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HW0414

8.6

a.

```
> # Output results
> cat("Variance estimate for males:", sigma2_males, "\n")
Variance estimate for males: 169.567
> cat("Variance estimate for females:", sigma2_females, "\n")
Variance estimate for females: 144.5766
> cat("F-statistic:", f_stat, "\n")
F-statistic: 1.172853
> cat("Upper critical F-value:", f_crit_upper, "\n")
Upper critical F-value: 1.196781
> cat("Lower critical F-value:", f_crit_lower, "\n")
Lower critical F-value: 0.837669
> cat("p-value:", p_value, "\n")
p-value: 0.08181964
>
> if (f_stat > f_crit_upper || f_stat < f_crit_lower) {
+   cat("Reject the null hypothesis: Variances are different.\n")
+ } else {
+   cat("Fail to reject the null hypothesis: Variances are equal.\n")
+ }
Fail to reject the null hypothesis: Variances are equal.
```

b.

```
> # Output results
> cat("Variance estimate for single individuals:", sigma2_single, "\n")
Variance estimate for single individuals: 142.3571
> cat("Variance estimate for married individuals:", sigma2_married, "\n")
Variance estimate for married individuals: 169.2488
> cat("F-statistic:", f_stat, "\n")
F-statistic: 1.188904
> cat("Critical F-value (one-sided):", f_crit, "\n")
Critical F-value (one-sided): 1.164705
> cat("p-value:", p_value, "\n")
p-value: 0.03102054
>
> if (f_stat > f_crit) {
+   cat("Reject the null hypothesis: Married individuals have greater variance.\n")
+ } else {
+   cat("Fail to reject the null hypothesis: No evidence that married individuals have greater variance.\n")
+ }
Reject the null hypothesis: Married individuals have greater variance.
```

c.

Yes, the Breusch-Pagan test results offer supporting evidence for the issue discussed in part (b) regarding differing error variances between married and unmarried individuals. When considered together, the two tests provide a fuller understanding:

The Goldfeld-Quandt test specifically indicates that marital status is linked to differences in error variances, with married individuals showing greater variance.

The Breusch-Pagan test verifies the presence of heteroskedasticity in the overall model, which aligns with and reinforces the specific observation related to marital status. Combined, these results enhance our confidence in concluding that wages are more variable for married individuals compared to singles. The hypothesis that married individuals—who may rely on spousal support—can pursue a broader range of employment opportunities, leading to greater wage variability, is supported by both tests. For accurate statistical inference in modeling wages, these findings imply that we should apply heteroskedasticity-robust standard errors or use weighted least squares to adjust for the unequal variances across groups.

```
> cat("NR^2 statistic:", NR2, "\n")
NR^2 statistic: 59.03
> cat("Degrees of freedom:", df, "\n")
Degrees of freedom: 4
> cat("Critical chi-squared value:", chi_squared_critical, "\n")
Critical chi-squared value: 9.487729
> cat("P-value:", p_value, "\n")
P-value: 4.637846e-12
>
> if (NR2 > chi_squared_critical) {
+   cat("Reject the null hypothesis: Evidence of heteroskedasticity.\n")
+ } else {
+   cat("Fail to reject the null hypothesis: No evidence of heteroskedasticity.\n")
+ }
Reject the null hypothesis: Evidence of heteroskedasticity.
```

d.

The auxiliary regression for the White test includes the following components:

The original explanatory variables (excluding the constant term):

• EDUC, EXPER, METRO, FEMALE (4 terms)

The squares of each explanatory variable:

• $EDUC^2$, $EXPER^2$, $METRO^2$, $FEMALE^2$ (4 terms)

Cross-products between all pairs of explanatory variables:

• EDUC×EXPER, EDUC×METRO, EDUC×FEMALE

• EXPER×METRO, EXPER×FEMALE

• METRO×FEMALE

• Total cross-product terms: 6

Thus, the total degrees of freedom for the test equal 4 (original variables) + 4 (squared terms) + 6 (interaction terms) = 14.

