

3.1

(a)  $n = 64$ .

$$H_0: b_2 = 0$$

$$H_1: b_2 > 0 \quad \#$$

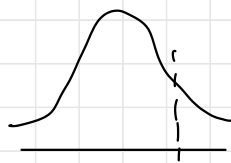
(b) test statistic:

$$T = \frac{\hat{\beta}_2}{\sqrt{\frac{\hat{\sigma}^2}{\sum (x_i - \bar{x})^2}}} \underset{H_0}{\sim} t(b_2) \quad \#$$

(c) if alternative hypothesis is true, the distribution will shift to the right relative to the usual  $t$ -distribution.

$H_0$  is true:  $b_2 = 0$ ,  $H_1$  is true:  $b_2 > 0$ , so the distribution will shift to the right,  $\#$

(d)  $\alpha = 1\%$



$$\alpha = 1\% : t(0.01, b_2)$$

$$= 2.388$$

we will reject  $H_0$  if

$t \geq 2.388$ , do not reject  $H_0$ .

when  $t < 2.388$

$\#$

$$(e) \quad T_0 = \frac{\hat{\beta}_2}{\text{se}(\hat{\beta}_2)} = \frac{0.01309}{0.00215} \approx 6.0884.$$

economic conclusion:  $T_0 \in \mathbb{R}$ , reject  $H_0$ , there is a positive relationship between the number of medals won and GDP.

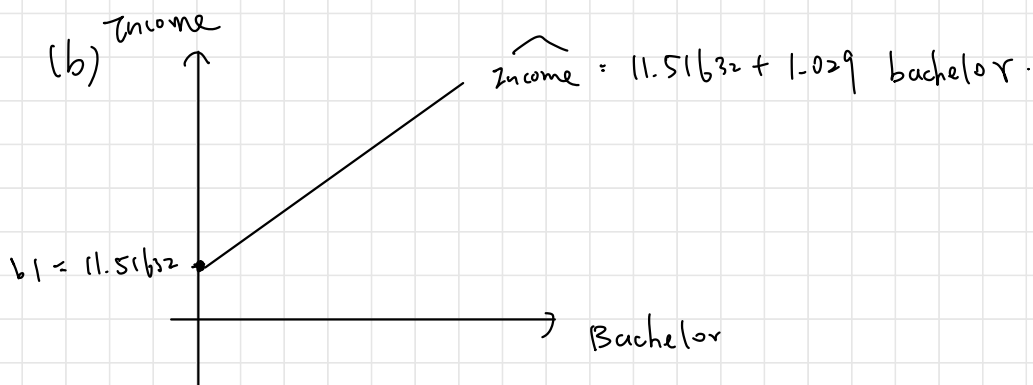
1% level of significance means the type I error,

the probability that when  $H_0$  is true, but reject  $H_0$  #

3.7

$$(a) \quad t = \frac{\hat{b}_1}{\text{se}(\hat{b}_1)}, \quad 4.31 = \frac{b_1}{2.672}, \quad b_1 = 11.51632.$$

$$\text{intercept} = 11.51632 \quad \#$$



Income and bachelor have a positive relationship and increasing at a constant rate because

the  $b_2$  coefficient is  $+1.029$  #

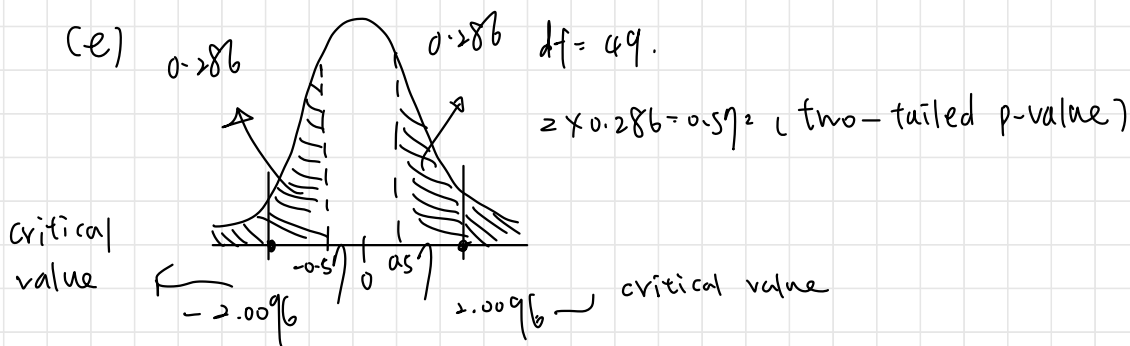
$$(c) \quad t = \frac{\hat{b}_2}{se(b_2)}, \quad 10.75 = \frac{1.029}{se(b_2)}$$

$$se(b_2) = 0.0959 \quad \#$$

$$(d) \quad H_0 : b_1 = 10$$

t-statistic:

$$\frac{\hat{b}_1 - b_1}{se(b_1)} = \frac{11.51632 - 10}{2.672} = 0.567 \quad \#$$



rejection region for a 5% test are t-values

greater than or equal to 2.0096 or less than

or equal to -2.0096 #

(f) interval estimate of the slope.

