8.6 Consider the wage equation

$$WAGE_i = \beta_1 + \beta_2 EDUC_i + \beta_3 EXPER_i + \beta_4 METRO_i + e_i \tag{XR8.6a}$$

where wage is measured in dollars per hour, education and experience are in years, and $METRO = 1$ if the person lives in a metropolitan area. We have $N = 1000$ observations from 2013.

a. We are curious whether holding education, experience, and *METRO* constant, there is the same amount of random variation in wages for males and females. Suppose $\text{var}(e_i|\mathbf{x}_i, FEMALE = 0) = \sigma_M^2$ and $\text{var}(e_i|\mathbf{x}_i, FEMALE = 1) = \sigma_F^2$. We specifically wish to test the null hypothesis $\sigma_M^2 = \sigma_F^2$ against $\sigma_M^2 \neq \sigma_F^2$. Using 577 observations on males, we obtain the sum of squared OLS residuals, $SSE_M = 97161.9174$. The regression using data on females yields $\hat{\sigma}_F = 12.024$. Test the null hypothesis at the 5% level of significance. Clearly state the value of the test statistic and the rejection region, along with your conclusion.

$$\begin{cases} H_0: \sigma_M^2 = \sigma_F^2 \\ H_a: \sigma_M^2 \neq \sigma_F^2 \end{cases} \qquad \hat{\sigma}_M^2 = \frac{SSE}{577-4} = 169.5670$$

$$F = \frac{\sigma_M^2}{\sigma_F^2} = \frac{169.5670}{12.024^2} = 1.1729$$

Reject region $\{F \mid F \leq 0.8377 \text{ or } F \geq 1.1968\}$

$F \notin RR$

There is no enough evidence at the 5% level to say that males and females have difference random variation of wage

b. We hypothesize that married individuals, relying on spousal support, can seek wider employment types and hence holding all else equal should have more variable wages. Suppose $\text{var}(e_i|\mathbf{x}_i, MARRIED = 0) = \sigma^2_{SINGLE}$ and $\text{var}(e_i|\mathbf{x}_i, MARRIED = 1) = \sigma^2_{MARRIED}$. Specify the null hypothesis $\sigma^2_{SINGLE} = \sigma^2_{MARRIED}$ versus the alternative hypothesis $\sigma^2_{MARRIED} > \sigma^2_{SINGLE}$. We add *FEMALE* to the wage equation as an explanatory variable, so that

$$WAGE_i = \beta_1 + \beta_2 EDUC_i + \beta_3 EXPER_i + \beta_4 METRO_i + \beta_5 FEMALE + e_i \tag{XR8.6b}$$

Using $N = 400$ observations on single individuals, OLS estimation of (XR8.6b) yields a sum of squared residuals is 56231.0382. For the 600 married individuals, the sum of squared errors is 100,703.0471. Test the null hypothesis at the 5% level of significance. Clearly state the value of the test statistic and the rejection region, along with your conclusion.

$$\begin{cases} H_0: \sigma^2_{single} = \sigma^2_{married} \\ H_a: \sigma^2_{single} < \sigma^2_{married} \end{cases} \qquad \sigma^2_{single} = \frac{56231.0382}{400-5} = 142.3571$$

$$\sigma^2_{married} = \frac{100703.0471}{600-5} = 169.2488$$

$$F = \frac{\sigma^2_{single}}{\sigma^2_{married}} = 0.8411$$

Reject Region: $\{F \mid F \leq F_{0.025, 395, 595} = 0.8338\}$

There is no enough evidence at the 5% level to say that married and singled female have difference random variation of wage

c. Following the regression in part (b), we carry out the NR^2 test using the right-hand-side variables in (XR8.6b) as candidates related to the heteroskedasticity. The value of this statistic is 59.03. What do we conclude about heteroskedasticity, at the 5% level? Does this provide evidence about the issue discussed in part (b), whether the error variation is different for married and unmarried individuals? Explain.

$\chi^2 = 59.03 \qquad \chi^2_{(0.95,4)} = 9.4877$

$\chi^2 > \chi^2_{0.95,4}$

We reject null hypothesis and conclude the heteroskedasticity exists.

