CH5 Q6

(a) $A = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, $c = \beta_2 = 0$, $b = \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}$ $df = 63 - 3 = 60$

$$t_0 = \frac{A^T b - c}{\sqrt{A^T \widehat{Cov}(b) A}} = \frac{3 - 0}{\sqrt{[0,1,0]\begin{bmatrix} 3 & -2 & 1 \\ -2 & 4 & 0 \\ 1 & 0 & 3 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}}} = \frac{3}{\sqrt{4}} = 1.5$$

$t_{0.05}(60) \approx 2$, $t_0 = 1.5 < 2$, Do not reject H_0

(b) $A = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$, $c = 5$, $b = \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}$

$$t_0 = \frac{A^T b - c}{\sqrt{A^T \widehat{Cov}(b) A}} = \frac{8 - 5}{\sqrt{[1,2,0]\begin{bmatrix} 3 & -2 & 1 \\ -2 & 4 & 0 \\ 1 & 0 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}}} = \frac{3}{\sqrt{11}}$$

$t_0 = \frac{3}{\sqrt{11}} \approx 0.905 < 2$, Do not reject H_0

(c) $A = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$, $c = 4$, $b = \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}$

$$t_0 = \frac{A^T b - c}{\sqrt{A^T \widehat{Cov}(b) A}} = \frac{-2 - 4}{\sqrt{[1,-1,1]\begin{bmatrix} 3 & -2 & 1 \\ -2 & 4 & 0 \\ 1 & 0 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}}} = \frac{-6}{\sqrt{16}} = -1.5$$

$t_0 = -1.5 > -2$, Do not reject H_0

CH5 Q 31

(a)

```
Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept)  20.8701     1.6758  12.454  < 2e-16 ***
depart        0.3681     0.0351  10.487  < 2e-16 ***
reds          1.5219     0.1850   8.225 1.15e-14 ***
trains        3.0237     0.6340   4.769 3.18e-06 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 6.299 on 245 degrees of freedom
Multiple R-squared:  0.5346,    Adjusted R-squared:  0.5289
F-statistic: 93.79 on 3 and 245 DF,  p-value: < 2.2e-16
```

β_1: Bill 在 6:30 出門且沒遇到紅燈 or 火車時需花費 20.8701 分鐘

β_2: 每晚出發 1 分鐘，通勤時間平均多 0.3681 分鐘.

β_3: 每遇到 1 個紅燈，〃 〃 1.5219 分鐘

β_4: 每等 1 班火車，〃 〃 3.0237 分鐘

(b)

```
> confint(model, level = 0.95)
                 2.5 %    97.5 %
(Intercept) 17.5694018 24.170871
depart       0.2989851  0.437265
reds         1.1574748  1.886411
trains       1.7748867  4.272505
```

信賴區間越窄越準確

(c)

$H_0: \beta_3 \geq 2$

$H_1: \beta_2 < 2$

$\alpha = 0.05$. $df = 245$. $t_{0.05}(245) \approx 1.65$

$$t = \frac{1.5219 - 2}{0.185} = -2.58$$

$-2.58 < -1.65$. Reject H_0

(d)

$H_0: \beta_4 = 3$

$H_1: \beta_4 \neq 3$

$\alpha = 0.1$, $t_{0.05}(241) \approx 1.65$

$$t = \frac{3.0237 - 3}{0.634} = 0.037$$

$0.037 < 1.65$, do not reject H_0

(e)

$H_0: 30\beta_2 \geq 10$

$H_1: 30\beta_2 < 10$

$\alpha = 0.05$, $t_{0.05}(241) \approx 1.65$

$$t = \frac{30 \times 0.3681 - 10}{30 \times 0.0351} = 0.9912$$

$0.9912 > -1.65$, do not reject H_0

(f)

$H_0: \beta_4 - 3\beta_3 \geq 0$

$H_1: \beta_4 - 3\beta_3 < 0$

$\alpha = 0.05$.

