5.6 Suppose that, from a sample of 63 observations, the least squares estimates and the corresponding estimated covariance matrix are given by

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix} \qquad \widehat{\operatorname{cov}}(b_1, b_2, b_3) = \begin{bmatrix} 3 & -2 & 1 \\ -2 & 4 & 0 \\ 1 & 0 & 3 \end{bmatrix}$$

Using a 5% significance level, and an alternative hypothesis that the equality does not hold, test each of the following null hypotheses:

a. $\beta_2 = 0$

$H_0: \beta_2 = 0 \qquad H_1: \beta_2 \neq 0$

$t = \frac{3-0}{\sqrt{4}} = 1.5 < t_{0.975,60} = 2.003 \Rightarrow$ not reject H_0

b. $\beta_1 + 2\beta_2 = 5$

$H_0: \beta_1 + 2\beta_2 = 5 \qquad H_1: \beta_1 + 2\beta_2 \neq 5$

$se(b_1 + 2b_2) = \sqrt{3 + 2^2 \cdot 4 + 2 \cdot 2 \cdot (-2)} = \sqrt{11}$

$t = \frac{(2+6)-5}{\sqrt{11}} = 0.9045 < t_{0.975,60} = 2.003 \Rightarrow$ not reject H_0

c. $\beta_1 - \beta_2 + \beta_3 = 4$

$H_0: \beta_1 - \beta_2 + \beta_3 = 4 \qquad H_1: \beta_1 - \beta_2 + \beta_3 \neq 4$

$se(b_1 - b_2 + b_3) = \sqrt{\widehat{var}(b_1) + \widehat{var}(b_2) + \widehat{var}(b_3) - 2cov(b_1,b_2) + 2cov(b_1,b_3) - 2cov(b_2,b_3)}$

$= \sqrt{3+4+3+4+2-0} = \sqrt{16} = 4$

$t = \frac{(2-3-1)-4}{4} = -1.5 > t_{0.025,60} = -2.003 \Rightarrow$ not reject H_0

5.31 Each morning between 6:30 AM and 8:00 AM Bill leaves the Melbourne suburb of Carnegie to drive to work at the University of Melbourne. The time it takes Bill to drive to work (*TIME*), depends on the departure time (*DEPART*), the number of red lights that he encounters (*REDS*), and the number of trains that he has to wait for at the Murrumbeena level crossing (*TRAINS*). Observations on these variables for the 249 working days in 2015 appear in the file *commute5*. *TIME* is measured in minutes. *DEPART* is the number of minutes after 6:30 AM that Bill departs.

a. Estimate the equation

$$TIME = \beta_1 + \beta_2 DEPART + \beta_3 REDS + \beta_4 TRAINS + e$$

Report the results and interpret each of the coefficient estimates, including the intercept β_1.

$\widehat{TIME} = 20.8701 + 0.3681\, DEPART + 1.5219\, REDS + 3.0237\, TRAINS$ ※

```
Call:
lm(formula = time ~ depart + reds + trains, data = commute5)

Residuals:
     Min       1Q   Median       3Q      Max 
-18.4389  -3.6774  -0.1188   4.5863  16.4986 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)  20.8701     1.6758  12.454  < 2e-16 ***
depart        0.3681     0.0351  10.487  < 2e-16 ***
reds          1.5219     0.1850   8.225 1.15e-14 ***
trains        3.0237     0.6340   4.769 3.18e-06 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 6.299 on 245 degrees of freedom
Multiple R-squared:  0.5346,	Adjusted R-squared:  0.5289 
F-statistic: 93.79 on 3 and 245 DF,  p-value: < 2.2e-16
```

b. Find 95% interval estimates for each of the coefficients. Have you obtained precise estimates of each of the coefficients?

β_1 : [17.5694, 24.1709]

β_2 : [0.2990, 0.4373]

β_3 : [1.1575, 1.886]

β_4 : [1.7749, 4.2725]

⇒ The CIs are narrow, espicially for depart and reds, indicating precise estimates. ※

c. Using a 5% significance level, test the null hypothesis that Bill's expected delay from each red light is 2 minutes or more against the alternative that it is less than 2 minutes.

$H_0: \beta_3 \geq 2 \quad H_1: \beta_3 < 2$

$t = \dfrac{1.5219 - 2}{0.1850} = -2.5843 < t_{0.05, 245} = -1.6511 \Rightarrow$ reject H_0

$\Rightarrow$ We reject H_0 at the 5% level. The expected delay from each red light is significantly less than 2 minutes.

d. Using a 10% significance level, test the null hypothesis that the expected delay from each train is 3 minutes against the alternative that it is not 3 minutes.

$H_0: \beta_4 = 3 \quad H_1: \beta_4 \neq 3$

$t = \dfrac{3.0237 - 3}{0.6340} = 0.0374 < t_{0.95, 245} = 1.6511 \Rightarrow$ not reject H_0

$\Rightarrow$ There is no significant evidence that the delay from each train differs from 3 minutes.

e. Using a 5% significance level, test the null hypothesis that Bill can expect a trip to be at least 10 minutes longer if he leaves at 7:30 AM instead of 7:00 AM, against the alternative that it will not be 10 minutes longer. (Assume other things are equal.)

$H_0: \beta_2 \geq \frac{1}{3} \quad H_1: \beta_2 < \frac{1}{3}$

$t = \dfrac{0.3681 - \frac{1}{3}}{0.0351} = 0.991 > t_{0.05, 245} = -1.6511 \Rightarrow$ not reject H_0

$\Rightarrow$ There is no signifant evidence that the extra day is less than 10 minutes.

f. Using a 5% significance level, test the null hypothesis that the expected delay from a train is at least three times greater than the expected delay from a red light against the alternative that it is less than three times greater.

$H_0: \beta_4 - 3\beta_3 \geq 0 \qquad H_1: \beta_4 - 3\beta_3 < 0$

$$t = \frac{3.2037 - 3 \cdot 1.5219}{0.844992} = -1.8249 < t_{0.05,245} = -1.6511 \Rightarrow \text{reject } H_0$$

$\Rightarrow$ The expected delay from a train is significantly less than three times the expected delay from a red light.

g. Suppose that Bill encounters six red lights and one train. Using a 5% significance level, test the null hypothesis that leaving Carnegie at 7:00 AM is early enough to get him to the university on or before 7:45 AM against the alternative that it is not. [Carry out the test in terms of the expected time $E(TIME|\mathbf{X})$ where $\mathbf{X}$ represents the observations on all explanatory variables.]

$H_0: \beta_1 + 3\beta_2 + 6\beta_3 + \beta_4 \leq 45 \qquad H_1: \beta_1 + 3\beta_2 + 6\beta_3 + \beta_4 > 45$

$$t = \frac{44.06924 - 45}{0.5392687} = -1.726 < 1.6511 \Rightarrow \text{not reject } H_0$$

$\Rightarrow$ There is no significant evidence that Bill will arrive after 7:45 AM

h. Suppose that, in part (g), it is imperative that Bill is not late for his 7:45 AM meeting. Have the null and alternative hypotheses been set up correctly? What happens if these hypotheses are reversed?

Yes, the hyphothesis in part (g) are set up correctly. Since it's critical that Bill is not late, the null hyphothesis should state that he arrives on time. If the hyphothesis are reversed, failing to reject the null would give no confidence that he'll be on time, which is risky given the importance of the meeting.