11.16 Consider the following supply and demand model

$$\text{Demand: } Q_i = \alpha_1 + \alpha_2 P_i + e_{di}, \quad \text{Supply: } Q_i = \beta_1 + \beta_2 P_i + \beta_3 W_i + e_{si}$$

where Q is the quantity, P is the price, and W is the wage rate, which is assumed exogenous. Data on these variables are in Table 11.7.

TABLE 11.7 **Data for Exercise 11.16**

Q	P	W
4	2	2
6	4	3
9	3	1
3	5	1
8	8	3

a. Derive the algebraic form of the reduced-form equations, $Q = \theta_1 + \theta_2 W + v_2$ and $P = \pi_1 + \pi_2 W + v_1$, expressing the reduced-form parameters in terms of the structural parameters.

$$Q = \alpha_1 + \alpha_2 P + e_d = \beta_1 + \beta_2 P + \beta_3 W + e_s$$

$$(\alpha_2 - \beta_2) P = (\beta_1 - \alpha_1) + \beta_3 W + e_s - e_d$$

$$P = \frac{\beta_1 - \alpha_1}{\alpha_2 - \beta_1} + \frac{\beta_3}{\alpha_2 - \beta_2} W + \frac{e_s - e_d}{\alpha_2 - \beta_2}$$

$$= \pi_1 + \pi_2 W + v_1$$

$$Q = \alpha_1 + \alpha_2(\pi_1 + \pi_2 W + v_1) + e_d = (\alpha_1 + \alpha_2 \pi_1) + \alpha_2 \pi_2 W + (\alpha_2 v_1 + e_d)$$

$$= \left(\alpha_1 + \alpha_2 \frac{\beta_1 \alpha_1}{\alpha_2 - \beta_2}\right) + \alpha_2 \frac{\beta_3}{\alpha_2 - \beta_2} W + \left(\alpha_2 \cdot \frac{e_s - e_d}{\alpha_2 - \beta_2} + e_d\right)$$

$$= \frac{\alpha_1 \beta_2 + \alpha_2 \beta_1}{\alpha_2 - \beta_2} + \frac{\alpha_2 \beta_3}{\alpha_2 - \beta_2} W + \frac{\alpha_2 e_s - \beta_2 e_d}{\alpha_2 - \beta_2}$$

$$= \theta_1 + \theta_2 W + v_2$$

b. Which structural parameters can you solve for from the results in part (a)? Which equation is "identified"?

$$\begin{cases} \theta_1 = \alpha_1 + \alpha_2 \pi_1 \\ \theta_2 = \alpha_2 \pi_2 \end{cases} \Rightarrow \text{Solve } \alpha_1, \alpha_2$$

$M = 2 \Rightarrow M-1$ must absent

$Q = \alpha_1 + \alpha_2 P + e_d \Rightarrow W$ is absent $\Rightarrow$ identified.

$Q = \beta_1 + \beta_2 P + \beta_3 W + e_s \Rightarrow$ No one is absent $\Rightarrow$ not identified.

c. The estimated reduced-form equations are $\hat{Q} = 5 + 0.5W$ and $\hat{P} = 2.4 + 1W$. Solve for the identified structural parameters. This is the method of **indirect least squares**.

$\hat{Q} = \underbrace{5}_{\hat{\theta}_1} + \underbrace{0.5}_{\hat{\theta}_2} W \qquad \hat{P} = \underbrace{2.4}_{\hat{\pi}_1} + \underbrace{1}_{\hat{\pi}_2} W$

$\theta_1 = \alpha_1 + \alpha_2 \pi_1 \Rightarrow 5 = \alpha_1 + \alpha_2 \times 2.4$

$\theta_2 = \alpha_2 \pi_2 \Rightarrow 0.5 = \alpha_2 \times 1 \Rightarrow \hat{\alpha}_2 = 0.5 \Rightarrow \hat{\alpha}_1 = 3.8$

d. Obtain the fitted values from the reduced-form equation for P, and apply 2SLS to obtain estimates of the demand equation.

TABLE 11.7 Data for Exercise 11.16

Q	P	W	$\hat{P}_i$	$\hat{P}_i - \bar{\hat{P}}_i$	$Q_i - \bar{Q}_i$
4	2	2	4.4	0	-2
6	4	3	5.4	1	0
9	3	1	3.4	-1	3
3	5	1	3.4	-1	3
8	8	3	5.4	1	2

$\bar{\hat{P}} = 4.4$

$\hat{P} = 2.4 + W$

$Q = \alpha_1 + \alpha_2 \hat{P}_i + e_i$

$$\alpha_2 = \frac{\sum(\hat{P}_i - \bar{\hat{P}}_i)(Q_i - \bar{Q}_i)}{\sum(\hat{P}_i - \bar{\hat{P}})^2} = \frac{2}{4} = \frac{1}{2}$$

$\alpha_1 = \bar{Q} - \alpha_2 \bar{P} = 6 - 0.5 \times 4.4 = 3.8$

$\Rightarrow \hat{Q} = 3.8 + 0.5\hat{P}$