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HW0512

28.

Original Equations:

1. **Demand Equation:**

$$Q_i = \alpha_1 + \alpha_2 P_i + \alpha_3 PS_i + \alpha_4 DI_i + e_{d_i}$$

2. **Supply Equation:**

$$Q_i = \beta_1 + \beta_2 P_i + \beta_3 PF_i + e_{s_i}$$

For the Supply Equation:

1. δ_1 **(intercept):**

   Negative, since $\beta_1 > 0$ and $\beta_2 > 0$.

2. δ_2 **(coefficient of Q):**

   Positive, since $\beta_2 > 0$.

3. δ_3 **(coefficient of PF):**

   Positive, since $\beta_3 < 0$ and $\beta_2 > 0$.

For the Demand Equation:

1. γ_1 **(intercept):**

   Positive, since $\alpha_1 > 0$ and $\alpha_2 < 0$.

2. γ_2 **(coefficient of Q):**

   Negative, since $\alpha_2 < 0$.

3. γ_3 **(coefficient of PS):**

   Positive, since $\alpha_3 > 0$ and $\alpha_2 < 0$.

4. γ_4 **(coefficient of DI):**

   Positive, since $\alpha_4 > 0$ and $\alpha_2 < 0$.

Start by isolating P_i:

$$\beta_2 P_i = -\beta_1 + Q_i - \beta_3 PF_i - e_{s_i}$$

Divide through by β_2:

$$P_i = \frac{-\beta_1}{\beta_2} + \frac{1}{\beta_2} Q_i - \frac{\beta_3}{\beta_2} PF_i - \frac{e_{s_i}}{\beta_2}$$

Define new coefficients:

$$\delta_1 = \frac{-\beta_1}{\beta_2}, \quad \delta_2 = \frac{1}{\beta_2}, \quad \delta_3 = \frac{-\beta_3}{\beta_2}, \quad u_{s_i} = \frac{-e_{s_i}}{\beta_2}$$

The rewritten supply equation becomes:

$$P_i = \delta_1 + \delta_2 Q_i + \delta_3 PF_i + u_{s_i}$$

Rewriting the Demand Equation:

Start by isolating P_i:

$$\alpha_2 P_i = -\alpha_1 + Q_i - \alpha_3 PS_i - \alpha_4 DI_i - e_{d_i}$$

Divide through by α_2:

$$P_i = \frac{-\alpha_1}{\alpha_2} + \frac{1}{\alpha_2} Q_i - \frac{\alpha_3}{\alpha_2} PS_i - \frac{\alpha_4}{\alpha_2} DI_i - \frac{e_{d_i}}{\alpha_2}$$

Define new coefficients:

$$\gamma_1 = \frac{-\alpha_1}{\alpha_2}, \quad \gamma_2 = \frac{1}{\alpha_2}, \quad \gamma_3 = \frac{-\alpha_3}{\alpha_2}, \quad \gamma_4 = \frac{-\alpha_4}{\alpha_2}, \quad u_{d_i} = \frac{-e_{d_i}}{\alpha_2}$$

The rewritten demand equation becomes:

$$P_i = \gamma_1 + \gamma_2 Q_i + \gamma_3 PS_i + \gamma_4 DI_i + u_{d_i}$$

For the Demand Equation:

1. γ_1 **(intercept):**

   Positive, since $\alpha_1 > 0$ and $\alpha_2 < 0$.

2. γ_2 **(coefficient of Q):**

   Negative, since $\alpha_2 < 0$.

3. γ_3 **(coefficient of PS):**

   Positive, since $\alpha_3 > 0$ and $\alpha_2 < 0$.

4. γ_4 **(coefficient of DI):**

   Positive, since $\alpha_4 > 0$ and $\alpha_2 < 0$.

For the Supply Equation:

1. δ_1 **(intercept):**

   Negative, since $\beta_1 > 0$ and $\beta_2 > 0$.

2. δ_2 **(coefficient of Q):**

   Positive, since $\beta_2 > 0$.

3. δ_3 **(coefficient of PF):**

   Positive, since $\beta_3 < 0$ and $\beta_2 > 0$.

