HW0331

1.

5.6 Suppose that, from a sample of 63 observations, the least squares estimates and the corresponding estimated covariance matrix are given by

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix} \qquad \widehat{\text{cov}}(b_1, b_2, b_3) = \begin{bmatrix} 3 & -2 & 1 \\ -2 & 4 & 0 \\ 1 & 0 & 3 \end{bmatrix}$$

Using a 5% significance level, and an alternative hypothesis that the equality does not hold, test each of the following null hypotheses:

a. $\beta_2 = 0$
b. $\beta_1 + 2\beta_2 = 5$
c. $\beta_1 - \beta_2 + \beta_3 = 4$

a. H0 : b2 = 0；H1 : b2 != 0

檢定統計量 = (b2 – 0) / $\sqrt{\text{SE(B2)}}$，SE(b2) = 2，因此檢定統計量 = 3/2 = 1.5，而 $t_{0.025, 60}$ = 2，因此沒有證據拒絕虛無假設。

b. H0 : b1 + 2b2 = 5；H1 : b1 + 2b2 != 5

令 θ = b1 + 2b2，則 var(θ) = var(b1 + 2b2) = var(b1) + 4var(b2) + 4cov(b1, b2) = 3 + 16 – 8 = 11，SE(θ) = $\sqrt{11}$，檢定統計量 = (2 + 2 * 3 – 5) / $\sqrt{11}$ = 0.904，亦沒有證據拒絕虛無假設。

c. H0 : b1 – b2 + b3 = 4；H1 : b1 – b2 + b3 != 4

令 θ = b1 – b2 + b3 = -2，var(θ) = var(b1) + var(b2) + var(b3) - 2cov(b1, b2) - 2cov(b2, b3) + 2cov(b1, b3) = 16，檢定統計量 = (-2 – 4) / 4 = -1.5，在 0.05 信心水準下，亦沒有證據拒絕虛無假設。

2.

5.31 Each morning between 6:30 AM and 8:00 AM Bill leaves the Melbourne suburb of Carnegie to drive to work at the University of Melbourne. The time it takes Bill to drive to work (*TIME*), depends on the departure time (*DEPART*), the number of red lights that he encounters (*REDS*), and the number of trains that he has to wait for at the Murrumbeena level crossing (*TRAINS*). Observations on these

[16] "Estimating the Economic Model of Crime with Panel Data," *Review of Economics and Statistics*, 76, 1994, 360–366. The data were kindly provided by the authors.

variables for the 249 working days in 2015 appear in the file *commute5*. *TIME* is measured in minutes. *DEPART* is the number of minutes after 6:30 AM that Bill departs.

a. Estimate the equation

$$TIME = \beta_1 + \beta_2 DEPART + \beta_3 REDS + \beta_4 TRAINS + e$$

Report the results and interpret each of the coefficient estimates, including the intercept β_1.

b. Find 95% interval estimates for each of the coefficients. Have you obtained precise estimates of each of the coefficients?

c. Using a 5% significance level, test the null hypothesis that Bill's expected delay from each red light is 2 minutes or more against the alternative that it is less than 2 minutes.

d. Using a 10% significance level, test the null hypothesis that the expected delay from each train is 3 minutes against the alternative that it is not 3 minutes.

e. Using a 5% significance level, test the null hypothesis that Bill can expect a trip to be at least 10 minutes longer if he leaves at 7:30 AM instead of 7:00 AM, against the alternative that it will not be 10 minutes longer. (Assume other things are equal.)

f. Using a 5% significance level, test the null hypothesis that the expected delay from a train is at least three times greater than the expected delay from a red light against the alternative that it is less than three times greater.

g. Suppose that Bill encounters six red lights and one train. Using a 5% significance level, test the null hypothesis that leaving Carnegie at 7:00 AM is early enough to get him to the university on or before 7:45 AM against the alternative that it is not. [Carry out the test in terms of the expected time $E(TIME|\mathbf{X})$ where **X** represents the observations on all explanatory variables.]

h. Suppose that, in part (g), it is imperative that Bill is not late for his 7:45 AM meeting. Have the null and alternative hypotheses been set up correctly? What happens if these hypotheses are reversed?

