EECS 182 HW4

ECE 182 Homework Assignment

Design a MIC single stage amplifier using SMC. Follow the steps below to do your design.

Step	Task
1	Check amplifier stability at the input and output; if the device is not unconditionally stable, draw the stability circles at the input and output.
2	If the device is conditionally stable, add loss at the output to make it unconditionally stable.
3	Compute the new scattering parameters for the device with the resistor added for stability.
4	Calculate maximum stable gain of the new device (original plus resistor).
5	Compute Γ_{ML} and Γ_{MS} and select the corresponding passive loads.
6	Use the Smith chart to design the input and output matching networks using discrete L, C networks.
7	Design the bias network for the device including the decoupling capacitors.
8	Provide a good draw to scale of the circuit layout.

Device Parameters and Dimensions

Note: Angles are in degrees and magnitude in linear scale.

S-Parameter Table (5.0 GHz)

freq	magS11	angS11	magS21	angS21	magS12	angS12	magS22	angS22
5.0	0.4	23	2.0	11	0.2	46	0.839	-66

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77 Package

- **Pin configuration (top-down):**
  - **Pin 4:** SOURCE (top)
  - **Pin 1:** GATE (left)
  - **Pin 3:** DRAIN (right)
  - **Pin 2:** SOURCE (bottom)

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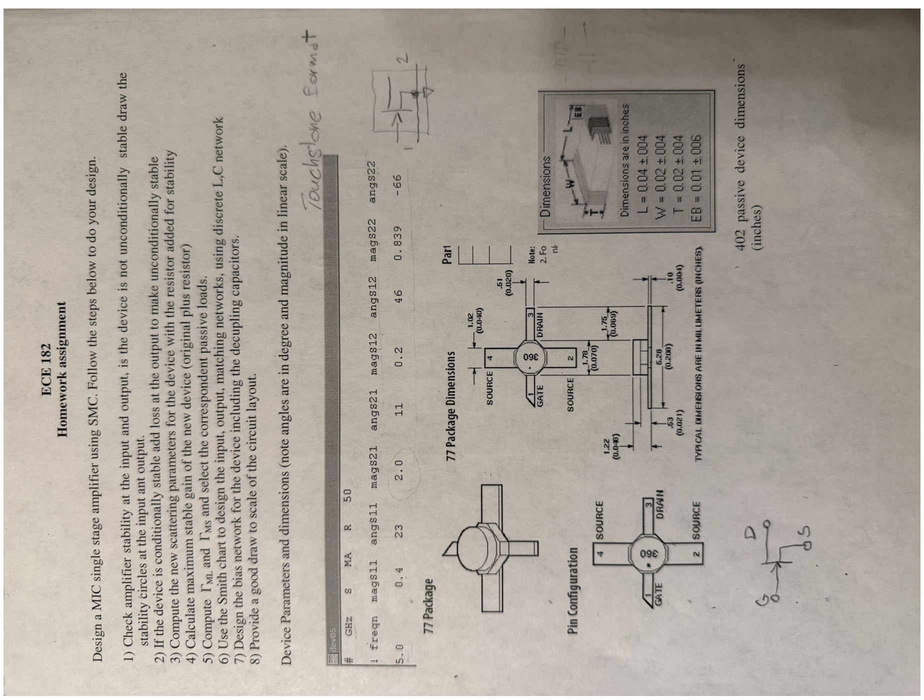
402 Passive Device Dimensions (inches)

Parameter	Value
L	0.04 ± 0.004
W	0.02 ± 0.004
T	0.02 ± 0.004
EB	0.01 ± 0.006



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1) Check amplifier stability at the input and output, is the device is not unconditionally stable draw the stability circles at the input ant output.
2) If the device is conditionally stable add loss at the output to make unconditionally stable
3) Compute the new scattering parameters for the device with the resistor added for stability
4) Calculate maximum stable gain of the new device (original plus resistor)
5) Compute Γ_{ML} and Γ_{MS} and select the correspondent passive loads.
6) Use the Smith chart to design the input, output, matching networks, using discrete L,C network
7) Design the bias network for the device including the decoupling capacitors.
8) Provide a good draw to scale of the circuit layout.

