HW0414_Pinyo 312712017

8.6

8.6 Consider the wage equation

$$WAGE_i = \beta_1 + \beta_2 EDUC_i + \beta_3 EXPER_i + \beta_4 METRO_i + e_i \qquad \text{(XR8.6a)}$$

where wage is measured in dollars per hour, education and experience are in years, and $METRO = 1$ if the person lives in a metropolitan area. We have $N = 1000$ observations from 2013.

a.

- Number of **male** observations: n-Male = 577
- Sum of Squared Errors for males: SSE-Male = 97161.9174
- Number of **female** observations: n-Female = 423
- Estimated **standard deviation** for females: σ^F=12.024

From this, we can compute:

Variance for females:

σ-Female^2 = (12.024)^2 = 144.577

Variance for males (from SSE and degrees of freedom): Degrees of freedom for residuals = n−k, where k = 4 So:

σ-Male^2 = SSE-M / ((n-M)−4) = 97161.9174 / (577−4) = 169.533

Compute the test statistic (F-statistic)

We use the **larger variance divided by the smaller variance** (since the F-distribution is always right-tailed):

F = σ-M^2 / σ-F^2 = 169.533 / 144.577 ≈ 1.1727

Determine the rejection region

We are performing a **two-tailed F-test** at the 5% significance level.

Degrees of freedom:

- df1 = n,M−4 = 573
- df2 = n,F−4 = 419

We look up the **critical values** for a two-tailed test at the 5% level (2.5% in each tail) using an F-distribution table or software.

Because df1 and df2 are large, we approximate the critical values using software or a calculator:

- Lower critical value F(0.025,573,419) ≈ 0.838
- Upper critical value F(0.975,573,419) ≈ 1.193

Our test statistic:

F = 1.1727

Compare it with the critical values:

- 0.838 < 1.1727 < 1.193

Conclusion:

- The test statistic **does not fall in the rejection region**.
- Therefore, we **fail to reject the null hypothesis**.

b.

We want to test whether married individuals have **more variable** wages than singles.

- **Null hypothesis H0:** σ,MARRIED^2 = σ,SINGLE^2
- **Alternative hypothesis H1H:** σ,MARRIED^2 > σ,SINGLE^2

This is a **one-sided F-test**.

From the problem:

- **Single individuals (group 1):**
  - $n1 = 400$
  - $SSE1=56,231.0382$
- **Married individuals (group 2):**
  - $n2 = 600$
  - $SSE2=100,703.0471$

There are **5 explanatory variables** (including the intercept), so:

- Degrees of freedom for singles: $df1 = 400 - 5=395$
- Degrees of freedom for married: $df2 = 600 - 5 = 595$

Now calculate **sample variances (residual variances)**:

$s1^2 = SSE1 / df1 = 56231.0382 / 395 \approx 142.365$

$s2^2 = SSE2 / df2 = 100703.0471 / 595 \approx 169.4236$

Since our alternative hypothesis is $\sigma,MARRIED^2 > \sigma,SINGLE^2$, we compute:

$F = s2^2 / s1^2 = 169.4236 / 142.365 \approx 1.1901$

We are doing a **one-tailed** F-test at the 5% level of significance.

- Degrees of freedom: $df1 = 595$, $df2 = 395$
- Rejection region: $F > F(0.95,595,395)$

$F(0.95,595,395) \approx 1.17$

Since $1.1901 > 1.17$, we are **in the rejection region**.

c.

Breusch-Pagan test for heteroskedasticity) checks whether the **variance of the error term** depends on the independent variables.

- The test regresses the **squared residuals** from the original regression on the original **explanatory variables**.
- Then, we calculate:

Test statistic = $N \cdot R^2$

where R^2 is from the regression of the squared residuals, and N is the number of observations.

