HW0512_313707002_羅芷羚

Q11.18:

a.

Demand: $Q_i = \alpha_1 + \alpha_2 P_i + \alpha_3 PS_i + \alpha_4 DI_i + e_{di}$ (11.11)

Supply: $Q_i = \beta_1 + \beta_2 P_i + \beta_3 PF_i + e_{si}$ (11.12)

(a) Demand Equation: $\alpha_2 P_i = Q_i - (\alpha_1 + \alpha_3 PS_i + \alpha_4 DI_i + e_{di})$

$\Rightarrow P_i = -\frac{\alpha_1}{\alpha_2} + \frac{1}{\alpha_2} Q_i - \frac{\alpha_3}{\alpha_2} PS_i - \frac{\alpha_4}{\alpha_2} DI_i + \frac{1}{\alpha_2} e_{di} = \delta_1 + \delta_2 Q_i + \delta_3 PS_i + \delta_4 DI + u^d$

$\delta_2 < 0$, law of demand

$\delta_3 > 0$, substitute goods

$\delta_4 > 0$, truffles are normal goods

Supply Equation: $\beta_2 P_i = Q_i - (\beta_1 + \beta_3 PF_i + e_{si})$

$\Rightarrow P_i = -\frac{\beta_1}{\beta_2} + \frac{1}{\beta_2} Q_i + \frac{\beta_3}{\beta_2} PF_i + \frac{1}{\beta_2} e_{si} = \pi_1 + \pi_2 Q_i + \pi_3 PF_i + u^s$

$\pi_2 > 0$, $P \uparrow \rightarrow S \uparrow$

$\pi_3 > 0$, cost of a factor of production

b. All of the coefficients are exactly as theory predicts.
Except for the intercept in the demand equation, all the other slope coefficients are statistically significant different from zero.

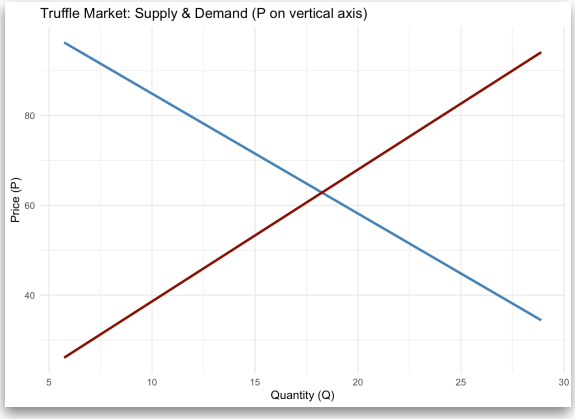
```
2SLS estimates for 'demand' (equation 1)
Model Formula: p ~ q + ps + di
Instruments: ~ps + di + pf

             Estimate Std. Error  t value   Pr(>|t|)
(Intercept) -11.42841   13.59161 -0.84084  0.4081026
q            -2.67052    1.17495 -2.27287  0.0315350 *
ps            3.46108    1.11557  3.10252  0.0045822 **
di           13.38992    2.74671  4.87490 4.6752e-05 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 13.165551 on 26 degrees of freedom
Number of observations: 30 Degrees of Freedom: 26
SSR: 4506.625289 MSE: 173.331742 Root MSE: 13.165551
Multiple R-Squared: 0.556717 Adjusted R-Squared: 0.505569
```

```
2SLS estimates for 'supply' (equation 2)
Model Formula: p ~ q + pf
Instruments: ~ps + di + pf

             Estimate Std. Error  t value   Pr(>|t|)
(Intercept) -58.798223   5.859161 -10.0353 1.3165e-10 ***
q             2.936711   0.215772  13.6103 1.3212e-13 ***
pf            2.958486   0.155964  18.9690 < 2.22e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 4.399078 on 27 degrees of freedom
Number of observations: 30 Degrees of Freedom: 27
SSR: 522.500877 MSE: 19.351884 Root MSE: 4.399078
Multiple R-Squared: 0.948605 Adjusted R-Squared: 0.944798
```

c.

```
> Pbar <- mean(truffles$p)
> Qbar <- mean(truffles$q)
> elas <- (1/lambda1) * (Pbar / Qbar)
> elas
        q
-1.272464
```

d.