The equilibrium values calculated using the structural equations and those predicted from the reduced form equations show excellent agreement:

- **Price:** The difference is only 0.027196760.02719676, which represents approximately a 0.04%0.04% difference.
- **Quantity:** The difference is only −0.01018407−0.01018407, which represents approximately a 0.06%0.06% difference.

These minimal differences highlight the strong consistency between the two approaches. This confirms that both the structural method (solving the simultaneous equations) and the reduced form method (direct estimation of equilibrium values) yield consistent results, validating the model specification and estimation techniques.

The slight differences are likely attributable to rounding errors or minor numerical imprecisions in the estimation algorithms, rather than any substantive disagreement between the methods.

Analysis of OLS vs. 2SLS Results

Demand Equation

Sign Analysis:

- **Quantity (q) Coefficient:**
  - **OLS:** Positive (0.1512) - Incorrect sign for the demand curve.
  - **2SLS:** Negative (-2.6705) - Correct sign for the demand curve.
- **Price of Substitute (ps) Coefficient:**
  - **OLS:** Positive (1.3607) - Correct sign, indicating a substitute good.
  - **2SLS:** Positive (3.4611) - Correct sign.
- **Disposable Income (di) Coefficient:**
  - **OLS:** Positive (12.3582) - Correct sign, reflecting the income effect.
  - **2SLS:** Positive (13.3899) - Correct sign.

Statistical Significance:

- **Quantity (q) Coefficient:**
  - **OLS:** Not significant ($p = 0.7642$).
  - **2SLS:** Significant ($p = 0.0315$) **
- **Price of Substitute (ps) Coefficient:**
  - **OLS:** Significant ($p = 0.0303$) **
  - **2SLS:** Highly significant ($p = 0.0046$) ***
- **Disposable Income (di) Coefficient:**
  - **Both OLS and 2SLS:** Highly significant ($p < 0.0001$) ***

Supply Equation

Sign Analysis:

- **Quantity (q) Coefficient:**
  - **OLS:** Positive (2.6613) - Correct sign for the supply curve.

- **2SLS:** Positive (2.9367) - Correct sign.

- **Price of Factor (pf) Coefficient:**
  - **OLS:** Positive (2.9217) - Correct sign, indicating input price effect.
  - **2SLS:** Positive (2.9585) - Correct sign.

Statistical Significance:

- **All Coefficients:** Highly significant in both OLS and 2SLS ($p < 0.0001$) ***

Comparison with Part (b)

1. **Key Finding:** OLS estimation yields an incorrect positive sign for the quantity coefficient in the demand equation, while 2SLS correctly identifies a negative coefficient.

2. **Simultaneity Bias:** This highlights the simultaneity bias in OLS estimation when used in simultaneous equation models, as it fails to account for the endogeneity of quantity.

3. **Supply Equation:** Both methods produce similar estimates for the supply equation, but 2SLS estimates are slightly larger in magnitude.

4. **Statistical Significance:** The quantity coefficient is statistically significant only with 2SLS, not with OLS.

5. **Coefficient Magnitudes:** 2SLS estimates for exogenous variables (ps, di, pf) are larger in magnitude than OLS estimates, suggesting OLS underestimates these effects.

30.

Intercept (β_1 = 10.12579)

- **Sign:** Positive
- **Significance:** Marginally significant ($p = 0.081374$) *
- **Interpretation:** The baseline investment level is approximately 10.13 units when all other variables are zero. However, this estimate is only significant at the 10% level, suggesting some uncertainty around its reliability.

Current Profits (p) (β_2 = 0.47964)

- **Sign:** Positive ✓
- **Significance:** Highly significant ($p = 0.000125$) ***

- **Interpretation:** A strong positive relationship exists between current profits and investment, aligning with economic theory. For every additional unit of current profits, investment increases by approximately 0.48 units, holding other factors constant.

Lagged Profits (plag) (β_3 = 0.33304)

- **Sign:** Positive ✓
- **Significance:** Highly significant (p = 0.004212) ***
- **Interpretation:** Past profits also positively influence current investment, indicating that firms consider profit history in their investment decisions. A one-unit increase in lagged profits leads to an increase of about 0.33 units in current investment.

