Q5. b

$$b = \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix} \quad Cov(b_1, b_2, b_3) = \begin{bmatrix} 3 & -2 & 1 \\ -2 & 4 & 0 \\ 1 & 0 & 3 \end{bmatrix}$$

a. $H_0: \beta_2 = 0$ v.s. $H_1: \beta_2 \neq 0$

$$\varphi = \frac{b_2}{SE(b_2)} \overset{A}{\sim} N(0,1),$$

$$RR = \{\varphi \mid |\varphi| > 1.96\}, \quad \varphi^{\#} = \frac{3}{\sqrt{4}} = 1.5 \notin RR$$

Doesn't reject $\beta_2 = 0$

b. $H_0: \beta_1 + 2\beta_2 = 5$ v.s. $H_1: \beta_1 + 2\beta_2 \neq 5$

$$\varphi = \frac{b_1 + 2b_2 - 5}{SE(b_1 + 2b_2)} \overset{A}{\sim} N(0,1)$$

$$RR = \{\varphi \mid |\varphi| > 1.96\},$$

$$SE(b_1 + 2b_2) = (3 + 4 \times 4 + 2 \times 2 \times -2)^{0.5} = 3.3166$$

$$\varphi^{\#} = \frac{2 + 2 \times 3 - 5}{3.3166} = 0.9045 \notin RR$$

Doesn't reject $\beta_1 + 2\beta_2 = 5$

c. $H_0: \beta_1 - \beta_2 + \beta_3 = 4$ v.s. $H_1: \beta_1 - \beta_2 + \beta_3 \neq 4$

$$\varphi = \frac{b_1 - b_2 + b_3 - 4}{SE(b_1 - b_2 + b_3)} \overset{A}{\sim} N(0,1)$$

$RR = \{\varphi \mid |\varphi| > 1.96\}$

$$SE(b_1 - b_2 + b_3) = (3 + 4 + 3 + 2\times -1\times -2 + 2\times -1\times 0 + 2\times 1)^{0.5} = 4$$

$$\varphi^{ob} = \frac{2-3-1}{4} = -\frac{1}{4} = -0.5 \notin RR$$

Doesn't reject $b_1 - b_2 + b_3 = 4$

Q.5.31

a.

$$\beta = \begin{bmatrix} 20.8701 \\ 0.3681 \\ 1.5291 \\ 3.0237 \end{bmatrix}$$

```
Call:
lm(formula = time ~ depart + reds + trains, data =
ta)

Residuals:
     Min       1Q   Median       3Q      Max
-18.4389  -3.6774  -0.1188   4.5863  16.4986

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  20.8701     1.6758  12.454  < 2e-16
depart        0.3681     0.0351  10.487  < 2e-16
reds          1.5219     0.1850   8.225 1.15e-14
trains        3.0237     0.6340   4.769 3.18e-06

(Intercept) ***
depart      ***
reds        ***
trains      ***
```

b.

$$CI_i @ 95\% = \left[\beta_i \pm SE(\beta_i)\, t_{0.025}(245)\right]$$

	2.5 %	97.5 %
(Intercept)	17.5694018	24.170871
depart	0.2989851	0.437265
reds	1.1574748	1.886411
trains	1.7748867	4.272505

c.

$H_0: \beta_3 = 2$ v.s. $H_A: \beta_3 < 2$

$$t = \frac{b_3 - 2}{SE(b_3)} = -2.53562$$

```
> #Q5.31.c H0: B3 = 2, H1: B3 < 2
> t_c <- (b[3]-2)/se[3]
> p_c <- pt(t_c , df = df.residual(mod1))
> t_c
      reds
-2.583562
> p_c
         reds
0.005179509
```

P-value = 0.00517 , reject that $\beta_3 = 2$
< 0.05

d. $H_0: \beta_4 = 3$ v.s. $H_A: \beta_4 \neq 3$

$$t = \frac{b_4 - 3}{SE(b_4)} = 0.03737444$$

```
> #Q5.31.d H0: B4 = 3, H1: B4 =/ 3
> t_d <- (b[4]-3)/se[4]
> p_d <- 2* pt(t_d , df = df.residual(mod1))
> t_d
    trains
0.03737444
> p_d
  trains
1.029783
```

