15.6 Using the NLS panel data on $N = 716$ young women, we consider only years 1987 and 1988. We are interested in the relationship between ln(*WAGE*) and experience, its square, and indicator variables for living in the south and union membership. Some estimation results are in Table 15.10.

TABLE 15.10 **Estimation Results for Exercise 15.6**

	(1) OLS 1987	(2) OLS 1988	(3) FE	(4) FE Robust	(5) RE
C	0.9348	0.8993	1.5468	1.5468	1.1497
	(0.2010)	(0.2407)	(0.2522)	(0.2688)	(0.1597)
EXPER	0.1270	0.1265	0.0575	0.0575	0.0986
	(0.0295)	(0.0323)	(0.0330)	(0.0328)	(0.0220)
$EXPER^2$	−0.0033	−0.0031	−0.0012	−0.0012	−0.0023
	(0.0011)	(0.0011)	(0.0011)	(0.0011)	(0.0007)
SOUTH	−0.2128	−0.2384	−0.3261	−0.3261	−0.2326
	(0.0338)	(0.0344)	(0.1258)	(0.2495)	(0.0317)
UNION	0.1445	0.1102	0.0822	0.0822	0.1027
	(0.0382)	(0.0387)	(0.0312)	(0.0367)	(0.0245)
N	716	716	1432	1432	1432

(standard errors in parentheses)

a. The OLS estimates of the ln(*WAGE*) model for each of the years 1987 and 1988 are reported in columns (1) and (2). How do the results compare? For these individual year estimations, what are you assuming about the regression parameter values across individuals (heterogeneity)?

b. The ln(*WAGE*) equation specified as a panel data regression model is

$$\ln(WAGE_{it}) = \beta_1 + \beta_2 EXPER_{it} + \beta_3 EXPER_{it}^2 + \beta_4 SOUTH_{it} + \beta_5 UNION_{it} + (u_i + e_{it}) \quad \text{(XR15.6)}$$

Explain any differences in assumptions between this model and the models in part (a).

c. Column (3) contains the estimated fixed effects model specified in part (b). Compare these estimates with the OLS estimates. Which coefficients, apart from the intercepts, show the most difference?

d. The *F*-statistic for the null hypothesis that there are no individual differences, equation (15.20), is 11.68. What are the degrees of freedom of the *F*-distribution if the null hypothesis (15.19) is true? What is the 1% level of significance critical value for the test? What do you conclude about the null hypothesis.

e. Column (4) contains the fixed effects estimates with cluster-robust standard errors. In the context of this sample, explain the different assumptions you are making when you estimate with and without cluster-robust standard errors. Compare the standard errors with those in column (3). Which ones are substantially different? Are the robust ones larger or smaller?

f. Column (5) contains the random effects estimates. Which coefficients, apart from the intercepts, show the most difference from the fixed effects estimates? Use the Hausman test statistic (15.36) to test whether there are significant differences between the random effects estimates and the fixed effects estimates in column (3) (Why that one?). Based on the test results, is random effects estimation in this model appropriate?

a.

1987 年與 1988 年相比，基準工資略降，經驗影響穩定，南方懲罰加劇，工會溢價減少。OLS 假設參數在個體間一致，忽略未觀察異質性，可能影響估計準確性。

b.

此模型是基於面板數據的對數工資回歸模型，通過經驗（及其平方）、南方居住和工會成員身份解釋工資差異。誤差項包含個體特定效應 u_i 和隨機誤差 e_{it}。單年OLS估計忽略 u_i，假設參數同質，可能導致估計偏差，而固定效應模型（如C3）則會控制 u_i。

c.

- **常數項 (C)**：
  - FE (1.5468) 高於C1 (0.9348) 和C2 (0.8993)。FE的常數項因吸收個體特定效應而較高，無法直接與OLS截距比較。
- **EXPER**：
  - FE (0.0575) 遠低於C1 (0.1270) 和C2 (0.1265)。這表明固定效應模型下經驗的工資回報顯著降低，可能因個體異質性（如能力）被移除，減弱了經驗效應。
- **EXPER²**：
  - FE (-0.0012) 比C1 (-0.0033) 和C2 (-0.0031) 的負值小。FE模型顯示經驗回報下降的幅度較小，反映個體效應校正後的非線性影響減弱。
- **SOUTH**：
  - FE (-0.3261) 比C1 (-0.2128) 和C2 (-0.2384) 更負。固定效應加劇了南方居住的負面影響，可能更精確地隔離了地區效應。
- **UNION**：
  - FE (0.0822) 低於C1 (0.1435) 且低於C2 (0.1102)。固定效應下工會溢價降低，表明個體效應可能誇大了OLS估計的工會影響。

總結

固定效應模型的係數顯示，控制個體特定效應後，經驗和工會溢價的影響顯著降低，而南方居住的負面影響加劇。與單年OLS相比，FE模型更能反映時間內變化的淨效應，因其消除了個體間未觀察的異質性。

d.

