11.28 Supply and demand curves as traditionally drawn in economics principles classes have price (P) on the vertical axis and quantity (Q) on the horizontal axis.

28

a. Rewrite the truffle demand and supply equations in (11.11) and (11.12) with price P on the left-hand side. What are the anticipated signs of the parameters in this rewritten system of equations?

b. Using the data in the file *truffles*, estimate the supply and demand equations that you have formulated in (a) using two-stage least squares. Are the signs correct? Are the estimated coefficients significantly different from zero?

c. Estimate the price elasticity of demand "at the means" using the results from (b).

d. Accurately sketch the supply and demand equations, with P on the vertical axis and Q on the horizontal axis, using the estimates from part (b). For these sketches set the values of the exogenous variables DI, PS, and PF to be $DI^* = 3.5$, $PF^* = 23$, and $PS^* = 22$.

e. What are the equilibrium values of P and Q obtained in part (d)? Calculate the predicted equilibrium values of P and Q using the estimated reduced-form equations from Table 11.2, using the same values of the exogenous variables. How well do they agree?

f. Estimate the supply and demand equations that you have formulated in (a) using OLS. Are the signs correct? Are the estimated coefficients significantly different from zero? Compare the results to those in part (b).

(a) Demand: $Q_i=\alpha_1+\alpha_2P_i+\alpha_3PS_i+\alpha_4DI_i+e_{di}$ (11.11)

Supply: $Q_i=\beta_1+\beta_2P_i+\beta_3PF_i+e_{si}$ (11.12)

11.11 ⇒ $\alpha_2P_i=Q_i-\alpha_1-\alpha_3PS_i-\alpha_4DI_i-e_{di}$

$$P_i=\frac{-\alpha_1}{\alpha_2}+\frac{1}{\alpha_2}Q_i+\frac{-\alpha_3}{\alpha_2}PS_i+\frac{-\alpha_4}{\alpha_2}DI_i+\frac{-e_{di}}{\alpha_2}$$

$$P_i=\gamma_1+\overset{-}{\gamma_2}Q_i+\overset{+}{\gamma_3}PS_i+\gamma_4\overset{+}{DI_i}+u_{di}$$

11.12 ⇒ $$P_i=\frac{-\beta_1}{\beta_2}+\frac{1}{\beta_2}Q_i+\frac{-\beta_3}{\beta_2}PF_i+\frac{-e_{si}}{\beta_2}$$

$$P_i=\delta_1+\overset{+}{\delta_2}Q_i+\overset{-}{\delta_3}PF_i+u_{si}$$

(b)

```
> summary(fit.2sls)

systemfit results
method: 2SLS

        N DF     SSR detRCov   OLS-R2 McElroy-R2
system 60 53 5029.13 3085.05 0.752661   0.928678

        N DF      SSR      MSE     RMSE       R2   Adj R2
demand 30 26 4506.625 173.3317 13.16555 0.556717 0.505569
supply 30 27  522.501  19.3519  4.39908 0.948605 0.944798

The covariance matrix of the residuals
          demand   supply
demand 173.3317 -16.4088
supply -16.4088  19.3519

The correlations of the residuals
          demand    supply
demand  1.000000 -0.283319
supply -0.283319  1.000000
```

```
2SLS estimates for 'demand' (equation 1)
Model Formula: P ~ Q + PS + DI
Instruments: ~PS + DI + PF

             Estimate Std. Error  t value   Pr(>|t|)
(Intercept) -11.42841   13.59161 -0.84084  0.4081026
Q            -2.67052    1.17495 -2.27287  0.0315350 *
PS            3.46108    1.11557  3.10252  0.0045822 **
DI           13.38992    2.74671  4.87490 4.6752e-05 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 13.165551 on 26 degrees of freedom
Number of observations: 30 Degrees of Freedom: 26
SSR: 4506.625289 MSE: 173.331742 Root MSE: 13.165551
Multiple R-Squared: 0.556717 Adjusted R-Squared: 0.505569
```

```
2SLS estimates for 'supply' (equation 2)
Model Formula: P ~ Q + PF
Instruments: ~PS + DI + PF

              Estimate Std. Error  t value   Pr(>|t|)
(Intercept) -58.798223   5.859161 -10.0353 1.3165e-10 ***
Q             2.936711   0.215772  13.6103 1.3212e-13 ***
PF            2.958486   0.155964  18.9690 < 2.22e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 4.399078 on 27 degrees of freedom
Number of observations: 30 Degrees of Freedom: 27
SSR: 522.500877 MSE: 19.351884 Root MSE: 4.399078
Multiple R-Squared: 0.948605 Adjusted R-Squared: 0.944798
```