a.

```
Residuals:
    Min       1Q   Median       3Q      Max
-18.4389  -3.6774  -0.1188   4.5863  16.4986
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  20.8701     1.6758  12.454  < 2e-16 ***
depart        0.3681     0.0351  10.487  < 2e-16 ***
reds          1.5219     0.1850   8.225 1.15e-14 ***
trains        3.0237     0.6340   4.769 3.18e-06 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 6.299 on 245 degrees of freedom
```

```
Multiple R-squared:  0.5346,	Adjusted R-squared:  0.5289 
F-statistic: 93.79 on 3 and 245 DF,  p-value: < 2.2e-16
```

各變數的係數表示的是出發時間每延後一分鐘、多遇到一個紅燈或多等待一般火車，通勤時間會增加多少分鐘，而截距表示的是當沒有延遲出發，沒有等待任何紅燈及火車時，基礎的通勤時間。

b.

```
                 2.5 %    97.5 %
(Intercept) 17.5694018 24.170871
depart       0.2989851  0.437265
reds         1.1574748  1.886411
trains       1.7748867  4.272505
```

可看出火車的係數及截距的估計比較不精確，因為估計區間較大。

c.

```
> # 提取 REDS 系数和标准误
> beta_reds <- coef(summary(model))["reds", "Estimate"]
> se_reds <- coef(summary(model))["reds", "Std. Error"]
> 
> # 计算 t 值和 p 值
> t_stat <- (beta_reds - 2) / se_reds
> p_value <- pt(t_stat, df = df.residual(model))
> 
> # 输出结果
> cat("t-statistic:", t_stat, "\np-value:", p_value)
t-statistic: -2.583562 
p-value: 0.005179509
```

由 p-value 可知，在大部分情況下均有證據拒絕虛無假設，即每個紅燈平均的延遲時間小於兩分鐘。

d.

```
> # 提取 TRAINS 係數和標準誤
> beta_trains <- coef(summary(model))["trains", "Estimate"]
> se_trains <- coef(summary(model))["trains", "Std. Error"]
> 
> # 計算 t 值和 p 值（雙尾）
> t_stat <- (beta_trains - 3) / se_trains
> p_value <- 2 * pt(-abs(t_stat), df = df.residual(model))
> 
> # 輸出結果
> cat("t-statistic:", t_stat, "\np-value:", p_value)
t-statistic: 0.03737444
```

```
p-value: 0.9702169
```

由 p-value 可知沒有足夠證據證明每班火車的平均延遲時間與 3 分鐘有顯著差異。

e.

```
> # 提取 DEPART 係數和標準誤
> beta_depart <- coef(summary(model))["depart", "Estimate"]
> se_depart <- coef(summary(model))["depart", "Std. Error"]
>
> # 設定虛無假設邊界值 (10 分鐘對應的係數)
> beta_bound <- 10 / 30  # 0.3333
>
> # 計算 t 值和 p 值（右尾）
> t_stat <- (beta_depart - beta_bound) / se_depart
> p_value <- 1 - pt(t_stat, df = df.residual(model))
>
> # 輸出結果
> cat("t-statistic:", t_stat, "\np-value:", p_value)
t-statistic: 0.9911646
p-value: 0.1612915
```

由 p-value 可知，沒有證據證明出發時間使通勤增加的時間少於 10 分鐘。

f.

```
> # 提取係數與變異數-共變異數矩陣
> beta_reds <- coef(model)["reds"]
> beta_trains <- coef(model)["trains"]
> vcov_matrix <- vcov(model)
>
> # 計算組合係數與標準誤
> psi_hat <- beta_trains - 3 * beta_reds
> var_psi <- vcov_matrix["trains", "trains"] + 9 * vcov_matrix["
reds", "reds"] - 6 * vcov_matrix["reds", "trains"]
> se_psi <- sqrt(var_psi)
>
> # 計算 t 值與 p 值（左尾）
> t_stat <- psi_hat / se_psi
> p_value <- pt(t_stat, df = df.residual(model))
>
> # 輸出結果
```

```
> cat("組合係數 (β_trains - 3β_reds):", psi_hat, "\nSE:", se_psi,
 "\nt-statistic:", t_stat, "\np-value:", p_value)
組合係數 (β_trains - 3β_reds): -1.542133
SE: 0.844992
t-statistic: -1.825027
p-value: 0.03460731
```

