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**Course:** Financial Econometrics

**HW0505**

**C11Q01, C11Q16, C11Q17, C11Q28, C11Q30**

# 1.

We have the simultaneous equations model:

y\_1 = α\_1 y\_2 + e\_1

y\_2 = α\_2 y\_1 + β\_1 x\_1 + β\_2 x\_2 + e\_2

Where x\_1 and x\_2 are exogenous and uncorrelated with error terms e\_1 and e\_2.

Part (a): Deriving the Reduced-Form Equation for y\_2

To find the reduced-form equation for y\_2, I need to substitute the first equation into the second.

First, I'll substitute the expression for y\_1 into the second equation:

y\_2 = α\_2(α\_1 y\_2 + e\_1) + β\_1 x\_1 + β\_2 x\_2 + e\_2

Expanding:

y\_2 = α\_2α\_1 y\_2 + α\_2 e\_1 + β\_1 x\_1 + β\_2 x\_2 + e\_2

Rearranging to isolate y\_2:

y\_2 - α\_2α\_1 y\_2 = α\_2 e\_1 + β\_1 x\_1 + β\_2 x\_2 + e\_2

y\_2(1 - α\_2α\_1) = α\_2 e\_1 + β\_1 x\_1 + β\_2 x\_2 + e\_2

y\_2 = (β\_1 x\_1 + β\_2 x\_2 + α\_2 e\_1 + e\_2)/(1 - α\_2α\_1)

This is the reduced-form equation for y\_2, which we can write as:

y\_2 = π\_1 x\_1 + π\_2 x\_2 + v\_2

Where:

π\_1 = β\_1/(1 - α\_2α\_1)

π\_2 = β\_2/(1 - α\_2α\_1)

v\_2 = (α\_2 e\_1 + e\_2)/(1 - α\_2α\_1)

Now, to show that y\_2 is correlated with e\_1, I'll examine the covariance:

Cov(y\_2, e\_1) = Cov(π\_1 x\_1 + π\_2 x\_2 + v\_2, e\_1)

Since x\_1 and x\_2 are exogenous and uncorrelated with e\_1:

Cov(y\_2, e\_1) = Cov(v\_2, e\_1)

Substituting the expression for v\_2:

Cov(y\_2, e\_1) = Cov((α\_2 e\_1 + e\_2)/(1 - α\_2α\_1), e\_1)

Since e\_2 is uncorrelated with e\_1:

Cov(y\_2, e\_1) = α\_2/(1 - α\_2α\_1) × Cov(e\_1, e\_1) = α\_2/(1 - α\_2α\_1) × Var(e\_1)

Since Var(e\_1) > 0 and assuming 1 - α\_2α\_1 ≠ 0, we have Cov(y\_2, e\_1) ≠ 0, which proves that y\_2 is correlated with e\_1.

Part (b): OLS Consistency

For OLS to consistently estimate parameters, the explanatory variables must be uncorrelated with the error term.

In the first equation: y\_1 = α\_1 y\_2 + e\_1

y\_2 is correlated with e\_1 as shown in part (a)

Therefore, OLS estimation of α\_1 will be inconsistent

In the second equation: y\_2 = α\_2 y\_1 + β\_1 x\_1 + β\_2 x\_2 + e\_2

y\_1 contains e\_1 (from first equation), which affects y\_2

y\_1 is correlated with e\_2 (can be shown similarly to part (a))

Therefore, OLS estimation of α\_2 will be inconsistent

However, if we could somehow isolate the effects, β\_1 and β\_2 could be consistently estimated since x\_1 and x\_2 are exogenous

Part (c): Identification

In a system of M simultaneous equations, at least (M-1) variables must be absent from each equation for it to be identified.

Here, M = 2, so at least 1 variable must be absent from each equation.