$$[\hat{b}_2 - t_{\frac{\alpha}{2}(n-2)} \text{se}(b_2), \hat{b}_2 + t_{\frac{\alpha}{2}(n-2)} \text{se}(b_2)]$$

$$[1.029 - 2.680 \cdot 0.0959, 1.029 + 2.680 \cdot 0.0959]$$

$$[0.7775, 1.2855] \quad \#$$

(g)  $H_0: b_2 = 1$

$$H_a: b_2 \neq 1$$

$$\alpha = 0.05$$

$$t\text{-statistic} = \frac{\hat{b}_2 - b_2}{\text{se}(b_2)} \underset{H_0}{\sim} t(n-2)$$

$$\text{Rejection region} = \{ |T| \geq t_{\frac{\alpha}{2}(n-2)} \} = |T| \geq 2.010$$

$$T = \frac{1.029 - 1}{0.0959} = 0.303$$

To  $\alpha$  RR, do not reject  $H_0: b_2 = 1$

即教育程度提升1%, 對應所得提升1000美元 #

3.17

$$(a) \quad H_0: b_2 \leq 1.8$$

$$H_a: b_2 > 1.8$$

②

$$\alpha = 0.05$$

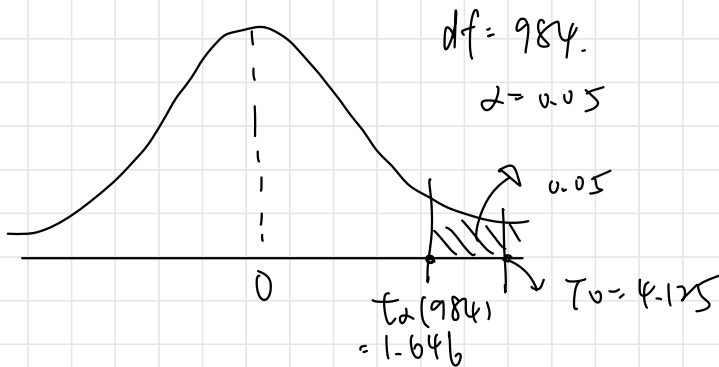
$$(3) \text{ test statistic: } \frac{\hat{b}_2 - b_2}{\text{se}(\hat{b}_2)} \underset{H_0}{\sim} t_{(n-2)}$$

$$(4) \text{ rejection region} = T \geq t_{\alpha}(n-2), T \geq t_{0.05}(984) \\ = T \geq 1.646$$

$$(5) T_0 = \frac{2.46 - 1.8}{0.16} = 4.125$$

$$4.125 \geq 1.646. \quad T_0 \in RR.$$

reject  $H_0: b_2 \leq 1.8$  and accept alternative  $\neq$



$$(b) \text{ EDUC} = 16, \text{ WAGE} = 23.92 \text{ (rural)}$$

$$t_{\frac{95}{2}}(212) \approx 1.971$$

interval estimate :

$$[23.92 - 1.971 \times 0.833, 23.92 + 1.971 \times 0.833]$$

$$= [22.278, 25.562]$$

use  $-0.761$  cov

$$\begin{aligned} \text{SE}_{\text{WAGE}} &= \sqrt{[\text{SE}(b_1)]^2 + (\text{EDUC})^2 [\text{SE}(b_2)]^2 + 2 \cdot \text{EDUC} \cdot \text{cov}(b_1, b_2)} \\ &= 1.1035. \end{aligned}$$

$$(c) \text{ EDUC} = 16, \text{ WAGE} = 28.6 \text{ (urban)}$$

$$\text{COV} = -0.345$$

$$\begin{aligned} \text{SE}_{\text{WAGE}} &= \sqrt{(\text{SE}(b_1))^2 + (\text{EDUC})^2 (\text{SE}(b_2))^2 + 2(\text{EDUC}) \cdot \text{cov}(b_1, b_2)} \\ &= 0.8164. \end{aligned}$$

$$\text{interval estimate : } t_{\frac{95}{2}}(984) \approx 1.962$$

$$[>8.6 - 0.8164 \cdot 1.962, >8.6 + 0.8164 \cdot 1.962]$$

$$= [26.998, 30.202]$$

urban's interval estimate is narrower than rural  
maybe because its sample is more large. #

