CH10 Q2

10.2 The labor supply of married women has been a subject of a great deal of economic research. Consider the following supply equation specification

$$HOURS = \beta_1 + \beta_2 WAGE + \beta_3 EDUC + \beta_4 AGE + \beta_5 KIDSL6 + \beta_6 NWIFEINC + e$$

where *HOURS* is the supply of labor, *WAGE* is hourly wage, *EDUC* is years of education, *KIDSL6* is the number of children in the household who are less than 6 years old, and *NWIFEINC* is household income from sources other than the wife's employment.

a. Discuss the signs you expect for each of the coefficients.
b. Explain why this supply equation cannot be consistently estimated by OLS regression.
c. Suppose we consider the woman's labor market experience *EXPER* and its square, $EXPER^2$, to be instruments for *WAGE*. Explain how these variables satisfy the logic of instrumental variables.
d. Is the supply equation identified? Explain.
e. Describe the steps [not a computer command] you would take to obtain IV/2SLS estimates.

(a)
β_2 (wage)：預期為正，工資越高，勞動供給應增加

β_3 (EDUC)：預期為正，教育程度越高，預期工資越高，勞動供給應增加

β_4 (AGE)：可能是負，年輕時勞動供給可能較高

β_5 (KIDSL6)：預期為負，家中有6歲以下小孩需照顧，可能減少婦女勞動供給

β_6 (NWIFEINC)：預期為負，家庭其他收入越高，經濟壓力小，可能減少勞動供給

(b) 因為可能有遺漏變數偏誤，如工作經驗 & 工作能力可能會同時影響 WAGE 和 HOURS，造成內生性問題

(c)
相關性：EXPER 和 $EXPER^2$ 和工資相關。

外生性：EXPER 和 $EXPER^2$ 只透過 WAGE 影響 HOURS，且與誤差項零相關 $(\text{corr}(z_i, e_i) = 0)$

(d) 因為我們有兩個工具變數 (EXPER, $EXPER^2$) 可以估計 1 個內生變數 (WAGE)，所以 identified

(c) 第一階段迴歸：

用工具變數 (EXPER, $EXPER^2$) 和其他外生變數迴歸 WAGE，得到估計值 $\widehat{WAGE}$

第二階段迴歸：

用第一階段預測的 $\widehat{WAGE}$，及其他外生變數去迴歸 HOURS

CH10 Q3

10.3 In the regression model $y = \beta_1 + \beta_2 x + e$, assume x is endogenous and that z is a valid instrument. In Section 10.3.5, we saw that $\beta_2 = \text{cov}(z, y)/\text{cov}(z, x)$.

a. Divide the denominator of $\beta_2 = \text{cov}(z, y)/\text{cov}(z, x)$ by $\text{var}(z)$. Show that $\text{cov}(z, x)/\text{var}(z)$ is the coefficient of the simple regression with dependent variable x and explanatory variable z, $x = \gamma_1 + \theta_1 z + v$. [*Hint*: See Section 10.2.1.] Note that this is the first-stage equation in two-stage least squares.

b. Divide the numerator of $\beta_2 = \text{cov}(z, y)/\text{cov}(z, x)$ by $\text{var}(z)$. Show that $\text{cov}(z, y)/\text{var}(z)$ is the coefficient of a simple regression with dependent variable y and explanatory variable z, $y = \pi_0 + \pi_1 z + u$. [*Hint*: See Section 10.2.1.]

c. In the model $y = \beta_1 + \beta_2 x + e$, substitute for x using $x = \gamma_1 + \theta_1 z + v$ and simplify to obtain $y = \pi_0 + \pi_1 z + u$. What are π_0, π_1, and u in terms of the regression model parameters and error and the first-stage parameters and error? The regression you have obtained is a **reduced-form** equation.

d. Show that $\beta_2 = \pi_1/\theta_1$.

e. If $\hat{\pi}_1$ and $\hat{\theta}_1$ are the OLS estimators of π_1 and θ_1, show that $\hat{\beta}_2 = \hat{\pi}_1/\hat{\theta}_1$ is a consistent estimator of $\beta_2 = \pi_1/\theta_1$. The estimator $\hat{\beta}_2 = \hat{\pi}_1/\hat{\theta}_1$ is an **indirect least squares** estimator.