Device Parameters and dimensions (note angles are in degree and magnitude in linear scale).

Touchstone format

```
# GHz S MA R 50
! freqn magS11 angS11 magS21 angS21 magS12 angS12 magS22 angS22
5.0 0.4 23 2.0 11 0.2 46 0.839 -66
```


Figure 1: HW4 assignment

All numerical values are from problem_calc.py.

Step 1: Stability and Stability Circles

1.1 Complex S-Parameters

Convert from magnitude and angle (linear scale, degrees) using $S_{ij} = |S_{ij}|\, e^{j\theta_{ij}}$ with angles in radians. At 5.0 GHz:

Parameter	Rectangular form $a + jb$
S_{11}	$0.368 + j0.156$
S_{21}	$1.963 + j0.382$
S_{12}	$0.139 + j0.144$
S_{22}	$0.341 - j0.766$

1.2 Stability Factor K

Definitions (Rollett):

$$\Delta = S_{11}S_{22} - S_{12}S_{21}$$

$$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2\,|S_{12}S_{21}|}$$

Numerical values: - $\Delta = S_{11}S_{22} - S_{12}S_{21} = 0.0276 - j0.564$ - $|\Delta| = 0.565$ - $K = 0.569$

Stability criterion (unconditional stability): $K > 1$ and $|\Delta| < 1$.

Here $K = 0.569 < 1$, so the device is **conditionally stable**. The input and output stability circles must be drawn to identify safe source and load terminations.

1.3 Input Stability Circle (Load Plane, Γ_L)

Locus of Γ_L for which $|\Gamma_{\text{in}}| = 1$.

Center:

$$C_L = \frac{(S_{22} - \Delta S_{11}^*)^*}{|S_{22}|^2 - |\Delta|^2}$$

Radius:

$$r_L = \frac{|S_{12}S_{21}|}{||S_{22}|^2 - |\Delta|^2|}$$

Numerical results: - $C_L = 1.09 + j1.44$ (or $|C_L| \approx 1.81$, angle $\approx 53°$) - $r_L = 1.04$

1.4 Output Stability Circle (Source Plane, Γ_S)

Locus of Γ_S for which $|\Gamma_{\text{out}}| = 1$.

Center:

$$C_S = \frac{(S_{11} - \Delta S_{22}^*)^*}{|S_{11}|^2 - |\Delta|^2}$$

Radius:

$$r_S = \frac{|S_{12}S_{21}|}{||S_{11}|^2 - |\Delta|^2|}$$

Numerical results: - $C_S = 0.463 + j2.06$ (or $|C_S| \approx 2.11$, angle $\approx 77°$) - $r_S = 2.51$

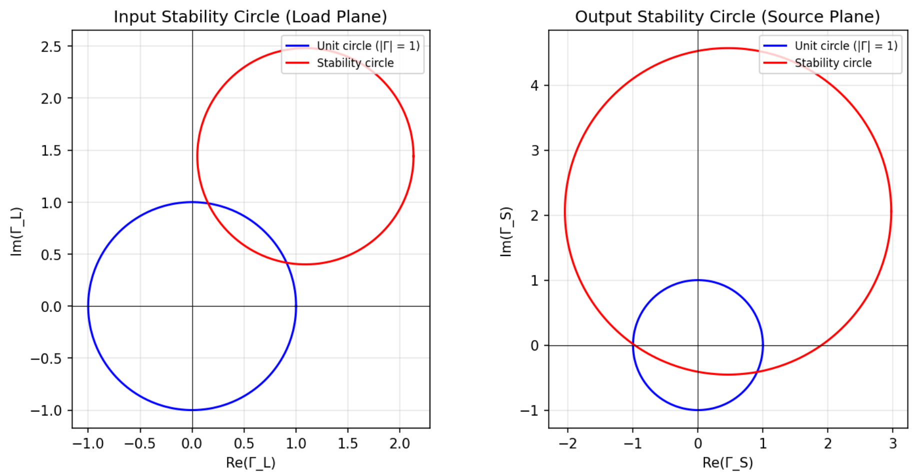


Figure 2: Stability circles (input and output)

