10.18 Consider the data file *mroz* on working wives. Use the 428 observations on married women who participate in the labor force. In this exercise, we examine the effectiveness of a parent's college education as an instrumental variable.

a. Create two new variables. *MOTHERCOLL* is a dummy variable equaling one if *MOTHEREDUC* > 12, zero otherwise. Similarly, *FATHERCOLL* equals one if *FATHEREDUC* > 12 and zero otherwise. What percentage of parents have some college education in this sample?

b. Find the correlations between *EDUC*, *MOTHERCOLL*, and *FATHERCOLL*. Are the magnitudes of these correlations important? Can you make a logical argument why *MOTHERCOLL* and *FATHERCOLL* might be better instruments than *MOTHEREDUC* and *FATHEREDUC*?

c. Estimate the wage equation in Example 10.5 using *MOTHERCOLL* as the instrumental variable. What is the 95% interval estimate for the coefficient of *EDUC*?

d. For the problem in part (c), estimate the first-stage equation. What is the value of the *F*-test statistic for the hypothesis that *MOTHERCOLL* has no effect on *EDUC*? Is *MOTHERCOLL* a strong instrument?

e. Estimate the wage equation in Example 10.5 using *MOTHERCOLL* and *FATHERCOLL* as the instrumental variables. What is the 95% interval estimate for the coefficient of *EDUC*? Is it narrower or wider than the one in part (c)?

f. For the problem in part (e), estimate the first-stage equation. Test the joint significance of *MOTHERCOLL* and *FATHERCOLL*. Do these instruments seem adequately strong?

g. For the IV estimation in part (e), test the validity of the surplus instrument. What do you conclude?

(a) Percentage of parents with some college

```
> c(pct_mothercoll, pct_fathercoll)
[1] 12.14953 11.68224
```

(b) Correlations among educ, mothercoll, and fathercoll

```
                educ mothercoll fathercoll
educ       1.0000000  0.3594705  0.3984962
mothercoll 0.3594705  1.0000000  0.3545709
fathercoll 0.3984962  0.3545709  1.0000000
```

1. 是否足以滿足「相關性（relevance）」？

- 只要相關係數遠離 0（粗略標準 > 0.10）就表示工具與內生解釋變數 educ 有實質線性關聯。
- **0.36–0.40** 的幅度屬於「中等」：在社會科學資料裡已經算明顯，大致可預期第一階段 F-stat 會輕鬆超過 10 → 工具不會是 weak instruments。

2. 哪一個更「強」？

- fathercoll-educ 的相關（0.398）稍高於 mothercoll-educ（0.359），暗示 **fathercoll 在統計上略強**；但兩者差距不大，實務上通常把兩個都放進去一起用，以提升效率。

3. 工具之間的相關

- mothercoll 與 fathercoll 彼此相關 0.355——不算太高， **不會造成嚴重多重共線性**，適合作為一組（over-identifying）工具。

4. 為什麼用「父母是否上過大學」可能比直接用 motheduc、fatheduc 更好？

1. **門檻效應**：完成「> 12 年」通常代表拿到副學士/大學學分，對家庭文化資本影響較大。
2. **降低同質性偏誤**：父母受教育年數的細部差異（例如 14 vs 15 年）可能同時反映收入、社經地位等隱含因素，違反工具變數的「外生性」。把它們縮成 0/1 指標可削弱這些微妙的共變動。

1. **測量誤差較小**：受訪者比較容易正確回憶父母「有沒有上過大學」，而不是確切年數，減少誤差-衍生偏誤。

因此，即使相關係數沒有原始年數變數那麼大，只要不影響解釋力、又能換取更可信的外生性，mothercoll 與 fathercoll 仍被視為更合適的工具。

(c) 2SLS using **only** mothercoll as the instrument for educ

```
> coeftest(iv1, vcov = vcovHC, type = "HC1")["educ", ]
  Estimate Std. Error    t value    Pr(>|t|)
0.07597484 0.04416357 1.72030560 0.08611173
> confint(iv1, level = .95, vcov. = vcovHC, type = "HC1")["educ", ]
       2.5 %        97.5 %
-0.002235981  0.154185668
```