Demand:

$$\hat{P}_i=-11.42841-2.67052Q_i+3.46108PS_i+13.38992DI_i$$

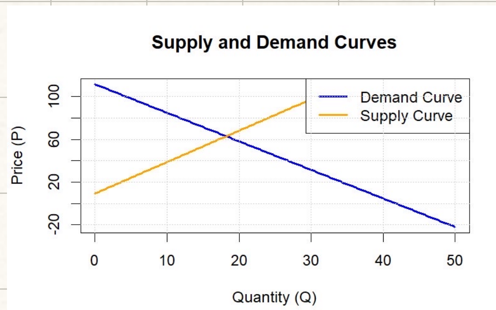
Supply:

$$\hat{P}_i=-58.798223+2.936711Q_i+2.958486PF_i$$

(c)

```
> #(c)
> # 取得需求方程中 P 的係數
> alpha2 <- coef(fit.2sls$eq[[1]])["P"]
>
> # 計算均值
> mean_P <- mean(truffles$P)
> mean_Q <- mean(truffles$Q)
>
> # 計算價格彈性
> price_elasticity <- as.numeric(alpha2 * mean_P / mean_Q)
> price_elasticity
[1] -1.272464
```

(d)



$DI^*=3.5,\ PF^*=23,\ PS^*=22$

(e)

```
> Q_star   # 均衡數量
[1] 18.2502
> P_star   # 均衡價格
[1] 62.84253
```

```
> # 計算預測值
> Q_hat <- coef_Q[1] + coef_Q["PS"]*PS_star
> P_hat <- coef_P[1] + coef_P["PS"]*PS_star
>
> Q_hat  # reduced form 預測的 Q
(Intercept)
    18.2604
> P_hat  # reduced form 預測的 P
(Intercept)
   62.81537
```

(f)

```
> summary(demand_ols)

Call:
lm(formula = P ~ Q + PS + DI, data = truffles)

Residuals:
     Min       1Q   Median       3Q      Max
-25.0753  -2.7742  -0.4097   4.7079  17.4979

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) -13.6195     9.0872  -1.499   0.1460
Q             0.1512     0.4988   0.303   0.7642
PS            1.3607     0.5940   2.291   0.0303 *
DI           12.3582     1.8254   6.770 3.48e-07 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 8.814 on 26 degrees of freedom
Multiple R-squared:  0.8013,    Adjusted R-squared:  0.7784
F-statistic: 34.95 on 3 and 26 DF,  p-value: 2.842e-09
```

```
> summary(supply_ols)

Call:
lm(formula = P ~ Q + PF, data = truffles)

Residuals:
    Min      1Q  Median      3Q     Max
-8.4721 -3.3287  0.1861  2.0785 10.7513

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) -52.8763     5.0238  -10.53 4.68e-11 ***
Q             2.6613     0.1712   15.54 5.42e-15 ***
PF            2.9217     0.1482   19.71  < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 4.202 on 27 degrees of freedom
Multiple R-squared:  0.9531,    Adjusted R-squared:  0.9496
F-statistic: 274.4 on 2 and 27 DF,  p-value: < 2.2e-16
```

```
> print(demand_df)
  Method Intercept       Q     PS      DI
1    OLS  -13.6195  0.1512 1.3607 12.3582
2   2SLS  -11.4284 -2.6705 3.4611 13.3899
> print(supply_df)
  Method Intercept      Q     PF
1    OLS  -52.8763 2.6613 2.9217
2   2SLS  -58.7982 2.9367 2.9585
```

11.30 Example 11.3 introduces Klein's Model I. Use the data file *klein* to answer the following questions.

a. Estimate the investment function in equation (11.18) by OLS. Comment on the signs and significance of the coefficients.

b. Estimate the reduced-form equation for profits, P_t, using all eight exogenous and predetermined variables as explanatory variables. Test the joint significance of all the variables except lagged profits, P_{t-1}, and lagged capital stock, K_{t-1}. Save the residuals, $\hat{v}_t$ and compute the fitted values, $\hat{P}_t$.

