11.28 Supply and demand curves as traditionally drawn in economics principles classes have price (P) on the vertical axis and quantity (Q) on the horizontal axis.

a. Rewrite the truffle demand and supply equations in (11.11) and (11.12) with price P on the left-hand side. What are the anticipated signs of the parameters in this rewritten system of equations?

$$\text{Demand: } Q_i = \alpha_1 + \alpha_2 P_i + \alpha_3 PS_i + \alpha_4 DI_i + e_{di} \quad (11.11)$$

$$\text{Supply: } Q_i = \beta_1 + \beta_2 P_i + \beta_3 PF_i + e_{si} \quad (11.12)$$

Demand : $P_i = \frac{1}{\alpha_2}(Q_i - \alpha_1 - \alpha_3 PS_i - \alpha_4 DI_i - e_{di})$

$P_i = -\frac{\alpha_1}{\alpha_2} + \frac{1}{\alpha_2} Q_i - \frac{\alpha_3}{\alpha_2} PS_i - \frac{\alpha_4}{\alpha_2} DI_i - \frac{1}{\alpha_2} e_{di}$

Supply : $P_i = \frac{1}{\beta_2}(Q_i - \beta_1 - \beta_3 PF_i - e_{si})$

$= -\frac{\beta_1}{\beta_2} + \frac{1}{\beta_2} Q_i - \frac{\beta_3}{\beta_2} PF_i - \frac{1}{\beta_2} e_{si}$

b. Using the data in the file *truffles*, estimate the supply and demand equations that you have formulated in (a) using two-stage least squares. Are the signs correct? Are the estimated coefficients significantly different from zero?

D:

```
Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept)  -11.428     13.592  -0.841  0.40810
q             -2.671      1.175  -2.273  0.03154 *
ps             3.461      1.116   3.103  0.00458 **
di            13.390      2.747   4.875 4.68e-05 ***
```

All of signs are correct.

All of estimated coefficients are significantly different from zero at 99% level

S:

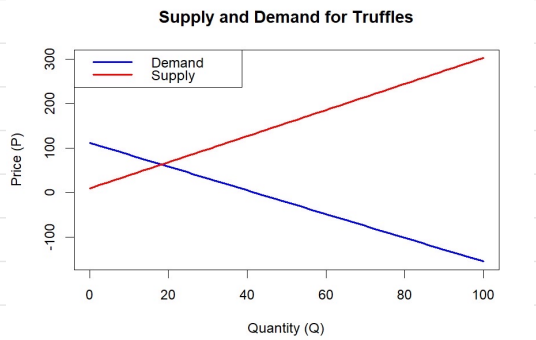
```
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) -58.7982     5.8592  -10.04 1.32e-10 ***
q             2.9367     0.2158   13.61 1.32e-13 ***
pf            2.9585     0.1560   18.97  < 2e-16 ***
```

c. Estimate the price elasticity of demand "at the means" using the results from (b).

$$\varepsilon_D = \frac{dQ}{dP} \cdot \frac{\bar{P}}{\bar{Q}} = \frac{1}{\delta_2} \cdot \frac{\bar{P}}{\bar{Q}}$$

```
Price Elasticity of Demand at the Means: -1.272464
```

d. Accurately sketch the supply and demand equations, with P on the vertical axis and Q on the horizontal axis, using the estimates from part (b). For these sketches set the values of the exogenous variables DI, PS, and PF to be $DI^* = 3.5$, $PF^* = 23$, and $PS^* = 22$.



e. What are the equilibrium values of P and Q obtained in part (d)? Calculate the predicted equilibrium values of P and Q using the estimated reduced-form equations from Table 11.2, using the same values of the exogenous variables. How well do they agree?

Equilibrium : $(Q^*, P^*) = (18.25, 62.84)$

```
> cat("Equilibrium Quantity (Q*):", round(eq_Q, 2), "\n")
Equilibrium Quantity (Q*): 18.25
> cat("Equilibrium Price (P*):", round(eq_P, 2), "\n")
Equilibrium Price (P*): 62.84
```

Reduced-form : $(Q, P) = (18.26, 62.82)$

```
> cat("Reduced-form predicted Q:", round(Q_rf, 2), "\n")
Reduced-form predicted Q: 18.26
> cat("Reduced-form predicted P:", round(P_rf, 2), "\n")
Reduced-form predicted P: 62.82
```

Two ways solutions are similar. We can say they are agree.

f. Estimate the supply and demand equations that you have formulated in (a) using OLS. Are the signs correct? Are the estimated coefficients significantly different from zero? Compare the results to those in part (b).

