1. Let $k=2$. $Y = x\beta + e$

$$Y = (y_1, y_2 \dots y_N), \quad \beta = (\beta_1, \beta_2 \dots \beta_k), \quad e = (e_1, e_2 \cdots e_n)$$

$$X = \begin{bmatrix} 1 & x_1 \\ \vdots & \vdots \\ 1 & x_n \end{bmatrix} \qquad Y = \begin{bmatrix} Y_1 \\ \vdots \\ Y_n \end{bmatrix}$$

$$X'X = \begin{bmatrix} 1 & 1 \\ x_1 & x_2 \end{bmatrix} \cdot \begin{bmatrix} 1 & x_1 \\ 1 & x_2 \end{bmatrix} = \begin{bmatrix} n & \Sigma x_i \\ \Sigma x_i & \Sigma x_i^2 \end{bmatrix}$$

$$X'Y = \begin{bmatrix} 1 & 1 \\ x_1 & x_2 \end{bmatrix} \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix} = \begin{bmatrix} \Sigma Y_i \\ \Sigma x_i Y_i \end{bmatrix}$$

$$b = (X'X)^{-1}(X'Y) \qquad (X'X)^{-1} = \frac{1}{n \cdot \Sigma x_i^2 - (\Sigma x_i)^2} \begin{bmatrix} \Sigma Y_i^2 & -\Sigma x_i \\ -\Sigma x_i & n \end{bmatrix}$$

$$\hat{b}_2^{ols} = \frac{n \cdot \Sigma x_i Y_i - \Sigma x_i Y_i}{n \cdot \Sigma x_i^2 - (\Sigma x_i)^2} = \frac{\Sigma x_i Y_i - \frac{\Sigma x_i Y_i}{n}}{\Sigma x_i^2 - \frac{\Sigma (x_i)^2}{n}}$$

$$= \frac{\sum_{i=1}^{n} (x - \bar{x})(Y_i - \bar{Y})}{\sum_{i=1}^{n} (x_i - \bar{x})^2}$$

$$\hat{b}_2^{OLS} = \frac{n \cdot \sum x_i y_i - \sum x_i y_i}{n \cdot \sum x_i^2 - (\sum x_i)^2} = \frac{\sum x_i y_i - \frac{\sum x_i y_i}{n}}{\sum x_i^2 - \frac{\sum (x_i)^2}{n}}$$

$$= \frac{\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n} (x_i - \bar{x})^2}$$

$$\hat{b}_1 = \frac{\sum x_i^2 y_i - \sum x_i \cdot \sum x_i y_i}{n \cdot \sum x_i^2 - (\sum x_i)^2} = \frac{\bar{y} \cdot n \cdot \sum x_i^2 - n \cdot \bar{x} \cdot \sum x_i y_i}{n \cdot \sum x_i^2 - n \cdot \bar{x}^2}$$

$$= \frac{\sum x_i \bar{y} - n \cdot \bar{x}^2 \cdot \bar{y}}{\sum (x_i - \bar{x})^2} - \bar{x} \frac{\sum x_i y_i - n \cdot \bar{x}\bar{y}}{\sum (x_i - \bar{x})^2}$$

$$= \bar{y} - \bar{x} \cdot \hat{b}_2$$

2. $k = 2$ $\quad Var(b) = \sigma^2 \cdot (x'x)^{-1}$

$$= \sigma^2 \frac{1}{n \cdot \sum x_i^2 - (\sum x_i)^2} \begin{bmatrix} \sum x_i^2 & -\sum x_i \\ -\sum x_i & n \end{bmatrix}$$

$$\operatorname{Var}(\hat{b}_1 \mid x) = \frac{\sigma^2 \cdot \sum x_i^2}{n \cdot \sum x_i^2 - (\sum x_i)^2} = \frac{\sigma^2 \cdot \sum x_i^2}{n \cdot \left(\sum x_i^2 - \frac{(\sum x_i)^2}{n}\right)}$$

$$= \frac{\sigma \cdot \sum x_i^2}{n \cdot \sum_{i=1}^{n} (x_i - \bar{x})^2}$$

$$\operatorname{Var}(\hat{b}_2 \mid x) = \frac{\sigma^2 \cdot n}{n \cdot \sum x_i^2 - (\sum x_i)^2} = \frac{\sigma^2 \cdot n}{n \cdot \sum (x_i - \bar{x})^2} = \frac{\sigma^2}{\sum (x_i - \bar{x})^2}$$

$$\operatorname{Cov}(\hat{b}_1, \hat{b}_2 \mid x) = \frac{\sigma^2 \cdot \sum x_i}{n \cdot \sum x_i^2 - (\sum x_i)^2} = \frac{-\sigma^2 \cdot n \cdot \bar{x}}{n \cdot \sum (x_i - \bar{x})^2}$$

$$\frac{-\sigma^2 \cdot \bar{x}}{\sum (x_i - \bar{x})^2} = \frac{-\bar{x}}{\sum (x_i - \bar{x})^2} \cdot \sigma^2$$

5.3 $WALL = \beta_1 + \beta_2 \ln(TOTEXP) + \beta_3 NK + \beta_4 AGE + e$

1. t for b_1 : $t = \dfrac{\hat{\beta}_1}{\text{St, Error}} = \dfrac{1.4515}{2.2019} = 0.6592$

Standard error for b_2 : $\dfrac{2.7648}{5.7103} = 0.4842$

$b_3 = 0.3695 \times (-3.9176) = -1.4549$

$R^2 = 1 - \dfrac{SSE}{SST} = \dfrac{SSR}{SST}$

$SST = (1200-1)(6.3954)^2 = 49041.5418$

$R^2 = 1 - \dfrac{46221.62}{49041.5418} = 0.0575$

$\hat{\sigma}^2 = \dfrac{SSE}{N-K} = \dfrac{46221.62}{1200-4} = 38.6468$

$\hat{\sigma} = \sqrt{38.6468} = 6.2169$

2.

b_2 總支出增加一單位，alcohol 增加 2.7648 單位

b_3 增加一個小孩，alcohol 減少 1.4549 單位

b_4 增加一歲，alcohol 減少 0.1503

3. 95% interval estimate for β_4

$[-0.1964, -0.1042]$

4.

b_1 不顯著，b_2、b_3、b_4 顯著

5. $H_0: \beta_3 = -2$

$H_1: \beta_3 \neq -2$

$$\frac{-1.45149-(-2)}{0.3695} = 1.4752$$

t value 1.9619

不拒絕 H_0