C11Q01

(a)

$$y_2 = \alpha_2(\alpha_1 y_2 + e_1) + \beta_1 x_1 + \beta_2 x_2 + e_2$$

$$(1-\alpha_2\alpha_1)y_2 = \beta_1 x_1 + \beta_2 x_2 + \alpha_2 e_1 + e_2$$

$$\pi_1 = \beta_1/(1-\alpha_2\alpha_1)$$

$$\Rightarrow \quad \pi_2 = \beta_2/(1-\alpha_2\alpha_1)$$

$v_2 = (\alpha_2 e_1 + e_2)/(1-\alpha_2\alpha_1) \Rightarrow y_2$ will be correlated with e_1, as long as $\alpha_2 \neq 0$.

(b)

π_1 and π_2 , since they are the coefficient of the exogenous variables.

(c)

equation 1, parameter α_1. Because 2IVs(> 2-1) omitted from equation 1.

C11Q01

(d)

The conditions express that the error term v_2 in part(a) are uncorrelated with the exogenous variables.

(e)

OLS: $SSE = \sum_{i=1}^{N}(y_{2i} - \pi_1 x_{i1} - \pi_2 x_{i2})^2$

first derivatives:

$$\frac{\partial SSE}{\partial \pi_1} = -2\sum_{i=1}^{N} x_{i1}(y_{2i} - \pi_1 x_{i1} - \pi_2 x_{i2})^2 = 0$$

$$\frac{\partial SSE}{\partial \pi_2} = -2\sum_{i=1}^{N} x_{i2}(y_{2i} - \pi_1 x_{i1} - \pi_2 x_{i2})^2 = 0$$

This two conditions are consistent with the conditions required by MOM.

C11Q01

(f)

$$\sum_{i=1}^{N} x_{i1}(y_{2i} - \pi_1 x_{i1} - \pi_2 x_{i2}) = 0$$
$$\sum_{i=1}^{N} x_{i2}(y_{2i} - \pi_1 x_{i1} - \pi_2 x_{i2}) = 0$$

$$\Rightarrow \ 3 - 1\pi_1 - 0\pi_2 = 0 \quad \Rightarrow \quad \pi_1 = 3$$
$$4 - 0\pi_1 - 0\pi_2 = 0 \qquad \pi_2 = 4$$

(g)

Because $\hat{y}_2$ is estimated by exogenous variables, it should be uncorrelated with the error term e_1.

$$\sum_{i=1}^{N} \hat{y}_{i2}\,(y_{i1} - \alpha_1 y_{i2}) = 0 \ \Rightarrow \ \alpha_1 = \sum_{i=1}^{N} \hat{y}_{i2}\, y_{i1} / \sum_{i=1}^{N} \hat{y}_{i2}\, y_{i2}$$

C11Q01

(h)

2nd stage regression equation: $y_1 = \alpha_1 \hat{y}_{i2} \ + e_1$

$$SSE = \ \sum_{i=1}^{N} (y_1 - \alpha_1 \hat{y}_{i2} \ )^2$$

first derivatives:

$$\frac{\partial SSE}{\partial \alpha_1} = -2 \sum_{i=1}^{N} \hat{y}_{i2} (y_1 - \alpha_1 \hat{y}_{i2} \ ) \ = 0$$

$\Rightarrow \alpha_1 = \sum_{i=1}^{N} \hat{y}_{i2} \, y_{i1} \Big/ \sum_{i=1}^{N} \hat{y}_{i2} \, y_{i2}$ consistent with the result in part (g).

C11Q16

(a)

$Q = \beta_1 + \beta_2(Q - e_{di} - \alpha_1)/\alpha_2 + \beta_3 W + e_{si}$ $\quad$ $\alpha_1 + \alpha_2 P + e_{di} = \beta_1 + \beta_2 P + \beta_3 W + e_{si}$

$$\Rightarrow \quad \begin{aligned} \theta_1 &= (\alpha_2\beta_1 - \alpha_1\beta_2)/(\alpha_2 - \beta_2) \\ \theta_2 &= \alpha_2\beta_3 / (\alpha_2 - \beta_2) \\ v_2 &= (\alpha_2 e_{si} + \beta_2 e_{di}) / (\alpha_2 - \beta_2) \end{aligned} \qquad \Rightarrow \quad \begin{aligned} \pi_1 &= (\beta_1 + \alpha_1)/(\alpha_2 - \beta_2) \\ \pi_2 &= \beta_3 / (\alpha_2 - \beta_2) \\ v_1 &= (e_{si} - e_{di})/(\alpha_2 - \beta_2) \end{aligned}$$

(b)

Demand can be solved (identified)

Supply can't be solved

C11Q16

(c)

$5 = (\alpha_2\beta_1 - \alpha_1\beta_2)/(\alpha_2 - \beta_2) = \alpha_2(\beta_1 - \alpha_1)/(\alpha_2 - \beta_2) + \alpha_1(\alpha_2 - \beta_2)/(\alpha_2 - \beta_2)$

$2.4 = (\beta_1 - \alpha_1)/(\alpha_2 - \beta_2)$

$0.5 = \alpha_2\beta_3 / (\alpha_2 - \beta_2)$

$1 = \beta_3 / (\alpha_2 - \beta_2)$

$\Rightarrow \alpha_2 = 0.5, \alpha_1 = 3.8$

(d)

fitted values for P: `4.4 5.4 3.4 3.4 5.4`

```
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)    3.800      6.481   0.586    0.599
P_hat          0.500      1.443   0.346    0.752
```

C11Q17

(a)

eq1 : 2 IVs(W_{2t}, P_{t-1}) eq2 : 2 IVs(P_{t-1}, K_{t-1})

eq3 : 2 IVs$(E_{t-1}, TIME_t)$ total: 5 IVs

Yes, we do.

We have an adequate number of Ivs to estimate each equation.

(b)

eq1 : 2 endo, 2 exo eq2 : 1 endo, 2 exo

eq3 : 1 endo, 2 exo

This condition is satisfied for each equation.

C11Q17

(c)

$$W_{1t} = \pi_1 + \pi_2 W_{2t} + \pi_3 P_{t-1} + \pi_4 K_{t-1} + \pi_5 E_{t-1} + \pi_6 TIME_t$$

(d)

Perform regression using the 1st stage equation :

$$W_{1t} = \pi_1 + \pi_2 W_{2t} + \pi_3 P_{t-1} + \pi_4 K_{t-1} + \pi_5 E_{t-1} + \pi_6 TIME_t$$

$$P_t = \theta_1 + \theta_2 W_{2t} + \theta_3 P_{t-1} + \theta_4 K_{t-1} + \theta_5 E_{t-1} + \theta_6 TIME_t$$

Perform regression using the estimates and 2nd stage equation:

$$CN_t = \alpha_1 + \alpha_2\left(\widehat{W}_{1t} + W_{2t}\right) + \alpha_3 \hat{P}_t + \alpha_4 P_{t-1}$$

(e)

No, the coefficient is the same, but the t-value isn’t, because the standard error in the hand calculation(2SLS) is incorrect compared to the IVREG.