c. The Hausman test for the presence of endogenous explanatory variables is discussed in Section 10.4.1. It is implemented by adding the reduced-form residuals to the structural equation and testing their significance, that is, using OLS estimate the model

$$I_t = \beta_1 + \beta_2 P_t + \beta_3 P_{t-1} + \beta_4 K_{t-1} + \delta \hat{v}_t + e_{2t}$$

Use a t-test for the null hypothesis $H_0: \delta = 0$ versus $H_1: \delta \neq 0$ at the 5% level of significance. By rejecting the null hypothesis, we conclude that P_t is endogenous. What do we conclude from the test? In the context of this simultaneous equations model what result should we find?

d. Obtain the 2SLS estimates of the investment equation using all eight exogenous and predetermined variables as IVs and software designed for 2SLS. Compare the estimates to the OLS estimates in part (a). Do you find any important differences?

e. Estimate the second-stage model $I_t = \beta_1 + \beta_2 \hat{P}_t + \beta_3 P_{t-1} + \beta_4 K_{t-1} + e_{2t}$ by OLS. Compare the estimates and standard errors from this estimation to those in part (d). What differences are there?

f. Let the 2SLS residuals from part (e) be $\hat{e}_{2t}$. Regress these residuals on all the exogenous and predetermined variables. If these instruments are valid, then the R^2 from this regression should be low, and none of the variables are statistically significant. The Sargan test for instrument validity is discussed in Section 10.4.3. The test statistic TR^2 has a chi-square distribution with degrees of freedom equal to the number of "surplus" IVs if the surplus instruments are valid. The investment equation includes three exogenous and/or predetermined variables out of the total of eight possible. There are $L = 5$ external instruments and $B = 1$ right-hand side endogenous variables. Compare the value of the test statistic to the 95th percentile value from the $\chi^2_{(4)}$ distribution. What do we conclude about the validity of the surplus instruments in this case?

$\hat{I}_t = \beta_1 + \beta_2 P_t + \beta_3 P_{t-1} + \beta_4 K_{t-1} + e_t$

(a)

OLS假設所有變數均外生 ⇒ 與error term不相關

```
> summary(investment_ols)

Call:
lm(formula = i ~ p + plag + klag, data = klein)

Residuals:
     Min       1Q   Median       3Q      Max 
-2.56562 -0.63169  0.03687  0.41542  1.49226 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 10.12579    5.46555   1.853 0.081374 .  
p            0.47964    0.09711   4.939 0.000125 ***
plag         0.33304    0.10086   3.302 0.004212 ** 
klag        -0.11179    0.02673  -4.183 0.000624 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 1.009 on 17 degrees of freedom
  (因為不存在，1 個觀察量被刪除了)
Multiple R-squared:  0.9313,	Adjusted R-squared:  0.9192 
F-statistic: 76.88 on 3 and 17 DF,  p-value: 4.299e-10
```

(b) 檢驗是否OLS有偏誤(P是內生?)，先用所有外生迴歸可能的內生P 取得 $\hat{v}$ (P不可解釋的部分)

```
> summary(profit_rf)

Call:
lm(formula = p ~ g + w2 + tx + time + plag + klag + elag, data = klein_complete)

Residuals:
    Min      1Q  Median      3Q     Max 
-3.9067 -1.3050  0.3226  1.3613  2.8881 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)  
(Intercept) 50.38442   31.63026   1.593   0.1352  
g            0.43902    0.39114   1.122   0.2820  
w2          -0.07961    2.53382  -0.031   0.9754  
tx          -0.92310    0.43376  -2.128   0.0530 .
time         0.31941    0.77813   0.410   0.6881  
plag         0.80250    0.51886   1.547   0.1459  
klag        -0.21610    0.11911  -1.814   0.0928 .
elag         0.02200    0.28216   0.078   0.9390  
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 2.183 on 13 degrees of freedom
Multiple R-squared:  0.8261,	Adjusted R-squared:  0.7324 
F-statistic: 8.821 on 7 and 13 DF,  p-value: 0.0004481
```