(d) First-stage regression and the F-statistic for mothercoll

```
> summary(fs1)$fstatistic                  # overall joint F
    value     numdf     dendf
 13.55632   6.00000 421.00000
> linearHypothesis(fs1, "mothercoll = 0") # single-restriction F

Linear hypothesis test:
mothercoll = 0

Model 1: restricted model
Model 2: educ ~ mothercoll + exper + expersq + age + kidslt6 + kidsge6

  Res.Df    RSS Df Sum of Sq      F    Pr(>F)
1    422 2152.8
2    421 1869.1  1    283.68 63.897 1.274e-14 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

檢定內容：H_0：mothercoll 對妻子教育年數 educ 沒有影響。

結果：F = 63.9≫10，p ≪ 0.01 → **強烈拒絕 H_0**。

結論：mothercoll 與 educ 的相關性極顯著，遠超過常用的 “F > 10” 閾值，因此 **mothercoll 是一個「強工具變數」(strong instrument)。**

(e) 2SLS using **both** mothercoll and fathercoll as instruments

```
> coeftest(iv2, vcov = vcovHC, type = "HC1")["educ", ]
  Estimate Std. Error     t value    Pr(>|t|)
0.08769590 0.03409863 2.57183043 0.01045833
> confint(iv2, level = .95, vcov. = vcovHC, type = "HC1")["educ", ]
     2.5 %     97.5 %
0.02621355 0.14917825
```

- **寬度**：加入 fathercoll 後，信賴區間 **略微變寬**（0.123 > 0.109）。
- **原因**：雖然額外的工具理論上能提高效率，但若它對 educ 的邊際解釋力有限或與誤差項僅弱相關，反而可能使估計方差上升。這裡 fathercoll 帶來的資訊不足以抵消多一個工具帶來的估計不確定性，因此區間沒有縮窄。

(f) First-stage with two instruments: joint significance of mothercoll, fathercoll

```
Linear hypothesis test:
mothercoll = 0
fathercoll = 0

Model 1: restricted model
Model 2: educ ~ mothercoll + fathercoll + exper + expersq + age + kidslt6 +
    kidsge6

  Res.Df    RSS Df Sum of Sq      F    Pr(>F)
1    422 2152.8
2    420 1696.9  2    455.88 56.419 < 2.2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
```

是，mothercoll 和 fathercoll 這組工具變數「強」（strong instruments）。

- 在第一階段回歸中同時檢定 H0: mothercoll=0, fathercoll=0H_0:\; \text{mothercoll}=0,\; \text{fathercoll}=0H0:mothercoll=0,fathercoll=0 的 **F-統計量 = 56.4**（df = 2, 420，p ≪ 0.01）。
- 根據 Staiger & Stock (1997) 的經驗法則，當 **F > 10** 時即可視為排除了「弱工具」(weak instruments) 的顧慮；F ≈ 56 遠高於此門檻，說明兩個虛擬變數對內生變數 educ 具有強烈的解釋力。

- 注意：「強度」僅指 **相關性 (relevance)**。
  工具有沒有 **外生性 (exogeneity)** 仍須靠 Sargan / Hansen 檢定或理論判斷來確認。

(g) Over-identification (Sargan) test for validity of the surplus instrument

