.157 1.01e-09 ***
educ_exper  -1.010e-03  3.791e-04  -2.665 0.007803 ** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.4638 on 1194 degrees of freedom
Multiple R-squared:  0.3227,    Adjusted R-squared:  0.3198 
F-statistic: 113.8 on 5 and 1194 DF,  p-value: < 2.2e-16
```

b. ∂ E [ln(WAGE) | EDUC, EXPER] /∂EDUC = β2 + 2×β3×EDUC + β6×EXPER
代入 b2, b3, b6
ME_EDUC = 0.08954 + 0.002916 EDUC − 0.001010 EXPER
隨著 EDUC 增加，邊際效應增加（因為 b3 = 0.001458 > 0），表明教育的邊際報酬隨著教育年數提高而增加。
隨著 EXPER 增加，邊際效應減少（因為 b6 = −0.001010 < 0），表明對於經驗更豐富的人，增加教育年數對工資的影響減弱。

c.



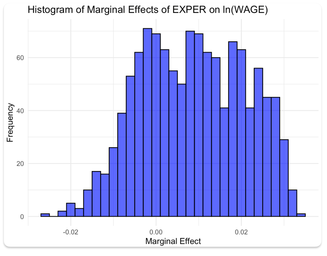
直方圖顯示邊際效應的範圍從 0.036 到 0.148，主體集中在 0.09 到 0.13 之間。

中位數: 0.1084
第 5 百分位數: 0.0801
第 95 百分位數: 0.1336

```
> quantile(data$marginal_effect, probs = c(0.05, 0.5, 0.95))
       5%       50%       95% 
0.08008187 0.10843125 0.13361880 
```

d. ∂ E [ln(WAGE) | EDUC, EXPER] /∂EXPER = β4 + 2×β5×EXPER + β6×EDUC
代入 b4, b5, b6
ME_EXPER = 0.04488 − 0.000936 EXPER − 0.001010 EDUC
隨著 EDUC 增加，邊際效應減少（因為 b6 = −0.001010 < 0），表明這意味著對於教育水平較高的人，增加工作經驗對工資的影響減弱。
隨著 EXPER 增加，邊際效應減少（因為 b5 = −0.000936 < 0），表明隨著經驗的增加，額外經驗對工資的影響減弱。

e.



直方圖顯示邊際效應的範圍從 − 0.025 到 0.034，主體集中在 -0.01 到 0.02 之間。

中位數: 0.0084
第 5 百分位數: −0.0104
第 95 百分位數: 0.0279

```
> quantile(data$marginal_effect, probs = c(0.05, 0.5, 0.95))
          5%          50%          95% 
-0.010376212  0.008418878  0.027931151 
```

f. David 的預期對數工資為：
E[ln(WAGE) | EDUC=17, EXPER=8] = β1 + β2×17 + β3×17² + β4×8 + β5×8² + β6×(17×8)
Svetlana 的預期對數工資為：
E[ln(WAGE) | EDUC=16, EXPER=18] = β1 + β2×16 + β3×16² + β4×18 + β5×18² + β6×(16×18)
原假設 H0: Svetlana 的預期對數工資大於或等於 David 的，即：
E[ln(WAGE) | EDUC=16, EXPER=18] ⩾ E[ln(WAGE) | EDUC=17, EXPER=8]
代入模型，並簡化：−β2 −33β3 +108β4 +2608β5 +1528β6 ⩾0

H0: −β2 −33β3 +108β4 +2608β5 +1528β6 ⩾0　　H1: −β2 −33β3 +108β4 +2608β5 +1528β6 < 0
α = 0.05，df = 1194，t_critical = −1.6461
t = 1.669902 < −1.6461，未落在拒絕域。
無法拒絕 H0，表示在 5% 顯著水準下，沒有足夠的證據支持 David 的預期對數工資大於 Svetlana 的。換句話說，Svetlana 的預期對數工資可能大於或等於 David 的。

g. 8 年後，David 的預期對數工資為：
E[ln(WAGE) | EDUC=17, EXPER=16] = β1 + β2×17 + β3×17² + β4×16 + β5×16² + β6×(17×16)
8 年後，Svetlana 的預期對數工資為：
E[ln(WAGE) | EDUC=16, EXPER=26] = β1 + β2×16 + β3×16² + β4×26 + β5×26² + β6×(16×26)
原假設 H0: Svetlana 的預期對數工資大於或等於 David 的，即：
E[ln(WAGE) | EDUC=16, EXPER=26] ⩾ E[ln(WAGE) | EDUC=17, EXPER=16]
代入模型，並簡化：−β2 −33β3 +108β4 +4208β5 +1448β6 ⩾0

H0: −β2 −33β3 +108β4 +4208β5 +1448β6 ⩾0　　H1: −β2 −33β3 +108β4 +4208β5 +1448β6 < 0
α = 0.05，df = 1194，t_critical = −1.6461
t = − 2.062365 > −1.6461，落在拒絕域。
拒絕 H0，表示在 5% 顯著水準下，有足夠的證據支持 8 年後 David 的預期對數工資大於 Svetlana 的。

h. ∂ E [ln(WAGE) | EDUC, EXPER] /∂EXPER = β4 + 2×β5×EXPER + β6×EDUC
Wendy 的邊際效應（EDUC=12，EXPER=17）：ME_Wendy = β4 + 28β5×17 + β6×12
Jill 的邊際效應（EDUC=16，EXPER=11）：ME_Jill = β4 + 28β5×11 + β6×16
原假設 H0：Wendy 和 Jill 的邊際效應相等，即：ME_Wendy = ME_Jill，簡化後：128β5 − 4β6 = 0

H0: 128β5 − 4β6 = 0　　H1: 128β5 − 4β6 ≠ 0
α = 0.05，df = 1194，t_critical = ±1.961953
t = −1.027304 ∈ [−1.961953, 1.961953]，未落在拒絕域。
無法拒絕 H0，表示在 5% 顯著水準下，沒有足夠的證據表明 Wendy 和 Jill 的額外經驗的邊際效應不同。

i. ∂ E [ln(WAGE) | EDUC, EXPER] /∂EXPER = β4 + 2×β5×EXPER + β6×EDUC
Jill 的邊際效應（EDUC=16）：ME_Jill = β4 + 28β5×EXPER + β6×16
找到 EXPER 的值，使 Jill 的邊際效應為 0：EXPER = −(β4 + 16β6) / 28β5

點估計
代入 b4 = 0.04488、b5 = −0.000468、b6 = −0.001010，得出 EXPER = 30.67706
Jill 目前的經驗是 11 年，因此邊際效應變為負值還需要：30.67706 − 11 = 19.67706 年

95% 區間估計
令 g = −(β4 + 16β6) / 28β5
偏導數：
∂g/∂β4 = −1 / 28β5
∂g/∂β5 = −(β4 + 16β6) / 28β5²
∂g/∂β6 = −16 / 28β5
Var(g) = g²×Var(b4) + g⁵²×Var(b5) + g6²×Var(b6)
+ 2×g4×g5×Cov(b4,b5) + 2×g4×g6×Cov(b4,b6) + 2×g5×g6×Cov(b5,b6) = 3.593728
SE(g) = √Var(g) = 1.895713
α = 0.05，df = 1194，t_critical = −1.6461
95% 信賴區間 = 19.67706 ± t_critical × SE(g) = [15.95776 , 23.39636]

結論
Jill 的經驗邊際效應在 19.68 年後變為負值。
額外年數的 95% 置信區間為 [15.96, 23.40] 年。