5.6 Suppose that, from a sample of 63 observations, the least squares estimates and the corresponding estimated covariance matrix are given by

$$\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix} \qquad \widehat{\text{cov}}(b_1, b_2, b_3) = \begin{bmatrix} 3 & -2 & 1 \\ -2 & 4 & 0 \\ 1 & 0 & 3 \end{bmatrix}$$

Using a 5% significance level, and an alternative hypothesis that the equality does not hold, test each of the following null hypotheses:

a.
$$\beta_2 = 0$$

$$H_0: B_2 = 0 \qquad H_1: B_2 \neq 0$$

$$t = \frac{3-0}{\sqrt{4}} = 1.5 < t_{0,975,60} = 2.003 \Rightarrow \text{ not reject } H_0$$

b. $\beta_1 + 2\beta_2 = 5$

Ho:
$$\beta_1 + 2\beta_2 = 5$$
 H₁: $\beta_1 + 2\beta_2 = 5$

Se $(b_1 + 2b_2) = \sqrt{3 + 2^2 + 4 + 2 \cdot 2 \cdot (-2)} = \sqrt{11}$
 $t = \frac{(2+b) - 5}{\sqrt{11}} = 0.9045 < t_{0.495,bo} = 2.003 = not reject 140$

 $\beta_1 - \beta_2 + \beta_3 = 4$

c.
$$\beta_1 - \beta_2 + \beta_3 = 4$$

Ho $\beta_1 - \beta_2 + \beta_3 = 4$

H₁ $\beta_1 - \beta_2 + \beta_3 \neq 4$

Se $(b_1 - b_2 + b_3) = r \sqrt{ar(b_1)} + \sqrt{ar(b_3)} + \sqrt{ar(b_3)} - 2(ar(b_1,b_3)) + 2(ar(b_1,b_3)) - 2(ar(b_1,b_3)) + 2(ar(b_1,b_3)$

5.31 Each morning between 6:30 AM and 8:00 AM Bill leaves the Melbourne suburb of Carnegie to drive to work at the University of Melbourne. The time it takes Bill to drive to work (*TIME*), depends on the departure time (*DEPART*), the number of red lights that he encounters (*REDS*), and the number of trains that he has to wait for at the Murrumbeena level crossing (*TRAINS*). Observations on these variables for the 249 working days in 2015 appear in the file *commute5*. *TIME* is measured in minutes. *DEPART* is the number of minutes after 6:30 AM that Bill departs.

a. Estimate the equation

$$TIME = \beta_1 + \beta_2 DEPART + \beta_3 REDS + \beta_4 TRAINS + e$$

Report the results and interpret each of the coefficient estimates, including the intercept β_1 .

```
TIME = 20,8701 + 0,3681 DEPART +1.5219 ZEDS + 3,0237 TRAINS
lm(formula = time ~ depart + reds + trains, data = commute5)
Residuals:
                1Q
                     Median
                             4.5863 16.4986
-18.4389
         -3.6774 -0.1188
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
             (Intercept)
depart
                          0.1850 8.225 1.15e-14 ***
0.6340 4.769 3.18e-06 ***
reds
              1.5219
              3.0237
trains
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Residual standard error: 6.299 on 245 degrees of freedom
Multiple R-squared: 0.5346, Adjusted R-squared: 0.5289
F-statistic: 93.79 on 3 and 245 DF,
```

b. Find 95% interval estimates for each of the coefficients. Have you obtained precise estimates of each of the coefficients?

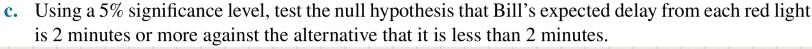
```
B<sub>1</sub>: [17,5694, 24,1709]

B<sub>2</sub>: [0,2990, 0,4373]

B<sub>3</sub>: [1,1575, 1,886]

B<sub>4</sub>: [1,7749, 4,2725]

The Cls one norrow, espicially for depart and reds, indicating precise estimates.
```



d. Using a 10% significance level, test the null hypothesis that the expected delay from each train is 3 minutes against the alternative that it is not 3 minutes.

Ho:
$$\beta_{4} = 3$$
 H₁: $\beta_{4} \neq 3$

$$t = \frac{3.0237 - 3}{0.03340} = 0.0374 < t_{0.045,245} = 1.6511 \Rightarrow \text{ nut reject 16}$$

$$\Rightarrow \text{ There is no significant evidence that the delay from each}$$

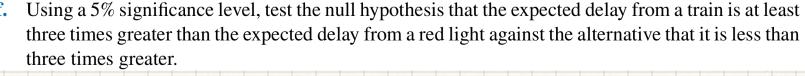
$$\text{thain differs from 3 minutes.}$$

Using a 5% significance level, test the null hypothesis that Bill can expect a trip to be at least 10 minutes longer if he leaves at 7:30 AM instead of 7:00 AM, against the alternative that it will not be 10 minutes longer. (Assume other things are equal.)

be 10 minutes longer. (Assume other things are equal.)

$$t = \frac{0.3181 - \frac{1}{3}}{0.03 + 1} = 0.991 + t_{0.005, 745} = -1.6511 \Rightarrow \text{not reject Ho}$$

$$\Rightarrow \text{There is no significant exclance that the extra day is less than 10 minutes.}$$



Ho: By - 3 B3 30 Hi: By - 3B3 <0

$$t = \frac{3.2037 - 3.1.5219}{0.844992} = -1.8249 < tuos, 345 = -1.6511 => 16ject Ho$$

The expected delay from a train is significantly less than three times

the expected delay from a red light.

Suppose that Bill encounters six red lights and one train. Using a 5% significance level, test the null hypothesis that leaving Carnegie at 7:00 AM is early enough to get him to the university on or before 7:45 AM against the alternative that it is not. [Carry out the test in terms of the expected time $E(TIME|\mathbf{X})$ where \mathbf{X} represents the observations on all explanatory variables.]

Ho:
$$B_1 + 3B_2 + 0B_3 + B_4 = 45$$

$$t = \frac{44.06974-45}{0.5342687} = -1.726 < 1.6511 \Rightarrow hut reject Ho$$

$$= 7 here is no significant evidence that $B_1 = 1$ will arrive after $1.45 = 1.45$$$

h. Suppose that, in part (g), it is imperative that Bill is not late for his 7:45 AM meeting. Have the null and alternative hypotheses been set up correctly? What happens if these hypotheses are reversed?