P-value = 1.0297 > 0.1 , Doesn't reject $\beta_4 = 3$

e. $H_0: \beta_2 = \frac{10}{30} \geq \frac{1}{3}$ v.s. $H_A: \beta_2 < \frac{1}{3}$

$$t = \frac{b_2 - \frac{1}{3}}{SE(b_2)} = 0.9911646$$

```
> #Q5.31.e H0: B2 > 1/3, H1: B4 < 1/3
> t_e <- (b[2]-(1/3))/se[2]
> p_e <- pt(t_e , df = df.residual(mod1))
> t_e
    depart
0.9911646
> p_e
    depart
0.8387085
```

P-value = 0.8387 > 0.05

, Doesn't reject $\beta_2 \geq \frac{1}{3}$

f. $H_0: \beta_4 - 3\beta_3 > 0$ v.s. $H_1: \beta_4 - 3\beta_3 < 0$

$$t = \frac{b_4 - 3b_3}{SE(b_4 - 3b_3)} = -1.825$$

```
> #Q5.31.f H0: B4 - 3B3 > 0, H1: B4 - 3B3 < 0
> vcov <- vcov(mod1)
> se_f <- (vcov[4,4] + 9*vcov[3,3] + 2*-3*vcov[3,4])^
0.5
> t_f <- (b[4]-3*b[3])/se_f
> p_f <- pt(t_f , df = df.residual(mod1))
> t_f
   trains
-1.825027
> p_f
    trains
0.03460731
```

P-value = 0.034 < 0.05

Reject $\beta_4 > 3\beta_3$

Note that,

$\sigma^2 = w' \Sigma w$

g. $H_0: \beta_1 + 30\beta_2 + 6\beta_3 + \beta_4 < 45$ v.s. $H_1: \beta_1 + 30\beta_2 + 6\beta_3 + \beta_4 > 45$

$$t = \frac{b_1 + 30b_2 + 6b_3 + b_4 - 45}{SE(b_1 + 30b_2 + 6b_3 + b_4)} = -1.726$$

```
> #Q5.31.g H0: B1 + 30B2 + 6B3 + B4 > 0
> w_g <- c(1,30,6,1)
> se_g <- sqrt(t(w_g) %*% vcov %*% w_g)
> t_g <- as.numeric((b[1]+ 30*b[2] + 6*b[3] + b[4] -
45)/se_g)
> p_g <- as.numeric(1-pt(t_g , df = df.residual(mod
1)))
> t_g
[1] -1.725964
> p_g
[1] 0.9571926
```

P-value = 0.95 > 0.05,

Doesn't reject H_0

h. $H_0: \beta_1 + 30\beta_2 + 6\beta_3 + \beta_4 > 45$ v.s. $H_1: \beta_1 + 30\beta_2 + 6\beta_3 + \beta_4 < 45$

$$t = \frac{b_1 + 30b_2 + 6b_3 + b_4 - 45}{SE(b_1 + 30b_2 + 6b_3 + b_4)} = -1.726$$

```
> #Q5.31.e H0: B1 + 30B2 + 6B3 + B4 > 45
> p_h <- as.numeric(pt(t_g , df = df.residual(mod1)))
> p_h
[1] 0.04280736
```