$$t = \frac{3.0237 - 3 \times 1.5219}{0.845} = -1.825$$

$$SE(\beta_4 - 3\beta_3) = \sqrt{Var(\beta_4) + 9\,Var(\beta_3) - 2 \times 3\,Cov(\beta_3, \beta_4)} = 0.845$$

$-1.825 < -1.65$, Reject H_0

(g) $H_0: E(TIME) \leq 45$

$H_1: E(TIME) > 45$

$\alpha = 0.05$

$time = 20.8701 + 0.3681 \times 30 + 1.5219 \times 6 + 3.0237 = 44.0682$

$SE(TIME) = 0.539$

$$t = \frac{44.0682 - 45}{0.539} = -1.726$$

$-1.726 < 1.65$. do not reject H_0

(h) $H_0: E(TIME) \geq 45$

$H_1: E(TIME) < 45$

$\alpha = 0.05$

$t = -1.726$

$-1.726 < -1.65$. Reject H_0 . 有較低的信心保證不遲到

CH5 Q33

(a) 若 p 值低於 α 則不顯著

而係數中最大的 p 值為 $EDUC^2$ 的 0.1149

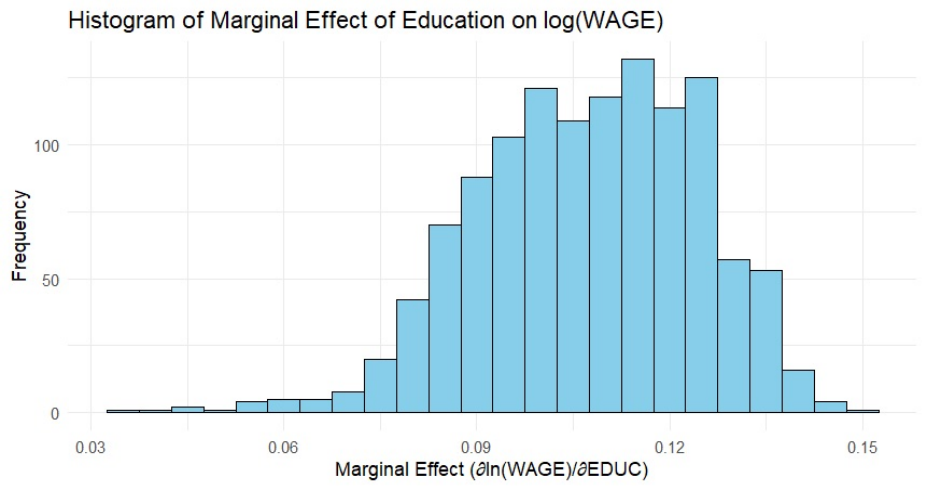
所以在 $\alpha > 0.1149$ 時. 所有係數均顯著異於 0

(b) $$\frac{\partial \ln(wage)}{\partial educ} = \beta_2 + 2 \times \beta_3 \times EDUC + \beta_6 \times EXPER$$

因為 $\beta_3 = 0.00146$ 為正. 所以隨著 EDUC 增加. EDUC 的邊際效應遞增

因為 $\beta_6 = -0.0001$ 為負. 所以隨著 EXPER 增加. EDUC 的邊際效應減少

(c)



5 th 百分位數： 0.08008

中位數： 0.10843

95 th 百分位數： 0.13362

可以觀察到教育的邊際效應為正且左偏

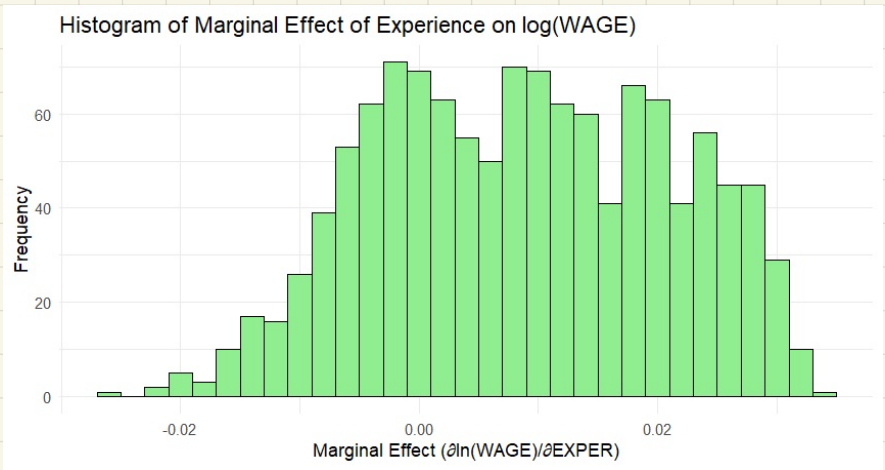
(d)