e. The reduced-form predicts $\hat{Q} = 18.26$ and $\hat{P} = 62.815$, comparing the result, we think the two approaches are in very good agreement."

```
> c(Q_eq = Q_eq, P_eq = P_eq)
Q_eq.(Intercept) P_eq.(Intercept)
        18.25021         62.84257
```

```
> c(Q_hat = Q_hat, P_hat = P_hat)
 Q_hat.1  P_hat.1
18.26040 62.81537
```

f. Except for the sign of demand estimated from OLS, all the other coefficient signs are correct. Except for the intercept and coefficients for Q of demand estimated from OLS, all the other coefficient are statistically significant different from zero.

```
# A tibble: 14 × 6
   model       term        estimate std.error statistic  p.value
   <chr>       <chr>          <dbl>     <dbl>     <dbl>    <dbl>
 1 Demand 2SLS (Intercept)  -11.4      13.6      -0.841 4.08e- 1
 2 Demand OLS  (Intercept)  -13.6       9.09     -1.50  1.46e- 1
 3 Supply 2SLS (Intercept)  -58.8       5.86    -10.0   1.32e-10
 4 Supply OLS  (Intercept)  -52.9       5.02    -10.5   4.68e-11
 5 Demand 2SLS di            13.4       2.75      4.87  4.68e- 5
 6 Demand OLS  di            12.4       1.83      6.77  3.48e- 7
 7 Supply 2SLS pf             2.96      0.156    19.0   3.88e-17
 8 Supply OLS  pf             2.92      0.148    19.7   1.47e-17
 9 Demand 2SLS ps             3.46      1.12      3.10  4.58e- 3
10 Demand OLS  ps             1.36      0.594     2.29  3.03e- 2
11 Demand 2SLS q             -2.67      1.17     -2.27  3.15e- 2
12 Demand OLS  q              0.151     0.499     0.303 7.64e- 1
13 Supply 2SLS q              2.94      0.216    13.6   1.32e-13
14 Supply OLS  q              2.66      0.171    15.5   5.42e-15
```

Q.11.30:

a.Both current profits (p)and lagged profits (plag) enter with positive coefficients: when firms earn higher profits and enjoy ample internal funds, they raise their investment spending, consistent with the expected positive profit–investment linkage. By contrast, the coefficient on lagged capital stock (klag) is negative: the larger the existing capital base, the lower the marginal need for additional capital, in line with the accelerator model.

All three core slope coefficients are significantly different from zero at the 1–5 percent levels.

```
> summary(ols_inv)

Call:
lm(formula = i ~ p + plag + klag, data = klein)

Residuals:
     Min       1Q   Median       3Q      Max
-2.56562 -0.63169  0.03687  0.41542  1.49226

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 10.12579    5.46555   1.853 0.081374 .
p            0.47964    0.09711   4.939 0.000125 ***
plag         0.33304    0.10086   3.302 0.004212 **
klag        -0.11179    0.02673  -4.183 0.000624 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

b.The value of F-test=1.93<3.025,so we fail to reject the null hypothesis that all coefficients of these variables are zero.

```
> summary(rf_P)

Call:
lm(formula = p ~ g + w2 + tx + time + plag + klag + elag, data = klein)

Residuals:
    Min      1Q  Median      3Q     Max
-3.9067 -1.3050  0.3226  1.3613  2.8881

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 50.38442   31.63026   1.593   0.1352
g            0.43902    0.39114   1.122   0.2820
w2          -0.07961    2.53382  -0.031   0.9754
tx          -0.92310    0.43376  -2.128   0.0530 .
time         0.31941    0.77813   0.410   0.6881
plag         0.80250    0.51886   1.547   0.1459
klag        -0.21610    0.11911  -1.814   0.0928 .
elag         0.02200    0.28216   0.078   0.9390
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

```
Model 1: restricted model
Model 2: p ~ g + w2 + tx + time + plag + klag + elag

  Res.Df RSS Df Sum of Sq    F Pr(>F)
1     18 108
2     13  62  5      46.1 1.93   0.16
```

```
> cat("Critical F(5,13;0.95) =", round(F_crit, 3), "\n")
Critical F(5,13;0.95) = 3.025
```

```
> df$phat
 [1] 13.255556 16.577368 19.282347 20.960143 19.766509 18.238731 17.573065 19.541720 20.375101
[10] 17.180415 12.705026  8.999780  9.054102 12.671263 14.421338 14.711907 19.796405 19.206691
[19] 17.419605 20.305654 22.657273
```

c.Since $\hat{v}$ are significantly at 0.001 levels, so we know P is endogenous.This is what we expected from the simultaneous equation model.

```
> summary(hausman)

Call:
lm(formula = i ~ p + plag + klag + vhat, data = df)

Residuals:
     Min       1Q   Median       3Q      Max
-1.04645 -0.56030  0.06189  0.25348  1.36700

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 20.27821    4.70179   4.313 0.000536 ***
p            0.15022    0.10798   1.391 0.183222
plag         0.61594    0.10147   6.070 1.62e-05 ***
klag        -0.15779    0.02252  -7.007 2.96e-06 ***
vhat         0.57451    0.14261   4.029 0.000972 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

d.The strong OLS effect appears to be simultaneity bias: high-investment years are also high-profit years, inflating the naive slope.
2SLS sacrifices some precision—especially for p—because it relies on variation supplied by the instruments, not by the endogenous regressor itself.

