Math 165 Midterm Exam   Section:_____   Name:_____

Part A: Pencil & Paper Only

This part of the exam has 10 problems on 3 pages. Each problem is worth 5 points. You may NOT use a calculator on this section. You must show all work. This part of the exam will be collected after 45 minutes.

1. Find dy/dx if
$$y = \frac{3x^3}{5x^2 + 2}.$$
   You do not need to simplify your answer.

   Solution:
   $$\frac{dy}{dx} = \frac{(9x^2)(5x^2 + 2) - (3x^3)(10x)}{(5x^2 + 2)^2}$$

2. Suppose
$$g(x) = \frac{2}{x^2}.$$
   Find $g''(2)$. Simplify your answer.

   Solution:
   $$g'(x) = -\frac{4}{x^3} \qquad g''(x) = \frac{12}{x^4} \qquad g''(2) = \frac{3}{4}$$

3. If
$$y = t^3 \ln t$$
   find dy/dt. Simplify your answer.

   Solution:
   $$\begin{aligned} \frac{dy}{dx}(t^3 \ln t) &= 3t^2 \ln t + t^3 \frac{1}{t} \\ &= 3t^2 \ln t + t^2 \\ &= t^2(3 \ln t + 1) \end{aligned}$$

4. Find
$$\lim_{x \to 1^-} \frac{x^2 + 4x + 3}{x^2 - 1}$$

Solution:

$$\lim_{x \to 1^-} \frac{x^2 + 4x + 3}{x^2 - 1} = \lim_{x \to 1^-} \frac{(x+1)(x+3)}{(x+1)(x-1)} = \lim_{x \to 1^-} \frac{x+3}{x-1} = -\infty$$

5. Let
$$y = \frac{3x^2 - 5x - 2}{x^2 - 4}.$$

Give the equations of all the horizontal asymptotes.

Solution:

$$\lim_{x \to \infty} \frac{3x^2 - 5x - 2}{x^2 - 4} = \lim_{x \to \infty} \frac{3 - 5/x - 2/x^2}{1 - 4/x^2} = \frac{3 - 0 - 0}{1 - 0} = 3$$

Therefore $y = 3$ is the horizontal asymptote.

6. Let
$$y = \frac{3x^2 - 5x - 2}{x^2 - 4}$$

Give the equations of all the vertical asymptotes.

Solution:
$$y = \frac{3x^2 - 5x - 2}{x^2 - 4} = \frac{(x-2)(3x+1)}{(x-2)(x+2)} = \frac{3x+1}{x+2}$$

The function is not defined when the denominator is 0, therefore, $x = -2$ is the vertical asymptote.

7. Find the derivative of $4x^3 \tan^2(x)$. You do not need to simplify your answer.

   Solution:

8. Find the slope of the line tangent to the graph of $y = \sqrt{x^2 + 4x - 5}$ at $x = 3$. Simplify your answer.

   Solution:

9. Suppose $f(x) = \sin^4(3e^x + 1)$. Find $f'(x)$. You do not need to simplify your answer.

   Solution:

10. Let $f(x) = x^3 + 2x^2 + 1$ for $x \geq 0$. Using that $f(1) = 4$, find $(f^{-1})'(4)$.

    Solution: