15.6 Using the NLS panel data on $N = 716$ young women, we consider only years 1987 and 1988. We are interested in the relationship between $\ln(WAGE)$ and experience, its square, and indicator variables for living in the south and union membership. Some estimation results are in Table 15.10.

TABLE 15.10 **Estimation Results for Exercise 15.6**

	(1) OLS 1987	(2) OLS 1988	(3) FE	(4) FE Robust	(5) RE
C	0.9348	0.8993	1.5468	1.5468	1.1497
	(0.2010)	(0.2407)	(0.2522)	(0.2688)	(0.1597)
$EXPER$	0.1270	0.1265	0.0575	0.0575	0.0986
	(0.0295)	(0.0323)	(0.0330)	(0.0328)	(0.0220)
$EXPER^2$	−0.0033	−0.0031	−0.0012	−0.0012	−0.0023
	(0.0011)	(0.0011)	(0.0011)	(0.0011)	(0.0007)
$SOUTH$	−0.2128	−0.2384	−0.3261	−0.3261	−0.2326
	(0.0338)	(0.0344)	(0.1258)	(0.2495)	(0.0317)
$UNION$	0.1445	0.1102	0.0822	0.0822	0.1027
	(0.0382)	(0.0387)	(0.0312)	(0.0367)	(0.0245)
N	716	716	1432	1432	1432

(standard errors in parentheses)

a. The OLS estimates of the $\ln(WAGE)$ model for each of the years 1987 and 1988 are reported in columns (1) and (2). How do the results compare? For these individual year estimations, what are you assuming about the regression parameter values across individuals (heterogeneity)?

兩年OLS的結果相當接近，各變數的係數估計值與標準誤差變化不大
OLS模型假設所有個體的母體參數值皆相同
忽略了異質性

b. The $\ln(WAGE)$ equation specified as a panel data regression model is

$$\ln(WAGE_{it}) = \beta_1 + \beta_2 EXPER_{it} + \beta_3 EXPER_{it}^2 + \beta_4 SOUTH_{it} + \beta_5 UNION_{it} + (u_i + e_{it}) \qquad \text{(XR15.6)}$$

Explain any differences in assumptions between this model and the models in part (a).

加入了個體固定效果 u_i，允許個體特質的差異，這些特質會影響工資結果

c. Column (3) contains the estimated fixed effects model specified in part (b). Compare these estimates with the OLS estimates. Which coefficients, apart from the intercepts, show the most difference?

固定效果的信賴區間：

EXPER: (-0.0085, 0.1235)

$EXPER^2$: (-0.0034, 0.001)

SOUTH: (-0.5777, -0.0745)

UNION: (0.0198, 0.1446)

只有OLS對EXPER的估計落在此區間之外

∴OLS估計值和固定效果模型有差異

d. The *F*-statistic for the null hypothesis that there are no individual differences, equation (15.20), is 11.68. What are the degrees of freedom of the *F*-distribution if the null hypothesis (15.19) is true? What is the 1% level of significance critical value for the test? What do you conclude about the null hypothesis.

$H_0: u_i = 0$

$H_1: u_i \neq 0$

test statistic: $F \sim F_{N-n,\, n-1}$

$N = 1432$

$n = 716$

$F = 11.68 > F_{0.99}(1432-716,\, 716-1) = 1.24$

∴ reject H_0, there are individual differences.

e. Column (4) contains the fixed effects estimates with cluster-robust standard errors. In the context of this sample, explain the different assumptions you are making when you estimate with and without cluster-robust standard errors. Compare the standard errors with those in column (3). Which ones are substantially different? Are the robust ones larger or smaller?

