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Deterministic models for opinion formation through communication: A survey[☆]

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ABSTRACT

A fundamental question in modeling opinion dynamics is to know how can opinion be formed and evolved in a social network? This is an thorny subject which has attracted a hulk of attitudes and whetted the curiosity of researchers from various disciplines. One of the major points of view rests on the fact that opinion can be formed and revised through a process called social influence. This latter lies at the heart of the opinion modeling process and it has two types: Informational social influence, where a user forms his opinion according to information he obtained from a certain number of agents in his friendship and neighborhood. normative social influence is the second type of social influence and it lead to conformity. A very few empirical studies indicate that, it is also important to consider the normative influence in the opinion modeling process. In contrary, informational Influence is one of the main underlying premises used by many well-known theoretical models of opinion dynamics

In the literature two main approaches have been adopted on how each individual updates her opinion: deterministic and probabilistic. Here, we focus only with deterministic models. We present various forms of modeling opinion dynamics in social networks and we show how opinions change following to social influence. Within the course of analysis, we point out both the strengths and weakness of many approaches. We aim to provide theoretical insight which may serve as guidelines for scientists, practitioners, researchers, consultants and developers who intend to design new methods in this area of interest.

1. Introduction

The rise of World Wide Web accelerated the development of large-scale social networks. In recent years, both social network and social media have become ubiquitous in our daily life and have allowed hundreds of millions of Internet users worldwide to produce and consume content. This is how the internet becomes a global incipient for discussion of topics, ideas and events.

Online social networks have offered an incredible platform for information exchange and have proved to be very powerful in many situations. As an example, we may mention Facebook during the 2010 Arab spring [1]. Various contents can be exchanged between Internet users such as photos, videos and articles. They are allowed also to express their opinions and give their hot takes, concerning many issues. During the past decades, much research attention has been drawn to understand how opinions change following to social influence. This latter lies at the heart of individuals opinion formation because users may form or update their opinion about a particular topic by learning from the information and opinions that their friends/neighbors share. There has been increasing interest to study how opinions are formed and evaluated over time and how they change following different social interactions.

In the literature, two main approaches have been adopted on how each individual updates her opinion: deterministic and probabilistic. The probabilistic approaches, named also the Bayesian approaches, have been widely used for managing uncertainty, and more recently, for opinion formation. The first work has been introduced by Bikhchandani et al. [2] and Banerjee [3], where they proposed to explain the uniformity in economic social behaviors. After that, similar ideas have been applied to Bayesian learning from observations of past sequential actions [4–6] and communication learning, in which individuals learn through communication other than the observation of others actions[7,8]. In this article, the part of our interest in opinion and belief dynamics is to understand the deterministic approaches, named also the non-Bayesian models.

☆ This document is a collaborative effort.

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In the past decades, the non-Bayesian models have been based in the empirical similarity [9,10]. Meaning that, individuals form beliefs about a situation based on their experiences in similar situations in the past. In particular, the tendency toward consensus and the possibility that dispersed information may not aggregate despite consensus are common features of these approaches. After that, several other forms of rule-of-thumb behavior are also used. Notable, among them there are the Ising model [11], the voter model [12,13] and others. After that, the trends of modeling opinion in social networks have been focused in social influence. The opinions of the agent for a single issue , evolve dynamically over time as a function of their neighbors opinions influence. Among these models there are Friedkin–Johnsen model [14], DeGroot model [15], the combination of DeGroot and FJ models [16,17]... In these years, much research attention has been drawn to develop a multidimensional model of opinion dynamics. In these models such as [18–20] opinions are multidimensional, representing the agents attitudes on several topics, which those topic-specific attitudes are interrelated.

In addition to this introductory section, the manuscript is divided into six distinct but complementary sections: The second section describes some concepts necessary to understand the rest of the manuscript. We start with the definition of opinion and where the beliefs and points of view came from. Because our visions of thing and the world can be influenced by our social environment, our emotional nature, our prejudices as usually factors, we propose to present the different forms of social influence.

To facilitate the construction of opinion dynamics models, we present in the third section, the key components in opinion formation proposed by Acemoglu and Ozdaglar [21]. Then, we present two groups of opinion dynamics models. In the first group, the opinions are considered discrete (often accepting only two different results (binary or ordinal/quantized)). However, the continuous opinions are classified as a second group, where they are modeled as continuous variables (which may correspond to beliefs about certain underlying variable state or the probability that a given statement is true).

An overview on the state of the art of opinion dynamics modeling methods will be presented in the fourth section. In the first sub-section, we will start by present the DeGroot model, which is a simple model of belief and consensus formation over social networks. We will see, however, that the specific assumptions it makes on how beliefs are updated may have certain non-desirable implications. Motivated by this, we will present, in the second and the third section, the different enhancements proposed to avoid limitations, respectively named the duplication of information and the disagreement/misinformation between agents.

Because there are several deterministic models, we introduce in section five a comparative table summarizing the different parameters used in most approaches. Section six concludes with a brief discussion and ours propositions of future work to improve some works. Finally a conclusion is given in the final section of this paper.

2. Background

As a result of social interactions with other people, opinions can be revised, changed and updated. So, we start by giving a definition of opinions, social influence and forms of influence. Then we present the term of opinion dynamics, known as the evolution of opinion over time and we investigate the key components in opinion formation.

2.1. Opinion

Opinion being increasingly used. It is extremely complex and we cannot provide it a single definition. In the most current sense, opinion may mean way of thinking about a topic or set of subjects, a personal judgment that is not necessarily true. Opinions are uttered spontaneously, directly, and brutally, that is to say without any definitions or explanations or demonstrations or arguments.

Acemoglu and Ozdaglar [21] treat the opinion as a reasoning product from ones context knowledge base where new knowledge fragments acquire through various types of learning experience. Schwitzgebel [22], definite opinion as a fact or proposition that an individual holds to be true. Opinions, in contrast, include both personal beliefs and attitudes or judgments that are not founded on proof or certainty. The question that needs to be answered here is where do these beliefs and opinions come from? It is true that certain phenotypic characteristics have biological and genetic bases. Yet, we do not think that our beliefs are imprinted on us by our genes, but they usually come from uncontrolled sources such as everyday experiences, media, education, interests, and passions. Actually, these phenomena influence our mind unconsciously.

Our opinions are acquired through various types of learning experiences [23]. Families play a main role in teaching some basic principles and beliefs to their children [21,22]. Much of it, however, much of opinion formation will take place in the social environment through a process of social learning. Banduras social learning theory [24], posits that people learn from one another, via observation, imitation, and modeling. Most human behavior is learned observationally through modeling: from observing others, one forms an idea of how new behaviors are performed, and on later occasions this coded information serves as a guide for action.

2.2. Social influence

The study of the influence of social backgrounds on opinion dates back to the early XX^{th} century. Charcot, who worked on hysteria, is the first one who has done these studies in this context [25]. He thought that these behaviors were there because of the heart impact of the social environment on people. Social influence is the influence exerted by an individual or a group on each of its members implying a change in behavior.

All impressions and changes in social life or relationships with others affect individuals or groups deeply, whether they are conscious of that or not. One of the central aspects of social influence is that we rely heavily on the ideas and opinions of others.

According to Edgar Morin [26], four forms of social influence can be outlined:

- Standardization: the group members influence each other
- Obedience: a person agrees to behave in accordance with the requests of an authority
- Innovation: a minority influences a majority
- The Conformism: also named as social pressure, is the influence of groups behavior that encourages an agent to change his behaviors to follow the group norms (a majority influence on a minority).

Two core conformity studies illustrating the effect of groups behaviors on individual's behaviors are Asch [27] and Sherif [28]. In these models, people were found to follow the rest of the group opinion. Sherif demonstrated that people tend to infer realities by referring to each other's judgments, resulting in private acceptance. Asch demonstrated that, even there are differences in two opinions, individuals still tend to accept opinions that, on their own, they would consider wrong. According to the study of Deutsch and Gerard [29], the social influences underlying conformity motivation can be categorized in two types of influence : Informational influence and Normative influence.

The first process, Informational influence, often leads to private acceptance (as Sherif has demonstrated). The individual user to update his opinion, on a particular topic, as a reaction to information received from his neighbors. [30] argue that Informational social influence occurs when individuals see other people as a source of information. In this model, agents refer to the behavior of others to figure out what is going on in the situation and what is right to do, because they are uncertain how to act or think. Informational social influence has been one of the main underlying premises used by many well-known theoretical models of opinion dynamics [12,13,15,31]. The latter process, i.e., normative influence, is the second psychological phenomena. As Asch has demonstrated, the purpose behind normative social influence is conformity which is opted for in order to be liked and accepted by others [29]. Individuals are likely to adjust their opinions and behaviors based on the statements of other members of a group to avoid being labeled as dissidents, even they have different views. The main aim of recent study proposed by Perfumi et al. [32] is to investigate the possible differences between a real and a virtual environment in eliciting conformity. To analyze the effects of a virtual environment on social conformity, they replicated the famous Aschs visual task and they created two new tasks of increasing ambiguity, assessed through the calculation of the items entropy. The experiment, conducted on 181 university students, emerged that conformity grows according to the ambiguity of the task and informational influence, induced with ambiguous stimuli, is still effective online but normative influence almost disappears. Normative influence is significantly weaker in virtual environments, if compared to face-to-face experiments. However, Informational influence is one of the main underlying premises used by many well-known theoretical models of opinion dynamics in social network [15,31,33] , normative influence has been ignored by models of opinion dynamics [34].

2.3. The concepts of trust

Trust is not a new research topic, a wide of variety of literature now exists on this concept. The trust concept originated in different disciplines: Psychology, Sociology and Computer Science. In social network, trust is based on the feedback on past interactions between members [35]. For instance, the relationship strengthens, and trust evolves when the two members interact with each other frequently. Models that exploit the social network structure, as the models that describe the evolution of opinion in social networks, are based on the concepts of Web of trust or FOAF (Friend-Of-A-Friend). The person in a social network is represented as nodes and the amount of trust they have for each friends is represented as the edges. The models presented in this paper, the amount of the confidence or the trust is a static value that depends only on the number of the neighbors.

