

# 計量經濟學\_HW5\_20250324

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1) Let  $K = 2$ , show that  $(b_1, b_2)$  in p.29 of slides in CH5 reduces to the formula of  $(b_1, b_2)$  in (2.7)~(2.8)

a)

2) Let  $K = 2$ , show that  $\text{COV}(b_1, b_2)$  in p.30 of slides in CH5 reduces to the formula of in (2.14)~(2.16)

a)

**5.3** Consider the following model that relates the percentage of a household's budget spent on alcohol  $WALC$  to total expenditure  $TOTEXP$ , age of the household head  $AGE$ , and the number of children in the household  $NK$ .

$$WALC = \beta_1 + \beta_2 \ln(TOTEXP) + \beta_3 NK + \beta_4 AGE + e$$

This model was estimated using 1200 observations from London. An incomplete version of this output is provided in Table 5.6.

**TABLE 5.6** Output for Exercise 5.3

Dependent Variable: $WALC$				
Included observations: 1200				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
$C$	1.4515	2.2019		0.5099
$\ln(TOTEXP)$	2.7648		5.7103	0.0000
$NK$		0.3695	-3.9376	0.0001
$AGE$	-0.1503	0.0235	-6.4019	0.0000
R-squared		Mean dependent var		6.19434
S.E. of regression		S.D. dependent var		6.39547
Sum squared resid	46221.62			

- Fill in the following blank spaces that appear in this table.
  - The  $t$ -statistic for  $b_1$ .
  - The standard error for  $b_2$ .
  - The estimate  $b_3$ .
  - $R^2$ .
  - $\hat{\sigma}$ .
- Interpret each of the estimates  $b_2$ ,  $b_3$ , and  $b_4$ .
- Compute a 95% interval estimate for  $\beta_4$ . What does this interval tell you?
- Are each of the coefficient estimates significant at a 5% level? Why?
- Test the hypothesis that the addition of an extra child decreases the mean budget share of alcohol by 2 percentage points against the alternative that the decrease is not equal to 2 percentage points. Use a 5% significance level.

3)

a)

**5.23** The file *cocaine* contains 56 observations on variables related to sales of cocaine powder in northeastern California over the period 1984–1991. The data are a subset of those used in the study Caulkins, J. P. and R. Padman (1993), “Quantity Discounts and Quality Premia for Illicit Drugs,” *Journal of the American Statistical Association*, 88, 748–757. The variables are

*PRICE* = price per gram in dollars for a cocaine sale  
*QUANT* = number of grams of cocaine in a given sale  
*QUAL* = quality of the cocaine expressed as percentage purity  
*TREND* = a time variable with 1984 = 1 up to 1991 = 8  
 Consider the regression model

$$PRICE = \beta_1 + \beta_2 QUANT + \beta_3 QUAL + \beta_4 TREND + e$$

4)

- a. What signs would you expect on the coefficients  $\beta_2$ ,  $\beta_3$ , and  $\beta_4$ ?
- b. Use your computer software to estimate the equation. Report the results and interpret the coefficient estimates. Have the signs turned out as you expected?
- c. What proportion of variation in cocaine price is explained jointly by variation in quantity, quality, and time?
- d. It is claimed that the greater the number of sales, the higher the risk of getting caught. Thus, sellers are willing to accept a lower price if they can make sales in larger quantities. Set up  $H_0$  and  $H_1$  that would be appropriate to test this hypothesis. Carry out the hypothesis test.
- e. Test the hypothesis that the quality of cocaine has no influence on expected price against the alternative that a premium is paid for better-quality cocaine.
- f. What is the average annual change in the cocaine price? Can you suggest why price might be changing in this direction?

- a)  $\beta_2$ : 對價格應該是「負面」影響, 因為量多使得物品價值減少, 故  $\beta_2 < 0$   
 $\beta_3$ : 對價格應該是「正面」影響, 因為品質越好, 理應價格越高, 故  $\beta_3 > 0$   
 $\beta_4$ : 對價格應該是「正面」影響, 因為物價隨時間會有通膨問題, 故  $\beta_4 > 0$

b) Model Summary 如下:

Variable	Estimate	Std. Error	t value	Pr(> t )	Significance
Intercept	90.84669	8.58025	10.588	1.39E-14	***
quant	-0.05997	0.01018	-5.892	2.85E-07	***
qual	0.11621	0.20326	0.572	0.57	
trend	-2.35458	1.38612	-1.699	0.0954	.

變數	Estimate	t-value	p-value	結論
quant	-0.06	-5.89	2.85E-07	非常顯著, 有影響
qual	+0.12	0.572	0.57	不顯著, 不一定有影響
trend	-2.35	-1.699	0.095	稍微顯著, 可能有影響

「Trend」跟我原本預期的有些許不同, 「Qual」也沒有我預期的那麼有顯著性。

- c) 模型的 Multiple  $R^2$  為 0.5097, 表示模型可解釋約 51% 的價格變異。但考慮變數個數與樣本數後, Adjusted  $R^2$  為 0.4814, 為更保守且真實的解釋力指標, 顯示模型仍具有中等程度的預測力。

d) 這題主要是想知道是否「Quant」與「Price」成負向關係，也就是檢測  $\beta_2 < 0$ ，下面為假設檢定流程：

i)  $H_0: \beta_2 \geq 0$

$H_1: \beta_2 < 0$

ii)  $\hat{\beta}_2 = -0.05997$

$SE = 0.01018$

$DF = 56 - 4 = 52$

iii)  $t = \frac{\hat{\beta}_2 - 0}{SE(\hat{\beta}_2)} \approx -5.892$

iv)  $pvalue_{\text{左尾}} = \frac{2.85 \times 10^{-7}}{2} = 1.425 \times 10^{-7} < 0.01$

v) 拒絕  $H_0$ ，數量增加會導致價格下降，支持「量多折扣」、「為了降低風險而大量銷售」的理論

e) 這題主要是想檢驗「Quality」與「Price」的關係，是否無影響，下面為檢定流程：

i)  $H_0: \beta_3 = 0$

$H_1: \beta_3 > 0$

ii)  $\hat{\beta}_3 = 0.11621$

$SE = 0.20326$

$DF = 52$

iii)  $t = \frac{\hat{\beta}_3 - 0}{SE(\hat{\beta}_3)} = 0.572$

iv)  $Pvalue_{\text{雙尾}} = 0.57$  (從 Summary 得知)

故  $Pvalue_{\text{右尾}} = 0.285$

v) 因為  $Pvalue_{\text{右尾}} = 0.285 \geq 0.05$ ，故我們無法拒絕  $H_0$

f) 年平均變化價格是指「Trend」的係數  $\beta_4 = -2.35458$ ，代表每一年平均價格會減少 2.35458 美元。

可能造成此現象的猜測：

i) 販毒集團擴張版圖，使得原物料以及製造成本降低

ii) 出現新型態的替代型毒品，使原本cocaine供給過剩