(d) rural:

$$\textcircled{1} H_0: b_1 \geq 4$$

$$H_a: b_1 < 4$$

$$\textcircled{2} \alpha = 0.01$$

$$\textcircled{3} \text{ test statistic: } \frac{\hat{b}_1 - b_1}{\text{se}(\hat{b}_1)} \underset{H_0}{\sim} t_{(212)}$$

$$\textcircled{4} \text{ rejection region: } T_0 < t_{0.01, (212)} = T_0 < 2.344$$

$$\textcircled{5} T_0 = \frac{-4.88 - 4}{3.29} = -2.699$$

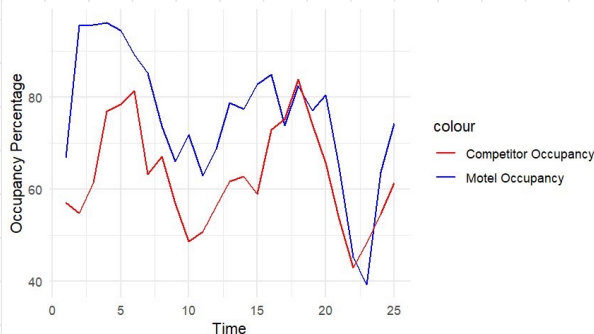
$T_0 \in RR$ , reject  $H_0$  and accept  $H_a$ . #

3. 19.

3.19 The owners of a motel discovered that a defective product was used during construction. It took 7 months to correct the defects during which approximately 14 rooms in the 100-unit motel were taken out of service for 1 month at a time. The data are in the file *motel*.

- Plot *MOTEL\_PCT* and *COMP\_PCT* versus *TIME* on the same graph. What can you say about the occupancy rates over time? Do they tend to move together? Which seems to have the higher occupancy rates? Estimate the regression model  $MOTEL\_PCT = \beta_1 + \beta_2 COMP\_PCT + e$ . Construct a 95% interval estimate for the parameter  $\beta_2$ . Have we estimated the association between *MOTEL\_PCT* and *COMP\_PCT* relatively precisely, or not? Explain your reasoning.
- Construct a 90% interval estimate of the expected occupancy rate of the motel in question, *MOTEL\_PCT*, given that *COMP\_PCT* = 70.
- In the linear regression model  $MOTEL\_PCT = \beta_1 + \beta_2 COMP\_PCT + e$ , test the null hypothesis  $H_0: \beta_2 \leq 0$  against the alternative hypothesis  $H_a: \beta_2 > 0$  at the  $\alpha = 0.01$  level of significance. Discuss your conclusion. Clearly define the test statistic used and the rejection region.
- In the linear regression model  $MOTEL\_PCT = \beta_1 + \beta_2 COMP\_PCT + e$ , test the null hypothesis  $H_0: \beta_2 = 1$  against the alternative hypothesis  $H_a: \beta_2 \neq 1$  at the  $\alpha = 0.01$  level of significance. If the null hypothesis were true, what would that imply about the motel's occupancy rate versus their competitor's occupancy rate? Discuss your conclusion. Clearly define the test statistic used and the rejection region.
- Calculate the least squares residuals from the regression of *MOTEL\_PCT* on *COMP\_PCT* and plot them against *TIME*. Are there any unusual features to the plot? What is the predominant sign of the residuals during time periods 17–23 (July, 2004 to January, 2005)?

(a)



their trend sometimes tend to move together, sometimes not (15-20 days), and motel seems to have the higher occupancy rate.

$\beta_2 = 0.8646$ , i.e. competitor  $\lambda$  佔率增加 1%, motel  $\lambda$  佔率在其他變數不變下會增加 0.8646%, 相對用圖形看, 用 regression model 會更加精確



95% interval estimate for  $\beta_2$ :

$$[0.4453, 1.2840] \neq$$

(b)  $\text{MOTEL} - \text{PCT} = 81.9247\%$

interval:

$$[77.3823, 86.4675]$$

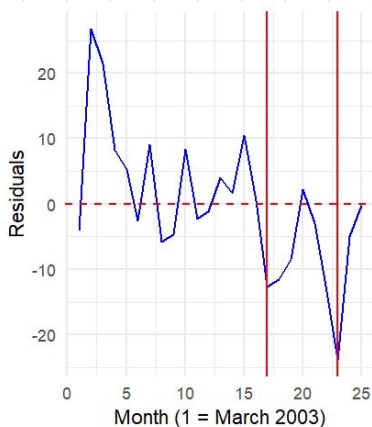
(c)  $\text{comp-pct}$ ,  $t = 4.27$ ,  $\in \text{RR}$ ,

reject  $H_0$ .

(d)  $t = -0.67$ ,  $\notin \text{RR}$ , do not reject  $H_0$ .

because  $-0.67$  is in the non-rejection-region  $\neq$

(e)



17~23. residuals almost  
are negative. this reflects  
maybe some factor not be  
captured by the regression  
model