3. Opinion dynamics models

This section starts by giving the definition of opinion dynamics and present the key components in opinion formation. Then, we present the models where the opinions are considered discrete then, the models where the opinions are considered continuous.

3.1. Opinion dynamics

Opinion was defined as drivers of human behavior and dynamic as the way things change over time. In this work, we use the term opinion, in view of their dynamic nature. That is to say, we define opinion dynamics as a continuous procedure featured by a sequence of varying opinions. Opinion dynamics, as a macroscopic collective social phenomenon, had been one of the active area research, which was the focus of interest for psychologists [36,37], statistical physicists [13], mathematicians [14,31]. The social psychologist Leon Festinger was the first to use the term Social Comparison. His theory, named social comparison theory, tackles the fundamentals of how a person forms beliefs and opinions about ones own capabilities. He has shown that human beings have the drive to assess their opinions and to know more about their abilities and when they are incapable of evaluating their opinions and abilities, they tend to compare themselves with others. Festingers theory of social comparison had significant implications for group formation and group structure. A lot of earlier research has extended the theory of social comparison such as Cartwright [38] facilitated the growth of group dynamics as a field of inquiry. Asch, [27], showed the power of groups to generate conformity. French [36] proposed a theory of social power that defines seven sources of power for changing conditions inside or outside a social group.

In social networks, people want to talk, share, post, tweet, like and perform any other kind of social action , so they influence others to do the same. In the interaction between two individuals, each can influence the other and gradually form common opinions by ignoring minority opinions and allowing opinion differences. Also, some users may express their opinions more frequently than others, thus trigger a greater number of follow-ups every time they express their opinion. This type of influence is one of the consequences of change and evolution of opinion. The first researcher, who started in this area, proposes to create a simple and intuitive model of interactions between people and information. This model was proposed by psychologist French in [36].

In the past, and because of the limitations in communication, conventional methods do not tackle the dynamics of opinion on large groups of people. Nowadays, in social networks people want to talk, share, post, tweet, exchange information and opinions with many others agents. Thanks to the development of the Internet people, it can be possible to exchange information and ideas more freely and frequently. Therefore, a calculation model will be a necessary element in the process of the dynamics of opinion. Building a model of opinion dynamics that is consistent with existing social theories and capable of handling large problem scalability is challenging. For reasons of simplification and extraction of the core parts of a difficult problem, Acemoglu and Ozdaglar [21] proposed to divide the formation of opinion into three key components: Prior opinions, information source and method of information processing.

(1) Prior opinions:
Any model of opinion formation has to start with some types of initial opinions (priors), determined by the knowledge base, which can be personal or public.

(2) Information sources:
One critical component of opinion dynamics is Information sources. By interacting with many fields of information acquisition, a person can update his knowledge basis and his/her opinion according to the new information that he receives. This might come from observing others actions and experiences, or from communication with others. An individual is much more likely to learn from and communicate with a certain category of people more than others: family members, friends, coworkers and other peers are much more likely to influence an individual by communicating with him or by providing information on the basis of their own experiences.

(3) Method of information processing:
It is based on showing how the individual will combine her initial opinions and the information he receives. There are models that similarly combine priors and information to

yield a new opinion while other models use the Bayes rule. The first models named non Bayesian models and the second models named Bayesian models Various factors are responsible for the way in which agents adopt the opinions of others. These include the degree of the kind of relationship closeness to that person and the matter of opinion similarity [39,40].

In the opinion dynamics model, each agent has an opinion described by a variable which can be continuous or discrete and it change in time. It is for this particular reason that we present, next, the models were the opinions are considered discrete than, the models were the opinions are considered continuous.

Let $G = (V, E)$ be an undirected (bidirectional) network with self-loops, where V and E are the sets of all nodes and links in the network, respectively. For a node $v \in V$, let $\Gamma(v)$ denote the set of neighbors of v in G, that is,

$$\Gamma(v) = u \in V; (u, v) \in E \tag{1}$$

3.2. Discrete opinion dynamics models

Discrete models assume that the opinions are discrete, that is the number of permitted values are finite. There are as many discrete opinion dynamics models, where binary values are selected to represent yes or no. Among that, we present opinion dynamics models which deal mostly with discrete states for agents and simple rules of interaction between them.

The voter model [12,13] is the first sociophysical approach to opinion dynamics. It was soon adapted to model an electoral competition between two candidates. Suppose that each node, in a connected graph represents a "voter". The connections indicate that there is some form of interaction between a pair of nodes and a voter's opinion at any given time can take one of two values, 0 or 1. Of course, the opinions of any given voter on some issue changes at random times under the influence of opinions of his neighbors. The elementary dynamical step consists to select a node at random, then, to choose one of its nearest neighbors including itself (since it connected by a direct link), also at random. Then, assign to both voter's opinions the same value if they have different values, and leaving them unchanged otherwise.

Another model that has received more attention is the Ising model [11]. It was used to describe the behavior of laborers on strike and the emergence of consensus [41,42]. Also, the Sznajd model which takes two agents to convince their neighbors of the correctness of their opinion [43,44]. Sznajd model based on the thesis of three people spreading reports of a tiger make you believe there is one around. At each time step, two agents are random select, agent i and agent $i+1$, to influence their nearest neighbors opinion, agent $i-1$ and agent $i+2$. Suppose that x_i is the opinion of agent i, the dynamic rules of this model are the following:

- If $x_i = x_{i+1}$, then they will affect their neighbors agent $i-1$ and $i+1$, $x_{i-1} = x_i = x_{i+1} = x_{i+2}$;
- If $x_i <> x_{i+1}$, then they can only affect each others neighbor, $x_{i-1} = x_{i+1}; x_i = x_{i+2}$

These models always lead to consensus when all the agents come to share the same opinion, which is rare in many social scenarios. A modified version of the voter model has been proposed to overcome this problem. It is called label propagation [45] where the node does not adopt the opinion of a single neighbor, but it adopts the majority opinion from among its neighbors. This allows limiting the arrival of consensus. This model leads to polarization instead of consensus. Serge Galam and his collaborators presented a bottom-up hierarchical voting model [46,47] . Agents are distributed randomly in a series of groups with finite size r, which constitute the hierarchy bottom. It is the level (0) of the hierar-

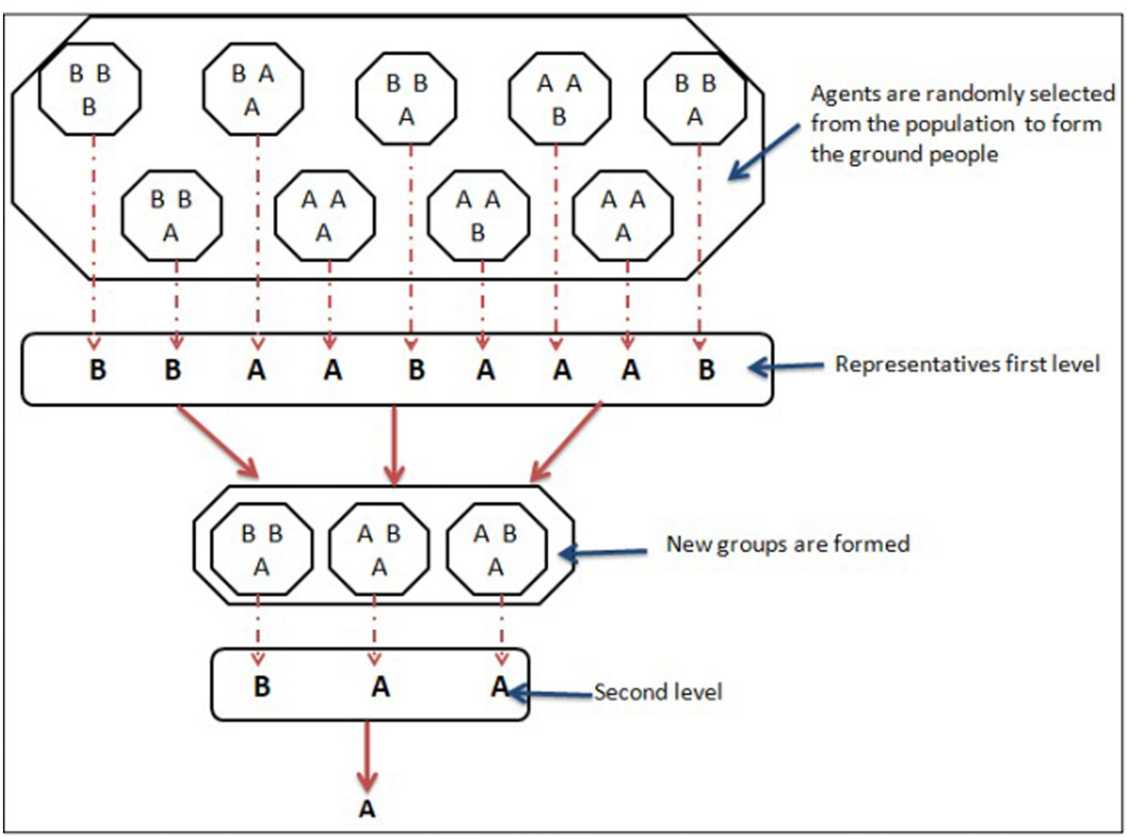

Fig. 3.2.1. Illustration of Galams local majority rule model.

chy. The majority rule is applied in each group so that the majority opinion is voted as the representative of each group and everyone in the group to replace their opinion with the majority opinion. This is the second level of the hierarchy. Likewise, the groups at the second level are recombined and the representatives are voted under the same rule. This is the third level of the hierarchy. This recombination process continues until the final representative(s) are obtained. Fig. 3.2.1 represents the Galams majority voting model where A and B are the choices of the agents' opinions.

Because, the forces that lead to consensus prevent the emergence of persistent disagreements, one solution proposes to incorporate stubborn agents [48]. Or another solution, [39] proposes to an agent the notion of adopting the neighbors opinion based on the similarity with his own. These models lead to polarization instead of consensus. The polarization can refer to such divergence within the group, that is, there's a certain divergence between opinions.

The discrete opinion dynamics models have been successfully studied when analyzing cases in which individuals are confronted with a limited number of options (a political election, for example) where one is forced to choose amongst a finite set of parties.