This result does not directly support (b) which was about whether the error variance differs between married and singled. This model does not include marital status as one of explanatory variables used to detect variance differences.

d. Following the regression in part (b) we carry out the White test for heteroskedasticity. The value of the test statistic is 78.82. What are the degrees of freedom of the test statistic? What is the 5% critical value for the test? What do you conclude?

$S = 5 + 2 + 6 = 13 \qquad df = 13 - 1 = 12$

4平方 → 交乘

$\chi^2 = 78.82 \qquad \chi^2_{(0.95,12)} = 21.0267$

$\chi^2 > \chi^2_{0.95,12}$

We reject null hypothesis and conclude the heteroskedasticity exists.

e. The OLS fitted model from part (b), with usual and robust standard errors, is

$$\widehat{WAGE} = -17.77 + 2.50EDUC + 0.23EXPER + 3.23METRO - 4.20FEMALE$$

(se)	(2.36)	(0.14)	(0.031)	(1.05)	(0.81)
(robse)	(2.50)	(0.16)	(0.029)	(0.84)	(0.80)

For which coefficients have interval estimates gotten narrower? For which coefficients have interval estimates gotten wider? Is there an inconsistency in the results?

Gotten narrower: EXPER. METRO. FEMALE

Gotten wider: Intercept. EDUC

This result is not inconsistent

f. If we add *MARRIED* to the model in part (b), we find that its t-value using a White heteroskedasticity robust standard error is about 1.0. Does this conflict with, or is it compatible with, the result in (b) concerning heteroskedasticity? Explain.

It is compatible. Both results suggest that marital status does not have a significant effect. (b) is test error variation between married and singled. (f) is test the relation between marital and mean wage.

8.16 A sample of 200 Chicago households was taken to investigate how far American households tend to travel when they take a vacation. Consider the model

$$MILES = \beta_1 + \beta_2 INCOME + \beta_3 AGE + \beta_4 KIDS + e$$

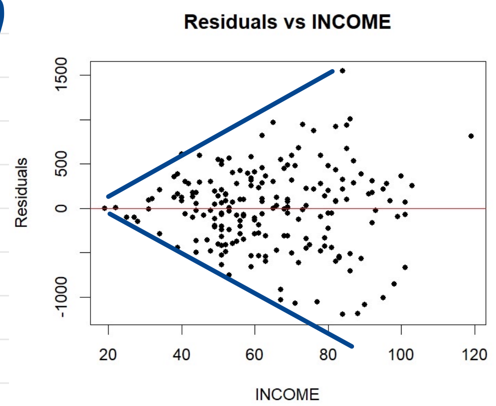
MILES is miles driven per year, *INCOME* is measured in \$1000 units, *AGE* is the average age of the adult members of the household, and *KIDS* is the number of children.

a. Use the data file *vacation* to estimate the model by OLS. Construct a 95% interval estimate for the effect of one more child on miles traveled, holding the two other variables constant.

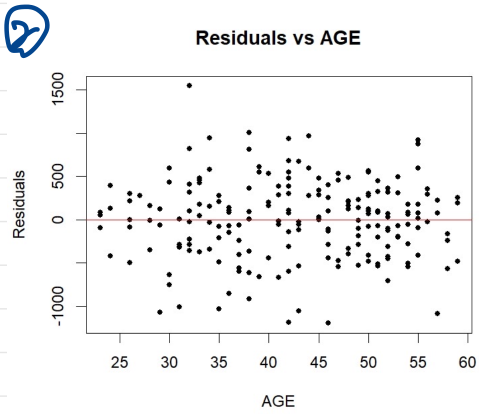
```
> confint(tab, "kids", level = 0.95)
          2.5 %     97.5 %
kids -135.3298 -28.32302
```

b. Plot the OLS residuals versus *INCOME* and *AGE*. Do you observe any patterns suggesting that heteroskedasticity is present?

①



②



① The pattern suggests heteroscedasticity, the variance of the residuals is not constant across the range of income.