Demand

```
Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) -13.6195     9.0872  -1.499   0.1460
q             0.1512     0.4988   0.303   0.7642
ps            1.3607     0.5940   2.291   0.0303 *
di           12.3582     1.8254   6.770 3.48e-07 ***
```

The estimated coefficient q is not correct and not significantly

Supply : similar with 2SLS.

```
Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) -52.8763     5.0238  -10.53 4.68e-11 ***
q             2.6613     0.1712   15.54 5.42e-15 ***
pf            2.9217     0.1482   19.71  < 2e-16 ***
```

11.30 Example 11.3 introduces Klein's Model I. Use the data file *klein* to answer the following questions.

a. Estimate the investment function in equation (11.18) by OLS. Comment on the signs and significance of the coefficients.

$$I_t = \beta_1 + \beta_2 P_t + \beta_3 P_{t-1} + \beta_4 K_{t-1} + e_{2t} \qquad (11.18)$$

```
Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) 10.12579    5.46555   1.853 0.081374 .
p            0.47964    0.09711   4.939 0.000125 ***
plag         0.33304    0.10086   3.302 0.004212 **
klag        -0.11179    0.02673  -4.183 0.000624 ***
```

The signs of the estimated coefficients are consistent with theoretical expectations, and the high statistical significance of the key variables indicates a well-specified model.

b. Estimate the reduced-form equation for profits, P_t, using all eight exogenous and predetermined variables as explanatory variables. Test the joint significance of all the variables except lagged profits, P_{t-1}, and lagged capital stock, K_{t-1}. Save the residuals, $\hat{v}_t$ and compute the fitted values, $\hat{P}_t$.

```
Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) 50.38442   31.63026   1.593   0.1352
g            0.43902    0.39114   1.122   0.2820
w2          -0.07961    2.53382  -0.031   0.9754
tx          -0.92310    0.43376  -2.128   0.0530 .
plag         0.80250    0.51886   1.547   0.1459
klag        -0.21610    0.11911  -1.814   0.0928 .
time         0.31941    0.77813   0.410   0.6881
elag         0.02200    0.28216   0.078   0.9390
```

```
Analysis of Variance Table

Model 1: p ~ plag + klag
Model 2: p ~ g + w2 + tx + plag + klag + time + elag
  Res.Df    RSS Df Sum of Sq      F Pr(>F)
1     18 108.04
2     13  61.95  5    46.093 1.9345 0.1566
```

```
Conclusion:
The result (F = 1.93, p = 0.1566) indicates that the additional varia
bles are not jointly statistically significant at the 5% level. Thus,
conditional on lagged profit and capital stock, these exogenous varia
bles do not significantly improve the explanatory power of the model.
```

	p	pt_hat	vt_hat
2	12.4	13.255556	-0.8555556
3	16.9	16.577368	0.3226319
4	18.4	19.282347	-0.8823465
5	19.4	20.960143	-1.5601433
6	20.1	19.766509	0.3334910
7	19.6	18.238731	1.3612688
8	19.8	17.573065	2.2269354
9	21.1	19.541720	1.5582796
10	21.7	20.375101	1.3248995
11	15.6	17.180415	-1.5804148
12	11.4	12.705026	-1.3050261
13	7.0	8.999780	-1.9997802
14	11.2	9.054102	2.1458976
15	12.3	12.671263	-0.3712632
16	14.0	14.421338	-0.4213385
17	17.6	14.711907	2.8880932
18	17.3	19.796405	-2.4964049
19	15.3	19.206691	-3.9066913
20	19.0	17.419605	1.5803947
21	21.1	20.305654	0.7943462
22	23.5	22.657273	0.8427268

c. The Hausman test for the presence of endogenous explanatory variables is discussed in Section 10.4.1. It is implemented by adding the reduced-form residuals to the structural equation and testing their significance, that is, using OLS estimate the model

$$I_t = \beta_1 + \beta_2 P_t + \beta_3 P_{t-1} + \beta_4 K_{t-1} + \delta \hat{v}_t + e_{2t}$$

