HW6

C05Q06

(a)

```
t-statistic: 1.5
critical value: ± 2.000298
```

=> Fail to Reject H0, there is insufficient evidence to conclude that β_2 is not 0.

(b)

```
t-statistic: 0.904534
critical value: ± 2.000298
```

=> Fail to Reject H0, there is insufficient evidence to conclude that $\beta_1+2\beta_2$ is not 5 .

(c)

```
t-statistic: -1.5
critical value: ± 2.000298
```

=> Fail to Reject H0, there is insufficient evidence to conclude that $\beta_1-\beta_2+\beta_3$ is not 4 .

C05Q31

(a)

```
Call:
lm(formula = time ~ depart + reds + trains, data = commute5)

Coefficients:
(Intercept)       depart         reds       trains
    20.8701       0.3681       1.5219       3.0237
```

(b)

```
b 1 :[ 17.5694 , 24.17087 ]
b 2 :[ 0.2989851 , 0.437265 ]
b 3 :[ 1.157475 , 1.886411 ]
b 4 :[ 1.774887 , 4.272505 ]
```

The estimates of parameters appear to be precise except for b_4.

C05Q31

(c)

H0: $\beta_3 \geq 2$ H1: $\beta_3 < 2$

```
t-statistic: -2.583562
critical value: -1.651097
```

=> Reject H0, it suggests that the expected delay from each red light is less than 2 minutes.

(d)

H0: $\beta_4 = 3$ H1: $\beta_4 \neq 3$

```
t-statistic: 1.614632
critical value: ± 1.651097
```

=> Fail to Reject H0, it suggests that the expected delay from a train is 3 minutes.

C05Q31

(e)

$$TIME_{7:30} = \beta_1 + 60\beta_2 + \beta_3 REDS + \beta_4 TRAINS$$

$$TIME_{7:00} = \beta_1 + 30\beta_2 + \beta_3 REDS + \beta_4 TRAINS$$

$$TIME_{7:30} - TIME_{7:00} = 30\beta_2$$

H0: $30\beta_2 \geq 10$ H1: $\beta_2 < {}^{1}/_{3}$

```
t-statistic: 0.9911646
critical value: -1.651097
```

=> Fail to Reject H0, it suggests that a trip to be at least 10 minutes longer if Bill leaves at 7:30 AM instead of 7:00 AM

C05Q31

(f)

$$TIME_{trains+1} = \beta_1 + 60\beta_2 + \beta_3 REDS + \beta_4(TRAINS + 1)$$

$$TIME_{reds+3} \quad = \beta_1 + 30\beta_2 + \beta_3(REDS + 3) + \beta_4 TRAINS$$

$$TIME_{trains+1} - TIME_{reds+3} = -3\beta_3 + \beta_4$$

H0: $-3\beta_3 + \beta_4 \geq 0$ H1: $-3\beta_3 + \beta_4 < 0$

```
t-statistic: -1.825027
critical value:  -1.651097
```

=> Reject H0, it suggests that the expected delay from a train is less than three times which from a red light.

C05Q31

(g)

$$TIME = \beta_1 + 30\beta_2 + 6\beta_3 + \beta_4$$

H0: $\beta_1 + 30\beta_2 + 6\beta_3 + \beta_4 \leq 45$ H1: $\beta_1 + 30\beta_2 + 6\beta_3 + \beta_4 > 45$

```
t-statistic: -1.725964
critical value: 1.651097
```

=> Fail to Reject H0, it suggests that 7:00 is early enough to get him to the university on 7:45.

C05Q31

(f)

No, they haven't.

The setting of hypotheses should be reversed so that when rejecting H0, the probability of Bill being late is sufficiently small (< Type I error α = 0.05).

$$TIME = \beta_1 + 30\beta_2 + 6\beta_3 + \beta_4$$

H0: $\beta_1 + 30\beta_2 + 6\beta_3 + \beta_4 \geq 45$ H1: $\beta_1 + 30\beta_2 + 6\beta_3 + \beta_4 < 45$

```
t-statistic: -1.725964
critical value: -1.651097
```

=> Reject H0, it suggests that Bill will not be late for his 7:45 meeting if he leaves Carnegie at 7:00.