The results from the White test provide strong evidence of heteroskedasticity in the wage equation model. This outcome is in line with earlier findings:

The Goldfeld-Quandt test in part (b) indicated that married individuals exhibit greater wage variance than singles.

The Breusch-Pagan test in part (c) also detected the presence of heteroskedasticity.

The White test result is particularly important because it is the most flexible among the three tests—it can detect a wide range of heteroskedasticity forms without needing to specify the source in advance.

Given the confirmation of heteroskedasticity, standard OLS inference is unreliable. To achieve valid statistical inference, we should:

Use heteroskedasticity-robust standard errors (White's correction),

Apply weighted least squares (WLS) estimation, or

Transform the variables to stabilize the variance.

Implementing these methods would lead to more accurate and trustworthy results when estimating the wage model.

```
> # Calculate p-value
> p_value <- 1 - pchisq(78.82, df = 14)
> print(p_value)
[1] 4.680689e-11
```

e.

Coefficients with Narrower Confidence Intervals:

EXPER: The robust standard error (0.029) is slightly smaller than the conventional standard error (0.031).

METRO: The robust standard error (0.84) is notably smaller than the usual standard error (1.05).

FEMALE: The robust standard error (0.80) is marginally smaller than the usual standard error (0.81).

Coefficients with Wider Confidence Intervals:

Intercept: The robust standard error (2.50) exceeds the usual standard error (2.36).

EDUC: The robust standard error (0.16) is larger than the usual standard error (0.14).

There is no contradiction in these results. The discrepancies between the conventional and robust standard errors reflect how heteroskedasticity affects the data. Under heteroskedasticity, OLS standard errors can either underestimate or overestimate the true variability depending on the relationship between the error variance and the regressors:

For coefficients with wider robust intervals (Intercept, EDUC): This suggests that error variances are smaller for extreme values of these variables than would be assumed under homoskedasticity. Hence, usual standard errors understate the true variability.

For coefficients with narrower robust intervals (EXPER, METRO, FEMALE): This indicates that error variances are larger for extreme values, meaning that conventional standard errors overstate variability.

These results are consistent with our earlier findings of heteroskedasticity in the model. By correcting for unequal error variances, robust standard errors offer more reliable inference.

f.

A t-statistic around 1.0 for the MARRIED coefficient implies that the estimated effect is roughly one standard error away from zero, and thus, not statistically significant under typical thresholds (e.g., $|t| > 1.96$ for 5% significance).

This outcome is fully compatible with the heteroskedasticity findings discussed in part (b) for several reasons:

Impact on Mean vs. Impact on Variance:

The t-value evaluates whether marital status influences the mean wage.

The heteroskedasticity tests investigated whether marital status affects the variance of wages.

These are distinct dimensions of the relationship.

Statistical Interpretation:

A variable can have no meaningful effect on the mean (low t-value) while having a significant impact on the variance.

The Goldfeld-Quandt test earlier confirmed that wage variability is significantly greater among married individuals.

Hypothesis Support:

Our earlier hypothesis stated that "married individuals, due to spousal support, might pursue a wider range of jobs, resulting in more wage variability."

The non-significant mean effect alongside significant variance differences perfectly supports this idea: marriage doesn't necessarily raise or lower average wages but does make wages more dispersed.

Practical Implication: The findings indicate that marriage does not systematically increase or decrease wage levels, but it does lead to greater wage dispersion. This aligns with the notion that spousal support allows married individuals to explore a broader set of employment opportunities, leading to more varied wage outcomes.

Conclusion:

The analysis of robust standard errors provides valuable insights into the wage equation:

Heteroskedasticity Evidence: Differences between usual and robust standard errors reinforce the presence of heteroskedasticity, with some variables exhibiting narrower and others wider intervals under robust estimation.

Effect of Marital Status: Marital status does not significantly influence mean wages (low t-value) but significantly affects wage variability, validating the earlier hypothesis.

Appropriate Statistical Methods: Given the consistent evidence from the Goldfeld-Quandt, Breusch-Pagan, and White tests, the use of heteroskedasticity-robust standard errors is fully warranted.

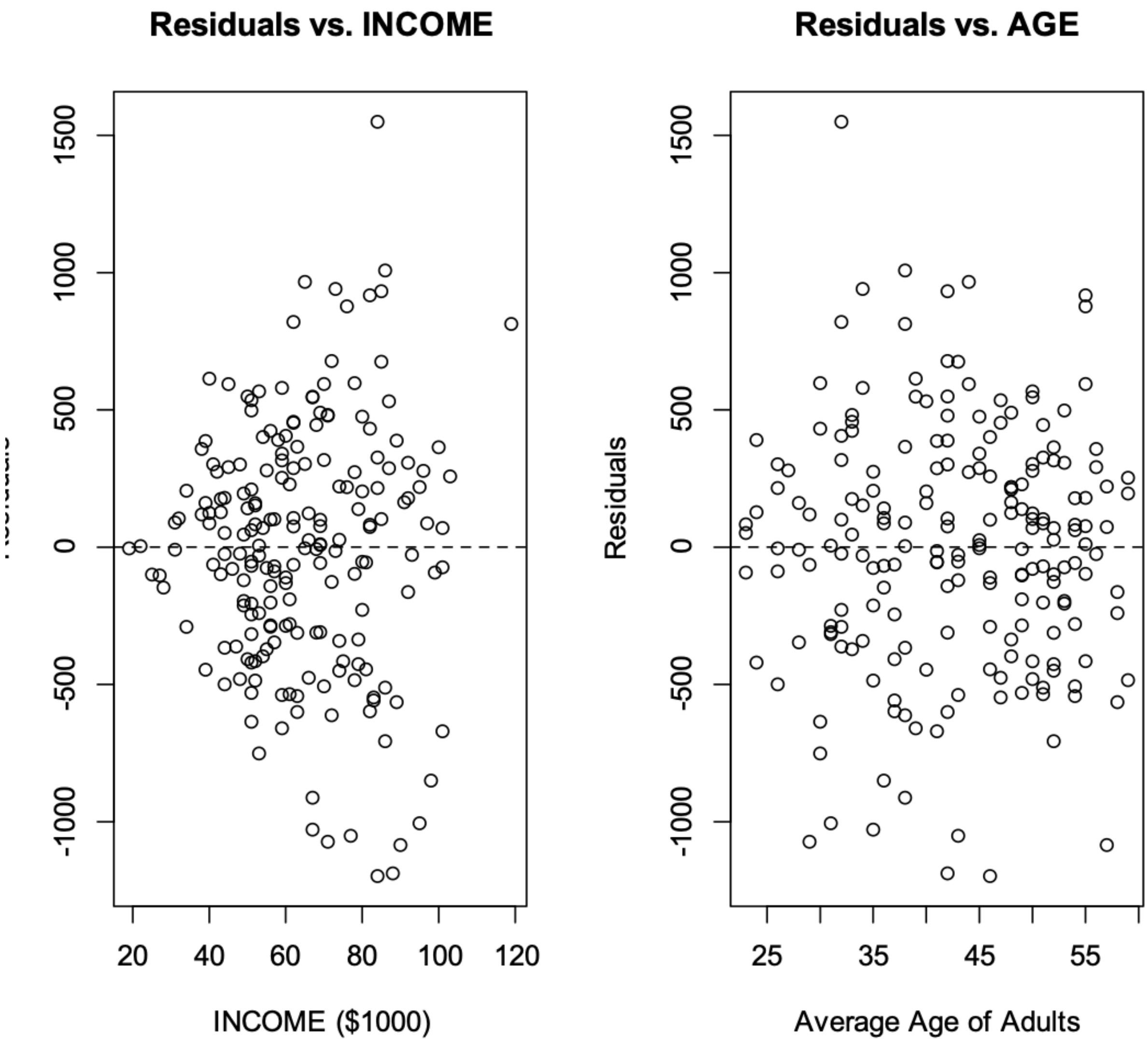
Overall, the findings underscore the importance of examining both average effects and variance effects in economic modeling, as variables may influence these two aspects differently. The non-significant effect of marital status on mean wages complements, rather than contradicts, the significant impact on wage variability.

