$\boxed{5.6}$ a. $H_0: \beta_2 = 0$

$H_a: \beta_2 \neq 0$, $df = N - K = 63 - 3 = 60$

$\alpha = 5\% \Rightarrow t_{crit_{0.025/df=60}} = -2.0003$, $t_{cr(0.975,60)} = 2.0003$

$-2.0003 < t = \frac{3-0}{\sqrt{4}} = 1.5 < 2.0003$

$\Rightarrow$ Do not reject H_0. There is insufficient evidence to conclude that the equality does not hold.

b. $H_0: \beta_1 + 2\beta_2 = 5$

$H_a: \beta_1 + 2\beta_2 \neq 5$

$var(\beta_1 + 2\beta_2)$

$= 1^2 var(b_1) + 2^2 var(b_2) + 2(1)(2) cov(b_1, b_2)$

$= 1^2(3) + 2^2(4) + 2(1)(2)(-2)$

$= 11$

$\Rightarrow t = \frac{2 + 2(3) - 5}{\sqrt{11}} = 0.9045 \Rightarrow$ p value $= 0.36933 > \alpha$

$\Rightarrow$ Do not reject H_0. There is insufficient evidence to conclude that the equality does not hold.

c. H_0 $\beta_1 - \beta_2 + \beta_3 = 4$

$H_a: \beta_1 - \beta_2 + \beta_3 \neq 4$

$var(b_1 - b_2 + b_3)$

$= 1^2 var(b_1) + (-1)^2 var(b_2) + 1^2 var(b_3) + 2(1)(-1) cov(b_1, b_2) + 2(1)(1) cov(b_1, b_3) + 2(-1)(1) cov(b_2, b_3)$

$= 1(3) + 1(4) + 1(3) + 2(-1)(-2) + 2(1) + 0$

$= 16$

$\Rightarrow t = \frac{2 - 3 + (-1) - 4}{\sqrt{16}} = -1.5 \Rightarrow$ p value $= 0.1388384 > \alpha$

We do not reject H_0.

There is insufficient evidence to conclude that the equality does not hold.

$\boxed{5.31}$ a. $\widehat{TIME} = \cancel{\beta}\ 20.8701 + 0.3681\ DEPART + 1.5219\ REDS + 3.0237\ TRAINS$

(se) $\quad (1.6758) \quad (0.0351) \quad (0.1850) \quad (0.6340)$

β_1: Bill's expected commute time when he leaves Carnegie at 6:30 AM and encounters no red lights and no trains is estimated to be 20.87 minutes

β_2: If Bill leaves later than 6:30 AM, the increase in his expected traveling time is estimated to be 3.7 minutes for every 10 minutes that his departure time is later than 6:30 AM (constant reds and trains).

β_3: The expected increase in traveling time from each red light, with departure time and trains held constant, is estimated to be 1.52 mins.

β_4: The expected increase in traveling time from each additional train is estimated to be 3.02 mins (constant departure time and reds).

b. 95% CIs: β_1 (17.57, 24.17), β_2 (0.30, 0.44), β_3 (1.16, 1.89)

β_4 (1.77, 4.27)

$\Rightarrow$ In the context of driving time, these intervals are relatively narrow ones. We have obtained precise estimates of each of the coefficents.

c. $H_0: \beta_3 \geq 2$ $\quad t_{cr(0.95, df=245)} = 1.651$, $\quad t = \dfrac{\overset{1.5219}{\cancel{3.0237}} - 2}{\underset{0.1850}{\cancel{0.6340}}}$

$H_a: \beta_3 < 2$

$(df = N - K = 249 - 4 = 245)$ $\quad = -2.5836$

$-2.5836 < -1.651 = t_{cr(0.05, 245)}$, we reject H_0.

There is sufficient evidence to conclude that the expected delay from each red light is less than 2 minutes.

d. $H_0: \beta_4 = 3$ $\quad \alpha = 10\%$, two-tailed $\Rightarrow \begin{cases} t_{cr(0.05, 245)} = -1.651 \\ t_{cr(0.95, 245)} = 1.651 \end{cases}$

$H_a: \beta_4 \neq 3$ $\quad (0.95 = 1 - \alpha/2)$

$t = (3.0237 - 3) / 0.6340$

$= 0.037$

Since $-1.651 < 0.037 < 1.651$, we do not reject H_0. There is insufficient evidence to conclude that the expected delay from each train is not 3 minutes.