Steps 2–3: Output Loss and New S-Parameters

Add loss at the output so the composite two-port is unconditionally stable, then compute its scattering parameters. The loss is modeled as a **shunt resistor** R at the device output (reference $Z_0 = 50\,\Omega$). The composite two-port is: [Device] → [Shunt R] → load.

2.1 Minimum loss for stability

Using `problem_calc.py`, I swept the shunt resistance to find the **largest** R (i.e., the **minimum loss**) such that the composite satisfies $K' \geq 1$ and $|\Delta'| < 1$.

At the stability boundary:

- $R \approx 299.3\,\Omega$ (marginally stable: $K' = 1$)
- $K' \approx 1.00$
- $|\Delta'| \approx 0.476$

I chose a different value for the design: $R = 50\,\Omega$, which gives $K' > 1$ and makes the composite **unconditionally stable**. All following calculations use $R = 50\,\Omega$.

2.2 Output Loss (Shunt Resistor)

With $Z_0 = 50\,\Omega$ and shunt resistance R, the two-port S-parameters of the resistor block are:

$$S_{11}^{(R)} = S_{22}^{(R)} = \frac{-Z_0}{2R + Z_0}, \qquad S_{12}^{(R)} = S_{21}^{(R)} = \frac{2R}{2R + Z_0}$$

Choice: $R = 50\,\Omega$ (chosen for unconditional stability; boundary is $R \approx 299.3\,\Omega$).

Numerical values: - $S_{11}^{(R)} = S_{22}^{(R)} = -\frac{1}{3} \approx -0.333$ - $S_{12}^{(R)} = S_{21}^{(R)} = \frac{2}{3} \approx 0.667$

2.3 Cascade via T-Parameters

Device output is connected to resistor input. The cascade is computed using T-parameters: $\mathbf{T}_{\text{cascade}} = \mathbf{T}_{\text{device}} \cdot \mathbf{T}_{\text{resistor}}$, then convert back to S.

S $\rightarrow$ T (for each two-port):

$$T_{11} = \frac{-\det(\mathbf{S})}{S_{21}}, \quad T_{12} = \frac{S_{11}}{S_{21}}, \quad T_{21} = \frac{-S_{22}}{S_{21}}, \quad T_{22} = \frac{1}{S_{21}}$$

T $\rightarrow$ S (for composite):

$$S'_{11} = \frac{T_{12}}{T_{22}}, \quad S'_{21} = \frac{1}{T_{22}}, \quad S'_{12} = \frac{\det(\mathbf{T})}{T_{22}}, \quad S'_{22} = \frac{-T_{21}}{T_{22}}, \quad \det(\mathbf{T}) = T_{11}T_{22} - T_{12}T_{21}$$

2.4 New S-Parameters (Composite: Device + Resistor)

Parameter	Rectangular form $a + jb$
S'_{11}	$0.328 + j0.047$
S'_{21}	$1.07 + j0.473$
S'_{12}	$0.060 + j0.100$

Parameter	Rectangular form $a + jb$
S'_{22}	$-0.137 - j0.261$

2.5 Stability Verification

$$\Delta' = S'_{11}S'_{22} - S'_{12}S'_{21}, \qquad K' = \frac{1 - |S'_{11}|^2 - |S'_{22}|^2 + |\Delta'|^2}{2\,|S'_{12}S'_{21}|}$$

Numerical values: - $\Delta' = -0.050 - j0.227$ - $|\Delta'| \approx 0.233$ - $K' \approx 3.15$

Conclusion: With $R = 50\,\Omega$, $K' > 1$ and $|\Delta'| < 1$, so the composite is **unconditionally stable**. (The boundary value is $R \approx 299.3\,\Omega$, where $K' = 1$.)