18.6

a.

```
lm(formula = MILES ~ INCOME + AGE + KIDS, data = vacation)

Residuals:
     Min       1Q   Median       3Q      Max 
-1198.14  -295.31    17.98   287.54  1549.41 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) -391.548    169.775  -2.306   0.0221 *  
INCOME        14.201      1.800   7.889 2.10e-13 ***
AGE           15.741      3.757   4.189 4.23e-05 ***
KIDS         -81.826     27.130  -3.016   0.0029 ** 
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 452.3 on 196 degrees of freedom
Multiple R-squared:  0.3406,	Adjusted R-squared:  0.3305 
F-statistic: 33.75 on 3 and 196 DF,  p-value: < 2.2e-16

> 
> # Construct 95% confidence interval for the coefficient of KIDS
> confint(model_ols, "KIDS", level = 0.95)
         2.5 %    97.5 %
KIDS -135.3298 -28.32302
```

b.



The Residuals vs. AGE plot does not reveal any clear signs of heteroskedasticity. The spread of residuals seems relatively constant across different AGE values.

Similarly, the Residuals vs. INCOME plot shows no strong visual indication of heteroskedasticity. Although the spread is not perfectly uniform, there is no obvious fanning pattern that would strongly suggest a variance problem associated with INCOME.

c.

```
Goldfeld-Quandt F-statistic: 3.104061
> cat("Critical F-value (5%):", f_critical, "\n")
Critical F-value (5%): 1.428617
> cat("p-value:", p_value, "\n")
p-value: 1.64001e-07
>
> # Decision
> if (p_value < 0.05) {
+   cat("Reject the null hypothesis: Evidence of heteroskedasticity.\n")
+ } else {
+   cat("Fail to reject the null hypothesis: No evidence of heteroskedasticity.\n")
+ }
Reject the null hypothesis: Evidence of heteroskedasticity.
>
```

d.

```
             Estimate Std. Error t value  Pr(>|t|)
(Intercept) -391.5480   142.6548 -2.7447 0.0066190 **
INCOME        14.2013     1.9389  7.3246 6.083e-12 ***
AGE           15.7409     3.9657  3.9692 0.0001011 ***
KIDS         -81.8264    29.1544 -2.8067 0.0055112 **
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

>
> # Calculate 95% confidence interval for KIDS using robust standard errors
> kids_coef <- coef(model_ols)["KIDS"]
> kids_robust_se <- robust_se["KIDS", "Std. Error"]
> t_critical <- qt(0.975, df = model_ols$df.residual)  # Using t-distribution
>
> kids_robust_ci <- c(kids_coef - t_critical * kids_robust_se,
+                     kids_coef + t_critical * kids_robust_se)
>
> cat("OLS Coefficient for KIDS:", kids_coef, "\n")
OLS Coefficient for KIDS: -81.82642
> cat("Robust 95% CI for KIDS:", kids_robust_ci[1], "to", kids_robust_ci[2], "\n")
Robust 95% CI for KIDS: -139.323 to -24.32986
>
> # Compare with original confidence interval
> kids_original_ci <- confint(model_ols, "KIDS", level = 0.95)
> cat("Original 95% CI for KIDS:", kids_original_ci[1], "to", kids_original_ci[2], "\n")
Original 95% CI for KIDS: -135.3298 to -28.32302
```

After adjusting for heteroskedasticity by using robust standard errors, the confidence intervals around the estimates become wider, which is consistent with the presence of heteroskedasticity in the residuals.

e.