e. $H_0: \beta_2 \geq 1/3$ $\quad t_{cr(0.05, 245)} = -1.651 \Rightarrow$ Rejection region: $t \leq -1.651$

$H_a: \beta_2 < 1/3$ $\quad t = (0.3681 - 1/3) / \cancel{1.67576}\ 0.0351 = 0.9912$

$0.9912 > -1.651$, we fail to reject Ho. There is insufficient evidence to conclude that delaying departure time by 30 minutes increases expected travel time by shorter than 10 minutes.

f. $H_0: \beta_4 \geq 3\beta_3$ $\quad t_{cr(0.05, 245)} = -1.651 \Rightarrow$ Rejection region: $t \leq -1.651$

$H_a: \beta_4 < 3\beta_3$ $\quad t = (3.0237 - 3 \times 1.5219) / 0.844992 = -1.825$

$$\left(se(b_4 - 3b_3) = \sqrt{var b_4 + 9 var b_3 - 6 cov(b_4, b_3)} = 0.844992 \right)$$

Since $-1.825 < -1.651$, reject Ho. The expected delay from a train is less than three times the delay from a red light.

g. $H_0: \beta_1 + 30\beta_2 + 6\beta_3 + \beta_4 \leq 45$ $\quad t_{cr(0.95, 245)} = 1.651 \Rightarrow$ Rejection region $t \geq 1.651$

$H_a: \beta_1 + 30\beta_2 + 6\beta_3 + \beta_4 > 45$

$se(\beta_1 + 30\beta_2 + 6\beta_3 + \beta_4)$

$$= \sqrt{\begin{aligned} & var b_1 + 900 var b_2 + 36 var b_3 + var b_4 \\ & + 60 cov(b_1, b_2) + 12 cov(b_1, b_3) + 2 cov(b_1, b_4) \\ & + 360 cov(b_2, b_3) + 60 cov(b_2, b_4) \\ & + 12 cov(b_3, b_4) \end{aligned}}$$

$= 0.5393$

$t = (20.8701 + 30 \times 0.3681 + 6 \times 1.5219 + 3.0237 - 45) / 0.5393$

$= -1.726 < 1.651$

$\Rightarrow$ Fail to reject Ho. There is insufficient ~~There is sufficient~~ evidence to conclude that Bill ~~can~~ will get to the university ~~on or before~~ after 7:45AM

h. Bill should set up the alternative hypothesis of having a commute time as less than 45 minutes because there is a high probability that his commute time will be less than 45 minutes. In (g), failing to reject Ho does not imply ~~does not imply~~ the commute time will necessarily be less than 45 minutes.

Therefore, if we reverse: $H_0: \beta_1 + 30\beta_2 + 6\beta_3 + \beta_4 \geq 45$ $\quad t = -1.726 < -1.651$

$t_{cr(0.05, 245)} = -1.651$ $\quad H_a: \beta_1 + 30\beta_2 + 6\beta_3 + \beta_4 < 45$

$\Rightarrow$ Reject Ho: He can expect to be on time for the meeting.

5.33 a. After runing the regression model in R, I check the p-value of each coefficient: Except for the coefficient on $EDUC^2$ (β_3) with a p-value of 0.114855, which is insignificant, all coefficients are significantly different from zero at the 1% level (p-value < 0.01).

$\Rightarrow$ β_3 is significant at a 12% level.

b. $ME_{EDUC} = \beta_2 + 2\beta_3 EDUC + \beta_6 EXPER$

$\Rightarrow$ Its estimate $= \widehat{ME}_{EDUC} = 0.08954 + 0.002916\, EDUC - 0.00101\, EXPER$

$\Rightarrow$ ME of education increases when EDUC $\uparrow$ and decreases when EXPER $\uparrow$

c. The ME range from 0.036 to 0.148 with most of them concentrated between 0.085 and 0.13. The 5^{th}, 50^{th} (median), and 95^{th} percentiles are 0.080, 0.108, and 0.134, respectively.

d. $ME_{EXPER} = \beta_4 + 2\beta_5 EXPER + \beta_6 EDUC$

$\Rightarrow \widehat{ME}_{EXPER} = 0.04488 - 0.000936\, EXPER - 0.00101\, EDUC$

$\Rightarrow$ ME of experience decreases when EDUC $\uparrow$ ~~and~~ as well as when EXPER $\uparrow$

e. Although most of the ME of experience are positive, there is a large proportion (26.3%) that are negative. Overall, the values range from -0.025 to 0.034. $\widehat{ME}_{(EXPER,\,0.05)} = -0.010$, $\widehat{ME}_{(EXPER,\,0.50)} = 0.008$, $\widehat{ME}_{(EXPER,\,0.95)} = 0.028$

f. $H_0: -\beta_2 - 33\beta_3 + 10\beta_4 + 260\beta_5 + 152\beta_6 \geq 0$ $\quad t_{cr(0.05,\,1194)} = -1.646$

