HWCODE	Assignment
HW0324Q1	Let $K=2$, show that (b1, b2) in p. 29 of slides in Ch 5 reduces to the formula of (b1, b2) in (2.7) - (2.8)
HW0324Q2	Let $K=2$, show that cov(b1, b2) in p. 30 of slides in Ch 5 reduces to the formula of in (2.14) - (2.16).
HW0324Q3	C05Q03(a,b,c)

1. Let $k=2$, $\quad Y = X\beta + e$

$$Y = (y_1, y_2, \cdots y_N)', \quad \beta = (\beta_1, \beta_2, \cdots, \beta_K), \quad e = (e_1, e_2 \cdots e_N)'$$

$$X = \begin{bmatrix} 1 & x_1 \\ 1 & x_2 \\ \vdots & \vdots \\ 1 & x_n \end{bmatrix} \qquad Y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{bmatrix}$$

$$X'X = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ x_1 & x_2 & \cdots & x_n \end{bmatrix} \cdot \begin{bmatrix} 1 & x_1 \\ 1 & x_2 \\ \vdots & \vdots \\ 1 & x_N \end{bmatrix} = \begin{bmatrix} N & \sum_{i=1}^N x_i \\ \sum_{i=1}^N x_i & \sum_{i=1}^N x_i^2 \end{bmatrix}$$

$$X'Y = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ x_1 & x_2 & \cdots & x_n \end{bmatrix} \cdot \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^N y_i \\ \sum_{i=1}^N x_i y_i \end{bmatrix}$$

套入公式 $\quad b = (X'X)^{-1} X'Y$

$$b = \left(\begin{bmatrix} N & \sum_{i=1}^N x_i \\ \sum_{i=1}^N x_i & \sum_{i=1}^N x_i^2 \end{bmatrix} \right)^{-1} \begin{bmatrix} \sum_{i=1}^N y_i \\ \sum_{i=1}^N x_i y_i \end{bmatrix}$$

$$= \frac{1}{N \sum_{i=1}^N x_i^2 - \left(\sum_{i=1}^N x_i\right)^2} \begin{bmatrix} \sum_{i=1}^N x_i^2 & -\sum_{i=1}^N x_i \\ -\sum_{i=1}^N x_i & N \end{bmatrix} \begin{bmatrix} \sum_{i=1}^N y_i \\ \sum_{i=1}^N x_i y_i \end{bmatrix}$$

$$= \frac{1}{N \sum_{i=1}^N x_i^2 - \left(\sum_{i=1}^N x_i\right)^2} \begin{bmatrix} \sum_{i=1}^N x_i^2 \sum_{i=1}^N y_i - \sum_{i=1}^N x_i \cdot \sum_{i=1}^N x_i y_i \\ -\sum_{i=1}^N x_i \sum_{i=1}^N y_i + N \sum_{i=1}^N x_i y_i \end{bmatrix}$$

$$b_1 = \frac{\sum_{i=1}^N x_i^2 \sum_{i=1}^N y_i - \sum_{i=1}^N x_i \sum_{i=1}^N x_i y_i}{N \sum_{i=1}^N x_i^2 - \left(\sum_{i=1}^N x_i\right)^2}$$

$$b_2 = \frac{N \sum_{i=1}^N x_i y_i - \sum_{i=1}^N x_i \sum_{i=1}^N y_i}{N \sum_{i=1}^N x_i^2 - \left(\sum_{i=1}^N x_i\right)^2}$$

△

$$b_2=\frac{\sum(x_i-\bar{x})(y_i-\bar{y})}{\sum(x_i-\bar{x})^2}$$

$$\sum(x_i-\bar{x})(y_i-\bar{y})$$

$$=\sum(x_iy_i-x_i\bar{y}-\bar{x}y_i+\bar{x}\bar{y})$$

分子 $=\sum x_iy_i-\sum x_i\bar{y}-\sum\bar{x}y_i+\sum\bar{x}\bar{y}$

$$=\sum x_iy_i-N\bar{x}\bar{y}-N\bar{x}\bar{y}+N\bar{x}\bar{y}$$

$$=\sum x_iy_i-N\bar{x}\bar{y} \quad \#$$

$$\sum(x_i-\bar{x})^2$$

$$=\sum(x_i^2-2x_i\bar{x}+\bar{x}^2)$$

分母 $=\sum x_i^2-2\bar{x}\sum x_i+N\bar{x}^2$

$$=\sum x_i^2-2N\bar{x}^2+N\bar{x}^2$$

$$=\sum x_i-N\bar{x}^2$$

$$\sum(x_i-\bar{x})^2=\sum(x_i^2-2\bar{x}x_i+\bar{x}^2)$$

$$=\sum x_i^2-2\sum\bar{x}x_i+\sum\bar{x}^2$$

$$=\sum x_i^2-2N\bar{x}^2+N\bar{x}^2$$

$$=\sum x_i^2-N\bar{x}^2$$

$b_2\Rightarrow$ 同除 N $\Rightarrow$ $\dfrac{\sum_{i=1}^{N}x_iy_i-\frac{1}{N}(N\bar{x})(N\bar{y})}{\sum_{i=1}^{N}x_i^2-\frac{1}{N}(N\bar{x})^2}=\dfrac{\sum_{i=1}^{N}x_iy_i-N\bar{x}\bar{y}}{\sum_{i=1}^{N}x_i^2-N\cdot(\bar{x})^2}$