C05Q33

(a)

```
Coefficients:
               Estimate Std. Error t value Pr(>|t|)    
(Intercept)  1.038e+00  2.757e-01   3.764 0.000175 ***
educ         8.954e-02  3.108e-02   2.881 0.004038 ** 
I(educ^2)    1.458e-03  9.242e-04   1.578 0.114855    
exper        4.488e-02  7.297e-03   6.150 1.06e-09 ***
I(exper^2)  -4.680e-04  7.601e-05  -6.157 1.01e-09 ***
educ:exper  -1.010e-03  3.791e-04  -2.665 0.007803 ** 
```

b_1 , b_2 , b_4 , b_5 and b_6 are significantly different from zero

(b)

b_2+2*b_3*EDUC+b_6*EXPER = $0.0895+2\times0.0015\times$EDUC$-0.001\times$EXPER

marginal effect increase as EDUC increase

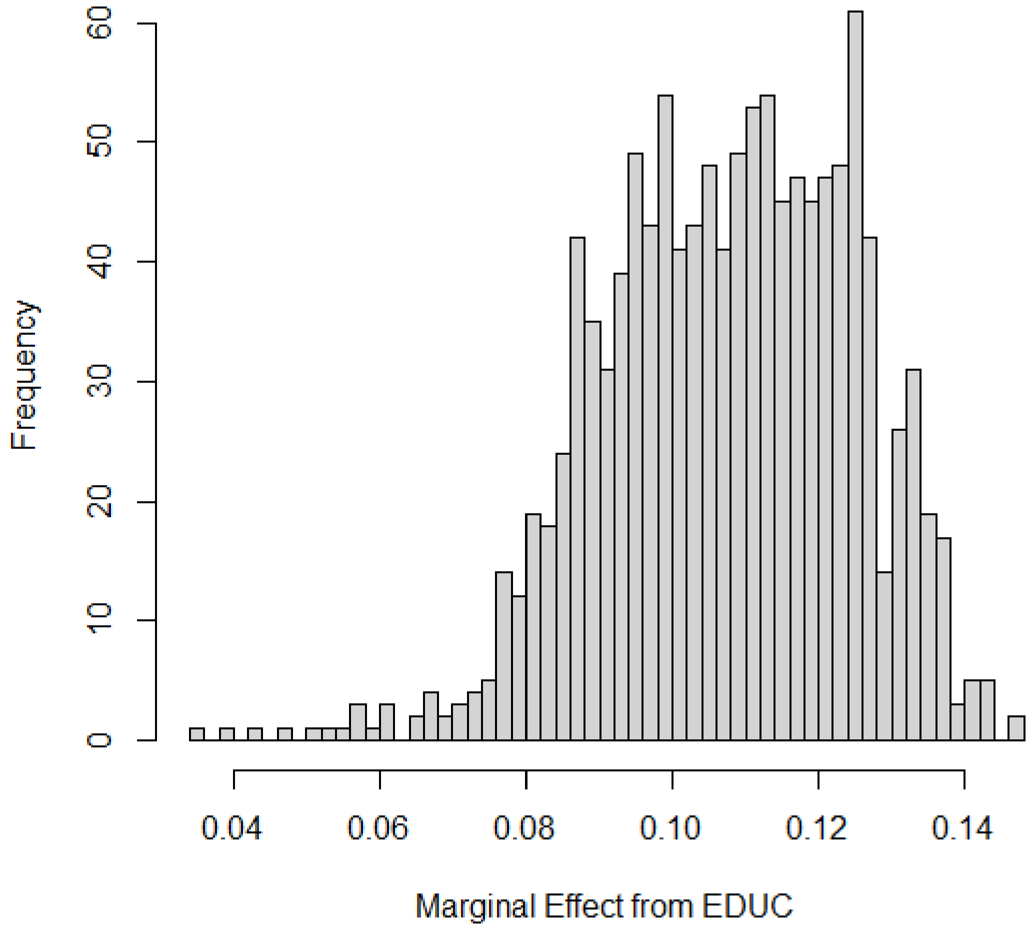
marginal effect decrease as EXPER increase

