2.11.1

a. $\bar{x} = (3+2+1-1+0)/5 = 1$

$\bar{y} = (4+2+3+1+0)/5 = 2$

$x-\bar{x}$	$(x-\bar{x})^2$	$y-\bar{y}$	$(x-\bar{x})(y-\bar{y})$
2	4	2	4
1	1	0	0
0	0	1	0
-2	4	-1	2
-1	1	-2	2

b.

$$b_2 = \frac{\Sigma(x_i - \bar{x})(y_i - \bar{y})}{\Sigma(x_i - \bar{x})^2} = \frac{8}{10} = \frac{4}{5}$$

When x increases by 1 unit,

y will increase $\frac{4}{5}$ unit

$$b_1 = \bar{y} - b_2\bar{x} = 2 - \frac{4}{5} \times 1 = \frac{6}{5}$$

c.

$$\sum_{i=1}^{5} x_i^2 = 3^2 + 2^2 + 1^2 + (-1)^2 + 0^2 = 15$$

$$\sum_{i=1}^{5} x_i y_i = 3 \times 4 + 2 \times 2 + 1 \times 3 + (-1) \times 1 + 0 = 18$$

$$\sum_{i=1}^{5} (x_i - \bar{x})^2 = 10$$

$$\sum_{i=1}^{5} (x_i - \bar{x})(y - \bar{y}) = 8$$

d.

x_i	y_i	$\hat{y}_i$	$\hat{e}_i$	$\hat{e}_i^2$	$x_i\hat{e}_i$
3	4	3.6	0.4	0.16	1.2
2	2	2.8	-0.8	0.64	-1.6
1	3	2	1	1	1
-1	1	0.4	0.6	0.36	-0.6
0	0	1.2	-1.2	1.44	0

Continue d.

$$S_y^2 = \frac{\sum (Y_i - \bar{y})^2}{N-1} = 2.5$$

$$S_x^2 = \frac{\sum_{i=1}^{N} (x_i - \bar{x})^2}{N-1} = 2.5$$

$$S_{xy} = \frac{\sum (Y_i - \bar{y})(x_i - \bar{x})}{N-1} = 2$$

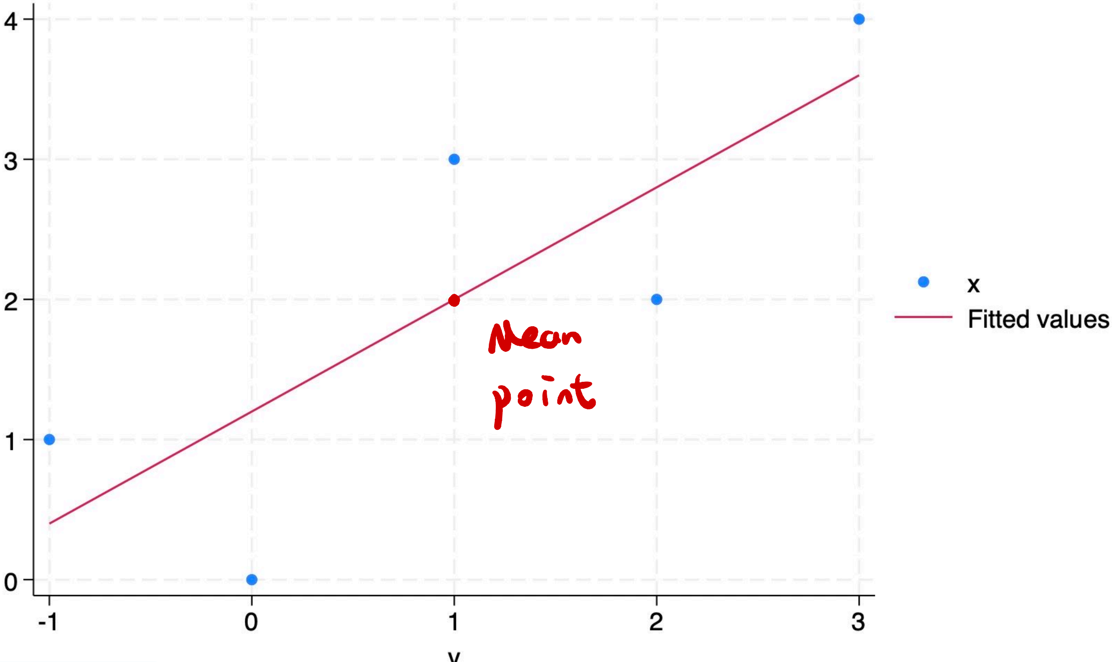
$$r_{xy} = \frac{S_{xy}}{S_x S_y} = \frac{2}{\sqrt{2.5} \times \sqrt{2.5}} = 0.8$$

