VV186 Honors Mathematics II

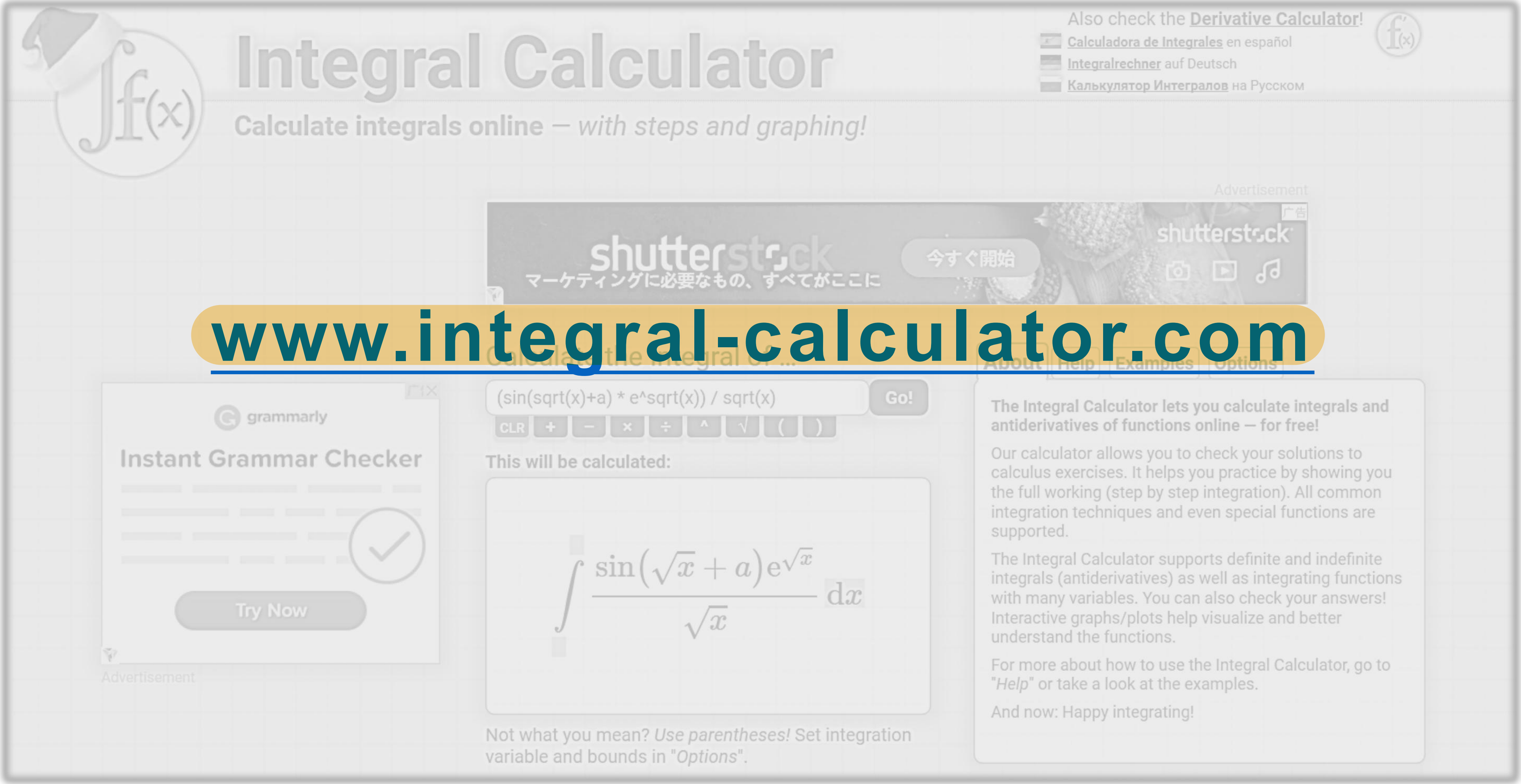
RC9 - The Last RC

KULU

Preview

1, Integration Calculation

2, Improper Integrals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Tactic 1 - Integration by Inspection / Recitation

