CH11

(a) $\begin{cases} y_1 = \alpha_1 y_2 + e_1 \\ y_2 = \alpha_2 y_1 + \beta_1 X_1 + \beta_2 X_2 + e_2 \end{cases}$

$$y_2 = \alpha_2(\alpha_1 y_2 + e_1) + \beta_1 X_1 + \beta_2 X_2 + e_2$$
$$y_2(1-\alpha_1\alpha_2) = \beta_1 X_1 + \beta_2 X_2 + (\alpha_2 e_1 + e_2)$$
$$y_2 = \frac{\beta_1}{1-\alpha_1\alpha_2} X_1 + \frac{\beta_2}{1-\alpha_1\alpha_2} X_2 + \frac{\alpha_2 e_1 + e_2}{1-\alpha_1\alpha_2}$$
$$\pi_1 = \frac{\beta_1}{1-\alpha_1\alpha_2}, \quad \pi_2 = \frac{\beta_2}{1-\alpha_1\alpha_2}, \quad v_2 = \frac{\alpha_2 e_1 + e_2}{1-\alpha_1\alpha_2}$$

$$\begin{aligned} cov(y_2, e_1 \mid x) &= E(y_2, e_1 \mid x) \\ &= E\left[\left(\frac{\beta_1}{1-\alpha_1\alpha_2} X_1 + \frac{\beta_2}{1-\alpha_1\alpha_2} X_2 + \frac{\alpha_2 e_1 + e_2}{1-\alpha_1\alpha_2}\right) e_1 \mid x\right] \\ &= E\left[\left(\frac{\beta_1}{1-\alpha_1\alpha_2} X_1 e_1 \mid x\right)\right] + E\left[\left(\frac{\beta_2}{1-\alpha_1\alpha_2} X_2 e_1 \mid x\right)\right] + E\left[\left(\frac{\alpha_2 e_1 + e_2}{1-\alpha_1\alpha_2} e_1 \mid x\right)\right] \\ &= E\left[\left(\frac{\alpha_2 e_1 + e_2}{1-\alpha_1\alpha_2} e_1 \mid x\right)\right] \\ &= \frac{\alpha_2 E(e_1^2 \mid x) + E(e_1 e_2 \mid x)}{1-\alpha_1\alpha_2} = \frac{\alpha_2 \sigma_1^2}{1-\alpha_1\alpha_2} > 0 \text{ unless } \alpha_2 = 0. \end{aligned}$$

(b) Since both equations have endogeneous variables, the OLS is biased and consistent.

(c) Since $M=2$ and $2-1=1$, at least 1 variable needs to be omitted from equations.
(1) It omitted two exogeneous varibables. ⟶ "identified"
(2) It omitted no variables. ⟶ "not identified"

(d) $E(X_{i1} V_{i2} \mid x) = E(X_{i2} V_{i2} \mid x) = 0$
Therefore, $E\left[X_{ik}\left(\frac{\alpha_2 e_1 + e_2}{1-\alpha_1\alpha_2} \mid x\right)\right] = E\left[\frac{\alpha_1}{1-\alpha_1\alpha_2} e_1 X_{ik} \mid x\right] + E\left[\frac{1}{1-\alpha_1\alpha_2} e_2 X_{ik} \mid x\right] = 0.$

(e) $\begin{cases} \frac{d}{d\pi_1} \Sigma(y_2 - \pi_1 X_1 - \pi_2 X_2)^2 = 2\Sigma(y_2 - \pi_1 X_1 - \pi_2 X_2) \times (-X_1) = 0 \longrightarrow N^{-1}\Sigma X_{i1}(y_2 - \pi_1 X_{i1} - \pi_2 X_{i2}) = 0 \\ \frac{d}{d\pi_2} \Sigma(y_2 - \pi_1 X_1 - \pi_2 X_2)^2 = 2\Sigma(y_2 - \pi_1 X_1 - \pi_2 X_2) \times (-X_2) = 0 \longrightarrow N^{-1}\Sigma X_{i2}(y_2 - \pi_1 X_{i1} - \pi_2 X_{i2}) = 0 \end{cases}$

(f) $\begin{cases} \Sigma X_{i1}(y_2 - \pi_1 X_{i1} - \pi_2 X_{i2}) = 0 \rightarrow \Sigma X_{i1} y_2 - \pi_1 \Sigma X_{i1}^2 - \pi_2 \Sigma X_{i1} X_{i2} = 0 \longrightarrow 3 - \pi_1 = 0, \ \pi_1 = 3 \\ \Sigma X_{i2}(y_2 - \pi_1 X_{i1} - \pi_2 X_{i2}) = 0 \rightarrow \Sigma X_{i2} y_2 - \pi_1 \Sigma X_{i1} X_{i2} - \pi_2 \Sigma X_{i2}^2 = 0 \longrightarrow 4 - \pi_2 = 0, \ \pi_2 = 4 \end{cases}$