$H_a: -\beta_2 - 33\beta_3 + 10\beta_4 + 260\beta_5 + 152\beta_6 < 0$ $\quad$ (df = 1200 − 6 = 1194) $\Rightarrow$ R.R.: $t \leq -1.646$

(~~David~~ Svetlana − David $= 16\beta_2 - 17\beta_2 + 16^2\beta_3 - 17^2\beta_3 + 18\beta_4 - 8\beta_4 + 18^2\beta_5 - 8^2\beta_5 + 16\times 18\,\beta_6 - 17\times 8\,\beta_6$

$= -\beta_2 - 33\beta_3 + 10\beta_4 + 260\beta_5 + 152\beta_6$)

$t = 0.0359 / 0.0215 = 1.669902 > -1.646$: Fail to reject H_0.

There is insufficient evidence to conclude that David's log-wage is greater.

g. After 8 years: Svetlana − David

$= -\beta_2 - 33\beta_3 + 10\beta_4 + 26^2\beta_5 - 16^2\beta_5 + 26\times 16\beta_6 - 16\times 17\beta_6$

$= -\beta_2 - 33\beta_3 + 10\beta_4 + 420\beta_5 + 144\beta_6$

$H_0: -\beta_2 - 33\beta_3 + 10\beta_4 + 420\beta_5 + 144\beta_6 \geq 0$

$H_a: -\beta_2 - 33\beta_3 + 10\beta_4 + 420\beta_5 + 144\beta_6 < 0$

$t\ \cancel{se} = \dfrac{-b_2 - 33b_3 + 10b_4 + 420b_5 + 144b_6}{se(-b_2 - 33b_3 + 10\beta_4 + 420b_5 + 144b_6)} = \dfrac{-0.0309}{0.0150} = -2.062$

$-2.062 < t_{cr(0.05,\,1194)} = -1.646$: Reject H_0

$\Rightarrow$ Not the same as in part (f). There is sufficient evidence to conclude that David's log-wage is greater.

Svetlana: 18 years of experience $\Rightarrow$ Extra years had a relatively small impact on her log-wage.

David: Only 8 years of experience

$\Rightarrow$ Extra years had a relatively large impact on his log-wage.

h. $ME_{EXPER_{Wendy}} - ME_{EXPER_{Jill}} = 2\beta_5(17-11) + \beta_6(12-16)$

$= 12\beta_5 - 4\beta_6$

$H_0: 12\beta_5 - 4\beta_6 = 0$

$H_a: 12\beta_5 - 4\beta_6 \neq 0$ $\quad t_{cr(0.975,\,1194)} = 1.962$

$\Rightarrow$ Rejection region: $|t| \geq 1.962$

$t = (12b_5 - 4b_6)/se(12b_5 - b_6) = -0.0016/0.0015 = -1.027$

$-1.962 < -1.027 < 1.962$: Fail to reject H_0. There is no evidence to suggest the ME from extra experience are different for Wendy and Jill.

i. $ME_{EXPER_{Jill}} = \beta_4 + 22\beta_5 + 16\beta_6$

$\Rightarrow \widehat{ME} = b_4 + 22b_5 + 16b_6 = 0.0184 \Rightarrow n = \dfrac{-(\beta_4 + 22\beta_5 + 16\beta_6)}{2\beta_5}$

After n years: $\widehat{ME} = 0.0184 + n\times 2b_5 = 0$

$\Rightarrow n = -0.0184/b_5 = -0.0184/(2\times -0.0005)$

$= 19.677$ years (before becoming negative)

$n = g(\beta_4, \beta_5, \beta_6) \Rightarrow \hat{g}_4 = -0.5/b_5 = 1066.325$

$\hat{g}_5 = (b_4 + 16b_6)/2b_5^2 = 65546.4$, $\hat{g}_6 = -8/b_5 = 17093.2$

$\Rightarrow se(n) = 1.896 \Rightarrow \begin{cases} 19.677 - 1.962\times 1.896 = 15.96 \\ 19.677 + 1.962\times 1.896 = 23.40 \end{cases} \Rightarrow$ 95% IE [15.96, 23.40]