C05Q33

(c)

the effect is left-skewed

and positive



5th: 0.08008187 median: 0.1084313 95th: 0.1336188

C05Q33

(d)

$b_4 + 2 * b_5 * EXPER + b_6 * EDUC$

$= 0.0449 - 2 \times 0.0004 \times EXPER - 0.001 \times EDUC$

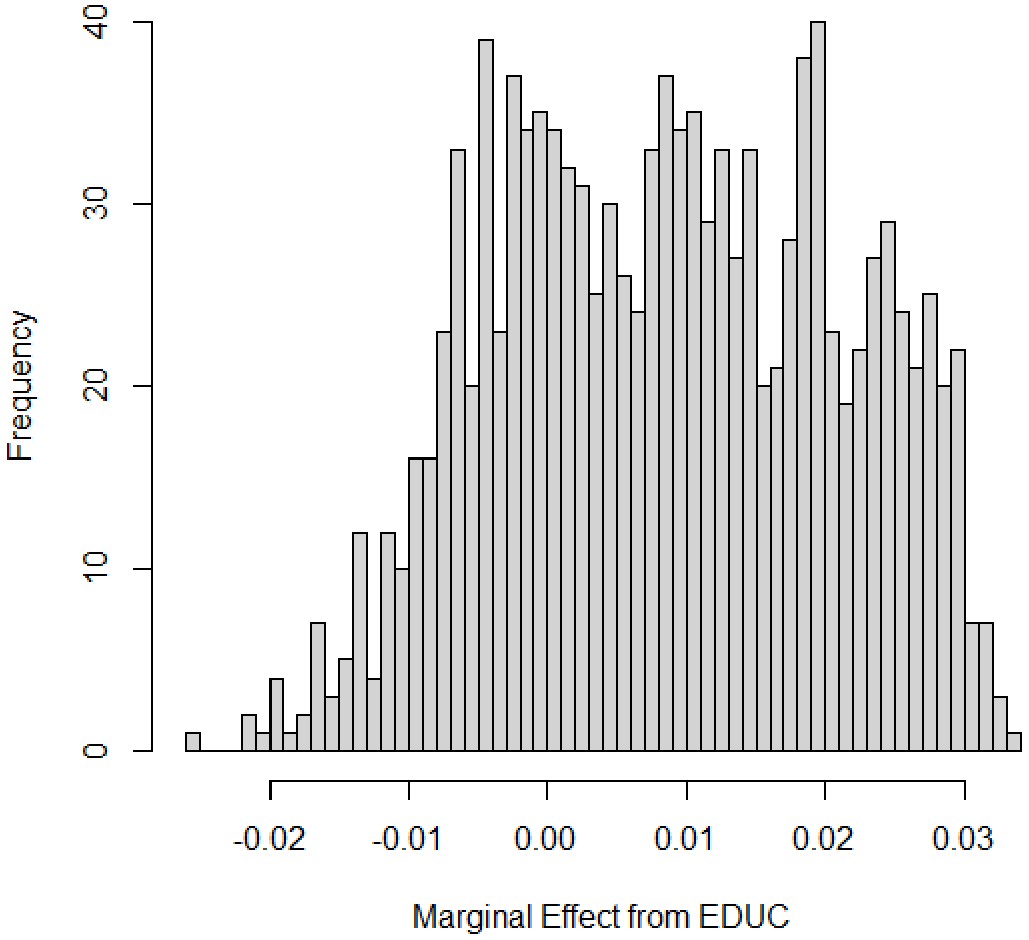
marginal effect decrease as EDUC increase

marginal effect decrease as EXPER increase

(e)

the effect is nearly symmetrical

and about 1/3 of data are negative



5th: -0.01037621 median: 0.008418878 95th: 0.02793115

C05Q33

(f)