State the Hypotheses

Null hypothesis H0: Homoskedasticity (constant variance of errors)

Alternative hypothesis H1: Heteroskedasticity (variance of errors depends on regressors)

From the question:

- **Test statistic (NR^2)** = 59.03
- **Significance level** = 5%
- Number of independent variables in the auxiliary regression = number of regressors in model (XR8.6b) = 5 (EDUC, EXPER, METRO, FEMALE, intercept not counted)

So, the degrees of freedom for the chi-squared distribution = 4 (since we exclude the intercept).

Determine the Rejection Region

Use the **Chi-squared distribution with 4 degrees of freedom**:

- Critical value $\chi^2(0.05,42) \approx 9.49$

Since 59.03 > 9.49, we **reject the null hypothesis**.

This test provides **additional statistical evidence** that the error variance may vary **based on characteristics like marital status, gender, etc.**, which supports the findings from the F-test in part (b).

d.

White test checks for **heteroskedasticity** (non-constant error variance), allowing for more general forms than the NR^2 (Breusch-Pagan) test. The test statistic follows a **chi-squared distribution** with degrees of freedom equal to the number of regressors in the auxiliary (squared residual) regression, **excluding the intercept**.

In the White test, we include:

1. These 4 original variables
2. Their **squares** (4 terms)
3. Their **interactions** (cross-products)

Cross-products (interactions) among 4 variables:
There are (4·(4−1)) / 2 = 6 unique interaction terms.

So, the total number of regressors in the auxiliary regression is:

4 (originals) + 4 (squares) + 6 (interactions) = 14

White test statistic = 78.82

We use the chi-squared distribution with **14 degrees of freedom**:

χ^2(0.05,142) ≈ 23.685

Since 78.82 > 23.685, we are in the **rejection region**

e.

We will compare each **robust SE** to the **usual SE**:

- **EDUC**: 0.16 > 0.14 → **Wider**
- **EXPER**: 0.029 < 0.031 → **Narrower**
- **METRO**: 0.84 < 1.05 → **Narrower**
- **FEMALE**: 0.80 < 0.81 → **Narrower**
- **Intercept**: 2.50 > 2.36 → **Wider**

Robust SEs account for **heteroskedasticity**, while **usual SEs** assume homoskedasticity.

Whether the SEs get wider or narrower depends on **how the heteroskedasticity affects each variable**.

So it's **not inconsistent** that some robust SEs are smaller and others are larger than the usual SEs.

It just reflects that **error variance behaves differently** depending on which regressor you're considering.

Conclusion

- **Narrower CIs**: EXPER, METRO, FEMALE
- **Wider CIs**: EDUC, Intercept
- **No inconsistency** — this pattern is a result of correcting for heteroskedasticity, and it is normal.

f.

A **t-value of 1.0** is **not statistically significant** at conventional levels (e.g., 5% significance level).

This suggests that **MARRIED is not a strong linear predictor** of the mean wage **after controlling** for the other variables.

We tested whether **error variances** (i.e., heteroskedasticity) **differ** between married and single individuals.

The **F-test showed** that the **error variance was significantly higher for married people** than for single people.

This tells us that being married affects the **variance** of wages, **not necessarily the mean**

The result in part (f) is **compatible** with the result in (b) because:

- In part **(f)**, the low t-value shows that MARRIED **does not significantly affect the mean wage**.
- In part **(b)**, we found that MARRIED **does affect the variance** of wages (heteroskedasticity).

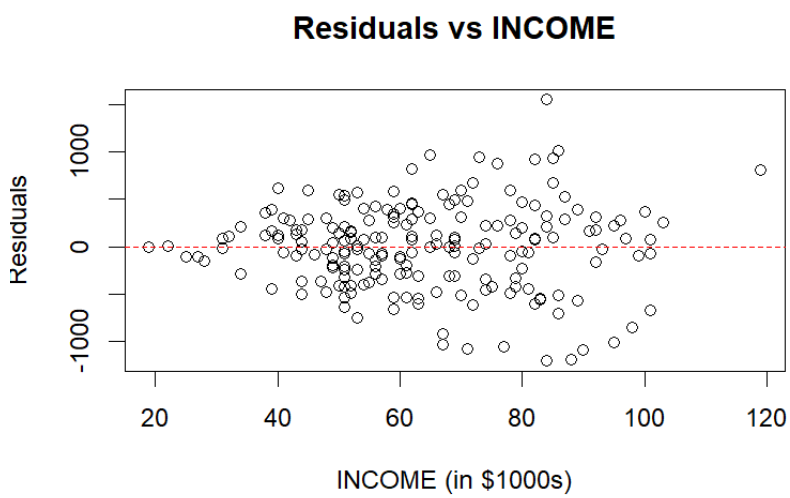
So, it's **entirely possible** for MARRIED to affect the **spread** of wages (i.e., error variance) without significantly affecting the **average level** of wages