(a) $E(x) = \gamma_1 + \theta_1 E(z) \Rightarrow x - E(x) = \theta_1 (z - E(z)) + v$

同乘 $(z - E(z))$

$$[z - E(z)][x - E(x)] = \theta_1 (z - E(z))^2 + (z - E(z))v$$

同取期望值，設 $E(z - E(z))v = 0$

$$E[(z - E(z))(x - E(x))] = \theta_1 E[(z - E(z))^2] + E(z - E(z))v$$

$$= \theta_1 E[(z - E(z))^2]$$

$$\theta_1 = \frac{E((z - E(z))(x - E(x)))}{E[(z - E(z))^2]} = \frac{\text{Cov}(z, x)}{\text{Var}(z)}$$

(b)

$E(y) = \pi_0 + \pi_1 E(z) \Rightarrow y - E(y) = \pi_1 (z - E(z)) + u$

同乘 $(z - E(z))$

$$(z - E(z))(y - E(y)) = \pi_1 (z - E(z))^2 + (z - E(z))u$$

同取期望值，設 $E(z - E(z))u = 0$

$$E[(z - E(z))(y - E(y))] = \pi_1 E[(z - E(z))^2]$$

$$\pi_1 = \frac{E[(z-E(z))(y-E(y))]}{E[(z-E(z))^2]} = \frac{\mathrm{Cov}(z,y)}{\mathrm{Var}(z)}$$

(c)

$$y = \beta_1 + \beta_2 x + e = \beta_1 + \beta_2(\gamma_1 + \theta_1 z + v) + e$$

$$= (\beta_1 + \beta_2\gamma_1) + \beta_2\theta_1 z + (\beta_2 v + e)$$

$$= \pi_0 + \pi_1 z + u$$

Thus $\pi_0 = (\beta_1 + \beta_2\gamma_1)$, $\pi_1 = \beta_2\theta_1$ and $u = (\beta_2 v + e)$

(d)

Solving $\pi_1 = \beta_2\theta_1$, we have $\beta_2 = \dfrac{\pi_1}{\theta_1}$

(e)

From (a)

$$\hat{\theta}_1 = \frac{\widehat{\mathrm{Cov}}(z,x)}{\widehat{\mathrm{Var}}(z)} = \frac{\Sigma(z-\bar{z})(x-\bar{x})/N}{\Sigma(z-\bar{z})^2/N} = \frac{\Sigma(z-\bar{z})(x-\bar{x})}{\Sigma(z-\bar{z})^2}$$

$$\hat{\pi}_1 = \frac{\widehat{\mathrm{Cov}}(z,y)}{\widehat{\mathrm{Var}}(z)} = \frac{\Sigma(z-\bar{z})(y-\bar{y})/N}{\Sigma(z-\bar{z})^2/N} = \frac{\Sigma(z-\bar{z})(y-\hat{y})}{\Sigma(z-\bar{z})^2}$$

Then

$$\hat{\beta}_2 = \frac{\hat{\pi}_1}{\hat{\theta}_1} = \frac{\dfrac{\Sigma(z-\bar{z})(y-\hat{y})}{\Sigma(z-\bar{z})^2}}{\dfrac{\Sigma(z-\bar{z})(x-\bar{x})}{\Sigma(z-\bar{z})^2}} = \frac{\Sigma(z-\bar{z})(y-\bar{y})}{\Sigma(z-\bar{z})(x-\bar{x})}$$

$$= \frac{\Sigma(z-\bar{z})(y-\bar{y})/N}{\Sigma(z-\bar{z})(x-\bar{x})/N} = \frac{\widehat{\mathrm{Cov}}(z,y)}{\widehat{\mathrm{Cov}}(z,x)}$$

$$\widehat{\mathrm{Cov}}(z,y) \xrightarrow{p} \mathrm{cov}(z,y), \quad \widehat{\mathrm{Cov}}(z,x) \xrightarrow{p} \mathrm{cov}(z,x)$$

$$\Rightarrow \hat{\beta}_2 = \frac{\hat{\pi}_1}{\hat{\theta}_1} = \frac{\widehat{\mathrm{Cov}}(z,y)}{\widehat{\mathrm{Cov}}(z,x)} \xrightarrow{p} \frac{\mathrm{cov}(z,y)}{\mathrm{cov}(z,x)} = \beta_2$$