P-value = 0.0428 < 0.05

Reject H_0

Q5.33.a

β_1 sig @ $\alpha = 0.001$

β_2 $= 0.01$

β_3 $= 0$

β_4 $= 0.001$

β_5 $= 0.001$

β_6 $= 0.01$

```
Call:
lm(formula = log(wage) ~ educ + I(educ^2) + exper + I
(exper^2) +
    I(educ * exper), data = data)

Residuals:
    Min      1Q  Median      3Q     Max
-1.6628 -0.3138 -0.0276  0.3140  2.1394

Coefficients:
                  Estimate Std. Error t value
(Intercept)      1.038e+00  2.757e-01   3.764
educ             8.954e-02  3.108e-02   2.881
I(educ^2)        1.458e-03  9.242e-04   1.578
exper            4.488e-02  7.297e-03   6.150
I(exper^2)      -4.680e-04  7.601e-05  -6.157
I(educ * exper) -1.010e-03  3.791e-04  -2.665
                Pr(>|t|)
(Intercept)     0.000175 ***
educ            0.004038 **
I(educ^2)       0.114855
exper           1.06e-09 ***
I(exper^2)      1.01e-09 ***
I(educ * exper) 0.007803 **
---
```

b.

$$\ln WAGE = \beta_1 + \beta_2 EDUC + \beta_3 EDUC^2 + \beta_4 EXPER + \beta_5 EXPER^2 + \beta_6 EDUC \times EXPER + e$$

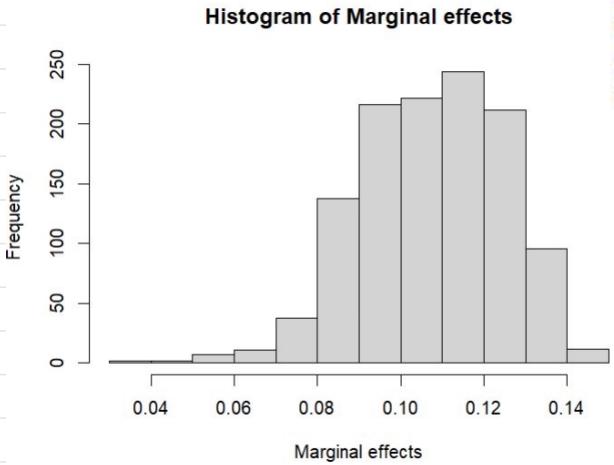
$$\frac{\partial E[\ln(WAGE) \mid EDUC, EXPER]}{\partial EDUC} = \beta_2 + 2\beta_3 EDUC + \beta_6 EXPER$$

$$\frac{\partial \widehat{\ln(WAGE)}}{\partial EDUC} = b_2 + 2b_3 EDUC + \beta_6 EXPER$$

$$= 0.08954 + 0.001458 \times 2\, EDUC - 0.00101\, EXPER$$

EDUC↑. ME↑ ; EXPERT. ME↓

c.




```
> cat(quantile(me,0.05),'\n')
0.08008187
> cat(quantile(me,0.5),'\n')
0.1084313
> cat(quantile(me,0.95),'\n')
0.1336188
```

Left skewed marginal effect and all positive

EDUC↑ → ln WAGE ↑

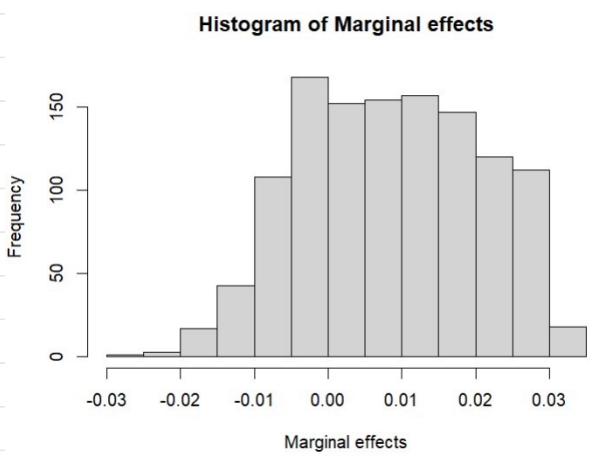
d.

$$\frac{\partial E[\ln(WAGE)|EXPER, EDUC]}{\partial EXPER} = \beta_4 + 2\beta_5 EXPER + \beta_6 EDUC$$

$$\frac{\partial \ln(\widehat{WAGE})}{\partial EXPER} = b_4 + 2b_5 EXPER + b_6 EDUC$$

$$= 0.04488 - 2 \times 0.000468\, EXPER - 0.00101\, EDUC$$

e.




```
> cat(quantile(me2,0.05),'\n')
-0.01037621
> cat(quantile(me2,0.5),'\n')
0.008418878
> cat(quantile(me2,0.95),'\n')
0.02793115
```