- **Null假設 (H_0)**：所有係數（$\beta_2, \beta_3, \beta_4, \beta_5$）均為零，即 $EXPER, EXPER^2, SOUTH, UNION$ 對 $\ln(WAGE)$ 無顯著影響。
- **備擇假設 (H_1)**：至少一個係數不為零。

假設自由度為 q = 4，n-k = 711, 5%顯著性水平的 F 臨界值約為 2.37。

各項係數除以標準誤皆大於該數字，拒絕 H0

e.

c, south, union 的 robust fe 的標準誤較 fe 高，robust 考慮了個體內殘差的相關性（例如同一女性的 1987 年和 1988 年觀測值可能相關）以及可能的異方差性

（不同個體的誤差方差不同），表明普通 FE 模型低估了這些相關性或異方差，導致其標準誤差偏小。

f.

south 係數相差最多

- **虛無假設 H_0**：隨機效果模型是適當的（RE 是一致且有效的）。
- **對立假設 H_1**：固定效果模型是適當的（RE 不一致，FE 是一致的）。

計算檢定統計量 Hausman χ^2：

$$H = (\beta_{RE} - \beta_{FE})' \cdot [\mathrm{Var}(\beta_{RE}) - \mathrm{Var}(\beta_{FE})]^{-1} \cdot (\beta_{RE} - \beta_{FE})$$

這個統計量在虛無假設下服從 卡方分布（Chi-square distribution），自由度為變數個數。

- **若 p 值 < 顯著水準（例如 0.05）** → 拒絕 H_0 → 使用固定效果模型（RE 有偏）
- **若 p 值 ≥ 顯著水準** → 不拒絕 H_0 → 可以使用隨機效果模型（RE 一致）

15.20 This exercise uses data from the STAR experiment introduced to illustrate fixed and random effects for grouped data. In the STAR experiment, children were randomly assigned within schools into three types of classes: small classes with 13–17 students, regular-sized classes with 22–25 students, and regular-sized classes with a full-time teacher aide to assist the teacher. Student scores on achievement tests were recorded as well as some information about the students, teachers, and schools. Data for the kindergarten classes are contained in the data file *star*.

a. Estimate a regression equation (with no fixed or random effects) where *READSCORE* is related to *SMALL*, *AIDE*, *TCHEXPER*, *BOY*, *WHITE_ASIAN*, and *FREELUNCH*. Discuss the results. Do students perform better in reading when they are in small classes? Does a teacher's aide improve scores? Do the students of more experienced teachers score higher on reading tests? Does the student's sex or race make a difference?

b. Reestimate the model in part (a) with school fixed effects. Compare the results with those in part (a). Have any of your conclusions changed? [*Hint*: specify *SCHID* as the cross-section identifier and *ID* as the "time" identifier.]

c. Test for the significance of the school fixed effects. Under what conditions would we expect the inclusion of significant fixed effects to have little influence on the coefficient estimates of the remaining variables?

d. Reestimate the model in part (a) with school random effects. Compare the results with those from parts (a) and (b). Are there any variables in the equation that might be correlated with the school effects? Use the LM test for the presence of random effects.

e. Using the *t*-test statistic in equation (15.36) and a 5% significance level, test whether there are any significant differences between the fixed effects and random effects estimates of the coefficients on *SMALL*, *AIDE*, *TCHEXPER*, *WHITE_ASIAN*, and *FREELUNCH*. What are the implications of the test outcomes? What happens if we apply the test to the fixed and random effects estimates of the coefficient on *BOY*?

f. Create school-averages of the variables and carry out the Mundlak test for correlation between them and the unobserved heterogeneity.

a.

```
Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) 437.76425    1.34622 325.180  < 2e-16 ***
small         5.82282    0.98933   5.886 4.19e-09 ***
aide          0.81784    0.95299   0.858    0.391
tchexper      0.49247    0.06956   7.080 1.61e-12 ***
boy          -6.15642    0.79613  -7.733 1.23e-14 ***
white_asian   3.90581    0.95361   4.096 4.26e-05 ***
freelunch   -14.77134    0.89025 -16.592  < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 30.19 on 5759 degrees of freedom
  (20 observations deleted due to missingness)
Multiple R-squared:  0.09685,   Adjusted R-squared:  0.09591
F-statistic: 102.9 on 6 and 5759 DF,  p-value: < 2.2e-16
```

b.

```
Coefficients:
              Estimate Std. Error  t-value  Pr(>|t|)
small         6.490231   0.912962   7.1090 1.313e-12 ***
aide          0.996087   0.881693   1.1297    0.2586
tchexper      0.285567   0.070845   4.0309 5.629e-05 ***
boy          -5.455941   0.727589  -7.4987 7.440e-14 ***
white_asian   8.028019   1.535656   5.2277 1.777e-07 ***
freelunch   -14.593572   0.880006 -16.5835 < 2.2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Total Sum of Squares:    4628000
Residual Sum of Squares: 4268900
R-Squared:      0.077592
Adj. R-Squared: 0.063954
F-statistic: 79.6471 on 6 and 5681 DF, p-value: < 2.22e-16
```

c.

這個子問題要求我們測試在 (b) 中加入的學校固定效應（由 `schid` 表示）是否顯著影響模型的係數估計。固定效應的顯著性表明學校之間存在未觀察到的異質性（例如教學質量、學校資源等），這些異質性會影響 readscore，並可能與自變量（如 `small`、`tchexper` 等）相關。如果固定效應不顯著，則學校差異對係數估計的影響可能很小，普通線性回歸（(a) 模型）可能已足夠。

使用 F 檢定（`pFtest`）比較固定效應模型（`model_b`）與無固定效應模型（`model_a`），檢查學校固定效應是否共同顯著。

- **p 值 < 0.05**：拒絕虛無假設（H0：固定效應不顯著），表示學校固定效應顯著，應保留固定效應模型，因為學校差異對係數估計有顯著影響。
- **p 值 ≥ 0.05**：接受虛無假設，固定效應不顯著，說明學校間的未觀察異質性對係數估計影響不大，(a) 的普通回歸結果可能已足夠。

```
data:  readscore ~ small + aide + tchexper + boy + white_asian + freelunch
F = 16.698, df1 = 78, df2 = 5681, p-value < 2.2e-16
alternative hypothesis: significant effects
```

d.

```
Random effects:
 Groups   Name        Variance Std.Dev.
 schid    (Intercept) 168.2    12.97
 Residual             751.5    27.41
Number of obs: 5766, groups:  schid, 79

Fixed effects:
            Estimate Std. Error t value
(Intercept) 436.1055     2.1053 207.149
small         6.4610     0.9121   7.083
aide          0.9925     0.8808   1.127
tchexper      0.3015     0.0703   4.288
boy          -5.5081     0.7273  -7.574
white_asian   7.3932     1.4374   5.144
freelunch   -14.5846     0.8746 -16.675
```

```
> VarCorr(model_d)
 Groups   Name        Std.Dev.
 schid    (Intercept) 12.968
 Residual             27.413
> lrtest(model_d, model_a)
Likelihood ratio test

Model 1: readscore ~ small + aide + tchexper + boy + white_asian + freelunch +
    (1 | schid)
Model 2: readscore ~ small + aide + tchexper + boy + white_asian + freelunch
  #Df LogLik Df  Chisq Pr(>Chisq)
1   9 -27377
2   8 -27825 -1 896.88  < 2.2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

假設 `lrtest` 結果顯示 p 值 < 0.05：

- **解釋**：拒絕虛無假設（H0：無隨機效應），表明學校隨機效應顯著，應使用隨機效應模型。
- 這與隨機效應方差（168.2）一致，學校間的變異不可忽略。

e.

```
        Hausman Test

data:  readscore ~ small + aide + tchexper + boy + white_asian + freelunch
chisq = 13.809, df = 6, p-value = 0.03184
```

假設 `phtest` 結果顯示 p 值 = 0.03（< 0.05，5% 顯著性水平）：

- **解釋**：拒絕虛無假設（H0：隨機效應假設成立），表明固定效應與隨機效應模型的係數存在顯著差異，隨機效應模型可能不合適，應使用固定效應模型。
- 這意味著某些自變量（如 `small`、`tchexper` 等）可能與學校層面未觀察因素（`schid` 的隨機效應）相關，違反隨機效應模型的假設。

boy在隨機效應模型中為 -5.5081，在固定效應模型中假設為 -5.455941，差異不大。這表明boy的係數估計受學校效應影響較小，性別效應在學校間較為一致。

f.

```
Fixed effects:
                Estimate Std. Error t value
(Intercept)     459.62070   19.65155  23.389
small             6.56177    0.92060   7.128
aide              1.09202    0.88783   1.230
tchexper          0.29460    0.07161   4.114
boy              -5.40859    0.73351  -7.374
white_asian       8.19637    1.54599   5.302
freelunch       -14.64249    0.88916 -16.468
mean_small      -18.49138   21.31830  -0.867
mean_aide        16.54857   19.90106   0.832
mean_tchexper     1.00302    0.59919   1.674
mean_boy        -53.46711   24.15103  -2.214
mean_white_asian  -6.72280    6.06500  -1.108
mean_freelunch    -3.35826    8.40734  -0.399
```

15.17 The data file *liquor* contains observations on annual expenditure on liquor (*LIQUOR*) and annual income (*INCOME*) (both in thousands of dollars) for 40 randomly selected households for three consecutive years.

a. Create the first-differenced observations on *LIQUOR* and *INCOME*. Call these new variables *LIQUORD* and *INCOMED*. Using OLS regress *LIQUORD* on *INCOMED* without a constant term. Construct a 95% interval estimate of the coefficient.

b. Estimate the model $LIQUOR_{it} = \beta_1 + \beta_2 INCOME_{it} + u_i + e_{it}$ using random effects. Construct a 95% interval estimate of the coefficient on *INCOME*. How does it compare to the interval in part (a)?

c. Test for the presence of random effects using the LM statistic in equation (15.35). Use the 5% level of significance.

d. For each individual, compute the time averages for the variable *INCOME*. Call this variable *INCOMEM*. Estimate the model $LIQUOR_{it} = \beta_1 + \beta_2 INCOME_{it} + \gamma INCOMEM_i + c_i + e_{it}$ using the random effects estimator. Test the significance of the coefficient γ at the 5% level. Based on this test, what can we conclude about the correlation between the random effect u_i and *INCOME*? Is it OK to use the random effects estimator for the model in (b)?

a.

```
Call:
lm(formula = LIQUORD ~ 0 + INCOMED, data = data_diff)

Residuals:
    Min      1Q  Median      3Q     Max
-3.8339 -1.1916  0.0221  1.1201  3.6778

Coefficients:
        Estimate Std. Error t value Pr(>|t|)
INCOMED 0.026017   0.008247   3.155  0.00204 **
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 1.684 on 118 degrees of freedom
Multiple R-squared:  0.07778,   Adjusted R-squared:  0.06996
F-statistic: 9.952 on 1 and 118 DF,  p-value: 0.002039
```

```
> # 構建 95% 信賴區間
> conf_int <- confint(model, level = 0.95)
> print(conf_int)
              2.5 %     97.5 %
INCOMED 0.009685207 0.04234903
```

b.

```
Effects:
                var std.dev share
idiosyncratic 0.9640  0.9819 0.571
individual    0.7251  0.8515 0.429
theta: 0.4459

Residuals:
     Min.   1st Qu.    Median   3rd Qu.      Max.
-2.263634 -0.697383  0.078697  0.552680  2.225798

Coefficients:
            Estimate Std. Error z-value  Pr(>|z|)
(Intercept) 0.9690324  0.5210052  1.8599 0.0628957 .
income      0.0265755  0.0070126  3.7897 0.0001508 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Total Sum of Squares:    126.61
Residual Sum of Squares: 112.88
```

```
                 2.5 %     97.5 %
(Intercept) -0.05211904 1.99018381
income       0.01283111 0.04031983
>
```

c.

```
> # 5% 顯著性水平判斷
> alpha <- 0.05
> p_value <- lm_test$p.value
> cat("p-value:", p_value, "\n")
p-value: 5.428756e-06
> if (p_value < alpha) {
+   cat("拒絕虛無假設，存在顯著的隨機效應（顯著性水平 5%）。\n")
+ } else {
+   cat("無法拒絕虛無假設，無顯著的隨機效應（顯著性水平 5%）。\n")
+ }
拒絕虛無假設，存在顯著的隨機效應（顯著性水平 5%）。
>
```

- 虛無假設 (H_0)：無隨機效應（池塘模型足夠）。
- 備擇假設 (H_1)：存在隨機效應。

d.

```
Coefficients:
             Estimate Std. Error z-value Pr(>|z|)
(Intercept) 0.9163337  0.5524439  1.6587  0.09718 .
income      0.0207421  0.0209083  0.9921  0.32117
incomem     0.0065792  0.0222048  0.2963  0.76700
```

```
> print(conf_int_re)
                 2.5 %     97.5 %
(Intercept) -0.16643649 1.99910382
income      -0.02023738 0.06172158
incomem     -0.03694132 0.05009980
>
```

無法拒絕虛無假設，γ 係數在 5% 顯著性水平下不顯著

結論：隨機效應 u 與 INCOME 之間無顯著相關性。