3.3. Continuous opinion dynamics models

In the second groups, we present the models where the opinions are considered continuous. In fact, continuous models are more suitable to analyze some issues (legalizing abortion, for example) because opinions can vary continuously from completely against to in complete agreement.

We will denote by x_i^t the number representing the opinion on a given topic that individual i has at time-step t. As all individuals interact and discuss together, the opinion will be more significative if it is represented as a real variable in a finite interval. So, we talk $x_i^t \in [0, 1]$. In all continuous opinion dynamics model, the values x_i^0 are randomly distributed in the interval [0, 1] for all $i = 1, \ldots, N$ with N is the number individuals in the population. Then, the dynamics is introd their opinions. The last decade has witnessed an increasing interest for continuous opinion dynamics systems [31,49–56]. The concept of bounded confidence was used in most models of this group. In these models, an individual can influence the opinion of his neighbor if these two opinions differ less than some given value. People holding too distant opinions on an issue will simply keep their original opinions and will ignore each other opinion. Among the models that implement the concept of bounded confidence we find:

A continuous model introduced by Deffuant and his collaborators [54] has received much attention. This model has been developed in a project about improving agri-environmental policies in the European Union and it has inspired a large number of extensions and modifications [57–60]...

Deffuant and his collaborators present a model, called DW model, in which an agent readjusts his opinion with other agent when the difference between his opinion and one of his neighbor's is smaller than a threshold. If the difference between his opinion and that of her neighbor exceeds her tolerance threshold, he will be unwilling to listen to her neighbor on the issue, and no change in one's opinion value will occur. we will denote by “d” the tolerance threshold which runs from 0 to 1. Suppose that, at time-step t, two individuals, say i and j, meet in random encounters in a given connectivity network. If their opinions satisfy $|x_i^t - x_j^t| < d$, so that they are close enough, they are adjusted according to:

$$\begin{aligned} x_i^{t+1} &= x_i^t + \mu(x_j^t - x_i^t) \\ x_j^{t+1} &= x_j^t + \mu(x_i^t - x_j^t) \end{aligned} \tag{2}$$

Where the convergence parameter μ is restricted to the interval (0, 0.5] during the simulations.

Deffuant el al, [51], propose that some agents (the extremists) do not have the same level of uncertainty as the no-extremists agents. So, they attribute another value. We denote by extremist agents those who have the most positive or negative opinions. A modification of the DW model which has received a lot of attention also, has been proposed by Hegselmann and his colleagues [31]. These researchers have developed a model (HK model) in which, in every iteration, agents take into account the opinion of all neighbors instead of one agent. The number of agents can interact with others at some point makes the basic difference between HK model and DW model. In the HK model, the opinions of agents influence each other when they are smaller than a given confidence level, that is, agent i only takes agent j into account if the difference of their opinions $|x_j(t) - x_i(t)|$ is less than a certain confidence level of agent i. Specifically, agent i update his opinion according to the following rule:

$$x_i(t+1) = a_{i1}x_1(t) + a_{i2}x_2(t) + \ldots + a_{iN}x_N(t) \tag{3}$$

Where a_{ij} denotes the weight of the neighboring agent j which can influence the opinion of agent i and which is calculated by

$$a_{ij} = \frac{1}{|I(i, x(t)|} \tag{4}$$

We denote by $|I(i, x(t)|$ the number neighbors of agent i whom they have an opinion difference not greater than the confidence level. Lanchier in [50], showed that with small threshold values lead to polarization with high probability. In other words the population splits into a finite number of groups such that all individuals in one group have exactly the same opinion. In contrast, with large threshold values lead to consensus with high probability.

The difference in opinion on a debated issue is indeed playing a crucial role. A mathematical model proposed by Guazzini et al. [61] present a DW model extension and it has incorporated the Cognitive Dissonance theory by Leon Festinger, in order to detail the mechanics of the evolution of the agents' parameters after the encounters.

At each time step t, two agents i and j, are selected according to a strategy based on their social distance, defined as

$$d_{ij}(t) = \Delta x_{ij}(t) \;\; (1 - \alpha_{ij(t)}(t)) \tag{5}$$

Where $\alpha_{ij}(t)$ represents the strength of the relation between the subjects i and j at the time t, $x_i(t)$ is the opinion of agent at the time t and $\Delta x_{ij}(t) = x_i(t) - x_j(t)$ represents the difference or distance in opinion or psychological State in a certain moment t. The two agents i and j meet and their opinions converge if and only if their affinity level is larger than the affinity threshold ($\alpha_{ij}(t) > \alpha_c(t)$). In this model the affinity is dynamically coupled to the opinion, and consequently it is translated into the concept of social distance. Agent interact updating opinion and affinity values according to the Eqs. (6) and (7):

$$x_i(t+1) = x_i(t) + \mu \Delta x_{ij}(t) \;\; \frac{tanh \;\; (\beta(\alpha_{ij}(t) - \alpha_c(t)))}{2} \tag{6}$$

The affinity level can increase or decrease according to the difference in opinion. The affinity threshold represents the average tolerance toward the others.

$$\alpha_{ij}(t) + 1 = \alpha_{ij}(t) + (1 - \alpha_{ij}(t)) \;\; \alpha_{ij}(t) \;\; tanh \;\; (\beta \;\; (\Delta x_c - \Delta x_{ij}^t)) \tag{7}$$

The threshold, Δx_c, is a constant parameter and it represents a sort of cultural related openness of mind. The role of this threshold is fundamental to mimic the effect of the cognitive dissonance on the evolution of the group. The convergence parameters, μ and β, set respectively to the values 0.5 and 1000. There are other interesting models that they incorporated the Cognitive Dissonance theory such as the work of Bagnoli et al. [62], Carletti et al. [63], Guazzini et al. [35].

Another variation of opinion dynamics models, proposes to define the neighbor sets in a different way. These works, [64,65], have extended the model which utilized the idea of confidence level. The neighbors of an individual in [64] are defined to be those individuals whose influence range contains this individual. Unlike to DW model, which proposes that the opinion of agent$i$ is affected by the opinion agent j only if their opinions satisfy $|x_i^t - x_j^t| < d$, in this model [64] the opinion of agent i is affected by the opinion agent j only if the difference between these two opinions is less than the influence bound of agent j. The works of [65] bring exogenous factors, such as the influence of media, into the model and each individual updates her opinion via the opinions of the population inside the individuals confidence range and the information from an exogenous input in that range.

The absolute consensus or the polarization is the convergence characteristics of bounded confidence dynamics. It has been the goal of a large body of work (see [66,67] and references). But other papers state that in many social settings consensus may never be attained, such as [68,69]. So, they characterize the cost of disagreements in a game theoretic setting.

A recent paper [34] proposes a hybrid model between discrete and continuous. It proposes a biased voter model, which is a unification of the voter model with flocking. Each agent is driven by a mix of three forces:

- Stubbornness users do not change opinion.
- Compromising users update their opinion with a convex combination of their initial opinion and their neighbors opinions.
- Conforming users simply take on the opinions of their neighbors, ignoring their own.

A preliminary data study is used to justify the tension between these forces, and two results have been obtained: On the one hand, even if an individual agent changes opinion continually, the relative frequencies of different opinions will converge. On the other hand, consensus still happens under certain conditions.

4. Modeling dynamics opinion with Non-Bayesian models

Most social decisions, ranging from product and occupational choices to voting and political behavior, rely on the information agents gather through communication with friends, neighbors, and co-workers as well as information obtained from news sources and prominent web pages. Among the research on belief and opinion dynamics in social networks, we find the non-Bayesian models of communication learning. These models focus on two implications. First, the communication that presents the sources of information. Individuals engage in communication with their neighbors in order to learn from their experiences. Second, the form of learning that is non-Bayesian. However, people start to update their opinions by specifying simple rules of thumb which always linearly combine their personal experience and the views of their neighbors.

The work provided by French [36] allows him to become the pioneer of this model family. Actually, individuals, in his/her works, form their new opinions by averaging other peoples opinions with whom they have directly communicated (at a time from their social neighborhood: friends, coworkers or peers). After that, several non-Bayesian approaches were developed around the same idea: the opinions evolve dynamically over time as a function of their neighbors opinions. For instance, DeGroot Model [15] proposes to replace the simple average function in French [36] with a weighted mean in order to assess opinion pooling of a dialogue among experts. The DeGroot model and their extensions have been studied extensively in the cooperative control literature as natural algorithms for achieving cooperative behavior with local information in networked-systems (see [70–74]).

In this section, we present the DeGroot model, which is a simple model of belief and consensus formation over social networks. We will see, however, that the specific assumptions it makes on how beliefs are updated may have certain non-desirable implications. Motivated by this, we will quote the different enhancements proposed to avoid some of these limitations.

Notation: Social structure is an important aspect of social dynamics which governs the interaction between two individuals, among individuals, and the way of interactions. So, we consider by $G = (N, E)$ a directed graph with a node set N and an edge set E. Each node of a graph represent an agent and the edges define the neighborhood set of an individual with whom it interacts. Consider a community of n agents , $N = (1,...,n)$, interacting in a social network. We define the set of neighbors of agent i, $i \in N$, as $N_i = j \in N{:}(i, j) \in E$ which means: If the agent j belongs to the community$j \in N$, and there exists a link between i and j defined in edge set E , $N{:}(i, j) \in E$, then j is a neighbor of i. We will denote by $x_i(t)$ the number representing the opinion on a given topic that individual i has at time-step t. Each agent$i$ starts with an initial belief (prejudices) about an underlying state, which we denote by $x_i(0) \in R$. Let $X = (x_1, x_n)^T$ is the vector of beliefs that contains all scalar opinions $x_i \in R$ of all agents and W is the matrix of interpersonal influences. W is a stochastic weight matrix that contains all trusts values of all agents (each row sum of W equals 1). The nonnegative inputs of the matrix W are (w_{ii}) and $(w_{ij}, \forall j \neq i)$, respectively correspond to the self-weight of agent$i$ and the weight allocates to the displayed opinions of others (the trust that an agent i places on agent j).

