411707009 林韋錚

11.1.

a. $y_2 = \alpha_2(\alpha_1 y_2 + e_1) + \beta_1 x_1 + \beta_2 x_2 + e_2$

$$\Rightarrow y_2 = \frac{\beta_1}{1-\alpha_2\cdot\alpha_1} x_1 + \frac{\beta_2}{1-\alpha_2\cdot\alpha_1} x_2 + \frac{\alpha_2\cdot e_1 + e_2}{1-\alpha_2\cdot\alpha_1}$$

$$= \pi_1 x_1 + \pi_2 x_2 + v_2, \text{ where } \pi_1 = \frac{\beta_1}{1-\alpha_2\cdot\alpha_1}, \pi_2 = \frac{\beta_2}{1-\alpha_2\cdot\alpha_1}$$

$$v_2 = \frac{\alpha_2\cdot e_1 + e_2}{1-\alpha_2\cdot\alpha_1}$$

$$\text{Corr}(y_2, e_1) = \sigma_{y_2}\cdot\sigma_{e_1}\cdot \text{Cor}(y_2, e_1) = \sigma_{y_2}\cdot\sigma_{e_1}\cdot\frac{\alpha_2}{1-\alpha_2\alpha_1}\cdot\sigma_{e}$$
$$\neq 0$$

b. Since both equation contain endogenous varlable(y_1 & y_2). The OLS estimator won't be consistent.

c. There are 2 equations, so there must be $2-1=1$ variable be omitted to make the equation identitied, in equation (1), there are 2 variables absent $\Rightarrow$ identified, while equation have no variable absent. Therefore, only $y_1 = \alpha_1 y_2 + e_1$ is identified.

d. Combine two equations we have $E(x_{i1}\cdot v_2 | x_1, x_2) = E(x_{i2}\cdot v_2 | x_1, x_2) = 0$

replace v_2 by $\frac{\alpha_2\cdot e_1 + e_2}{1-\alpha_2\cdot\alpha_1}$ we have

$$\begin{cases} \frac{\alpha_2}{1-\alpha_2\cdot\alpha_1}\cdot E(x_{i1}\cdot e_1 | x_1, x_2) + \frac{1}{1-\alpha_2\cdot\alpha_1} E(x_{i1}\cdot e_1 | x_1, x_2) = 0 \\ \frac{\alpha_2}{1-\alpha_2\cdot\alpha_1}\cdot E(x_{i2}\cdot e_1 | x_1, x_2) + \frac{1}{1-\alpha_2\alpha_1}\cdot E(x_{i2}\cdot e_1 | x_1, x_2) = 0 \end{cases}$$

while $E(x_{ik}\cdot e_1 | x_1 \cdots x_k) = 0 = E(x_{ik}\cdot e_2 | x_1 \cdots x_k)$

makes x_1, x_2 consistent

e. OLS : $\min \sum (y_2 - \pi_1 x_1 - \pi_2 x_2)^2$ by F.O.C $\begin{cases} -2\cdot \sum x_1 (y_2 - \pi_1 x_1 - \pi_2 x_2) = 0 \\ -2\cdot \sum x_2 (y_2 - \pi_1 x_1 - \pi_2 x_2) = 0 \end{cases}$

are equivalent to 2 equations in part (d)

f. Equations in part d $\begin{cases} \sum x_1 y_2 - \pi_1 \cdot \sum x_1^2 - \pi_2 \cdot \sum x_1 x_2 = 0 \\ \sum x_2 y_2 - \pi_1 \cdot \sum x_2 x_1 - \pi_2 \cdot \sum x_2^2 = 0 \end{cases} \Rightarrow \begin{cases} \pi_1 + 0\pi_2 = 3 \Rightarrow \pi_1 = 3 \\ 0\pi_1 + \pi_2 = 4 \Rightarrow \pi_2 = 4 \end{cases}$

g. $\sum \hat{y}_2 (y_1 - \alpha_1 y_2) = 0 \Rightarrow \alpha_1 = \dfrac{\sum \hat{y}_2 \cdot y_1}{\sum \hat{y}_2 \cdot y_2}$, plug $\hat{y}_2 = \hat{\pi}_1 x_1 + \hat{\pi}_2 x_2$ into it

$$\Rightarrow \hat{\alpha}_1 = \frac{\sum(\hat{\pi}_1 \cdot x_1 + \hat{\pi}_2 x_2) y_1}{\sum(\hat{\pi}_1 \cdot x_1 + \hat{\pi}_2 x_2) y_2} = \frac{3\sum(x_1 y_1) + 4\sum(x_2 \cdot y_1)}{3\cdot\sum(x_1 \cdot y_2) + 4\sum(x_2 \cdot y_2)} = \frac{3\times 2 + 4\times 3}{3\times 3 + 4\times 4} = \frac{18}{25}$$