$$y_{David} = \beta_1 + 17\beta_2 + 17^2\beta_3 + 8\beta_4 + 8^2\beta_5 + 17 \times 8\beta_6$$

$$y_{Svetlana} = \beta_1 + 16\beta_2 + 16^2\beta_3 + 18\beta_4 + 18^2\beta_5 + 16 \times 18\beta_6$$

$$y_{S-D} = -\beta_2 - 33\beta_3 + 10\beta_4 + 260\beta_5 + 152\beta_6$$

H0: $-\beta_2 - 33\beta_3 + 10\beta_4 + 260\beta_5 + 152\beta_6 \geq 0$

H1: $-\beta_2 - 33\beta_3 + 10\beta_4 + 260\beta_5 + 152\beta_6 < 0$

```
t-statistic: 1.669902
critical value:  -1.646131
```

=> Fail to Reject H0, it suggests that the Svetlana's expected log-wage is equal to or greater than David's.

C05Q33

(g)

$$y_{David} = \beta_1 + 17\beta_2 + 17^2\beta_3 + 16\beta_4 + 16^2\beta_5 + 17 \times 16\beta_6$$

$$y_{Svetlana} = \beta_1 + 16\beta_2 + 16^2\beta_3 + 26\beta_4 + 26^2\beta_5 + 16 \times 26\beta_6$$

$$y_{S-D} = -\beta_2 - 33\beta_3 + 10\beta_4 + 420\beta_5 + 144\beta_6$$

H0: $y_{S-D} \geq 0$ H1: $y_{S-D} < 0$

```
t-statistic: -2.062365
critical value:  -1.646131
```

=> Reject H0, it suggests that the David's expected log-wage is greater after 8 years. The result is different from (f), since the marginal effect of EXPER diminishes to WAGE as EXPER increase. Besides, both coefficients of EDUC are positive, so we can expect that David, who has a larger EDUC, will earn more wages than Svetlana someday.

C05Q33

(h)

$$y_{Wendy} = \beta_4 + 2 \times 17 \ \ \beta_5 + 12\beta_6$$

$$y_{Jill} \quad = \beta_4 + 2 \times 11 \ \ \beta_5 + 16\beta_6$$

$$y_{W-J} \quad = 12\beta_5 \ - \ 4\beta_6$$

H0: $12\beta_5 \ - \ 4\beta_6 = 0$

H1: $12\beta_5 \ - \ 4\beta_6 \neq 0$

```
t-statistic: -1.027304
critical value: ± 1.961953
```

=> Fail to Reject H0, it suggests that their marginal effects of extra experience are equal.

C05Q33

(i)

point estimate:

$$b_4 + b_5 \times 2 \times EXPER + b_6 \times EDUC \ = \ 0$$

$$EXPER \ = \ (-b_4 - b_6 \times 16)/(2b_5) \ =$$

```
   exper
30.67706
```

interval estimate:

$$EXPER \ = \frac{-b_4 - b_6 \times 16}{2b_5} \sim ?$$

The distribution of EXPER is not a linear combination of the estimators, which causes its distribution to be unidentified.

We use point estimate(pe) to calculate the interval of marginal effect.

C05Q33

(i)

$$y = b4 + b5 \times 2 \times EXPER + b6 \times EDUC\ , \ (y-0)/se(y) \sim t$$

interval of $y/se(y)$: $\pm t \times se(y)$

interval of $EXPER$:

$$(-b_4 - b_6 EDUC \pm t \times se(y))/(2b_5) = pe \pm (t \times se(y))/(2b_5)$$

If we consider that Jill has had experience for 11 years already,

```
EXPER interval: [ 26.95776 , 34.39636 ]
Jill remains: [ 15.95776 , 23.39636 ]
```

We can also use the delta method and get the same results:

```
by delta method:
EXPER interval: [ 26.95776 , 34.39636 ]
Jill remains: [ 15.95776 , 23.39636 ]
```