Both Generalized Least Squares (GLS) confidence intervals are narrower than their Ordinary Least Squares (OLS) counterparts, as GLS effectively down-weights the heteroskedasticity associated with INCOME. The robust GLS confidence interval (–121.41, –32.20) is slightly wider than the conventional GLS interval (–119.90, –33.72), yet it remains significantly tighter than the robust OLS interval (–139.32, –24.33). In all cases, the intervals lie entirely below zero, indicating a statistically significant negative effect of having an additional child on miles traveled.

```
> # GLS confidence interval for KIDS (conventional)
> kids_gls_coef <- coef(model_gls)["KIDS"]
> kids_gls_se <- summary_gls$coefficients["KIDS", "Std. Error"]
> t_critical <- qt(0.975, df = model_gls$df.residual)
>
> kids_gls_ci <- c(kids_gls_coef - t_critical * kids_gls_se,
+                  kids_gls_coef + t_critical * kids_gls_se)
>
> cat("GLS Coefficient for KIDS:", kids_gls_coef, "\n")
GLS Coefficient for KIDS: -76.80629
> cat("GLS 95% CI for KIDS:", kids_gls_ci[1], "to", kids_gls_ci[2], "\n")
GLS 95% CI for KIDS: -119.8945 to -33.71808
>
> # Robust GLS standard errors
> robust_se_gls <- coeftest(model_gls, vcov = vcovHC(model_gls, type = "HC1"))
> kids_robust_gls_se <- robust_se_gls["KIDS", "Std. Error"]
>
> # Robust GLS confidence interval
> kids_robust_gls_ci <- c(kids_gls_coef - t_critical * kids_robust_gls_se,
+                         kids_gls_coef + t_critical * kids_robust_gls_se)
>
> cat("Robust GLS 95% CI for KIDS:", kids_robust_gls_ci[1], "to", kids_robust_gls_ci[2], "\n")
Robust GLS 95% CI for KIDS: -121.4134 to -32.19919
```

8.18

a.

```
> # Output results
> cat("Variance estimate for males:", var_males, "\n")
Variance estimate for males: 0.2207836
> cat("Variance estimate for females:", var_females, "\n")
Variance estimate for females: 0.2101181
> cat("F-statistic:", f_stat, "\n")
F-statistic: 1.05076
> cat("Upper critical F-value:", f_crit_upper, "\n")
Upper critical F-value: 1.058097
> cat("Lower critical F-value:", f_crit_lower, "\n")
Lower critical F-value: 0.9452566
> cat("p-value:", p_value, "\n")
p-value: 0.08569168
>
> if (f_stat > f_crit_upper || f_stat < f_crit_lower) {
+   cat("Reject the null hypothesis: Variances differ between males and females.\n")
+ } else {
+   cat("Fail to reject the null hypothesis: No evidence of different variances.\n")
+ }
Fail to reject the null hypothesis: No evidence of different variances.
```

b.

```
> # Method 1: Using bptest function
> bp_test1 <- bptest(full_model, ~ METRO + FEMALE + BLACK, data = cps5)
> print(bp_test1)

    studentized Breusch-Pagan test

data:  full_model
BP = 23.557, df = 3, p-value = 3.091e-05
```

```
> # Output results
> cat("NR² statistic (specific variables):", nr_squared1, "\n")
NR² statistic (specific variables): 23.55681
> cat("Critical chi-squared value (1%):", chi_crit1, "\n")
Critical chi-squared value (1%): 11.34487
> cat("p-value:", p_value1, "\n")
p-value: 3.0909e-05
>
> if (p_value1 < 0.01) {
+   cat("Reject the null hypothesis: Evidence of heteroskedasticity at 1% level.\n")
+ } else {
+   cat("Fail to reject the null hypothesis: No evidence of heteroskedasticity at 1% level.\n")
+ }
Reject the null hypothesis: Evidence of heteroskedasticity at 1% level.
```

```
> # Output results
> cat("\nNR² statistic (all variables):", nr_squared2, "\n")

NR² statistic (all variables): 109.4243
> cat("Critical chi-squared value (1%):", chi_crit1, "\n")
Critical chi-squared value (1%): 11.34487
> cat("p-value:", p_value2, "\n")
p-value: 0
>
> if (p_value2 < 0.01) {
+   cat("Reject the null hypothesis: Evidence of heteroskedasticity at 1% level.\n")
+ } else {
+   cat("Fail to reject the null hypothesis: No evidence of heteroskedasticity at 1% level.\n")
+ }
Reject the null hypothesis: Evidence of heteroskedasticity at 1% level.
```