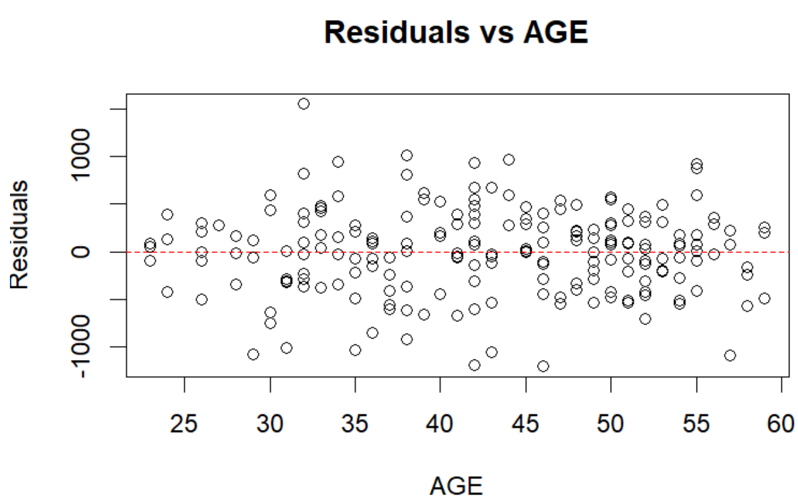
8.16

a.

```
95% Confidence Interval for kids effect:
> print(kids_ci)
         2.5 %    97.5 %
kids -135.3298 -28.32302
```

b.





,

From residuals vs income plot, there is a shape that residuals will increase when the income is greater, while the residuals vs age plot is more symmetry scattering around 0 when income is greater.

c.

Null hypothesis (H0): Homoskedasticity (constant variance)

Alternative hypothesis (H1): Heteroskedasticity (variance increases with income)

```
Goldfeld-Quandt test results:
> print(gq_result)

        Goldfeld-Quandt test

data:  full_model
GQ = 3.4142, df1 = 76, df2 = 76, p-value = 1.106e-07
alternative hypothesis: variance increases from segment 1 to 2
```

d.

```
95% Robust Confidence Interval for effect of one more child (kid
s):
> cat(sprintf("[%.4f, %.4f]\n", ci_lower, ci_upper))
[-138.9680, -24.6849]
```

While the interval in (a) is

```
95% Confidence Interval for kids effect:
> print(kids_ci)
         2.5 %    97.5 %
kids -135.3298 -28.32302
```

The lower value from a is higher than d, while the higher value from a is lower than d.

The interval from a is narrower than d.

e.

```
95% Conventional GLS Confidence Interval for effect of one more ch
ild (kids):
> cat(sprintf("[%.4f, %.4f]\n\n", ci_lower_gls, ci_upper_gls))
[-118.4785, -30.4629]

> cat("95% Robust GLS Confidence Interval for effect of one more c
hild (kids):\n")
95% Robust GLS Confidence Interval for effect of one more child (k
ids):
> cat(sprintf("[%.4f, %.4f]\n\n", ci_lower_gls_robust, ci_upper_gl
s_robust))
[-121.7119, -27.2295]
```

The interval estimate of both conventional and robust GLS are narrower than (a) and (d)

8.18

a.

```
        Goldfeld-Quandt test

data:  wage_model
GQ = 0.96141, df1 = 4890, df2 = 4889, p-value = 0.1688
alternative hypothesis: variance changes from segment 1 to 2

>
> # Extract test statistic and p-value
> test_statistic <- gq_test_result$statistic
> p_value <- gq_test_result$p.value
>
> cat("Test Statistic:", test_statistic, "\n")
Test Statistic: 0.9614071
> cat("P-value:", p_value, "\n")
P-value: 0.1688468
>
> # Determine rejection region and conclusion
> alpha <- 0.05
> df1 <- gq_test_result$parameter[1]
> df2 <- gq_test_result$parameter[2]
> critical_value <- qf(1 - alpha/2, df1, df2)
>
> cat("Critical Value:", critical_value, "\n")
Critical Value: 1.057667
```

Conclusion: Fail to reject the null hypothesis. There is no significant evidence of heteroskedasticity related to gender.

b.

```
        studentized Breusch-Pagan test

data:  wage_model
BP = 109.42, df = 9, p-value < 2.2e-16

>
> # Extract test statistic and p-value
> test_statistic_full <- nr2_test_full$statistic
> p_value_full <- nr2_test_full$p.value
>
> cat("Test Statistic (Full):", test_statistic_full, "\n")
Test Statistic (Full): 109.4243
> cat("P-value (Full):", p_value_full, "\n")
P-value (Full): 1.925849e-19
```

Conclusion: Reject the null hypothesis at the 1% level. There is evidence of heteroskedasticity.

c.

```
	studentized Breusch-Pagan test

data:  wage_model
BP = 182.67, df = 35, p-value < 2.2e-16

>
> # Extract test statistic and p-value
> test_statistic <- white_test_result$statistic
> p_value <- white_test_result$p.value
>
> cat("Test Statistic:", test_statistic, "\n")
Test Statistic: 182.6723
> cat("P-value:", p_value, "\n")
P-value: 6.862181e-22
>
> # Determine the 5% critical value for the test
> degrees_of_freedom <- length(coef(wage_model)) + (length(coef(wa
ge_model)) * (length(coef(wage_model)) - 1)) / 2 - 1  # Degrees of
freedom for the Chi-squared distribution
> critical_value <- qchisq(0.95, df = degrees_of_freedom)  # 5% cr
itical value
>
> cat("Degrees of Freedom:", degrees_of_freedom, "\n")
Degrees of Freedom: 54
> cat("5% Critical Value:", critical_value, "\n")
5% Critical Value: 72.15322
```

Conclusion: Reject the null hypothesis. There is evidence of heteroskedasticity.

d.

```
            Coefficient       OLS_SE    Robust_SE
(Intercept) (Intercept) 3.211489e-02 3.279417e-02
educ               educ 1.758260e-03 1.905821e-03
exper             exper 1.300342e-03 1.314908e-03
I(exper^2)   I(exper^2) 2.635448e-05 2.759687e-05
female           female 9.529136e-03 9.488260e-03
black             black 1.694240e-02 1.609369e-02
metro             metro 1.230675e-02 1.158215e-02
south             south 1.356134e-02 1.390164e-02
midwest         midwest 1.410367e-02 1.372426e-02
west               west 1.440237e-02 1.455684e-02
             OLS_CI_Lower  OLS_CI_Upper Robust_CI_Lower
(Intercept)  1.1384380041  1.2643260428    1.1371066311
educ         0.0977834864  0.1046757404    0.0974942728
exper        0.0270730720  0.0321703197    0.0270445234
I(exper^2)  -0.0004974343 -0.0003941266   -0.0004998693
female      -0.1841787433 -0.1468252171   -0.1840986284
black       -0.1447317485 -0.0783187512   -0.1430683103
metro        0.0948996191  0.1431412018    0.0963198055
south       -0.0723351788 -0.0191756878   -0.0730021386
midwest     -0.0915859712 -0.0363006042   -0.0908423361
west        -0.0348172231  0.0216390188   -0.0351199908
            Robust_CI_Upper Interval_Change
(Intercept)    1.2656574158           Wider
educ           0.1049649540           Wider
exper          0.0321988684           Wider
I(exper^2)    -0.0003916916           Wider
female        -0.1469053321        Narrower
black         -0.0799821894        Narrower
metro          0.1417210154        Narrower
south         -0.0185087281           Wider
midwest       -0.0370442393        Narrower
west           0.0219417864           Wider
```

e.