4.1. DeGroot model

In the DeGroot Model [15] agents exchange information about their beliefs with their neighbors. At each discrete time instance, $t \geq 0$, agents update their beliefs to a convex combination of their current beliefs and the beliefs of their neighbors. So, agent i updated his belief according to the relation:

$$x_i(t+1) = \sum_{j=1}^{n} w_{ij} x_j(t) \tag{8}$$

We denote by $w_{ij} = 0$ the lack of trust between agent i and agent j, or equivalently, agent "i" does not get direct information from agent "j" regarding his belief. In a general way, the evolution of the beliefs can be expressed as

$$X(t+1) = W \times X(t) \quad For \quad all \quad t \geq 0 \tag{9}$$

We can see that this rule is quite reasonable, if we think of a one-step update. At the first moment, an individual will update her beliefs to be closer to those of the agents whom he trusts. However, when this update rule is applied dynamically, it may not be as compelling. For example, agent i will always update his opinion after each communication with agent j. But, when agent j does not update his beliefs (Agent j named stubborn agent), agent i will keep on updating its own information and creating an extreme form of duplication of information. We can say, DeGroot update rule might be too myopic, when individuals interact in the same manner in each period.

An attractive feature in the DeGroot Model, besides its simplicity and intuitiveness, is that the analysis of consensus is straightforward as well. As the weighting matrix W in DeGroot is stochastic and static, the sufficient conditions to ensure the convergence can be easily derived from Markov Chain Theory. If matrix W is irreducible and aperiodic then, agent's beliefs reach a consensus in the limit.

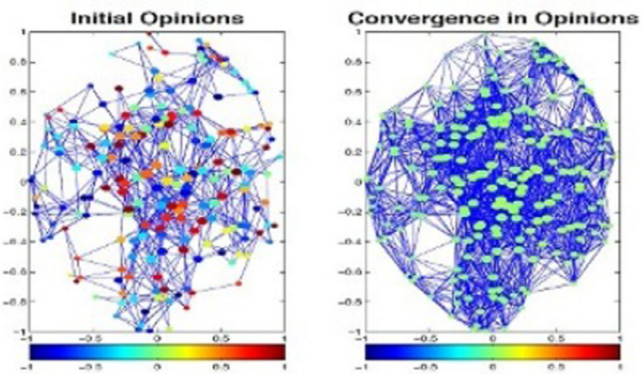

Fig. 4.1.1. Each color in the nodes represents a different opinion value. Reach a consensus after 300 interactions, using DeGroot Model [75].

- Aperiodicity
  The aperiodicities are guaranteed in most of the literature in this area. If some or all agents assign positive weight to their own belief, w_{ii}, in the update relation, the aperiodicity is guaranteed. Or equivalent, the matrix W is aperiodic if the greatest common divisor of the lengths of its simple cycles is 1.
- Irreducibility
  The irreducibility of the Markov chain is equivalent to the underlying social network being strongly connected. That is to say, there is a directed path from every node to every other node. The focus of this model is on consensus and how to reach it.

Consequently, the forces that lead to consensus preclude the emergence of persistent disagreements. We can note that DeGroot model not concerned with establishing how close the agents will come to the right decision, but rather if they all will eventually agree.

In Fig. 4.1.1, the initial opinion of each agent is represented on the left side and the final opinions are represented on the right side. One can immediately note that in the DeGroot model analysis all nodes have the same color after 300 interactions. This is illustrating the imminent convergence in opinion.

The disagreements are pervasive in our life and in many economic, political and social agendas such as Tax increases, death penalty (should it be legalized or not), unemployment benefits... So, the importance of modelling disagreement beside consensus was pointed for especially by many researchers. Many factors combine to make the disagreement between us a natural and logical thing such as the circumstances of agents, their environments, their levels of consciousness and culture, in addition to the differences in age, life experiences, the temperaments and the psychologists. So, when we tackle certain issues, all of us have a point of view, an opinion or impression which might correspond to an agreement with the other, or may be different with them.

The absence of this feature, (disagreement) can impose limits on the extent of misinformation. We will quote, now, the proposed solutions to overcome these limits which is particularly: the duplication of information and the disagreement/misinformation between agents.

4.2. Duplication of information based models

To overcome this problem, Acemoglu et al. [76] propose to define three possibilities in order to update the beliefs of agents. Following a meeting between i and j, there is a potential exchange of information and the agents update their beliefs according to one of the following three possibilities:

(1) Agents i and$j$ reach pairwise consensus and update the beliefs according to

$$x_i(t) = x_j(t) = \frac{x_i(t) + x_j(t)}{2} \tag{10}$$

(2) Agent j influences agent i, in which case for some $\epsilon \in (0, 1/2]$, but their beliefs do not change

$$\begin{aligned} x_i(t+1) &= \epsilon x_i(t) + (1-\epsilon)x_j(t) \\ x_j(t+1) &= x_j(t) \end{aligned} \tag{11}$$

(3) Agents i and j do not agree and stick to their beliefs.

$$\begin{aligned} x_j(t+1) &= x_j(t) \\ x_i(t+1) &= x_i(t) \end{aligned} \tag{12}$$

With these three propositions, the problem of duplication of information, highlighted in the context of the DeGroot model, is almost solved (it is still present but is now less severe). In fact, if we ignore the second possibility (2) and we focus on two agents who have communicated in the recent past, then as they communicated again, they cannot exchange any information since the consensus has already been reached. Thus, one more round of updating is either inconsequential or would not run into the problem that there is a state of duplication of information. There will be communication between these two agents, only if there is relevant information to be exchanged. That is to say, if one of these two agents has met with somebody else and has updated his beliefs in the process. Like this, the problem of duplication of information is resolved, but there will still be some amount of replication of information.

Another solution has been proposed in [6] to account for the possible duplication of information. The idea is to label the information sent between agents, to let not confusing between new information and previously communicated information. This approach presents a Bayesian model of communication learning. For this moment, we consider n agents situated in a communication network represented by a directed graph $G^n = (N^n, E^n)$, where $N^n = 1, \dots, n$ is the set of agents and E^n is the set of directed edges with which agents are linked.

Agent j forms beliefs about the state of the world from her private signal s_j, as well as information she obtains from other agents through a given communication network G^n. That is, agent i can send information/ message from j (or equivalently j receives information from i) if there is an edge from i to j in graph G^n, that is, $(i, j)/inE^n$. The following mapping defines this communication:

$$m^n_{ij,t} = \mathcal{I}^n_{i,t} \rightarrow M^n_{ij,t} for(i,j) \in E \tag{13}$$

Let $I^n_{i,t}$ denote the information set of agent i at time$t$ and $\mathcal{I}^n_{i,t}$ denote the set of all possible information sets and $M^n_{ij,t}$ denotes the set of messages that agent i can send to agent j at time t. This mapping makes it clear that the messages that i can send to j could in principle depend on the information set of agent i as well as the identity of agent j . if agent i sharing all her information with agent j, that is, the cardinality of $M^n_{ij,t}$ is no less than that of $I^n_{i,t}$ then the mechanical duplication of information can be avoided. As well, an agent can communicate with the neighbors of her neighbors in two time periods.

As the Fig. 4.2.1 shows, at t=0, each agent forms beliefs about the state of the world from her private signal. So, the information set of agent 1 at time 0 is only his private signal s_1.

At time t=1, all agents communicate directly with his immediate neighbors and they obtain from each agents the message sent in the form of a signal. Thus, the information set of each agent is composed of his private signal and the received information from

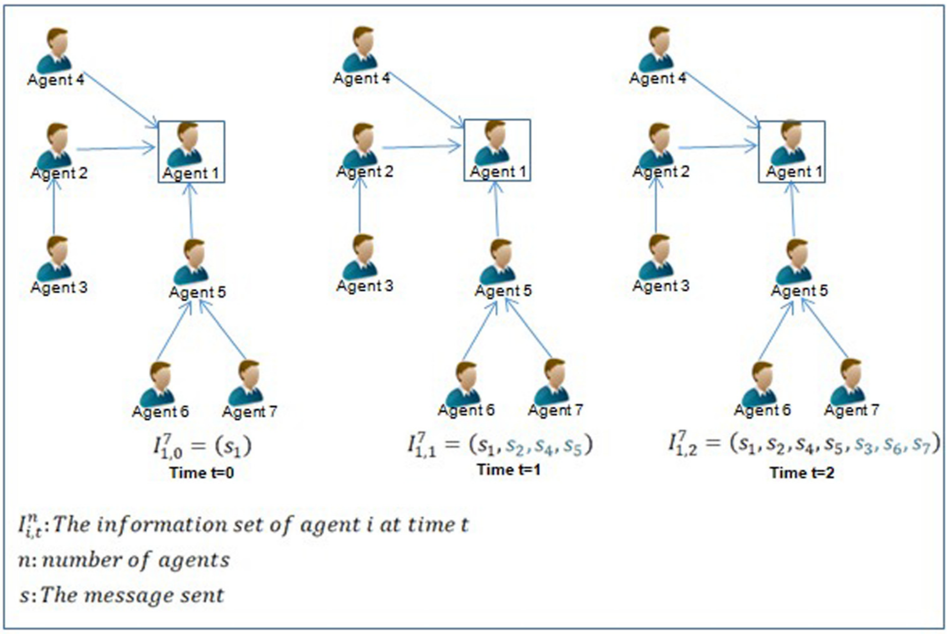

Fig. 4.2.1. The information set of Agent 1 under truthful communication.

his neighbors. As an example, the information set of agent 1 is s_1, s_2, s_4, s_5, the information set of agent 2 is s_2, s_3.

At t=2, all agents communicate directly with her immediate neighbors too. Agent 1 can communicate again with agent 2 if he has updated his beliefs with somebody (agent 3 at t=1), or else, when agent 2 repeats his original signal, his message will not be recorded as an additional piece of information by agent 1. This is exactly the case of agent 4 that he has not updated his beliefs so, his message will not be recorded as an additional piece of information, since given the size of the message space $M^n_{ij,t}$, each piece of information is tagged. This ensures that the mechanical duplication of information is avoided. Thus, an agent can communicate (indirectly) with a much larger set of agents than just her immediate neighbors.

