10.2 The labor supply of married women has been a subject of a great deal of economic research. Consider the following supply equation specification

$$HOURS = \beta_1 + \beta_2 WAGE + \beta_3 EDUC + \beta_4 AGE + \beta_5 KIDSL6 + \beta_6 NWIFEINC + e$$

where *HOURS* is the supply of labor, *WAGE* is hourly wage, *EDUC* is years of education, *KIDSL6* is the number of children in the household who are less than 6 years old, and *NWIFEINC* is household income from sources other than the wife's employment.

a. Discuss the signs you expect for each of the coefficients.
b. Explain why this supply equation cannot be consistently estimated by OLS regression.
c. Suppose we consider the woman's labor market experience *EXPER* and its square, *EXPER*^{2}, to be instruments for *WAGE*. Explain how these variables satisfy the logic of instrumental variables.
d. Is the supply equation identified? Explain.
e. Describe the steps [not a computer command] you would take to obtain IV/2SLS estimates.

a. 預期各係數正負：

WAGE、EDUC、AGE ：正
KIDSL6、NWIFEINC：負

b. ∵ HOURS和WAGE具內生性 ∴ 會造成估計失效

c.

① Relevance：

$WAGE = \beta_1' + \beta_2' EXPER + \beta_3' EXPER^2 + e'$ ∴ EXPER和EXPER² 和WAGE相關性強

② Exogeneity：

$Cov(EXPER, e') = 0$、$Cov(EXPER^2, e') = 0$ ∴ 外生性

d. 用工具變數替換WAGE後，不再具內生性問題

e. $WAGE = \alpha_1 + \alpha_2 EDUC + \alpha_3 AGE + \alpha_4 KIDSL6 + \alpha_5 NWIFEINC + \alpha_7 EXPER + \alpha_8 EXPER^2 + u$ ⇒ 迴歸得到 $\widehat{WAGE}$

得到 $\widehat{WAGE}$ 取代 WAGE，再用OLS

10.3 In the regression model $y = \beta_1 + \beta_2 x + e$, assume x is endogenous and that z is a valid instrument. In Section 10.3.5, we saw that $\beta_2 = \text{cov}(z, y)/\text{cov}(z, x)$.

a. Divide the denominator of $\beta_2 = \text{cov}(z, y)/\text{cov}(z, x)$ by $\text{var}(z)$. Show that $\text{cov}(z, x)/\text{var}(z)$ is the coefficient of the simple regression with dependent variable x and explanatory variable z, $x = \gamma_1 + \theta_1 z + v$. [*Hint*: See Section 10.2.1.] Note that this is the first-stage equation in two-stage least squares.

b. Divide the numerator of $\beta_2 = \text{cov}(z, y)/\text{cov}(z, x)$ by $\text{var}(z)$. Show that $\text{cov}(z, y)/\text{var}(z)$ is the coefficient of a simple regression with dependent variable y and explanatory variable z, $y = \pi_0 + \pi_1 z + u$. [*Hint*: See Section 10.2.1.]

c. In the model $y = \beta_1 + \beta_2 x + e$, substitute for x using $x = \gamma_1 + \theta_1 z + v$ and simplify to obtain $y = \pi_0 + \pi_1 z + u$. What are π_0, π_1, and u in terms of the regression model parameters and error and the first-stage parameters and error? The regression you have obtained is a **reduced-form** equation.

d. Show that $\beta_2 = \pi_1/\theta_1$.

e. If $\hat{\pi}_1$ and $\hat{\theta}_1$ are the OLS estimators of π_1 and θ_1, show that $\hat{\beta}_2 = \hat{\pi}_1/\hat{\theta}_1$ is a consistent estimator of $\beta_2 = \pi_1/\theta_1$. The estimator $\hat{\beta}_2 = \hat{\pi}_1/\hat{\theta}_1$ is an **indirect least squares** estimator.

a. $x = \gamma_1 + \theta_1 z + v \Rightarrow x - \gamma_1 - \theta_1 z = v$

$$\Rightarrow E(x - \gamma_1 - \theta_1 z) = \underbrace{E(v)}_{=0} = 0 \quad — ①$$

$$\text{Cov}(z, v) = 0 \Rightarrow E(zv) - E(z)\underbrace{E(v)}_{=0} = 0$$

$$\Rightarrow E(zv) = 0 \Rightarrow E(z(x - \gamma_1 - \theta_1 z)) = 0$$

$$\Rightarrow E(xz) - \gamma_1 E(z) - \theta_1 E(z^2) = 0 \quad — ②$$

Summary:

$$\begin{cases} E(x) - \gamma_1 - \theta_1 E(z) = 0 \\ E(xz) - \gamma_1 E(z) - \theta_1 E(z^2) = 0 \end{cases}$$

