Neutron EDM

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1 Quark EDM

Lagrangain

$$\mathcal{L} = -\frac{1}{\sqrt{2}} Y^{dd} \bar{d} P_R d(H + iA) - \frac{1}{\sqrt{2}} Y^{sd} \bar{s} P_R d(H + iA) + \text{H.C} \tag{1}$$

Y^{dd} is a complex number, it can be written as $Y^{dd} e^{i\theta}$.

Propagator is d and H.

$$\mathcal{L}_{dH} = -\frac{1}{\sqrt{2}} \bar{d}(Y^{dd} cos\theta + iY^{dd} sin\theta\gamma_5) dH - Q_d A_\mu \bar{d}\gamma^\mu d \tag{2}$$

Coupling Vertex. p_1, p_2 are the momentum of the d quark, p_3 is the momentum of the phothon, k is the loop momentum. We have the relation $p_1 = p_2 + p_3$.

$$\begin{aligned}&\bar{u}(p_2) \int \frac{d^4k}{(2\pi)^4} \left[-\frac{i}{\sqrt{2}} \left(Y^{dd} c_\theta + iY^{dd} s_\theta \gamma_5 \right) \right] \frac{i(\not{p}_1 - \not{k} - \not{p}_3 + m_d)}{(p_1 - k - p_3)^2 - m_d^2} (-iQ_d \gamma^\mu) \\ &\frac{i(\not{p}_1 - \not{k} + m_d)}{(p_1 - k)^2 - m_d^2} \left[-\frac{i}{\sqrt{2}} \left(Y^{dd} c_\theta + iY^{dd} s_\theta \gamma_5 \right) \right] \frac{i}{k^2 - m_H^2} u(p_1)\end{aligned} \tag{3}$$

The term which has only one γ_5 will has the contribution to EDM.

$$\begin{aligned}
&\Rightarrow \frac{iQ_d(Y^{dd})^2}{2}c_\theta s_\theta \int \frac{d^4k}{(2\pi)^4}\frac{(\not{p}_1-\not{k}-\not{p}_3+m_d)\gamma^\mu(\not{p}_1-\not{k}+m_d)\gamma_5}{[(p_1-k-p_3)^2-m_d^2]\,[(p_1-k)^2-m_d^2]\,[k^2-m_H^2]} \\
&+\frac{iQ_d(Y^{dd})^2}{2}c_\theta s_\theta \int \frac{d^4k}{(2\pi)^4}\frac{\gamma_5(\not{p}_1-\not{k}-\not{p}_3+m_d)\gamma^\mu(\not{p}_1-\not{k}+m_d)}{[(p_1-k-p_3)^2-m_d^2]\,[(p_1-k)^2-m_d^2]\,[k^2-m_H^2]}
\end{aligned} \tag{4}$$

Denominator. Using Feynmann parameter.

$$\begin{aligned}
&\int \frac{d^4k}{(2\pi)^4}\frac{Numerator}{[(p_1-k-p_3)^2-m_d^2]\,[(p_1-k)^2-m_d^2]\,[k^2-m_H^2]} \\
&=2\int_0^1 dx_i \int \frac{d^4k}{(2\pi)^4}\frac{N}{D^3}
\end{aligned} \tag{5}$$

where

$$\begin{aligned}
D &= \left[(p_1-k-p_3)^2-m_d^2\right]x_1+\left[(p_1-k)^2-m_d^2\right]x_2+\left[k^2-m_H^2\right]x_3 \\
&= \left[p_1^2+k^2+p_3^2-2p_1\cdot k-2p_1\cdot p_3+2k\cdot p_3-m_d^2\right]x_1+ \\
&\quad \left[p_1^2+k^2-2p_1\cdot k-m_d^2\right]x_2+\left[k^2-m_H^2\right]x_3 \\
&= \left[k^2+p_3^2-2p_1\cdot k-2p_1\cdot p_3+2k\cdot p_3\right]x_1+\left[k^2-2p_1\cdot k\right]x_2+\left[k^2-m_H^2\right]x_3 \\
&= k^2+2k\cdot(x_1p_3-x_1p_1-x_2p_1)+x_1p_3^2-2x_1p_1\cdot p_3-x_3m_H^2
\end{aligned} \tag{6}$$

Define $l=k+(x_1p_3-x_1p_1-x_2p_1)$.

$$\begin{aligned}
D &= l^2-(x_1p_3-x_1p_1-x_2p_1)^2+x_1p_3^2-2x_1p_1\cdot p_3-x_3m_H^2 \\
&= l^2-x_1(x_1-1)p_3^2-(x_1+x_2)^2m_d^2+2x_1(x_1+x_2-1)p_3\cdot p_1-x_3m_H^2 \\
&= l^2-x_1(x_1-1)p_3^2-(x_1+x_2)^2m_d^2-2x_1x_3p_3\cdot p_1-x_3m_H^2 \\
&= l^2-x_1(x_1-1)p_3^2-(x_1+x_2)^2m_d^2+x_1x_3(p_1-p_3)^2-x_3m_H^2 \\
&\quad -x_1x_3(p_1)^2-x_1x_3(p_3)^2 \\
&= l^2-x_1(x_1-1)p_3^2-(x_1+x_2)^2m_d^2-x_1x_3p_3^2-x_3m_H^2 \\
&= l^2+x_1x_2p_3^2-(x_1+x_2)^2m_d^2-x_3m_H^2
\end{aligned} \tag{7}$$

Numerator.Using Dirac equation and the anti-commutation relation between γ matrix.

$$\begin{aligned}
&\gamma_5(\not{p}_1-\not{k}-\not{p}_3+m_d)\gamma^\mu(\not{p}_1-\not{k}+m_d)+(\not{p}_1-\not{k}-\not{p}_3+m_d)\gamma^\mu(\not{p}_1-\not{k}+m_d)\gamma_5 \\
&=(-\not{p}_1+\not{l}-x_1\not{p}_3+x_1\not{p}_1+x_2\not{p}_1+\not{p}_3+m_d)\gamma_5\gamma^\mu(\not{p}_1-\not{l}+x_1\not{p}_3-x_1\not{p}_1-x_2\not{p}_2+m_d) \\
&\quad +(\not{p}_1-\not{l}+x_1\not{p}_3-x_1\not{p}_1-x_2\not{p}_1-\not{p}_3+m_d)\gamma^\mu\gamma_5(-\not{p}_1+\not{l}-x_1\not{p}_3+x_1\not{p}_1+x_2\not{p}_1+m_d) \\
&=(\not{l}+x_1\not{p}_2+x_2\not{p}_1-\not{p}_2+m_d)\gamma_5\gamma^\mu(\not{p}_1-\not{l}-x_1\not{p}_2-x_2\not{p}_1+m_d) \\
&\quad +(-\not{l}-x_1\not{p}_2-x_2\not{p}_1+\not{p}_2+m_d)\gamma^\mu\gamma_5(-\not{p}_1+\not{l}+x_1\not{p}_2+x_2\not{p}_1+m_d) \\
&=(-\not{l}-x_1m_d-x_2\not{p}_1)\gamma^\mu\gamma_5(-\not{l}-x_1\not{p}_2-x_2m_d+2m_d) \\
&\quad +(-\not{l}-x_1m_d-x_2\not{p}_1+2m_d)\gamma^\mu\gamma_5(\not{l}+x_1\not{p}_2+x_2m_d)
\end{aligned} \tag{8}$$

Because

$$\begin{aligned}
&\int \frac{d^4k}{(2\pi)^4}\frac{l^\mu}{D^3}=0, \\
&\int \frac{d^4k}{(2\pi)^4}\frac{l^\mu l^\nu}{D^3}=\frac{1}{4}g^{\mu\nu}\int \frac{d^4k}{(2\pi)^4}\frac{l^2}{D^3}=0
\end{aligned} \tag{9}$$