The Breusch-Pagan tests challenge the conclusion in part (a), indicating that even if male and female variances are comparable, heteroskedasticity persists because of other factors.

c.

```
> # Output results
> cat("White test NR² statistic:", white_nr_squared, "\n")
White test NR² statistic: 194.4447
> cat("Degrees of freedom:", white_df, "\n")
Degrees of freedom: 54
> cat("Critical chi-squared value (5%):", white_crit, "\n")
Critical chi-squared value (5%): 72.15322
> cat("p-value:", white_p_value, "\n")
p-value: 0
>
> if (white_p_value < 0.05) {
+   cat("Reject the null hypothesis: Evidence of heteroskedasticity at 5% level.\n")
+ } else {
+   cat("Fail to reject the null hypothesis: No evidence of heteroskedasticity at 5% level.\n")
+ }
Reject the null hypothesis: Evidence of heteroskedasticity at 5% level.
```

d.

```
> print(ci_comparison)
      Variable Conventional_SE    Robust_SE Conv_CI_Lower Conv_CI_Upper
1  (Intercept)    3.211489e-02 3.279417e-02  1.1384302204  1.2643338265
2         EDUC    1.758260e-03 1.905821e-03  0.0977830603  0.1046761665
3        EXPER    1.300342e-03 1.314908e-03  0.0270727569  0.0321706349
4       EXPER2    2.635448e-05 2.759687e-05 -0.0004974407 -0.0003941203
5       FEMALE    9.529136e-03 9.488260e-03 -0.1841810529 -0.1468229075
6        BLACK    1.694240e-02 1.609369e-02 -0.1447358548 -0.0783146449
7        METRO    1.230675e-02 1.158215e-02  0.0948966363  0.1431441846
8        SOUTH    1.356134e-02 1.390164e-02 -0.0723384657 -0.0191724010
9      MIDWEST    1.410367e-02 1.372426e-02 -0.0915893895 -0.0362971859
10        WEST    1.440237e-02 1.455684e-02 -0.0348207138  0.0216425095
   Robust_CI_Lower Robust_CI_Upper Width_Change
1      1.137098683    1.2656653641    2.1151700
2      0.097493811    0.1049654160    8.3924256
3      0.027044205    0.0321991870    1.1201599
4     -0.000499876   -0.0003916849    4.7141298
5     -0.184100928   -0.1469030324   -0.4289553
6     -0.143072211   -0.0799782888   -5.0093755
7      0.096316998    0.1417238226   -5.8878100
8     -0.073005508   -0.0185053588    2.5092781
9     -0.090845662   -0.0370409129   -2.6901695
10    -0.035123519    0.0219453146    1.0725746
```

```
Coefficients with wider intervals using robust SE:
> print(wider)
[1] "(Intercept)" "EDUC"        "EXPER"       "EXPER2"      "SOUTH"
[6] "WEST"
>
> cat("\nCoefficients with narrower intervals using robust SE:\n")

Coefficients with narrower intervals using robust SE:
> print(narrower)
[1] "FEMALE"  "BLACK"   "METRO"   "MIDWEST"
```

