Problem Set

The problem set is due on Monday April 16^{th}. Please, submit your codes by email, and type your answers in LaTeX.

Warm up. Consider the function $f(x, y) = e^x e^y$. Approximate this function on $\mathcal{X} = [0, 1] \times [-1, 0]$ using the projection algorithm discussed in class (use collocation with Chebyshev polynomials). Now approximate the function $g(x) = \max\{0, x - 1\}$ on $\mathcal{X} = [0, 2]$ using the same algorithm. Consider $n = p = 3$, $n = p = 5$, $n = p = 10$, $n = p = 20$. Plot these four approximations as a function of x, along with the true function.

The neoclassical growth model. Consider the neoclassical growth model. Final good firms use the production function $y_t = \exp\{z_t\} k_t^\alpha l_t^{1-\alpha}$. Households utility function is $u(c_t, l_t) = \log(c_t) - \chi l^{1+\nu^{-1}} / (1 + \nu^{-1})$.

1. Set $\alpha = 0.3$, $\nu = 2$, $\beta = 0.99$, $\delta = 0.025$, $\xi = 0.5$, $\rho_z = 0.95$ and $\sigma_z = 0.007$. Choose χ so that $l^{ss} = 0.33$.

2. Solve the model using a first order perturbation.

3. Write down a projection algorithm to solve the model. Please, comment on the state and control variables, bounds and order of the Chebyshev polynomials you use, initial guesses, the residual function, and the algorithm you use to find the zeros of the residual function. Implement your algorithm. Generate two plots: $\hat{c}(\hat{k}, z)$ as a function of $\hat{k}$ for $z = 0$, and $\hat{c}(\hat{k}, z)$ as a function of z, for $k = k^{ss}$.[1] Compare this policy function with the one you got in the perturbation solution.

The neoclassical growth model with occasionally binding financial constraints. Consider now the same model, but introduce a financial sector as in Gertler and Karadi (2011). For part 1-4, assume inelastic labor supply ($\chi = 0$), $\sigma = 0$ and full depreciation.

1. Set $\alpha = 0.3$, $\beta = 0.99$, $\psi = 0.98$, $\rho_z = 0.95$ and $\sigma_z = 0.007$. Choose (λ, ω) so that the Lagrange multiplier in a deterministic steady state equals 0.0001, and leverage of financial intermediaries equals 3.30.

[1] The notation $\hat{x}$ means log-deviation from the deterministic steady state of variable x.

2. Solve the model using a first order perturbation.

3. Write down a projection algorithm to solve the model. Please, comment on the state and control variables, bounds and order of the Chebyshev polynomials you use, the residual function, initial guesses, and the algorithm you use to find the zeros of the residual function. Implement your algorithm.

4. Perform a simulation from your model. Compute business cycle statistics, including statistics regarding the Lagrange multiplier on the bankers' leverage constraint (fraction of time it binds, etc.). Report statistical properties for the Euler Equation Errors.

5. Consider now the case in which $\sigma = 1$ and $\delta = 0.025$. Explain what you would change in your algorithm to solve this version of the model. Implement these changes and repeat part 4.