(g) $\because \hat{y}_2 = \hat{\pi}_1 X_1 + \hat{\pi}_2 X_2$
$\therefore \Sigma \hat{y}_2 (y_1 - \alpha_1 y_2) = 0, \ \Sigma \hat{y}_2 y_1 - \alpha_1 \Sigma \hat{y}_2 y_2 = 0, \ \alpha_1 = \frac{\Sigma \hat{y}_2 y_1}{\Sigma \hat{y}_2 y_2}$
$\rightarrow \hat{\alpha}_1 = \frac{\Sigma(\hat{\pi}_1 X_1 + \hat{\pi}_2 X_2) y_1}{\Sigma(\hat{\pi}_1 X_1 + \hat{\pi}_2 X_2) y_2} = \frac{3\Sigma X_1 y_1 + 4\Sigma X_2 y_1}{3\Sigma X_1 y_2 + 4\Sigma X_2 y_2} = \frac{3\times 2 + 4\times 3}{3\times 3 + 4\times 4} = \frac{18}{25}$

(h) To prove $\hat{\alpha}_{1,2SLS} = \frac{\Sigma \hat{y}_2 y_1}{\Sigma \hat{y}_2^2} = \hat{\alpha}_1$, we need to prove $\Sigma \hat{y}_2^2 = \Sigma \hat{y}_2 y_2$.
And, $\Sigma \hat{y}_2^2 = \Sigma \hat{y}_2(y_2 - \hat{v}_2) = \Sigma \hat{y}_2 y_2 - \Sigma \hat{y}_2 \hat{v}_2 = \Sigma \hat{y}_2 y_2$.

16.

(a) $\alpha_1 + \alpha_2 P_i + e_{di} = \beta_1 + \beta_2 P_i + \beta_3 W_i + e_{si}$

$(\alpha_2 - \beta_2) P_i = (\beta_1 - \alpha_1) + \beta_3 W_i + (e_{si} - e_{di})$

$P_i = \frac{\beta_1 - \alpha_1}{\alpha_2 - \beta_2} + \frac{\beta_3}{\alpha_2 - \beta_2} W_i + \frac{e_{si} - e_{di}}{\alpha_2 - \beta_2}$

$Q_i = \alpha_1 + \alpha_2 \left(\frac{\beta_1 - \alpha_1}{\alpha_2 - \beta_2} + \frac{\beta_3}{\alpha_2 - \beta_2} W_i + \frac{e_{si} - e_{di}}{\alpha_2 - \beta_2} \right) + e_{di}$

$Q_i = \alpha_1 + \frac{\beta_1 - \alpha_1}{\alpha_2 - \beta_2} \alpha_2 + \frac{\beta_3}{\alpha_2 - \beta_2} W_i \alpha_2 + \frac{e_{si} - e_{di}}{\alpha_2 - \beta_2} \alpha_2 + e_{di}$

(b) Since $M = 2$ and $2 - 1 = 1$, at least 1 variable needs to be omitted from equations.

(1) It omitted two exogeneous varibables. $\longrightarrow$ "identified"

(2) It omitted no variables. $\longrightarrow$ "not identified"

(c) $\begin{cases} \hat{Q} = 5 + 0.5W & \longrightarrow \quad 5 + 0.5W = \alpha_1 + \alpha_2 (2.4 + W) = (\alpha_1 + 2.4\alpha_2) + \alpha_2 W \\ \hat{P} = 2.4 + W & \longrightarrow \quad \alpha_1 = 3.8,\ \alpha_2 = 0.5 \end{cases}$

(d) $\hat{P} = 2.4 + W$

W	$\hat{P}$	$\hat{P} - \bar{\hat{P}}$	$Q - \bar{Q}$
2	4.4	0	-2
3	5.4	1	0
1	3.4	-1	3
1	3.4	-1	-3
3	5.4	1	2

$\longrightarrow \bar{\hat{P}} = 4.4,\ \bar{Q} = 6$

$Q = \alpha_1 + \alpha_2 \hat{P} + e_i$

$\hat{\alpha}_2 = \frac{\Sigma(\hat{P}_i - \bar{\hat{P}})(Q_i - \bar{Q})}{\Sigma(\hat{P}_i - \bar{\hat{P}})^2} = \frac{1}{2}$

$\hat{\alpha}_1 = \bar{Q} - \alpha_2 \bar{\hat{P}} = 6 - 0.5 \times 4.4 = 3.8$

$\longrightarrow \hat{Q} = 3.8 + 0.5P$

17.

(a) Since M=8 and 8-1=7. at least 7 variable needs to be omitted from equations.

Consumption — It omitted 10 exogeneous varibables.

Investment — It omitted 11 exogeneous varibables.

Wage ——— It omitted 11 exogeneous varibables.

⟶ all functions are "identified."

(b)

	endogeneous variables		exogeneous variables
Consumption —	2	<	5
Investment —	1	<	5
Wage ———	1	<	5

⟶ all functions are satisfied.

(c) $W_{1t} = \pi_1 + \pi_2 G_t + \pi_3 W_{2t} + \pi_4 Tx_t + \pi_5 TIME_t + \pi_6 P_{t-1} + \pi_7 K_{t-1} + \pi_8 E_{t-1} + V$

(d) 1. Get $\hat{W}_{1t}$ from (c)

2. Use the same method as $\hat{P}_t$.

3. Create $W_t^* = \hat{W}_{1t} + W_{2t}$

4. Regress CN_t by OLS.

(e) Two coefficient will be the same, but t-value will be different.