4.3. Misinformation and disagreement integration in the communication models

The advent of social networking has made on effective and important and incredible platform for information sharing and communication. But, the lines between what is true and what is false are getting more blurred every day.As the usage of social networks increased, the misinformation also increased many fold, which may lead to the pernicious influence on individuals or society. We define the misinformation as the false or inaccurate information, especially that which is deliberately intended to deceive. Also, when the misinformation spreads to some parts of the society, individuals that communicate and share the same sources of information might nonetheless disagree significantly even in the very long run. Disagreement among individuals in a society is the norm and the agreement is the rare exception. Notably, such disagreement is not a consequence of lack of communication, but it remains even as individuals communicate and sometimes change their opinions following an influence. It would be particularly important to take account of of disagreement.

It has been shown that in a strongly connected network, some of existing models of communication and learning, based on Bayesian or non-Bayesian updating mechanisms (e.g.,Golub, Benjamin and Jackson, Matthew [74] Acemoglu, Dahleh, Lobel and Ozdaglar [4], Acemoglu, Ozdaglar and ParandehGheibi [76]) are unable to explain persistent disagreements because consensus are typically achieved. Consequently, the forces that lead to consensus preclude the emergence of persistent disagreements, since all agents converge to the same opinions. This feature might put limits on the extent of misinformation.

It would be particularly important to take into consideration how such long-run disagreement is possible in opinion dynamics models. So, in order to incorporate the disagreement, we will present two types of solutions.

The first solution is provided by models that incorporate a form of homophily mechanism in communication, whereby individuals are more likely to exchange opinions or communicate with others that have similar beliefs, and fail to interact with agents whose beliefs differ from theirs by more than some given confidence threshold. The presence of various types of agents (regular, forceful and stubborn agents) is the second solution proposed in several approaches in order to lead the persistent disagreements. we refer by Regular agents as the agents who exchange information with their neighbors (when they meet); Forceful agents who influence others disproportionately and Stubborn agents are similar to the forceful agents, but never update their opinions and continually influence those of the rest of the society.

4.3.1. Solution 1: Use of bounded confidence

To add the aspect of disagreement, one notable exception is provided by models that incorporate a form of homophily mechanism in communication, whereby individuals are more likely to exchange opinions or communicate with individuals whose not have significantly different opinions.

A situation referred as bounded confidence (BC) is when agents interact with each other only if their opinions are sufficiently close to each other. In contrast, the absence of bounded confidence allows each agent can interact with every other agent, regardless of their opinions. The bounded confidence model was first proposed by Axelrod [39] in the discrete opinion dynamics setting, and then Deffuant and Weisbuch (DW model) [54] and by Hegselmann and Krause (HK model)[31] , in the continuous opinion dynamics framework (Section 3.3).

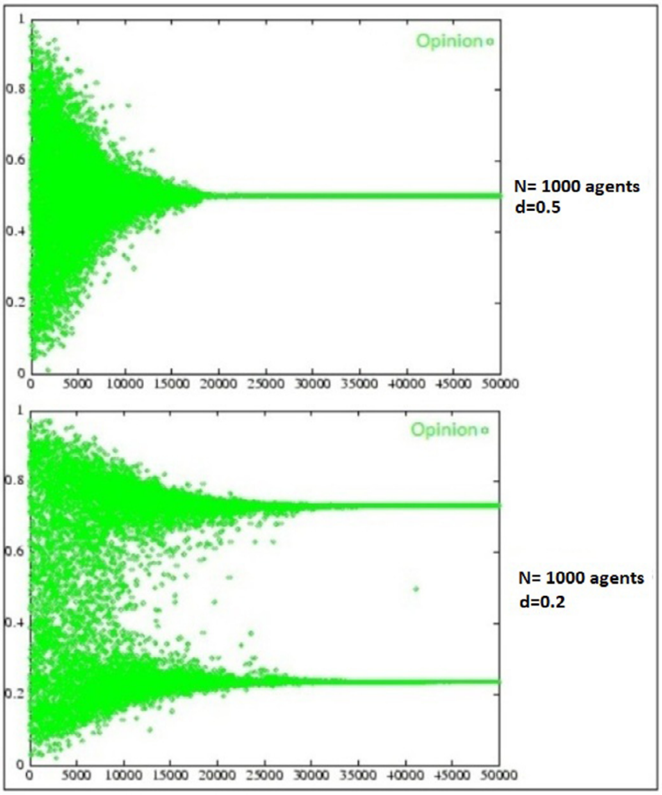

Fig. 4.3.1.1. Time chart of opinions [54].

These models try to make an assessment of the ideal conditions that typically lead to the emergence of different asymptotic opinion clusters. For example, lead an interacting group of agents to polarisation (two clusters), fragmentation (more clusters)and in some cases to consensus. So, each agent in each cluster has an opinion totally different in relation to another agent belonging to the other cluster. In this case, the situation of desaccort between individuals can be present. Fig. 4.3.1.1 display the convergence of opinions among a population of N=1000 agents for two values of the threshold d=0.5 and d=0.2. As we say in Section 3.3, initial opinions were randomly generated across a uniform distribution on [0,1]. According to Eq. (2) and (3), in each time step a random pair is chosen and agents re-adjust their opinion. Convergence of opinions is observed, but disagreement among agents is only achieved for the smallest value of d.

DW model [54] and HK model [31] used only one confidence threshold which corresponds to some openness character or, with another interpretation, uncertainty character. That is to say, when agents have some initial opinions with some degree of uncertainty and they would adjust their opinion without take other opinions outside their uncertainty range. Improvements to these models have been assuming that uncertainty as well as opinion can be modified by interactions. So, each social agent may have diverse confidence levels. Among these, we find the heterogeneous DW model [77], the heterogeneous HK model

In heterogeneous DW model [77], each agent i is characterized by his opinion $x_i \in (0, 1]$ and his uncertainty $u_i \in (0, 1]$. An opinion segment $s_i = [x_i - u_i, x_i + u_i]$ is assigned to each agent i. The way change of uncertainty in opinion dynamics is the main difference between DW model [54] and heterogeneous DW model [77].

For two agents, i and j, their opinion segments overlaps if and only if

$$h_{ij} = min(x_i + u_i, x_j + u_j) - max(x_i - u_i, x_j - u_j) > 0 \quad (14)$$

where h_{ij} is called an opinion overlap.

If $h_{ij} < u_i$, there is no influence of i on j. So there is no change in the opinion and uncertainty of agent j. Else, if $h_{ij} > u_i$, then the opinion and uncertainty of the passive agent are updated according to the following rule:

$$x_j = x_j + \mu\left(\frac{h_{ij}}{u_i} - 1\right)(x_i - x_j) \quad (15)$$

$u_j = u_j + \mu(\frac{h_{ij}}{u_i} - 1)(u_i - u_j)$ where μ is a constant parameter which amplitude controls the speed of the dynamics; $\mu \in [0, 1]$.

According to Eq. (12), the change in opinion x_j of agent j under the influence of agent i is proportional to the overlap between both segments, divided by the uncertainty of the influencing segment. In this model, both the opinion and uncertainty of passive agent (with low uncertainty) get closer to those of the active agent. In other words, if there is an interaction between two agents, then the active agent convinces the passive one, and there are no possibilities of disagreement on within a group but there is disagreement between groups.

The simulation results in [78], show that, in HK model, when all agents, in a given society have the same confidence level, the larger the confidence, the higher the probability of reaching consensus. Suppose there is a social group with N=200 agents. According to (4), each agent updated his opinion under confidence level. Fig. 4.3.1.2 shows three patterns of the final opinion with three homogeneous confidence levels. The first, second, and third figure denote the opinions of agents with low, middle, and high confidence levels, respectively. The x-axis represents simulation time step and the y-axis represents the evolutions of opinions.

From the Fig. 4.3.1.2, it can be seen that if confidence level is very small the final opinions are fragments and from 39 distinct final opinions. When the homogeneous confidence level increases to 0.15, the number of the final opinions decreases to 2 and a polarization pattern emerges. Finally, if confidence level is very big, the 200 agents easily reach consensus.

To ensure the disagreement between the opinion of agents, a similar form with the HK model has been proposed in [78]. This model proposes to differentiate agents into some opinion subgroups according to their different confidence levels. For that, it was called a heterogeneous HK model or multi-level opinion model.

Consider a social system with N agents which have m heterogeneous confidence levels. The multi-agent group can be divided into m opinion subgroups, which have confidence levels for Q^k, respectively k= 1,2,3,..., m. Agents with confidence level Q^k belong to the k^{th} subgroup. During the evolution of the collective opinions, at each time step, each agent i firstly searches his neighbors according to his own confidence level. In other words, the neighbors of agent i who they have a opinion difference less than the confidence level Q^k. Then, a agent i updates his opinion according to the following rule:

$$x_i(t+1) = \sum_{j \in I_k(i,x(t))} a_{ij} x_j(t) \quad (16)$$

The set $I_k(i, x(t)) = \{j| \ ||x_j(t) - x_i(t)|| < Q^k\}$ is the neighbor set of agent i with confidence level less than Q^k and a_{ij} is the weight associated with these neighbors and it is calculated according to (5) by replacing $I(i, x(t))$ with $I_k(i, x(t))$.

G. Kou et al. [78] show that, the possibility of the fragmentation of collective opinions, similarly the disagreement between the opinion of agents, will increase as the fraction of agents with small confidence level increases. From the Fig. 4.3.1.3, it is clear that when the fraction of close-minded agents are small, the number of final opinions are also small.