For the first equation: y\_1 = α\_1 y\_2 + e\_1

Both exogenous variables x\_1 and x\_2 are absent

Therefore, this equation is identified (over-identified since 2 > M-1)

For the second equation: y\_2 = α\_2 y\_1 + β\_1 x\_1 + β\_2 x\_2 + e\_2

No exogenous variables are absent

Therefore, this equation is not identified (under-identified)

In summary: The first equation's parameter α\_1 is identified and can be consistently estimated using 2SLS. The second equation's parameters α\_2, β\_1, and β\_2 are not identified and cannot be consistently estimated using any method.

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# 16.

**Part (a): Deriving Reduced-Form Equations**

Given model:

* Demand: Qi = α1 + **α2**Pi + edi
* Supply: Qi = β1 + β2Pi + β3Wi + esi

Setting demand equal to supply in equilibrium: α1 + α2Pi + edi = β1 + β2Pi + β3Wi + esi

Solving for Pi: Pi(α2 - β2) = β1 - α1 + β3Wi + esi - edi Pi = (β1 - α1 + β3Wi + esi - edi)/(α2 - β2)

This gives the reduced-form equation for P: Pi = π1 + π2Wi + v1i

Where:

* π1 = (β1 - α1)/(α2 - β2)
* **π2 = β3/(α2 - β2)**
* v1i = (esi - edi)/(α2 - β2)

Substituting this expression for Pi into the demand equation: Qi = α1 + α2[(β1 - α1 + β3Wi + esi - edi)/(α2 - β2)] + edi

After simplification: Qi = (α2β1 - α1β2)/(α2 - β2) + [α2β3/(α2 - β2)]Wi + (α2esi - β2edi)/(α2 - β2)

This gives the reduced-form equation for Q: Qi = θ1 + θ2Wi + v2i

Where:

* θ1 = (α2β1 - α1β2)/(α2 - β2)
* **θ2 = α2β3/(α2 - β2)**
* v2i = (α2esi - β2edi)/(α2 - β2)

**Part (b): Identifying Structural Parameters**

From the reduced-form parameters:

* We can identify α2 = θ2/π2 = (α2β3/(α2-β2))/(β3/(α2-β2)) = α2

The demand equation is identified because:

* The wage rate W appears in the supply equation but not in the demand equation
* This satisfies the order condition for identification (at least M-1 variables absent)

The supply equation is not identified because:

* There are no exogenous variables that appear in the demand equation but not in the supply equation

**Part (c): Indirect Least Squares**

Given estimated reduced-form equations:

* Q̂ = 5 + 0.5W
* P̂ = 2.4 + 1W

We can identify:

* α2 = θ2/π2 = 0.5/1 = 0.5

For α1 (intercept in demand equation):

* α1 = 5 - 0.5(2.4) = 5 - 1.2 = 3.8

Therefore, the identified demand equation is: Q = 3.8 + 0.5P̂ + ed

**Part (d)**

**Step 1: Calculate fitted values from the reduced-form equation for P**

Using the estimated reduced-form equation for P: P̂ = 2.4 + 1W

For each observation in the data:

| **Observation** | **W** | **P̂ = 2.4 + 1W** |
| --- | --- | --- |
| 1 | 2 | 2.4 + 1(2) = 4.4 |
| 2 | 3 | 2.4 + 1(3) = 5.4 |
| 3 | 1 | 2.4 + 1(1) = 3.4 |
| 4 | 1 | 2.4 + 1(1) = 3.4 |
| 5 | 3 | 2.4 + 1(3) = 5.4 |

**Step 2: Apply 2SLS to estimate the demand equation**

The demand equation is: Q = α₁ + α₂P + ed

For 2SLS, we replace P with P̂ in the second stage: Q = α₁ + α₂P̂ + ed

Creating a table with the original Q and the fitted P̂:

| **Observation** | **Q** | **P̂** |
| --- | --- | --- |
| 1 | 4 | 4.4 |
| 2 | 6 | 5.4 |
| 3 | 9 | 3.4 |
| 4 | 3 | 3.4 |
| 5 | 8 | 5.4 |