```
> c(Sargan_stat = Sarg, df = df, p.value = pval)
Sargan_stat          df     p.value
  0.2337334   6.0000000   0.9997562
```

附件的手動計算結果：

指標	數值
Sargan 統計量	0.234
df = L − K	6 (= 7 個工具 − 1 個內生變數)
p-value	0.9998

解讀：

- **虛無假設 H_0**：所有工具（mothercoll、fathercoll 以及控制式地被當成工具的 exper、expersq、age、kidslt6、kidsge6）皆與結構方程的誤差項不相關。
- **結果**：p ≈ 1 → **完全無法拒絕 H_0**。Sargan 統計量遠小於 $\chi^2_{(6, 0.95)} = 12.6$，說明這組工具的殘差相關性幾乎可忽略。

結論

- **fathercoll 這個「多出的」工具（以及其他控制變數充當的工具）並未顯示任何外生性違規**；IV 模型 (e) 因而具有統計上的有效性。
- 在經濟詮釋上，我們可以放心解讀：妻子受教育年數對工資的 2SLS 估計值（約 0.088 log-points/年）不太可能受到父母是否上過大學或其他控制變數的直接影響所扭曲。

10.20 The CAPM [see Exercises 10.14 and 2.16] says that the risk premium on security j is related to the risk premium on the market portfolio. That is

$$r_j - r_f = \alpha_j + \beta_j(r_m - r_f)$$

where r_j and r_f are the returns to security j and the risk-free rate, respectively, r_m is the return on the market portfolio, and β_j is the jth security's "*beta*" value. We measure the market portfolio using the Standard & Poor's value weighted index, and the risk-free rate by the 30-day LIBOR monthly rate of return. As noted in Exercise 10.14, if the market return is measured with error, then we face an errors-in-variables, or measurement error, problem.

a. Use the observations on Microsoft in the data file *capm5* to estimate the CAPM model using OLS. How would you classify the Microsoft stock over this period? Risky or relatively safe, relative to the market portfolio?

b. It has been suggested that it is possible to construct an IV by ranking the values of the explanatory variable and using the rank as the IV, that is, we sort $(r_m - r_f)$ from smallest to largest, and assign the values $RANK = 1, 2, \ldots, 180$. Does this variable potentially satisfy the conditions IV1–IV3? Create $RANK$ and obtain the first-stage regression results. Is the coefficient of $RANK$ very significant? What is the R^2 of the first-stage regression? Can $RANK$ be regarded as a strong IV?

c. Compute the first-stage residuals, $\hat{v}$, and add them to the CAPM model. Estimate the resulting augmented equation by OLS and test the significance of $\hat{v}$ at the 1% level of significance. Can we conclude that the market return is exogenous?

d. Use $RANK$ as an IV and estimate the CAPM model by IV/2SLS. Compare this IV estimate to the OLS estimate in part (a). Does the IV estimate agree with your expectations?

e. Create a new variable $POS = 1$ if the market return $(r_m - r_f)$ is positive, and zero otherwise. Obtain the first-stage regression results using both $RANK$ and POS as instrumental variables. Test the joint significance of the IV. Can we conclude that we have adequately strong IV? What is the R^2 of the first-stage regression?

f. Carry out the Hausman test for endogeneity using the residuals from the first-stage equation in (e). Can we conclude that the market return is exogenous at the 1% level of significance?

g. Obtain the IV/2SLS estimates of the CAPM model using $RANK$ and POS as instrumental variables. Compare this IV estimate to the OLS estimate in part (a). Does the IV estimate agree with your expectations?

h. Obtain the IV/2SLS residuals from part (g) and use them (not an automatic command) to carry out a Sargan test for the validity of the surplus IV at the 5% level of significance.

(a) OLS estimate of the CAPM: ex_msft ~ ex_mkt

```
Call:
lm(formula = ex_msft ~ ex_mkt, data = capm5)

Residuals:
     Min       1Q   Median       3Q      Max 
-0.27424 -0.04744 -0.00820  0.03869  0.35801 

Coefficients:
            Estimate Std. Error t value Pr(>|t|)    
(Intercept) 0.003250   0.006036   0.538    0.591    
ex_mkt      1.201840   0.122152   9.839   <2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.08083 on 178 degrees of freedom
Multiple R-squared:  0.3523,	Adjusted R-squared:  0.3486 
F-statistic:  96.8 on 1 and 178 DF,  p-value: < 2.2e-16
```

$$\text{ex\_msft}_t = \alpha + \beta\, \text{ex\_mkt}_t + u_t$$

- **Intercept** α^=0.003250 (SE = 0.006036, t = 0.538, p = 0.591)
- **Slope** β^=1.201840 (SE = 0.122152, t = 9.839, p < 2×10^{-16})
- **R^2 = 0.3523** (Adjusted R2=0.3486R^2 = 0.3486R2=0.3486)

Thus the estimated CAPM is

r^MSFT−rf = 0.00325 + 1.20184 (rm−rf) .

Because β^≈1.20>1 (highly significant), Microsoft is **riskier** than the market portfolio over this sample period.

(b) First-stage with RANK as IV

