11.1

a.

$$y_2 = \alpha_2 y_1 + \beta_1 x_1 + \beta_2 x_2 + e_2 \quad \text{已知 } y_1 = \alpha_1 y_2 + e_1$$

$$y_2 = \alpha_2(\alpha_1 y_2 + e_1) + \beta_1 x_1 + \beta_2 x_2 + e_2$$

$$= \alpha_1 \alpha_2 y_2 + \alpha_2 e_1 + \beta_1 x_1 + \beta_2 x_2 + e_2$$

$$(1 - \alpha_1 \alpha_2) y_2 = \beta_1 x_1 + \beta_2 x_2 + \alpha_2 e_1 + e_2$$

$$y_2 = \frac{\beta_1}{1 - \alpha_1 \alpha_2} x_1 + \frac{\beta_2}{1 - \alpha_1 \alpha_2} x_2 + \frac{\alpha_2 e_1 + e_2}{1 - \alpha_1 \alpha_2}$$

$$= \pi_1 x_1 + \pi_2 x_2 + v_2$$

$$\therefore \pi_1 = \frac{\beta_1}{1 - \alpha_1 \alpha_2} \quad \pi_2 = \frac{\beta_2}{1 - \alpha_1 \alpha_2} \quad v_2 = \frac{\alpha_2 e_1 + e_2}{1 - \alpha_1 \alpha_2}$$

$$cov(y_2, e_1) = cov(y_2, y_1 - \alpha_1 y_2)$$

$$= cov(y_1, y_2) - \alpha_1 \sigma_{y_2}^2 \neq 0$$

b.

$\pi_1 \ \pi_2$ are consisent because $x_1 \ x_2$ are exogenous variables.

c.

Equation 1 is identified, while equation 2 is not. Because equation 1 excludes 2 exogenous variables

but equation 2 doesn't exclude any exogenous variables

d、

$\because$ x_1, x_2 為 exogenous variables，應與誤差項無關.

所以 $E(x_1, v_2) = 0$ $E(x_2, v_2) = 0$

$E[(y_2 - \pi_1 x_1 - \pi_2 x_2) x_1] = 0$ $E[(y_2 - \pi_1 x_1 - \pi_2 x_2) x_2] = 0$

e、

$y_2 = \pi_1 x_1 + \pi_2 x_2 + v_2$

$\min S(\pi_1, \pi_2) = \Sigma (y_2 - \pi_1 x_1 - \pi_2 x_2)^2$

F.O.C

$\frac{\partial S}{\partial \pi_1} = 0$ $-2 \Sigma x_1 (y_2 - \pi_1 x_1 - \pi_2 x_2) = 0$

$\frac{\partial S}{\partial \pi_2} = 0$ $-2 \Sigma x_2 (y_2 - \pi_1 x_1 - \pi_2 x_2) = 0$

同除以 N

$\frac{1}{N} \Sigma x_1 (y_2 - \pi_1 x_1 - \pi_2 x_2) = 0$

$\frac{1}{N} \Sigma x_2 (y_2 - \pi_1 x_1 - \pi_2 x_2) = 0$

跟 (d) 一模一樣

1.

$$\nexists \, \sum x_1(y_2 - \hat{\pi}_1 x_1 - \hat{\pi}_2 x_2) = 0$$

$$\sum x_1 y_2 - \hat{\pi}_1 \sum x_1^2 - \hat{\pi}_2 \sum x_1 x_2 = 0$$

$$\Rightarrow 3 - \hat{\pi}_1 = 0 \Rightarrow \hat{\pi}_1 = 3$$

$$\nexists \, \sum x_2(y_2 - \hat{\pi}_1 x_1 - \hat{\pi}_2 x_2) = 0$$

$$\sum x_2 y_2 - \hat{\pi}_1 \sum x_1 x_2 - \hat{\pi}_2 \sum x_2^2 = 0$$

$$4 - \hat{\pi}_2 = 0 \Rightarrow \hat{\pi}_2 = 4$$

2.

∵ y_2 具有内生性，∴ 不能直接使用 OLS.

$$\hat{y}_2 = \hat{\pi}_1 x_1 + \hat{\pi}_2 x_2$$

$$E[(\hat{\pi}_1 x_1 + \hat{\pi}_2 x_2)(y_1 - \alpha_1 y_2) | X] = 0$$

$$\nexists \, \sum (\hat{\pi}_1 x_1 + \hat{\pi}_2 x_2)(y_1 - \alpha_1 y_2) = 0$$

$$\Rightarrow \sum \hat{y}_2(y_1 - \alpha_1 y_2) = 0$$

$$\Rightarrow \sum y_1 \hat{y}_2 - \alpha_1 \sum \hat{y}_2 y_2 = 0$$

$$\Rightarrow \alpha_1 = \frac{\sum y_1 \hat{y}_2}{\sum \hat{y}_2 y_2}$$

$$= \frac{\sum y_1(\hat{\pi}_1 x_1 + \hat{\pi}_2 x_2)}{\sum y_2(\hat{\pi}_1 x_1 + \hat{\pi}_2 x_2)}$$

$$= \frac{\hat{\pi}_1 \sum x_1 y_1 + \hat{\pi}_2 \sum x_2 y_1}{\hat{\pi}_1 \sum x_1 y_2 + \hat{\pi}_2 \sum x_2 y_2} = \frac{3 \times 2 + 4 \times 3}{3 \times 3 + 4 \times 4} = \frac{18}{25} \quad \#$$

h.

$\hat{\alpha_1} = \dfrac{\Sigma \hat{y_2} y_1}{\Sigma \hat{y_2} y_2} = \dfrac{\Sigma \hat{y_2} y_1}{\Sigma \hat{y_2}^2}$

$\hat{y_2} = y_2 - v_i \rightleftharpoons y_2 = \hat{y_2} + v_2$

$\Sigma \hat{y_2} y_2 = \Sigma \hat{y_2}(\hat{y_2} + v_2) = \Sigma \hat{y_2}^2 + \Sigma \hat{y_2} v_2$

$\because cov(\hat{y_2}, v_2) = 0$

$\therefore \Sigma \hat{y_2} v_2 = 0$

$\Sigma \hat{y_2}^2 + \Sigma \hat{y_2} v_2 = \Sigma \hat{y_2}^2$