$$\frac{\partial \ln(wage)}{\partial EXPER} = 0.04488 - 0.000936\,EXPER - 0.0010\,EDUC$$

因為為負，隨 EXPER 增加，邊際效應遞減

因為為負，隨 EDUC 增加，EXPER 邊際效應減少

(e)



5 th 百分位數： -0.01038

中位數： 0.00842

95 th 百分位數： 0.02193

(f) $H_0: E(\ln(wage)|S) - E(\ln(wage)|D) \geq 0$

$H_1: E(\ln(wage)|S) - E(\ln(wage)|D) < 0$

$E(\ln(wage)|S) = 3.209$

$E(\ln(wage)|D) = 3.194$

$$t = \frac{3.209 - 3.194}{0.02149} = 1.6699$$

$1.6699 > -1.65$, do not reject H_0

(g)

$H_0: E(\ln(wage)|S) - E(\ln(wage)|D) \geq 0$

$H_1: E(\ln(wage)|S) - E(\ln(wage)|D) < 0$

$\alpha = 0.05$

$$t = \frac{-0.03092}{0.01499} = -2.0624$$

$-2.0624 < -1.65$, Reject H_0. 8年後 David 工資大於 Svetlana 有信心說

(h) $H_0: 12\beta_5 - 4\beta_6 = 0 \Rightarrow 3\beta_5 - \beta_6 = 0$

$H_1: 3\beta_5 - \beta_6 \neq 0$

$\alpha = 0.05$

$$t = \frac{-0.00158}{0.00153} = -1.0293$$

$-1.0293 > -1.98$, do not reject H_0. 並無顯著差異

(i) $\frac{\partial E[\ln(wage)]}{\partial EXPER} = \beta_3 + 2\beta_4 x + \beta_5 \times 16 = 0$

$x = \frac{-(\beta_3 + 16\beta_5)}{2\beta_4} = \frac{-0.02871}{-0.000936} = 30.7$, $30.7 - 11 = 19.7$ yr

$h(\beta) = \frac{-\beta_3 - 16\beta_5}{2\beta_4}$, $\frac{\partial h}{\partial \beta_3} = \frac{-1}{2\beta_4}$, $\frac{\partial h}{\partial \beta_4} = \frac{\beta_3 + 16\beta_5}{2\beta_4^2}$, $\frac{\partial h}{\partial \beta_5} = \frac{-16}{2\beta_4}$

$Var(x) = \left(\frac{-1}{2\beta_4}\right)^2 Var(\beta_3) + \left(\frac{\beta_3 + 16\beta_5}{2\beta_4^2}\right)^2 Var(\beta_4) + \left(\frac{-16}{2\beta_4}\right)^2 Var(\beta_5)$

$+ 2\left[\left(\frac{-1}{2\beta_4}\right)\left(\frac{\beta_3 + 16\beta_5}{2\beta_4^2}\right) Cov(\beta_3, \beta_4) + \left(\frac{-1}{2\beta_4}\right)\left(\frac{-16}{2\beta_4}\right) Cov(\beta_3, \beta_5) + \left(\frac{\beta_3 + 16\beta_5}{2\beta_4^2}\right)\left(\frac{-16}{2\beta_4}\right) Cov(\beta_4, \beta_5)\right]$

$\Rightarrow 1.896$

$19.7 \pm 1.896 \times 1.96 = [15.958, 23.396]$