Q1.

$$k=2 \longrightarrow y_i = b_1 + b_2 x_i + e_i$$

$$Y = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \end{bmatrix} \qquad X = \begin{bmatrix} 1 & x_1 \\ 1 & \vdots \\ 1 & x_n \end{bmatrix}$$

$$X'X = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ x_1 & x_2 & & x_N \end{bmatrix} \begin{bmatrix} 1 & x_1 \\ 1 & x_2 \\ \vdots & \vdots \\ 1 & x_n \end{bmatrix} = \begin{bmatrix} n & \sum_i x_i \\ \sum_i x_i & \sum_i x_i^2 \end{bmatrix}$$

$$(X'X)^{-1} = \frac{1}{n\sum_i x_i^2 - [\sum_i x_i]^2} \begin{bmatrix} \sum_i x_i^2 & -\sum_i x_i \\ -\sum_i x_i & n \end{bmatrix}$$

$$X'Y = \begin{bmatrix} 1 & 1 & \cdots & 1 \\ x_1 & x_2 & \cdots & x_n \end{bmatrix} = \begin{bmatrix} \sum_i y_i \\ \sum_i x_i y_i \end{bmatrix}$$

$$b = (X'X)^{-1}(X'Y) = \frac{1}{n\sum_i x_i^2 - (\sum_i x_i)^2} \begin{bmatrix} \sum_i x_i^2 & -\sum_i x_i \\ -\sum_i x_i & n \end{bmatrix} \begin{bmatrix} \sum_i y_i \\ \sum_i x_i y_i \end{bmatrix}$$

$$= \frac{1}{n\sum_i x_i^2 - (\sum_i x_i)^2} \begin{bmatrix} \sum_i x_i^2 \sum_i y_i - \sum_i x_i \sum_i x_i y_i \\ -\sum_i x_i \sum_i y_i + n\sum_i x_i y_i \end{bmatrix}$$

$$b_2 = \frac{-\sum_i x_i \sum_i y_i + n\sum_i x_i y_i}{n\sum_i x_i^2 - (\sum_i x_i)^2} = \frac{-n\bar{x}\bar{y} + \sum_i x_i y_i}{\sum_i x_i^2 - n\bar{x}^2}$$

$$= \frac{\sum_i (x_i - \bar{x})(y_i - \bar{y})}{\sum_i (x_i - \bar{x})^2}$$

$$b_1 = \frac{\sum_i x_i^2 \sum_i y_i - \sum_i x_i \sum_i x_i y_i}{n\sum_i x_i^2 - (\sum_i x_i)^2} = \frac{\bar{y}\sum_i (x_i - \bar{x})^2 - \bar{x}\sum (x_i - \bar{x})(y_i - \bar{x})}{\sum (x_i - \bar{x})^2}$$

$$= \bar{y} - b_1 \bar{x}.$$

Q2. $$Var(b) = \sigma^2 (X'X)^{-1} = \sigma^2 \frac{1}{n\sum x_i^2 - (\sum x_i)^2} \begin{bmatrix} \sum x_i^2 & -\sum x_i \\ -\sum x_i & n \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\sigma^2 \sum x_i^2}{n\sum (x_i - \bar{x})^2} & \frac{\sigma^2 (-\bar{x})}{\sum (x_i - \bar{x})^2} \\ \frac{\sigma^2 (-\bar{x})}{\sum (x_i - \bar{x})^2} & \frac{\sigma^2}{\sum (x_i - \bar{x})^2} \end{bmatrix}$$

So $var(b_1|x) = \frac{\sigma^2 \sum x_i^2}{n \sum (x_i - \bar{x})^2}$ $\quad Var(b_2|x) = \frac{\sigma^2}{\sum (x_i - \bar{x})^2}$

$$cov(b_1, b_2) = \sigma^2 \frac{-\sum x_i}{n\sum x_i^2 - (\sum x_i)^2} = \frac{-\sigma^2 \bar{x}}{\sum (x_i - x)^2}$$

5.3

$$t(b_1) = \frac{b_1}{Se(b_1)} = \frac{1.4515}{2.2019} = 0.659.$$

$$Se(b_2) = \frac{b_2}{t(b_2)} = \frac{2.7648}{5.7163} = 0.484$$

$$b_3 = t(b_3) Se(b_3) = -3.9376 \times 0.3595 = -1.4549.$$

$$R = 1 - \frac{SSE}{SST} = 1 - \frac{SSE}{(n-1)S_y} = 1 - \frac{46221.62}{(1200-1)6.39547}$$

$$= 0.8575.$$

$$\hat{\sigma}^2 = \sqrt{\frac{SSE}{n-k}} = \sqrt{\frac{46221.62}{1200-4}} = 6.217$$

b. $b_2 = 2.875$ in log, when expenditure increase 1%, spending increase $\frac{2.675}{100} = 0.02675$ %

$b_3 = -1.4549$. The number of children increase 1, spending on alcohol decreases by 1.4589%

$b_4 = -0.1503$. Age increase by 1 year, the spending on alcohol will decrease 0.1503%

c. $b_4 = t_{(0.975, 1196)} se(b_4) = (-0.1503 \pm 1.96 \times 0.0235)$

$= [-0.1964, -0.1042]$

The age of household head increase 1 year, the spending on alcohol will decrease by 0.1042 to 0.1964 %

e) $b_3 \neq -2$

$H_0: b_3 = -2$; $H_1 = b_3 = -2$ $\quad t = \frac{1.4549 - 2}{0.3695} = 1.475 < 1.96$.

$\rightarrow$ reject H_0.

d) p-value < 5% $\longrightarrow$ all coefs are significant

5.23 a) $\beta_2 < 0$; $\beta_3 > 0$; $\beta_4 < 0$

b) PRICE = 108 − 0.059 Quant + 0.116 qual − 2.354 Trend

The quantity increase 1 unit → price will increase

Quanlity increase 1 unit → price increase 0.059 unit 0.116 unit

times increase 1 unit → price decrease 2.354

c) $R^2 = 0.5097$

d) $H_0\ \beta_2 \geq 0$; $H_1\ \beta < 0$. $t = -5.892 < t = -1.96$.
$H_0\ \beta_2 < 0$ is true

e) $H_0: \beta_3 = 0$ or $H_1 = \beta_3 > 0$ $t = 0.572$
→ cannot reject H_0. → quality does not affect price.

f) $b_4 = -2.35458$. The price will decrease overtime