1. Write down the primitives of the following functions.

$e^{\alpha x}$, α is a constant

x^{α}, $\alpha \in \mathbb{Q} \setminus \{-1\}$ is a constant

lnx

$sinx$ & $cosx$

$\frac{1}{1+x^2}$ & $\frac{1}{\sqrt{1-x^2}}$

$\frac{x}{\sqrt{x^2-a^2}}$

DIY : $\frac{x}{(\sqrt{x^2+a^2})^n}$

$\int e^{\alpha x} = \frac{1}{\alpha} e^{\alpha x}$

$\frac{1}{\alpha+1} x^{\alpha+1}$

$x \ln x - x \int \frac{1}{x^2+\lambda}$ → $\frac{1}{\lambda} \int \frac{1}{(\sqrt{\lambda}x)^2+1}$

$\int \ln x dx = \int (x)' \cdot \ln x \, dx = x \cdot \ln x - \int (\ln x)' dx$

→ arctanx

→ arcsinx.

$\int \sin x \to -\cos x$

$\int \cos x \to \sin x$.

→ $(x^2-a^2)^{\frac{1}{2}}$ '

→ $\cancel{\frac{1}{2}} (x^2-a^2)^{-\frac{1}{2}} \cdot \cancel{2}x$

$\frac{(x^2+a^2)^{-\frac{n}{2}+1}}{2-n}$

$[(x^2+a^2)^{-\frac{n}{2}+1}]'$

$x \cdot (x^2+a^2)^{-\frac{n}{2}}$.

$= (-\frac{n}{2}+1) \cdot (x^2+a^2)^{-\frac{n}{2}} \cdot 2x$.

Tactic 2 - Substitution Rule

4.2.4. Substitution Rule. Let $f \in \text{Reg}([\alpha, \beta])$ and $g: [a, b] \to [\alpha, \beta]$ continuously differentiable. Then

$$\int_a^b (f \circ g)(x) g'(x)\, dx = \int_{g(a)}^{g(b)} f(y)\, dy.$$

$1 + \tan^2\theta = \frac{1}{\cos^2\theta}$

Expression	Substitution	Comment
$\sqrt{\beta^2 - x^2} = \beta\cos\theta$ (>0)	$x = \beta\sin(\theta)$	$-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$
$\sqrt{\beta^2 + x^2}$ $\sqrt{\beta^2(1+\tan^2\theta)}$	$x = \beta\tan(\theta)$	$-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$
$\sqrt{x^2 - \beta^2} = \beta \cdot \frac{1}{\cos\theta}$ (>0)	$x = \frac{\beta}{\cos(\theta)}$	$0 \le \theta < \pi/2$ or $\pi \le \theta < \pi \cdot 3/2$
$\frac{\sin(\theta)}{\cos(\theta)}$	$t = \cos(\theta)$	$t \in (-1,0) \cup (0,1)$
$\frac{\cos(\theta)}{\sin(\theta)}$	$t = \sin(\theta)$	$t \in (-1,0) \cup (0,1)$
$f(\sin(x))\cos(x)$	$t = \sin x$	
$f(\cos(x))\sin(x)$	$t = \cos(x)$	

Tactic 2 - Substitution Rule

1, Integrate $\int \frac{1}{cosx}$

2, Show that $dx = \frac{2}{1+t^2}dt$, if we substitute $t = \tan\left(\frac{x}{2}\right)$ in the integral

3, $\int \left(\frac{1-\cos(2x)}{2}\right)^2 \sin(x)\cos(x)dx$

DIY : $\int \frac{1}{\sqrt{a^2-x^2}}$, $a > 0$

$\int$

$\sin x \to \frac{2t}{1+t^2}$

$\cos x \to \frac{1-t^2}{1+t^2}$

$\tan x$

$\tan\frac{x}{2}$

t

$\frac{dx}{dt}$ $x = 2\arctan t$. $\frac{1}{1+t^2}$

$\frac{dx}{dt} = \frac{2}{1+t^2}$

$\int \sin x \, dx$.

