5.6 Suppose that, from a sample of 63 observations, the least squares estimates and the corresponding estimated covariance matrix are given by

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix} \qquad \widehat{\text{cov}}(b_1, b_2, b_3) = \begin{bmatrix} 3 & -2 & 1 \\ -2 & 4 & 0 \\ 1 & 0 & 3 \end{bmatrix}$$

Using a 5% significance level, and an alternative hypothesis that the equality does not hold, test each of the following null hypotheses:

a. $\beta_2 = 0$
b. $\beta_1 + 2\beta_2 = 5$
c. $\beta_1 - \beta_2 + \beta_3 = 4$

α = 0.05, df = n - k = 63 - 3 = 60

a. H0: β2 = 0 H1: β2 ≠ 0
t = (b2-0) / se(b2) = 3 / √4 = 3 / 2 = 1.5
For α = 0.05, the critical t-value (t_critical) for a two-tailed test with 60 degrees of freedom is approximately ±2.0.
| t | = 1.5 < 2.0.

```
> qt(1-0.05/2,60)
[1] 2.000298
```

We fail to reject H0 because the t-statistic is not in the rejection region.
Conclusion: There's not enough evidence at the 5% level to say β2 ≠ 0.

b. H0: β1 + 2×β2 = 5 H1: β1 + 2×β2 ≠ 5
Var(b1 + 2×b2) = 1^2×Var(b1) + 2^2×Var(b2) + 2×1×2×Cov(b1, b2)
= 1^2×3 + 2^2×4 + 2×1×2×(-2) = 11
t = ((b1 + 2×b2)-5)) / se(b1 + 2×b2) = ((2 + 2×3) - 5) / √11
= (8 - 5) / √11 = 3 / √11 = 0.905
For α = 0.05, the critical t-value (t_critical) for a two-tailed test with 60 degrees of freedom is approximately ±2.0.
| t | = 0.905 < 2.0.

```
> qt(1-0.05/2,60)
[1] 2.000298
```

We fail to reject H0 because the t-statistic is not in the rejection region.
Conclusion: No evidence at 5% level that β1 + 2×β2 ≠ 5.

c. H0: β1 - β2 + β3 = 4 H1: β1 - β2 + β3 ≠ 4
Var(b1 - b2 + b3) = 1^2×3 + $(-1)^2$×4 + 1^2×3 + 2×1×(-1)×(-2) + 2×1×1×1 + 2×(-1)×1×0
= 3 + 4 + 3 + 4 + 2 + 0 = 16
t = ((b1 - b2 + b3)-4)) / se(b1 - b2 + b3) = ((2 - 3 + (-1)) - 4) / √16
= ((-2) - 4) / √16 = (-6) / √16 = -1.5
For α = 0.05, the critical t-value (t_critical) for a two-tailed test with 60 degrees of freedom is approximately ±2.0.
| t | = 1.5 < 2.0.

```
> qt(1-0.05/2,60)
[1] 2.000298
```

We fail to reject H0 because the t-statistic is not in the rejection region.
Conclusion: No evidence at 5% level that β1 - β2 + β3 ≠ 4.