② This looks reasonable with no obvious pattern or heteroscedasticity.

c. Sort the data according to increasing magnitude of income. Estimate the model using the first 90 observations and again using the last 90 observations. Carry out the Goldfeld–Quandt test for heteroskedastic errors at the 5% level. State the null and alternative hypotheses.

$$\begin{cases} H_0: \sigma^2_{first\ 90} = \sigma^2_{last\ 90} \\ H_a: \sigma^2_{first\ 90} \neq \sigma^2_{last\ 90} \end{cases}$$

```
> cat("F-statistic:", F_stat, "\n")
F-statistic: 0.3221587
> cat("Critical value:", c(F_crit1 ,F_crit2), "\n")
Critical value: 0.6534355 1.530373
> if (F_stat > F_crit2 | F_stat < F_crit1) {
+   cat("Reject the null hypothesis: evidence of heteroskedasticity.\n")
+ } else {
+   cat("Fail to reject the null hypothesis: no strong evidence of heteroskedasticit
y.\n")
+ }
Reject the null hypothesis: evidence of heteroskedasticity.
```

d. Estimate the model by OLS using heteroskedasticity robust standard errors. Construct a 95% interval estimate for the effect of one more child on miles traveled, holding the two other variables constant. How does this interval estimate compare to the one in (a)?

```
(2.5%) kids  (97.5%) kids
-139.32297   -24.32986
```

(a)

```
> confint(tab, "kids", level = 0.95)
          2.5 %     97.5 %
kids -135.3298 -28.32302
```

The effect of KIDS is still statistically significant.

e. Obtain GLS estimates assuming $\sigma_i^2 = \sigma^2 INCOME_i^2$. Using both conventional GLS and robust GLS standard errors, construct a 95% interval estimate for the effect of one more child on miles traveled, holding the two other variables constant. How do these interval estimates compare to the ones in (a) and (d)?

```
> # 95% CI for KIDS in GLS
> confint(tab_gls, "kids", level = 0.95)
          2.5 %     97.5 %
kids -119.8945 -33.71808
```

(a)

```
> confint(tab, "kids", level = 0.95)
          2.5 %     97.5 %
kids -135.3298 -28.32302
```

```
> ci_robust_gls
(2.5% kids  (97.5%)kids
-121.41339   -32.19919
```

(d)

```
(2.5%) kids  (97.5%) kids
-139.32297   -24.32986
```

All intervals are negative and statistically significant from 0

CIs of GLS model and robust GLS model are narrower than those of OLS.

8.18 Consider the wage equation,

$$\ln(WAGE_i) = \beta_1 + \beta_2 EDUC_i + \beta_3 EXPER_i + \beta_4 EXPER_i^2 + \beta_5 FEMALE_i + \beta_6 BLACK + \beta_7 METRO_i + \beta_8 SOUTH_i + \beta_9 MIDWEST_i + \beta_{10} WEST + e_i$$

where *WAGE* is measured in dollars per hour, education and experience are in years, and $METRO = 1$ if the person lives in a metropolitan area. Use the data file *cps5* for the exercise.

a. We are curious whether holding education, experience, and *METRO* equal, there is the same amount of random variation in wages for males and females. Suppose $\text{var}(e_i|\mathbf{x}_i, FEMALE = 0) = \sigma_M^2$ and $\text{var}(e_i|\mathbf{x}_i, FEMALE = 1) = \sigma_F^2$. We specifically wish to test the null hypothesis $\sigma_M^2 = \sigma_F^2$ against $\sigma_M^2 \neq \sigma_F^2$. Carry out a Goldfeld–Quandt test of the null hypothesis at the 5% level of significance. Clearly state the value of the test statistic and the rejection region, along with your conclusion.

$$F = \frac{\sigma_M^2}{\sigma_F^2}$$

```
> cat("F statistic:", F_stat, "\n")
F statistic: 1.04809
> cat("Critical region: F <", round(F_critical_lower, 4), "or F >", round(F_critical_upper, 4), "\n")
Critical region: F < 0.9451 or F > 1.0581
```

