**2.1** Consider the following five observations. You are to do all the parts of this exercise using only a calculator.

| $x$ | $y$ | $x - \bar{x}$ | $(x - \bar{x})^2$ | $y - \bar{y}$ | $(x - \bar{x})(y - \bar{y})$ |
|---|---|---|---|---|---|
| 3 | 4 | 2 | 4 | 2 | 4 |
| 2 | 2 | 1 | 1 | 0 | 0 |
| 1 | 3 | 0 | 0 | 1 | 0 |
| −1 | 1 | −2 | 4 | −1 | 2 |
| 0 | 0 | −1 | 1 | −2 | 2 |
| $\sum x_i = 5$ | $\sum y_i = 10$ | $\sum(x_i - \bar{x}) = 0$ | $\sum(x_i - \bar{x})^2 = 10$ | $\sum(y_i - \bar{y}) = 0$ | $\sum(x_i - \bar{x})(y_i - \bar{y}) = 8$ |

a. Complete the entries in the table. Put the sums in the last row. What are the sample means $\bar{x}$ and $\bar{y}$?
b. Calculate $b_1$ and $b_2$ using (2.7) and (2.8) and state their interpretation.
c. Compute $\sum_{i=1}^{5} x_i^2, \sum_{i=1}^{5} x_i y_i$. Using these numerical values, show that $\sum(x_i - \bar{x})^2 = \sum x_i^2 - N\bar{x}^2$ and $\sum(x_i - \bar{x})(y_i - \bar{y}) = \sum x_i y_i - N\bar{x}\bar{y}$.
d. Use the least squares estimates from part (b) to compute the fitted values of $y$, and complete the remainder of the table below. Put the sums in the last row.
   Calculate the sample variance of $y$, $s_y^2 = \sum_{i=1}^{N}(y_i - \bar{y})^2/(N-1)$, the sample variance of $x$, $s_x^2 = \sum_{i=1}^{N}(x_i - \bar{x})^2/(N-1)$, the sample covariance between $x$ and $y$, $s_{xy} = \sum_{i=1}^{N}(y_i - \bar{y})(x_i - \bar{x})/(N-1)$, the sample correlation between $x$ and $y$, $r_{xy} = s_{xy}/(s_x s_y)$ and the coefficient of variation of $x$, $CV_x = 100(s_x/\bar{x})$. What is the median, 50th percentile, of $x$?

$\hat{y} = 1.2 + 0.8x$

| $x_i$ | $y_i$ | $\hat{y}_i$ | $\hat{e}_i$ | $\hat{e}_i^2$ | $x_i \hat{e}_i$ |
|---|---|---|---|---|---|
| 3 | 4 | 3.6 | 0.4 | 0.16 | 1.2 |
| 2 | 2 | 2.8 | −0.8 | 0.64 | −1.6 |
| 1 | 3 | 2 | 1 | 1 | 1 |
| −1 | 1 | 0.4 | 0.6 | 0.36 | −0.6 |
| 0 | 0 | 1.2 | −1.2 | 1.44 | 0 |
| $\sum x_i = 5$ | $\sum y_i = 10$ | $\sum \hat{y}_i = 10$ | $\sum \hat{e}_i = 0$ | $\sum \hat{e}_i^2 = 3.6$ | $\sum x_i \hat{e}_i = 0$ |

e. On graph paper, plot the data points and sketch the fitted regression line $\hat{y}_i = b_1 + b_2 x_i$.
f. On the sketch in part (e), locate the point of the means $(\bar{x}, \bar{y})$. Does your fitted line pass through that point? If not, go back to the drawing board, literally.
g. Show that for these numerical values $\bar{y} = b_1 + b_2\bar{x}$.
h. Show that for these numerical values $\bar{\hat{y}} = \bar{y}$, where $\bar{\hat{y}} = \sum \hat{y}_i/N$.
i. Compute $\hat{\sigma}^2$.
j. Compute $\widehat{\text{var}}(b_2|\mathbf{x})$ and $\text{se}(b_2)$.

