10.18 Consider the data file *mroz* on working wives. Use the 428 observations on married women who participate in the labor force. In this exercise, we examine the effectiveness of a parent's college education as an instrumental variable.

a. Create two new variables. *MOTHERCOLL* is a dummy variable equaling one if *MOTHEREDUC* > 12, zero otherwise. Similarly, *FATHERCOLL* equals one if *FATHEREDUC* > 12 and zero otherwise. What percentage of parents have some college education in this sample?

```
> cat("母親有部分大學教育的比例：", round(mother_coll_pct, 2), "%\n")
母親有部分大學教育的比例： 12.15 %
> cat("父親有部分大學教育的比例：", round(father_coll_pct, 2), "%\n")
父親有部分大學教育的比例： 11.68 %
```

b. Find the correlations between *EDUC*, *MOTHERCOLL*, and *FATHERCOLL*. Are the magnitudes of these correlations important? Can you make a logical argument why *MOTHERCOLL* and *FATHERCOLL* might be better instruments than *MOTHEREDUC* and *FATHEREDUC*?

```
> print(correlations)
                educ MOTHERCOLL FATHERCOLL
educ       1.0000000  0.3594705  0.3984962
MOTHERCOLL 0.3594705  1.0000000  0.3545709
FATHERCOLL 0.3984962  0.3545709  1.0000000
```

這兩個變數與 educ 有部分的關聯，可能是具關聯性的工具變數

具有大學學歷更能反映家庭資本、地位或教育觀念，影響子女的教育程度，但不直接影響勞動收入

c. Estimate the wage equation in Example 10.5 using *MOTHERCOLL* as the instrumental variable. What is the 95% interval estimate for the coefficient of *EDUC*?

```
> confint(iv_model, level = 0.95)["educ", ]
       2.5 %       97.5 %
-0.001219763  0.153255678
> cat('95% Confidence Interval: [',confint(iv_model, leve
95% Confidence Interval: [ -0.001219763 0.1532557 ]
>
```

d. For the problem in part (c), estimate the first-stage equation. What is the value of the *F*-test statistic for the hypothesis that *MOTHERCOLL* has no effect on *EDUC*? Is *MOTHERCOLL* a strong instrument?

```
Linear hypothesis test:
MOTHERCOLL = 0

Model 1: restricted model
Model 2: educ ~ MOTHERCOLL + exper + exper2

  Res.Df    RSS Df Sum of Sq      F    Pr(>F)
1    425 2219.2
2    424 1929.9  1    289.32 63.563 1.455e-14 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
>
```

F>10➔Strong instrument

e. Estimate the wage equation in Example 10.5 using *MOTHERCOLL* and *FATHERCOLL* as the instrumental variables. What is the 95% interval estimate for the coefficient of *EDUC*? Is it narrower or wider than the one in part (c)?

```
95% Confidence Interval for EDUC (with 2 IVs): [ 0.02751845 , 0.1481769 ]
```

➔變窄了

f. For the problem in part (e), estimate the first-stage equation. Test the joint significance of *MOTHERCOLL* and *FATHERCOLL*. Do these instruments seem adequately strong?

```
Linear hypothesis test:
MOTHERCOLL = 0
FATHERCOLL = 0

Model 1: restricted model
Model 2: educ ~ exper + exper2 + MOTHERCOLL + FATHERCOLL

  Res.Df    RSS Df Sum of Sq      F    Pr(>F)    
1    425 2219.2                                  
2    423 1748.3  2    470.88 56.963 < 2.2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

F>10 且 P<0.05➔強工具變數

g. For the IV estimation in part (e), test the validity of the surplus instrument. What do you conclude?

```
> cat("Sargan statistic:", sargan_stat, "\n")
Sargan statistic: 0.2375851
> cat("p-value:", p_value, "\n")
p-value: 0.6259557
```

➔工具變數有效

10.20 The CAPM [see Exercises 10.14 and 2.16] says that the risk premium on security j is related to the risk premium on the market portfolio. That is

$$r_j - r_f = \alpha_j + \beta_j(r_m - r_f)$$

where r_j and r_f are the returns to security j and the risk-free rate, respectively, r_m is the return on the market portfolio, and β_j is the jth security's "*beta*" value. We measure the market portfolio using the Standard & Poor's value weighted index, and the risk-free rate by the 30-day LIBOR monthly rate of return. As noted in Exercise 10.14, if the market return is measured with error, then we face an errors-in-variables, or measurement error, problem.

a. Use the observations on Microsoft in the data file *capm5* to estimate the CAPM model using OLS. How would you classify the Microsoft stock over this period? Risky or relatively safe, relative to the market portfolio?