$\int \frac{1}{2-\sin^2 x} dx$ $\frac{d\tan x}{dx} = \frac{1}{\cos^2 x}$

$t = \tan x$ $u = \frac{1}{\sqrt{2}}t$ $\frac{du}{dt} = \frac{1}{\sqrt{2}}$

$= \int \frac{1}{2\cos^2 x + \sin^2 x} dx = \int \frac{\frac{1}{\cos^2 x} dx}{2+\tan^2 x} = \int \frac{d\tan x}{2+\tan^2 x} \Rightarrow \int \frac{dt}{t^2+2} = \frac{1}{2}\int \frac{\sqrt{2}du}{1+u^2}$ $\arctan u$

$$\int \frac{1}{\cos x} dx$$

$$\begin{cases} d\sin x \\ d\cos x \end{cases}$$

$$\frac{d\sin x}{dx} = \cos x$$

$$\int \frac{\cos x}{\cos^2 x} dx$$

$$\int \frac{1}{\cos x} dx \to \frac{d\sin x}{\cos x}$$

$$= \int \frac{d\sin x}{1-\sin^2 x} \overset{t=\sin x}{=} \int \frac{dt}{1-t^2} = \int \left(\frac{1}{1-t} + \frac{1}{1+t}\right) \times \frac{1}{2}\, dt$$

部分分式

ln

$$\int \frac{1}{1-t} \quad -\ln(1-t)$$

$$\frac{1}{2}\ln \frac{1+\sin x}{1-\sin x}$$

$$\int \frac{1}{1+t} \quad \ln(1+t)$$

$$\int \left(\frac{1-\cos(2x)}{2}\right)^2 \sin(x)\cos(x) dx$$

$$\frac{1}{2}\int \left(\frac{1-\cos 2x}{2}\right)^2 \sin 2x\, dx$$

$$u = \cos 2x$$

$$\frac{du}{dx} = -2\sin 2x$$

$$\frac{1}{2}\int \left(\frac{1-u}{2}\right)^2 \left(-\frac{1}{2}\right) du$$

$$-\frac{1}{16}\int (u^2-2u+1)\, du$$

$$\frac{u^3}{3} - \cdots$$

$$\int \frac{1}{\sqrt{a^2-x^2}} dx \quad a > 0$$

$$x = a\sin t \qquad t \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

$$\frac{dx}{dt} = a\cos t \qquad \int \frac{a\cos t\, dt}{a\cos t} = t = \arcsin\frac{x}{a}$$

Tactic 3 - Integration by Parts

4.2.8. Theorem. Let $f, g \in C^1([a, b], \mathbb{C})$. Then

$$\int_a^b f'(x)g(x)\,dx = f(x)g(x)\Big|_a^b - \int_a^b f(x)g'(x)\,dx.$$

Tactic 3 - Integration by Parts

1, Calculate $\int e^{ax} sin(bx)$ and $\int e^{ax} cos(bx)$
2, Calculate $\int e^{ax} P_n(x)$ (e.g. $\int e^{x} \cdot x^4$)

$-\sin^{n-1} x \cdot (\cos x)'$

Exercise 11.9

Let $a_n := \int_0^{\pi/2} \sin^n x\, dx$. $I_n \int \sin^n x\, dx$ RC8.

i) Show that $\{a_n\}_{n\in\mathbb{N}}$ is a convergent sequence by establishing that it is bounded below and decreasing. **(1 Mark)**

I_{n-1} I_{n-2} . .