$$CV_x = 100 \times \frac{\sqrt{2.5}}{1} = 158.144$$

$x = \{-1, 0, 1, 2, 3\}$ Medin of $x = 1$

e. use stata, I plot graph

f. Yes, the regression pass Mean point (1, 2)


4
3
2
1
0
-1
0
1
2
3
y
x
Fitted values
Mean point

```
. reg y x

      Source |       SS           df       MS      Number of obs   =         5
-------------+----------------------------------   F(1, 3)         =      5.33
       Model |         6.4         1         6.4   Prob > F        =    0.1041
    Residual |         3.6         3         1.2   R-squared       =    0.6400
-------------+----------------------------------   Adj R-squared   =    0.5200
       Total |          10         4         2.5   Root MSE        =    1.0954

------------------------------------------------------------------------------
           y | Coefficient  Std. err.      t    P>|t|     [95% conf. interval]
-------------+----------------------------------------------------------------
           x |         .8   .3464102     2.31   0.104    -.3024317    1.902432
       _cons |        1.2         .6     2.00   0.139    -.7094678    3.109468
------------------------------------------------------------------------------
```

g. $\bar{y} = 2$

$b_1 + b_2\bar{x} = \frac{6}{5} + \frac{4}{5} \times 1 = 2$

h. $\bar{\hat{y}} = \frac{1}{5}\left(\frac{16}{5} + \frac{14}{5} + 2 + \frac{2}{5} + \frac{6}{5}\right) = \frac{1}{5} \times \frac{50}{5} = 2 = \bar{y}$

i. $\hat{\sigma}^2 = \frac{\sum \hat{e}_i^2}{N-2} = \frac{\frac{18}{5}}{3} = \frac{6}{5}$

j. $\widehat{Var}(b_2 | x) = \frac{\hat{\sigma}^2}{\sum(x_i - \bar{x})^2} = \frac{3}{25}$

$se(b_2) = \sqrt{\widehat{Var}(b_2 | x)} = \sqrt{\frac{3}{25}} = \frac{\sqrt{3}}{5}$

2.14

a.

$$-4.88 + 1.8 \times \overline{EDUC} = 19.74$$

$$\overline{EDUC} = 13.6778$$

$$\varepsilon = \beta_2 \frac{x}{E(y|x)} = 1.8 \times \frac{13.6778}{19.74} = 1.247$$

b.

$$-10.76 + 2.46 \times 13.68 = 22.8928$$

$$se(\varepsilon) = 0.16 \times \frac{13.68}{22.8928} \approx 0.0956$$

c.