So we can drop the term which only has one l.

$$\begin{aligned} Numerator \Rightarrow & (-x_1 m_d - x_2 \not{p}_1)\gamma^\mu\gamma_5(-x_1\not{p}_2 - x_2 m_d + 2m_d) \\ & + (-x_1 m_d - x_2\not{p}_1 + 2m_d)\gamma^\mu\gamma_5(x_1\not{p}_2 + x_2 m_d) \\ = & 2(x_2 - x_1)m_d^2\gamma^\mu\gamma_5 + 2x_1 m_d\gamma^\mu\gamma_5\not{p}_2 - 2x_2 m_d\not{p}_1\gamma^\mu\gamma_5 \end{aligned} \tag{10}$$

Using

$$\begin{aligned} \not{p}_1\gamma^\mu\gamma_5 &= 2p_1^\mu\gamma_5 + m_d\gamma^\mu\gamma_5 \\ \gamma^\mu\gamma_5\not{p}_2 &= -2p_2^\mu\gamma_5 + m_d\gamma^\mu\gamma_5 \end{aligned} \tag{11}$$

We have

$$\begin{aligned} Numerator \Rightarrow & 2(x_2 - x_1)m_d^2\gamma^\mu\gamma_5 - 4x_1 m_d p_2^\mu\gamma_5 + 2x_1 m_d^2\gamma^\mu\gamma_5 \\ & - 4x_2 m_d p_1^\mu\gamma_5 - 2x_2 m_d^2\gamma^\mu\gamma_5 \\ = & -4x_1 m_d p_2^\mu\gamma_5 - 4x_2 m_d p_1^\mu\gamma_5 \\ = & -2(x_1 + x_2)m_d(p_1 + p_2)^\mu\gamma_5 - 2(x_2 - x_1)(p_1 - p_2)^\mu m_d\gamma_5 \end{aligned} \tag{12}$$

Because

$$\bar{u}(p_2)\sigma_{\mu\nu}(p_1 - p_2)^\nu\gamma_5 u(p_1) = -i\bar{u}(p_2)(p_1 + p_2)^\mu\gamma_5 u(p_1) \tag{13}$$

We have

$$Numerator \Rightarrow -2(x_1 + x_2)m_d i\sigma^{\mu\nu}p_{3\nu}\gamma_5 \tag{14}$$

So we can change (4) to

$$-2iQ_d(Y^{dd})^2 c_\theta s_\theta m_d \int_0^1 dx_i \int \frac{d^4 l}{(2\pi)^4}\frac{x_1 + x_2}{D^3}\bar{u}(p_2)i\sigma^{\mu\nu}p_{3\nu}\gamma_5 u(p_1) \tag{15}$$

Using

$$\int \frac{d^4 l}{(2\pi)^4}\frac{1}{(l^2 - \Delta)^3} = \frac{-i}{(4\pi)^2}\frac{1}{2\Delta} \tag{16}$$

We have EDM term($P_3 \to 0$)

$$\begin{aligned} d_{dH} &= -2iQ_d(Y^{dd})^2 c_\theta s_\theta m_d \int_0^1 dx_i \int \frac{d^4 l}{(2\pi)^4}\frac{x_1 + x_2}{D^3} \\ &= -\frac{Q_d(Y^{dd})^2 c_\theta s_\theta m_d}{(4\pi)^2}\int_0^1 \frac{x_1 + x_2}{\Delta} \\ &= -\frac{Q_d(Y^{dd})^2 c_\theta s_\theta m_d}{(4\pi)^2}\int_0^1 \frac{x_1 + x_2}{(x_1 + x_2)^2 m_d^2 + (1 - x_1 - x_2)m_H^2} \\ &= -\frac{Q_d(Y^{dd})^2 c_\theta s_\theta m_d}{(4\pi)^2 m_H^2}\int_0^1 \frac{x_1 + x_2}{(x_1 + x_2)^2 m_d^2/m_H^2 + (1 - x_1 - x_2)} \end{aligned} \tag{17}$$

Define $a = m_d/m_H$.The integration is

$$\begin{aligned} &\int_0^1 dx_1 \int_0^{1-x_1} dx_2 \frac{x_1 + x_2}{(x_1 + x_2)^2 a^2 + (1 - x_1 - x_2)} = \frac{1}{a^2} + \frac{ln(a^2)}{2a^4} \\ &+ \frac{1 - 2a^2}{2a^4}\frac{1}{\sqrt{1 - 4a^2}}\left[ln\left(\frac{1 + \sqrt{1 - 4a^2} - 2a^2}{1 + \sqrt{1 - 4a^2}}\right) - ln\left(\frac{1 - \sqrt{1 - 4a^2} - 2a^2}{1 - \sqrt{1 - 4a^2}}\right)\right] \\ &= \frac{1}{a^2} + \frac{ln(a^2)}{2a^4} + \frac{1 - 2a^2}{2a^4}\frac{1}{\sqrt{1 - 4a^2}}ln\left(\frac{1 + \sqrt{1 + 4a^2}}{1 - \sqrt{1 - 4a^2}}\right) \\ &= \frac{1}{a^2} + \frac{ln(a^2)}{2a^4} + \frac{1 - 2a^2}{2a^4}\frac{1}{\sqrt{1 - 4a^2}}ln\left(\frac{1 - 2a^2 + \sqrt{1 - 4a^2}}{2a^2}\right) \end{aligned} \tag{18}$$

Beacus $m_d \sim MeV$,$m_H \sim GeV$.a is a small quantity. Using Taylor expansion

$$\begin{aligned} (1-4a^2)^{-\frac{1}{2}} &= 1+2a^2+6a^4+\mathcal{O}(a^6) \\ ln\left(\frac{1-2a^2+\sqrt{1-4a^2}}{2}\right) &= -2a^2-3a^4+\mathcal{O}(a^6) \end{aligned} \tag{19}$$

So we can get

$$\begin{aligned} &\int_0^1 dx_1 \int_0^{1-x_1} dx_2 \frac{x_1+x_2}{(x_1+x_2)^2a^2+(1-x_1-x_2)} = \frac{1}{a^2}+\frac{ln(a^2)}{2a^4} \\ &= \frac{1}{a^2}+\frac{ln(a^2)}{2a^4}+\frac{1-2a^2}{2a^4}(1+2a^2+6a^4+\mathcal{O}(a^6)) \\ &\quad \left(-2a^2-3a^4+\mathcal{O}(a^6)-\ln(a^2)\right) \\ &= \frac{1}{a^2}+\frac{ln(a^2)}{2a^4}+\left(\frac{1}{2a^4}+1+\mathcal{O}(a^2)\right)\left(-2a^2-3a^4+\mathcal{O}(a^6)-\ln(a^2)\right) \\ &= -\frac{3}{2}-\ln(a^2)+\mathcal{O}(a^2)+\ln(a^2)\mathcal{O}(a^2) \end{aligned} \tag{20}$$

We only keep the zero order term of a. So we have

$$d_{dH} = \frac{Q_d(Y^{dd})^2 c_\theta s_\theta m_d}{(4\pi)^2 m_H^2}\left(\frac{3}{2}+\ln\left(\frac{m_d^2}{m_H^2}\right)\right) \tag{21}$$

The Lagrangain of the dA term is

$$\mathcal{L}_{dA} = -\frac{1}{\sqrt{2}}\bar{b}(iY^{dd}cos\theta\gamma_5 - Y^{dd}sin\theta)dA - Q_d A_\mu \bar{d}\gamma^\mu dd(Y^{dd}) \tag{22}$$

