**2.28** How much does education affect wage rates? The data file *cps5_small* contains 1200 observations on hourly wage rates, education, and other variables from the 2013 Current Population Survey (CPS). [Note: *cps5* is a larger version.]

**a.** Obtain the summary statistics and histograms for the variables *WAGE* and *EDUC*. Discuss the data characteristics.

**b.** Estimate the linear regression $WAGE = \beta_1 + \beta_2 EDUC + e$ and discuss the results.

**c.** Calculate the least squares residuals and plot them against *EDUC*. Are any patterns evident? If assumptions SR1–SR5 hold, should any patterns be evident in the least squares residuals?

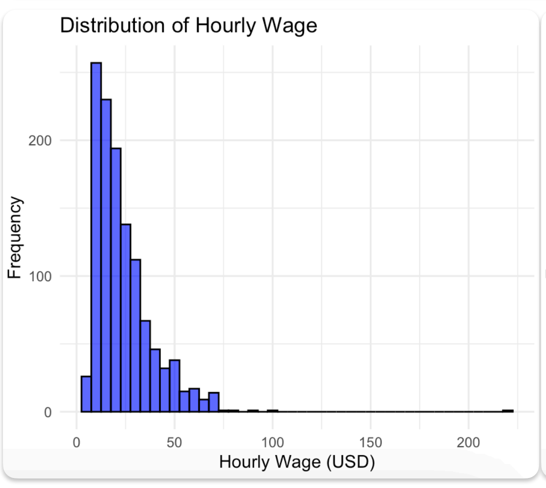
**d.** Estimate separate regressions for males, females, blacks, and whites. Compare the results.

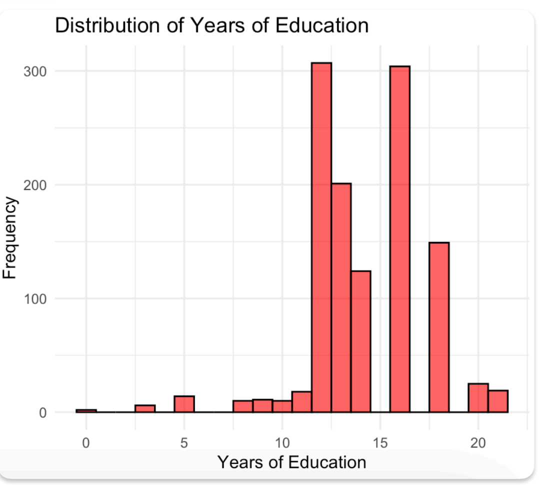
**e.** Estimate the quadratic regression $WAGE = \alpha_1 + \alpha_2 EDUC^2 + e$ and discuss the results. Estimate the marginal effect of another year of education on wage for a person with 12 years of education and for a person with 16 years of education. Compare these values to the estimated marginal effect of education from the linear regression in part (b).

**f.** Plot the fitted linear model from part (b) and the fitted values from the quadratic model from part (e) in the same graph with the data on *WAGE* and *EDUC*. Which model appears to fit the data better?

a.

```
# 摘要統計
summary(cps5_small$wage)
 Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
 3.94   13.00   19.30   23.64   29.80  221.10
summary(cps5_small$educ)
 Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
  0.0    12.0    14.0    14.2    16.0    21.0
```




WAGE（工資）：呈右偏分佈（skewed right），表示少數人擁有極高的工資，而大部分人的工資相對較低。
EDUC（教育年數）：受教育年數多集中在 12-16 年，顯示多數人至少完成了高中或部分大學教育。
綜上，教育程度與工資之間可能存在正向關係。