Step 4: Maximum Stable Gain of the Composite Device

For the stabilized device (original plus shunt resistor), the **maximum stable gain** is

$$G_{\mathrm{MSG}} = \frac{|S'_{21}|}{|S'_{12}|}\left(K' - \sqrt{K'^2 - 1}\right)$$

with $K' > 1$ and $|\Delta'| < 1$. Using the composite S-parameters and K' from Steps 2–3,

- $G_{\mathrm{MSG}} = 1.63$ (linear)
- $G_{\mathrm{MSG}} \approx 4.25\,\mathrm{dB}$

Step 5: $\Gamma_{\mathbf{ML}}$ and $\Gamma_{\mathbf{MS}}$ (Simultaneous Conjugate Match)

For the unconditionally stable composite device, the simultaneous conjugate match gives the source and load reflection coefficients that maximize transducer gain. Using the composite S-parameters S'_{ij} and $\Delta' = S'_{11}S'_{22} - S'_{12}S'_{21}$:

Source side (input match):

$$B_1 = 1 + |S'_{11}|^2 - |S'_{22}|^2 - |\Delta'|^2, \qquad C_1 = S'_{11} - \Delta'(S'_{22})^*$$

$$\Gamma_{\mathrm{MS}} = \frac{B_1 - \sqrt{B_1^2 - 4|C_1|^2}}{2C_1}$$

(The minus sign in front of the square root is chosen so that $|\Gamma_{\mathrm{MS}}| < 1$ for a passive source termination.)

Load side (output match):

$$B_2 = 1 + |S'_{22}|^2 - |S'_{11}|^2 - |\Delta'|^2, \qquad C_2 = S'_{22} - \Delta'(S'_{11})^*$$

$$\Gamma_{\mathrm{ML}} = \frac{B_2 - \sqrt{B_2^2 - 4|C_2|^2}}{2C_2}$$

(The minus sign gives $|\Gamma_{\mathrm{ML}}| < 1$ for a passive load.)

Numerical values (from `problem_calc.py`, composite with $R = 50\,\Omega$):

Quantity	Value	$\lvert\Gamma\rvert$
Γ_{MS}	$0.294 - j0.032$	0.296
Γ_{ML}	$-0.127 + j0.217$	0.252

Conjugate-match impedances ($Z = Z_0(1+\Gamma)/(1-\Gamma)$ with $Z_0 = 50\,\Omega$):

Quantity	Value (Ohm)	Normalized $z = Z/Z_0$
Z_{MS}	$91.38 - j6.41$	$1.828 - j0.128$
Z_{ML}	$35.54 + j16.50$	$0.711 + j0.330$

Both $|\Gamma_{\mathrm{MS}}|$ and $|\Gamma_{\mathrm{ML}}|$ are less than 1, so the corresponding terminations are **passive** and realizable. I select these as the design values for the input and output matching networks: the source matching network should present Γ_{MS} to the composite device input, and the load matching network should present Γ_{ML} to the composite device output (or equivalently, the load impedance that gives reflection Γ_{ML} when looking into the output port).