---

**(a)** $\bar{x} = 1, \ \bar{y} = 2$

**(b)** $b_2 = \dfrac{\sum(x-\bar{x})(y-\bar{y})}{\sum(x-\bar{x})^2}$

$= \dfrac{8}{10} = 0.8$ ✗

$b_1 = \bar{y} - b_2\bar{x}$

$= 2 - 0.8 \times 1 = 1.2$ ✗

**(c)** $\sum x^2 = 15, \ \sum xy = 18$

$\sum(x-\bar{x})^2 = 10$

$\sum x^2 - N\bar{x}^2 = 15 - 5 \times 1^2 = 10$

$\Rightarrow \sum(x-\bar{x})^2 = \sum x^2 - N\bar{x}^2$

$\sum(x-\bar{x})(y-\bar{y}) = 8$

$\sum xy - N\bar{x}\bar{y} = 18 - 5 \times 1 \times 2$

$= 8$

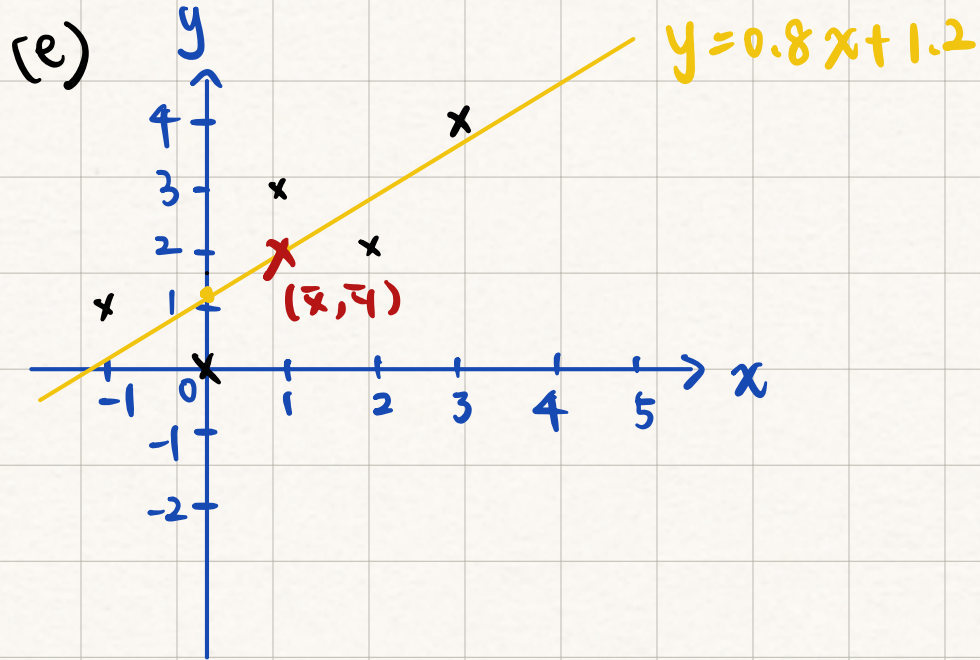$\Rightarrow \sum(x-\bar{x})(y-\bar{y}) = \sum xy - N\bar{x}\bar{y}$

**(D)** $s_x^2 = \dfrac{10}{4} = 2.5, \ s_y^2 = \dfrac{10}{4} = 2.5$

$s_{xy} = \dfrac{8}{4} = 2$

$r_{xy} = \dfrac{2}{2.5} = 0.8$

$CV_x = 100\left(\dfrac{\sqrt{2.5}}{1}\right) = 158.114$

median $= 1$

(e)



$y=0.8x+1.2$

$(\bar{x}, \bar{y})$

(f) Yes, it passes through the point.