```
Call:
lm(formula = excess_msft ~ excess_mkt, data = capm5)

Residuals:
     Min       1Q   Median       3Q      Max 
-0.27424 -0.04744 -0.00820  0.03869  0.35801 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 0.003250   0.006036   0.538    0.591    
excess_mkt  1.201840   0.122152   9.839   <2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.08083 on 178 degrees of freedom
Multiple R-squared:  0.3523,	Adjusted R-squared:  0.3486 
F-statistic:  96.8 on 1 and 178 DF,  p-value: < 2.2e-16
```

$\beta_{msft} = 1.20184 > 1$➔波動相較於$r_m$大➔Risky

b. It has been suggested that it is possible to construct an IV by ranking the values of the explanatory variable and using the rank as the IV, that is, we sort $(r_m - r_f)$ from smallest to largest, and assign the values $RANK = 1, 2, \ldots, 180$. Does this variable potentially satisfy the conditions IV1–IV3? Create *RANK* and obtain the first-stage regression results. Is the coefficient of *RANK* very significant? What is the R^2 of the first-stage regression? Can *RANK* be regarded as a strong IV?

```
Call:
lm(formula = excess_mkt ~ RANK, data = capm5)

Residuals:
      Min        1Q    Median        3Q       Max 
-0.110497 -0.006308  0.001497  0.009433  0.029513 

Coefficients:
              Estimate Std. Error t value Pr(>|t|)    
(Intercept) -7.903e-02  2.195e-03   -36.0   <2e-16 ***
RANK         9.067e-04  2.104e-05    43.1   <2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.01467 on 178 degrees of freedom
Multiple R-squared:  0.9126,	Adjusted R-squared:  0.9121 
F-statistic:  1858 on 1 and 178 DF,  p-value: < 2.2e-16
```

Rsquare0.9126，表示 Rank 對$r_m - r_f$有極高解釋力/相關性，另外 Rank 與 error 無關且並無直接解釋 y 變數，因此是不錯的工具變數

c. Compute the first-stage residuals, $\hat{v}$, and add them to the CAPM model. Estimate the resulting augmented equation by OLS and test the significance of $\hat{v}$ at the 1% level of significance. Can we conclude that the market return is exogenous?

```
Call:
lm(formula = excess_msft ~ excess_mkt + v_hat, data = capm5)

Residuals:
     Min       1Q   Median       3Q      Max 
-0.27140 -0.04213 -0.00911  0.03423  0.34887 

Coefficients:
             Estimate Std. Error t value Pr(>|t|)    
(Intercept)  0.003018   0.005984   0.504   0.6146    
excess_mkt   1.278318   0.126749  10.085   <2e-16 ***
v_hat       -0.874599   0.428626  -2.040   0.0428 *  
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.08012 on 177 degrees of freedom
Multiple R-squared:  0.3672,	Adjusted R-squared:   0.36 
F-statistic: 51.34 on 2 and 177 DF,  p-value: < 2.2e-16
```

V_{hat}的 P value=0.0428，在 1%顯著水準下不顯著，無法拒絕市場報酬是外生的假設

d. Use *RANK* as an IV and estimate the CAPM model by IV/2SLS. Compare this IV estimate to the OLS estimate in part (a). Does the IV estimate agree with your expectations?

```
Call:
ivreg(formula = excess_msft ~ excess_mkt | RANK, data = capm5)

Residuals:
      Min        1Q    Median        3Q       Max 
-0.271625 -0.049675 -0.009693  0.037683  0.355579 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 0.003018   0.006044   0.499    0.618    
excess_mkt  1.278318   0.128011   9.986   <2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.08092 on 178 degrees of freedom
Multiple R-Squared: 0.3508,	Adjusted R-squared: 0.3472 
Wald test: 99.72 on 1 and 178 DF,  p-value: < 2.2e-16
```

正常來說 measurement error 會使β被低估，而$\beta_{2SLS} = 1.278318 > \beta_{msft} = 1.20184$符合被低估得觀點，因此加入 IV 後的結果與預期一致

e. Create a new variable $POS = 1$ if the market return $(r_m - r_f)$ is positive, and zero otherwise. Obtain the first-stage regression results using both *RANK* and *POS* as instrumental variables. Test the joint significance of the IV. Can we conclude that we have adequately strong IV? What is the R^2 of the first-stage regression?