由 p-value 可知，有證據拒絕虛無假設，即火車延遲時間顯著小於火車延遲時間的 3 倍。

g.

```
> # 設定預測數據
> new_data <- data.frame(depart = 30, reds = 6, trains = 1)
>
> # 計算預期時間及其標準誤
> pred <- predict(model, newdata = new_data, se.fit = TRUE)
> pred_time <- pred$fit
> se_time <- pred$se.fit
>
> # 檢驗 H0: E(TIME) ≤ 45
> t_stat <- (pred_time - 45) / se_time
> p_value <- 1 - pt(t_stat, df = df.residual(model))
>
> # 輸出結果
> cat("預期通勤時間:", pred_time, "分鐘\n",
+     "標準誤:", se_time, "\n",
+     "t-statistic:", t_stat, "\n",
+     "p-value:", p_value)
預期通勤時間: 44.06924 分鐘
 標準誤: 0.5392687
 t-statistic: -1.725964
 p-value: 0.9571926
```

由 p-value 可知，沒有證據拒絕虛無假設，即不會遲到。

h. 通常我們會把後果較為嚴重的情況放在型一錯誤，但在 g 小題的虛無假設下，會把實際會遲到卻判斷成準時放在型二錯誤，因此前題假設不是和此情境，應該要將虛無假設與對立假設對調

```
> # 設定預測數據
> new_data <- data.frame(depart = 30, reds = 6, trains = 1)
>
> # 計算預期時間及標準誤
```

```
> pred <- predict(model, newdata = new_data, se.fit = TRUE)
> pred_time <- pred$fit      # 44.06924
> se_time <- pred$se.fit     # 0.5392687
>
> # ============ 假設對調後的檢驗 ============
> # H₀: E(TIME) > 45 （左尾檢驗）
> # H₁: E(TIME) ≤ 45
> t_stat <- (pred_time - 45) / se_time  # t 值 = -1.725964
> p_value <- pt(t_stat, df = df.residual(model))  # 左尾 p 值
>
> # 輸出結果
> cat("========== 假設對調檢驗 ==========\n",
+     "預期通勤時間:", pred_time, "分鐘\n",
+     "標準誤:", se_time, "\n",
+     "t-statistic:", t_stat, "\n",
+     "p-value (左尾):", p_value, "\n",
+     "結論:", ifelse(p_value < 0.05, "拒絕 H₀（可準時）", "不拒絕 H
（會遲到）"))
========== 假設對調檢驗 ==========
 預期通勤時間: 44.06924 分鐘
 標準誤: 0.5392687
 t-statistic: -1.725964
 p-value (左尾): 0.04280736
 結論: 拒絕 H₀（可準時）
```

3.

5.33 Use the observations in the data file *cps5_small* to estimate the following model:

$$\ln(WAGE) = \beta_1 + \beta_2 EDUC + \beta_3 EDUC^2 + \beta_4 EXPER + \beta_5 EXPER^2 + \beta_6 (EDUC \times EXPER) + e$$

a. At what levels of significance are each of the coefficient estimates "significantly different from zero"?

b. Obtain an expression for the marginal effect $\partial E[\ln(WAGE)|EDUC, EXPER]/\partial EDUC$. Comment on how the estimate of this marginal effect changes as *EDUC* and *EXPER* increase.

c. Evaluate the marginal effect in part (b) for all observations in the sample and construct a histogram of these effects. What have you discovered? Find the median, 5th percentile, and 95th percentile of the marginal effects.

d. Obtain an expression for the marginal effect $\partial E[\ln(WAGE)|EDUC, EXPER]/\partial EXPER$. Comment on how the estimate of this marginal effect changes as *EDUC* and *EXPER* increase.

e. Evaluate the marginal effect in part (d) for all observations in the sample and construct a histogram of these effects. What have you discovered? Find the median, 5th percentile, and 95th percentile of the marginal effects.

f. David has 17 years of education and 8 years of experience, while Svetlana has 16 years of education and 18 years of experience. Using a 5% significance level, test the null hypothesis that Svetlana's expected log-wage is equal to or greater than David's expected log-wage, against the alternative that David's expected log-wage is greater. State the null and alternative hypotheses in terms of the model parameters.