UNION are substantially different

robust SE are larger

f. Column (5) contains the random effects estimates. Which coefficients, apart from the intercepts, show the most difference from the fixed effects estimates? Use the Hausman test statistic (15.36) to test whether there are significant differences between the random effects estimates and the fixed effects estimates in column (3) (Why that one?). Based on the test results, is random effects estimation in this model appropriate?

variable	FE	RE	RE−FE	
EXPER	0.0575	0.0986	+0.0411	
$EXPER^2$	−0.0012	−0.0023	−0.0011	
SOUTH	−0.3261	−0.2326	+0.0935	← most difference
UNION	0.0822	0.1027	+0.0205	

Hausman test!

$$t = \frac{b_{FE,k} - b_{RE,k}}{\left[\widehat{\operatorname{var}}(b_{FE,k}) - \widehat{\operatorname{var}}(b_{RE,k})\right]^{1/2}} = \frac{b_{FE,k} - b_{RE,k}}{\left[\operatorname{se}(b_{FE,k})^2 - \operatorname{se}(b_{RE,k})^2\right]^{1/2}}$$

$H_0: \beta_{FE,k} = \beta_{RE,k}$

$H_1: \beta_{FE,k} \neq \beta_{RE,k}$

$\alpha = 5\%$

$RR = \{|t| \geq t_{0.025} = 1.96\}$

$$t_{EXPER} = \frac{0.0575 - 0.0986}{\sqrt{(0.0337)^2 - (0.022)^2}} = -1.67$$

$$t_{EXPER^2} = \frac{-0.0012 - (-0.0023)}{\sqrt{(0.0011)^2 - (0.0007)^2}} = 1.296$$

$$t_{SOUTH} = \frac{-0.3261 - (-0.2326)}{\sqrt{(0.1258)^2 - (0.0317)^2}} = -0.77$$

$$t_{UNION} = \frac{0.0822 - 0.1027}{\sqrt{(0.0312)^2 - (0.0245)^2}} = -1.06$$

$t \notin RR$ ∴ don't reject H_0

radom effects estimation is appropriate.

15.17 The data file *liquor* contains observations on annual expenditure on liquor (*LIQUOR*) and annual income (*INCOME*) (both in thousands of dollars) for 40 randomly selected households for three consecutive years.

a. Create the first-differenced observations on *LIQUOR* and *INCOME*. Call these new variables *LIQUORD* and *INCOMED*. Using OLS regress *LIQUORD* on *INCOMED* without a constant term. Construct a 95% interval estimate of the coefficient.

FD ⇒ 消除個體固定效應

```
Call:
lm(formula = LIQUORD ~ INCOMED - 1, data = liquor_fd)

Residuals:
    Min      1Q  Median      3Q     Max
-3.6852 -0.9196 -0.0323  0.9027  3.3620

Coefficients:
        Estimate Std. Error t value Pr(>|t|)
INCOMED  0.02975    0.02922   1.018    0.312

Residual standard error: 1.417 on 79 degrees of freedom
  (因為不存在，40 個觀察量被刪除了)
Multiple R-squared:  0.01295,   Adjusted R-squared:  0.0004544
F-statistic: 1.036 on 1 and 79 DF,  p-value: 0.3118

> confint(fd_mod, level = 0.95)
              2.5 %     97.5 %
INCOMED -0.02841457 0.08790818
>
```

b. Estimate the model $LIQUOR_{it} = \beta_1 + \beta_2 INCOME_{it} + u_i + e_{it}$ using random effects. Construct a 95% interval estimate of the coefficient on *INCOME*. How does it compare to the interval in part (a)?

```
Call:
plm(formula = liquor ~ income, data = pdata, model = "random")

Balanced Panel: n = 40, T = 3, N = 120

Effects:
                 var std.dev share
idiosyncratic 0.9640  0.9819 0.571
individual    0.7251  0.8515 0.429
theta: 0.4459

Residuals:
     Min.   1st Qu.    Median   3rd Qu.      Max.
-2.263634 -0.697383  0.078697  0.552680  2.225798

Coefficients:
             Estimate Std. Error z-value  Pr(>|z|)
(Intercept) 0.9690324  0.5210052  1.8599 0.0628957 .
income      0.0265755  0.0070126  3.7897 0.0001508 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Total Sum of Squares:    126.61
Residual Sum of Squares: 112.88
R-Squared:      0.1085
Adj. R-Squared: 0.10095
Chisq: 14.3618 on 1 DF, p-value: 0.00015083
```

```
> confint(re_mod)["income", ]
     2.5 %     97.5 %
0.01283111 0.04031983
>
```

narrower

c. Test for the presence of random effects using the LM statistic in equation (15.35). Use the 5% level of significance.

```
Lagrange Multiplier Test - (Breusch-Pagan)

data:  liquor ~ income
chisq = 20.68, df = 1, p-value = 5.429e-06
alternative hypothesis: significant effects
```

