

5.23 The file *cocaine* contains 56 observations on variables related to sales of cocaine powder in northeastern California over the period 1984–1991. The data are a subset of those used in the study Caulkins, J. P. and R. Padman (1993), “Quantity Discounts and Quality Premia for Illicit Drugs,” *Journal of the American Statistical Association*, 88, 748–757. The variables are

- PRICE* = price per gram in dollars for a cocaine sale
 - QUANT* = number of grams of cocaine in a given sale
 - QUAL* = quality of the cocaine expressed as percentage purity
 - TREND* = a time variable with 1984 = 1 up to 1991 = 8
- Consider the regression model

$$PRICE = \beta_1 + \beta_2 QUANT + \beta_3 QUAL + \beta_4 TREND + e$$

- a. What signs would you expect on the coefficients β_2 , β_3 , and β_4 ?
- b. Use your computer software to estimate the equation. Report the results and interpret the coefficient estimates. Have the signs turned out as you expected?
- c. What proportion of variation in cocaine price is explained jointly by variation in quantity, quality, and time?
- d. It is claimed that the greater the number of sales, the higher the risk of getting caught. Thus, sellers are willing to accept a lower price if they can make sales in larger quantities. Set up H_0 and H_1 that would be appropriate to test this hypothesis. Carry out the hypothesis test.
- e. Test the hypothesis that the quality of cocaine has no influence on expected price against the alternative that a premium is paid for better-quality cocaine.
- f. What is the average annual change in the cocaine price? Can you suggest why price might be changing in this direction?

- a.
 - β_2 (for QUANT): Negative. We expect a negative sign because larger quantities (QUANT) are likely associated with a lower price per gram (PRICE) due to quantity discounts in the cocaine market.
 - β_3 (for QUAL): Positive. We expect a positive sign because higher quality (greater purity, QUAL) should increase the price per gram (PRICE), as buyers are willing to pay a premium for purer cocaine.
 - β_4 (for TREND): Negative. We expect a negative sign because, over the period from 1984 to 1991, the price per gram of cocaine likely decreased due to increased supply, market saturation, and competition.

b.

Call:
lm(formula = price ~ quant + qual + trend, data = cocaine)

Residuals:
Min 1Q Median 3Q Max
-43.479 -12.014 -3.743 13.969 43.753

Coefficients:
Estimate Std. Error t value Pr(>|t|)
(Intercept) 90.84669 8.58025 10.588 1.39e-14 ***
quant -0.05997 0.01018 -5.892 2.85e-07 ***
qual 0.11621 0.20326 0.572 0.5700
trend -2.35458 1.38612 -1.699 0.0954 .

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 20.06 on 52 degrees of freedom
Multiple R-squared: 0.5097, Adjusted R-squared: 0.4814
F-statistic: 18.02 on 3 and 52 DF, p-value: 3.806e-08

- Coefficient for QUANT ($\beta_2 = -0.05997$)
Interpretation: For each additional gram of cocaine in a sale (QUANT increases by 1), the price per gram decreases by 0.05997 dollars (about 6 cents), holding QUAL and TREND constant.
 - Coefficient for QUAL ($\beta_3 = 0.11621$)
Interpretation: For each 1% increase in the purity of the cocaine (QUAL), the price per gram increases by 0.11621 dollars (about 12 cents), holding QUANT and TREND constant.
 - Coefficient for TREND ($\beta_4 = -2.35458$)
Interpretation: For each year increase in the time trend (TREND), the price per gram decreases by 2.35458 dollars (about \$2.35), holding QUANT and QUAL constant.
 - Conclusion on Signs: The actual signs of all coefficients (β_2 , β_3 , and β_4) match our expectations from part a. This alignment suggests that our economic intuition about quantity discounts, quality premia, and time trends in the cocaine market is consistent with the data.

- c. The proportion of variation in cocaine price (PRICE) explained jointly by variation in quantity (QUANT), quality (QUAL), and time (TREND) is given by the R^2 :
 $R^2 = 0.5097$
This means 50.97% of the variation in cocaine price is explained by the model.

- d. $H_0: \beta_2 \geq 0$ (there is no quantity discount or the price increases with quantity).
 $H_1: \beta_2 < 0$ (larger quantities lead to a lower price per gram).
This is a one-tailed test (left-tailed) because alternative hypothesis specifies a negative direction.
Significance level: $\alpha=0.05$.
The critical t-value for a one-tailed test (left-tailed) at $\alpha=0.05$ with 52 degrees of freedom is approximately -1.675.
The calculated t-value is $t = -0.05997 \div 0.01018 = -5.8910$
Since $-5.892 < -1.675$, we can reject H_0 based on the critical value.
Conclusion: At the 5% significance level, we reject the null hypothesis and conclude that there is significant evidence to support the claim that sellers accept a lower price per gram for larger quantities.

> qt(0.05,52)

[1] -1.674689

- e. $H_0: \beta_3 = 0$ (Quality has no influence on price).
 $H_1: \beta_3 > 0$ (A premium is paid for better-quality cocaine, meaning higher quality increases the price).
This is a one-tailed test (right-tailed) because the alternative hypothesis specifies a positive direction.
Significance level: $\alpha=0.05$.
The critical t-value for a one-tailed test (right-tailed) at $\alpha=0.05$ with 52 degrees of freedom is approximately 1.675.
The calculated t-value is $t = 0.11621 \div 0.20326 = 0.5717$
Since $0.5717 < 1.675$, we do not reject H_0 based on the critical value.
Conclusion: At the 5% significance level, we fail to reject the null hypothesis. There is not sufficient evidence to conclude that the quality of cocaine (QUAL) has a significant positive influence on the expected price (PRICE). We cannot confirm that a premium is paid for better-quality cocaine in this sample.

> qt(1-0.05,52)

[1] 1.674689

- f. The average annual change in the cocaine price is a decrease of 2.35458 dollars per gram, as indicated by the coefficient $b_4 = -2.35458$ for TREND. This means that, holding quantity and quality constant, the price per gram decreases by about 2.35 dollars each year from 1984 to 1991.
A possible reason for a decreasing price is expansion of supply chains. Improved trafficking or production efficiency could increase supply, pushing prices down.