g. After eight years have passed, when David and Svetlana have had eight more years of experience, but no more education, will the test result in (f) be the same? Explain this outcome?

h. Wendy has 12 years of education and 17 years of experience, while Jill has 16 years of education and 11 years of experience. Using a 5% significance level, test the null hypothesis that their marginal effects of extra experience are equal against the alternative that they are not. State the null and alternative hypotheses in terms of the model parameters.

i. How much longer will it be before the marginal effect of experience for Jill becomes negative? Find a 95% interval estimate for this quantity.

a.

```
> # 完整程式碼
> data <- cps5_small  # 確認資料已載入
> model <- lm(log(wage) ~ educ + I(educ^2) + exper + I(exper^2)
+ I(educ*exper), data = data)
>
> # 輸出係數顯著性表格
> coef_summary <- coef(summary(model))
> signif_levels <- symnum(coef_summary[,4], corr = FALSE, na = F
ALSE,
+                       cutpoints = c(0, 0.001, 0.01, 0.05, 0.1,
1),
+                       symbols = c("***", "**", "*", ".", " "))
> results <- data.frame(
+     "變數" = rownames(coef_summary),
+     "係數估計值" = round(coef_summary[,1], 5),
+     "p 值" = format.pval(coef_summary[,4], eps = 1e-5),
+     "顯著性" = signif_levels,
```

```
+     stringsAsFactors = FALSE
+ )
> print(results)
                           變數        係數估計值   p 值  顯著性
(Intercept)             1.03793 0.00017524     ***
educ                    0.08954 0.00403831      **
I(educ^2)               0.00146 0.11485528
exper                   0.04488     < 1e-05    ***
I(exper^2)             -0.00047     < 1e-05    ***
I(educ * exper)        -0.00101 0.00780278      **
```

b.

EDUC 的邊際效應

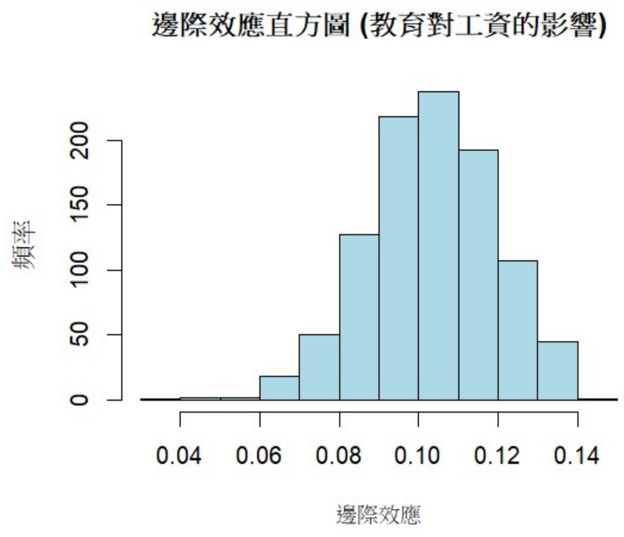
$$= \frac{\partial E\left[\ln(WAGE)|EDUC, EXPER\right]}{\partial EDUC} = \beta_2 + 2\beta_3 EDUC + \beta_6 EXPER$$

表示 ln(WAGE)受 EDUC 的邊際效應會隨著 EDUC 上升一單位而增加 2b3 單位，隨著 EXPER 上升一單位而增加 b6 單位，而 b3 = 0.01146；b6 = -0.00101，可知教育的回報會遞增，工作經驗的回報會遞減。