We have

$$\begin{aligned} &\Rightarrow -\frac{iQ_d(Y^{dd})^2}{2}c_\theta s_\theta \int \frac{d^4k}{(2\pi)^4}\frac{(\not{p}_1-\not{k}-\not{p}_3+m_d)\gamma^\mu(\not{p}_1-\not{k}+m_d)\gamma_5}{[(p_1-k-p_3)^2-m_d^2]\,[(p_1-k)^2-m_d^2]\,[k^2-m_A^2]} \\ &- \frac{iQ_d(Y^{dd})^2}{2}c_\theta s_\theta \int \frac{d^4k}{(2\pi)^4}\frac{\gamma_5(\not{p}_1-\not{k}-\not{p}_3+m_d)\gamma^\mu(\not{p}_1-\not{k}+m_d)}{[(p_1-k-p_3)^2-m_d^2]\,[(p_1-k)^2-m_d^2]\,[k^2-m_A^2]} \end{aligned} \tag{23}$$

Except for a minus sign and changing H to A, it is the same as dH condition.So we have

$$d_{dA} = -\frac{Q_d(Y^{dd})^2 c_\theta s_\theta m_d}{(4\pi)^2 m_A^2}\left(\frac{3}{2}+\ln\left(\frac{m_d^2}{m_A^2}\right)\right) \tag{24}$$

The Lagrangain of sH term is

$$\mathcal{L}_{sH} = -\frac{1}{2}Y^{sd}(\bar{s}P_R d+\bar{d}P_L s)H \tag{25}$$

We have

$$\begin{aligned} &\bar{u}(p_2)\int \frac{d^4k}{(2\pi)^4}\left[-\frac{i}{\sqrt{2}}Y^{sd}P_L\right]\frac{i(\not{p}_1-\not{k}-\not{p}_3+m_s)}{(p_1-k-p_3)^2-m_s^2}(-iQ_s\gamma^\mu) \\ &\frac{i(\not{p}_1-\not{k}+m_s)}{(p_1-k)^2-m_s^2}\left[-\frac{i}{\sqrt{2}}Y^{sd}P_R\right]\frac{i}{k^2-m_H^2}u(p_1) \end{aligned} \tag{26}$$

Only one γ_5 term.

$$
\begin{aligned}
&\Rightarrow \frac{Q_s(Y^{sd})^2}{8}\int\frac{d^4k}{(2\pi)^4}\frac{(\not{p}_1-\not{k}-\not{p}_3+m_s)\gamma^\mu(\not{p}_1-\not{k}+m_s)\gamma_5}{[(p_1-k-p_3)^2-m_s^2]\,[(p_1-k)^2-m_s^2]\,[k^2-m_H^2]}\\
&-\frac{Q_s(Y^{sd})^2}{8}\int\frac{d^4k}{(2\pi)^4}\frac{\gamma_5(\not{p}_1-\not{k}-\not{p}_3+m_s)\gamma^\mu(\not{p}_1-\not{k}+m_s)}{[(p_1-k-p_3)^2-m_s^2]\,[(p_1-k)^2-m_d^2]\,[k^2-m_H^2]}
\end{aligned}
\tag{27}
$$

Denominator.

$$
\begin{aligned}
D &= \left[(p_1-k-p_3)^2-m_s^2\right]x_1+\left[(p_1-k)^2-m_s^2\right]x_2+\left[k^2-m_H^2\right]x_3\\
&= \left[p_1^2+k^2+p_3^2-2p_1\cdot k-2p_1\cdot p_3+2k\cdot p_3-m_s^2\right]x_1+\\
&\quad \left[p_1^2+k^2-2p_1\cdot k-m_s^2\right]x_2+\left[k^2-m_H^2\right]x_3\\
&= \left[k^2+p_3^2-2p_1\cdot k-2p_1\cdot p_3+2k\cdot p_3-\Delta m_{sd}^2\right]x_1+\left[k^2-2p_1\cdot k-\Delta m_{sd}^2\right]x_2\\
&\quad +\left[k^2-m_H^2\right]x_3\\
&= k^2+2k\cdot(x_1p_3-x_1p_1-x_2p_1)+x_1p_3^2-2x_1p_1\cdot p_3-x_3m_H^2-(x_1+x_2)\Delta m_{sd}^2
\end{aligned}
\tag{28}
$$

where $\Delta m_{sd}^2=m_s^2-m_d^2$.Define $l=k+(x_1p_3-x_1p_1-x_2p_1)$.

$$
\begin{aligned}
D &= l^2-(x_1p_3-x_1p_1-x_2p_1)^2+x_1p_3^2-2x_1p_1\cdot p_3-x_3m_H^2-(x_1+x_2)\Delta m_{sd}^2\\
&= l^2+x_1x_2p_3^2-(x_1+x_2)^2m_d^2-x_3m_H^2-(x_1+x_2)\Delta m_{sd}^2
\end{aligned}
\tag{29}
$$

Numerator.

$$
\begin{aligned}
N &= (\not{p}_2-\not{k}+m_s)\gamma^\mu(\not{p}_1-\not{k}+m_s)\gamma_5-\gamma_5(\not{p}_2-\not{k}+m_s)\gamma^\mu(\not{p}_1-\not{k}+m_s)\\
&= 2(m_s^2-m_d^2)\gamma^\mu\gamma_5+2m_d\gamma^\mu\gamma_5\not{k}+2m_d\not{k}\gamma^\mu\gamma_5-2\not{k}\gamma^\mu\gamma_5\not{k}\\
&\supset 2m_d\gamma^\mu\gamma_5(x_1\not{p}_2+x_2\not{p}_1)+2m_d(x_1\not{p}_2+x_2\not{p}_1)\gamma^\mu\gamma_5-2(x_1\not{p}_2+x_2\not{p}_1)\gamma^\mu\gamma_5(x_1\not{p}_2+x_2\not{p}_1)\\
&\supset 2x_1m_d\gamma^\mu\gamma_5\not{p}_2+2x_2m_d\not{p}_1\gamma^\mu\gamma_5-2x_1^2m_d\gamma^\mu\gamma_5\not{p}_2-2x_2^2m_d\not{p}_1\gamma^\mu\gamma_5-2x_1x_2\not{p}_1\gamma^\mu\gamma_5\not{p}_2\\
&= 2(x_1-x_1^2)m_d\gamma^\mu\gamma_5\not{p}_2+2(x_2-x_2^2)\not{p}_1\gamma^\mu\gamma_5-2x_1x_2\not{p}_1\gamma^\mu\gamma_5\not{p}_2
\end{aligned}
\tag{30}
$$

At the third and forth step, I drop $\gamma^\mu\gamma_5$ term. Using

$$
\begin{aligned}
\not{p}_1\gamma^\mu\gamma_5 &= 2p_1^\mu\gamma_5+m_d\gamma^\mu\gamma_5\\
\gamma^\mu\gamma_5\not{p}_2 &= -2p_2^\mu\gamma_5+m_d\gamma^\mu\gamma_5\\
\not{p}_1\gamma^\mu\gamma_5\not{p}_2 &= -2m_dp_1^\mu\gamma_5-p_3^2\gamma^\mu\gamma_5+2m_dp_2^\mu\gamma_5+m_d^2\gamma^\mu\gamma_5
\end{aligned}
\tag{31}
$$

We have

$$
\begin{aligned}
N &\supset 2(x_1-x_1^2)m_d\gamma^\mu\gamma_5\not{p}_2+2(x_2-x_2^2)\not{p}_1\gamma^\mu\gamma_5-4x_1x_2m_d(p_2^\mu-p_1^\mu)\gamma_5\\
&\supset -4(x_1-x_1^2)m_dp_2^\mu\gamma_5+4(x_2-x_2^2)m_dp_1^\mu\gamma_5-4x_1x_2m_d(p_2^\mu-p_1^\mu)\gamma_5\\
&\supset 2m_d\left[(x_2-x_2^2)-(x_1-x_1^2)\right](p_1+p_2)^\mu\gamma_5
\end{aligned}
\tag{32}
$$