ii) Prove the recursion formula

$$\int_0^{\pi/2} \sin^n x\, dx = \frac{n-1}{n} \int_0^{\pi/2} \sin^{n-2} x\, dx \qquad (*)$$

and calculate $\int_0^{\pi/2} \sin^2 x\, dx$, $\int_0^{\pi/2} \sin x\, dx$.
(2 Marks)

i) Prove (e.g., using integration by parts) that

$$\int_0^{\pi} \cos^{2m}\theta \sin^{2n}\theta\, d\theta = \frac{(2m)!}{2^{2m} m!} \frac{(2n)!}{2^{2n} n!} \frac{\pi}{(n+m)!}, \qquad n, m \in \mathbb{N}.$$

(2 Marks)

Calculate $\int e^{ax}sin(bx)$ and $\int e^{ax}cos(bx)$

$$\int e^x \sin 2x \, dx.$$

① $\int (e^x)' \sin 2x \, dx$

$= e^x \sin 2x - 2\int e^x \cos 2x \, dx.$ ①

$\int (e^x)' \cos 2x = e^x \cos 2x + 2\int e^x \sin 2x$ ②

$\int e^x \sin 2x \, dx = \frac{1}{5} e^x (\sin 2x - 2\cos 2x).$

② Ansatz 猜解.

$F(x) = e^x (A \sin 2x + B \cos 2x).$ ($A = \frac{1}{5}$, $B = -\frac{2}{5}$)

$F'(x) = \int e^x \sin 2x.$

$e^x (A \sin 2x + B \cos 2x) + e^x (2A \cos 2x - 2B \sin 2x).$

$A - 2B = 1$

$B + 2A = 0.$

$(e^x)'$

$\int e^x x^4$

Ansatz $F(x) = e^x (a_4 x^4 + a_3 x^3 + a_2 x^2 + a_1 x + a_0)$

① $e^x x^4 - \int e^x 4x^3.$ $F'(x) = e^x x^4$

$\int e^x x$ $F'(x) = e^x (a_4 x^4 + a_3 x^3 + a_2 x^2 + a_1 x + a_0)$

$+ e^x (4a_4 x^3 + 3a_3 x^2 + 2a_2 x + a_1)$

$$\begin{cases} a_4 = 1 \\ a_3 + 4a_4 = 0 \\ a_2 + 3a_3 = 0 \\ \dots \end{cases}$$

$a_4 = 1$ $a_1 = -24$

$a_3 = -4$ $a_0 = 24$

$a_2 = 12.$

(i) $\int_0^{\pi} \underline{\cos^{2m}\theta \cdot \sin^{2n}\theta \, d\theta} = \frac{(2m)!}{2^{2m} \cdot m!} \; \frac{(2n)!}{2^{2n} \cdot n!} \; \frac{\pi}{(n+m)!} \quad (n, m \in N)$

$$\int_0^{\pi} (\sin\theta)' \cos^{2m-1}\theta \cdot \sin^{2n}\theta \, d\theta.$$

$$= \sin^{2n+1}\theta \cos^{2m-1}\theta \Big|_0^{\pi} - \int_0^{\pi} \sin\theta \cdot (2m-1)\cos^{2m-2}\theta \, (-\sin\theta) \sin^{2n}\theta + \cos^{2m-1}\theta \cdot 2n \cdot \sin^{2n-1}\theta \cdot \cos\theta \sin\theta \, d\theta$$

$$= \int_0^{\pi} (2m-1)\sin^{2n+2}\theta \cos^{2m-2} - \cos^{2m}\theta \sin^{2n}\theta \, (2n) \, d\theta$$

$$(2n+1)\int_0^{\pi} \cos^{2m}\theta \sin^{2n}\theta \, d\theta = \int_0^{\pi} (2m-1)\sin^{2n+2}\theta \cos^{2m-2}\theta \, d\theta.$$

$$= \int_0^{\pi} (2m-1)\sin^{2n}\theta \cos^{2m-2}\theta \, (1-\cos^2\theta) \, d\theta.$$