(g) $\bar{y} = 0.8 + 1.2 \times 1 = 2$

(h) $\hat{\bar{y}} = \dfrac{3.6 + 2.8 + 2 + 0.4 + 1.2}{5} = \dfrac{10}{5} = 2$

$\hat{\bar{y}} = \bar{y} = 2$

(i) $\hat{\sigma}^2 = \dfrac{\Sigma(y-\hat{y})^2}{n-2} = \dfrac{3.6}{3} = 1.2$

$\rightarrow \hat{se}_i$

(j) $\widehat{Var}(b_2|x) = \dfrac{\hat{\sigma}^2}{S_{xx}} = \dfrac{1.2}{10} = 0.12$

(h) $se(b_2) = \sqrt{\dfrac{\hat{\sigma}^2}{S_{xx}}} = \sqrt{0.12} \approx 0.3464$

**2.14** Consider the regression model $WAGE = \beta_1 + \beta_2 EDUC + e$, where $WAGE$ is hourly wage rate in U.S. 2013 dollars and $EDUC$ is years of education, or schooling. The regression model is estimated twice using the least squares estimator, once using individuals from an urban area, and again for individuals in a rural area.

Urban $\quad \widehat{WAGE} = -10.76 + 2.46\ EDUC, \quad N = 986$
$\qquad\qquad$ (se) $\qquad$ (2.27) $\quad$ (0.16)

Rural $\quad \widehat{WAGE} = -4.88 + 1.80\ EDUC, \quad N = 214$
$\qquad\qquad$ (se) $\qquad$ (3.29) $\quad$ (0.24)

a. Using the estimated rural regression, compute the elasticity of wages with respect to education at the "point of the means." The sample mean of $WAGE$ is \$19.74.

b. The sample mean of $EDUC$ in the urban area is 13.68 years. Using the estimated urban regression, compute the standard error of the elasticity of wages with respect to education at the "point of the means." Assume that the mean values are "givens" and not random.

c. What is the predicted wage for an individual with 12 years of education in each area? With 16 years of education?

(a) $\quad \varepsilon = \dfrac{\Delta E(y)/E(y)}{\Delta x/x} = \dfrac{\Delta E(y)x}{\Delta x E(y)} = \dfrac{\Delta E(y)}{\Delta x} \times \dfrac{x}{y} = \beta_2 \times \dfrac{x}{\beta_1+\beta_2 x}$

$19.74 = -4.88 + 1.8\ \overline{EUDC}$

$\overline{EUDC} = 13.6778$

$\hat{\varepsilon} = 1.8 \times \dfrac{13.6778}{19.74} = 1.2472$ ✳

(b) $\quad \overline{EDUC} = 13.68$

$SE(\hat{\varepsilon}) = 0.16 \times \dfrac{13.68}{22.8928} = 0.0956$ ✳

(c) $EDUC=12$ $\Big\{$ Urban: $-10.76+2.46 \times 12 = 18.76$ ✳
$\qquad\qquad\qquad$ Rural: $-4.88 + 1.8 \times 12 = 16.72$ ✳

$EDUC=16$ $\Big\{$ Urban: $-10.76+2.46\times16 = 28.6$ ✳
$\qquad\qquad\qquad$ Rural: $-4.88+1.8\times16 = 23.92$ ✳

**2.16** The capital asset pricing model (CAPM) is an important model in the field of finance. It explains variations in the rate of return on a security as a function of the rate of return on a portfolio consisting of all publicly traded stocks, which is called the *market* portfolio. Generally, the rate of return on any investment is measured relative to its opportunity cost, which is the return on a risk-free asset. The resulting difference is called the *risk premium*, since it is the reward or punishment for making a risky investment. The CAPM says that the risk premium on security $j$ is *proportional* to the risk premium on the market portfolio. That is,

$$r_j - r_f = \beta_j(r_m - r_f)$$

where $r_j$ and $r_f$ are the returns to security $j$ and the risk-free rate, respectively, $r_m$ is the return on the market portfolio, and $\beta_j$ is the $j$th security's "*beta*" value. A stock's *beta* is important to investors since it reveals the stock's volatility. It measures the sensitivity of security $j$'s return to variation in the whole stock market. As such, values of *beta* less than one indicate that the stock is "defensive" since its variation is less than the market's. A *beta* greater than one indicates an "aggressive stock." Investors usually want an estimate of a stock's *beta* before purchasing it. The CAPM model shown above is the "economic model" in this case. The "econometric model" is obtained by including an intercept in the model (even though theory says it should be zero) and an error term

$$r_j - r_f = \alpha_j + \beta_j(r_m - r_f) + e_j$$

a. Explain why the econometric model above is a simple regression model like those discussed in this chapter.