e.f

```
> # Display results
> print(ci_comparison_fgls)
      Variable      FGLS_SE    Robust_SE FGLS_CI_Lower FGLS_CI_Upper
1  (Intercept) 3.159320e-02 3.279417e-02  1.1302695254  1.2541279001
2         EDUC 1.764615e-03 1.905821e-03  0.0982024458  0.1051204663
3        EXPER 1.297517e-03 1.314908e-03  0.0275467064  0.0326335081
4       EXPER2 2.678918e-05 2.759687e-05 -0.0005086498 -0.0004036251
5       FEMALE 9.480830e-03 9.488260e-03 -0.1847977976 -0.1476290326
6        BLACK 1.699247e-02 1.609369e-02 -0.1441623553 -0.0775448504
7        METRO 1.145945e-02 1.158215e-02  0.0953066354  0.1402324253
8        SOUTH 1.352230e-02 1.390164e-02 -0.0713493481 -0.0183363498
9      MIDWEST 1.398389e-02 1.372426e-02 -0.0906033967 -0.0357807800
10        WEST 1.437651e-02 1.455684e-02 -0.0336747637  0.0226870709
   Robust_CI_Lower Robust_CI_Upper Width_Change
1      1.137098683    1.2656653641  -3.66215144
2      0.097493811    0.1049654160  -7.40917939
3      0.027044205    0.0321991870  -1.32261698
4     -0.000499876   -0.0003916849  -2.92675532
5     -0.184100928   -0.1469030324  -0.07831277
6     -0.143072211   -0.0799782888   5.58466271
7      0.096316998    0.1417238226  -1.05938782
8     -0.073005508   -0.0185053588  -2.72870976
9     -0.090845662   -0.0370409129   1.89177946
10    -0.035123519    0.0219453146  -1.23885273
```

```
> cat("\nCoefficients with wider intervals using FGLS vs. OLS robust:\n")

Coefficients with wider intervals using FGLS vs. OLS robust:
> print(fgls_wider)
[1] "BLACK"   "MIDWEST"
>
> cat("\nCoefficients with narrower intervals using FGLS vs. OLS robust:\n")

Coefficients with narrower intervals using FGLS vs. OLS robust:
> print(fgls_narrower)
[1] "(Intercept)" "EDUC"        "EXPER"       "EXPER2"      "FEMALE"
[6] "METRO"       "SOUTH"       "WEST"
>
```

g.

```
--- Robust FGLS vs. FGLS ---
> cat("Wider under Robust FGLS:\n");    print(wider_rf_vs_f)
Wider under Robust FGLS:
[1] "(Intercept)" "EDUC"        "EXPER"       "EXPER2"      "METRO"
[6] "SOUTH"       "WEST"
> cat("Narrower under Robust FGLS:\n"); print(narrower_rf_vs_f)
Narrower under Robust FGLS:
[1] "FEMALE"  "BLACK"   "MIDWEST"
>
> cat("\n--- Robust FGLS vs. Robust OLS ---\n")

--- Robust FGLS vs. Robust OLS ---
> cat("Wider under Robust FGLS:\n");    print(wider_rf_vs_o)
Wider under Robust FGLS:
character(0)
> cat("Narrower under Robust FGLS:\n"); print(narrower_rf_vs_o)
Narrower under Robust FGLS:
 [1] "(Intercept)" "EDUC"        "EXPER"       "EXPER2"      "FEMALE"
 [6] "BLACK"       "METRO"       "SOUTH"       "MIDWEST"     "WEST"
>
```

We should rely on the Feasible GLS (FGLS) estimates paired with heteroskedasticity-consistent ("robust") standard errors:

We have confirmed the presence of heteroskedasticity (through the White test and NR^2 tests), meaning the conventional standard errors from either OLS or FGLS are invalid. To ensure correct inference, robust standard errors are necessary.

FGLS reweights the observations based on the estimated variance structure, and if the model closely approximates the true variance, it produces more efficient (lower variance) coefficient estimates than OLS. As shown in the table, the FGLS point estimates shift slightly (e.g., intercept from 1.201 to 1.192, EDUC from 0.1012 to 0.1017), and the FGLS robust standard errors are generally slightly smaller than the OLS robust ones.

Using FGLS with robust standard errors gives us the "best of both worlds":

Efficiency gains from FGLS reweighting.

Reliable inference even if the variance model is not perfectly specified

Consequently, in empirical research, it is common practice to report "FGLS with White (or Huber-White) standard errors" as the main specification once heteroskedasticity has been identified.