$$= \int_0^{\pi} -(2m-1)\sin^{2n}\theta \cos^{2m}\theta \, d\theta. + \int_0^{\pi} (2m-1)\sin^{2n}\theta \cos^{2m-2}\theta \, d\theta.$$

$$\int_0^{\pi} \cos^{2m}\theta \sin^{2n}\theta \, d\theta = \frac{2m-1}{2(m+n)} \int_0^{\pi} \sin^{2n}\theta \cos^{2m-2}\theta \, d\theta.$$

$$= \frac{(2m-1)!!\,(n)!}{2^m (m+n)!} \int_0^{\pi} \sin^{2n}\theta \, d\theta.$$

$$\int_0^{\pi} \sin^{2n}\theta \, d\theta = -\int_0^{\pi} \sin^{2n-1}\theta \cdot (\cos\theta)' \, d\theta$$

$$= -\cos\theta \sin^{2n-1}\theta \Big|_0^{\pi} + \int_0^{\pi} \cos\theta \cdot (2n-1)\sin^{2n-2}\theta \cos\theta \, d\theta$$

$$= \int_0^{\pi} (2n-1)\sin^{2n-2}\theta \, (1-\sin^2\theta) \, d\theta$$

$$= -\int_0^{\pi} (2n-1)\sin^{2n}\theta \, d\theta + \int_0^{\pi} (2n-1)\sin^{2n-2}\theta \, d\theta$$

$$\int_0^{\pi} \sin^{2n}\theta \, d\theta = \frac{2n-1}{2n} \int_0^{\pi} \sin^{2n-2}\theta \, d\theta$$

$$= \frac{(2n-1)!!}{2^n \cdot n!} \cdot \pi.$$

So $$\int_0^{\pi} \cos^{2m}\theta \sin^{2n}\theta \, d\theta = \frac{(2m-1)!!\,(2n-1)!!}{2^m \cdot 2^n \cdot (m+n)!} \pi$$

$$\begin{cases} (2m-1)!! = \dfrac{(2m)!}{2^m \cdot m!} \\ (2n-1)!! = \dfrac{(2n)!}{2^n \cdot n!} \end{cases}$$

$$\int_0^{\pi} \cos^{2m}\theta \sin^{2n}\theta \, d\theta = \frac{(2m)!\,(2n)! \;\; \pi}{2^{2m} \cdot 2^{2n} \cdot m!\, n!\, (n+m)!}$$

$$\frac{1}{ax^2+bx+c} = \frac{1}{a(x-x_1)(x-x_2)} = \frac{1}{a}\cdot\left(\frac{1}{x-x_1}-\frac{1}{x-x_2}\right)\cdot\frac{1}{x_1-x_2}.$$

"$\int \frac{1}{ax^2+bx+c}$, $a > 0$": this case should be divided by three subcases ln

$ax^2+bx+c=0$. x_1 x_2

If $\Delta > 0$, use factorization of the denominator.

If $\Delta = 0$, use factorization and substitution rule and the primitive $\frac{1}{x}$.

$$\int \frac{1}{a(x-t)^2}$$

$$\frac{1}{x-t}.$$

If $\Delta < 0$, change the denominator into the form $a\left(x+\frac{b}{2a}\right)^2 + E$, $E > 0$, then use substitution rule and the primitive $\arctan x$

复分析

$t = x + \frac{b}{2a}$.

$$\frac{dx+e}{ax^2+bx+c}.$$

$\ln(At^2+B)$

$$\int \frac{Ct+D}{At^2+B}\,dt.$$

$$\int \frac{Ct}{At^2+B} + \int \frac{D}{At^2+B}$$

arctan

Tactic 4 - Comprehensive Methods

1, $sin\,(mx)\,cos(nx)$ $= \frac{1}{2}\sin((m+n)x) + \frac{1}{2}\sin((m-n)x)$.