In these models, the belief dynamics typically lead to the disagreement between the different opinion clusters, but fail to

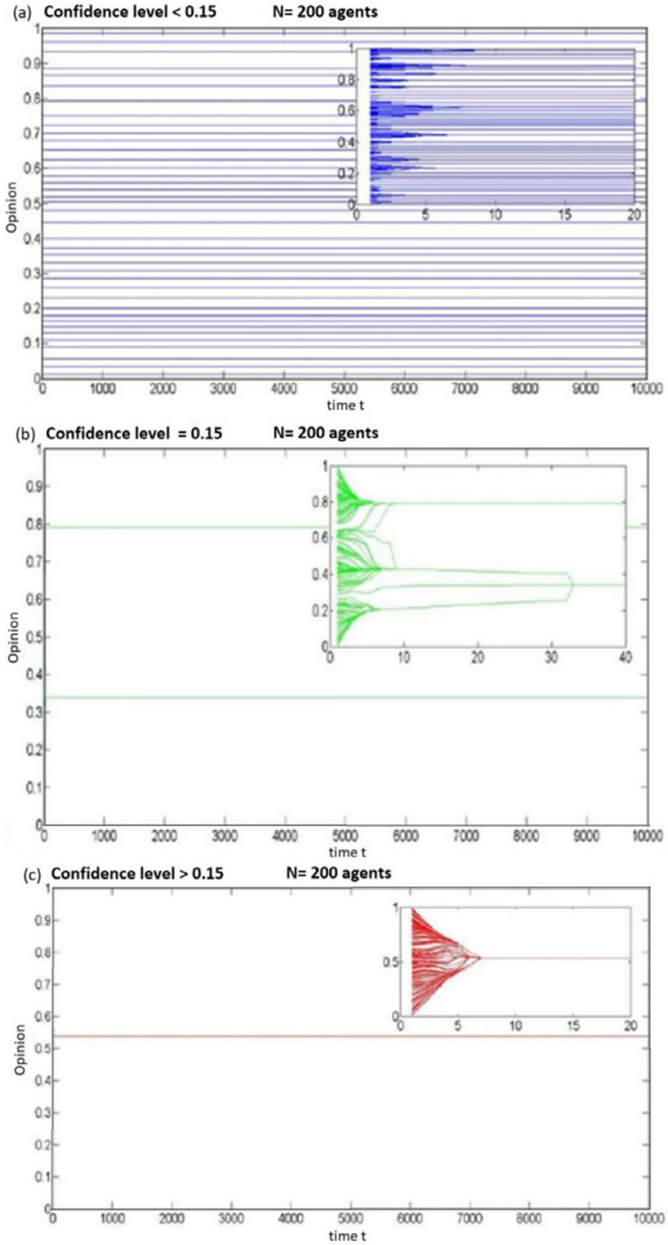

Fig. 4.3.1.2. Plot of opinion evolution of HK model [78].

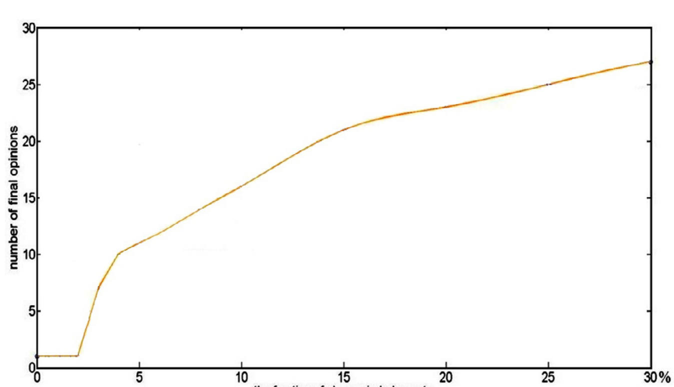

Fig. 4.3.1.3. The relationship between the number of final opinions and the fraction of agents with small confidence level [78].

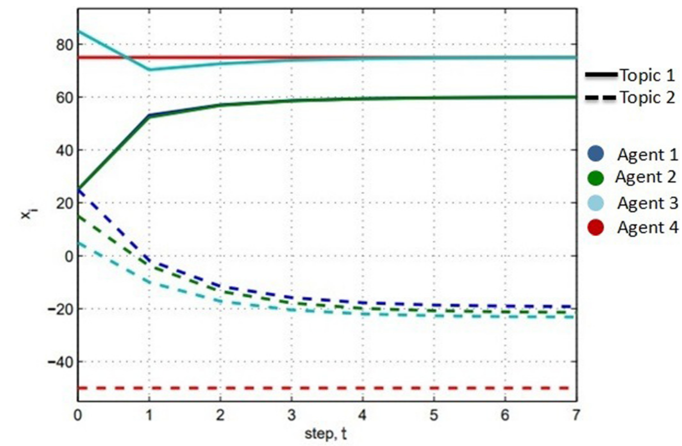

Fig. 4.3.2.1. The convergence of the F-J model [18].

explain the role of influential agents in the opinion formation process. We can say that, the presence of stubborn agents with competing opinions is the second solution that leads to disagreement among agents, and it can also explain the role of influential agents.

4.3.2. Solution 2: Distinction between agents

–The Friedkin–Johnsen (FJ) model:

For modelling disagreement beside consensus on DeGroot model, an attractive generalization, employing such heterogeneity, was suggested by Friedkin and Johnsen [14,79], henceforth referred to as the Friedkin–Johnsen (FJ) model. Unlike the DeGroot scheme, where each actor updates his opinion according to his own and as well as his neighbors opinions, in the model FJ some level of stubbornness has been added to each agent. The latter is supposed to adhere to its initial opinion or prejudice to some degree. In other words, the agent is stubborn never forgetting their prejudices, and thus remain persistently influenced by exogenous conditions under which those prejudices were formed [14,80].

The Friedkin–Johnsen (FJ) model of opinions evolution is determined by two matrices, that is a row-stochastic matrix of interpersonal influences$W \in R^{(n \times n)}$ and a diagonal matrix of actors susceptibilities to neighbors opinions $I_n \geq \Lambda \geq 0$ (we follow the notations from [81,82]). In fact, to determine the updated opinion $x_i(t+1)$, on each stage of opinion iteration the agent i calculates the weighted average of its own opinions and opinions of its neighbors $\Sigma_j w_{ij} x_j(t)$; along with the agents prejudice u_i.

At any time $t \geq 0$, agent$i$ updates his belief according to the relation:

$$x_i(t+1) = \lambda_{ii} \sum_{j=1}^{n} w_{ij} x_j(t) + (1 - \lambda_{ii}) u_i \;\; where \;\; u_i = x_i(0) \qquad (17)$$

The self-weight w_{ii} plays a special role, considered to be a measure of stubbornness or closure of the i^{th} agent to interpersonal influence. If $w_{ii} = 1$ and w_{ij}=0, $\forall j < > i$, then he is maximally stubborn and completely ignores opinions of his neighbors. Conversely, if $w_{ii} = 0$ (and thus his susceptibility is maximal $\lambda_{ii} = 1$), then the agent is completely open to interpersonal influence, attaches no weight to his own opinion (and thus forgets its initial conditions), relying fully on others opinions. The susceptibility of the i^{th} agent $\lambda_{ii} = 1 - w_{ii}$ varies between 0 and 1, where the extreme values correspond respectively to maximally stubborn and open-minded agents. The Fig. 4.3.2.1 shows the evolution of beliefs of 4 agents on 2 different subjects. Full and dotted lines of the same color correspond to the same agent on two different subjects. Three agents are open-minded people, so, theirs susceptibility value is higher and lower strictly to 0 and 1, respectively. The fourth agent (red curve) never changed his opinion, he is maximally stubborn and his susceptibility value equals to 1. We can notice that the three agents are completely open to interpersonal influence, and attach no weight (blue curve) or a very low weight to their own opinion. Their attitudes toward subject one become more positive and those toward another subject become less positive, compared to the initial values. We can conclude that stubborn agent influences others disproportionately.

–The extensions of the FJ model:

We refer by opinions of agents as the levels of certainty of belief of the truth about t one topic. The Friedkin–Johnsen model assumes that a change of belief on the truth of one topic does not affect beliefs on the truth of other topics. In revenge, the levels of certainty of belief about one topic are some mixture of that individuals certainty of beliefs about other topics. That is, if an individuals certainty of belief of the truth of one topic is altered, the alteration may propagate changes of the individuals certainties of beliefs on the truth of other topics.

In this context, a significant extension of the classical Friedkin–Johnsen model has been proposed by Sergey et all in 2017 [18] that represent the dynamics of agents opinions on two or more topics, in which those topics-specific opinions are interdependent.

Unlike too many models of opinion dynamics that its are focus on scalar opinions, this extension deals the influence that may modify opinions on several topics. This makes it natural to consider vector-valued opinions [55,83,84]; where each opinion vector is constituted by m >1 topic-specific scalar opinions.

Let $x_1(t)$, $x_n(t) \in R^m$ opinion vectors. The elements of each vector $x_i(t) = (x_i^1(t), x_i^m(t))$ stand for the opinions of the i^{th} agent on m different issues. Contrariwise to FJ model that present X as opinion vectors, in this extension [18] X represents a $n * m$ matrix of n individuals and m truth statements with truth values (true or false). Each individual has a certainty of belief in the [0,1] interval. For example, if $x_i^j = 0.5$ then agent i has maximum uncertainty on the truth value of subject j of the m topics. $x_i^j = 0$, $x_i^j = 1$ corresponds to an agent i with maximum certainty that the truth value of topic j is true, false respectively.

Two matrices were inherited by the structure of the usual FJ dynamics: The W is a $n{*}n$ matrix of social influences and the Λ is a $n * n$ diagonal matrix of agents susceptibilities. The C is, a $m * m$ matrix of interdependencies among the m truth topics. The dynamics of this n-individual belief system on m truth topics is defined by

$$x(t+1) = [(\Lambda W) \otimes C]x(t) + [(I_n - \Lambda) \otimes I_m]x(0) \tag{18}$$

Similar to (15), at any time $t \geq 0$, agent i updates his belief according to the relation:

$$x_i(t+1) = \lambda_{ii} C \sum_{j=1}^{n} w_{ij} x_j(t) + (I_n - \lambda_{ii}) u_i \;\; where \;\; u_i = x_i(0) \tag{19}$$

The crucial difference with the FJ model is the presence of additional introspective transformation, described by a constant coupling matrix $C \in R^{(m \times m)}$ called the matrix of multi-issues dependence structure (MiDS), adjusting and mixing the averaged topic-specific opinions.

To estimate C, an experiment can be performed where a group of individual communicates on interdependent issues, then they record their initial opinions. After that one of two natural types of methods, namely Finite-horizon identification procedure or Infinite-horizon identification procedure, is used for estimate the matrix C. This model [18] which was inherited from the original Friedkin–Johnsen model has received great importance and a numerical analysis of a system with two different MiDS matrices was made in [85]. This last article show how the 1992–2003 fluctuations of the U.S. populations certainties of belief on truth statements involved in the decision to invade Iraq may be understood.

