

15.17

The data file *liquor* contains observations on annual expenditure on liquor (*LIQUOR*) and annual income (*INCOME*) (both in thousands of dollars) for 40 randomly selected households for three consecutive years.

- b. Estimate the model  $LIQUOR_{it} = \beta_1 + \beta_2 INCOME_{it} + u_i + e_{it}$  using random effects. Construct a 95% interval estimate of the coefficient on INCOME. How does it compare to the interval in part (a)?

Ans.

$$\widehat{LIQUOR}_{it} = 0.96903 + 0.02658 \cdot INCOME_{it}$$

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Oneway (individual) effect Random Effect Model
(Swamy-Arora's transformation)

Call:
plm(formula = liquor ~ income, data = pdata, model = "random")

Balanced Panel: n = 40, T = 3, N = 120

Effects:
              var std.dev share
idiosyncratic 0.9640  0.9819 0.571
individual    0.7251  0.8515 0.429
theta: 0.4459

Residuals:
      Min.   1st Qu.   Median   3rd Qu.   Max.
-2.263634 -0.697383  0.078697  0.552680  2.225798

Coefficients:
              Estimate Std. Error z-value Pr(>|z|)
(Intercept) 0.9690324  0.5210052  1.8599 0.0628957 .
income       0.0265755  0.0070126  3.7897 0.0001508 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Total Sum of Squares: 126.61
Residual Sum of Squares: 112.88
R-Squared: 0.1085
Adj. R-Squared: 0.10095
Chisq: 14.3618 on 1 DF, p-value: 0.00015083
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|             | 2.5 %       | 97.5 %     |
|-------------|-------------|------------|
| (Intercept) | -0.05211904 | 1.99018381 |
| income      | 0.01283111  | 0.04031983 |

使用 random effects model 進行估計後，結果顯示 income 對 liquor 支出具有正向且統計上顯著的影響 (估計值為 0.0266，p 值 < 0.001)。每當 household 年收入增加 \$1,000，預期酒類支出會增加 \$26.6 元。INCOME 95% 信賴區間 [0.0128, 0.0403] 不包含 0，結果顯著，因此可以拒絕虛無假設，認為收入變化會顯著影響酒類支出。

Recall: (a) 的結果是，信賴區間涵蓋了 0，結果不顯著，因此無法拒絕虛無假設，無法說明收入變化顯著影響酒類支出。因此，(a) 與 (b) 得出相反的推論。

- c. Test for the presence of random effects using the LM statistic in equation (15.35). Use the 5% level of significance.

Ans. LM test

$$H_0: \sigma_u^2 = 0 \text{ (No random effects)} \quad H_1: \sigma_u^2 > 0 \text{ (Random effects exist)}$$

LM 統計量 (Chi-squared) 為 20.68 > 臨界值 3.84，且 p 值遠小於 0.05 顯著水準。

因此，拒絕虛無假設  $H_0: \sigma_u^2 = 0$ ，表示模型中存在顯著的 random effects。

| Lagrange Multiplier Test - (Breusch-Pagan)  |                            |
|---|----------------------------|
| data: liquor ~ income                       |                            |
| chisq = 20.68, df = 1, p-value = 5.429e-06  | > qchisq(p = 0.95, df = 1) |
| alternative hypothesis: significant effects | [1] 3.841459               |

- d. For each individual, compute the time averages for the variable *INCOME*. Call this variable *INCOMEM*. Estimate the model  $LIQUOR_{it} = \beta_1 + \beta_2 INCOME_{it} + \gamma INCOMEM_i + c_i + e_{it}$  using the random effects estimator. Test the significance of the coefficient  $\gamma$  at the 5% level. Based on this test, what can we conclude about the correlation between the random effect  $u_i$  and *INCOME*? Is it OK to use the random effects estimator for the model in (b)?

Ans. Mundlak test

$H_0: \gamma = 0$  (No endogeneity)     $H_1: \gamma \neq 0$  (Endogeneity)

INCOMEM 的 p 值 = 0.7670 > 0.05，表示我們無法拒絕虛無假設  $H_0: \gamma = 0$

換句話說，「戶別的平均收入與個體隨機效果  $u_i$  之間沒有統計上的顯著相關」。

沒有證據顯示  $u_i$  與 INCOME 有相關性。

根據此檢定結果，使用 random effects 模型是合理的（沒有違反其關鍵假設：隨機效果與自變數無關）。

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Oneway (individual) effect Random Effect Model
(Swamy-Arora's transformation)

Call:
plm(formula = liquor ~ income + INCOMEM, data = pdata_m, model = "random")

Balanced Panel: n = 40, T = 3, N = 120

Effects:
              var std.dev share
idiosyncratic 0.9640  0.9819 0.571
individual    0.7251  0.8515 0.429
theta: 0.4459

Residuals:
      Min.   1st Qu.   Median   3rd Qu.    Max.
-2.300955 -0.703840  0.054992  0.560255  2.257325

Coefficients:
              Estimate Std. Error z-value Pr(>|z|)
(Intercept)  0.9163337  0.5524439  1.6587  0.09718 .
income       0.0207421  0.0209083  0.9921  0.32117
INCOMEM      0.0065792  0.0222048  0.2963  0.76700
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Total Sum of Squares: 126.61
Residual Sum of Squares: 112.79
R-Squared: 0.10917
Adj. R-Squared: 0.093945
Chisq: 14.3386 on 2 DF, p-value: 0.00076987
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