5.6 Suppose that, from a sample of 63 observations, the least squares estimates and the corresponding estimated covariance matrix are given by

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix} \qquad \widehat{\operatorname{cov}}(b_1, b_2, b_3) = \begin{bmatrix} 3 & -2 & 1 \\ -2 & 4 & 0 \\ 1 & 0 & 3 \end{bmatrix}$$

Using a 5% significance level, and an alternative hypothesis that the equality does not hold, test each of the following null hypotheses:

a. $\beta_2 = 0$

b. $\beta_1 + 2\beta_2 = 5$

c. $\beta_1 - \beta_2 + \beta_3 = 4$

a. $t^* = \dfrac{3-0}{\sqrt{4}} = 1.5$, $|t^*| < t_{0.25,60} \approx 1.96$, do not reject H_0 #

b. $E(b_1+2b_2) = 8$, $Var(b_1+2b_2) = Var(b_1) + 4 \cdot Var(b_2) + 4\,Cov(b_1,b_2)$

$= 3 + 16 + (-8) = 11$

$t^* = \dfrac{8-5}{\sqrt{11}} = 0.9045$, $|t^*| < t_{0.25,60} \approx 1.96$, do not reject H_0 #

c. $E(b_1-b_2+b_3) = 6$, $Var(b_1-b_2+b_3) = Var(b_1) + Var(b_2) + Var(b_3)$

$- 2Cov(b_1,b_2) + 2Cov(b_1,b_3) - 2Cov(b_2,b_3)$

$= 3+4+3+4+2-0 = 16$

$t^* = \dfrac{6-4}{\sqrt{16}} = 0.5$, $|t^*| < t_{0.25,60} \approx 1.96$ do not reject H_0

Q31

(a)

```
Residuals:
     Min       1Q   Median       3Q      Max 
-18.4389  -3.6774  -0.1188   4.5863  16.4986 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept)  20.8701     1.6758  12.454  < 2e-16 ***
depart        0.3681     0.0351  10.487  < 2e-16 ***
reds          1.5219     0.1850   8.225 1.15e-14 ***
trains        3.0237     0.6340   4.769 3.18e-06 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

(b)

```
> confint(model, level = 0.95)
                 2.5 %    97.5 %
(Intercept) 17.5694018 24.170871
depart       0.2989851  0.437265
reds         1.1574748  1.886411
trains       1.7748867  4.272505
```

(c) p-value<0.05，拒絕H0，不會因為紅燈停超過兩分鐘

```
> t_c <- (reds_coef - 2) / reds_se
> p_c <- pt(t_c, df = model$df.residual)  # 單尾
> 
> t_c; p_c
[1] -2.583562
[1] 0.005179509
```

(d)無法拒絕H0，沒有足夠證據拒絕因火車多停三分鐘

```
> trains_coef <- summary(model)$coefficients["trains", "Estimate"]
> trains_se <- summary(model)$coefficients["trains", "Std. Error"]
> 
> t_d <- (trains_coef - 3) / trains_se
> p_d <- 2 * pt(-abs(t_d), df = model$df.residual)  # 雙尾
> 
> t_d; p_d
[1] 0.03737444
[1] 0.9702169
```

(e)無法拒絕H0，沒有足夠證據說明7:30出發會比7:00不晚到10分鐘

```
> depart_coef <- summary(model)$coefficients["depart", "Estimate"]
> depart_se <- summary(model)$coefficients["depart", "Std. Error"]
> # (e) 不同時間點出發的差距
> t_e <- (30 * depart_coef - 10) / (30 * depart_se)
> p_e <- pt(t_e, df = model$df.residual)
> 
> t_e; p_e
[1] 0.9911646
[1] 0.8387085
```

(f)火車延遲是否為紅燈的三倍(H0: $\beta_4 \geq 3\times\beta_3$)，無法拒絕H0

