

Example 11.3 introduces Klein's Model I.

- a. Do we have an adequate number of IVs to estimate each equation? Check the necessary condition for the identification of each equation. The necessary condition for identification is that in a system of M equations at least $M - 1$ variables must be omitted from each equation.

Ans.

Klein's Model I:

$$\text{consumption function: } CN_t = \alpha_1 + \alpha_2(W_{1t} + W_{2t}) + \alpha_3P_t + \alpha_4P_{t-1} + e_{1t}$$

$$\text{investment equation: } I_t = \beta_1 + \beta_2P_t + \beta_3P_{t-1} + \beta_4K_{t-1} + e_{2t}$$

$$\text{wage equation: } W_{1t} = \gamma_1 + \gamma_2E_t + \gamma_3E_{t-1} + \gamma_4TIME_t + e_{3t}$$

$$W_t = W_{1t} + W_{2t} \quad E_t = CN_t + I_t + (G_t - W_{2t}) \quad TIME_t = YEAR_t - 1931$$

8 endogenous variables: $CN_t, I_t, W_{1t}, W_t, P_t, E_t, Y_t, K_t$

8 exogenous variables: $G_t, W_{2t}, TX_t, TIME_t, X_{1t}, P_{t-1}, K_{t-1}, E_{t-1}$

模型中總共有 $M=8$ 條結構方程（包含三條估計式與五條定義式），

每條結構方程至少需排除 $M-1=7$ 個外生變數，才能滿足「最小識別條件」。

方程	使用變數數量	被排除變數數量	結論
consumption	使用 6 個變數	排除 10 個變數	滿足
investment	使用 5 個變數	排除 11 個變數	滿足
wage	使用 5 個變數	排除 11 個變數	滿足

結論：三條方程都符合，皆可識別。

- b. An equivalent identification condition is that the number of excluded exogenous variables from the equation must be at least as large as the number of included right-hand side endogenous variables. Check that this condition is satisfied for each equation.

Ans.

要識別，方程排除的外生變數數量必須 \geq 右邊的內生變數數量。

方程	RHS 內生變數數量	被排除變數數量	結論
consumption	W_{1t}, P_t : 2 個	$G_t, TX_t, TIME_t, K_{t-1}, E_{t-1}$: 5 個	滿足
investment	P_t : 1 個	$G_t, TX_t, TIME_t, K_{t-1}, E_{t-1}$: 5 個	滿足
wage	E_t : 1 個	$G_t, W_{2t}, TX_t, P_{t-1}, K_{t-1}$: 5 個	滿足

結論：三條方程都符合。

- c. Write down in econometric notation the first-stage equation, the reduced form, for W_{1t} , wages of workers earned in the private sector. Call the parameters π_1, π_2, \dots

Ans.

W_{1t} 的 reduced form 會是所有外生變數和預先決定變數的線性組合：

$$W_{1t} = \pi_1 + \pi_2G_t + \pi_3W_{2t} + \pi_4TX_t + \pi_5TIME_t + \pi_6P_{t-1} + \pi_7K_{t-1} + \pi_8E_{t-1} + \nu$$

- d. Describe the two regression steps of 2SLS estimation of the consumption function. This is not a question about a computer software command.

Ans.

第一步：取得內生變數的預測值

$$\widehat{W}_{1t} = \pi_{11} + \pi_{12}G_t + \pi_{13}W_{2t} + \pi_{14}TX_t + \pi_{15}TIME_t + \pi_{16}P_{t-1} + \pi_{17}K_{t-1} + \pi_{18}E_{t-1}$$

$$\widehat{P}_t = \pi_{21} + \pi_{22}G_t + \pi_{23}W_{2t} + \pi_{24}TX_t + \pi_{25}TIME_t + \pi_{26}P_{t-1} + \pi_{27}K_{t-1} + \pi_{28}E_{t-1}$$

第二步：用 OLS 估計結構式（代入預測值）

$$CN_t = \alpha_0 + \alpha_1(\widehat{W}_{1t} + W_{2t}) + \alpha_2\widehat{P}_t + \alpha_3P_{t-1} + u_t$$

- e. Does following the steps in part (d) produce regression results that are identical to the 2SLS estimates provided by software specifically designed for 2SLS estimation? In particular, will the t -values be the same?

Ans.

在 (d) 步驟中，使用 2SLS 估計消費方程時，第一階段以外生變數預測內生變數 W_{1t}, P_t ，得到預測值 $\widehat{W}_{1t}, \widehat{P}_t$ ，然後以這些預測值代入原始方程，使用 OLS 估計係數。

手動 2SLS 與軟體套件結果是否相同？

- 參數估計值將會相同
- t 值將會不同：手動 2SLS 時，由於 $\widehat{W}_{1t}, \widehat{P}_t$ 本身含有估計誤差，OLS 並未考慮這些誤差，導致標準誤 SE 被低估，進而導致 t 值 (t -statistics) 與信賴區間錯誤。

若要進行有效的推論，應該使用真正的 2SLS 軟體或套件來自動計算正確 SE。