10.2 The labor supply of married women has been a subject of a great deal of economic research. Consider the following supply equation specification

$$HOURS = \beta_1 + \beta_2 WAGE + \beta_3 EDUC + \beta_4 AGE + \beta_5 KIDSL6 + \beta_6 NWIFEINC + e$$

where *HOURS* is the supply of labor, *WAGE* is hourly wage, *EDUC* is years of education, *KIDSL6* is the number of children in the household who are less than 6 years old, and *NWIFEINC* is household income from sources other than the wife's employment.

a. Discuss the signs you expect for each of the coefficients.
b. Explain why this supply equation cannot be consistently estimated by OLS regression.
c. Suppose we consider the woman's labor market experience *EXPER* and its square, $EXPER^2$, to be instruments for *WAGE*. Explain how these variables satisfy the logic of instrumental variables.
d. Is the supply equation identified? Explain.
e. Describe the steps [not a computer command] you would take to obtain IV/2SLS estimates.

a. $\beta_2 > 0$ ∵ Wage ↑ 可以增加誘因 ⇒ Supply of Labor ↑

β_3 uncertain ∵ Education ↑ ⇒ productivity ↑ ⇒ supply of Labor ↑ / Education ↑ ⇒ efficient ↑ ⇒ work hours ↓ ⇒ Supply of Labor ↓

β_4 uncertain ∵ Age ↑ ⇒ EXPER ↑ ⇒ Supply of Labor ↑ / Age ↑ ⇒ Spend more time on family ⇒ supply of Labor ↓

$\beta_5 < 0$ ∵ KID ↑ ⇒ Spend more time on taking care of children ⇒ Supply of Labor ↓

$\beta_6 < 0$ ∵ Househol income ↑ ⇒ The need of wowen income ↓ ⇒ Supply of Labor ↓

b. ① Wage and Supply of Labor 皆由供需市場決定 ⇒ Wage 屬於內生變數

② 遺漏變數: ability ⇒ ability 的影響被歸為誤差 ⇒ ability 與 EDUC 及 Wage 相關 ⇒ EDUC 及 wage 與誤差相關 ⇒ 違反假設

c. 工具變數需符合 ① 工具變數需與內生變數高度相關
② 工具變數需與誤差無關

① EXPER 和 $EXPER^2$ 與 wage 呈正相關 ∵ 有越多經驗的勞工可以要求越高的薪資

② EXPER 和 $EXPER^2$ 是勞動需求方的要求，與婦女是否投入職場的個人決定無關.

⇒ EXPER 和 $EXPER^2$ 適合作為 IV.

d. 方程中有1個內生變數(Wage)和2個IV(EXPER, $EXPER^2$)

⇒ # of IV > # of endogenous variable

⇒ Indentified.

e. Step 1: 估計 $\widehat{Wage}$

$$\widehat{Wage} = \gamma_1 + \gamma_2 EDUC + \gamma_3 AGE + \gamma_4 KIDSL6 + \gamma_5 NWIFEINC + \theta_1 EXPER + \theta_2 EXPER^2 + e$$

Step 2: 迴歸原方程，用 $\widehat{Wage}$ 取代 Wage

$$Hours = \beta_1 + \beta_2 \widehat{Wage} + \beta_3 EDUC + \beta_4 AGE + \beta_5 KIDSL6 + \beta_6 NWIFEINC + e$$

10.3 In the regression model $y = \beta_1 + \beta_2 x + e$, assume x is endogenous and that z is a valid instrument. In Section 10.3.5, we saw that $\beta_2 = \text{cov}(z, y)/\text{cov}(z, x)$.

a. Divide the denominator of $\beta_2 = \text{cov}(z, y)/\text{cov}(z, x)$ by $\text{var}(z)$. Show that $\text{cov}(z, x)/\text{var}(z)$ is the coefficient of the simple regression with dependent variable x and explanatory variable z, $x = \gamma_1 + \theta_1 z + v$. [*Hint*: See Section 10.2.1.] Note that this is the first-stage equation in two-stage least squares.

b. Divide the numerator of $\beta_2 = \text{cov}(z, y)/\text{cov}(z, x)$ by $\text{var}(z)$. Show that $\text{cov}(z, y)/\text{var}(z)$ is the coefficient of a simple regression with dependent variable y and explanatory variable z, $y = \pi_0 + \pi_1 z + u$. [*Hint*: See Section 10.2.1.]