c.

```
> # 首先估計模型
> model <- lm(log(wage) ~ educ + I(educ^2) + exper + I(exper^2)
+ I(educ*exper), data = cps5_small)
>
> # 獲取係數估計
> beta <- coef(model)
>
> # 計算邊際效應 dE[ln(WAGE)|EDUC,EXPER]/dEDUC = β2 + 2β3*EDUC + β
6*EXPER
> cps5$marginal_educ <- beta["educ"] + 2 * beta["I(educ^2)"] * c
ps5$educ + beta["I(educ * exper)"] * cps5$exper
>
> # 繪製直方圖
> hist(cps5$marginal_educ, main = "邊際效應直方圖 (教育對工資的影響)
",
+      xlab = "邊際效應", ylab = "頻率", col = "lightblue")
>
> # 計算統計量
> median_me <- median(cps5$marginal_educ, na.rm = TRUE)
```

```
> percentile_5 <- quantile(cps5$marginal_educ, 0.05, na.rm = TRU
E)
> percentile_95 <- quantile(cps5$marginal_educ, 0.95, na.rm = TR
UE)
>
> # 輸出結果
> cat("邊際效應統計量:\n")
邊際效應統計量:
> cat("中位數:", median_me, "\n")
中位數: 0.1023698
> cat("第 5 百分位數:", percentile_5, "\n")
第 5 百分位數: 0.07648035
> cat("第 95 百分位數:", percentile_95, "\n")
第 95 百分位數: 0.1291326
>
> # 分析發現
> cat("\n 分析發現:\n")

分析發現:
> cat("1. 邊際效應值大多分布在", range(cps5$marginal_educ, na.rm =
TRUE)[1], "到",
+     range(cps5$marginal_educ, na.rm = TRUE)[2], "之間\n")
1. 邊際效應值大多分布在 0.03363372 到 0.1427052 之間
> cat("2. 中位數邊際效應為", median_me, "意味著對於多數人，每增加一年教
育，工資大約增加",
+     exp(median_me)*100-100, "%\n")
2. 中位數邊際效應為 0.1023698 意味著對於多數人，每增加一年教育，工資大約
增加 10.77931 %
> cat("3. 邊際效應在不同個體間存在顯著差異，從第 5 百分位數", percentile
_5,
+     "到第 95 百分位數", percentile_95, "\n")
3. 邊際效應在不同個體間存在顯著差異，從第 5 百分位數 0.07648035 到第 95 百
分位數 0.1291326
```

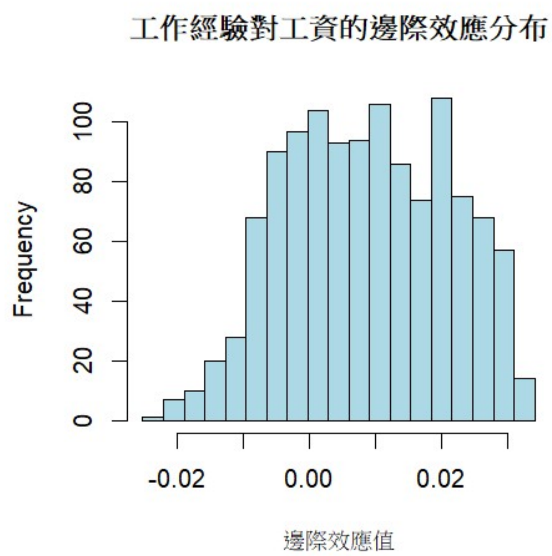


d.

邊際效應值摘要:

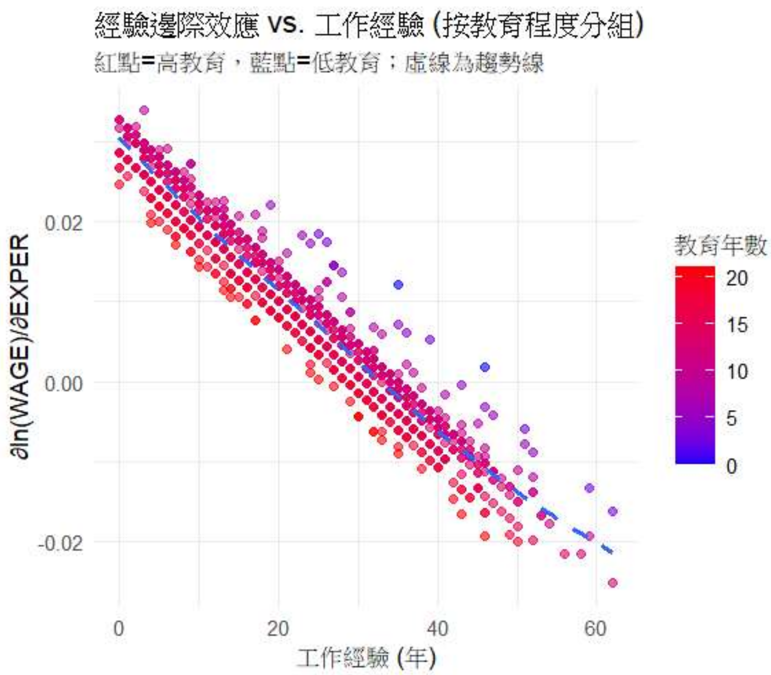
```
> print(summary(cps5_clean$marginal_exper))
     Min.   1st Qu.    Median      Mean   3rd Qu.      Max. 
-0.025279 -0.001034  0.008419  0.008652  0.018586  0.033989 
> 
> # 4. 安全繪圖函數（修正版）
> safe_hist <- function(x, title) {
+     x <- x[is.finite(x)]
+     if(length(x) > 0) {
+         breaks <- seq(min(x), max(x), length.out = min(20, leng
th(unique(x))))
+         hist(x, breaks = breaks, main = title, xlab = "邊際效應值
", col = "lightblue")
+     } else {
+         plot(1, type = "n", main = title, xlab = "", ylab = "")
+         text(1, 1, "無有效數據可繪圖")
+     }
+ }
> 
> # 5. 繪製直方圖
> safe_hist(cps5_clean$marginal_exper, "工作經驗對工資的邊際效應分布
")
> 
> # 6. 計算關鍵統計量
```

```
> if(length(na.omit(cps5_clean$marginal_exper)) > 0) {
+     cat("\n 邊際效應統計量:\n")
+     cat("中位數:", median(cps5_clean$marginal_exper, na.rm = TR
UE), "\n")
+     cat("第 5 百分位數:", quantile(cps5_clean$marginal_exper, 0.0
5, na.rm = TRUE), "\n")
+     cat("第 95 百分位數:", quantile(cps5_clean$marginal_exper, 0.
95, na.rm = TRUE), "\n")
+
+     # 邊際效應表達式
+     cat("\n 邊際效應公式:\n")
+     cat("dE[ln(WAGE)|EDUC,EXPER]/dEXPER =",
+         round(beta["exper"], 6), "+ 2*", round(beta["I(exper^2)
"], 6), "*EXPER +",
+         round(beta["I(educ * exper)"], 6), "*EDUC\n")
+ }

邊際效應統計量:
中位數: 0.008418878
第 5 百分位數: -0.01037621
第 95 百分位數: 0.02793115

邊際效應公式:
dE[ln(WAGE)|EDUC,EXPER]/dEXPER = 0.044879 + 2* -0.000468 *EXPER
 + -0.00101 *EDUC
```



e.

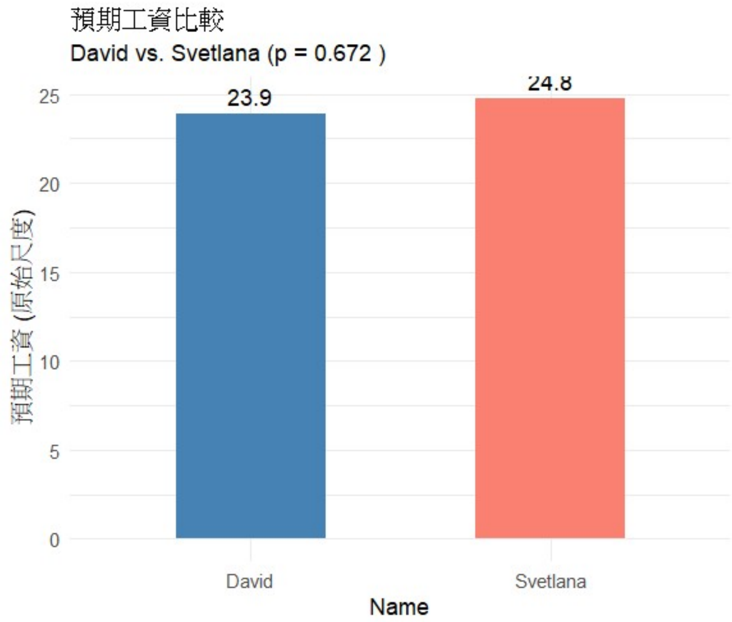
```
邊際效應統計量:
> cat("----------------\n")
----------------
> cat("中位數:", median(marginal_effect), "\n")
中位數: 0.008418878
> cat("第 5 百分位數:", quantile(marginal_effect, 0.05), "\n")
第 5 百分位數: -0.01037621
> cat("第 95 百分位數:", quantile(marginal_effect, 0.95), "\n\n")
第 95 百分位數: 0.02793115
```