The integral of the Feynman parameter is zero because of the symmetry between the x_1 and x_2.So we have

$$
d_{sH}=0 \tag{33}
$$

The Langrangain of sA term is

$$
\mathcal{L}_{sA}=-\frac{i}{\sqrt{2}}Y^{sd}(\bar{s}P_Rd-\bar{d}P_Ls)A-Q_sA_\mu\bar{s}\gamma^\mu s \tag{34}
$$

We have

$$\bar{u}(p_2)\int\frac{d^4k}{(2\pi)^4}\left[-\frac{1}{\sqrt{2}}Y^{sd}P_L\right]\frac{i(\not{p}_1-\not{k}-\not{p}_3+m_s)}{(p_1-k-p_3)^2-m_s^2}(-iQ_s\gamma^\mu)$$
$$\frac{i(\not{p}_1-\not{k}+m_s)}{(p_1-k)^2-m_s^2}\left[\frac{1}{\sqrt{2}}Y^{sd}P_R\right]\frac{i}{k^2-m_A^2}u(p_1) \tag{35}$$

Only one γ_5 term.

$$\Rightarrow\frac{Q_s(Y^{sd})^2}{8}\int\frac{d^4k}{(2\pi)^4}\frac{(\not{p}_1-\not{k}-\not{p}_3+m_s)\gamma^\mu(\not{p}_1-\not{k}+m_s)\gamma_5}{[(p_1-k-p_3)^2-m_s^2]\,[(p_1-k)^2-m_s^2]\,[k^2-m_A^2]}$$
$$-\frac{Q_s(Y^{sd})^2}{8}\int\frac{d^4k}{(2\pi)^4}\frac{\gamma_5(\not{p}_1-\not{k}-\not{p}_3+m_s)\gamma^\mu(\not{p}_1-\not{k}+m_s)}{[(p_1-k-p_3)^2-m_s^2]\,[(p_1-k)^2-m_d^2]\,[k^2-m_A^2]} \tag{36}$$

The only difference with (27) is m_A.So we have

$$d_{sA}=0 \tag{37}$$

EDM.Sum(21)(24)

$$d=d_{dH}+d_{dA}$$
$$=\frac{Q_d(Y^{dd})^2c_\theta s_\theta m_d}{(4\pi)^2m_H^2}\left[\frac{3}{2}\left(1-\frac{m_H^2}{m_A^2}\right)+\ln\left(\frac{m_d^2}{m_H^2}\right)-\frac{m_H^2}{m_A^2}\ln\left(\frac{m_d^2}{m_A^2}\right)\right] \tag{38}$$

Because

$$Y^{dd}=1.6\times10^{-1}\frac{1}{|\lambda_3|}\left(\frac{m_H}{TeV}\right)^2 \tag{39}$$

We have

$$d=\frac{Q_d(1.6\times10^{-1})^2c_\theta s_\theta m_d}{(4\pi)^2(TeV)^2}\left(\frac{m_H}{TeV}\right)^2\frac{1}{\lambda_3^2}\left[\frac{3}{2}\left(1-\frac{m_H^2}{m_A^2}\right)\right.$$
$$\left.+\ln\left(\frac{m_d^2}{m_H^2}\right)-\frac{m_H^2}{m_A^2}\ln\left(\frac{m_d^2}{m_A^2}\right)\right] \tag{40}$$

Using the $B-\bar{B}$ mixing equation.

$$\Delta m_{Bs}=K\frac{1}{{\lambda_3}^2}\left(\frac{m_H}{\mathrm{TeV}}\right)^2\left(1-\frac{m_H^2}{m_A^2}\right)\,\mathrm{ps}^{-1} \tag{41}$$

where $K=3.055\times10^{-3}$. We can get

$$d=\frac{Q_d(1.6\times10^{-1})^2c_\theta s_\theta m_d}{(4\pi)^2(TeV)^2}\frac{\Delta m_{Bs}\,\mathrm{ps}^{-1}}{K\,\mathrm{ps}^{-1}}\left[\left(\frac{3}{2}+\ln\left(\frac{m_d^2}{m_H^2}\right)\right)\right.$$
$$\left.-2\frac{m_H^2}{m_A^2-m_H^2}\ln\left(\frac{m_H}{m_A}\right)\right] \tag{42}$$

$Q_d=Q_s=-\frac{1}{3}e$,$m_d=4.70\,\mathrm{MeV}$,$m_s=93\,\mathrm{MeV}$

$$\frac{Q_d(1.6\times10^{-1})^2m_d}{2(4\pi)^2(TeV)^2K}=-8.2024\times10^{-25}\,e\cdot\mathrm{cm} \tag{43}$$

d quark edm is

$$d_d = \left(-8.2020 \times 10^{-25}\, e \cdot \mathrm{cm}\right) s_{2\theta} \frac{\Delta m_{Bs}}{\mathrm{ps}^{-1}} \left[\left(\frac{3}{2} + \ln\left(\frac{m_d^2}{m_H^2}\right)\right)\right.$$
$$\left. -2\frac{m_H^2}{m_A^2 - m_H^2} \ln\left(\frac{m_H}{m_A}\right)\right] \tag{44}$$

For u quark, the Lagrangain is

$$\mathcal{L} = -\frac{1}{\sqrt{2}} Y^{uu} \bar{u} P_L u (H + iA) + \mathrm{H.C} \tag{45}$$

u quark edm is similar with eq(38),expcept a minus sign.

$$\begin{aligned} d_u &= d_{uH} + d_{uA} \\ &= -\frac{Q_u (Y^{uu})^2 c_\beta s_\beta m_u}{(4\pi)^2 m_H^2} \left[\frac{3}{2}\left(1 - \frac{m_H^2}{m_A^2}\right) + \ln\left(\frac{m_u^2}{m_H^2}\right) - \frac{m_H^2}{m_A^2} \ln\left(\frac{m_u^2}{m_A^2}\right)\right] \end{aligned} \tag{46}$$

where

$$Y^{uu} = 1.2 \times 10^{-1} \frac{1}{\lambda_3} \left(\frac{m_H}{\mathrm{TeV}}\right)^2 \tag{47}$$

So we have

$$\begin{aligned} d_u = &-\frac{Q_u (1.2 \times 10^{-1})^2 c_\beta s_\beta m_u}{(4\pi)^2 (TeV)^2} \left(\frac{m_H}{TeV}\right)^2 \frac{1}{\lambda_3^2} \left[\frac{3}{2}\left(1 - \frac{m_H^2}{m_A^2}\right)\right. \\ &\left. + \ln\left(\frac{m_u^2}{m_H^2}\right) - \frac{m_H^2}{m_A^2} \ln\left(\frac{m_u^2}{m_A^2}\right)\right] \\ = &-\frac{Q_u (1.2 \times 10^{-1})^2 c_\beta s_\beta m_u}{(4\pi)^2 (TeV)^2} \frac{\Delta m_{Bs}\, \mathrm{ps}^{-1}}{K\, \mathrm{ps}^{-1}} \left[\left(\frac{3}{2} + \ln\left(\frac{m_u^2}{m_H^2}\right)\right)\right. \\ &\left. -2\frac{m_H^2}{m_A^2 - m_H^2} \ln\left(\frac{m_H}{m_A}\right)\right] \\ = &(-4.2406 \times 10^{-25}\ e \cdot \mathrm{cm}) s_{2\beta} \frac{\Delta m_{Bs}}{\mathrm{ps}^{-1}} \left[\left(\frac{3}{2} + \ln\left(\frac{m_u^2}{m_H^2}\right)\right)\right. \\ &\left. -2\frac{m_H^2}{m_A^2 - m_H^2} \ln\left(\frac{m_H}{m_A}\right)\right] \end{aligned} \tag{48}$$