```
Call:
lm(formula = excess_mkt ~ RANK + POS, data = capm5)

Residuals:
      Min        1Q    Median        3Q       Max 
-0.109182 -0.006732  0.002858  0.008936  0.026652 

Coefficients:
              Estimate Std. Error t value Pr(>|t|)    
(Intercept) -0.0804216  0.0022622  -35.55   <2e-16 ***
RANK         0.0009819  0.0000400   24.55   <2e-16 ***
POS         -0.0092762  0.0042156   -2.20   0.0291 *  
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.01451 on 177 degrees of freedom
Multiple R-squared:  0.9149,	Adjusted R-squared:  0.9139 
F-statistic: 951.3 on 2 and 177 DF,  p-value: < 2.2e-16
```

```
Linear hypothesis test:
RANK = 0
POS = 0

Model 1: restricted model
Model 2: excess_mkt ~ RANK + POS

  Res.Df     RSS Df Sum of Sq      F    Pr(>F)    
1    179 0.43784                                  
2    177 0.03727  2   0.40057 951.26 < 2.2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

p-value < 0.01 拒絕 H_o表示兩個工具變數至少一個是顯著可用的，$R^2 = 0.9149$，另外 F 檢定 951>10 表示為強工具變數

f. Carry out the Hausman test for endogeneity using the residuals from the first-stage equation in (e). Can we conclude that the market return is exogenous at the 1% level of significance?

```
Call:
lm(formula = excess_msft ~ excess_mkt + v_hat2, data = capm5)

Residuals:
     Min       1Q   Median       3Q      Max 
-0.27132 -0.04261 -0.00812  0.03343  0.34867 

Coefficients:
             Estimate Std. Error t value Pr(>|t|)    
(Intercept)  0.003004   0.005972   0.503   0.6157    
excess_mkt   1.283118   0.126344  10.156   <2e-16 ***
v_hat2      -0.954918   0.433062  -2.205   0.0287 *  
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.07996 on 177 degrees of freedom
Multiple R-squared:  0.3696,	Adjusted R-squared:  0.3625 
F-statistic: 51.88 on 2 and 177 DF,  p-value: < 2.2e-16
```

V_hat2的 P value 為 0.0287>0.01➔無法拒絕 market return 是外生的

g. Obtain the IV/2SLS estimates of the CAPM model using *RANK* and *POS* as instrumental variables. Compare this IV estimate to the OLS estimate in part (a). Does the IV estimate agree with your expectations?

```
Call:
ivreg(formula = excess_msft ~ excess_mkt | RANK + POS, data = capm5)

Residuals:
     Min       1Q   Median       3Q      Max 
-0.27168 -0.04960 -0.00983  0.03762  0.35543 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 0.003004   0.006044   0.497     0.62    
excess_mkt  1.283118   0.127866  10.035   <2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.08093 on 178 degrees of freedom
Multiple R-Squared: 0.3507,	Adjusted R-squared: 0.347 
Wald test: 100.7 on 1 and 178 DF,  p-value: < 2.2e-16 
```

$\beta_{2iv2SLS} = 1.283118 > \beta_{msft} = 1.20184$，符合原先模型的$\beta$低估的預期

h. Obtain the IV/2SLS residuals from part (g) and use them (not an automatic command) to carry out a Sargan test for the validity of the surplus IV at the 5% level of significance.