$EDUC = 12$

Urban $\quad Wage = -10.76 + 2.46 \times 12 = 18.76$

Rural $\quad Wage = -4.88 + 1.8 \times 12 = 16.72$

$EDUC = 16$

Urban $\quad Wage = -10.76 + 2.46 \times 16 = 28.6$

Rural $\quad Wage = -4.88 + 1.8 \times 16 = 23.92$

2.16

a. $r_j - r_f = \alpha_i + \beta_j (r_m - r_f) + e_j$

$r_i - r_f$ like return, means y,

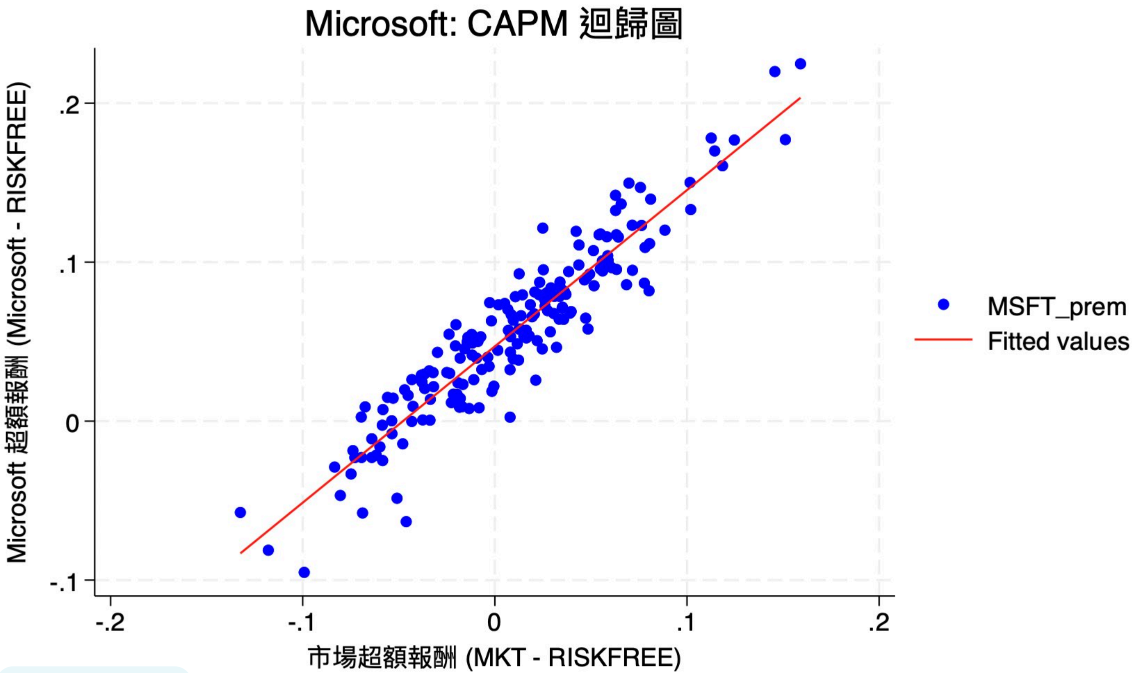
α_i is Intercept term

β_j is Coefficient, usually we check this Number

e_j is error term

b. I didn't found capm5, so I use random to Creating data, Below is my code and graph. to answer b to d.

```
. clear all

. set seed 12345

. set obs 180   // 180 個月份的資料 (例如：1998/01 ~ 2012/12)
Number of observations (_N) was 0, now 180.

.
. * 產生無風險利率 (假設每月常數 0.002，即約0.2%)
. gen RISKFREE = 0.002

.
. * 產生市場報酬率 (MKT)，假設均值為 0.01，標準差 0.05
. gen MKT = 0.01 + rnormal(0, 0.05)

.
. * 假設六家公司的真實 CAPM 參數：
. * 固定值 (例如：額外常數項固定為 0.05) ，以及不同的 beta 值
. * GE: beta = 1.1, IBM: beta = 0.9, Ford: beta = 1.2,
. * Microsoft: beta = 1.0, Disney: beta = 0.8, Exxon-Mobil: beta = 1.3
. * 並加入隨機誤差 (標準差 0.02)
.
. gen GE          = RISKFREE + 0.05 + 1.1*(MKT-RISKFREE) + rnormal(0,0.02)

. gen IBM         = RISKFREE + 0.05 + 0.9*(MKT-RISKFREE) + rnormal(0,0.02)

. gen Ford        = RISKFREE + 0.05 + 1.2*(MKT-RISKFREE) + rnormal(0,0.02)

. gen Microsoft   = RISKFREE + 0.05 + 1.0*(MKT-RISKFREE) + rnormal(0,0.02)

. gen Disney      = RISKFREE + 0.05 + 0.8*(MKT-RISKFREE) + rnormal(0,0.02)

. gen Exxon_Mobil = RISKFREE + 0.05 + 1.3*(MKT-RISKFREE) + rnormal(0,0.02)

.
. *-----------------------------------------------------------
. * 2. 計算市場超額報酬與各股票的超額報酬
. gen mkt_prem = MKT - RISKFREE

. gen GE_prem        = GE         - RISKFREE

. gen IBM_prem       = IBM        - RISKFREE

. gen Ford_prem      = Ford       - RISKFREE

. gen MSFT_prem      = Microsoft  - RISKFREE

. gen Disney_prem    = Disney     - RISKFREE

. gen Exxon_prem     = Exxon_Mobil- RISKFREE

.
. *-----------------------------------------------------------
. * 3. 使用完整模型 (含截距項) 分別估計各股票的 CAPM 模型
. di "=== CAPM 迴歸 (含截距) ==="
=== CAPM 迴歸 (含截距) ===

. * GE
. reg GE_prem mkt_prem

      Source |       SS           df       MS      Number of obs   =       180
-------------+----------------------------------   F(1, 178)       =   1424.72
       Model |  .606780569         1  .606780569   Prob > F        =    0.0000
    Residual |   .07580944       178  .000425896   R-squared       =    0.8889
-------------+----------------------------------   Adj R-squared   =    0.8883
       Total |   .68259001       179  .003813352   Root MSE        =    .02064

------------------------------------------------------------------------------
     GE_prem | Coefficient  Std. err.      t    P>|t|     [95% conf. interval]
-------------+----------------------------------------------------------------
    mkt_prem |   1.130185   .0299423    37.75   0.000     1.071097    1.189272
       _cons |   .0521487   .0015676    33.27   0.000     .0490553    .0552422
------------------------------------------------------------------------------

. * IBM
. reg IBM_prem mkt_prem

      Source |       SS           df       MS      Number of obs   =       180
-------------+----------------------------------   F(1, 178)       =    806.67
       Model |  .348248968         1  .348248968   Prob > F        =    0.0000
    Residual |  .076844967       178  .000431713   R-squared       =    0.8192
-------------+----------------------------------   Adj R-squared   =    0.8182
       Total |  .425093935       179  .002374826   Root MSE        =    .02078