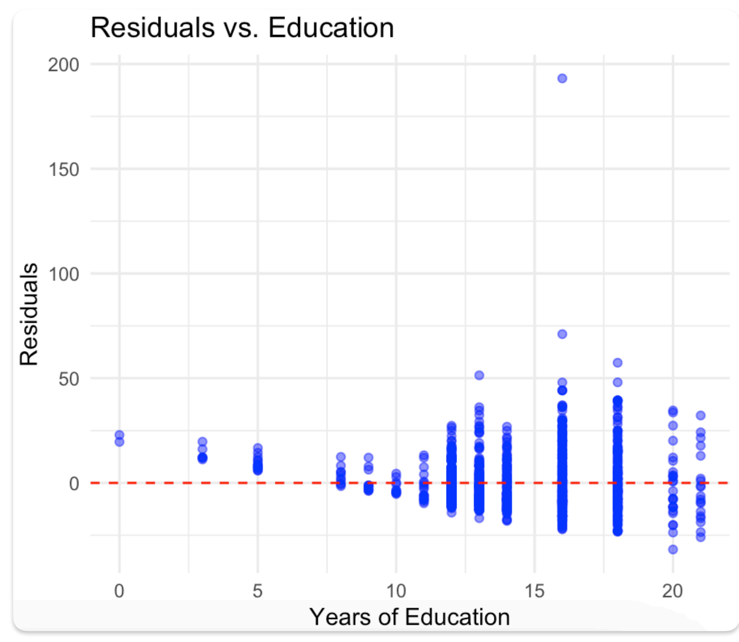
b.

```
Call:
lm(formula = wage ~ educ, data = cps5_small)

Residuals:
    Min      1Q  Median      3Q     Max
-31.785  -8.381  -3.166   5.708 193.152

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) -10.4000     1.9624    -5.3 1.38e-07 ***
educ          2.3968     0.1354    17.7  < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 13.55 on 1198 degrees of freedom
Multiple R-squared:  0.2073,    Adjusted R-squared:  0.2067
F-statistic: 313.3 on 1 and 1198 DF,  p-value: < 2.2e-16
```

$\widehat{wage} = -10.4 + 2.3968 educ$

- 教育年數的係數 β2 =2.3968
  p 值 < 0.01（非常顯著），表示教育年數對工資的影響統計顯著。
  t 值高達 17.7，進一步確認教育對工資有強烈影響。
- R-squared = 0.2073（約 20.73%）：解釋變異較低，表示 教育年數只能解釋 20.73% 的工資變異，還有其他因素影響工資（如經驗、產業、地區等）。
- 調整後 R-squared=0.2067（略低）：表示模型的適配度仍然有限，添加更多變數可能提高預測能力。
- 殘差標準誤=13.55（表示預測工資與實際工資的誤差平均約 13.55 美元）。
  可能存在 Heteroskedasticity 或遺漏變數問題 Omitted Variable Bias。
- F 檢定：F 值 = 313.3，p 值 < 2.2e-16
  表示 整體回歸模型顯著，即 EDUC 在模型中具有統計顯著性。
- 結論：
  教育年數對工資有顯著正向影響（每多 1 年教育，工資約增加 2.40 美元）。
  但R-squared低，表示工資還受到許多其他因素影響。

c.




由上圖可知：
- 殘差的變異數隨 EDUC 增加而變大。
- 存在極端值（Outliers）。
- 殘差未完全隨機分佈，在特定教育年數（如 12-16 年）可能較集中。

模型可能違反 OLS 假設（SR1-SR5），特別是：
- SR2：E(e | X) = 0。
  由上圖可知，殘差在某些教育年數範圍內明顯偏離 0。
- SR3：Homoskedasticity，Var(e | X) = $\sigma^2$。
  由上圖可知，殘差的變異數隨 EDUC 增加，表示存在 Heteroskedasticity。

d. 男性：

```
Call:
lm(formula = wage ~ educ, data = cps5_small, subset = (female ==
    0))

Residuals:
    Min      1Q  Median      3Q     Max
-27.643  -9.279  -2.957   5.663 191.329

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  -8.2849     2.6738  -3.099  0.00203 **
educ          2.3785     0.1881  12.648  < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 14.71 on 670 degrees of freedom
Multiple R-squared:  0.1927,    Adjusted R-squared:  0.1915
F-statistic:   160 on 1 and 670 DF,  p-value: < 2.2e-16
```

女性：

```
Call:
lm(formula = wage ~ educ, data = cps5_small, subset = (female ==
    1))

Residuals:
    Min      1Q  Median      3Q     Max
-30.837  -6.971  -2.811   5.102  49.502

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) -16.6028     2.7837  -5.964 4.51e-09 ***
educ          2.6595     0.1876  14.174  < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 11.5 on 526 degrees of freedom
Multiple R-squared:  0.2764,    Adjusted R-squared:  0.275
F-statistic: 200.9 on 1 and 526 DF,  p-value: < 2.2e-16
```

黑人：

```
Call:
lm(formula = wage ~ educ, data = cps5_small, subset = (black ==
    1))

Residuals:
    Min      1Q  Median      3Q     Max
-15.673  -6.719  -2.673   4.321  40.381

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  -6.2541     5.5539  -1.126    0.263
educ          1.9233     0.3983   4.829 4.79e-06 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 10.51 on 103 degrees of freedom
Multiple R-squared:  0.1846,    Adjusted R-squared:  0.1767
F-statistic: 23.32 on 1 and 103 DF,  p-value: 4.788e-06
```