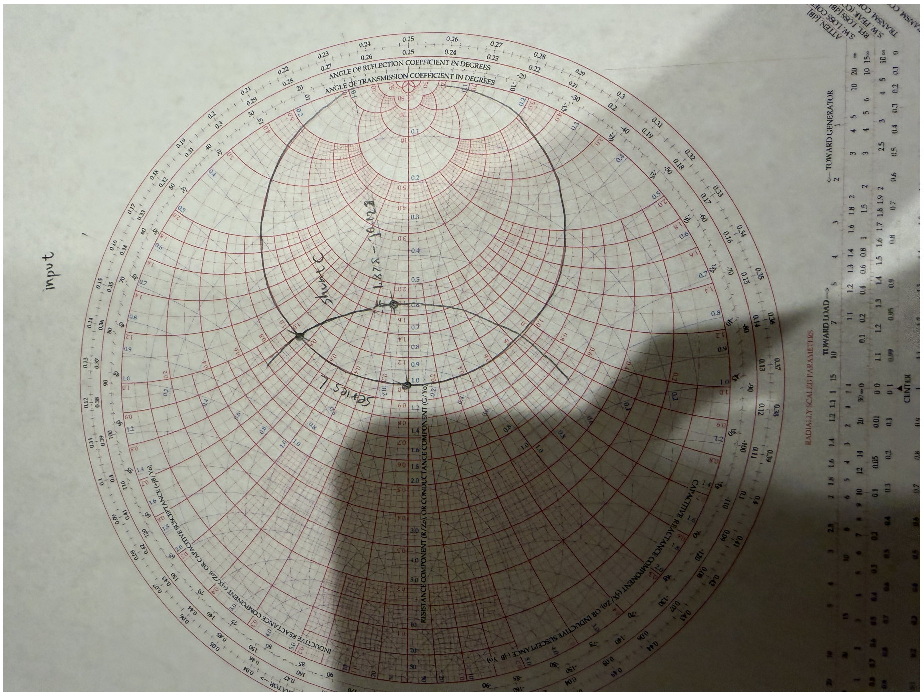
Step 6: Input and Output Matching Networks (Smith Chart)

Design frequency $f = 5\,\mathrm{GHz}$, $Z_0 = 50\,\Omega$. The matching networks transform 50 Ω to the conjugate-match impedances Z_{MS} (input) and Z_{ML} (output) using two-element L-sections.

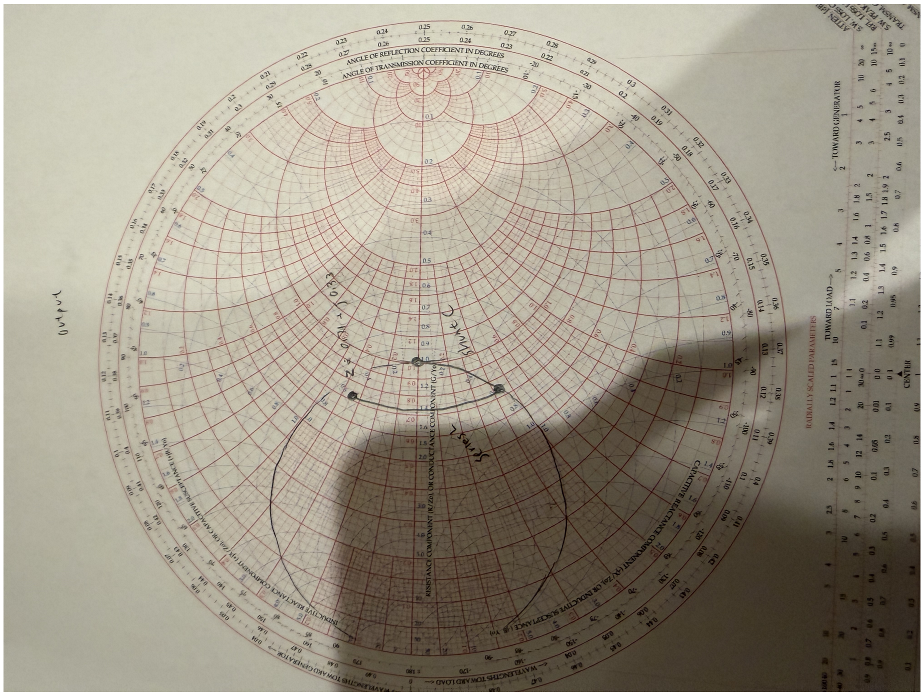
Target impedances (for the chart)

- **Input:** Match 50 Ω to $Z_{\mathrm{MS}} = 91.38 - j6.41\,\Omega$. Normalized $z_{\mathrm{MS}} = 1.828 - j0.128$.
- **Output:** Match 50 Ω to $Z_{\mathrm{ML}} = 35.54 + j16.50\,\Omega$. Normalized $z_{\mathrm{ML}} = 0.711 + j0.330$.