------------------------------------------------------------------------------
    IBM_prem | Coefficient  Std. err.      t    P>|t|     [95% conf. interval]
-------------+----------------------------------------------------------------
    mkt_prem |   .8562064   .0301461    28.40   0.000     .7967166    .9156961
       _cons |   .0509167   .0015783    32.26   0.000     .0478021    .0540312
------------------------------------------------------------------------------

. * Ford
. reg Ford_prem mkt_prem

      Source |       SS           df       MS      Number of obs   =       180
-------------+----------------------------------   F(1, 178)       =   1532.72
       Model |  .688129466         1  .688129466   Prob > F        =    0.0000
    Residual |  .079914698       178  .000448959   R-squared       =    0.8960
-------------+----------------------------------   Adj R-squared   =    0.8954
       Total |  .768044164       179   .00429075   Root MSE        =    .02119

------------------------------------------------------------------------------
   Ford_prem | Coefficient  Std. err.      t    P>|t|     [95% conf. interval]
-------------+----------------------------------------------------------------
    mkt_prem |   1.203562   .0307423    39.15   0.000     1.142896    1.264229
       _cons |   .0510046   .0016095    31.69   0.000     .0478285    .0541808
------------------------------------------------------------------------------

. * Microsoft
. reg MSFT_prem mkt_prem

      Source |       SS           df       MS      Number of obs   =       180
-------------+----------------------------------   F(1, 178)       =   1292.17
       Model |  .459257189         1  .459257189   Prob > F        =    0.0000
    Residual |  .063263998       178  .000355416   R-squared       =    0.8789
-------------+----------------------------------   Adj R-squared   =    0.8782
       Total |  .522521187       179  .002919113   Root MSE        =    .01885

------------------------------------------------------------------------------
   MSFT_prem | Coefficient  Std. err.      t    P>|t|     [95% conf. interval]
-------------+----------------------------------------------------------------
    mkt_prem |   .9832445   .0273528    35.95   0.000      .929267    1.037222
       _cons |   .0470294    .001432    32.84   0.000     .0442035    .0498554
------------------------------------------------------------------------------

. * Disney
. reg Disney_prem mkt_prem

      Source |       SS           df       MS      Number of obs   =       180
-------------+----------------------------------   F(1, 178)       =    717.67
       Model |  .312099374         1  .312099374   Prob > F        =    0.0000
    Residual |  .077408517       178  .000434879   R-squared       =    0.8013
-------------+----------------------------------   Adj R-squared   =    0.8001
       Total |   .38950789       179  .002176022   Root MSE        =    .02085

------------------------------------------------------------------------------
 Disney_prem | Coefficient  Std. err.      t    P>|t|     [95% conf. interval]
-------------+----------------------------------------------------------------
    mkt_prem |   .8105503   .0302564    26.79   0.000     .7508428    .8702578
       _cons |   .0509637   .0015841    32.17   0.000     .0478378    .0540897
------------------------------------------------------------------------------

. * Exxon-Mobil
. reg Exxon_prem mkt_prem

      Source |       SS           df       MS      Number of obs   =       180
-------------+----------------------------------   F(1, 178)       =   1875.25