Neutron edm

$$d_n \approx \frac{4}{3} d_d - \frac{1}{3} d_u \tag{49}$$

2 Quark CDM

Quark chromo-dipole moment(CDM).

$$\mathcal{L}_g = -\frac{i}{2} g_s f_q \bar{q} \sigma^{\mu\nu} \gamma_5 t^A q G^A_{\mu\nu} \tag{50}$$

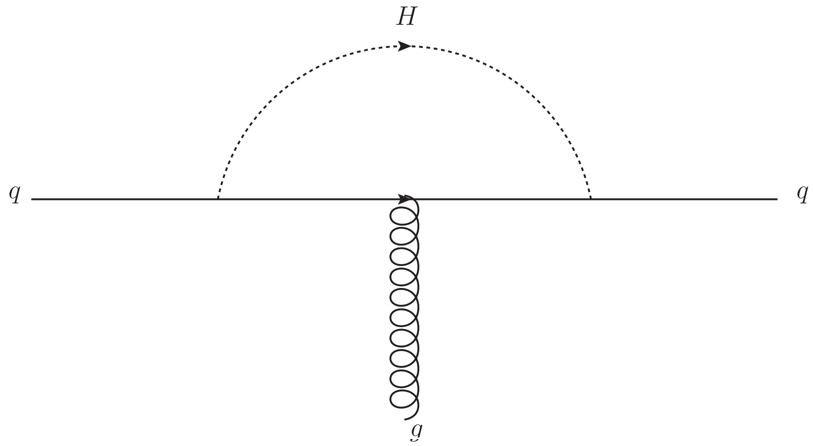

Figure 1: The one loop contribution to the CDM of quark q

f_q is CDM, T^A is $SU(2)$ generator.

Lagrangain

$$\mathcal{L} = -\frac{1}{\sqrt{2}}(Y_2^{D,dd}\bar{d}P_R d + Y_2^{U,uu}\bar{u}P_L u)(H + iA) + H.C - g_s\bar{q}_a\gamma^\mu t^A_{ab}q_b A^A_\mu. \tag{51}$$

a is the color index,$a = 1, 2, 3$.

Let's see the result of the d quark CEDM when the particle in the loop is H as an example, other case is the similar.The Lagrangain is

$$\mathcal{L}_{dH} = -\frac{1}{\sqrt{2}}\bar{d}(Y^{dd}cos\theta + iY^{dd}sin\theta\gamma_5)dH - g_s A^A_\mu \bar{d}\gamma^\mu t^a d \tag{52}$$

The one loop contribution is FIG1. So we have

$$\begin{aligned}
i\mathcal{M}^{\mu,A}_{dH} &= \bar{u}(p_2)\int\frac{d^4k}{(2\pi)^4}\left[-\frac{i}{\sqrt{2}}\left(Y^{dd}c_\theta + iY^{dd}s_\theta\gamma_5\right)\right]\frac{i(\not{p}_1 - \not{k} - \not{p}_3 + m_d)}{(p_1 - k - p_3)^2 - m_d^2}(-ig_s\gamma^\mu t^A) \\
&\quad \frac{i(\not{p}_1 - \not{k} + m_d)}{(p_1 - k)^2 - m_d^2}\left[-\frac{i}{\sqrt{2}}\left(Y^{dd}c_\theta + iY^{dd}s_\theta\gamma_5\right)\right]\frac{i}{k^2 - m_H^2}u(p_1) \\
&= \frac{g_s t^A_{ab}(Y^{dd})^2}{2}\bar{u}_a(p_2)\int\frac{d^4k}{(2\pi)^4}\left[(c_\theta + is_\theta\gamma_5)\right]\frac{(\not{p}_1 - \not{k} - \not{p}_3 + m_d)}{(p_1 - k - p_3)^2 - m_d^2}(\gamma^\mu) \\
&\quad \frac{(\not{p}_1 - \not{k} + m_d)}{(p_1 - k)^2 - m_d^2}\left[(c_\theta + is_\theta\gamma_5)\right]\frac{1}{k^2 - m_H^2}u_b(p_1)
\end{aligned} \tag{53}$$

This is the same with eq(3) if we chang $g_s t^A_{ab}$ to Q_d. So the result is similar with eq(21).

$$i\mathcal{M}^{\mu,A}_{dH} \supset g_s f_{dH}\bar{u}(p_2)i\sigma^{\mu\nu}t^A\gamma_5 u(p_1)p_{3\nu} \tag{54}$$

where

$$f_{dH} = \frac{(Y^{dd})^2 c_\theta s_\theta m_d}{(4\pi)^2 m_H^2}\left(\frac{3}{2} + \ln\left(\frac{m_d^2}{m_H^2}\right)\right). \tag{55}$$

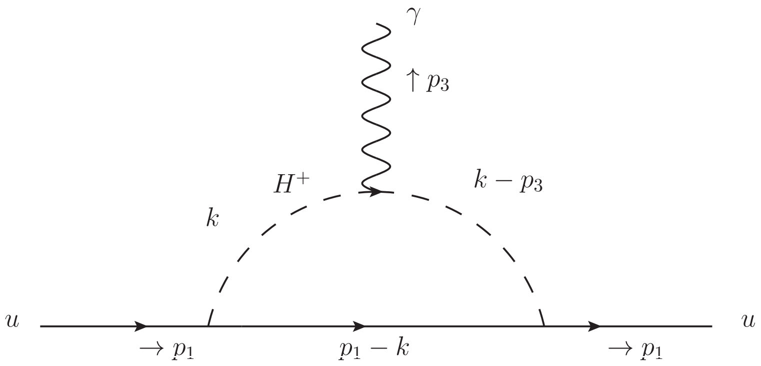

Figure 2: The one loop contribution to the u quark EDM from the charged scalar

Other case is same because we just need to change Q_q to $g_s t^A$ during the calculation so we have

$$\begin{aligned} f_u &= f_{uH} + f_{uA} \\ &= -\frac{(Y^{uu})^2 c_\beta s_\beta m_u}{(4\pi)^2 m_H^2}\left[\frac{3}{2}\left(1-\frac{m_H^2}{m_A^2}\right)+\ln\left(\frac{m_u^2}{m_H^2}\right)-\frac{m_H^2}{m_A^2}\ln\left(\frac{m_u^2}{m_A^2}\right)\right] \\ &= d_u/Q_u \\ f_d &= d_{dH} + d_{dA} \\ &= \frac{(Y^{dd})^2 c_\theta s_\theta m_d}{(4\pi)^2 m_H^2}\left[\frac{3}{2}\left(1-\frac{m_H^2}{m_A^2}\right)+\ln\left(\frac{m_d^2}{m_H^2}\right)-\frac{m_H^2}{m_A^2}\ln\left(\frac{m_d^2}{m_A^2}\right)\right] \\ &= d_d/Q_d \end{aligned} \tag{56}$$

just like eq(46) and (38). The contribution to neutron EDM is

$$d_n^{qCDM} = -\frac{1}{3}(4Q_d f_d - Q_u f_u) = -\frac{1}{3}(4d_d - d_u) = -d_n^{qEDM} \tag{57}$$

3 quark EDM from charged Higgs

The interaction between charged scalar particle and the photon is

$$\mathcal{L}_{H\gamma} = -iQeA_\mu(H^-\partial H^+ - \partial H^- H^+). \tag{58}$$

This term is come from $(D_\mu H_2)^\dagger(D^\mu H_2)$ where

$$D_\mu = \partial_\mu + ig' B_\mu Y + igW_\mu^i \sigma^i/2, \tag{59}$$

and

$$H_2 = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}(H+iA) \end{pmatrix}. \tag{60}$$

The correspond vertex rule is $-iQe(p_\mu + q_\mu)$.

