6.

f. Column (5) contains the random effects estimates. Which coefficients, apart from the intercepts, show the most difference from the fixed effects estimates? Use the Hausman test statistic (15.36) to test whether there are significant differences between the random effects estimates and the fixed effects estimates in column (3) (Why that one?). Based on the test results, is random effects estimation in this model appropriate?

(f)

$$t_{Exper} : \frac{0.0575 - 0.0786}{\sqrt{0.033^2 - 0.022^2}} = -1.67$$

$$t_{Exper^2} = \frac{-0.0012 + 0.0023}{\sqrt{0.0011^2 - 0.0007^2}} = 1.3$$

$$t_{south} = \frac{-0.3261 + 0.2326}{\sqrt{0.1258^2 - 0.0317^2}} = -0.77$$

$$t_{union} = \frac{0.0822 - 0.1027}{\sqrt{0.0312^2 - 0.0245^2}} = -1.06$$

⇒ EXPER t 值最大，但其結果在統計上不顯著，FE、RE 無明顯差異.

17.

✓ **15.17** The data file *liquor* contains observations on annual expenditure on liquor (*LIQUOR*) and annual income (*INCOME*) (both in thousands of dollars) for 40 randomly selected households for three consecutive years.

a. Create the first-differenced observations on *LIQUOR* and *INCOME*. Call these new variables *LIQUORD* and *INCOMED*. Using OLS regress *LIQUORD* on *INCOMED* without a constant term. Construct a 95% interval estimate of the coefficient.

b. Estimate the model $LIQUOR_{it} = \beta_1 + \beta_2 INCOME_{it} + u_i + e_{it}$ using random effects. Construct a 95% interval estimate of the coefficient on *INCOME*. How does it compare to the interval in part (a)?

c. Test for the presence of random effects using the LM statistic in equation (15.35). Use the 5% level of significance.

d. For each individual, compute the time averages for the variable *INCOME*. Call this variable *INCOMEM*. Estimate the model $LIQUOR_{it} = \beta_1 + \beta_2 INCOME_{it} + \gamma INCOMEM_i + c_i + e_{it}$ using the random effects estimator. Test the significance of the coefficient γ at the 5% level. Based on this test, what can we conclude about the correlation between the random effect u_i and *INCOME*? Is it OK to use the random effects estimator for the model in (b)?

b.

```
> # 取出95%信賴區間
> confint(re_model)
                  2.5 %     97.5 %
(Intercept) -0.05211904 1.99018381
income       0.01283111 0.04031983
>
```

The RE coefficient estimate is slightly smaller than the difference estimator coefficient, but the standard error of the random effects estimator is about 75% of the se of the difference estimator's standard error, yielding a statistical significance

c.

```
        Lagrange Multiplier Test - (Breusch-Pagan)

data:  liquor ~ income
chisq = 20.68, df = 1, p-value = 5.429e-06
alternative hypothesis: significant effects
```

reject the null hypothesis that $\sigma_u^2 = 0$ and accept the alternative that $\sigma_u^2 > 0$. indicating that there is statistically significant unobserved heterogeneity.

d.

```
Coefficients:
            Estimate  Std. Error z-value Pr(>|z|)
(Intercept) 0.9163337 0.5524439 1.6587  0.09718 .
income      0.0207421 0.0209083 0.9921  0.32117
income_mean 0.0065792 0.0222048 0.2963  0.76700
```

income_m t-value is 0.3, ⇒ statistically insignificance.

There is no evidence for correlation between income and the unobserved heterogeneity.

2.