$\boxed{}\cos(m+n)x$. $\boxed{}\cos(m-n)x$.

2, $\int \frac{1}{x^2+4x-5}$

3, $\int \frac{x}{3x^2+6x+10}$

$\int \frac{1}{x^4-16}$, $\int \frac{x}{x^2-1}$

$$\int \frac{1}{(x^2+4)(x^2-4)}\,dx.$$

$\frac{A}{x^2+4}$ $\frac{B}{x^2-4}$ $\frac{C}{x+2}$ $\frac{D}{x-2}$

arctan

$d\cos x$ $\quad$ $d\sin x$

Integrate $\int \frac{1}{\sin(x)\cos^3(x)}dx$ (SJTU Math textbook, P191)

$$= \int \frac{\sin x}{\sin^2 x \cos^3 x} dx.$$

$$= \int \frac{-d\cos x}{(1-\cos^2 x)\cos^3 x} \qquad \int \frac{dt}{(1-t^2)t^3}$$

$$\frac{1-t+t}{(1-t)t^3} = \frac{1}{t^3} + \frac{1}{(1-t)t^2}.$$

$$= \int \frac{[(1-t)+(1+t)]\times\frac{1}{2}dt}{(1-t)(1+t)\,t^3} = A\int \frac{1\,dt}{(1+t)t^3} + B\int \frac{1\,dt}{(1-t)t^3}.$$

$$\frac{1+t-t}{(1+t)t^3} = \frac{1}{t^3} - \frac{1+t-t}{(1+t)t^2}$$

$$\frac{1}{t^2} - \frac{1}{(1+t)t} \qquad \frac{1}{1+t} - \frac{1}{t}$$

$$(\sin\ln x)' = \cos\ln x \cdot \frac{1}{x}.$$

Find: $\int \cos \ln x \, dx = \int x \cdot (\sin \ln x)' \, dx = x \sin \ln x - \int \sin \ln x \, dx$ ①

对偶 $\int \sin \ln x \, dx = -\int x \cdot (\cos \ln x)' \, dx = -x \cos \ln x + \int \cos \ln x \, dx$ ②

$$\Rightarrow \begin{cases} \int \cos \ln x \cdot dx = \frac{x}{2}(\sin \ln x + \cos \ln x) \\ \int \sin \ln x \, dx = \frac{x}{2}(\sin \ln x - \cos \ln x) \end{cases}$$

$\int \sin \, dx$ $\qquad$ $\int \cos dx$

Tactic 5 - Extra!

$\iiint$ dx dy dz.

Multi-variable Method (VV285)

Complex Plane (VV286)

etc...

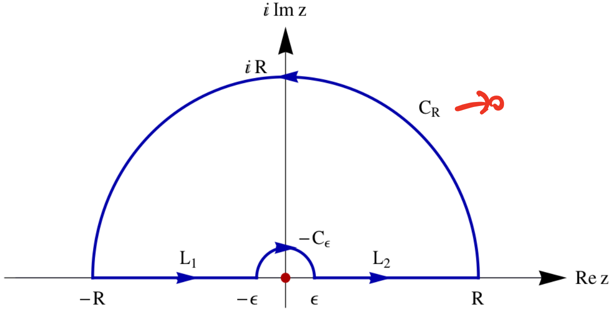
20.6. Example. We will show that

$$\int_0^\infty \frac{1-\cos x}{x^2}\,dx = \frac{\pi}{2}.$$

We integrate the function

$$f : \mathbb{C}\setminus\{0\} \to \mathbb{C}, \qquad f(z) = \frac{1-e^{iz}}{z^2}$$

along the indented semi-circle:



Improper Integrals - Definition

4.2.10. Definition. Assume that $b \le \infty$ and that $f \colon [a, b) \to \mathbb{C}$ is regulated on any closed subinterval $[a, x]$, $x < b$. Then

$$\int_a^b f(t)\,dt$$

is called an ***improper integral*** and is said to ***converge*** or ***exist*** if

$$\lim_{x \nearrow b} \int_a^x f(t)\,dt =: I$$

exists.