Running the regression of Q on P̂ gives us: Q = 3.8 + 0.5P̂ + ed

**Verification using the data:** To verify this result, we can calculate the predicted Q values using our 2SLS equation:

| **Observation** | **P̂** | **Q̂ = 3.8 + 0.5P̂** | **Actual Q** | **Residual** |
| --- | --- | --- | --- | --- |
| 1 | 4.4 | 3.8 + 0.5(4.4) = 6.0 | 4 | -2.0 |
| 2 | 5.4 | 3.8 + 0.5(5.4) = 6.5 | 6 | -0.5 |
| 3 | 3.4 | 3.8 + 0.5(3.4) = 5.5 | 9 | 3.5 |
| 4 | 3.4 | 3.8 + 0.5(3.4) = 5.5 | 3 | -2.5 |
| 5 | 5.4 | 3.8 + 0.5(5.4) = 6.5 | 8 | 1.5 |

**Comparison with OLS:** For comparison, if we had incorrectly used OLS directly (regressing Q on P), we would get: Q = 4.75 + 0.28P

This differs from our 2SLS estimate because OLS doesn't account for the endogeneity between P and Q in the simultaneous equations system.

**Conclusion:**

The 2SLS estimate of the demand equation is: Q = 3.8 + 0.5P̂ + ed

This matches our result from the indirect least squares method in part (c), confirming that both methods yield the same estimates for the identified equation when applied correctly.

# 17.

(a) There are M = 8 equations requiring 7 omitted variables in each equation. There is a total of 16 variables in the system. The consumption equation includes 6 variables and omits 10. The necessary condition is satisfied. The investment equation includes 5 variables and omits 11. The necessary condition is satisfied. The private sector wage equation includes 5 variables and omits 11. The necessary condition is satisfied.

(b) The consumption equation has 2 RHS endogenous variables and excludes 5 exogenous variables. The investment and private wage equations have 1 RHS endogenous variable and omit 5 exogenous variables.

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# 28.

**Part (a): Rewriting the Equations with P on the Left-Hand Side**

Original equations:

* Demand: Qᵢ = α₁ + α₂Pᵢ + α₃PSᵢ + α₄DIᵢ + edᵢ
* Supply: Qᵢ = β₁ + β₂Pᵢ + β₃PFᵢ + eₛᵢ

Rewritten with P on the left-hand side:

**Demand Equation:** α₂Pᵢ = -α₁ + Qᵢ - α₃PSᵢ - α₄DIᵢ - eₐᵢ Pᵢ = (-α₁/α₂) + (1/α₂)Qᵢ - (α₃/α₂)PSᵢ - (α₄/α₂)DIᵢ - (edᵢ/α₂) Pᵢ = γ₁ + γ₂Qᵢ + γ₃PSᵢ + γ₄DIᵢ + udᵢ

Where:

* γ₁ = -α₁/α₂
* γ₂ = 1/α₂
* γ₃ = -α₃/α₂
* γ₄ = -α₄/α₂
* uₐᵢ = -edᵢ/α₂

**Supply Equation:** β₂Pᵢ = -β₁ + Qᵢ - β₃PFᵢ - eₛᵢ = (-β₁/β₂) + (1/β₂)Qᵢ - (β₃/β₂)PFᵢ - (eₛᵢ/β₂) = δ₁ + δ₂Qᵢ + δ₃PFᵢ + uₛᵢ

Where:

* δ₁ = -β₁/β₂
* δ₂ = 1/β₂
* δ₃ = -β₃/β₂
* uₛᵢ = -eₛᵢ/β₂

**Anticipated Signs:**

For the demand equation:

* γ₁ (intercept): Positive (since α₁ is expected to be positive and α₂ negative in the original demand equation)
* γ₂ (coefficient of Q): Negative (since α₂ is expected to be negative in the original demand equation)
* γ₃ (coefficient of PS): Positive (since α₃ is expected to be positive and α₂ negative)
* γ₄ (coefficient of DI): Positive (since α₄ is expected to be positive and α₂ negative)

For the supply equation:

* δ₁ (intercept): Negative (since β₁ is expected to be positive and β₂ positive in the original supply equation)
* δ₂ (coefficient of Q): Positive (since β₂ is expected to be positive in the original supply equation)
* δ₃ (coefficient of PF): Positive (since β₃ is negative and β₂ is positive)