Use a t-test for the null hypothesis $H_0: \delta = 0$ versus $H_1: \delta \neq 0$ at the 5% level of significance. By rejecting the null hypothesis, we conclude that P_t is endogenous. What do we conclude from the test? In the context of this simultaneous equations model what result should we find?

```
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 20.27821    4.70179   4.313 0.000536 ***
p            0.15022    0.10798   1.391 0.183222
plag         0.61594    0.10147   6.070 1.62e-05 ***
klag        -0.15779    0.02252  -7.007 2.96e-06 ***
vt_hat       0.57451    0.14261   4.029 0.000972 ***
```

```
Conclusion:
Since the p-value is well below 0.05, we reject the null hypothesis H0:delta=0.
This means:The residual vhat_t from the reduced-form equation is statistically
significant in the structural equation, providing evidence that current profits
P_t are endogenous in the investment equation.This result validates the concern
that OLS estimates in the structural equation may be biased, because P_t is cor
related with the error term.
In the context of this simultaneous equations model, we should expect to find t
hat P_t  is endogenous.
```

d. Obtain the 2SLS estimates of the investment equation using all eight exogenous and predetermined variables as IVs and software designed for 2SLS. Compare the estimates to the OLS estimates in part (a). Do you find any important differences?

```
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 20.27821    8.38325   2.419  0.02707 *
p            0.15022    0.19253   0.780  0.44598
plag         0.61594    0.18093   3.404  0.00338 **
klag        -0.15779    0.04015  -3.930  0.00108 **
```

```
Conclusion:
The 2SLS estimation shows that the coefficient on current profits (P_t ) drops
from 0.48 (OLS, significant at 1%) to 0.15 (2SLS, not significant), indicating
that the OLS estimate was likely biased due to endogeneity. In contrast, the co
efficients on lagged profits and lagged capital remain significant and even inc
rease in magnitude under 2SLS. This confirms the result of the Hausman test and
supports using 2SLS for consistent estimation in the Klein model.
```

e. Estimate the second-stage model $I_t = \beta_1 + \beta_2 \hat{P}_t + \beta_3 P_{t-1} + \beta_4 K_{t-1} + e_{2t}$ by OLS. Compare the estimates and standard errors from this estimation to those in part (d). What differences are there?

```
Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept) 20.27821    9.97663   2.033  0.05802 .
pt_hat       0.15022    0.22913   0.656  0.52084
plag         0.61594    0.21531   2.861  0.01083 *
klag        -0.15779    0.04778  -3.302  0.00421 **
```

```
Conclusion:
The second-stage OLS regression using the fitted values Phat_t  reproduces
the exact same point estimates as the ivreg 2SLS. However, the standard err
ors are noticeably larger in the manual two-stage OLS , leading to higher p
-values. This discrepancy arises because naive second-stage OLS underestima
tes the sampling variability of Phat_t, whereas ivreg reports the correct 2
SLS standard errors.
```

f. Let the 2SLS residuals from part (e) be $\hat{e}_{2t}$. Regress these residuals on all the exogenous and predetermined variables. If these instruments are valid, then the R^2 from this regression should be low, and none of the variables are statistically significant. The Sargan test for instrument validity is discussed in Section 10.4.3. The test statistic TR^2 has a chi-square distribution with degrees of freedom equal to the number of "surplus" IVs if the surplus instruments are valid. The investment equation includes three exogenous and/or predetermined variables out of the total of eight possible. There are $L = 5$ external instruments and $B = 1$ right-hand side endogenous variables. Compare the value of the test statistic to the 95th percentile value from the $\chi^2_{(4)}$ distribution. What do we conclude about the validity of the surplus instruments in this case?

```
Conclusion:
The Sargan test indicates that the surplus instruments used in the 2SLS est
imation of the investment equation are statistically valid. The test statis
tic (1.28) is well below the 95% critical value from the chi-square distrib
ution with 4 degrees of freedom (9.49). Therefore, we do not reject the nul
l hypothesis that the instruments are uncorrelated with the structural erro
r term, supporting the appropriateness of the chosen instrumental variable
s.
```