H_0: no individual random effect
H_1: have individual random effect
∵ p-value < 0.05, reject H_0
there have individual random effect

d. For each individual, compute the time averages for the variable *INCOME*. Call this variable *INCOMEM*. Estimate the model $LIQUOR_{it} = \beta_1 + \beta_2 INCOME_{it} + \gamma INCOMEM_i + c_i + e_{it}$ using the random effects estimator. Test the significance of the coefficient γ at the 5% level. Based on this test, what can we conclude about the correlation between the random effect u_i and *INCOME*? Is it OK to use the random effects estimator for the model in (b)?

```
Call:
plm(formula = liquor ~ income + INCOMEM, data = pdata, model = "random")

Balanced Panel: n = 40, T = 3, N = 120

Effects:
                 var std.dev share
idiosyncratic 0.9640  0.9819 0.571
individual    0.7251  0.8515 0.429
theta: 0.4459

Residuals:
     Min.   1st Qu.    Median   3rd Qu.      Max.
-2.300955 -0.703840  0.054992  0.560255  2.257325

Coefficients:
             Estimate Std. Error z-value Pr(>|z|)
(Intercept) 0.9163337  0.5524439  1.6587  0.09718 .
income      0.0207421  0.0209083  0.9921  0.32117
INCOMEM     0.0065792  0.0222048  0.2963  0.76700
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Total Sum of Squares:    126.61
Residual Sum of Squares: 112.79
R-Squared:      0.10917
Adj. R-Squared: 0.093945
Chisq: 14.3386 on 2 DF, p-value: 0.00076987
```

p-value = 0.767 > α = 0.05
don't reject H_0
Income與個體效果無顯著相關，隨機效果模型是適用的

15.17 R

```
#(a)
summary(liquor5)
liquor_fd <- liquor5 %>%
  group_by(hh) %>%
  arrange(year) %>%
  mutate(
    LIQUORD  = liquor  - lag(liquor),
    INCOMED  = income - lag(income)
  ) %>%
  filter(!is.na(liquor) & !is.na(income)) %>%
  ungroup()

fd_mod <- lm(LIQUORD ~ INCOMED - 1, data = liquor_fd)

summary(fd_mod)
confint(fd_mod, level = 0.95)

#(b)
# 載入套件
library(plm)
# 宣告 panel 資料結構
pdata <- pdata.frame(liquor5, index = c("hh", "year"))

# 隨機效果模型估計
re_mod <- plm(liquor ~ income, data = pdata, model = "random")
summary(re_mod)
confint(re_mod)["income", ]
#(c)
library(plm)
# 用 pooled OLS 模型作為基礎檢定
pooled_mod <- plm(liquor ~ income, data = pdata, model = "pooling")

# Breusch-Pagan LM 檢定 (檢測是否需要隨機效果)
plmtest(pooled_mod, type = "bp")

#(d)
library(dplyr)
# 資料中每戶收入的平均值
pdata$INCOMEM <- ave(pdata$income, pdata$hh)
re_mundlak <- plm(liquor ~ income + INCOMEM, data = pdata, model = "random")
summary(re_mundlak)
coeftest(re_mundlak)
```

15.20 This exercise uses data from the STAR experiment introduced to illustrate fixed and random effects for grouped data. In the STAR experiment, children were randomly assigned within schools into three types of classes: small classes with 13–17 students, regular-sized classes with 22–25 students, and regular-sized classes with a full-time teacher aide to assist the teacher. Student scores on achievement tests were recorded as well as some information about the students, teachers, and schools. Data for the kindergarten classes are contained in the data file *star*.