       Model |  .825027395         1  .825027395   Prob > F        =    0.0000
    Residual |  .078312242       178  .000439956   R-squared       =    0.9133
-------------+----------------------------------   Adj R-squared   =    0.9128
       Total |  .903339637       179   .00504659   Root MSE        =    .02098

------------------------------------------------------------------------------
  Exxon_prem | Coefficient  Std. err.      t    P>|t|     [95% conf. interval]
-------------+----------------------------------------------------------------
    mkt_prem |   1.317855   .0304326    43.30   0.000       1.2578     1.37791
       _cons |    .052582   .0015933    33.00   0.000     .0494379    .0557261
------------------------------------------------------------------------------

.
. *-----------------------------------------------------------
. * 從結果中，你可以檢查各股票的 beta 值 (mkt_prem 的係數) ：
. * - beta 大於1的股票視為積極型，
. * - beta 小於1的股票視為防禦型。
. * 例如：若 Exxon-Mobil 的 beta 最大，則它為最積極的股票，
. * 而 Disney 的 beta 較低則可能屬於較防禦的股票。
.
. *-----------------------------------------------------------
. * 4. 對 Microsoft 畫出資料散佈圖與迴歸線
. twoway (scatter MSFT_prem mkt_prem, mcolor(blue)) ///
>        (lfit MSFT_prem mkt_prem, lcolor(red)), ///
>        title("Microsoft: CAPM 迴歸圖") ///
>        xtitle("市場超額報酬 (MKT - RISKFREE)") ///
>        ytitle("Microsoft 超額報酬 (Microsoft - RISKFREE)")

.
. *-----------------------------------------------------------
. * 5. 在假設截距項 a = 0 的條件下重新估計模型
. di "=== CAPM 迴歸 (不含截距) ==="
=== CAPM 迴歸 (不含截距) ===

. * GE
. reg GE_prem mkt_prem, noconstant

      Source |       SS           df       MS      Number of obs   =       180
-------------+----------------------------------   F(1, 179)       =    282.17
       Model |  .862495175         1  .862495175   Prob > F        =    0.0000
    Residual |  .547131177       179  .003056599   R-squared       =    0.6119
-------------+----------------------------------   Adj R-squared   =    0.6097
       Total |  1.40962635       180  .007831258   Root MSE        =    .05529

------------------------------------------------------------------------------
     GE_prem | Coefficient  Std. err.      t    P>|t|     [95% conf. interval]
-------------+----------------------------------------------------------------
    mkt_prem |    1.32218   .0787103    16.80   0.000      1.16686    1.477499
------------------------------------------------------------------------------

. * IBM
. reg IBM_prem mkt_prem, noconstant

      Source |       SS           df       MS      Number of obs   =       180
-------------+----------------------------------   F(1, 179)       =    182.82
       Model |   .53740082         1   .53740082   Prob > F        =    0.0000
    Residual |  .526159022       179  .002939436   R-squared       =    0.5053
-------------+----------------------------------   Adj R-squared   =    0.5025
       Total |  1.06355984       180  .005908666   Root MSE        =    .05422

------------------------------------------------------------------------------
    IBM_prem | Coefficient  Std. err.      t    P>|t|     [95% conf. interval]
-------------+----------------------------------------------------------------
    mkt_prem |   1.043665    .077187    13.52   0.000     .8913519    1.195979
------------------------------------------------------------------------------

. * Ford
. reg Ford_prem mkt_prem, noconstant

      Source |       SS           df       MS      Number of obs   =       180
-------------+----------------------------------   F(1, 179)       =    322.09