c. In the model $y = \beta_1 + \beta_2 x + e$, substitute for x using $x = \gamma_1 + \theta_1 z + v$ and simplify to obtain $y = \pi_0 + \pi_1 z + u$. What are π_0, π_1, and u in terms of the regression model parameters and error and the first-stage parameters and error? The regression you have obtained is a **reduced-form** equation.

d. Show that $\beta_2 = \pi_1/\theta_1$.

e. If $\hat{\pi}_1$ and $\hat{\theta}_1$ are the OLS estimators of π_1 and θ_1, show that $\hat{\beta}_2 = \hat{\pi}_1/\hat{\theta}_1$ is a consistent estimator of $\beta_2 = \pi_1/\theta_1$. The estimator $\hat{\beta}_2 = \hat{\pi}_1/\hat{\theta}_1$ is an **indirect least squares** estimator.

a. $x = \gamma_1 + \theta_1 z + v$ ↓ 同減 $E(x)$

$\Rightarrow x - E(x) = \gamma_1 + \theta_1 z + v - E(\gamma_1 + \theta_1 z + v) = \cancel{\gamma_1} + \theta_1 z + v - \cancel{\gamma_1} - \theta E(z) - 0$

$\Rightarrow x - E(x) = \theta_1 (z - E(z)) + v$ ↓ 同乘 $(z - E(z))$

$\Rightarrow [x - E(x)][z - E(z)] = \theta_1 (z - E(z))^2 + v(z - E(z))$ ↓ 同取期望值

$\Rightarrow E[(x - E(x))(z - E(z))] = \theta_1 E[(z - E(z))^2] + E[(z - E(z)) \cdot v]$ (∵ IV(z)與 v 無關 $\Rightarrow E[(z - E(z)) \cdot v] = 0$)

$\Rightarrow E[(x - E(x))(z - E(z))] = \theta_1 E[(z - E(z))^2]$

$$\Rightarrow \theta = \frac{E[(x - E(x))(z - E(z))]}{E[(z - E(z))^2]} = \frac{\text{Cov}(z, x)}{\text{Var}(z)}$$

b. $y = \pi_0 + \pi_1 z + u$

$\Rightarrow y - E(y) = \pi_1 (z - E(z)) + u$

$\Rightarrow [y - E(y)][z - E(z)] = \pi_1 (z - E(z))^2 + (z - E(z)) u$

$\Rightarrow E[(y - E(y))(z - E(z))] = \pi_1 E[(z - E(z))^2] + E[(z - E(z)) \cdot u]$

$\Rightarrow E[(y - E(y))(z - E(z))] = \pi_1 E[(z - E(z))^2]$

$$\Rightarrow \pi_1 = \frac{E[(y - E(y))(z - E(z))]}{E[(z - E(z))^2]} = \frac{\text{Cov}(y, z)}{\text{Var}(z)}$$

c. $y = \beta_1 + \beta_2 x + e$

$\Rightarrow y = \beta_1 + \beta_2(\gamma_1 + \theta_1 z + v) + e$

$= \beta_1 + \beta_2\gamma_1 + \beta_2\theta_1 z + \beta_2 v + e$

$= (\beta_1 + \beta_2\gamma_1) + (\beta_2\theta_1) z + (\beta_2 v + e)$

$= \pi_0 + \pi_1 z + u$

$\Rightarrow \pi_0 = \beta_1 + \beta_2\gamma_1$

$\pi_1 = \beta_2\theta_1$

$u = \beta_2 v + e$

d. $\pi_1 = \beta_2 \theta$

$\Rightarrow \beta_2 = \dfrac{\pi_1}{\theta}$

e. $\hat{\theta} = \dfrac{cov(z,x)}{var(z)}$

$\hat{\pi}_1 = \dfrac{cov(z,y)}{var(z)}$

$\because \beta_2 = \dfrac{\pi_1}{\theta_1} \Rightarrow \hat{\beta}_2 = \dfrac{\hat{\pi}_1}{\hat{\theta}_1} = \dfrac{\frac{cov(z,y)}{var(z)}}{\frac{cov(z,x)}{var(z)}} = \dfrac{cov(y,z)}{cov(x,z)}$

又 $\widehat{cov}(x,z) \xrightarrow{p} cov(x,z)$

$\widehat{cov}(y,z) \xrightarrow{p} cov(y,z)$

$\therefore \hat{\beta}_1 \xrightarrow{p} \dfrac{cov(y,z)}{cov(x,z)} = \beta_1$