白人：

```
Call:
lm(formula = wage ~ educ, data = cps5_small, subset = (black ==
    0))

Residuals:
    Min      1Q  Median      3Q     Max
-32.131  -8.539  -3.119   5.960 192.890

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  -10.475      2.081  -5.034  5.6e-07 ***
educ           2.418      0.143  16.902  < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 13.79 on 1093 degrees of freedom
Multiple R-squared:  0.2072,    Adjusted R-squared:  0.2065
F-statistic: 285.7 on 1 and 1093 DF,  p-value: < 2.2e-16
```

回歸結果摘要：

| 族群 | 截距 ($\beta_1$) | 教育係數 ($\beta_2$) | $R^2$ | p 值 |
|---|---|---|---|---|
| 男性 | -8.2849 | **2.3785** | 0.1927 | < 2.2e-16 |
| 女性 | -16.6028 | **2.6595** | 0.2764 | < 2.2e-16 |
| 黑人 | -6.2541 | **1.9233** | 0.1846 | 4.79e-06 |
| 白人 | -10.475 | **2.418** | 0.2072 | < 2.2e-16 |

e.

```
Call:
lm(formula = wage ~ I(educ^2), data = cps5_small)

Residuals:
    Min      1Q  Median      3Q     Max
-34.820  -8.117  -2.752   5.248 193.365

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 4.916477   1.091864   4.503 7.36e-06 ***
I(educ^2)   0.089134   0.004858  18.347  < 2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 13.45 on 1198 degrees of freedom
Multiple R-squared:  0.2194,    Adjusted R-squared:  0.2187
F-statistic: 336.6 on 1 and 1198 DF,  p-value: < 2.2e-16
```

$\widehat{wage} = 4.9165 + 0.0891 educ^2$

Marginal Effect (ME): $\frac{d(wage)}{d(educ)} = 2 \times 0.0891 educ$

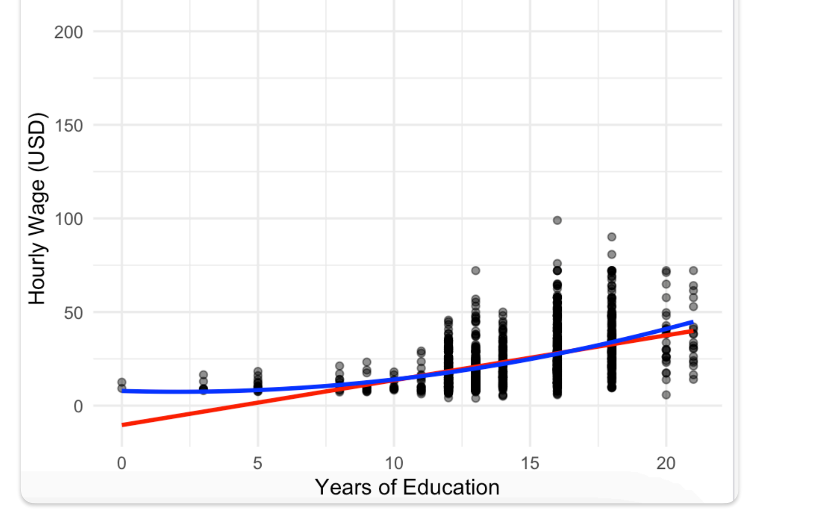
when $educ = 12$, $ME = 2 \times 0.0891 \times 12 = 2.1384$

when $educ = 16$, $ME = 2 \times 0.0891 \times 16 = 2.8512$

與 (b) 相比，在 線性回歸 中，Marginal Effect 是常數，ME = 2.3968
但在 二次回歸 中，Marginal Effect 會變動：
- 當 EDUC = 12 時，ME = 2.1384（比線性回歸小）。
- 當 EDUC = 16 時，ME = 2.8512（比線性回歸大）。

線性回歸假設固定回報，但二次回歸顯示教育對工資的影響可能是遞增的。

f.




二次回歸（藍色）比線性回歸（紅色）更能捕捉數據的變化趨勢：
在低教育水準（EDUC < 10），線性回歸（紅色）對低教育年數的擬合偏低，特別是在 EDUC < 5 時，預測的工資接近 0 或負數，不合理。二次回歸（藍色）曲線更貼合數據點。