FIG2 is the one loop diagram of the u quark EDM due to the charged Higgs. The Yukawa interaction is

$$\mathcal{L}_Y \supset -\overline{U_j}\Big[\left(VY_2^D\right)_{jk}P_R - \left(Y_2^{U\dagger}V\right)_{jk}P_L\Big]D_k H^+ + H.c. \tag{61}$$

Only $Y_2^{D,bs,sd,dd}$ and $Y_2^{U,uu}$ to be none-zero.

3.1 usu Condition

When the intermediate propagator in FIG2 is an s-quark.

$$\begin{aligned}\mathcal{L} \supset &-[V_{ub}Y_2^{D,bs}\bar{u}P_R s - Y_2^{U,uu*}V_{us}\bar{u}P_L s]H^+ + H.C\\ &= -\bar{u}\left(V_{ub}Y_2^{D,bs}P_R - Y_2^{U,uu*}V_{us}P_L\right)sH^+\\ &\quad - \bar{s}\left(V_{ub}^*Y_2^{D,bs*}P_L - Y_2^{U,uu}V_{us}^*P_R\right)uH^-\\ &\quad - iQeA_\mu(H^-\partial H^+ - \partial H^- H^+).\end{aligned} \tag{62}$$

So for the feyman diagram like FIG2 we have

$$\begin{aligned}&i\mathcal{M}\\ &= \bar{u}(p_2)\left[-i\left(V_{ub}Y_2^{D,bs}P_R - Y_2^{U,uu*}V_{us}P_L\right)\right]\frac{i(\not{p}_1 - \not{k} + m_s)}{(p_1-k)^2 - m_s^2}\left[-i\left(V_{ub}^*Y_2^{D,bs*}P_L - Y_2^{U,uu}V_{us}^*P_R\right)\right]\\ &u(p_1)\frac{i}{k^2 - m_{H^+}^2}\frac{i}{(k-p_3)^2 - m_{H^+}^2}(-iQe)(2k-p_3)_\mu\int\frac{d^4k}{(2\pi)^4}\epsilon_\mu^*(p_3)\\ &= Qe\bar{u}(p_2)\int\frac{d^4k}{(2\pi)^4}\frac{\left(V_{ub}Y_2^{D,bs}P_R - Y_2^{U,uu*}V_{us}P_L\right)(\not{p}_1 - \not{k} + m_s)\left(V_{ub}^*Y_2^{D,bs*}P_L - Y_2^{U,uu}V_{us}^*P_R\right)}{[(p_1-k)^2 - m_s^2]\,[k^2 - m_{H^+}^2]\,[(k-p_3)^2 - m_{H^+}^2]}\\ &\quad (2k-p_3)_\mu u(p_1)\epsilon^{\mu,*}\end{aligned} \tag{63}$$

Define

$$\Gamma^\mu = \int\frac{d^4k}{(2\pi)^4}\frac{(a+b\gamma^5)(\not{p}_1 - \not{k} + m_s)(a^* - b^*\gamma^5)}{[(p_1-k)^2 - m_s^2]\,[k^2 - m_{H^+}^2]\,[(k-p_3)^2 - m_{H^+}^2]}(2k-p_3)_\mu \tag{64}$$

where

$$\begin{aligned}a &= V_{ub}Y_2^{D,bs} - Y_2^{U,uu*}V_{us}\\ b &= V_{ub}Y_2^{D,bs} + Y_2^{U,uu*}V_{us}\end{aligned} \tag{65}$$

We have

$$i\mathcal{M} = \frac{Qe}{4}\bar{u}(p_2)\Gamma^\mu u(p_1)\epsilon_\mu^*(p_3) \tag{66}$$

Using Feynman parameter the denominator becomes

$$\begin{aligned}&x_1[(p_1-k)^2 - m_s^2] + x_2[k^2 - m_{H^+}^2] + x_3[(k-p_3)^2 - m_{H^+}^2]\\ &= k^2 - 2(x_1p_1 + x_3p_3)\cdot k + x_1(m_u^2 - m_s^2) - (x_2+x_3)m_{H^+}^2 + x_3p_3^2\\ &= l^2 - (x_1p_1 + x_3p_3)^2 + x_1(m_u^2 - m_s^2) - (x_2+x_3)m_{H^+}^2 + x_3p_3^2\\ &= l^2 - \Delta\end{aligned} \tag{67}$$

where

$$\begin{aligned}l &= k - (x_1p_1 + x_3p_3),\\ \Delta &= (x_1p_1 + x_3p_3)^2 - x_1(m_u^2 - m_s^2) + (x_2+x_3)m_{H^+}^2 - x_3p_3^2\end{aligned} \tag{68}$$

So we have

$$\Gamma^\mu = \int_0^1 dx_i\delta(x_i - 1)2\int\frac{d^4k}{(2\pi)^4}\frac{(a+b\gamma^5)(\not{p}_1 - \not{k} + m_s)(a^* - b^*\gamma^5)}{(l^2-\Delta)^3}(2k-p_3)^\mu \tag{69}$$

The numerator the becomes

$$
\begin{aligned}
\text{Num} &= (\not{p}_1 - \not{k} + m_s)(2k - p_3)_\mu \\
&= (\not{p}_1 - x_1\not{p}_1 - x_3\not{p}_3 + m_s - \not{l})(2l + 2x_1p_1 + 2x_3p_3 - p_3) \\
&= -2\not{l}l + (\not{p}_1 - x_1\not{p}_1 - x_3\not{p}_3 + m_s)(2x_1p_1 + 2x_3p_3 - p_3)
\end{aligned} \tag{70}
$$

We have drop the term which has only one l because

$$
\begin{aligned}
&\int \frac{d^4\ell}{(2\pi)^4}\frac{\ell^\mu}{D^3} = 0; \\
&\int \frac{d^4\ell}{(2\pi)^4}\frac{\ell^\mu\ell^\nu}{D^3} = \int \frac{d^4\ell}{(2\pi)^4}\frac{\frac{1}{4}g^{\mu\nu}\ell^2}{D^3}.
\end{aligned} \tag{71}
$$

Only the second term will have a contribution to de EDM so

$$
\Gamma^\mu \subset \int_0^1 dx_i\delta(x_i - 1)2\int \frac{d^4k}{(2\pi)^4}\frac{(a + b\gamma^5)(\not{p}_1 - x_1\not{p}_1 - x_3\not{p}_3 + m_s)(2x_1p_1 + 2x_3p_3 - p_3)(a^* - b^*\gamma^5)}{(l^2 - \Delta)^3} \tag{72}
$$

The term which has only one γ^5 has the contribution to EDM, because other term is proportional to γ^μ.

$$
\begin{aligned}
&- ab^*(\not{p}_1 - x_1\not{p}_1 - x_3\not{p}_3 + m_s)(2x_1p_1 + 2x_3p_3 - p_3)_\mu\gamma^5 \\
&= -ab^*[(x_3 - x_2)m_u + m_s][(2x_1 + 2x_3 - 1)p_1 + (1 - 2x_3)p_2]_\mu\gamma^5 \\
&= -ab^*[(x_3 - x_2)m_u + m_s][x_1(p_1 + p_2) + (x_1 + 2x_3 - 1)(p_1 - p_2)]_\mu\gamma^5
\end{aligned} \tag{73}
$$

where we have used Dirac equation and $p_3 = p_1 - p_2$.Similarly we have

$$
\begin{aligned}
&a^*b\gamma^5(\not{p}_1 - x_1\not{p}_1 - x_3\not{p}_3 + m_s)(2x_1p_1 + 2x_3p_3 - p_3)_\mu \\
&= a^*b[(x_2 - x_3)m_u + m_s][x_1(p_1 + p_2) + (x_1 + 2x_3 - 1)(p_1 - p_2)]_\mu\gamma^5
\end{aligned} \tag{74}
$$