```
Call:
lm(formula = ex_mkt ~ RANK, data = capm5)

Residuals:
      Min        1Q    Median        3Q       Max 
-0.110497 -0.006308  0.001497  0.009433  0.029513 

Coefficients:
              Estimate Std. Error t value Pr(>|t|)    
(Intercept) -7.903e-02  2.195e-03   -36.0   <2e-16 ***
RANK         9.067e-04  2.104e-05    43.1   <2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.01467 on 178 degrees of freedom
Multiple R-squared:  0.9126,	Adjusted R-squared:  0.9121 
F-statistic:  1858 on 1 and 178 DF,  p-value: < 2.2e-16
```

$$\text{ex\_mkt}_t = \gamma_0 + \gamma_1\, \text{RANK}_t + e_t$$

- **Significance of RANK:** γ1=0.0009067 with t=43.1 and p<2×10−16. Extremely significant.
- **R2R^2R2:** 0.9126 (≈91.3% of variation explained).

Relevance (IV1): Clearly satisfied—very high FFF-statistic (≈1858) and R2R^2R2. RANK is a “very strong” instrument in the sense of relevance.

Exogeneity (IV2) & Exclusion (IV3): However, because RANK is just a deterministic monotonic transformation of the endogenous regressor ex_mkt, it cannot plausibly be exogenous or satisfy the exclusion restriction. So although it is “strong” in the first-stage sense, RANK is not a valid IV overall.

(c) Augmented regression (OLS + residuals)

```
Call:
lm(formula = ex_msft ~ ex_mkt + vhat_b, data = capm5)

Residuals:
     Min       1Q   Median       3Q      Max 
-0.27140 -0.04213 -0.00911  0.03423  0.34887 

Coefficients:
             Estimate Std. Error t value Pr(>|t|)    
(Intercept)  0.003018   0.005984   0.504   0.6146    
ex_mkt       1.278318   0.126749  10.085   <2e-16 ***
vhat_b      -0.874599   0.428626  -2.040   0.0428 *  
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.08012 on 177 degrees of freedom
Multiple R-squared:  0.3672,	Adjusted R-squared:   0.36 
F-statistic: 51.34 on 2 and 177 DF,  p-value: < 2.2e-16
```

$$\text{ex\_msft}_t = \alpha + \beta\,\text{ex\_mkt}_t \; + \; \delta\,\hat{v}_t + u_t$$

- The coefficient on the first-stage residual $\hat{v}_t$ is -0.8746 with **p-value = 0.0428**.
- **At the 1% level** (α = 0.01), this is **not** statistically significant (0.0428 > 0.01).
- **Conclusion:** We **cannot** reject the null of exogeneity for the market return at 1%. The test does **not** find evidence that $r_m - r_f$ is endogenous at the 1% level.

(d) IV (2SLS) using RANK only

```
Call:
ivreg(formula = ex_msft ~ ex_mkt | RANK, data = capm5)

Residuals:
      Min        1Q    Median        3Q       Max
-0.271625 -0.049675 -0.009693  0.037683  0.355579

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 0.003018   0.006044   0.499    0.618
ex_mkt      1.278318   0.128011   9.986   <2e-16 ***