a. Estimate a regression equation (with no fixed or random effects) where *READSCORE* is related to *SMALL*, *AIDE*, *TCHEXPER*, *BOY*, *WHITE_ASIAN*, and *FREELUNCH*. Discuss the results. Do students perform better in reading when they are in small classes? Does a teacher's aide improve scores? Do the students of more experienced teachers score higher on reading tests? Does the student's sex or race make a difference?

```
Call:
lm(formula = readscore ~ small + aide + tchexper + boy + white_asian +
    freelunch, data = star)

Residuals:
     Min       1Q   Median       3Q      Max
-107.220  -20.214   -3.935   14.339  185.956

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) 437.76425    1.34622 325.180  < 2e-16 ***
small         5.82282    0.98933   5.886 4.19e-09 ***
aide          0.81784    0.95299   0.858    0.391
tchexper      0.49247    0.06956   7.080 1.61e-12 ***
boy          -6.15642    0.79613  -7.733 1.23e-14 ***
white_asian   3.90581    0.95361   4.096 4.26e-05 ***
freelunch   -14.77134    0.89025 -16.592  < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 30.19 on 5759 degrees of freedom
  (因為不存在，20 個觀察量被刪除了)
Multiple R-squared:  0.09685,    Adjusted R-squared:  0.09591
F-statistic: 102.9 on 6 and 5759 DF,  p-value: < 2.2e-16
```

+5.82 ⇒ Yes, perform better in small classes

p-value = 0.391, not significant

+0.49 ⇒ Yes, more experienced teachers score higher

Yes, make difference

b. Reestimate the model in part (a) with school fixed effects. Compare the results with those in part (a). Have any of your conclusions changed? [*Hint*: specify *SCHID* as the cross-section identifier and *ID* as the "time" identifier.]

```
Call:
plm(formula = readscore ~ small + aide + tchexper + boy + white_asian +
    freelunch, data = star_panel, model = "within")

Unbalanced Panel: n = 79, T = 34-137, N = 5766

Residuals:
     Min.   1st Qu.    Median   3rd Qu.      Max.
-102.6381  -16.7834   -2.8473   12.7591  198.4169

Coefficients:
              Estimate Std. Error  t-value  Pr(>|t|)
small         6.490231   0.912962   7.1090 1.313e-12 ***
aide          0.996087   0.881693   1.1297    0.2586
tchexper      0.285567   0.070845   4.0309 5.629e-05 ***
boy          -5.455941   0.727589  -7.4987 7.440e-14 ***
white_asian   8.028019   1.535656   5.2277 1.777e-07 ***
freelunch   -14.593572   0.880006 -16.5835 < 2.2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Total Sum of Squares:    4628000
Residual Sum of Squares: 4268900
R-Squared:      0.077592
Adj. R-Squared: 0.063954
F-statistic: 79.6471 on 6 and 5681 DF, p-value: < 2.22e-16
>
```

small 效果↑

aide 仍不顯著

tchexper 效果下降

boy 效果上升

white_asian 效果上升

freelunch 一致

c. Test for the significance of the school fixed effects. Under what conditions would we expect the inclusion of significant fixed effects to have little influence on the coefficient estimates of the remaining variables?

```
        F test for individual effects

data:  readscore ~ small + aide + tchexper + boy + white_asian + freelunch
F = 16.698, df1 = 78, df2 = 5681, p-value < 2.2e-16
alternative hypothesis: significant effects
```

p-value < 0.05 學校固定效果顯著

當學校固定效果與主解釋變數幾乎不相關，主要變數的迴歸係數不會有明顯改變。

d. Reestimate the model in part (a) with school random effects. Compare the results with those from parts (a) and (b). Are there any variables in the equation that might be correlated with the school effects? Use the LM test for the presence of random effects.

```
Effects:
                  var std.dev share
idiosyncratic 751.43   27.41 0.829
individual    155.31   12.46 0.171
theta:
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
 0.6470  0.7225  0.7523  0.7541  0.7831  0.8153

Residuals:
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
-97.483 -17.236  -3.282   0.037  12.803 192.346

Coefficients:
              Estimate Std. Error  z-value  Pr(>|z|)
(Intercept) 436.126774   2.064782 211.2217 < 2.2e-16 ***
small         6.458722   0.912548   7.0777 1.466e-12 ***
aide          0.992146   0.881159   1.1260    0.2602
tchexper      0.302679   0.070292   4.3060 1.662e-05 ***
boy          -5.512081   0.727639  -7.5753 3.583e-14 ***
white_asian   7.350477   1.431376   5.1353 2.818e-07 ***
freelunch   -14.584332   0.874676 -16.6740 < 2.2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Total Sum of Squares:    6158000
Residual Sum of Squares: 4332100
R-Squared:      0.29655
Adj. R-Squared: 0.29582
Chisq: 493.205 on 6 DF, p-value: < 2.22e-16
```

```
Lagrange Multiplier Test - (Breusch-Pagan)

data:  readscore ~ small + aide + tchexper + boy + white_asian + freelunch
chisq = 6677.4, df = 1, p-value < 2.2e-16
alternative hypothesis: significant effects
```