再把 $\hat{v}$ 放回OLS model，若 δ 顯著，代表 $\hat{v}_t$ 和 I_t 顯著相關 ⇒ P是內生

(c) $I_t = \beta_1 + \beta_2 P_t + \beta_3 P_{t-1} + \beta_4 K_{-1} + \delta \hat{v}_t + e_{2t}$

```
> summary(investment_hausman)

Call:
lm(formula = i ~ p + plag + klag + v_hat, data = klein_complete)

Residuals:
     Min       1Q   Median       3Q      Max 
-1.04645 -0.56030  0.06189  0.25348  1.36700 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 20.27821    4.70179   4.313 0.000536 ***
p            0.15022    0.10798   1.391 0.183222    
plag         0.61594    0.10147   6.070 1.62e-05 ***
klag        -0.15779    0.02252  -7.007 2.96e-06 ***
v_hat        0.57451    0.14261   4.029 0.000972 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.7331 on 16 degrees of freedom
Multiple R-squared:  0.9659,	Adjusted R-squared:  0.9574 
F-statistic: 113.4 on 4 and 16 DF,  p-value: 1.588e-11
```

$\begin{cases} H_0: \delta = 0 \ (P_t \text{ 外生}) \\ H_1: \delta \neq 0 \ (P_t \text{ 內生}) \end{cases}$

p-value = 0.000972 < 0.05

⇒ $\hat{v}_t$ 在迴歸中高度顯著，也表示P和 e_{2t} 相關

⇒ P是 endogenous.

(d)

通常手動standerror會低估，沒考慮stage1的uncertainty

Sargan用來檢驗IV是否合理，有無過度識別問題

```
> summary(investment_2sls)

Call:
ivreg(formula = i ~ p + plag + klag | g + w2 + tx + time + plag + 
    klag + elag, data = klein)

Residuals:
    Min      1Q  Median      3Q     Max 
-3.2909 -0.8069  0.1423  0.8601  1.7956 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)   
(Intercept) 20.27821    8.38325   2.419  0.02707 * 
p            0.15022    0.19253   0.780  0.44598   
plag         0.61594    0.18093   3.404  0.00338 **
klag        -0.15779    0.04015  -3.930  0.00108 **

Diagnostic tests:
                 df1 df2 statistic  p-value    
Weak instruments   5  13     1.934 0.156630    
Wu-Hausman         1  16    16.230 0.000972 ***
Sargan             4  NA     1.815 0.769743    
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 1.307 on 17 degrees of freedom
Multiple R-Squared: 0.8849,	Adjusted R-squared: 0.8646 
Wald test:  41.2 on 3 and 17 DF,  p-value: 5.148e-08
```

P是內生

⇒ p在2SLS中，變不顯著，表示P有在嚴重內生性

(e)

```
> summary(investment_2nd_stage)

Call:
lm(formula = i ~ fitted(profit_rf) + plag + klag, data = klein_complete)

Residuals:
    Min      1Q  Median      3Q     Max 
-3.8778 -1.0029  0.3058  0.7275  2.1831 

Coefficients:
                  Estimate Std. Error t value Pr(>|t|)   
(Intercept)       20.27821    9.97663   2.033  0.05802 . 
fitted(profit_rf)  0.15022    0.22913   0.656  0.52084   
plag               0.61594    0.21531   2.861  0.01083 * 
klag              -0.15779    0.04778  -3.302  0.00421 **
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 1.556 on 17 degrees of freedom
Multiple R-squared:  0.837,	Adjusted R-squared:  0.8082 
F-statistic: 29.09 on 3 and 17 DF,  p-value: 6.393e-07
```

$\hat{P}$

(f) 用Sargan檢驗IV有效性 ⇒ 將2SLS的殘差和外生變數跑OLS，得到 R^2

$TR^2 = n \times R^2$

```
> # 6. 輸出檢定結果
> cat("Sargan test statistic (TR^2):", TR2, "\n")
Sargan test statistic (TR^2): 1.814965
> cat("Critical value (chi-squared, df =", df, "):", critical_value, "\n")
Critical value (chi-squared, df = 6 ): 12.59159
> if (TR2 < critical_value) {
+   cat("結論：無法拒絕工具變數有效性假設，工具變數設定合理。\n")
+ } else {
+   cat("結論：拒絕工具變數有效性假設，工具變數設定可能有問題。\n")
+ }
結論：無法拒絕工具變數有效性假設，工具變數設定合理。
```

☆ df = ? 4 or 6

(5-1) (7-1)