More like bell shape and some ME's are negative.

f.

$$\beta_1 + 17\beta_2 + 17^2\beta_3 + 8\beta_4 + 8^2\beta_5 + 136\beta_6 \leq$$

$$\beta_1 + 16\beta_2 + 16^2\beta_3 + 18\beta_4 + 18^2\beta_5 + 288\beta_6$$

$H_0: \beta_2 + 33\beta_3 - 10\beta_4 - 260\beta_5 - 152\beta_6 \leq 0$ v.s.

$H_A: \beta_2 + 33\beta_3 - 10\beta_4 - 260\beta_5 - 152\beta_6 > 0$

$t = -1.669902$

P-value $= 0.9523 >$

0.05

```
> vcov <- vcov(mod1)
> w <- c(0,1,33,-10,-260,-152)
> se <- sqrt(t(w) %*% vcov %*% w)
> t <- (b[2] + 33*b[3] - 10*b[4] - 260*b[5] -152*b[6])/se
> p <- 1-pt(t , df = df.residual(mod1))
> cat(t,'\n')
-1.669902
> cat(p,'\n')
0.9523996
```

Doesn't reject H_0

g. $H_0: \beta_2 + 33\beta_4 - 10\beta_4 - 420\beta_5 - 144\beta_6 \leq 0$

$H_1: \beta_2 + 33\beta_4 - 10\beta_4 - 420\beta_5 - 144\beta_6 \geq 0$

$t = 2.062365$

P-value $= 0.019694$

< 0.05

Reject H_0

```
> #Q5.33.g
> w2 <- c(0,1,33,-10,-420,-144)
> se2 <- sqrt(t(w2) %*% vcov %*% w2)
> t2 <- (b[2] + 33*b[3] - 10*b[4] - 420*b[5] -144*b
[6])/se2
> p2 <- 1-pt(t2 , df = df.residual(mod1))
> cat(t2,'\n')
2.062365
> cat(p2,'\n')
0.01969445
```

h.

$H_0: 12\beta_5 - 4\beta_6 = 0$

$H_1: 12\beta_5 - 4\beta_6 \neq 0$

```
> #Q5.33.h
> w3 <- c(0,0,0,0,12,-4)
> se3 <- sqrt(t(w3) %*% vcov %*% w3)
> t3 <- (12*b[5] -4*b[6])/se3
> p3 <- 2*pt(t3 , df = df.residual(mod1))
> cat(t3,'\n')
-1.027304
> cat(p3,'\n')
0.3044854
```

$t = -1.0273$

P-value = 0.3044

> 0.05 Doesn't reject H_0

i.

$$ME = \frac{\partial \ln(\widehat{WAGE})}{\partial EXPER} = b_4 + 2b_5 EXPER + b_6 EDUC$$

For Jill, EXPER=11, EDUC=16

When

$$ME = b_4 + 2b_5(11 + \Delta EXPER) + b_6 EDUC = 0,$$

$$\Delta EXPER = \frac{-b_4 - b_6 \times 16}{2b_5} - 11 = 19.67706$$

CI: [15.96146, 23.39265]

```
> delta <- -(b[4] + 16 * b[6]) / (2 * b[5])-11
> w4 <- c(0, 0, 0, -1 / (2 * b[5]), (b[4] + 16 * b
[6]) / (2 * b[5]^2), -8 / b[5])
> se4 <- sqrt(t(w4) %*% vcov %*% w4)
> ci_lower <- delta - 1.96 * se4
> ci_upper <- delta + 1.96 * se4
>
> # 輸出結果
> cat('Still ',delta, 'years so that me become negati
ve', "\n")
Still  19.67706 years so that me become negative
> cat("95% CI: [", ci_lower, ", ", ci_upper, "]\n")
95% CI: [ 15.96146 ,  23.39265 ]
```