$F_{(0.025, 5414, 4365)} < F < F_{(0.975, 5414, 4365)}$

```
Conclusion: Fail to reject the null hypothesis. There is no significant difference in error variances between males and females.
```

b. Estimate the model by OLS. Carry out the NR^2 test using the right-hand-side variables *METRO*, *FEMALE*, *BLACK* as candidates related to the heteroskedasticity. What do we conclude about heteroskedasticity, at the 1% level? Do these results support your conclusions in (a)? Repeat the test using all model explanatory variables as candidates related to the heteroskedasticity.

```
> cat("NR² test statistic (selected vars):", NR2_1, "\n")
NR² test statistic (selected vars): 23.55681
> cat("Critical value (1% level, df = 3):", crit_val_1pct, "\n")
Critical value (1% level, df = 3): 11.34487
> if (NR2_1 > crit_val_1pct) {
+   cat("Conclusion: Reject H0: Evidence of heteroskedasticity.\n")
+ } else {
+   cat("Conclusion: Fail to reject H0: No evidence of heteroskedasticity.\n")
+ }
Conclusion: Reject H0: Evidence of heteroskedasticity.
```

The NR² test suggests that heteroskedasticity exists and may be influenced by variables like FEMALE, METRO, or BLACK, even if we do not observe a significant variance difference between male and female groups alone. Hence, part (b) provides broader evidence of heteroskedasticity beyond what was tested in (a).

```
NR² test statistic (all vars): 109.4243
> cat("Critical value (1% level, df = 9):", crit_val_2, "\n")
Critical value (1% level, df = 9): 21.66599
> if (NR2_2 > crit_val_2) {
+   cat("Conclusion: Reject H0: Evidence of heteroskedasticity with all variables.\n")
+ } else {
+   cat("Conclusion: Fail to reject H0: No evidence of heteroskedasticity.\n")
+ }
Conclusion: Reject H0: Evidence of heteroskedasticity with all variables.
```

c. Carry out the White test for heteroskedasticity. What is the 5% critical value for the test? What do you conclude?

```
> cat("White test statistic:", white_stat, "\n")
White test statistic: 143.6927
> cat("Critical value at 5% level (df =", df, "):", critical_val, "\n")
Critical value at 5% level (df = 13 ): 22.36203
> if (white_stat > critical_val) {
+   cat("Conclusion: Reject the null hypothesis. Evidence of heteroskedasticity.\n")
+ } else {
+   cat("Conclusion: Fail to reject the null hypothesis. No evidence of heteroskedasticit
y.\n")
+ }
Conclusion: Reject the null hypothesis. Evidence of heteroskedasticity.
```

d. Estimate the model by OLS with White heteroskedasticity robust standard errors. Compared to OLS with conventional standard errors, for which coefficients have interval estimates gotten narrower? For which coefficients have interval estimates gotten wider? Is there an inconsistency in the results?

```
> print(result_compare)
                OLS_Lower     OLS_Upper  Robust_Lower  Robust_Upper     OLS_Width Robust_Width   Change
(Intercept)  1.1384302204  1.2643338265  1.1371382559  1.2656257910 0.1259036061 0.1284875351    Wider
educ         0.0977830603  0.1046761665  0.0974961106  0.1049631162 0.0068931063 0.0074670055    Wider
exper        0.0270727569  0.0321706349  0.0270457914  0.0321976003 0.0050978780 0.0051518089    Wider
I(exper^2)  -0.0004974407 -0.0003941203 -0.0004998427 -0.0003917182 0.0001033205 0.0001081245    Wider
female      -0.1841810529 -0.1468229075 -0.1840894784 -0.1469144820 0.0373581454 0.0371749964 Narrower
black       -0.1447358548 -0.0783146449 -0.1430527905 -0.0799977092 0.0664212100 0.0630550813 Narrower
metro        0.0948966363  0.1431441846  0.0963309747  0.1417098462 0.0482475482 0.0453788716 Narrower
south       -0.0723384657 -0.0191724010 -0.0729887326 -0.0185221340 0.0531660647 0.0544665986    Wider
midwest     -0.0915893895 -0.0362971859 -0.0908291013 -0.0370574741 0.0552922035 0.0537716271 Narrower
west        -0.0348207138  0.0216425095 -0.0351059530  0.0219277486 0.0564632233 0.0570337016    Wider
```