**Part (b):**

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**Part (c):**

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**Part (d):**

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**Part (e):**

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The equilibrium values calculated from the structural equations and those predicted from the reduced form equations agree very well:

* **Price**: The difference is only 0.02719676 (about 0.04% difference)
* **Quantity**: The difference is only -0.01018407 (about 0.06% difference)

These extremely small differences indicate excellent agreement between the two methods. This confirms that both the structural approach (solving the simultaneous equations) and the reduced form approach (direct estimation of the equilibrium values) produce consistent results, which validates the model specification and estimation technique. The slight differences are likely due to rounding errors in the calculations or minor numerical imprecisions in the estimation algorithms, rather than any substantive disagreement between the methods.

**Part (f):**

**OLS:**

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**Analysis of OLS vs 2SLS Results**

**Demand Equation**

**Sign Analysis**

* **q coefficient**:
  + OLS: Positive (0.1512) - **incorrect sign** for demand curve
  + 2SLS: Negative (-2.6705) - **correct sign** for demand curve
* **ps coefficient**:
  + OLS: Positive (1.3607) - **correct sign** (substitute good price)
  + 2SLS: Positive (3.4611) - **correct sign**
* **di coefficient**:
  + OLS: Positive (12.3582) - **correct sign** (income effect)
  + 2SLS: Positive (13.3899) - **correct sign**

**Statistical Significance**

* **q coefficient**:
  + OLS: Not significant (p=0.7642)
  + 2SLS: Significant (p=0.0315) \*\*
* **ps coefficient**:
  + OLS: Significant (p=0.0303) \*\*
  + 2SLS: Highly significant (p=0.0046) \*\*\*
* **di coefficient**:
  + OLS: Highly significant (p<0.0001) \*\*\*
  + 2SLS: Highly significant (p<0.0001) \*\*\*

**Supply Equation**

**Sign Analysis**

* **q coefficient**:
  + OLS: Positive (2.6613) - **correct sign** for supply curve
  + 2SLS: Positive (2.9367) - **correct sign**
* **pf coefficient**:
  + OLS: Positive (2.9217) - **correct sign** (input price effect)
  + 2SLS: Positive (2.9585) - **correct sign**

**Statistical Significance**

* All coefficients in both OLS and 2SLS supply equations are highly significant (p<0.0001) \*\*\*

**Comparison with Part (b)**

1. **Key Finding**: OLS estimation of the demand equation yields an **incorrect positive sign** for the quantity coefficient, while 2SLS correctly produces a negative coefficient.
2. **Simultaneity Bias**: This demonstrates the simultaneity bias in OLS estimation when applied to simultaneous equation models. The OLS estimate fails to account for the endogeneity of quantity.
3. **Supply Equation**: Both methods produce similar estimates for the supply equation, but 2SLS estimates are slightly larger in magnitude.
4. **Statistical Significance**: The quantity coefficient in the demand equation is only statistically significant with 2SLS, not with OLS.
5. **Coefficient Magnitudes**: The 2SLS estimates for the exogenous variables (ps, di, pf) are larger in magnitude than their OLS counterparts, suggesting that OLS underestimates these effects.

In conclusion, the 2SLS results from part (b) correctly identify the structural parameters of the model, while OLS suffers from simultaneity bias, particularly in the demand equation where it fails to capture the negative relationship between price and quantity.

# 30.

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**Intercept (β₁ = 10.12579)**

* **Sign**: Positive
* **Significance**: Marginally significant (p = 0.081374) \*
* **Interpretation**: When all other variables are zero, the baseline investment level is estimated at about 10.13 units, though this is only significant at the 10% level.

**Current Profits (p) (β₂ = 0.47964)**

* **Sign**: Positive ✓
* **Significance**: Highly significant (p = 0.000125) \*\*\*
* **Interpretation**: This positive relationship aligns with economic theory - higher current profits lead to increased investment. For each additional unit of profit, investment increases by approximately 0.48 units, holding other factors constant.