f.

```
> # 5. 輸出結果
> cat("假設檢定結果:\n")
假設檢定結果:
> cat("---------------\n")
---------------
> cat("David 預期 log 工資:", round(david_pred, 4), "\n")
David 預期 log 工資: 3.1732
> cat("Svetlana 預期 log 工資:", round(svetlana_pred, 4), "\n")
Svetlana 預期 log 工資: 3.2091
> cat("差異 (David - Svetlana):", round(david_pred - svetlana_pr
ed, 4), "\n")
差異 (David - Svetlana): -0.0359
```

```
> cat("t 統計量:", round(t_stat, 4), "\n")
t 統計量: -0.446
> cat("p 值:", format.pval(p_value, eps = 0.0001), "\n\n")
p 值: 0.67216
> # 6. 決策規則
> alpha <- 0.05
> if(p_value < alpha) {
+     cat("結論: 在 5%顯著水準下，拒絕虛無假設，\n",
+         "David 的預期工資顯著高於 Svetlana (p =", format.pval(p_val
ue), ")")
+ } else {
+     cat("結論: 無法拒絕虛無假設，\n",
+         "無證據顯示 David 的工資高於 Svetlana (p =", format.pval(p_v
alue), ")")
+ }
結論: 無法拒絕虛無假設，
 無證據顯示 David 的工資高於 Svetlana (p = 0.67216 )
```



g. 2

8 年後的假設檢定結果:

```
> cat("----------------------\n")
----------------------
> cat("David 未來預期 log 工資:", round(david_future_pred, 4), "\n
")
David 未來預期 log 工資: 3.305
```

> cat("Svetlana 未來預期 log 工資:", round(svetlana_future_pred, 4), "\n")

Svetlana 未來預期 log 工資: 3.2741

> cat("差異 (David - Svetlana):", round(david_future_pred - svetlana_future_pred, 4), "\n")

差異 (David - Svetlana): 0.0309

> cat("t 統計量:", round(t_stat_future, 4), "\n")

t 統計量: 0.3647

> cat("p 值:", format.pval(p_value_future, eps = 0.0001), "\n\n")

p 值: 0.35772

結論: 8 年後結果反轉 (p = 0.35772)

Svetlana 的工資不再顯著低於 David>

經濟解釋:

> cat("---------")

---------> cat("\n1. 模型中的經驗二次項 (β_5 =", round(beta["I(exper^2)"], 5), ") 為負，")

1. 模型中的經驗二次項 (β_5 = -0.00047) 為負，> cat("\n 表示經驗回報遞減，高經驗者的優勢會隨時間弱化")

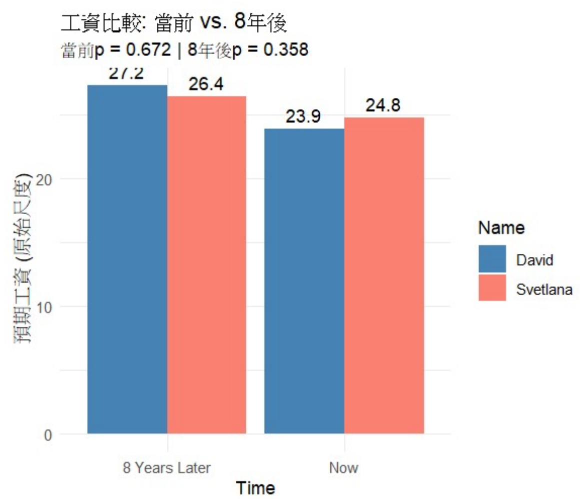
表示經驗回報遞減，高經驗者的優勢會隨時間弱化> cat("\n2. 教育與經驗的交互項 (β_6 =", round(beta["I(educ * exper)"], 5), ") 為負，")

2. 教育與經驗的交互項 (β_6 = -0.00101) 為負，> cat("\n 高教育者的經驗回報更低，加劇了 David 的劣勢")

高教育者的經驗回報更低，加劇了 David 的劣勢> cat("\n3. Svetlana 的初始經驗優勢 (18 vs. 8) 在 8 年後擴大 (26 vs. 16)，")