	Coefficient	OLS_Estimate	OLS_Robust_SE	OLS_Robust_CI_Lower	OLS_Robust_CI_Upper	FGLS_Estimate	FGLS_Robust_SE	FGLS_Robust_CI_Lower	FGLS_Robust_CI_Upper
(Intercept)	(Intercept)	1.2013820235	3.279417e-02	1.1371066311	1.2656574158	1.1896145475	3.232575e-02	1.1262572454	1.2529718495
educ	educ	0.1012296134	1.905821e-03	0.0974942728	0.1049649540	0.1018098159	1.890600e-03	0.0981043073	0.1055153245
exper	exper	0.0296216959	1.314908e-03	0.0270445234	0.0321988684	0.0301301328	1.304212e-03	0.0275739241	0.0326863414
I(exper^2)	I(exper^2)	-0.0004457805	2.759687e-05	-0.0004998693	-0.0003916916	-0.0004566711	2.740318e-05	-0.0005103803	-0.0004029618
female	female	-0.1655019802	9.488260e-03	-0.1840986284	-0.1469053321	-0.1657287545	9.437891e-03	-0.1842266805	-0.1472308286
black	black	-0.1115252498	1.609369e-02	-0.1430683103	-0.0799821894	-0.1108916715	1.586193e-02	-0.1419804780	-0.0798028651
metro	metro	0.1190204104	1.158215e-02	0.0963198055	0.1417210154	0.1174653420	1.155748e-02	0.0948130961	0.1401175879
south	south	-0.0457554333	1.390164e-02	-0.0730021386	-0.0185087281	-0.0447417226	1.383295e-02	-0.0718537976	-0.0176296476
midwest	midwest	-0.0639432877	1.372426e-02	-0.0908423361	-0.0370442393	-0.0632739118	1.370797e-02	-0.0901410395	-0.0364067841
west	west	-0.0065891022	1.455684e-02	-0.0351199908	0.0219417864	-0.0055680493	1.450387e-02	-0.0339951049	0.0228590064