**Lagged Profits (plag) (β₃ = 0.33304)**

* **Sign**: Positive ✓
* **Significance**: Highly significant (p = 0.004212) \*\*\*
* **Interpretation**: Past profits also positively affect current investment, suggesting firms use profit history in investment decisions. Each additional unit of last period's profit increases current investment by about 0.33 units.

**Lagged Capital Stock (klag) (β₄ = -0.11179)**

* **Sign**: Negative ✓
* **Significance**: Highly significant (p = 0.000624) \*\*\*
* **Interpretation**: This negative relationship suggests a capital adjustment process - firms with higher existing capital stock tend to invest less in the current period, consistent with diminishing returns to capital. For each additional unit of last period's capital stock, current investment decreases by about 0.11 units.

**Part (b)**:

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The joint hypothesis test fails to reject the null hypothesis that g, tx, w2, time, and elag are all simultaneously equal to zero (p-value = 0.1566), suggesting these variables do not collectively have a statistically significant effect on the dependent variable at conventional significance levels.

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**Results Interpretation: Hausman Test for Endogeneity**

**Key Statistics**

* Residual coefficient: 0.57451
* Standard error: 0.14261
* t-value: 4.029
* p-value: 0.000972 (highly significant)

**Conclusion**

We reject the null hypothesis that δ = 0 at the 5% significance level (and even at the 0.1% level). This provides strong evidence that p (profits) is indeed endogenous in the investment equation.

**Context in Simultaneous Equations Model**

This result aligns with what we would expect in Klein's Model I where:

* Consumption (CN) affects profits (P) through equation 11.17
* Investment (I) affects profits (P) through national income identity
* Profits (P) affects investment (I) through equation 11.18

In this simultaneous equations system, profits cannot be treated as exogenous because they are jointly determined with investment and consumption. The significant residual coefficient confirms this theoretical expectation, indicating that:

1. OLS estimates would be biased and inconsistent
2. Alternative estimation methods like 2SLS or IV are more appropriate
3. The simultaneous nature of the relationship between investment and profits is empirically validated

The high R-squared (0.9659) indicates the model explains most of the variation in investment, and the significant F-statistic confirms the overall model fit.

**Part (d):**

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The substantial differences between OLS and 2SLS estimates confirm the presence of endogeneity in the investment equation. The most striking finding is that current profits (p) appear to have a much smaller and statistically insignificant effect on investment when estimated with 2SLS, while lagged profits have a much stronger effect than OLS suggested.

These differences highlight the importance of addressing endogeneity in this model. The OLS estimates were biased due to the simultaneous relationship between investment and profits, and the 2SLS method has helped correct this bias by using instrumental variables. The results suggest that investment decisions are influenced more by past profits than by current profits, which makes economic sense as investment planning typically relies on historical performance.

**Part (e):**

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**Coefficients:** The point estimates are identical between the two models.

**Standard Errors:** The manual 2SLS approach consistently produces larger standard errors (about 19% higher) compared to the automated approach. This suggests the manual approach might be less efficient in its estimation.

The differences observed are likely due to how the **standard errors** are calculated in each approach. The automated 2SLS implementation in the ivreg function might use more efficient methods for computing standard errors, possibly accounting for heteroskedasticity or using different degrees of freedom adjustments.

These findings highlight the importance of using specialized software for 2SLS estimation rather than manually implementing the procedure, as the specialized software may incorporate refinements that lead to more efficient estimates and more accurate inference. While the **point estimates** are identical, the inference drawn from them could differ, especially in borderline cases of statistical significance.

**Part (f):**

**Sargan Test Results Summary**

The Sargan test for instrument validity yields:

* Test statistic (TR²): 1.2815
* Critical value (χ²₄,₀.₉₅): 9.4877
* p-value: 0.8645

We fail to reject the null hypothesis of valid instruments. The R² is very low (0.061) and none of the instruments are statistically significant in the residual regression (all p-values > 0.05). This confirms that the surplus instruments used in the 2SLS estimation appear to be valid for the investment equation.

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