與 part(a)、(b) 估計類似
所有變數變化不大，跟 school effects 相關的可能性小

LM test

$H_0: \sigma_u^2 = 0$ (No random effect)

$H_1: \sigma_u^2 > 0$ (Random effect exist)

∵ p-value = 2.2e-16 < 0.05

∴ reject H_0, random effect exist

存在 school-level unobserved heterogeneity

e. Using the t-test statistic in equation (15.36) and a 5% significance level, test whether there are any significant differences between the fixed effects and random effects estimates of the coefficients on *SMALL*, *AIDE*, *TCHEXPER*, *WHITE_ASIAN*, and *FREELUNCH*. What are the implications of the test outcomes? What happens if we apply the test to the fixed and random effects estimates of the coefficient on *BOY*?

```
        Hausman Test

data:  readscore ~ small + aide + tchexper + boy + white_asian + freelunch
chisq = 13.809, df = 6, p-value = 0.03184
alternative hypothesis: one model is inconsistent
```

Hausman test

$H_0: \beta_{FE,k} = \beta_{RE,k}$

$H_1: \beta_{FE,k} \neq \beta_{RE,k}$

$\because$ p-value $= 0.032 < 0.05$

$\therefore$ reject RE model

優先選擇 FE model

```
small        : t =  1.146, p-value = 0.2518 => 無顯著差異
aide         : t =  0.128, p-value = 0.8978 => 無顯著差異
tchexper     : t = -1.938, p-value = 0.0527 => 無顯著差異
white_asian  : t =  1.218, p-value = 0.2232 => 無顯著差異
freelunch    : t = -0.096, p-value = 0.9239 => 無顯著差異
boy          : t =    NaN, p-value = NaN => NA
```

$$t = \frac{b_{FE,k} - b_{RE,k}}{\left[\widehat{\operatorname{var}}(b_{FE,k}) - \widehat{\operatorname{var}}(b_{RE,k})\right]^{1/2}} = \frac{b_{FE,k} - b_{RE,k}}{\left[\operatorname{se}(b_{FE,k})^2 - \operatorname{se}(b_{RE,k})^2\right]^{1/2}}$$

係數 boy 分母是負數 → Nan，不適用 t 檢定

f. Create school-averages of the variables and carry out the Mundlak test for correlation between them and the unobserved heterogeneity.

```
Oneway (individual) effect Random Effect Model
   (Swamy-Arora's transformation)

Call:
plm(formula = readscore ~ small + aide + tchexper + boy + white_asian +
    freelunch + small_m + aide_m + tchexper_m + boy_m + white_asian_m +
    freelunch_m, data = star_panel_clean, model = "random")

Unbalanced Panel: n = 79, T = 34-136, N = 5745

Effects:
                 var std.dev share
idiosyncratic 752.52   27.43 0.819
individual    166.40   12.90 0.181
theta:
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
 0.6574  0.7311  0.7585  0.7613  0.7879  0.8206

Residuals:
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
-99.048 -16.980  -3.167   0.039  12.837 193.411

Coefficients:
                Estimate Std. Error  z-value  Pr(>|z|)
(Intercept)    460.99680   20.16960  22.8560 < 2.2e-16 ***
small            6.56507    0.91441   7.1796 6.994e-13 ***
aide             1.06482    0.88338   1.2054   0.22805
tchexper         0.28019    0.07099   3.9469 7.918e-05 ***
boy             -5.43457    0.72913  -7.4535 9.090e-14 ***
white_asian      7.91483    1.53980   5.1402 2.745e-07 ***
freelunch      -14.64874    0.88290 -16.5915 < 2.2e-16 ***
small_m        -19.28544   22.02019  -0.8758   0.38113
aide_m          14.49123   20.15601   0.7190   0.47217
tchexper_m       0.96814    0.61464   1.5751   0.11523
boy_m          -52.30481   24.94549  -2.0968   0.03601 *
white_asian_m   -6.92933    6.21515  -1.1149   0.26489
freelunch_m     -3.90646    8.65144  -0.4515   0.65160
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Total Sum of Squares:    6055700
Residual Sum of Squares: 4309400
R-Squared:      0.28843
Adj. R-Squared: 0.28694
Chisq: 504.325 on 12 DF, p-value: < 2.22e-16
```