Others researches were not in agreement with the simultaneous communication because in a large-scale of social network, this type of communication can hardly be implemented. Note that in a model where the communication is simultaneous, the actors communicate with all of their neighbors at each step. In contrast, an asynchronous version of the FJ model was proposed in [81,82] and it assumes that at each step there are only one pair of agents interacts. This more realistic model is based on asynchronous gossip-based communication [86,87]. Although in model previously presented [18] a comparative study has shown that the same final opinions can be reached by using an asynchronous protocol-based gossip and it is much faster compared to this last model.

In all models presented above, the self-weight of an individual is fixed and never changes.This point was a disadvantage for some researchers, so, they proposed in this work [16] to explain via a reflected appraisal mechanism the evolution of individuals self-weights, i.e., how confident an individual is about his opinions on a sequence of issues. This dynamical model [16], named DeGroot–friedkin model, contains two stages: In the first stage, individuals update their opinions for a particular issue, according to the averaging rule by DeGroot, and in the second stage, the self-confidence for the next issue is governed by the reflected appraisal mechanism. This mechanism, mathematized by [56], described in simple words the phenomenon that individuals self-appraisals on some dimension (e.g., self-confidence, self-esteem) are influenced by the appraisals of other individuals on them. For example, if ones opinion is frequently accepted by others and hence influences the entire set of opinions within the social network, he will very likely be more confident about himself in the future. In this article, [16], the self-confidence level for the next issue takes place only after the opinions of actual issue converges, which takes many or infinite number of discussions. A recent article [17] proposes to modify the DeGroot–Friedkin model to overcome this limit. The objective is to know the self-confidence level for the next issue without waiting that long. This recent article has shown that to succeed in overcoming this limit, one only needs to know the self-confidence levels of her neighbors and the interpersonal weight from her neighbors.

Motivated by the DeGroot–Friedkin model, another extended DeGroot–Friedkin model which includes stubborn individuals has been investigated in [88]. The stubbornness of an individual is introduced in the first stage of the DeGroot–Friedkin model, and the individual adheres to his initial opinion to a certain degree when updating the opinion for any particular issue.

Usually, the presence of stubborn individuals in opinion dynamics models entails long-run disagreement because different individuals are influenced by of distinct stubborn agents to varying degrees. The presence of this kind of agents has received increasing attention and it did not touch only FJ model and its variations but it was considered in a lot of others work [19,48,89]. In [19] and [48], the effects of stubborn individuals are investigated carefully in a randomized gossiping process. In [89], the opinion formation process is regarded as a local interaction game and the concept of the stubbornness of an individual regarding his initial opinion is introduced. Besides, other models propose to generalize certain aspects of the Degroot model for reproducing the persistent disagreements and the spread of misinformation. Among the variations of this model we cite: the models proposed by DeMarzo et al. [90], Ellison et al. [73], Golub and Jackson [74], and Acemoglu et al.[76], Ali et al.[91].

Acemoglu, and al. [76] deals at a first attempt with modeling the possible spread of misinformation. They proposed to make a distinction between regular and forceful agents. As noted above, this model has managed to find a solution to the problem of duplication of information using the three propositions. Also, according to the second proposition, it can be possible that some agents forcefully influence others, without being influenced to the, same degree. This presents the second advantage that enables to model some agents manipulating the beliefs of others or spreading misinformation.

This model of forceful agents proposes to separate the evolution of beliefs in the society into two components, one representing the underlying social network, on the other representing the influence structure in the society, called the influence matrix. The main result obtained in this article, [76], is that, the quantification of the extent of misinformation in the society is linked to the number and the location of forceful agents and the mixing properties of the Markov chain induced by the social network matrix. However, in this benchmark model, even though misinformation might spread, persistent disagreement is not possible. So a generalization of this model has been proposed in which both misinformation and persistent disagreement among agents can coexist [92].

5. Comparison between the different models

We below introduce a comparative table (Table 1) summarizing the different parameters used in the presented approaches, for the evolution of opinion dynamics in social networks. We note: - Initial opinion: Agents have some initial opinions and iteratively update their opinions based on their own initial opinions and the opinions of their neighbors.

- Previous opinion: Agents update their opinions based on their previous opinions (the opinion at time $t= t$-1) and the opinions of their neighbors.

- The weight allocated to the displayed opinions of others : influence and weight accorded by agent to others

- The opinion of all neighbors: Each agent obtains information from a certain number of agents in his friendly neighborhood, and adapts his opinion by taking into account the views of all his neighbors to form the new opinion and increase one's benefit.

- The opinion of a single agent: Only two agents communicate and shall exchange information between them.

- Agent's behavior: Stubbornness users, Compromising users and Conforming users.

- The confidence level : An agent takes into account only those agents whose opinions differ from his own with no more than a certain confidence level to adjust his opinion in period t +1.

- The degree of the user node in the graph: the number of neighbors

From this comparatif table (Table 1), we can notice that all approaches need the previous opinions of an agent to adjust his view in period t +1. Nevertheless, not all models take into account the initial opinion of the individual. [14,18,89], assume that any agent is adhering to his initial opinion to a certain degree, so, they use his original view, into every iteration of opinion. However, in other works, only the stubborn agents never forget their initial opinion and thus remain persistently influenced by those prejudices. These models, [34,73], only use the initial opinion of stubborn agents. In addition to this, [73] shows that when an agent updated his point of view with the most similar neighbor's view, his opinion is more likely to be closer to the initial opinion.

The concept of bounded confidence was used in many models. The idea is to consider only the opinions of neighbors that are similar, ie, the difference of opinions is no larger than some threshold. The number of agents can interact with others at some point, is the basic difference between these models.

In [54] an agent can readjust his opinion only with one other agent that has a similar opinion. Contrariwise, in [31], agents take into account the opinion of many neighbors that have similar opinions with him. In parenthesis, it may be possible, but not usually,

Table 1
Comparison of some approaches.

	Agent updated his/her opinion based on							
	Initial opinion	Previous opinion	The weight allocates to the opinion of others	The opinion of all these neighbors	The opinion of a single agent	Social behavior	The confidence level	Dgree of the node in the graph
[15]		×	×	×				
[14]	×	×	×	×		×		
[89]	×	×	×	×		×		×
[76]		×	×		×			
[54]		×			×		×	
[73]	×	×		×		×		
[72]		×	×	×				
[18]	×	×	×	×		×		
[31]		×	×	×			×	
[34]	×	×	×		×	×		×
[36]		×		×				
[12]		×			×			
[77]		×	×	×			×	

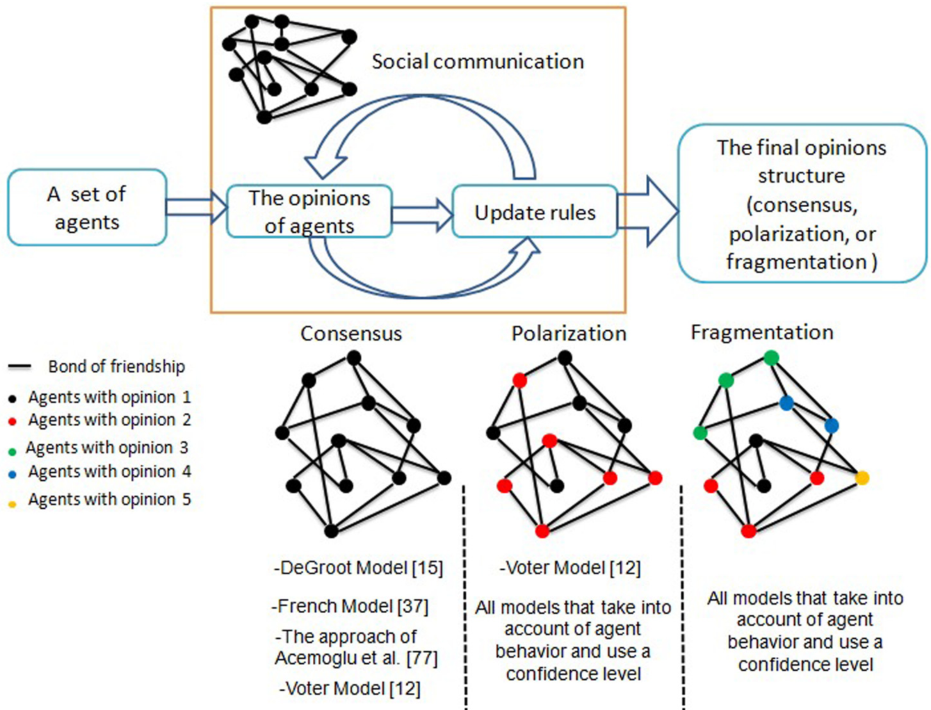

Fig. 5.1. Graphical comparison of the final opinions structure for different models.

that there are only one neighbor who has a similar opinion. For this reason we have associated with the model [31], and the same for [89], two settings: The opinion of all these neighbors and The opinion of a single agent.

If an agent receives some information of a neighboring and he believed that these bits information are useful, he will accord a large weight to his neighbor. Alternatively, the agent may wish to assign a large weight to his own opinion and small weight to others.

The weight associated with the neighbor's opinions should be chosen on the basis of the relative importance that the agent assigns to the opinions of the various members including himself. It is important to note that the weights may change with time or with the opinion. This strategy was used in many models, such as [14,15,18] , to describe the weight allocated to the displayed opinions of others. Another approach, [89], proposed to assign equal weight to the opinion of the neighbor and greater weight to one's initial opinion. This weight depends on the number of neighbors and the measure of stubbornness. On the other hand, in [34] the weight depends only on the number of neighbors.

Fig. 5.1 shows, a graphical comparison of the final opinions structure for different models. In fact, opinion dynamic is a fusion process of individual opinions, in which a group of interacting agents continuously combine their opinions on the same issue based on established update rules. In each model of opinion formation, there is a process of opinion dynamics that may lead to a consensus among the agents or to a polarization between the agents or, more general, which results in a certain fragmentation of the opinions.