Improper Integrals - Practise

Whether $\int_1^{\infty} \frac{\ln x}{x^2+1}$ converge or not?

Whether $\int_1^{\infty} \frac{\sin x}{x\sqrt[3]{x}}$ converge or not?

$$\int_1^{\infty} \left|\frac{\sin x}{x\sqrt[3]{x}}\right| dx \leq \int_1^{a} \frac{1}{x^{\frac{7}{2}}}\, dx$$

$$\int_1^{a} \frac{1}{x^{\frac{7}{2}}}\, dx = \int_1^{a} x^{-\frac{7}{2}}\, dx = -\frac{2}{5} x^{-\frac{5}{2}}\Big|_1^{a} = -\frac{2}{5} a^{-\frac{5}{2}} + \frac{2}{5} \xrightarrow{a \to +\infty} \frac{2}{5}.$$

i) Show that $\int_1^{\infty} \frac{\ln x}{x^2+1}$ is integrable

ii) Show that $\int_0^{1} \frac{\ln x}{x^2+1} = -\int_1^{\infty} \frac{\ln x}{x^2+1}$

iii) Calculate the value of the integral $\int_0^{\infty} \frac{\ln x}{x^2+1}$

$$= \int_0^1 \frac{\ln x}{x^2+1} + \int_1^{\infty} \frac{\ln x}{x^2+1}$$

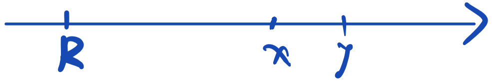
$u\,dv = uv - v\,du.$

4.2.13. Cauchy Criterion. Let $a \in \mathbb{R}$ and $f : [a, \infty) \to \mathbb{R}$ be integrable on every interval $[a, x]$, $x \in \mathbb{R}$. The improper integral

$$\int_a^{\infty} f(x)\, dx$$

converges if and only if

$$\forall_{\varepsilon>0}\ \exists_{R>0}\ \forall_{x,y>R} \quad \left|\int_x^y f(t)\, dt\right| < \varepsilon.$$



4.2.16. Comparison Test. Let $I \subset \mathbb{R}$ and $f : I \to \mathbb{C}$, $g : I \to [0, \infty)$. Suppose that $|f(t)| \leq g(t)$ for $t \in I$ and $\int_I g(t)\, dt$ converges. Then $\int_I f(t)\, dt$ also converges.

$$\left|\int f(t)\right| \qquad \int g(t)$$

$$\underset{\triangle}{\int_1^{\infty} \frac{\ln x}{x^2+1}\,dx} < \overset{=0.}{\int_1^{\infty} \frac{\ln x}{x^2}\,dx} = -\int_1^{\infty} \ln x \cdot d\left(\frac{1}{x}\right) = \frac{\ln x}{-x} + \int \frac{1}{x^2}\,dx.$$

$$= \frac{\ln x}{-x} - \frac{1}{x}.$$

$$\frac{d\left(\frac{1}{x}\right)}{dx} = \frac{1}{-x^2}.$$

$t = \frac{1}{x}$ $\quad t \quad 1 \sim \infty.$

$$\int_0^1 \frac{\ln x}{x^2+1}\,dx = \int_{\infty}^{1} \frac{\ln\left(\frac{1}{t}\right)}{\frac{1}{t^2}+1}\,d\frac{1}{t}.$$

$$= \int_{\infty}^{1} \frac{-t^2 \ln t}{t^2+1}\,d\frac{1}{t}$$

$$= -\int_{\infty}^{1} \frac{t^2 \ln t}{t^2+1}\,\frac{1}{t^2}\,dt.$$

$$= \int_1^{\infty} \frac{\ln t}{t^2+1}\,dt.$$

What's left after integration.....

