5.6 Suppose that, from a sample of 63 observations, the least squares estimates and the corresponding estimated covariance matrix are given by

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix} \qquad \widehat{\operatorname{cov}}(b_1, b_2, b_3) = \begin{bmatrix} 3 & -2 & 1 \\ -2 & 4 & 0 \\ 1 & 0 & 3 \end{bmatrix}$$

Using a 5% significance level, and an alternative hypothesis that the equality does not hold, test each of the following null hypotheses:

a. $\beta_2 = 0$
b. $\beta_1 + 2\beta_2 = 5$
c. $\beta_1 - \beta_2 + \beta_3 = 4$

(a.) $se(b_2) = \sqrt{\widehat{var}(b_2)} = \sqrt{4} = 2.$

$\Rightarrow t = \dfrac{b_2 - 0}{2} = \dfrac{3-0}{2} = 1.5 \sim t_{(63-3)} = t_{(60, 0.05)} = 2.$

$\because 0.5 < 2 \Rightarrow$ don't reject $H_0 \Rightarrow \beta_2 = 0$ at 95% level.

(b) $se(b_1 + 2b_2) = \sqrt{\widehat{var}(b_1 + 2b_2)} = \sqrt{3 + 4 \cdot 4 + 2 \cdot 2 \cdot (-2)} = \sqrt{11}$

$\Rightarrow t = \dfrac{b_1 + 2b_2 - 5}{se(b_1 + 2b_2)} = \dfrac{3}{\sqrt{11}} < 2 \Rightarrow$ don't reject $H_0 \Rightarrow \beta_1 + 2\beta_2 = 5$ at 95% level.

(c) $se(b_1 - b_2 + b_3) = \sqrt{\widehat{var}(b_1 - b_2 + b_3)} = \sqrt{3+4+3-2(-2+1+0)} = \sqrt{8}.$

$\Rightarrow t = \dfrac{b_1 - b_2 + b_3 - 4}{se(b_1 - b_2 + b_3)} = \dfrac{-6}{\sqrt{8}} = -\dfrac{3}{\sqrt{2}} < -t_{60, 0.05} \Rightarrow$ don't reject H_0.

$\Rightarrow b_1 - b_2 + b_3 = 4$ at 95% level.