$$=\frac{\sum(x_i-\bar{x})(y_i-\bar{y})}{\sum(x_i-\bar{x})^2}$$

得證

$$b_1=\frac{\sum_{i=1}^{N}x_i^2\sum_{i=1}^{N}y_i-\sum_{i=1}^{N}x_i\sum_{i=1}^{N}x_iy_i}{N\sum_{i=1}^{N}x_i^2-(\sum_{i=1}^{N}x_i)^2}$$

$$=\frac{\sum_{i=1}^{N}x_i^2(N\cdot\bar{y})-N\bar{x}\cdot\sum_{i=1}^{N}x_iy_i}{N\sum_{i=1}^{N}x_i^2-(N\bar{x})^2}$$

$$=\frac{\sum_{i=1}^{N}x_i^2\bar{y}-\bar{x}\cdot\sum_{i=1}^{N}x_iy_i}{\sum_{i=1}^{N}x_i^2-N\bar{x}^2}$$

$$\bar{x}b_2=\frac{\bar{x}\sum_{i=1}^{N}x_iy_i-N\bar{x}^2\bar{y}}{\sum_{i=1}^{N}x_i^2-N\cdot(\bar{x})^2}$$

$$\bar{y}-\bar{x}b_2=\frac{\bar{y}\sum_{i=1}^{N}x_i^2-\cancel{N(\bar{x})^2\bar{y}}-\bar{x}\sum_{i=1}^{N}x_iy_i+\cancel{N\bar{x}^2\bar{y}}}{\sum_{i=1}^{N}x_i^2-N(\bar{x})^2}$$

$$=\frac{\bar{y}\sum_{i=1}^{N}x^2-\bar{x}\sum_{i=1}^{N}x_iy_i}{\sum_{i=1}^{N}x_i^2-N\bar{x}^2}$$

故得證 $b_1=\bar{y}-b_2\bar{x}$.

2. $k=2$

$$X=\begin{bmatrix}1 & x_1\\ 1 & x_2\\ \vdots & \vdots\\ 1 & x_n\end{bmatrix} \qquad X'X=\begin{bmatrix}1 & 1 & \cdots & 1\\ x_1 & x_2 & \cdots & x_n\end{bmatrix}\begin{bmatrix}1 & x_1\\ 1 & x_2\\ \vdots & \vdots\\ 1 & x_n\end{bmatrix}=\begin{bmatrix}N & \Sigma x_i\\ \Sigma x_i & \Sigma x_i^2\end{bmatrix}$$

$$(X'X)^{-1}=\frac{1}{N\Sigma x_i^2-\Sigma x_i\Sigma x_i}\begin{bmatrix}\Sigma x_i^2 & -\Sigma x_i\\ -\Sigma x_i & N\end{bmatrix}$$

$$\mathrm{var}(b)=\sigma^2(X'X)^{-1}=\sigma^2\cdot\frac{1}{N\Sigma x_i^2-\Sigma x_i\Sigma x_i}\begin{bmatrix}\Sigma x_i^2 & -\Sigma x_i\\ -\Sigma x_i & N\end{bmatrix}$$

$$N\Sigma x_i^2-\Sigma x_i\Sigma x_i$$

$$=N\Sigma x_i^2-N\bar{x}\cdot N\bar{x}$$

$$N\cdot\Sigma(x_i-\bar{x})^2$$

$$=N\cdot(\Sigma x_i^2-2\Sigma\bar{x}x_i+\Sigma\bar{x}^2$$

$$=N\cdot\Sigma x_i^2-2N\bar{x}\bar{x}+N\bar{x}^2$$

$$=N\cdot\Sigma x_i^2-N^2\bar{x}^2$$

所以 $\mathrm{var}(b)=\sigma^2\cdot\dfrac{1}{n\cdot(\Sigma(x_i-\bar{x})^2)}\begin{bmatrix}\Sigma x_i^2 & -\Sigma x_i\\ -\Sigma x_i & N\end{bmatrix}$

$$\mathrm{var}(b_1)=\sigma^2\cdot[(x'x)^{-1}]_{11}=\sigma^2\left[\frac{\Sigma x_i^2}{N(\Sigma(x_i-\bar{x})^2)}\right]$$ (2.14) 故得證.