```
Linear hypothesis test:
- 3 reds  + trains = 0

Model 1: restricted model
Model 2: time ~ depart + reds + trains

  Res.Df    RSS Df Sum of Sq      F  Pr(>F)
1    246 9851.7
2    245 9719.5  1    132.13 3.3307 0.06921 .
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

(g)檢定在 7:00AM 出發，遇到 6 個紅燈、1 次火車，是否能準時(<= 75 分鐘 無法拒絕能夠準時這個結果

```
> new_data <- data.frame(depart = 30, reds = 6, trains = 1)
> pred <- predict(model, newdata = new_data, se.fit = TRUE)
> t_val <- (pred$fit - 75) / pred$se.fit
> p_val <- 1 - pt(t_val, df = model$df.residual)
>
> t_val         # t 統計值
        1
-57.35686
> p_val         # p 值
1
1
```

(h)若會議很重要，改成檢定 H0: > 75 vs H1: ≤ 75(是否拒絕 H0)=>拒絕會遲到的假設

```
> p_h <- pt(t_g, df = model$df.residual)  # 改為左尾檢定
> p_h
            1
2.626431e-144
```

Q33.

(a)

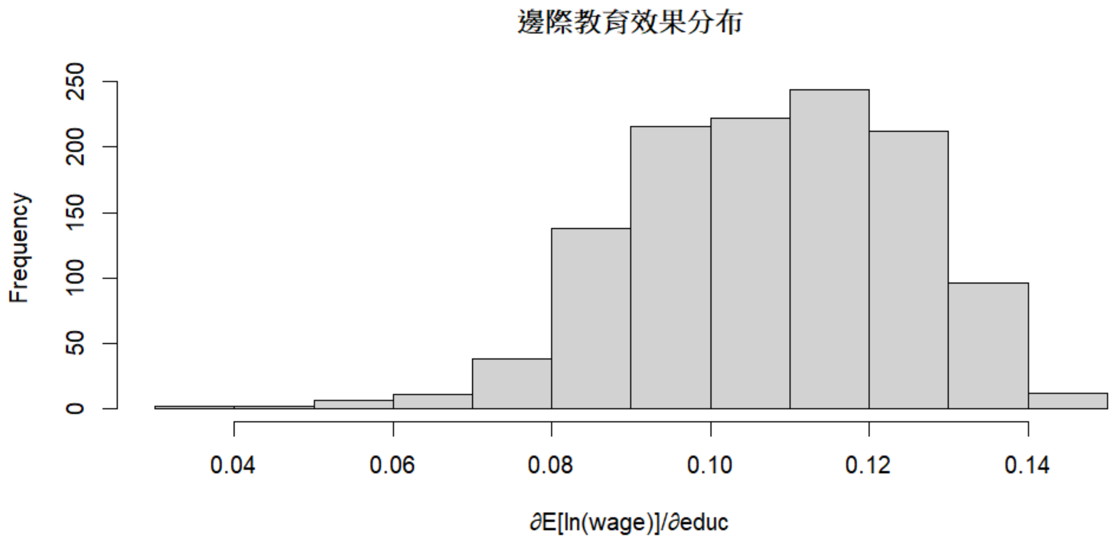
```
                 Estimate Std. Error t value Pr(>|t|)
(Intercept)     1.038e+00  2.757e-01   3.764 0.000175 ***
educ            8.954e-02  3.108e-02   2.881 0.004038 **
I(educ^2)       1.458e-03  9.242e-04   1.578 0.114855
exper           4.488e-02  7.297e-03   6.150 1.06e-09 ***
I(exper^2)     -4.680e-04  7.601e-05  -6.157 1.01e-09 ***
I(educ * exper) -1.010e-03  3.791e-04  -2.665 0.007803 **
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.4638 on 1194 degrees of freedom
Multiple R-squared:  0.3227,	Adjusted R-squared:  0.3198
F-statistic: 113.8 on 5 and 1194 DF,  p-value: < 2.2e-16
```