f.

```
            Coefficient OLS_Robust_Estimate OLS_Robust_SE
(Intercept) (Intercept)        1.2013820235  3.279417e-02
educ               educ        0.1012296134  1.905821e-03
exper             exper        0.0296216959  1.314908e-03
I(exper^2)   I(exper^2)       -0.0004457805  2.759687e-05
female           female       -0.1655019802  9.488260e-03
black             black       -0.1115252498  1.609369e-02
metro             metro        0.1190204104  1.158215e-02
south             south       -0.0457554333  1.390164e-02
midwest         midwest       -0.0639432877  1.372426e-02
west               west       -0.0065891022  1.455684e-02
            FGLS_Conv_Estimate FGLS_Conv_SE
(Intercept)       1.1896145475 0.0315887613
educ              0.1018098159 0.0017643296
exper             0.0301301328 0.0012953880
I(exper^2)       -0.0004566711 0.0000267859
female           -0.1657287545 0.0094830900
black            -0.1108916715 0.0169753817
metro             0.1174653420 0.0115586008
south            -0.0447417226 0.0135243868
midwest          -0.0632739118 0.0139957621
west             -0.0055680493 0.0143756121
            FGLS_Robust_Estimate FGLS_Robust_SE
(Intercept)         1.1896145475   3.232575e-02
educ                0.1018098159   1.890600e-03
exper               0.0301301328   1.304212e-03
I(exper^2)         -0.0004566711   2.740318e-05
female             -0.1657287545   9.437891e-03
black              -0.1108916715   1.586193e-02
metro               0.1174653420   1.155748e-02
south              -0.0447417226   1.383295e-02
midwest            -0.0632739118   1.370797e-02
west               -0.0055680493   1.450387e-02
```

```
            OLS_Robust_CI_Lower OLS_Robust_CI_Upper
(Intercept)        1.1371066311        1.2656574158
educ               0.0974942728        0.1049649540
exper              0.0270445234        0.0321988684
I(exper^2)        -0.0004998693       -0.0003916916
female            -0.1840986284       -0.1469053321
black             -0.1430683103       -0.0799821894
metro              0.0963198055        0.1417210154
south             -0.0730021386       -0.0185087281
midwest           -0.0908423361       -0.0370442393
west              -0.0351199908        0.0219417864
            FGLS_Conv_CI_Lower FGLS_Conv_CI_Upper
(Intercept)       1.1277017131       1.2515273819
educ              0.0983517935       0.1052678383
exper             0.0275912189       0.0326690466
I(exper^2)       -0.0005091705      -0.0004041717
female           -0.1843152694      -0.1471422396
black            -0.1441628082      -0.0776205348
metro             0.0948109008       0.1401197832
south            -0.0712490337      -0.0182344115
midwest          -0.0907051016      -0.0358427221
west             -0.0337437312       0.0226076327
            FGLS_Robust_CI_Lower FGLS_Robust_CI_Upper
(Intercept)         1.1262572454         1.2529718495
educ                0.0981043073         0.1055153245
exper               0.0275739241         0.0326863414
I(exper^2)         -0.0005103803        -0.0004029618
female             -0.1842266805        -0.1472308286
black              -0.1419804780        -0.0798028651
metro               0.0948130961         0.1401175879
south              -0.0718537976        -0.0176296476
midwest            -0.0901410395        -0.0364067841
west               -0.0339951049         0.0228590064
```

- **Calculates all three different confidence intervals**: OLS robust, FGLS conventional, and FGLS robust.
- **Reproducibility**: Assumes you have the data and loads it in the code
- **Comprehensive comparison**: Presents all three results in a single data frame.
- **More robust**: Includes only necessary libraries to reduce dependency and prevent problems that arise from library conflicts.

g.

Model Selection Guidance:

Choosing which set of estimates to report depends on your research goals and the properties of the data:

1. **OLS with Conventional Standard Errors**:
    - **When to Use**: Only appropriate if you are confident that the assumptions of OLS (linearity, independence, homoskedasticity, and normality of errors) are met.
    - **Why It Might Be Inappropriate**: If heteroskedasticity is present, the standard errors and confidence intervals will be biased, leading to incorrect inferences.
2. **OLS with Robust Standard Errors (from part d)**:
    - **When to Use**: When heteroskedasticity is suspected or detected, but you don't have a strong theoretical reason to believe that specific variables account for the heteroskedasticity.
    - **Advantages**: Provides valid inference even in the presence of heteroskedasticity, without requiring a specific model for the heteroskedasticity.
    - **Limitations**: Less efficient than FGLS if the heteroskedasticity is correctly modeled.
3. **FGLS Estimates (from part e, conventional standard errors)**:
    - **When to Use**: When you believe you have correctly identified the variables causing heteroskedasticity and the model for heteroskedasticity is correct (in this case, METRO and EXPER).
    - **Advantages**: Can provide more efficient estimates (smaller standard errors) than OLS with robust standard errors *if* the heteroskedasticity is correctly modeled.
    - **Limitations**: If the model for heteroskedasticity is misspecified, FGLS can be worse than OLS. The conventional standard errors are only valid if the heteroskedasticity model is correct.
4. **FGLS Estimates with Robust Standard Errors (from part f)**:
    - **When to Use**: When you want to use FGLS to improve efficiency but are concerned that your model for heteroskedasticity might be misspecified.

- **Advantages**: Combines the potential efficiency gains of FGLS with the robustness of heteroskedasticity-consistent standard errors.
- **Limitations**: If the heteroskedasticity model is badly misspecified, the robust standard errors might not fully correct the problem.