10.2 The labor supply of married women has been a subject of a great deal of economic research. Consider the following supply equation specification

$$HOURS = \beta_1 + \beta_2 WAGE + \beta_3 EDUC + \beta_4 AGE + \beta_5 KIDSL6 + \beta_6 NWIFEINC + e$$

where *HOURS* is the supply of labor, *WAGE* is hourly wage, *EDUC* is years of education, *KIDSL6* is the number of children in the household who are less than 6 years old, and *NWIFEINC* is household income from sources other than the wife's employment.

a. Discuss the signs you expect for each of the coefficients.

expect $\beta_2 > 0$，工資↑，更願意投入工資時間，與勞動供給正相關

expect $\beta_3 > 0$，教育程度↑，可能有更高薪資的工作，更可能參與勞動市場，工時可能↑

expect β_4 not sure，年齡較大可能工時更長，但可能因家庭或健康問題減少工作時間

expect $\beta_5 < 0$，6歲以下幼兒↑，要負擔更多家務，導致工時↓

expect $\beta_6 < 0$，家庭其他來源收入↑，經濟壓力↓，勞動供給↓

b. Explain why this supply equation cannot be consistently estimated by OLS regression.

∵ WAGE變數具有內生性，可能受到未觀察到的個人特質影響，這些因素同時也會影響被解釋變數(HOURS)。

(WAGE與 e 相關)

c. Suppose we consider the woman's labor market experience *EXPER* and its square, $EXPER^2$, to be instruments for *WAGE*. Explain how these variables satisfy the logic of instrumental variables.

工具變數須滿足兩個條件!

1. 相關性:工具變數與內生變數WAGE有關

2. 外生性:......不能與誤差項e相關

∵ EXPER, $EXPER^2$ 與WAGE高度相關,且可合理假設只影響工資,不直接影響勞動供給決策

d. Is the supply equation identified? Explain.

∵ 工具變數數量:2 > 1:內生變數數量

∴ 乃 identified

e. Describe the steps [not a computer command] you would take to obtain IV/2SLS estimates.

1. 用所有外生變數和工具變數去迴歸內生變數,取得預測值 $\widehat{WAGE}$

2. 在原本的勞動供給方程式中以 $\widehat{WAGE}$ 代替 WAGE,然後用OLS估計這個新方程式

3. 所得估計值即為2SLS的一致估計結果

10.3 In the regression model $y = \beta_1 + \beta_2 x + e$, assume x is endogenous and that z is a valid instrument. In Section 10.3.5, we saw that $\beta_2 = \text{cov}(z, y)/\text{cov}(z, x)$.

a. Divide the denominator of $\beta_2 = \text{cov}(z, y)/\text{cov}(z, x)$ by $\text{var}(z)$. Show that $\text{cov}(z, x)/\text{var}(z)$ is the coefficient of the simple regression with dependent variable x and explanatory variable z, $x = \gamma_1 + \theta_1 z + v$. [*Hint*: See Section 10.2.1.] Note that this is the first-stage equation in two-stage least squares.

$$x = \gamma_1 + \theta_1 z + v$$

$$-)\ E(X) = \gamma_1 + \theta_1 E(Z)$$

$$\Rightarrow x - E(X) = \theta_1 (Z - E(Z)) + v$$

同乘$(Z-E(Z))$

$$\Rightarrow (Z-E(Z))(x-E(X)) = \theta_1 (Z-E(Z))^2 + (Z-E(Z))v$$

取期望值

$$E[(Z-E(Z))(x-E(X))] = \theta_1 E[(Z-E(Z))^2] + E[(Z-E(Z))v]$$

$\because$ By assumption, $\text{cov}(Z, v) = 0$ $(\because E(v)=0)$

$$\text{cov}(z, v) = E[(Z-E(Z))(v-E(v))] = E[(Z-E(Z))v] = 0$$

$$\therefore \theta_1 = \frac{E[(Z-E(Z))(x-E(X))]}{E[(Z-E(Z))^2]} = \frac{\text{cov}(Z, X)}{\text{Var}(Z)}$$