```
> summary(iv_inv)

Call:
ivreg(formula = i ~ p + plag + klag | g + w2 + tx + time + plag +
    klag + elag, data = df)

Residuals:
    Min      1Q  Median      3Q     Max
-3.2909 -0.8069  0.1423  0.8601  1.7956

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 20.27821    8.38325   2.419  0.02707 *
p            0.15022    0.19253   0.780  0.44598
plag         0.61594    0.18093   3.404  0.00338 **
klag        -0.15779    0.04015  -3.930  0.00108 **
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

```
OLS vs 2SLS coefficients:
> print(compare_slopes, n = Inf)
# A tibble: 8 × 6
  model term        estimate std.error statistic  p.value
  <chr> <chr>          <dbl>     <dbl>     <dbl>    <dbl>
1 OLS   (Intercept)   10.1       5.47       1.85  0.0814
2 OLS   p              0.480     0.0971     4.94  0.000125
3 OLS   plag           0.333     0.101      3.30  0.00421
4 OLS   klag          -0.112     0.0267    -4.18  0.000624
5 2SLS  (Intercept)   20.3       8.38       2.42  0.0271
6 2SLS  p              0.150     0.193      0.780 0.446
7 2SLS  plag           0.616     0.181      3.40  0.00338
8 2SLS  klag          -0.158     0.0402    -3.93  0.00108
```

e. All slopes keep their signs and magnitudes; only their standard errors change.

```
> summary(stage2)

Call:
lm(formula = i ~ phat + plag + klag, data = df)

Residuals:
    Min      1Q  Median      3Q     Max
-3.8778 -1.0029  0.3058  0.7275  2.1831

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 20.27821    9.97663   2.033  0.05802 .
phat         0.15022    0.22913   0.656  0.52084
plag         0.61594    0.21531   2.861  0.01083 *
klag        -0.15779    0.04778  -3.302  0.00421 **
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

f.

