Math 165 Midterm Exam Section:_________ Name:_______________

**Part A: Pencil & Paper Only**

This part of the exam has 10 problems on 3 pages. Each problem is worth 5 points. You may NOT use a calculator on this section. You must show all work. This part of the exam will be collected after 45 minutes.

1. Find $dy/dx$ if
$$y = \frac{3x^3}{5x^2+2}.$$
You do not need to simplify your answer.

**Solution:**

$$\frac{dy}{dx} = \frac{(9x^2)(5x^2+2) - (3x^3)(10x)}{(5x^2+2)^2}$$

2. Suppose
$$g(x) = \frac{2}{x^2}.$$
Find $g''(2)$. Simplify your answer.

**Solution:**
$$g'(x) = -\frac{4}{x^3} \qquad g''(x) = \frac{12}{x^4} \qquad g''(2) = \frac{3}{4}$$

3. If
$$y = t^3 \ln t$$
find $dy/dt$. Simplify your answer.

**Solution:**
$$\begin{aligned}\frac{dy}{dx}(t^3 \ln t) &= 3t^2 \ln t + t^3 \frac{1}{t} \\ &= 3t^2 \ln t + t^2 \\ &= t^2(3\ln t + 1)\end{aligned}$$

4. Find
$$\lim_{x\to 1^-}\frac{x^2+4x+3}{x^2-1}$$

**Solution:**
$$\lim_{x\to 1^-}\frac{x^2+4x+3}{x^2-1}=\lim_{x\to 1^-}\frac{(x+1)(x+3)}{(x+1)(x-1)}=\lim_{x\to 1^-}\frac{x+3}{x-1}=-\infty$$

5. Let
$$y=\frac{3x^2-5x-2}{x^2-4}.$$
Give the equations of all the horizontal asymptotes.

**Solution:**
$$\lim_{x\to\infty}\frac{3x^2-5x-2}{x^2-4}=\lim_{x\to\infty}\frac{3-5/x-2/x^2}{1-4/x^2}=\frac{3-0-0}{1-0}=3$$
Therefore $y=3$ is the horizontal asymptote.

6. Let
$$y=\frac{3x^2-5x-2}{x^2-4}$$
Give the equations of all the vertical asymptotes.

**Solution:**
$$y=\frac{3x^2-5x-2}{x^2-4}=\frac{(x-2)(3x+1)}{(x-2)(x+2)}=\frac{3x+1}{x+2}$$
The function is not defined when the denominator is 0, therefore, $x=-2$ is the vertical asymptote.

7. Find the derivative of $4x^3 \tan^2(x)$. You do not need to simplify your answer.

   **Solution:**

8. Find the slope of the line tangent to the graph of $y = \sqrt{x^2 + 4x - 5}$ at $x = 3$. Simplify your answer.

   **Solution:**

9. Suppose $f(x) = \sin^4(3e^x + 1)$. Find $f'(x)$. You do not need to simplify your answer.

   **Solution:**

10. Let $f(x) = x^3 + 2x^2 + 1$ for $x \geq 0$. Using that $f(1) = 4$, find $(f^{-1})(4)$.

    **Solution:**