This is consistent because we have $y_1 = \alpha_1 y_2 + e_1$, and the moment condition of (y_2, e_1) make $\hat{\alpha}_1$ consistent

h. $\hat{v}_2 = y_2 - \hat{y}_2 \Rightarrow \hat{y}_2 = y_2 - \hat{v}_2$, $\hat{\alpha}_{1,2SLS} = \dfrac{\sum \hat{y}_2 \cdot y_1}{\sum \hat{y}_2^2}$, so we need

to prove $\sum \hat{y}_2^2 = \sum \hat{y}_2 \cdot y_2 \Rightarrow \sum \hat{y}_2 (y_2 - \hat{y}_2) = \sum \hat{y}_2 \cdot y_2$

$\Rightarrow \sum \hat{y}_2 y_2 - \sum \hat{y}_2 \cdot \hat{y}_2 = \sum \hat{y}_2 \cdot y_2$,

note that $\sum \hat{y}_2 \cdot \hat{y}_2 = 0$

if $\mathrm{cor}(\hat{y}_2, \hat{y}_2) = 0$

$\Rightarrow \sum \hat{y}_2 y_2 - 0 = \sum \hat{y}_2 \cdot y_2$ #

11.16.

a. $Q_i = \alpha_1 + \alpha_2 P_i + e_{di} = \beta_1 + \beta_2 P_i + \beta_3 W_i + e_{si}$

$$\Rightarrow P_i = \frac{\beta_1 - \alpha_1}{\alpha_2 - \beta_2} + \frac{\beta_3}{\alpha_2 + \beta_2} W_i + \frac{e_{di} + e_{si}}{\alpha_2 - \beta_2} = \pi_1 + \pi_2 W + V_1$$

$$Q_1 = \alpha_1 + \alpha_2 (\pi_1 + \pi_2 W + V_1) + e_{di}$$

$$= (\alpha_1 + \frac{\alpha_2(\beta_1 - \alpha_1)}{\alpha_2 - \beta_2}) + \frac{\alpha_2 \cdot \beta_3}{\alpha_2 - \beta_2} W_i + (e_{di} + \frac{\alpha_2 (e_{di} + e_{si})}{\alpha_2 - \beta_2})$$

$$= \theta_1 + \theta_2 W_i + V_2$$

b. Only Demand Equation is "identified" because $M = 2$, and there is zero variable being omitted in Supply equation, which require at least $2-1=1$ variable being omitted to make equation "identified" $\Rightarrow \alpha_1, \alpha_2$ can be solved

c. $5 + 0.5W = \alpha_1 + \alpha_2 (2.4 + W) \Rightarrow \begin{cases} 2.4\alpha_2 + \alpha_1 = 5 \\ \alpha_2 = 0.5 \end{cases} \Rightarrow \alpha_1 = 3.8, \alpha_2 = 0.5$

d. $\hat{P} = 2.4 + W$, $\bar{\hat{P}} = 4.4$, $\bar{Q} = 6$

W	$\hat{P}$	$\hat{P} - \bar{\hat{P}}$	$Q - \bar{Q}$	$(\hat{P} - \bar{\hat{P}})^2$	$(\hat{P} - \bar{\hat{P}}) \cdot (Q - \bar{Q})$
2	4.4	0	-2	-2	0
3	5.4	1	0	0	1
1	3.4	-1	3	3	1
1	3.4	-1	-3	-3	1
3	5.4	1	2	2	1
SUM 10	22	0	0	4	2

$Q_1 = \alpha_1 - \alpha_2 \hat{P} + e_i$

$$\alpha_2 = \frac{\sum(\hat{P}_i - \bar{\hat{P}}_i)(Q_i - \bar{Q})}{\sum(\hat{P}_i - \bar{\hat{P}})^2} = \frac{2}{4} = \frac{1}{2}$$

$$\Rightarrow \hat{\alpha}_1 = \bar{Q} - \alpha_2 \bar{\hat{P}} = 6 - \frac{1}{2} \cdot 4.4 = 3.8$$

$$\Rightarrow \hat{Q} = 3.8 + 0.5P$$

11.17.

a. $M=8$, Endogenous $=8$, Exogenous $=8$, at least $8-1=7$ variable should be omitted to make equation identified.

Consumption: 5 variable included, 11 omitted ⇒ identified

Investment: 4 " 12 " ⇒ identified

Wage : 4 " 11 " ⇒ identified

b. consumption: 2 endogenous variables included and exclude 5 exogenous

Investment : 1 " " 5 "

Wage : 1 " " 5 "

⇒ all satisfied

c. $W_{1t} = \pi_1 + \pi_2 G_t + \pi_3 W_{2t} + \pi_4 + x_t + \pi_6 TIME_t + \pi_6 P_{t-1} + \pi_7 K_{t-1} + \pi 8 E_{t-1}$

d. from (c), we get $\hat{W}_{1t}$, and apply same method to obtain $\hat{P}_t$, then regress CN_t by OLS with $\hat{W}_{1t}$ and $\hat{P}_t$

e. Coefficient will be the same, but t-values won't.