Using

$$
\bar{u}(p_2)\sigma_{\mu\nu}(p_1 - p_2)^\nu\gamma_5u(p_1) = -i\bar{u}(p_2)(p_1 + p_2)^\mu\gamma_5u(p_1) \tag{75}
$$

We have

$$
\begin{aligned}
\Gamma_\mu \subset &\int_0^1 dx_i\delta(x_i - 1)2\int \frac{d^4k}{(2\pi)^4}\frac{1}{(l^2 - \Delta)^3} \\
&\{a^*b[(x_2 - x_3)m_u + m_s]x_1 - ab^*[(x_3 - x_2)m_u + m_s]x_1\}\, i\sigma_{\mu\nu}p_3^\nu\gamma_5
\end{aligned} \tag{76}
$$

So we have u quark EDM is

$$
d_{us} = \frac{Qe}{4}\int_0^1 dx_i2\int \frac{d^4k}{(2\pi)^4}\frac{1}{(l^2 - \Delta)^3}\{a^*b[(x_2 - x_3)m_u + m_s]x_1 - ab^*[(x_3 - x_2)m_u + m_s]x_1\} \tag{77}
$$

Because

$$
\int \frac{d^4\ell}{(2\pi)^4}\frac{1}{[\ell^2 - \Delta]^m} = \frac{i(-1)^m}{(4\pi)^2}\frac{1}{(m-1)(m-2)}\frac{1}{\Delta^{m-2}}. \tag{78}
$$

So we have

$$
d_{us} = \frac{-i}{(4\pi)^2}\frac{Qe}{4}\int_0^1 dx_i\frac{2}{\Delta}x_1\{Re(a^*b)[(x_2 - x_3)m_u] + iIm(a^*b)m_s\} \tag{79}
$$

where

$$\begin{aligned}\Delta &= (x_1p_1 + x_3p_3)^2 - x_1(m_u^2 - m_s^2) + (x_2 + x_3)m_{H^+}^2 - x_3p_3^2 \\ &= x_1^2m_u^2 + 2x_1x_3p_1\cdot p_3 - x_1(m_{us}^2) + (1 - x_1)m_{H^+}^2 + x_3^2p_3^2 - x_3p_3^2 \\ &= x_1^2m_u^2 - x_1(m_{us}^2) + (1 - x_1)m_{H^+}^2\end{aligned} \tag{80}$$

At the last step we let $p_3 \to 0$.Because

$$\int_0^1 dx_1 \int_0^{1-x_1} dx_2(2x_2 + x_1 - 1) = 0 \tag{81}$$

So RE(a^*b) term is 0, we have

$$\begin{aligned}d_{us} &= \frac{1}{(4\pi)^2}\frac{Qe}{2}\int_0^1 dx_i \frac{1}{\Delta}x_1\mathrm{Im}(a^*b)m_s \\ &= \frac{\mathrm{Im}(a^*b)m_s}{(4\pi)^2}\frac{Qe}{2}\int_0^1 dx_1\int_0^{1-x_1} dx_2\frac{x_1}{x_1^2m_u^2 - x_1(m_{us}^2) + (1 - x_1)m_{H^+}^2} \\ &= \frac{\mathrm{Im}(a^*b)m_s}{(4\pi)^2}\frac{Qe}{2m_{H^+}^2}\int_0^1 dx_1\int_0^{1-x_1} dx_2\frac{x_1}{x_1^2A^2 - x_1B + 1}\end{aligned} \tag{82}$$

where

$$A^2 = \frac{m_{u^2}}{m_{H^+}^2}, \qquad B - 1 = \frac{m_{us}^2}{m_{H^+}^2} = \frac{m_u^2 - m_s^2}{m_{H^+}^2} = A^2 - C^2 \tag{83}$$

are small quantities because the mass of H^+ is GeV scale.

$$\begin{aligned}&\int_0^1 dx\frac{x(1-x)}{x_1^2A^2 - x_1B + 1} \\ &= -\frac{1}{A^2} + \frac{C^2 - 1}{2A^4}\ln(C^2) + \frac{1}{A^2}\left[1 + \frac{(C^2 - 1)(A^2 - C^2 + 1)}{2A^2}\right]\frac{1}{\sqrt{1 - 2(A^2 + C^2) + (A^2 - C^2)^2}} \\ &\quad \ln\left(\frac{1 + (A^2 - C^2)^2 - 2A^2 + (1 - A^2 + C^2)\sqrt{B^2 - 4A^2}}{2C^2}\right)\end{aligned} \tag{84}$$

Taylor expansion

$$\begin{aligned}&\frac{1}{A^2}\left[1 + \frac{(C^2 - 1)(A^2 - C^2 + 1)}{2A^2}\right] = -\frac{1}{2A^4} + \frac{1}{2A^2} + \frac{C^2}{2A^2} - \frac{C^4}{2A^4} + \frac{C^2}{A^4} \\ &\frac{1}{\sqrt{1 - 2(A^2 + C^2) + (A^2 - C^2)^2}} = 1 + C^2 + A^2 + A^4 + 4A^2C^2 + C^4 + O(A^4) \\ &\ln\left(\frac{1 + (A^2 - C^2)^2 - 2A^2 + (1 - A^2 + C^2)\sqrt{B^2 - 4A^2}}{2}\right) = -2A^2 - A^4 - 2A^2C^2 + O(A^4)\end{aligned} \tag{85}$$

So we have

$$\frac{1}{A^2}\left[1 + \frac{(C^2 - 1)(A^2 - C^2 + 1)}{2A^2}\right]\frac{1}{\sqrt{1 - 2(A^2 + C^2) + (A^2 - C^2)^2}} = -\frac{1}{2A^4} + \frac{C^2}{2A^4} + O(1) \tag{86}$$

So we have

$$
\begin{aligned}
&\int_0^1 dx \frac{x(1-x)}{x_1^2 A^2 - x_1 B + 1} \\
&= -\frac{1}{A^2} + \frac{C^2-1}{2A^4}\ln(C^2) + \frac{1}{A^2}\left[1 + \frac{(C^2-1)(A^2-C^2+1)}{2A^2}\right] \frac{1}{\sqrt{1-2(A^2+C^2)+(A^2-C^2)^2}} \\
&\quad \ln\left(\frac{1+(A^2-C^2)^2 - 2A^2 + (1-A^2+C^2)\sqrt{B^2-4A^2}}{2C^2}\right) \\
&= \frac{1}{2} + O(1) + O(1)ln(C^2)
\end{aligned}
\tag{87}
$$

So we have

$$
d_{us} = \frac{\mathrm{Im}(a^* b) m_s}{(4\pi)^2} \frac{Qe}{4m_{H^+}^2} \tag{88}
$$

where

$$
\begin{aligned}
a &= V_{ub} Y_2^{D,bs} - Y_2^{U,uu*} V_{us} \\
b &= V_{ub} Y_2^{D,bs} + Y_2^{U,uu*} V_{us}
\end{aligned}
\tag{89}
$$