```
(f)  Sargan test:
> cat("   TR^2  =", round(TR2, 3), "\n")
   TR^2  = 1.815
> cat("   χ²_0.95(df=4) =", round(crit95, 3), "\n")
   χ²_0.95(df=4) = 9.488
> if (TR2 < crit95) {
+   cat("   → Fail to reject H0 : surplus instruments appear valid.\n")
+ } else {
+   cat("   → Reject H0 : at least one surplus instrument may be invalid.\n")
+ }
   → Fail to reject H0 : surplus instruments appear valid.
```

Q15.6

15.6 Using the NLS panel data on $N = 716$ young women, we consider only years 1987 and 1988. We are interested in the relationship between ln(*WAGE*) and experience, its square, and indicator variables for living in the south and union membership. Some estimation results are in Table 15.10.

a. The OLS estimates of the ln(*WAGE*) model for each of the years 1987 and 1988 are reported in columns (1) and (2). How do the results compare? For these individual year estimations, what are you assuming about the regression parameter values across individuals (heterogeneity)?

(a) OLS 在 1987年和 1988年的估計差異不大.

→個体間無異質性.

b. The ln(*WAGE*) equation specified as a panel data regression model is

$$\ln(WAGE_{it}) = \beta_1 + \beta_2 EXPER_{it} + \beta_3 EXPER_{it}^2 + \beta_4 SOUTH_{it} + \beta_5 UNION_{it} + (u_i + e_{it}) \qquad \text{(XR15.6)}$$

Explain any differences in assumptions between this model and the models in part (a).

(b) 加了時間和個体的下標

假設每個個体都會有自己隨時間變動的殘差 e_{it}

允許 u_i 和解釋變數相關但不隨時間改變 u_i

c. Column (3) contains the estimated fixed effects model specified in part (b). Compare these estimates with the OLS estimates. Which coefficients, apart from the intercepts, show the most difference?

(c) 斜率從 0.9 → 1.5 變化最大

EXPER、EXPER²的效果剛好一半.

SOUTH、UNION 係數變小.

FE 95% confidence interval:

→ EXPER (−0.0085, 0.1235) SOUTH (−0.5999, −0.0945)

EXPER² (−0.0034, 0.001) UNION (0.0198, 0.1446)

EXPER使用 OLS 的估計值不在區間裡

所以可以知道 OLS估計值和FE估計值

有顯著差異.

TABLE 15.10 Estimation Results for Exercise 15.6

	(1) OLS 1987	(2) OLS 1988	(3) FE	(4) FE Robust	(5) RE
C	0.9348	0.8993	1.5468	1.5468	1.1497
	(0.2010)	(0.2407)	(0.2522)	(0.2688)	(0.1597)
EXPER	0.1270	0.1265	0.0575	0.0575	0.0986
	(0.0295)	(0.0323)	(0.0330)	(0.0328)	(0.0220)
$EXPER^2$	−0.0033	−0.0031	−0.0012	−0.0012	−0.0023
	(0.0011)	(0.0011)	(0.0011)	(0.0011)	(0.0007)
SOUTH	−0.2128	−0.2384	−0.3261	−0.3261	−0.2326
	(0.0338)	(0.0344)	(0.1258)	(0.2495)	(0.0317)
UNION	0.1445	0.1102	0.0822	0.0822	0.1027
	(0.0382)	(0.0387)	(0.0312)	(0.0367)	(0.0245)
N	716	716	1432	1432	1432

(standard errors in parentheses)

d. The F-statistic for the null hypothesis that there are no individual differences, equation (15.20), is 11.68. What are the degrees of freedom of the F-distribution if the null hypothesis (15.19) is true? What is the 1% level of significance critical value for the test? What do you conclude about the null hypothesis.

(d) $F = 11.68 > 1\% = F_{0.01}(715, 711)$

→ 拒絕虛無假設(個体無差異)，支持使用固定效果

e. Column (4) contains the fixed effects estimates with cluster-robust standard errors. In the context of this sample, explain the different assumptions you are making when you estimate with and without cluster-robust standard errors. Compare the standard errors with those in column (3). Which ones are substantially different? Are the robust ones larger or smaller?

(e) within transformation $\tilde{e}_{it} = e_{it} - \bar{e}_{it}$

欄四標準誤普遍變大

f. Column (5) contains the random effects estimates. Which coefficients, apart from the intercepts, show the most difference from the fixed effects estimates? Use the Hausman test statistic (15.36) to test whether there are significant differences between the random effects estimates and the fixed effects estimates in column (3) (Why that one?). Based on the test results, is random effects estimation in this model appropriate?

TABLE 15.10 Estimation Results for Exercise 15.6

	(1) OLS 1987	(2) OLS 1988	(3) FE	(4) FE Robust	(5) RE
C	0.9348	0.8993	1.5468	1.5468	1.1497
	(0.2010)	(0.2407)	(0.2522)	(0.2688)	(0.1597)
EXPER	0.1270	0.1265	0.0575	0.0575	0.0986
	(0.0295)	(0.0323)	(0.0330)	(0.0328)	(0.0220)
$EXPER^2$	−0.0033	−0.0031	−0.0012	−0.0012	−0.0023
	(0.0011)	(0.0011)	(0.0011)	(0.0011)	(0.0007)
SOUTH	−0.2128	−0.2384	−0.3261	−0.3261	−0.2326
	(0.0338)	(0.0344)	(0.1258)	(0.2495)	(0.0317)
UNION	0.1445	0.1102	0.0822	0.0822	0.1027
	(0.0382)	(0.0387)	(0.0312)	(0.0367)	(0.0245)
N	716	716	1432	1432	1432

(standard errors in parentheses)

(f) ① $EXPER^2$ 係數差 $\frac{0.00023}{0.00012} = 1.9$ 倍.

② Hausman test $t_j = \frac{\hat{\beta}_{FE} - \hat{\beta}_{RE}}{\sqrt{Var(\hat{\beta}_{FE}) - Var(\hat{\beta}_{RE})}}$

$t_{EXPER} = \frac{0.0575 - 0.0986}{\sqrt{0.033^2 - 0.022^2}} = -1.067$ $\quad t_{SOUTH} = \frac{-0.3261 - (-0.2326)}{\sqrt{0.1258^2 - 0.0317^2}} = -0.77$

$t_{EXPER^2} = \frac{-0.0012 - (-0.0023)}{\sqrt{0.0011^2 - 0.0007^2}} = 1.296$ $\quad t_{EXPER} = \frac{0.0822 - 0.1027}{\sqrt{0.0312^2 - 0.0245^2}} = -1.06$

只有 EXPER 的係數在 10% 顯著水準上有顯著差異 → random effects estimation is appropriate.

Q.15.17

a. $\widehat{LIQUORD}_{it} = 0.02975 INCOMED_{it}$