b. In the data file *capm5* are data on the monthly returns of six firms (GE, IBM, Ford, Microsoft, Disney, and Exxon-Mobil), the rate of return on the market portfolio (*MKT*), and the rate of return on the risk-free asset (*RISKFREE*). The 180 observations cover January 1998 to December 2012. Estimate the CAPM model for each firm, and comment on their estimated *beta* values. Which firm appears most aggressive? Which firm appears most defensive?

c. Finance theory says that the intercept parameter $\alpha_j$ should be zero. Does this seem correct given your estimates? For the Microsoft stock, plot the fitted regression line along with the data scatter.

d. Estimate the model for each firm under the assumption that $\alpha_j = 0$. Do the estimates of the *beta* values change much?

---

### (Handwritten annotations)

(a) Simple Regression Model:

$$Y = \beta_1 + \beta_2 X + \varepsilon$$

$$\underset{\approx}{} Y = r_j - r_f, \quad X = r_m - r_f$$

$$r_j - r_f = \alpha_j + \beta_j(r_m - r_f) + e_j$$

$$\Rightarrow \quad Y = \alpha_j + \beta_j X + e_j$$

---

(b)

```
> summary(ge_capm)

Call:
lm(formula = GE_excess ~ MKT_excess, data = capm_data)

Residuals:
     Min       1Q   Median       3Q      Max
-0.173801 -0.033907 -0.003789 0.038858 0.202748

Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) -0.0009587  0.0044244  -0.217    0.829
MKT_excess   1.1479521  0.0895394  12.821   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.05925 on 178 degrees of freedom
Multiple R-squared:  0.4801,    Adjusted R-squared:  0.4772
F-statistic: 164.4 on 1 and 178 DF,  p-value: < 2.2e-16
```

```
> summary(ibm_capm)

Call:
lm(formula = IBM_excess ~ MKT_excess, data = capm_data)

Residuals:
     Min       1Q   Median       3Q      Max
-0.254880 -0.035266 -0.005322 0.031490 0.274520

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 0.006053   0.004834   1.252    0.212
MKT_excess  0.976890   0.097839   9.985   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.06474 on 178 degrees of freedom
Multiple R-squared:  0.359,     Adjusted R-squared:  0.3554
F-statistic: 99.69 on 1 and 178 DF,  p-value: < 2.2e-16
```

```
> summary(ford_capm)

Call:
lm(formula = Ford_excess ~ MKT_excess, data = capm_data)

Residuals:
     Min       1Q   Median       3Q      Max
-0.27315 -0.07875 -0.01198 0.04720 1.08874

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 0.003779   0.010225   0.370    0.712
MKT_excess  1.662031   0.206937   8.032  1.27e-13 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.1369 on 178 degrees of freedom
Multiple R-squared:  0.266,     Adjusted R-squared:  0.2619
F-statistic: 64.51 on 1 and 178 DF,  p-value: 1.271e-13
```

```
> summary(microsoft_capm)

Call:
lm(formula = Microsoft_excess ~ MKT_excess, data = capm_data)

Residuals:
     Min       1Q   Median       3Q      Max
-0.27424 -0.04744 -0.00820 0.03869 0.35801

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 0.003250   0.006036   0.538    0.591
MKT_excess  1.201840   0.122152   9.839   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.08083 on 178 degrees of freedom
Multiple R-squared:  0.3523,    Adjusted R-squared:  0.3486
F-statistic:  96.8 on 1 and 178 DF,  p-value: < 2.2e-16
```

```
> summary(disney_capm)

Call:
lm(formula = Disney_excess ~ MKT_excess, data = capm_data)

Residuals:
     Min       1Q   Median       3Q      Max
-0.176233 -0.030021 -0.004232 0.029179 0.280528

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 0.001047   0.004676   0.224    0.823
MKT_excess  1.011521   0.094638  10.688   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.06262 on 178 degrees of freedom
Multiple R-squared:  0.3909,    Adjusted R-squared:  0.3875
F-statistic: 114.2 on 1 and 178 DF,  p-value: < 2.2e-16
```

```
> summary(exxon_capm)

Call:
lm(formula = ExxonMobil_excess ~ MKT_excess, data = capm_data)

Residuals:
     Min       1Q   Median       3Q      Max
-0.114474 -0.030596 -0.001884 0.026849 0.215396

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 0.005284   0.003535   1.494    0.137
MKT_excess  0.456521   0.071550   6.380  1.48e-09 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.04734 on 178 degrees of freedom
Multiple R-squared:  0.1861,    Adjusted R-squared:  0.1816
F-statistic: 40.71 on 1 and 178 DF,  p-value: 1.48e-09
```
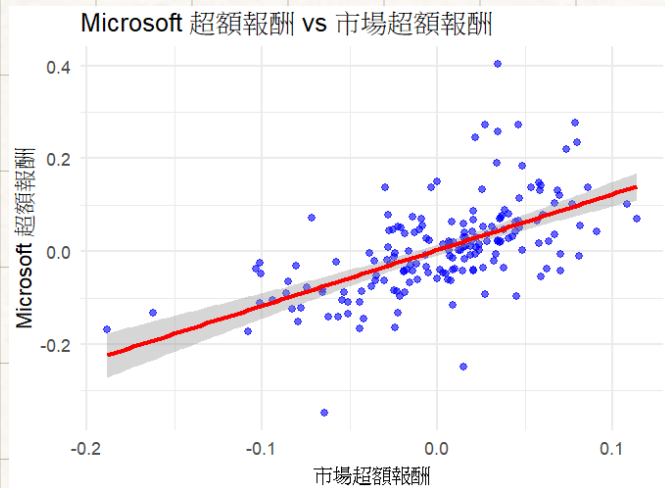
$\beta_{Ford} = 1.662$
$\Rightarrow$ most aggressive