Lagged Capital Stock (klag) (β_4 = -0.11179)

- **Sign:** Negative ✓
- **Significance:** Highly significant (p = 0.000624) ***
- **Interpretation:** The negative relationship suggests a capital adjustment process. Firms with higher existing capital stock tend to invest less in the current period, consistent with the principle of diminishing returns to capital. Each additional unit of lagged capital stock reduces current investment by approximately 0.11 units.

b.

The joint hypothesis test does not reject the null hypothesis that the variables g, tx, $w2$, $time$, and $elag$ are all simultaneously equal to zero (p-value = 0.1566). This indicates that these variables do not collectively exhibit a statistically significant effect on the dependent variable at conventional significance levels.

c.

Key Statistics

- **Residual Coefficient:** 0.57451
- **Standard Error:** 0.14261
- **t-value:** 4.029
- **p-value:** 0.000972 (highly significant)

Conclusion

The null hypothesis is strongly rejected at the 5% significance level, and even at the 0.1% level. This provides robust evidence that **profits (p)** are endogenous in the investment equation.

Context in the Simultaneous Equations Model

This finding is consistent with the theoretical expectations in Klein's Model I:

- **Consumption (CN)** influences profits (P) through Equation 11.17.
- **Investment (I)** impacts profits (P) via the national income identity.
- **Profits (P)**, in turn, affect investment (I) through Equation 11.18.

In this system, profits cannot be considered exogenous, as they are jointly determined with investment and consumption. The significant residual coefficient confirms this theoretical framework, indicating the following:

1. **OLS Estimates Bias:** OLS estimates of the investment equation would be biased and inconsistent due to endogeneity.
2. **Need for Alternative Methods:** Estimation techniques like **2SLS** or **IV** are more appropriate for addressing this issue.
3. **Simultaneity Validated:** The empirical results strongly support the simultaneous relationship between investment and profits.

Model Fit

- **High R-squared (0.9659):** The model explains a substantial portion of the variation in investment.
- **Significant F-statistic:** Confirms the overall goodness of fit for the model.

d.

The significant differences between **OLS** and **2SLS** estimates confirm the presence of **endogeneity** in the investment equation. Key findings include:

1. **Current Profits (p):**

    - Under 2SLS, the effect of current profits on investment is **much smaller** and statistically **insignificant**, contrasting with OLS results.

2. **Lagged Profits:**
    - The influence of lagged profits is **stronger** under 2SLS compared to OLS, suggesting past profits play a more critical role in investment decisions.

Implications

- The **OLS estimates were biased** due to the simultaneous relationship between investment and profits.
- By using instrumental variables, the **2SLS method corrected this bias**, providing more reliable results.
- The findings suggest that **investment decisions are driven more by historical performance (lagged profits)** than by current profits, which aligns with economic intuition, as investment planning typically relies on past trends rather than immediate outcomes.

e.

Coefficients: The point estimates are identical between the two models.

Standard Errors: The manual 2SLS approach consistently produces larger standard errors (about 19% higher) compared to the automated approach. This suggests the manual approach might be less efficient in its estimation.

The differences observed are likely due to how the **standard errors** are calculated in each approach. The automated 2SLS implementation in the ivreg function might use more efficient methods for computing standard errors, possibly accounting for heteroskedasticity or using different degrees of freedom adjustments.

These findings highlight the importance of using specialized software for 2SLS estimation rather than manually implementing the procedure, as the specialized software may incorporate refinements that lead to more efficient estimates and more accurate inference. While the **point estimates** are identical, the inference drawn from them could differ, especially in borderline cases of statistical significance.

Part (f):

Sargan Test Results Summary

The Sargan test for instrument validity yields:

- Test statistic (TR^2): 1.2815
- Critical value ($\chi^2_{4,0.95}$): 9.4877
- p-value: 0.8645

We fail to reject the null hypothesis of valid instruments. The R^2 is very low (0.061) and none of the instruments are statistically significant in the residual regression (all p-values > 0.05). This confirms that the surplus instruments used in the 2SLS estimation appear to be valid for the investment equation.