An understanding of the involved process of opinion formation is hardly possible without an explicit formulation of a mathematical model. An early formulation of such a model was given by J.R.P. French in 1956, [36], in order to understand complex phenomena found empirically about groups. After that, another model was presented by M.H. DeGroot in 1974,[15], which studies in a fixed network how consensus is formed when individual opinions are updated using the average of the neighborhood. The focus of these works and others [12,76] was on consensus and how to reach it. In reality, consensus is rare in society, and opinions tend to typically be in a state of persistent disagreement, often in highly polarized states. Consequently, the forces that lead to consensus preclude the emergence of persistent disagreements, since all agents converge to the same opinions.

To achieve polarization or fragmentation, some models require users to ignore opinions that are less related to their own, in their opinion formation process. These models use a confidence threshold and that we can name them non-linear models [31,54,77,78]. These models try to make an assessment of the ideal conditions that typically lead to the emergence of two or more opinion clusters. As we showed above, the number of clusters depends on the number of agents, the confidence level. In fact, if confidence level is very small the final opinions are fragments, if confidence level is very big, the agents easily reach consensus and at certain threshold level, a polarization pattern can emerges.

In order to achieve polarization or fragmentation, other models require that some users are always being stubborn regardless of the information they receive. This idea was pointed out especially by Friedkin and Johnsen [14]. In this model, some level of stubbornness has been added to each agent. The latter is supposed to adhere to its initial opinion or prejudice to some degree. After that a significant extensions of this classical Friedkin–Johnsen model has been proposed such as [18,34,73,89].

6. Discussion and future direction

We presented in the preceding sections some approaches of modeling opinion dynamics. Now, we are intending to conduct a discussion that we point out both the strengths and weakness of existing approaches.

For that, we focus in four main criteria that we allow us to carry out a discussion: (i) How opinion is presented, (ii) How some approaches take into account the possible duplication of information, (iii) how to describe the disagreement, and (iv) how others represent the dynamics of agents opinions on two or more topics, where those topics-specific opinions are interdependent.

In non-Bayesian models of opinion formation with learning through communication, each agent has an opinion described by a variable which can be continuous or discrete and it change in time. We found approaches that used Non Bayesian updating of continuous beliefs as [14,15,17,31,49–56,76,79,86] and the other one based on non-Bayesian updating of discrete beliefs as [12,13,39,41–45,48]. When the communications take place over a strongly connected network, some existing models, typically tend toward a consensus opinion, that is, that all agents eventually hold the same opinion about any specific issue. Among these models we can mention: Discrete models, like the Voter [12], [13], Ising [41,42], Sznajd [43,44]. However, the forces that lead to consensus also preclude the emergence of persistent disagreements and this feature might put limits on the extent of misinformation. Typically, as Acemoglu and Ozdaglar [21] show, the persistent disagreements, misinformation and belief manipulation might be better analyzed using non-Bayesian approaches.

Concerning the non-Bayesian model, a few amendments have been suggested which are capable of producing disagreement among agents. We propose to classify these amendments in two solutions: The first solution presents the models including a homophily mechanism. In other words, the models where agents limit their communication to individuals whose opinions are not too different from their own. That way, patterns of opinion diversity and disagreement such as [31,54], more recently [64,65], can be reproduced. In another strand, the second solution proposed in several approaches, for incorporating the disagreement, is to distinguish between the various types of agents: regular, forceful and stubborn agents. In [14,79], some level of stubbornness has been added to each agent. So, we can distinguish between two types of agents, stubborn and regular, whereby the latter (stubborn) never update their opinions but stubbornly retain their old beliefs. In Yildiz et al. [48], a discrete version of the DeGroot model with stubborn agents is analyzed in which regular agents randomly adopt one of their neighbors binary opinions.

The presence of stubborn individuals has received increasing attention and it has touched a lot of others work [17,19,34,48,82,88].However, Acemoglu et al. [76] proposed to make a distinction between regular and forceful agents . Regular agents exchange information with their neighbors (when they meet). In contrast, forceful agents influence others disproportionately.

The objective of this work, [76], is to overcome both limitations: the first, as already discussed, the emergence of persistent disagreements. The second limit is the mechanical duplication of information. In Deterministic approaches such as [15,72], people continue to use the simple rules of thumb, which treats all information as new for updating opinions. In this situation, updating opinions with repetitive information may establish an extreme form of duplication of information. [76] has highlighted this problem in the context of the DeGroot model. Though this problem is still present, but it is less severe. The idea is to reach the consensus the second time where the agent communicates again with another agent.

The studies of the multidimensional opinion dynamics are rare. In contrast to the most models those describe the evolution of opinion about a particular topic, [86] offer a novel multidimensional extension, which represents the dynamics of agents opinions on several topics, and those topics-specific opinions are interdependent. This model, [86], is a significant extension of the classical Friedkin–Johnsen model [14,79] to the case where agents opinions on two or more interdependent topics are being influenced. Instead to deal opinion dynamics with scalar opinions, this article consider vector-valued opinions [55,83,84], where each opinion vector in such a model is constituted by topic specific scalar opinions. Thus, Jager et al., [93]; Urbig et al., [94] made interesting studies with two dimensional opinions but focusing on specific interplays between two opinion dimensions. [95] used and extended the opinion dynamics models of Deffuant–Weisbuch and Hegselmann–Krause in to more dimensional opinions.

Sensitivity analysis is aimed at understanding the conditions under which the model yields the expected results. For example, with the bounded confidence models described above, one might ask, what is the perfect degree to which the agent can get closer to the views of others [54,77]. Or, how extreme do the extremists have to be for all the agents eventually to join one of the extreme parties [51]. To find out, one needs to run the simulation for a series of values of the uncertainty parameter, perhaps ranging from 0.5 to 1 in steps of 0.1. But in the model [51], every agent starts with an opinion taken from a uniform random distribution and with a common level of uncertainty fixed randomly, with the exception a few extremists, those who have the most positive or negative opinions. It should perform a number of runs for each value of the uncertainty parameter to obtain a mean and variance. The confidence value used in some models, namely the weight allocated to the opinion of others, depends only on the number of friends who are in contact with him. For example if an agent has four friends then the weight allocated of all friends is 1/4. In some others models, the weight allocated to the displayed opinions of others is randomly and nonnegative. All this weights are representing in a weighting matrix that must be stochastic and static. The social behavior of an agent strongly depends on the trust value assigned to others. If an agent is open to the influence of others then he attaches no weight to his own opinion and he relying fully on others opinions. So, the sum of the weight allocated to the displayed opinions of others is equals to one. Contrary, if an agent is maximally stubborn then the sum of the weight allocated is equals to zero.

The dataset comprised of user opinions extracted from different topics. The choice of subjects was based on the knowledge of the agents. That is, most agents did not have a strong prior opinion, but had some knowledge of the selected subject. The agents refined their opinions continuously by communicating only with their neighbors. At each time, the authors attempted to capture every opinion change of all participants, the agents who were told to form opinions base solely on discussions with designated social network neighbors. Others models used a uniform opinion distribution in an interval determined. Model validation with real data, which are starting to become available, is still largely lacking and should in our opinion be the main ambition of future investigations.

As an idea of future work, we propose to improve some works such as: the approaches that use a confidence threshold. In all these works, the trust value of each agent depends only on the number of friends who are in contact with him. For more accurate results, we propose to extend this value which, considering other factors such as the duration of friendship, personal similarity (age, sex, city,...), frequency of contact. Also, we can take into account the dynamics of trust in relationships which depend on several variables and evolve also at the time. As a first idea, we can apply a dynamic Bayesian network which relates variables to each other over adjacent time steps. In the same way, we can take account of the dynamics of stubbornness in the approaches which propose to perform distinction between agents in order to lead the persistent disagreements. In these approaches, this value of stubbornness is a constant value and it differs from one agent to another. To make this more realistic value, we propose that it will be dynamic and that it will change not only from agent to other but also change according to the subject and to time.

The common point between these works is the version of network used. In other words most studies of opinion dynamics have been induced with a static version of the network. Or, the study of the properties and characteristics of social networks has been considered as an important research track. So, in our opinion, it is essentially interesting to analyze the dynamics of the structure of the social network next to the dynamism of the dynamics of opinion. However, a temporal criterion can be taken into account to understand the evolutions taking place in the network. For example, the dynamics of the interactions of individuals in networks (deletions or creation of links). Also, analysis of the dynamics of information in networks is an important factor and can in turn be an important element of structural change. We can give importance to the freshness of the information that individuals share when modeling of opinion.

Many future advancements of the field, in short and long terms, stem from this endeavor.

First, it could be made more important to have some approaches in which learning and opinion updating are, at least partly, Bayesian. It is clear that in non-Bayesian approaches, the simplicity of the models and the ease of adding the notion misinformation allow researchers to develop several approaches. But, the game theoretic models of communication can be used, in Bayesian models, for adding misinformation. These Game theoretic models allow analyzing situations in which a sender may explicitly try to mislead one or several receivers. As a result, the notion of misinformation can be added.

Second, the approaches can be extended to time-varying network. So, the number of friends, the bonds of friendship between the agents will be invariant in time. In other words, the network structure changes in every time step.

Thirdly, the model validation with real data is starting to become available but it is still largely lacking. So, it should be the main ambition of future investigations.

Fourth, an important area for future research is to have models in which individuals update their beliefs and opinions after obtaining information both from their observations of the actions/experiences of others and from their communication with others.

7. Conclusion

In this paper, we have provided an overview of work on modeling opinion dynamics in social networks. The part of our interest in opinion and belief dynamics is to present Non-Bayesian models of communication learning where people start to form their opinions by specifying simple rules of thumb. For their simplicity, the literature has considered several non-Bayesian approaches. The simplest one is DeGroot learning, where each individual updates her beliefs as a weighted average of the beliefs of his social neighbors, with weights given by the trust he has for those neighbors. Another example will be one in which individuals meet one person at a time from their social neighborhood (friends, coworkers or peers) and update their opinion to a weighted average of their initial opinion and the opinion of the person they have just met. There are many interesting variations of the DeGroot model for overcoming some limitations. Among them worth mentioning are the bounded confidence models where agents are more likely to exchange opinion or communicate with others that have similar beliefs. Others variations propose to distinguish between the various types of agents. This distinction is modeled by a measure of stubbornness or closure of the agent to interpersonal influence. The diversity of non-Bayesian learning communication models led us to make a comparison between the different parameters used in certain approaches.

Conflict of Interests

None.

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