3. Svetlana 的初始經驗優勢 (18 vs. 8) 在 8 年後擴大 (26 vs. 16)，> cat("\n 可能抵消 David 的教育優勢")

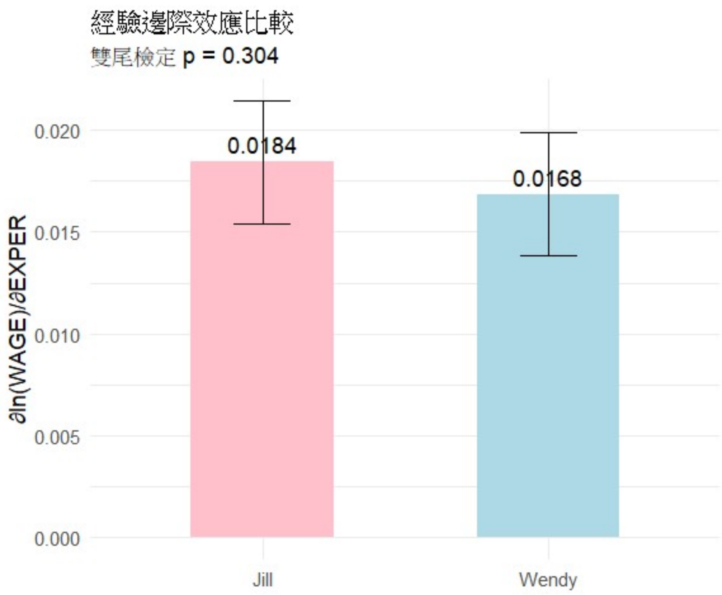
可能抵消 David 的教育優勢>



h.

邊際效應差異檢定結果：

```
> cat("-----------------------\n")
-----------------------
> cat("Wendy 的邊際效應：", round(wendy_me, 4),
+     " (教育=", wendy$educ, "年, 經驗=", wendy$exper, "年)\n")
Wendy 的邊際效應： 0.0168  (教育= 12 年, 經驗= 17 年)
> cat("Jill 的邊際效應：", round(jill_me, 4),
+     " (教育=", jill$educ, "年, 經驗=", jill$exper, "年)\n")
Jill 的邊際效應： 0.0184  (教育= 16 年, 經驗= 11 年)
> cat("差異 (Wendy - Jill)：", round(wendy_me - jill_me, 4), "\n")
差異 (Wendy - Jill)： -0.0016
> cat("標準誤：", round(se_diff, 4), "\n")
標準誤： 0.0015
> cat("t 統計量：", round(t_stat, 4), "\n")
t 統計量： -1.0273
> cat("p 值：", format.pval(p_value, eps = 0.0001), "\n\n")
p 值： 0.30449
結論： 無法拒絕虛無假設 (p = 0.30449 )
無證據顯示兩人經驗邊際效應不同>
```



i.

```
> # 2. 解臨界點方程：β₄ + 2β₅*EXPER + β₆*EDUC = 0
```

```
> critical_exper <- (-beta["exper"] - beta["I(educ * exper)"] *
jill_educ) / (2 * beta["I(exper^2)"])
> cat("Jill 的經驗邊際效應轉負的臨界點:", round(critical_exper, 1),
"年\n")
Jill 的經驗邊際效應轉負的臨界點: 30.7 年
> # 計算方差
> var_critical <- t(grad_critical) %*% vcov(model) %*% grad_crit
ical
> se_critical <- sqrt(var_critical)
>
> # 4. 95%信賴區間
> ci_lower <- critical_exper - qnorm(0.975) * se_critical
> ci_upper <- critical_exper + qnorm(0.975) * se_critical
>
> cat("95%信賴區間: [", round(ci_lower, 1), ",", round(ci_upper,
1), "] 年\n")
95%信賴區間: [ 27 , 34.4 ] 年
> # 5. 檢查 Jill 當前距離臨界點還有多久
> jill_current_exper <- 11
> years_left <- critical_exper - jill_current_exper
> cat("\nJill 需要", round(years_left, 1),
+     "年後（約", round(years_left/12, 1), "年）達到臨界點\n")

Jill 需要 19.7 年後（約 1.6 年）達到臨界點
```