# HW3

# Q01

If $k = 2$ :

$$X = \begin{bmatrix} 1 & x_1 \\ 1 & x_2 \\ \vdots & \vdots \\ 1 & x_N \end{bmatrix} \quad Y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_3 \end{bmatrix}$$

$$b = (X'X)^{-1}(X'Y)$$

$$= \begin{bmatrix} N & \sum x \\ \sum x & \sum x^2 \end{bmatrix}^{-1} \begin{bmatrix} \sum y \\ \sum xy \end{bmatrix}$$

$$= \frac{1}{N \sum x^2 - N^2 \bar{x}^2} \begin{bmatrix} \sum x^2 & -N\bar{x} \\ -N\bar{x} & N \end{bmatrix} \begin{bmatrix} N\bar{y} \\ \sum xy \end{bmatrix}$$

$$= \frac{N}{N \sum x^2 - N^2 \bar{x}^2} \begin{bmatrix} \bar{y} \sum x^2 - \bar{x} \sum xy \\ -N\bar{x}\bar{y} + \sum xy \end{bmatrix}$$

$$= \frac{1}{\sum (x - \bar{x})^2} \begin{bmatrix} \bar{y} \sum x^2 - \bar{x}(\sum (x - \bar{y})(y - \bar{y}) + N\bar{x}\bar{y}) \\ \sum (x - \bar{y})(y - \bar{y}) \end{bmatrix}$$

Notice that :

$$\sum x = N\bar{x}$$

$$\begin{aligned} \sum (x - \bar{x})(y - \bar{y}) &= \sum (xy - x\bar{y} - \bar{x}y + \bar{y}\bar{x}) \\ &= \sum xy - \bar{y} \sum x - \bar{x} \sum y + N\bar{x}\bar{y} \\ &= \sum xy - 2N\bar{x}\bar{y} + N\bar{x}\bar{y} \\ &= \sum xy - N\bar{x}\bar{y} \end{aligned}$$

$$\Rightarrow \quad \sum (x - \bar{x})^2 = \sum x^2 - N\bar{x}^2$$

# Q01

from the previous page:

$$\frac{1}{\sum(x-\bar{x})^2}\begin{bmatrix}\bar{y}\sum x^2-\bar{x}\left(\sum(x-\bar{y})(y-\bar{y})+N\bar{x}\bar{y}\right)\\ \sum(x-\bar{y})(y-\bar{y})\end{bmatrix}$$

$$=\frac{1}{\sum(x-\bar{x})^2}\begin{bmatrix}\bar{y}\left(\sum x^2-N\bar{x}^2\right)-\bar{x}\sum(x-\bar{y})(y-\bar{y})\\ \sum(x-\bar{y})(y-\bar{y})\end{bmatrix}$$

$$=\begin{bmatrix}\bar{y}-\bar{x}\frac{\sum(x-\bar{y})(y-\bar{y})}{\sum(x-\bar{x})^2}\\ \frac{\sum(x-\bar{y})(y-\bar{y})}{\sum(x-\bar{x})^2}\end{bmatrix}$$

$$=\begin{bmatrix}b_1\\ b_2\end{bmatrix}$$ from eq.(2.7) (2.8)

# Q02

If $k = 2$ :

$$var(b) = \sigma^2 (X'X)^{-1}$$

$$= \frac{\sigma^2}{\sum(x-\bar{x})^2} \begin{bmatrix} \frac{\sum x^2}{N} & -\bar{x} \\ -\bar{x} & 1 \end{bmatrix}$$

$$= \sigma^2 \begin{bmatrix} \frac{\sum x^2}{N\sum(x-\bar{x})^2} & \frac{-\bar{x}}{\sum(x-\bar{x})^2} \\ \frac{-\bar{x}}{\sum(x-\bar{x})^2} & \frac{1}{\sum(x-\bar{x})^2} \end{bmatrix}$$

$$= \begin{bmatrix} var(b_1) & cov(b_1, b_2) \\ cov(b_2, b_1) & var(b_2) \end{bmatrix}$$

from eq.(2.14)~(2.16)

From Q01 we have:

$$(X'X)^{-1} = \frac{1}{N\sum x^2 - N^2\bar{x}^2} \begin{bmatrix} \sum x^2 & -N\bar{x} \\ -N\bar{x} & N \end{bmatrix}$$

$$= \frac{1}{\sum(x-\bar{x})^2} \begin{bmatrix} \frac{\sum x^2}{N} & -\bar{x} \\ -\bar{x} & 1 \end{bmatrix}$$

# C05Q03

(a)

```
The t-statistic for b1:  0.659191
The std for b2:  0.4841777
The estimate for b3:  -1.454943
R-squared:  0.05750068
σ_hat:  6.216658
```

(b)

$b_2$ : Holding other variables constant, a 1% increase TOTEXP is expected to change the percentage of the household's budget spent on alcohol (WALC) by $b_2$ percentage points.

$b_3$ : Holding other variables constant, for each additional child in the household, WALC is expected to change by $b_3$ percentage points.

$b_4$ : Holding other variables constant, for each +1 year in the age of the household head, WALC is expected to change by $b_4$ percentage points.

# C05Q03

(c)

```
95%CI of β4: [ -0.1041942 ,  -0.1964058 ]
```

The interval suggests that the mean budget share of alcohol decrease by 0.1 to 0.2 percentage points for each +1 year in the age of the household head.

(d)

The estimates b2, b3, and b4 are significant, as their p-values < 0.05.

However, b1 is not significant, as its p-value > 0.05

# C05Q03

(e)

H0: $\beta_3 = -2$ H1: $\beta_3 \neq -2$

```
t-value:  1.47512
critical value: ± 1.961949
```

=> Fail to reject H0, it suggests that the addition of an extra child decreases the mean budget share of alcohol by 2 percentage points .

# C05Q23

(a)

$\beta_2$ : -  $\beta_3$ : +  $\beta_4$ : +

(b)

```
Call:
lm(formula = price ~ quant + qual + trend, data = cocaine)

Coefficients:
(Intercept)        quant         qual        trend
   90.84669     -0.05997      0.11621     -2.35458
```

The signs of the estimates for $\beta_2$ and $\beta_3$ have turned out as expected, while the sign for $\beta_4$ has not.

# C05Q23

(c)

```
R-squared:   0.50965
```

(d)

H0: $\beta_2 \geq 0$        H1: $\beta_2 < 0$

set $\alpha$ = 0.05

```
t-value:   -5.891936
critical value:   -1.674689
```

=> Reject H0, it suggests that sellers are willing to accept a lower price if they can make sales in larger quantities.

# C05Q23

(e)

H0: $\beta_3 \leq 0$ H1: $\beta_3 > 0$

set α = 0.05

```
t-value:  0.5716946
critical value:  1.674689
```

=> Fail to Reject H0, it suggests that the quality of cocaine has no influence on expected price .

# C05Q23

(f)

```
b4 = -2.354579
```

The price per gram of cocaine decreases by an average of $2.35 every year, which may be caused by either an increase in supply, a decrease in demand or both over time.