```
> summary(fd_mod)

Call:
lm(formula = LIQUORD ~ INCOMED - 1, data = liquor_fd)

Residuals:
    Min      1Q  Median      3Q     Max
-3.6852 -0.9196 -0.0323  0.9027  3.3620

Coefficients:
        Estimate Std. Error t value Pr(>|t|)
INCOMED  0.02975    0.02922   1.018    0.312

Residual standard error: 1.417 on 79 degrees of freedom
  (40 observations deleted due to missingness)
Multiple R-squared:  0.01295,   Adjusted R-squared:  0.0004544
F-statistic: 1.036 on 1 and 79 DF,  p-value: 0.3118
```

```
> confint(fd_mod, level = 0.95)
                2.5 %     97.5 %
INCOMED -0.02841457 0.08790818
```

Q.15.20.a.

Small-class effect (SMALL) significant.

Coefficient ≈ +5.82. Small-class instruction has a statistically significant positive impact on reading performance.

Teacher's aide effect (AIDE) not statistically significant.

Coefficient ≈ +0.82. There is no evidence that having a teacher's aide significantly improves reading scores.

Teacher experience (TCHEXPER) significant

Coefficient ≈ +0.49. More experienced teachers are associated with better student reading outcomes.

Gender difference (BOY) significant

Coefficient ≈ –6.16. Girls outperform boys in this reading assessment.

Race/ethnicity (WHITE_ASIAN) significant

Coefficient ≈ +3.91. White and Asian students achieve higher average reading scores.

Economic disadvantage (FREELUNCH) significant

Coefficient ≈ –14.77. Economic disadvantage is strongly associated with lower reading performance.

```
> summary(ols_a)

Call:
lm(formula = readscore ~ small + aide + tchexper + boy + white_asian +
    freelunch, data = star)

Residuals:
     Min       1Q   Median       3Q      Max
-107.220  -20.214   -3.935   14.339  185.956

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) 437.76425    1.34622 325.180  < 2e-16 ***
small         5.82282    0.98933   5.886 4.19e-09 ***
aide          0.81784    0.95299   0.858    0.391
tchexper      0.49247    0.06956   7.080 1.61e-12 ***
boy          -6.15642    0.79613  -7.733 1.23e-14 ***
white_asian   3.90581    0.95361   4.096 4.26e-05 ***
freelunch   -14.77134    0.89025 -16.592  < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

b.

The small-class advantage actually grows once we control for school-specific factors, and remains highly significant

The experience effect falls in magnitude—suggesting some of the OLS effect was driven by differences across schools—but remains positive and highly significant.

Boys continue to score about 5–6 points below girls, a highly significant gap that persists within schools.

Controlling for school heterogeneity roughly doubles the race/ethnicity premium, indicating that within-school differences are even larger than the pooled estimate suggested.

```
> summary(fe_b)
Oneway (individual) effect Within Model

Call:
plm(formula = readscore ~ small + aide + tchexper + boy + white_asian +
    freelunch, data = pdata, model = "within")

Unbalanced Panel: n = 79, T = 34-137, N = 5766

Residuals:
     Min.   1st Qu.    Median   3rd Qu.      Max.
-102.6381  -16.7834   -2.8473   12.7591  198.4169

Coefficients:
             Estimate Std. Error  t-value  Pr(>|t|)
small         6.490231   0.912962   7.1090 1.313e-12 ***
aide          0.996087   0.881693   1.1297    0.2586
tchexper      0.285567   0.070845   4.0309 5.629e-05 ***
boy          -5.455941   0.727589  -7.4987 7.440e-14 ***
white_asian   8.028019   1.535656   5.2277 1.777e-07 ***
freelunch   -14.593572   0.880006 -16.5835 < 2.2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

c.

The F-test statistic is 16.70 larger than 1.2798, so we reject the null hypothesis that there are no significant differences between schools.

If the between-school heterogeneity is negligible, then absorbing school-specific intercepts merely shifts the overall level (intercept) without altering the slope estimates.

Similarly, if most of the variation in the key regressors comes from within-school differences rather than across-school differences, the inclusion of fixed effects has little impact on the estimated slopes.

```
> # F 検定：FE vs pooled
> pFtest(fe_b, pool_c)

        F test for individual effects

data:  readscore ~ small + aide + tchexper + boy + white_asian + freelunch
F = 16.698, df1 = 78, df2 = 5681, p-value < 2.2e-16
alternative hypothesis: significant effects
```