11.16

Consider the following supply and demand model

$$\text{Demand: } Q_i = \alpha_1 + \alpha_2 P_i + e_{di}, \quad \text{Supply: } Q_i = \beta_1 + \beta_2 P_i + \beta_3 W_i + e_{si}$$

where Q is the quantity, P is the price, and W is the wage rate, which is assumed exogenous. Data on these variables are in Table 11.7.

TABLE 11.7 Data for Exercise 11.16

Q	P	W
4	2	2
6	4	3
9	3	1
3	5	1
8	8	3

a. Derive the algebraic form of the reduced-form equations, $Q = \theta_1 + \theta_2 W + v_2$ and $P = \pi_1 + \pi_2 W + v_1$, expressing the reduced-form parameters in terms of the structural parameters.

Ans.

市場在均衡時 供給 = 需求：$\alpha_1 + \alpha_2 P_i + e_{di} = \beta_1 + \beta_2 P_i + \beta_3 W_i + e_{si}$

$$(\alpha_2 - \beta_2) P_i = \beta_1 - \alpha_1 + \beta_3 W_i + e_{si} - e_{di}$$

$$P_i = \frac{\beta_1 - \alpha_1}{\alpha_2 - \beta_2} + \frac{\beta_3}{\alpha_2 - \beta_2} W_i + \frac{e_{si} - e_{di}}{\alpha_2 - \beta_2}$$

表示為 reduced-form：$P_i = \pi_1 + \pi_2 W_i + v_{1i}$，其中：$\pi_1 = \dfrac{\beta_1 - \alpha_1}{\alpha_2 - \beta_2}$，$\pi_2 = \dfrac{\beta_3}{\alpha_2 - \beta_2}$，$v_{1i} = \dfrac{e_{si} - e_{di}}{\alpha_2 - \beta_2}$

將 P_i 帶入需求式得 Q_i 的 reduced-form：

$$\begin{aligned} Q_i &= \alpha_1 + \alpha_2 P_i + e_{di} \\ &= \alpha_1 + \alpha_2(\pi_1 + \pi_2 W_i + v_{1i}) + e_{di} \\ &= (\alpha_1 + \alpha_2 \pi_1) + \alpha_2 \pi_2 W_i + (\alpha_2 v_{1i} + e_{di}) \end{aligned}$$

整理後得：$Q_i = \theta_1 + \theta_2 W_i + v_{2i}$，其中：$\theta_1 = \alpha_1 + \alpha_2 \pi_1$，$\theta_2 = \alpha_2 \pi_2$，$v_{2i} = \alpha_2 v_{1i} + e_{di}$

b. Which structural parameters can you solve for from the results in part (a)? Which equation is "identified"?

Ans.

需求 和 供給 各有一條方程式，共有 M = 2 個方程式，因此每個方程式要被識別，至少需要有 M–1 = 1 個外生變數被排除。

需求方程式中，外生變數 W 被排除。排除的外生變數數量為 1，等於所需的 M–1 = 1，因此需求方程式 "identified"，可以解出 α_1, α_2。

供給方程式中，所有外生變數 W 都被包含，沒有任何外生變數被排除。排除的外生變數數量為 0，小於所需的 M–1 = 1，因此供給方程式 (2) "not identified"，無法解出 $\beta_1, \beta_2, \beta_3$。

c. The estimated reduced-form equations are $\hat{Q} = 5 + 0.5W$ and $\hat{P} = 2.4 + 1W$. Solve for the identified structural parameters. This is the method of indirect least squares.

Ans.

$\hat{Q} = 5 + 0.5W \quad \Rightarrow \quad \theta_1 = 5, \theta_2 = 0.5 \qquad ; \qquad \hat{P} = 2.4 + 1W \quad \Rightarrow \quad \pi_1 = 2.4, \pi_2 = 1$

根據(a)

$$\theta_2 = \alpha_2 \pi_2 \quad \Rightarrow \quad \alpha_2 = \frac{\theta_2}{\pi_2} = \frac{0.5}{1} = 0.5$$

$$\theta_1 = \alpha_1 + \alpha_2 \pi_1 \quad \Rightarrow \quad \alpha_1 = \theta_1 - \alpha_2 \cdot \pi_1 = 5 - 0.5 \cdot 2.4 = 5 - 1.2 = 3.8$$

結論：透過 reduced-form 參數可識別需求方程中的結構參數，可得出 $\alpha_1 = 3.8, \quad \alpha_2 = 0.5$

供給方程未被識別，故無法估計其結構參數。

d. Obtain the fitted values from the reduced-form equation for P, and apply 2SLS to obtain estimates of the demand equation.

Ans.

第一階段：使用 reduced-form 價格方程 $\hat{P}_i = 2.4 + 1 \cdot W_i$ 計算每筆觀察值的擬合價格：

Q_i	P_i	W_i	$\hat{P}_i = 2.4 + W_i$
4	2	2	4.4
6	4	3	5.4
9	3	1	3.4
3	5	1	3.4
8	8	3	5.4

第二階段：以 $\hat{P}_i$ 作為工具變數進行需求方程的估計：$Q_i = \alpha_1 + \alpha_2 \hat{P}_i + u_i$

計算斜率 $\alpha_2 = \dfrac{\sum (Q_i - \bar{Q})(\hat{P}_i - \bar{\hat{P}})}{\sum (\hat{P}_i - \bar{\hat{P}})^2}$

計算平均值：$\bar{Q} = \dfrac{4+6+9+3+8}{5} = \dfrac{30}{5} = 6, \quad \bar{\hat{P}} = \dfrac{4.4+5.4+3.4+3.4+5.4}{5} = \dfrac{22}{5} = 4.4$

計算分子：$\begin{aligned}&(4-6)(4.4-4.4) + (6-6)(5.4-4.4) + (9-6)(3.4-4.4) + (3-6)(3.4-4.4) + (8-6)(5.4-4.4)\\ &= (-2)(0) + (0)(1) + (3)(-1) + (-3)(-1) + (2)(1)\\ &= 0 + 0 - 3 + 3 + 2 = 2\end{aligned}$

計算分母：$(4.4-4.4)^2 + (5.4-4.4)^2 + (3.4-4.4)^2 + (3.4-4.4)^2 + (5.4-4.4)^2 = 0 + 1 + 1 + 1 + 1 = 4$

因此，斜率 $\alpha_2 = \dfrac{2}{4} = 0.5$

計算截距 $\alpha_1 = \bar{Q} - \alpha_2 \cdot \bar{P} = 6 - 0.5 \cdot 4.4 = 6 - 2.2 = 3.8$

因此，使用 2SLS 估計的需求方程為：$Q_i = 3.8 + 0.5 P_i$