15.20 This exercise uses data from the STAR experiment introduced to illustrate fixed and random effects for grouped data. In the STAR experiment, children were randomly assigned within schools into three types of classes: small classes with 13–17 students, regular-sized classes with 22–25 students, and regular-sized classes with a full-time teacher aide to assist the teacher. Student scores on achievement tests were recorded as well as some information about the students, teachers, and schools. Data for the kindergarten classes are contained in the data file *star*.

a. Estimate a regression equation (with no fixed or random effects) where *READSCORE* is related to *SMALL*, *AIDE*, *TCHEXPER*, *BOY*, *WHITE_ASIAN*, and *FREELUNCH*. Discuss the results. Do students perform better in reading when they are in small classes? Does a teacher's aide improve scores? Do the students of more experienced teachers score higher on reading tests? Does the student's sex or race make a difference?

b. Reestimate the model in part (a) with school fixed effects. Compare the results with those in part (a). Have any of your conclusions changed? [*Hint*: specify *SCHID* as the cross-section identifier and *ID* as the "time" identifier.]

c. Test for the significance of the school fixed effects. Under what conditions would we expect the inclusion of significant fixed effects to have little influence on the coefficient estimates of the remaining variables?

d. Reestimate the model in part (a) with school random effects. Compare the results with those from parts (a) and (b). Are there any variables in the equation that might be correlated with the school effects? Use the LM test for the presence of random effects.

e. Using the *t*-test statistic in equation (15.36) and a 5% significance level, test whether there are any significant differences between the fixed effects and random effects estimates of the coefficients on *SMALL*, *AIDE*, *TCHEXPER*, *WHITE_ASIAN*, and *FREELUNCH*. What are the implications of the test outcomes? What happens if we apply the test to the fixed and random effects estimates of the coefficient on *BOY*?

f. Create school-averages of the variables and carry out the Mundlak test for correlation between them and the unobserved heterogeneity.

```
Effects:
                 var std.dev share
idiosyncratic 751.43   27.41 0.829
individual    155.31   12.46 0.171
theta:
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
 0.6470  0.7225  0.7523  0.7541  0.7831  0.8153

Residuals:
   Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
-97.483 -17.236  -3.282   0.037  12.803 192.346

Coefficients:
              Estimate Std. Error  z-value  Pr(>|z|)
(Intercept) 436.126774   2.064782 211.2217 < 2.2e-16 ***
small         6.458722   0.912548   7.0777 1.466e-12 ***
aide          0.992146   0.881159   1.1260    0.2602
tchexper      0.302679   0.070292   4.3060 1.662e-05 ***
boy          -5.512081   0.727639  -7.5753 3.583e-14 ***
white_asian   7.350477   1.431376   5.1353 2.818e-07 ***
freelunch   -14.584332   0.874676 -16.6740 < 2.2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Total Sum of Squares:    6158000
Residual Sum of Squares: 4332100
R-Squared:      0.29655
Adj. R-Squared: 0.29582
Chisq: 493.205 on 6 DF, p-value: < 2.22e-16
```

```
        Lagrange Multiplier Test - (Breusch-Pagan)

data:  readscore ~ small + aide + tchexper + boy + white_asian + freelunch
chisq = 6677.4, df = 1, p-value < 2.2e-16
alternative hypothesis: significant effects
```

d. 與 OLS & fixed effect 相比，RE 的估計與其皆相似。

在 LM 假定下，reject H0，代表學校存在異質性，

應使用隨機效果模型。

e.

```
        Hausman Test

data:  readscore ~ small + aide + tchexper + boy + white_asian + freelunch
chisq = 13.809, df = 6, p-value = 0.03184
alternative hypothesis: one model is inconsistent
```

⇒ reject H0，使用 FE 較適合

f.

```
(f) Mundlak 檢定結果（學校平均變數是否顯著）：
> print(mundlak_test)

Linear hypothesis test:
small_avg = 0
aide_avg = 0
tchexper_avg = 0
boy_avg = 0
white_asian_avg = 0
freelunch_avg = 0

Model 1: restricted model
Model 2: readscore ~ small + aide + tchexper + boy + white_asian + freelunch +
    small_avg + aide_avg + tchexper_avg + boy_avg + white_asian_avg +
    freelunch_avg

Note: Coefficient covariance matrix supplied.

  Res.Df Df      F  Pr(>F)
1   5695
2   5689  6 2.2541 0.03557 *
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
> 
```

因 p-value < 0.05，reject H0

因此不適用隨機效果模型