5.23 The file *cocaine* contains 56 observations on variables related to sales of cocaine powder in northeastern California over the period 1984–1991. The data are a subset of those used in the study Caulkins, J. P. and R. Padman (1993), "Quantity Discounts and Quality Premia for Illicit Drugs," *Journal of the American Statistical Association*, 88, 748–757. The variables are

PRICE = price per gram in dollars for a cocaine sale
QUANT = number of grams of cocaine in a given sale
QUAL = quality of the cocaine expressed as percentage purity
TREND = a time variable with 1984 = 1 up to 1991 = 8
Consider the regression model

$$PRICE = \beta_1 + \beta_2 QUANT + \beta_3 QUAL + \beta_4 TREND + e$$

a. What signs would you expect on the coefficients β_2, β_3, and β_4?

b. Use your computer software to estimate the equation. Report the results and interpret the coefficient estimates. Have the signs turned out as you expected?

c. What proportion of variation in cocaine price is explained jointly by variation in quantity, quality, and time?

d. It is claimed that the greater the number of sales, the higher the risk of getting caught. Thus, sellers are willing to accept a lower price if they can make sales in larger quantities. Set up H_0 and H_1 that would be appropriate to test this hypothesis. Carry out the hypothesis test.

e. Test the hypothesis that the quality of cocaine has no influence on expected price against the alternative that a premium is paid for better-quality cocaine.

f. What is the average annual change in the cocaine price? Can you suggest why price might be changing in this direction?

a.

預期所有係數除了 beta2 皆為正，數量越多價格越低(beta2)，品質越高價格越高(beta3)，因有通膨，所以價格隨時間會越來越貴(beta4)。

b.

```
Residuals:
    Min      1Q  Median      3Q     Max
-43.479 -12.014  -3.743  13.969  43.753

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 90.84669    8.58025  10.588 1.39e-14 ***
quant       -0.05997    0.01018  -5.892 2.85e-07 ***
qual         0.11621    0.20326   0.572   0.5700
trend       -2.35458    1.38612  -1.699   0.0954 .
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Trend 的係數我們預估錯誤，價格隨時間越來越低。

當所有變數為 0 時 price 為 90.84669

每上升一單位的 quant 會使 price 下降 0.05997

每上升一單位的 qual 會使 price 上升 0.11621

每上升一單位的 trend 會使 price 下降 2.35458

但結果只有 beta1 與 beta2 為顯著

c.

```
Residual standard error: 20.06 on 52 degrees of freedom
Multiple R-squared:  0.5097,    Adjusted R-squared:  0.4814
F-statistic: 18.02 on 3 and 52 DF,  p-value: 3.806e-08
```

可卡因價格的變異中有 50.97%（即 R^2 = 0.5097）可以由數量（QUANT）、品質（QUAL）和時間（TREND）共同解釋。

d.

- 原假設（H0）：$\beta 2 \geq 0$ 這表示交易量（QUANT）對價格沒有負向影響，或者有正向影響（即交易量增加時，價格不下降或上升）。
- 對立假設（H1）：$\beta 2 < 0$
  這表示交易量（QUANT）增加時，價格下降，符合題目中的說法（賣家願意以較低價格出售較大數量的可卡因以降低風險）。

這是一個單尾檢驗（左尾），因為我們關心 $\beta 2$ 是否顯著小於 0。
由前面所得資訊可知 t=-5.892，se=0.01018，coefficient=-0.05997
P 值為 1.39e-14，單尾為 7e-15<0.05，因此我們拒絕原假設 H0。

e.

原假設（H0）：$\beta 3 = 0$
對立假設（H1）：$\beta 3 \neq 0$
T 值為 0.572，P 值為 0.57>0.05，無法拒絕 H0

f.

平均每年下降 2.35458，推測原因為隨著執法力度加強，賣家面臨更高的被抓風險。這種風險可能會增加交易成本（例如需要更多賄賂、更隱秘的運輸方式），從而推高價格。