與 OLS 估計迴歸係數公式一致

b. Divide the numerator of $\beta_2 = \text{cov}(z, y)/\text{cov}(z, x)$ by $\text{var}(z)$. Show that $\text{cov}(z, y)/\text{var}(z)$ is the coefficient of a simple regression with dependent variable y and explanatory variable z, $y = \pi_0 + \pi_1 z + u$. [*Hint*: See Section 10.2.1.]

similar to (a)

$$y=\pi_0+\pi_1 z+u$$

$$\to E(y)=\pi_0+\pi_1 E(z)$$

$$\Rightarrow y-E(y)=\pi_1(z-E(z))+u$$

同乘 $(z-E(z))$

$$(z-E(z))(y-E(y))=\pi_1(z-E(z))^2+(z-E(z))u$$

取期望值

$$E[(z-E(z))(y-E(y))]=\pi_1 E(z-E(z))^2+\underbrace{E[(z-E(z))u]}_{=0}$$

$$\Rightarrow \pi_1=\frac{E[(z-E(z))(y-E(y))]}{E(z-E(z))^2}=\frac{cov(z,y)}{Var(z)}$$

與 OLS 估計迴歸係數公式一致

c. In the model $y=\beta_1+\beta_2 x+e$, substitute for x using $x=\gamma_1+\theta_1 z+v$ and simplify to obtain $y=\pi_0+\pi_1 z+u$. What are π_0, π_1, and u in terms of the regression model parameters and error and the first-stage parameters and error? The regression you have obtained is a **reduced-form** equation.

$y=\beta_1+\beta_2 x+e$ (x 用 $r_1+\theta_1 z+v$ 代入)

$$\Rightarrow y=\beta_1+\beta_2(r_1+\theta_1 z+v)+e$$

$$=(\beta_1+\beta_2 r)+\beta_2\theta_1 z+(\beta_2 v+e)$$

$$=\pi_0+\pi_1 z+u$$

$\pi_0=\beta_1+\beta_2 r$, $\pi_1=\beta_2\theta_1$, $u=\beta_2 v+e$

得到 y 只跟 z 有關 fcn, 內生變數 x 被解掉

d. Show that $\beta_2 = \pi_1/\theta_1$.

by (c), $\because \pi_1 = \beta_2\theta_1$, $\beta_2 = \dfrac{\pi_1}{\theta_1}$

e. If $\hat{\pi}_1$ and $\hat{\theta}_1$ are the OLS estimators of π_1 and θ_1, show that $\hat{\beta}_2 = \hat{\pi}_1/\hat{\theta}_1$ is a consistent estimator of $\beta_2 = \pi_1/\theta_1$. The estimator $\hat{\beta}_2 = \hat{\pi}_1/\hat{\theta}_1$ is an **indirect least squares** estimator.

by (a), (b)

$\hat{\theta}_1$是 $x = r_1 + \theta_1 z + v$ 的迴歸係數估計量

$\hat{\theta}_1 = \dfrac{cov(z,x)}{var(z)}$ 且 $\hat{\theta}_1 \xrightarrow{P} \theta_1$ (若z與v不相關)

$\hat{\pi}_1$是 $y = \pi_0 + \pi_1 z + u$ 的迴歸係數估計量

$\hat{\pi}_1 = \dfrac{cov(z,y)}{Var(z)}$ 且 $\hat{\pi}_1 \xrightarrow{P} \pi_1$ (若z與u不相關)

consider $\hat{\beta}_2 = \dfrac{\hat{\pi}_1}{\hat{\theta}_1}$ (Assume $\theta_1 \neq 0$)

$\because \hat{\pi}_1 \xrightarrow{P} \pi_1$, $\hat{\theta}_1 \xrightarrow{P} \theta_1$, $g(a,b) = \dfrac{a}{b}$ is continuous

By continuous mapping Thm

$\therefore \hat{\beta}_2 = \dfrac{\hat{\pi}_1}{\hat{\theta}_1} \xrightarrow{P} \dfrac{\pi_1}{\theta_1} = \beta_2$, 故為 called ILS estimator (間接最小平方估計)

* prior condition $\begin{cases} \theta_1 \neq 0 \text{ (工具變數與內生變數有關)} \\ cov(z,v) = 0 \\ cov(z,u) = 0 \end{cases}$