Input matching (50 $\Omega \to Z_{\mathrm{MS}}$)



Output matching (50 Ω → $Z_{\mathbf{ML}}$)



Component values (from `problem_calc.py`, $f = 5\,$GHz)

Network	Topology	Element 1	Element 2
Input (50 Ω → Z_{MS})	series L, shunt C	$L = 1.46\,$nH (series)	$C = 0.34\,$pF (shunt)
Output (50 Ω → Z_{ML})	shunt C, series L	$C = 0.41\,$pF (shunt)	$L = 1.25\,$nH (series)

Step 7: Bias Network and Decoupling Capacitors

The bias network supplies DC gate bias to the device while isolating RF at the bias ports. Decoupling capacitors are used to short RF to ground at the bias nodes so that the matching networks see only the intended impedances. In practice, the bias **C** and **L** values are chosen so that their reactances at 5.0 GHz are extreme compared to 50 Ω (e.g., $|X_C| \ll 50\,\Omega$ for RF bypass capacitors and $|X_L| \gg 50\,\Omega$ for RF chokes), so they do not significantly disturb the small-signal matching.

Step 8: Circuit Layout (Draw to Scale)

The figure below shows a block diagram of the microwave amplifier with input matching, output matching (including the stability resistor), and the bias network with quarter-wave transformer and DC/RF blocking elements, plus the common-source MOSFET schematic with gate bias V_G, drain resistor R, and load Z_L. At the design frequency 5.0 GHz, the free-space quarter-wave transformer length is

$$\ell_{\lambda/4} = \frac{\lambda_0}{4} = \frac{c}{4f} \approx 15\,\text{mm} \approx 0.59\,\text{in},$$

where $c \approx 3.0 \times 10^8$ m/s is the speed of light. On a MIC substrate with effective permittivity ε_{eff}, the physical line length is shorter by $\ell_{\lambda/4}/\sqrt{\varepsilon_{\text{eff}}}$.

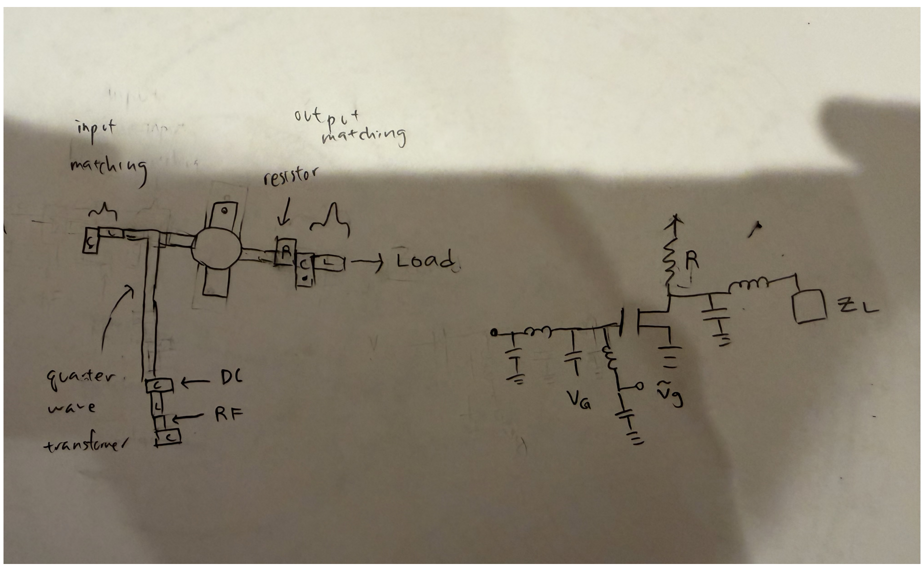


Figure 3: Bias network and circuit layout