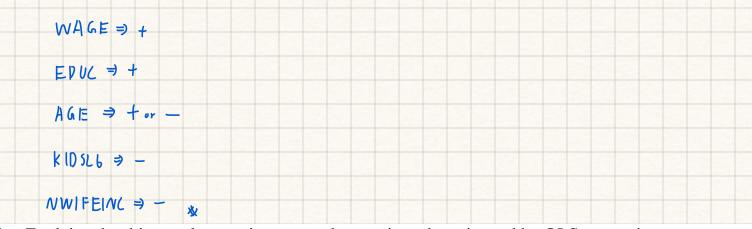
**10.2** The labor supply of married women has been a subject of a great deal of economic research. Consider the following supply equation specification

$$HOURS = \beta_1 + \beta_2 WAGE + \beta_3 EDUC + \beta_4 AGE + \beta_5 KIDSL6 + \beta_6 NWIFEINC + e$$

where *HOURS* is the supply of labor, *WAGE* is hourly wage, *EDUC* is years of education, *KIDSL6* is the number of children in the household who are less than 6 years old, and *NWIFEINC* is household income from sources other than the wife's employment.

**a.** Discuss the signs you expect for each of the coefficients.



**b.** Explain why this supply equation cannot be consistently estimated by OLS regression.

Because WAGE is endogenous and likely curelated with the error term,

the DLS estimator will be biased and inconsistent. This violets the

exogeneity assumption regained for consistent DLS estimation.

**c.** Suppose we consider the woman's labor market experience *EXPER* and its square, *EXPER*<sup>2</sup>, to be instruments for *WAGE*. Explain how these variables satisfy the logic of instrumental variables.

EXPER and EXPER? we valid instruments for WHGE because they one concluded with WAGE (relevence) but do not directly affect HUNRS (exogeneity), only through their import on wages.

d. Is the supply equation identified? Explain.

Yes, the supply equation is identified.

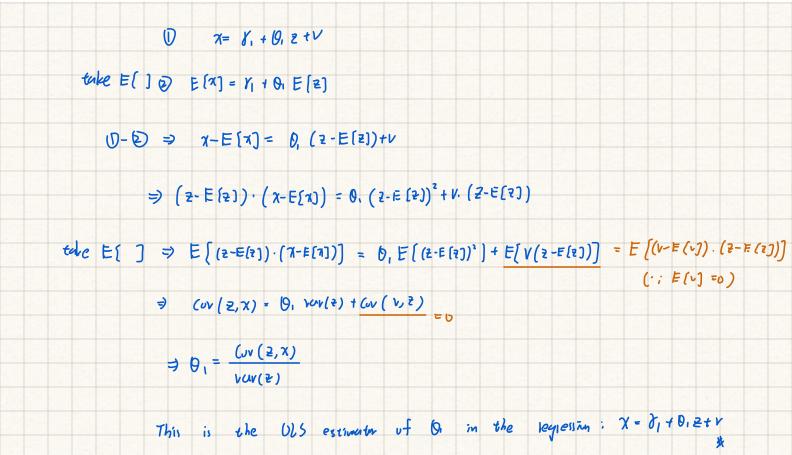
Since there are valid instruments for the andogenous variable WIGE—

spacially EXPER and EXPER—— and there are more instruments than

en degenous regressors, the equation satisfies the order condition for identification.

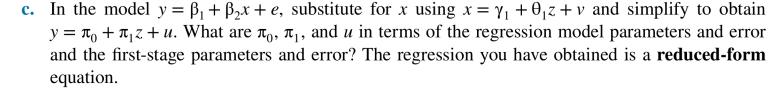
	1.	First	stage:	Regres	WAG	Eun	the	instru	ments	(EXP	512	und	EXP	EK')	und	
	Z.	Second	other ex		wgenous	genous variab		to u	brain	the	fitte	2	values	υf	WA	6E.
			stage:	: Regress	Hou	RS un	th	e fit	1ed	vulues	of	W	96E	and	the	oth
				exogenou									atim.			
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- 10.3 In the regression model  $y = \beta_1 + \beta_2 x + e$ , assume x is endogenous and that z is a valid instrument. In Section 10.3.5, we saw that  $\beta_2 = \cos(z, y)/\cos(z, x)$ .
  - a. Divide the denominator of  $\beta_2 = \text{cov}(z, y)/\text{cov}(z, x)$  by var(z). Show that cov(z, x)/var(z) is the coefficient of the simple regression with dependent variable x and explanatory variable z,  $x = \gamma_1 + \theta_1 z + v$ . [Hint: See Section 10.2.1.] Note that this is the first-stage equation in two-stage least squares.



**b.** Divide the numerator of  $\beta_2 = \cos(z, y)/\cos(z, x)$  by  $\sin(z)$ . Show that  $\cos(z, y)/\sin(z)$  is the coefficient of a simple regression with dependent variable y and explanatory variable z,  $y = \pi_0 + \pi_1 z + u$ . [Hint: See Section 10.2.1.]

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**d.** Show that  $\beta_2 = \pi_1/\theta_1$ .

$$\overline{\Lambda}_1 = B_2 O_1 \Rightarrow B_2 = \frac{\overline{\Lambda}_1}{D_1}$$

e. If  $\hat{\pi}_1$  and  $\hat{\theta}_1$  are the OLS estimators of  $\pi_1$  and  $\theta_1$ , show that  $\hat{\beta}_2 = \hat{\pi}_1/\hat{\theta}_1$  is a consistent estimator of  $\beta_2 = \pi_1/\theta_1$ . The estimator  $\hat{\beta}_2 = \hat{\pi}_1/\hat{\theta}_1$  is an **indirect least squares** estimator.

$$\hat{Q}_{1} = \frac{Cov(2, x)}{V\hat{\omega}(2)} = \frac{\Sigma(2_{1} - \overline{z})(x_{1} - \overline{z})}{\Sigma(2_{1} - \overline{z})^{2}}$$

$$\hat{A}_{1} = \frac{(\hat{v}(2, x))}{V\hat{\omega}v^{2}} = \frac{\Sigma(2_{1} - \overline{z})(y_{1} - \overline{z})}{\Sigma(2_{1} - \overline{z})^{2}}$$

$$\hat{A}_{2} = \frac{\hat{A}_{1}}{\hat{A}_{1}} = \frac{\Sigma(2_{1} - \overline{z})(y_{1} - \overline{z})}{\Sigma(2_{1} - \overline{z})(x_{1} - \overline{z})} = \frac{\hat{Cov}(2, x)}{\hat{Cov}(2, x)}$$

$$\hat{B}_{2} = \frac{\hat{Cov}(2, x)}{\hat{Cov}(2, x)} + \frac{\hat{Cov}(2, x)}{\hat{Cov}(2, x)} = \hat{B}_{2}$$

$$\hat{Cov}(2, x) = \hat{B}_{2}$$

$$\hat{Cov}(2, x) = \hat{B}_{2}$$