- Integral test for series.
- Function series.
- Taylor's theorem.
- Riemann Zeta Function
- Stirling's Formula

Important Detail 1:Taylor's Theorem

Taylor's Theorem

4.3.5. Taylor's Theorem. Let $I \subset \mathbb{R}$ an open interval and $f \in C^k(I)$. Let $x \in I$ and $y \in \mathbb{R}$ such that $x + y \in I$. Then for all $p \leq k$,

$$f(x+y) = f(x) + \frac{1}{1!}f'(x)y + \cdots + \frac{1}{(p-1)!}f^{(p-1)}(x)y^{p-1} + R_p \quad (4.3.5)$$

$x \to 0$.

with the remainder term

$$R_p := \int_0^1 \frac{(1-t)^{p-1}}{(p-1)!} f^{(p)}(x+ty)y^p \, dt.$$

Very important to remember! 6 points in last years' final!

But how to use?

Just remember the formula and
Use it in approximation and inequality!

Important Detail 2:Stirling's Formula

4.3.15. Stirling's Formula. $n! \sim \sqrt{2\pi n}\left(\frac{n}{e}\right)^n$ as $n \to \infty$.

The proof is not important! Remember the conclusion!

When to use? Use it in approximation when n goes to infinity!

Important Detail 2:Stirling's Formula

Exercise 1. Comphrehensive Exercise

$n! \sim \sqrt{2\pi n}\cdot\left(\frac{n}{e}\right)^n$

Prove the following:

- $\frac{e^n\cdot n!}{n^n}=\infty$ as n goes to infinity

$\frac{e^n\cdot n!}{n^n} \sim \frac{\cancel{e^n}}{\cancel{n^n}}\cdot\sqrt{2\pi n}\cdot\frac{\cancel{n^n}}{\cancel{e^n}} \to \infty.$

- $\frac{n!}{n^n}=0$ as n goes to infinity
- $\frac{n!}{(n/3)^n}=\infty$ as n goes to infinity

$\frac{n!}{(n/3)^n} \sim \frac{1}{(n/3)^n}\cdot\sqrt{2\pi n}\cdot\frac{n^n}{e^n} = \sqrt{2\pi n}\cdot\left(\frac{3}{e}\right)^n \to \infty.$

- $\frac{n!}{(n/2)^n}=0$ as n goes to infinity

$\underbrace{\sqrt{2\pi n}}_{n^{\frac{1}{2}}}\cdot\big(\underbrace{\tfrac{2}{e}}_{<1}\big)^n \to 0.$

- Find a positive number $\alpha\in\mathbb{R}$, such that when $x>\alpha$, $\frac{n!}{(n/x)^n}=0$

  when 0<x<α, $\frac{n!}{(n/x)^n}=\infty$

$\alpha = e.$

Note: remind yourself of stirling's formula whenever you see n factorial ! ! !

See you!

Thanks for your coming!
I am very glad to be your taa. Thanks for your support and companion!
Perhaps I am not going to apply for TA of any courses in the future...
But wish you the best and find your own beauty in your life!

VV186 is a start, which opens the huge door of Math for you...
Honors Mathematics: VV186 - VV285 - VV286
Discrete Mathematics: VE203
186 285 286.
Linear Algebra: VV214 / VV417
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Reflection

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Find the true beauty in your life.

Find your passion, and flourish.

Be brave, try something new in your youth.

近维声 ⟷ bili bili.

(上),

离散数学.

屈婉玲.

复分析.

复变函数.

Reference

2021 & 2022 VV186 Lecture Slides Horst Hohberger

2019 VV186 TA Zhang Leyang

2021 VV186 TA Ni Yinchen

2021 VV186 TA Huang Yue

2022 VV186 TA Sun Meng

2022 VV186 TA Ding zizhao

2022 VV186 TA Ma tianyi

Thanks and Have Fun!

This is not the end... See you in big RC!