       Model |  .955092723         1  .955092723   Prob > F        =    0.0000
    Residual |   .53078216       179  .002965263   R-squared       =    0.6428
-------------+----------------------------------   Adj R-squared   =    0.6408
       Total |  1.48587488       180   .00825486   Root MSE        =    .05445

------------------------------------------------------------------------------
   Ford_prem | Coefficient  Std. err.      t    P>|t|     [95% conf. interval]
-------------+----------------------------------------------------------------
    mkt_prem |   1.391345   .0775254    17.95   0.000     1.238364    1.544326
------------------------------------------------------------------------------

. * Microsoft
. reg MSFT_prem mkt_prem, noconstant

      Source |       SS           df       MS      Number of obs   =       180
-------------+----------------------------------   F(1, 179)       =    264.44
       Model |  .659759709         1  .659759709   Prob > F        =    0.0000
    Residual |  .446590679       179   .00249492   R-squared       =    0.5963
-------------+----------------------------------   Adj R-squared   =    0.5941
       Total |  1.10635039       180  .006146391   Root MSE        =    .04995

------------------------------------------------------------------------------
   MSFT_prem | Coefficient  Std. err.      t    P>|t|     [95% conf. interval]
-------------+----------------------------------------------------------------
    mkt_prem |   1.156392   .0711116    16.26   0.000     1.016067    1.296717
------------------------------------------------------------------------------

. * Disney
. reg Disney_prem mkt_prem, noconstant

      Source |       SS           df       MS      Number of obs   =       180
-------------+----------------------------------   F(1, 179)       =    166.79
       Model |  .491581746         1  .491581746   Prob > F        =    0.0000
    Residual |   .52755346       179  .002947226   R-squared       =    0.4824
-------------+----------------------------------   Adj R-squared   =    0.4795
       Total |  1.01913521       180  .005661862   Root MSE        =    .05429

------------------------------------------------------------------------------
 Disney_prem | Coefficient  Std. err.      t    P>|t|     [95% conf. interval]
-------------+----------------------------------------------------------------
    mkt_prem |   .9981827   .0772892    12.91   0.000     .8456674    1.150698
------------------------------------------------------------------------------

. * Exxon-Mobil
. reg Exxon_prem mkt_prem, noconstant

      Source |       SS           df       MS      Number of obs   =       180
-------------+----------------------------------   F(1, 179)       =    361.88
       Model |  1.12709551         1  1.12709551   Prob > F        =    0.0000
    Residual |  .557498156       179  .003114515   R-squared       =    0.6691
-------------+----------------------------------   Adj R-squared   =    0.6672
       Total |  1.68459367       180  .009358854   Root MSE        =    .05581

------------------------------------------------------------------------------
  Exxon_prem | Coefficient  Std. err.      t    P>|t|     [95% conf. interval]
-------------+----------------------------------------------------------------
    mkt_prem |   1.511446   .0794525    19.02   0.000     1.354662     1.66823
------------------------------------------------------------------------------
```


Microsoft: CAPM 迴歸圖
Microsoft 超額報酬 (Microsoft - RISKFREE)
.2
.1
0
-.1
-.2
-.1
0
.1
.2
市場超額報酬 (MKT - RISKFREE)
MSFT_prem
Fitted values