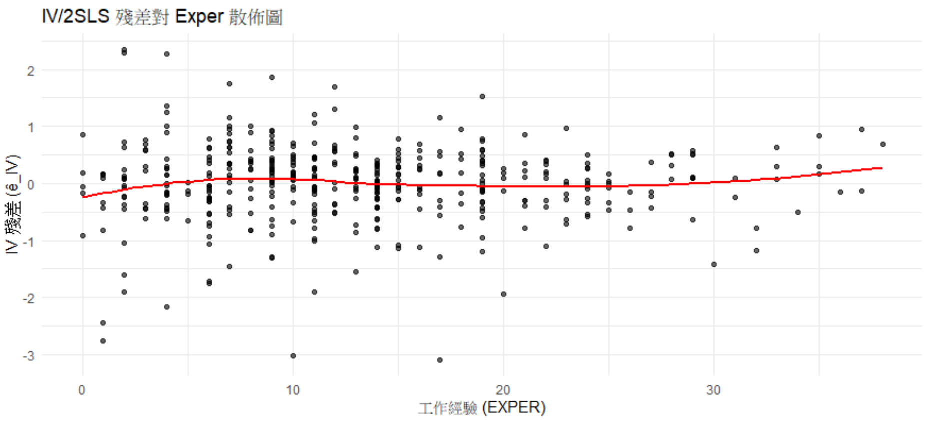
```
Call:
lm(formula = resid_iv ~ capm5$RANK + capm5$POS)

Residuals:
     Min       1Q   Median       3Q      Max 
-0.26914 -0.04702 -0.00801  0.03771  0.35674 

Coefficients:
              Estimate Std. Error t value Pr(>|t|)
(Intercept) -0.0022220  0.0126326  -0.176    0.861
capm5$RANK   0.0001370  0.0002234   0.613    0.540
capm5$POS   -0.0174499  0.0235409  -0.741    0.460

Residual standard error: 0.08103 on 177 degrees of freedom
Multiple R-squared:  0.003103,	Adjusted R-squared:  -0.008162 
F-statistic: 0.2754 on 2 and 177 DF,  p-value: 0.7596
```

n	180L
p_value	0.454880022254587
R2_sargan	0.00310257448755873
resid_iv	Named num [1:180] 0.14674 0.04036 -0.0115…
sargan_stat	0.558463407760572

Sargan test 結果中，P value 大於 0.05➔無法拒絕工具變數有效的虛無假設

10.24 Consider the data file *mroz* on working wives. Use the 428 observations on married women who participate in the labor force. In this exercise, we examine the effectiveness of alternative standard errors for the IV estimator. Estimate the model in Example 10.5 using IV/2SLS using both *MOTHEREDUC* and *FATHEREDUC* as IV. These will serve as our baseline results.

a. Calculate the IV/2SLS residuals, $\hat{e}_{IV}$. Plot them versus *EXPER*. Do the residuals exhibit a pattern consistent with homoskedasticity?



殘差呈現漏斗形➔heteroskedasticity

b. Regress $\hat{e}_{IV}^2$ against a constant and *EXPER*. Apply the NR^2 test from Chapter 8 to test for the presence of heteroskedasticity.

n	428L
NR2	7.43855244235516
p_value	0.00638412191828993
R2	0.0173797954260635

P value<0.05➔拒絕虛無假設(homoskedasticity) ➔heteroskedasticity

c. Obtain the IV/2SLS estimates with the software option for Heteroskedasticity Robust Standard Errors. Are the robust standard errors larger or smaller than those for the baseline model? Compute the 95% interval estimate for the coefficient of *EDUC* using the robust standard error.

```
> # 顯示結果
> cat("基準模型中的 EDUC 係數標準誤：", base_se_educ, "\n")
基準模型中的 EDUC 係數標準誤： 0.0314367
> cat("穩健模型中的 EDUC 係數標準誤：", robust_se_educ, "\n")
穩健模型中的 EDUC 係數標準誤： 0.03365975
```

➔穩健模型中 EDUC 係數的標準誤較大

```
EDUC 係數的 95% 區間估計： -0.00457648 到 0.1273697
```

d. Obtain the IV/2SLS estimates with the software option for Bootstrap standard errors, using $B = 200$ bootstrap replications. Are the bootstrap standard errors larger or smaller than those for the baseline model? How do they compare to the heteroskedasticity robust standard errors in (c)? Compute the 95% interval estimate for the coefficient of *EDUC* using the bootstrap standard error.

```
EDUC 係數的 95% 信賴區間（Bootstrap）： -0.8414525 到 0.8751997
```

```
> # 比較 Bootstrap 標準誤差與基準模型及穩健模型的差異
> if (bootstrap_se_educ > base_se_educ) {
+   cat("Bootstrap 標準誤差較大\n")
+ } else {
+   cat("Bootstrap 標準誤差較小\n")
+ }
Bootstrap 標準誤差較大
>
> if (bootstrap_se_educ > robust_se_educ) {
+   cat("Bootstrap 標準誤差較大\n")
+ } else {
+   cat("Bootstrap 標準誤差較小\n")
+ }
Bootstrap 標準誤差較大
```