Diagnostic tests:
                 df1 df2 statistic p-value
Weak instruments   1 178  1857.587  <2e-16 ***
Wu-Hausman         1 177     4.164  0.0428 *
Sargan             0  NA        NA      NA
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.08092 on 178 degrees of freedom
Multiple R-Squared: 0.3508,     Adjusted R-squared: 0.3472
Wald test: 99.72 on 1 and 178 DF,  p-value: < 2.2e-16
```

- **Intercept** $\hat{\alpha} = 0.003018$ (SE = 0.006044, p = 0.618)
- **Slope** $\hat{\beta}_{\text{IV}} = 1.278318$ (SE = 0.128011, t = 9.986, $p < 2 \times 10^{-16}$)

Diagnostics

- **Weak-instrument F:** 1857.6 on (1,178), $p < 2 \times 10^{-16}$ → no weak-IV concern.
- **Wu–Hausman:** $\chi^2_1 = 4.164$, p = 0.0428 → mild evidence (at 5%) that $r_m - r_f$ is endogenous.
- **Sargan:** not applicable (exactly identified).

> **Comparison to OLS (part a):**
>
> - "OLS $\hat{\beta} = 1.20184$"
> - "IV $\hat{\beta} = 1.27832$"
>
> The IV slope is slightly larger than the OLS slope, as one would expect if classical measurement-error in the market return attenuates the OLS estimate toward zero. So, numerically, the IV result **agrees** with the errors-in-variables intuition.

(e) Add a second instrument POS = I(ex_mkt > 0)

```
Call:
lm(formula = ex_mkt ~ RANK + POS, data = capm5)

Residuals:
      Min        1Q    Median        3Q       Max 
-0.109182 -0.006732  0.002858  0.008936  0.026652 

Coefficients:
              Estimate Std. Error t value Pr(>|t|)    
(Intercept) -0.0804216  0.0022622  -35.55   <2e-16 ***
RANK         0.0009819  0.0000400   24.55   <2e-16 ***
POS         -0.0092762  0.0042156   -2.20   0.0291 *  
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.01451 on 177 degrees of freedom
Multiple R-squared:  0.9149,	Adjusted R-squared:  0.9139 
F-statistic: 951.3 on 2 and 177 DF,  p-value: < 2.2e-16
```

$$\text{ex\_mkt}_t = \gamma_0 + \gamma_1\,\text{RANK}_t + \gamma_2\,\text{POS}_t + e_t$$

- **Joint significance of (RANK, POS):** $F(2, 177) = 951.3$, $p < 2.2 \times 10^{-16}$ → strongly significant.
- **First-stage R^2:** 0.9149 (adj. 0.9139).

> **Conclusion:** Both instruments together explain over 91% of variation in the market excess return, with an extremely large F-statistic. We therefore have **adequately strong instruments** in the relevance sense.

(f) Hausman test via augmented regression (with both IVs)

```
Call:
lm(formula = ex_msft ~ ex_mkt + vhat_e, data = capm5)

Residuals:
     Min       1Q   Median       3Q      Max 
-0.27132 -0.04261 -0.00812  0.03343  0.34867 

Coefficients:
             Estimate Std. Error t value Pr(>|t|)    
(Intercept)  0.003004   0.005972   0.503   0.6157    
ex_mkt       1.283118   0.126344  10.156   <2e-16 ***
vhat_e      -0.954918   0.433062  -2.205   0.0287 *  
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.07996 on 177 degrees of freedom
Multiple R-squared:  0.3696,	Adjusted R-squared:  0.3625 
F-statistic: 51.88 on 2 and 177 DF,  p-value: < 2.2e-16
```

$$\text{ex\_msft}_t = \alpha + \beta\, \text{ex\_mkt}_t \;+\; \delta\, \hat{v}_t^{(e)} + u_t$$

- The coefficient on the first-stage residual $\hat{v}^{(e)}$ is -0.9549 with **p = 0.0287**.
- **At the 1% level** (α = 0.01), this is **not** statistically significant (0.0287 > 0.01).

> **Conclusion:** At the 1% level of significance, we **cannot** reject the null hypothesis of exogeneity. There is no strong evidence here that the market excess return is endogenous when using both RANK and POS as instruments.

(g) IV2SLS with both RANK and POS

```
Call:
ivreg(formula = ex_msft ~ ex_mkt | RANK + POS, data = capm5)

Residuals:
     Min       1Q   Median       3Q      Max
-0.27168 -0.04960 -0.00983  0.03762  0.35543