只有 boy_m 顯著

boy 變數與學校效果相關

應使用 FE model

15.20R

```r
head(star)
# a
ols_a <- lm(readscore ~ small + aide + tchexper + boy + white_asian + freelunch,
            data = star)
summary(ols_a)

#b
star$studentid <- star$id
star_panel <- pdata.frame(star, index = c("schid", "studentid"))
fe_model <- plm(
  readscore ~ small + aide + tchexper + boy + white_asian + freelunch,
  data = star_panel,
  model = "within"
)
summary(fe_model)
#c
# pooled OLS model (沒有固定效果)
ols_mod <- plm(
  readscore ~ small + aide + tchexper + boy + white_asian + freelunch,
  data = star_panel,
  model = "pooling"
)

# fixed effects model (已經估計過)
fe_mod <- plm(
  readscore ~ small + aide + tchexper + boy + white_asian + freelunch,
  data = star_panel,
  model = "within"
)

# F-test for fixed effects (檢定學校固定效果是否顯著)
pFtest(fe_mod, ols_mod)

#(d)
# 隨機效果模型
re_model <- plm(
  readscore ~ small + aide + tchexper + boy + white_asian + freelunch,
  data = star_panel,
  model = "random"
)

summary(re_model)
# LM 檢定: 隨機效果 vs 無效果模型 (pooled OLS)
plm::plmtest(ols_mod, type = "bp")

#(e)
# 提取 FE 與 RE 模型係數及標準誤
fe_coef <- summary(fe_model)$coefficients
re_coef <- summary(re_model)$coefficients

# 欲檢定的變數
vars <- c("small", "aide", "tchexper", "white_asian", "freelunch", "boy")


cat("=== FE vs RE 模型逐變數 t 檢定結果 ===\n")
for (v in vars) {
  # 確保變數存在
  if (!(v %in% rownames(fe_coef)) | !(v %in% rownames(re_coef))) {
    cat(sprintf("%-12s : 變數名稱不存在, 請確認\n", v))
    next
  }

  b_fe <- fe_coef[v, "Estimate"]
  se_fe <- fe_coef[v, "Std. Error"]

  b_re <- re_coef[v, "Estimate"]
  se_re <- re_coef[v, "Std. Error"]

  # 計算 t 統計量 (分母為標準誤平方和)
  denom_sq <- se_fe^2 - se_re^2
  t_value <- (b_fe - b_re) / sqrt(denom_sq)
  p_value <- 2 * (1 - pnorm(abs(t_value)))

  cat(sprintf("%-12s : t = %6.3f, p-value = %.4f => %s\n",
              v, t_value, p_value,
              ifelse(p_value < 0.05, "有顯著差異", "無顯著差異")))
}
phtest(fe_model, re_model)

#(f)
library(dplyr)


# 建立 school-level means (以 schid 為單位)
star <- star %>%
  group_by(schid) %>%
  mutate(
    small_m       = mean(small, na.rm = TRUE),
    aide_m        = mean(aide, na.rm = TRUE),
    tchexper_m    = mean(tchexper, na.rm = TRUE),
    boy_m         = mean(boy, na.rm = TRUE),
    white_asian_m = mean(white_asian, na.rm = TRUE),
    freelunch_m   = mean(freelunch, na.rm = TRUE)
  ) %>%
  ungroup()

# 建立 panel 結構資料
star_panel <- pdata.frame(star, index = c("schid", "studentid"))

# 移除 NA 值 (防止估計錯誤)
star_panel_clean <- na.omit(star_panel)

# 執行 Mundlak 模型 (RE 模型加入 school-level means)
mundlak_model <- plm(
  readscore ~ small + aide + tchexper + boy + white_asian + freelunch +
    small_m + aide_m + tchexper_m + boy_m + white_asian_m + freelunch_m,
  data = star_panel_clean,
  model = "random"
)

# 查看結果
summary(mundlak_model)
```