$\beta_{Exxon} = 0.4565$
$\Rightarrow$ most defensive

**(c)**



Microsoft 超額報酬 vs 市場超額報酬

Yes, we don't reject $a_3=0$,
because the p-value of all company
are not significant.

**(b)** 有截距

```
> # 提取 Beta 值
> betas <- c(
+     GE = coef(ge_capm)[2],
+     IBM = coef(ibm_capm)[2],
+     Ford = coef(ford_capm)[2],
+     Microsoft = coef(microsoft_capm)[2],
+     Disney = coef(disney_capm)[2],
+     ExxonMobil = coef(exxon_capm)[2]
+ )
>
> print(betas)
        GE.MKT_excess          IBM.MKT_excess          Ford.MKT_excess    Microsoft.MKT_excess
            1.1479521               0.9768898               1.6620307               1.2018398
    Disney.MKT_excess ExxonMobil.MKT_excess
            1.0115207               0.4565208
```

$\Rightarrow$ beta 差異不大

無截距

```
> # 提取 Beta 值
> betas <- c(
+     GE = coef(ge_capm)[1],
+     IBM = coef(ibm_capm)[1],
+     Ford = coef(ford_capm)[1],
+     Microsoft = coef(microsoft_capm)[1],
+     Disney = coef(disney_capm)[1],
+     ExxonMobil = coef(exxon_capm)[1]
+ )
>
> print(betas)
        GE.MKT_excess          IBM.MKT_excess          Ford.MKT_excess    Microsoft.MKT_excess
            1.1467633               0.9843954               1.6667168               1.2058695
    Disney.MKT_excess ExxonMobil.MKT_excess
            1.0128190               0.4630727
```