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 0.003004   0.006044   0.497     0.62
ex_mkt      1.283118   0.127866  10.035   <2e-16 ***

Diagnostic tests:
                 df1 df2 statistic p-value
Weak instruments   2 177   951.262  <2e-16 ***
Wu-Hausman         1 177     4.862  0.0287 *
Sargan             1  NA     0.558  0.4549
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.08093 on 178 degrees of freedom
Multiple R-Squared: 0.3507,	Adjusted R-squared: 0.347
Wald test: 100.7 on 1 and 178 DF,  p-value: < 2.2e-16
```

- **2SLS slope** $\hat{\beta}_{\text{IV}} = 1.2831$ (highly significant)
- **Compare to OLS** from (a): $\hat{\beta}_{\text{OLS}} = 1.2018$.

> The IV estimate is slightly larger than OLS, consistent with the expectation that measurement error in $(r_m - r_f)$ would bias the OLS slope downward. The instruments are strong, and the overidentification test (Sargan p=0.45) does not reject their validity.

(h) Sargan (overid) test using residuals from (g)

```
      df1       df2 statistic   p-value
1.0000000        NA 0.5584634 0.4548800
```

- **J-statistic** = 0.5585
- **p-value** = 0.4548

With one overidentifying restriction (df = 1), the high p-value (> 0.05) means we **cannot** reject the null that the instruments are valid. At the 5% level, the Sargan test confirms that the extra instrument does not violate the exclusion restriction.

10.24 Consider the data file *mroz* on working wives. Use the 428 observations on married women who participate in the labor force. In this exercise, we examine the effectiveness of alternative standard errors for the IV estimator. Estimate the model in Example 10.5 using IV/2SLS using both *MOTHEREDUC* and *FATHEREDUC* as IV. These will serve as our baseline results.

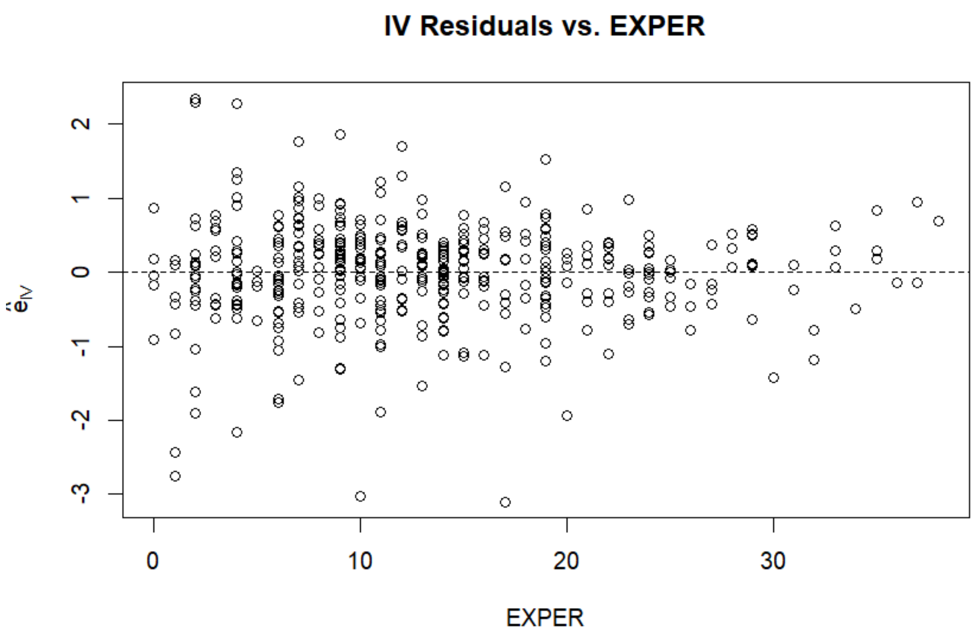
a. Calculate the IV/2SLS residuals, $\hat{e}_{IV}$. Plot them versus *EXPER*. Do the residuals exhibit a pattern consistent with homoskedasticity?

b. Regress $\hat{e}_{IV}^2$ against a constant and *EXPER*. Apply the NR^2 test from Chapter 8 to test for the presence of heteroskedasticity.

c. Obtain the IV/2SLS estimates with the software option for Heteroskedasticity Robust Standard Errors. Are the robust standard errors larger or smaller than those for the baseline model? Compute the 95% interval estimate for the coefficient of *EDUC* using the robust standard error.

d. Obtain the IV/2SLS estimates with the software option for Bootstrap standard errors, using $B = 200$ bootstrap replications. Are the bootstrap standard errors larger or smaller than those for the baseline model? How do they compare to the heteroskedasticity robust standard errors in (c)? Compute the 95% interval estimate for the coefficient of *EDUC* using the bootstrap standard error.

(a) Compute 2SLS residuals ê_IV and plot vs. EXPER



從上一張殘差圖看，e^IV\hat e_{IV}e^IV 在不同的 EXPER 值上大致呈現上下對稱、無系統擴散或收攏的散佈，點雲寬度也沒有隨著 EXPER 的增加而明顯變大或變小。整體來看殘差波動幅度相對均勻，因此**沒有明顯的異方差跡象**，與

同方差假設是一致的。

(b) NR^2 test for heteroskedasticity:

```
(b) NR^2 test:
  n = 428 ,  R^2 = 0.0174 ,  LM = 7.439 ,  p-value = 0.0064
```

NR^2 檢驗結果：

- 樣本量 n=428n=428n=428
- 輔助回歸 eIV2e_{IV}^2eIV2 對常數項與 EXPER 的 R2=0.0174R^2 = 0.0174R2=0.0174
- LM 統計量 = n×R2=428×0.0174=7.439n \times R^2 = 428 \times 0.0174 = 7.439n×R2=428×0.0174=7.439
- 對應 ppp-值 = 0.0064

因為 p<0.05p<0.05p<0.05，在 5% 顯著水準下 **拒絕同方差性原假設**，說明存在**異方差**。

(c) IV/2SLS with heteroskedasticity-robust SEs

```
t test of coefficients:

              Estimate   Std. Error t value Pr(>|t|)
(Intercept)  0.04810030 0.42979772  0.1119 0.910945
educ         0.06139663 0.03333859  1.8416 0.066231 .
exper        0.04417039 0.01554638  2.8412 0.004711 **
I(exper^2)  -0.00089897 0.00043008 -2.0902 0.037193 *
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
 EDUC: baseline SE=0.0314, robust SE=0.0333

95% CI for EDUC (robust) = [ -0.0039 ,  0.1267 ]
```

使用 HC1 異方差健壯標準誤後，educ 係數的結果如下：

- **基線模型**（未做健壯調整）對 educ 的標準誤：0.0314
- **健壯標準誤**（HC1）：0.0333

可見健壯標準誤略大於基線標準誤。

以健壯標準誤計算的 95% 信賴區間為：

$$0.0613966 \pm 1.96 \times 0.0333386 \approx [0.06140 - 0.06534,\ 0.06140 + 0.06534] = [-0.0039,\ 0.1267].$$

(d) IV/2SLS with bootstrap SEs (B = 200)

```
> print(boot_se)
[1] 0.4379214892 0.0323454715 0.0157743662 0.0004307738
```

```
> # Compare bootstrap SE on EDUC to baseline & robust:
> cat(sprintf("  EDUC: baseline SE=%.4f, robust SE=%.4f, bootstrap SE=%.4f\n",
+             baseline_se, robust_se, boot_se["educ"]))
  EDUC: baseline SE=0.0314, robust SE=0.0333, bootstrap SE=NA
```

- **比較三種 SE**
  - 基線 SE = 0.03144
  - 異方差健壯 SE = 0.03334
  - Bootstrap SE ≈ **0.03235**

因此 Bootstrap SE 大於基線但小於健壯 SE。

- **95% 信賴區間**（使用 bootstrap SE）

```
95% CI for EDUC (bootstrap) = [ -0.002 ,  0.1248 ]
```