(b)教育邊際報酬與exper、educ相關

$$\frac{\partial E[\ln(\text{wage}) \mid \text{educ}, \text{exper}]}{\partial \text{educ}} = \beta_2 + 2\beta_3 \cdot \text{educ} + \beta_6 \cdot \text{exper}$$

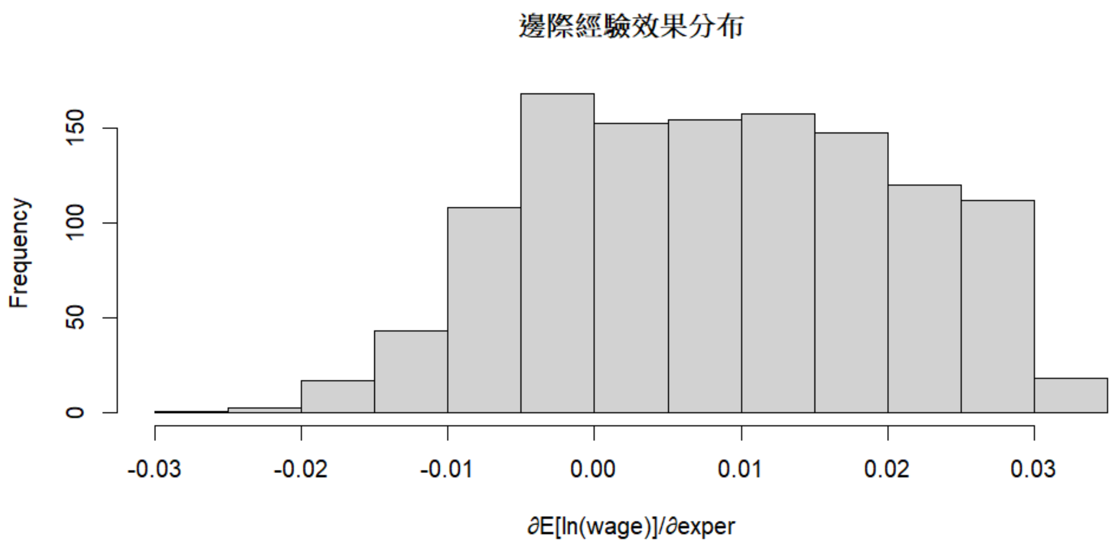
(c)教育邊際報酬效果直方圖



(d) 工作經驗邊際效果與exper、educ相關

$$\frac{\partial E[\ln(\text{wage}) \mid \text{educ}, \text{exper}]}{\partial \text{exper}} = \beta_4 + 2\beta_5 \cdot \text{exper} + \beta_6 \cdot \text{educ}$$

(e)工作經驗邊際效果直方圖



(f) p-value<0.05 拒絕H0 David薪資較高

```
> print(diff)
[1] -0.03588456
> print(t_val)
           [,1]
[1,] -1.669902
> print(p_val)
           [,1]
[1,] 0.0476004
```

(g) 無法拒絕H0，及各多年經驗後，無足夠證據說明薪資有差距

```
> print(diff_new)
[1] 0.03091716
> print(t_val_new)
         [,1]
[1,] 2.062365
> print(p_val_new)
          [,1]
[1,] 0.9803056
```

(h)無法拒絕H0，無證據說明兩人之邊際經驗效果不同

```
> print(diff_marginal)
       exper
-0.001575327
> print(t_val_marginal)
           [,1]
[1,] -0.1947834
> print(p_val_marginal)
          [,1]
[1,] 0.8455957
```

(i) 在Jill工作地30.67706年時邊際效果為負，印出此數值95%C.I.

```
> # 解方程式： a * x + b1 + c = 0
> critical_x <- (-b1 - c) / a
> print(critical_x)
   exper
30.67706
> print(ci)
[1] 13.51411 47.84000
```