3.2 udu Condition

$$
\begin{aligned}
\mathcal{L} \supset &- [V_{us} Y_2^{D,sd} \bar{u} P_R d + V_{ud} Y_2^{dd} \bar{u} P_R d - Y_2^{U,uu*} V_{ud} \bar{u} P_L d] H^+ + H.C \\
&= -\bar{u}\left(V_{us} Y_2^{D,sd} P_R + V_{ud} Y_2^{dd} P_R - Y_2^{U,uu*} V_{ud} P_L\right) dH^+ \\
&\quad - \bar{d}\left(V_{us}^* Y_2^{D,sd*} P_L + V_{ud}^* Y_2^{dd*} P_L - Y_2^{U,uu} V_{ud}^* P_R\right) uH^- \\
&\quad - iQeA_\mu (H^- \partial H^+ - \partial H^- H^+).
\end{aligned}
\tag{90}
$$

So we have

$$
d_{ud} = \frac{\mathrm{Im}(a^* b) m_d}{(4\pi)^2} \frac{Qe}{4m_{H^+}^2} \tag{91}
$$

where

$$
\begin{aligned}
a &= V_{us} Y_2^{D,sd} + V_{ud} Y_2^{dd} - Y_2^{U,uu*} V_{ud} \\
b &= V_{us} Y_2^{D,sd} + V_{ud} Y_2^{dd} + Y_2^{U,uu*} V_{ud}
\end{aligned}
\tag{92}
$$

3.3 dud Condition

$$
\begin{aligned}
\mathcal{L} \supset &- [V_{us} Y_2^{D,sd} \bar{u} P_R d + V_{ud} Y_2^{dd} \bar{u} P_R d - Y_2^{U,uu*} V_{ud} \bar{u} P_L d] H^+ + H.C \\
&= -\bar{u}\left(V_{us} Y_2^{D,sd} P_R + V_{ud} Y_2^{dd} P_R - Y_2^{U,uu*} V_{ud} P_L\right) dH^+ \\
&\quad - \bar{d}\left(V_{us}^* Y_2^{D,sd*} P_L + V_{ud}^* Y_2^{dd*} P_L - Y_2^{U,uu} V_{ud}^* P_R\right) uH^- \\
&\quad - iQeA_\mu (H^- \partial H^+ - \partial H^- H^+).
\end{aligned}
\tag{93}
$$

The Lagrangain is the same with udu condition. So we have

$$
d_{du} = \frac{\mathrm{Im}(ab^*) m_u}{(4\pi)^2} \frac{Qe}{4m_{H^+}^2} = -\frac{\mathrm{Im}(a^* b) m_u}{(4\pi)^2} \frac{Qe}{4m_{H^+}^2} \tag{94}
$$

where

$$
\begin{aligned}
a &= V_{us} Y_2^{D,sd} + V_{ud} Y_2^{dd} - Y_2^{U,uu*} V_{ud} \\
b &= V_{us} Y_2^{D,sd} + V_{ud} Y_2^{dd} + Y_2^{U,uu*} V_{ud}
\end{aligned}
\tag{95}
$$

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$$
\begin{aligned}
d_n &= \frac{4}{3}d_d - \frac{1}{3}d_u \\
&= -\frac{4}{3}\frac{\mathrm{Im}(a^*b)m_u}{(4\pi)^2}\frac{Qe}{4m_{H^+}^2} - \frac{1}{3}\left(\frac{\mathrm{Im}(a^*b)m_d}{(4\pi)^2}\frac{Qe}{4m_{H^+}^2} + \frac{\mathrm{Im}(a_s^*b_s)m_s}{(4\pi)^2}\frac{Qe}{4m_{H^+}^2}\right) \\
&= \frac{Qe}{4(4\pi)^2 m_{H^+}^2}\left[-\frac{4}{3}\mathrm{Im}(a^*b)m_u - \frac{1}{3}\mathrm{Im}(a^*b)m_d - \frac{1}{3}\mathrm{Im}(a_s^*b_s)m_s\right]
\end{aligned}
\tag{96}
$$

where

$$
\begin{aligned}
a &= V_{us}Y_2^{D,sd} + V_{ud}Y_2^{dd} - Y_2^{U,uu*}V_{ud} \\
b &= V_{us}Y_2^{D,sd} + V_{ud}Y_2^{dd} + Y_2^{U,uu*}V_{ud} \\
a_s &= V_{ub}Y_2^{D,bs} - Y_2^{U,uu*}V_{us} \\
b_s &= V_{ub}Y_2^{D,bs} + Y_2^{U,uu*}V_{us}
\end{aligned}
\tag{97}
$$

Calculate Im part.

$$
\begin{aligned}
\mathrm{Im}(a_s^*b) &= 2\mathrm{Im}(V_{ub}^*V_{us}Y^{bs*}Y^{uu*}) \\
\mathrm{Im}(a^*b) &= 2\mathrm{Im}(V_{us}^*V_{ud}Y^{sd*}Y^{uu*}) + 2|V_{ud}|^2\mathrm{Im}(Y^{dd*}Y^{uu*})
\end{aligned}
\tag{98}
$$

We have

$$
|Y_2^{D,bs}| \approx 2.3\times 10^{-4}\frac{1}{|\lambda_3|}\left(\frac{m_H}{TeV}\right)^2 \ , \quad |Y_2^{D,sd}| \approx 5.7\times 10^{-8}\frac{1}{|\lambda_3|}\left(\frac{m_H}{TeV}\right)^2 \ ,
$$
$$
|Y_2^{D,dd}| \approx 1.6\times 10^{-1}\frac{1}{|\lambda_3|}\left(\frac{m_H}{TeV}\right)^2 \ , \quad |Y_2^{U,uu}| \approx 1.2\times 10^{-1}\frac{1}{|\lambda_3|}\left(\frac{m_H}{TeV}\right)^2 \ .
$$

For CKM matrix V_{ud}, V_{us} are real. And

$$
|V_{ud}| = 0.97367\pm 0.00032, \quad |V_{us}| = 0.22431\pm 0.00085, \quad |V_{ub}| = (3.82\pm 0.20)\times 10^{-3} \tag{99}
$$

Set $Y^{dd} = |Y^{dd}|e^{i\theta}, Y^{uu} = |Y^{uu}|e^{i\beta}, Y^{bs} = |Y^{bs}|e^{i\alpha}, Y^{sd} = |Y^{sd}|e^{i\gamma}$. So we have

$$
\begin{aligned}
d_n &= -\frac{1}{3}\frac{Qe}{4(4\pi)^2 m_{H^+}^2}\frac{1}{\lambda_3^2}\left(\frac{m_H}{TeV}\right)^4 \\
&\Big[(4m_u+m_d)(2V_{us}V_{ud}(5.7\times 1.2\times 10^{-9})sin(-\gamma-\beta) + 2|V_{ud}|^2(1.6\times 1.2\times 10^{-2})sin(-\theta-\beta)) \\
&+2V_{ub}V_{us}(2.3\times 1.2\times 10^{-5})sin(\delta-\alpha-\beta)m_s\Big] \\
&= \frac{1}{3}\frac{Qe}{4(4\pi)^2 m_{H^+}^2}\frac{\Delta m_{Bs}}{3.055\times 10^{-3}\mathrm{ps}{-1}}\left(\frac{m_H}{TeV}\right)^2\frac{m_A^2}{m_A^2 - m_{H^2}} \\
&\Big[(4m_u+m_d)(2V_{us}V_{ud}(5.7\times 1.2\times 10^{-9})sin(\gamma+\beta) + 2|V_{ud}|^2(1.6\times 1.2\times 10^{-2})sin(\theta+\beta)) \\
&+2V_{ub}V_{us}(2.3\times 1.2\times 10^{-5})sin(-\delta+\alpha+\beta)m_s\Big]
\end{aligned}
\tag{100}
$$

At the last step we used eq(41).