e. Obtain FGLS estimates using candidate variables *METRO* and *EXPER*. How do the interval estimates compare to OLS with robust standard errors, from part (d)?

```
> print(comparison_df)
               FGLS_Lower    FGLS_Upper  Robust_Lower  Robust_Upper    FGLS_Width Robust_Width   Change
(Intercept)  1.1302695254  1.2541279001  1.1371382559  1.2656257910 0.1238583747 0.1284875351 Narrower
educ         0.0982024458  0.1051204663  0.0974961106  0.1049631162 0.0069180205 0.0074670055 Narrower
exper        0.0275467064  0.0326335081  0.0270457914  0.0321976003 0.0050868017 0.0051518089 Narrower
I(exper^2)  -0.0005086498 -0.0004036251 -0.0004998427 -0.0003917182 0.0001050246 0.0001081245 Narrower
female      -0.1847977976 -0.1476290326 -0.1840894784 -0.1469144820 0.0371687649 0.0371749964 Narrower
black       -0.1441623553 -0.0775448504 -0.1430527905 -0.0799977092 0.0666175049 0.0630550813    Wider
metro        0.0953066354  0.1402324253  0.0963309747  0.1417098462 0.0449257899 0.0453788716 Narrower
south       -0.0713493481 -0.0183363498 -0.0729887326 -0.0185221340 0.0530129983 0.0544665986 Narrower
midwest     -0.0906033967 -0.0357807800 -0.0908291013 -0.0370574741 0.0548226168 0.0537716271    Wider
west        -0.0336747637  0.0226870709 -0.0351059530  0.0219277486 0.0563618347 0.0570337016 Narrower
```

f. Obtain FGLS estimates with robust standard errors using candidate variables *METRO* and *EXPER*. How do the interval estimates compare to those in part (e) and OLS with robust standard errors, from part (d)?

OLS - FGLS

```
> print(comparison_df3)
            RobustOLS_Lower RobustOLS_Upper FGLSRobust_Lower FGLSRobust_Upper OLSRobust_Width FGLSRobust_Width   Change
(Intercept)    1.1371382559    1.2656257910     1.1288062434     1.2555911821    0.1284875351     0.1267849388 Narrower
educ           0.0974961106    0.1049631162     0.0979535403     0.1053693718    0.0074670055     0.0074158315 Narrower
exper          0.0270457914    0.0321976003     0.0275343640     0.0326458505    0.0051518089     0.0051114864 Narrower
I(exper^2)    -0.0004998427   -0.0003917182    -0.0005098303    -0.0004024447    0.0001081245     0.0001073856 Narrower
female        -0.1840894784   -0.1469144820    -0.1847026015    -0.1477242287    0.0371749964     0.0369783728 Narrower
black         -0.1430527905   -0.0799977092    -0.1419404578    -0.0797667479    0.0630550813     0.0621737099 Narrower
metro          0.0963309747    0.1417098462     0.0951178555     0.1404212052    0.0453788716     0.0453033498 Narrower
south         -0.0729887326   -0.0185221340    -0.0719445076    -0.0177411903    0.0544665986     0.0542033173 Narrower
midwest       -0.0908291013   -0.0370574741    -0.0900552675    -0.0363289092    0.0537716271     0.0537263583 Narrower
west          -0.0351059530    0.0219277486    -0.0339164878     0.0229287950    0.0570337016     0.0568452827 Narrower
```

g. If reporting the results of this model in a research paper which one set of estimates would you present? Explain your choice.

I present (f) FGLS with robust standard errors. These estimates take advantage of a structured variance model based on METRO and EXPER, while using heteroskedasticity-consistent standard errors to protect against potential misspecification. Compared to OLS and FGLS alone, this approach offers a more efficient and robust inference framework under confirmed heteroskedasticity.