$$\mathrm{var}(b_2)=\sigma^2\cdot[(x'x)^{-1}]_{22}=\sigma^2\left[\frac{N}{N(\Sigma(x_i-\bar{x})^2)}\right]=\frac{\sigma^2}{\Sigma(x_i-\bar{x})^2}$$ (2.15) 故得證.

$$\mathrm{cov}(b_1,b_2|x)=\sigma^2[(x'x)^{-1}]_{12}=\sigma^2\frac{-\Sigma x_i}{N(\Sigma(x_i-\bar{x})^2)}=\sigma^2\frac{-N\bar{x}}{N(\Sigma(x_i-\bar{x})^2)}$$

$$=\sigma^2\left[\frac{-\bar{x}}{\Sigma(x_i-\bar{x})^2}\right]$$ (2.16) 故得證.

5.3 Consider the following model that relates the percentage of a household's budget spent on alcohol *WALC* to total expenditure *TOTEXP*, age of the household head *AGE*, and the number of children in the household *NK*.

$$WALC = \beta_1 + \beta_2 \ln(TOTEXP) + \beta_3 NK + \beta_4 AGE + e$$

This model was estimated using 1200 observations from London. An incomplete version of this output is provided in Table 5.6.

TABLE 5.6 **Output for Exercise 5.3**

Dependent Variable: *WALC*
Included observations: 1200

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	1.4515	2.2019		0.5099
ln(*TOTEXP*)	2.7648		5.7103	0.0000
NK		0.3695	−3.9376	0.0001
AGE	−0.1503	0.0235	−6.4019	0.0000
R-squared		Mean dependent var		6.19434
S.E. of regression		S.D. dependent var		6.39547
Sum squared resid SSR	46221.62			

a. Fill in the following blank spaces that appear in this table.
  i. The *t*-statistic for b_1.
  ii. The standard error for b_2.
  iii. The estimate b_3.
  iv. R^2.
  v. $\hat{\sigma}$.

b. Interpret each of the estimates b_2, b_3, and b_4.

c. Compute a 95% interval estimate for β_4. What does this interval tell you?

d. Are each of the coefficient estimates significant at a 5% level? Why?

e. Test the hypothesis that the addition of an extra child decreases the mean budget share of alcohol by 2 percentage points against the alternative that the decrease is not equal to 2 percentage points. Use a 5% significance level.

5.3 $WALC = \beta_1 + \beta_2 \ln(TOTEXP) + \beta_3 NK + \beta_4 AGE + e$

(i) t-statistic for b_1: $t = \dfrac{\hat{\beta}_1}{\text{Std. Error}} \cdot \dfrac{1.4515}{2.2019} = 0.6592$

(ii) standard error for b_2: $\dfrac{2.7648}{5.7103} = 0.4842$

(iii) b_3: $0.3695 \times (-3.9376) = -1.4549$

$R^2 = 1 - \dfrac{SSE}{SST} = \dfrac{SSR}{SST}$

$SST = (1200-1) \cdot (6.39547)^2 = 49041.5418$

(iv) $R^2 = 1 - \dfrac{46221.62}{49041.5418} = 1 - 0.9425 = 0.0575$

(v) $\hat{\sigma}^2 = \dfrac{SSE}{N-K} = \dfrac{46221.62}{1200-4} = 38.6468$

$\hat{\sigma} = \sqrt{38.6468} = 6.2169$

b、

b_2: 2.7648 → 當總支出每增加一單位，alcohol 的支出會增加 2.7648 單位
(總支出)

b_3: -1.4549 → 當每增加一個小孩，alcohol 的支出會減少 1.4549 單位
(有幾個小孩)

b_4: -0.1503 → 當每增加一歲，alcohol 支出會減少 0.1503 單位.
(年齡)

c、 95% interval estimate for β_4

$-0.1503 \pm 1.9619 \times 0.0235$

-0.1503 ± 0.0461

$[-0.1964, -0.1042]$

年齡每增加1歲，那對酒料的支出會減少 0.1042 單位到 0.1964 單位.

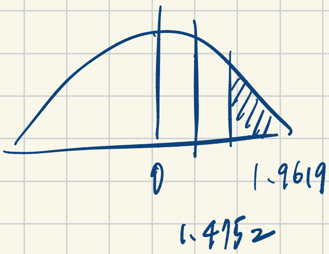
d、 p-value < 0.05 顯著.

	p-value	
b_1	0.5099	不顯著
b_2	0.0000	顯著
b_3	0.0001	顯著
b_4	0.0000	顯著.

e、 $H_0: \beta_3 = -2$

$H_1: \beta_3 \neq -2$

$$\frac{-1.4549-(-2)}{0.3695} = 1.4752$$



t值: 1.9619

不拒絕 H_0，沒有足夠的證據去證明每多增加一個小孩對酒精支出的影響不是2個單位.