代入

$$\Rightarrow E(xz) - [E(x) - \theta_1 E(z)]E(z) - \theta_1 E(z^2) = 0$$

$$\Rightarrow E(xz) - E(x)E(z) + \theta_1 [E(z)]^2 - \theta_1 E(z^2) = 0$$

$$\Rightarrow \underbrace{E(xz) - E(x)E(z)}_{= Cov(x,z)} = \theta_1 \left\{ \underbrace{[E(z)]^2 - E(z^2)}_{= Var(z)} \right\}$$

$$\Rightarrow Cov(x,z) = \theta_1 Var(z)$$

$$\Rightarrow \theta_1 = \frac{Cov(x,z)}{Var(z)} \quad \#$$

b.

$$y = \pi_0 + \pi_1 z + u \Rightarrow E(y - \pi_0 - \pi_1 z) = E(u) = 0$$

$$Cov(z,u) = 0 \Rightarrow E(zu) = 0 \Rightarrow E[z(y - \pi_0 - \pi_1 z)] = 0$$

$$\Rightarrow E(zy) - \pi_0 E(z) - \pi_1 E(z^2) = 0$$

Summary:

$$\begin{cases} E(y) - \pi_0 - \pi_1 E(z) = 0 \\ \qquad \text{代入} \\ E(zy) - \pi_0 E(z) - \pi_1 E(z^2) = 0 \end{cases}$$

$$\Rightarrow E(zy) - [E(y) - \pi_1 E(z)] E(z) - \pi_1 E(z^2) = 0$$

$$\Rightarrow E(zy) - E(z)E(y) = \pi_1 \left\{ [E(z)]^2 - E(z^2) \right\}$$

$$\Rightarrow Cov(z,y) = \pi_1 Var(z) \Rightarrow \pi_1 = \frac{Cov(z,y)}{Var(z)} \quad \#$$

c.

$$\begin{cases} x = \gamma_1 + \theta_1 z + v \\ y = \beta_1 + \beta_2 x + e \end{cases} \Rightarrow y = \beta_1 + \beta_2(\gamma_1 + \theta_1 z + v) + e$$

$$\Rightarrow y = (\beta_1 + \beta_2\gamma_1) + \beta_2\theta_1 z + (\beta_2 v + e)$$

$$\Updownarrow$$

reduced form : $y = \pi_0 + \pi_1 z + u$

$$\Rightarrow \begin{cases} \pi_0 = \beta_1 + \beta_2\gamma_1 \\ \pi_1 = \beta_2\theta_1 \\ u = \beta_2 v + e \end{cases}$$

d. $\pi_1 = \beta_2\theta_1 \Rightarrow \beta_2 = \dfrac{\pi_1}{\theta_1}$ #

e. 由 (a) : $\hat{\theta}_1 = \dfrac{\widehat{Cov}(z,x)}{\widehat{Var}(z)} = \dfrac{\sum(z_i - \bar{z})(x_i - \bar{x})}{\sum(z_i - \bar{z})^2}$

由 (b) : $\hat{\pi}_1 = \dfrac{\widehat{Cov}(z,y)}{\widehat{Var}(z)} = \dfrac{\sum(z_i - \bar{z})(y_i - \bar{y})}{\sum(z_i - \bar{z})^2}$

$$\hat{\beta}_2 = \frac{\hat{\pi}_1}{\hat{\theta}_1} = \frac{\sum(z_i - \bar{z})(y_i - \bar{y})}{\sum(z_i - \bar{z})(x_i - \bar{x})} = \frac{\widehat{Cov}(z,y)}{\widehat{Cov}(z,x)}$$

$\because$ $\widehat{Cov}(z, y) \xrightarrow{p} Cov(z, y)$

$\widehat{Cov}(z, x) \xrightarrow{p} Cov(z, x)$

$\therefore$ by 連續映射，$\hat{\beta}_\gamma = \frac{\hat{\pi}_1}{\hat{\theta}_1} = \frac{\widehat{Cov}(z, y)}{\widehat{Cov}(z, x)} \xrightarrow{p} \frac{Cov(z, y)}{Cov(z, x)} = \beta_\gamma$