

VV186 Honors Mathematics II

RC9 - The Last RC

KULU

Preview

1, Integration Calculation

2, Improper Integrals



Integral Calculator

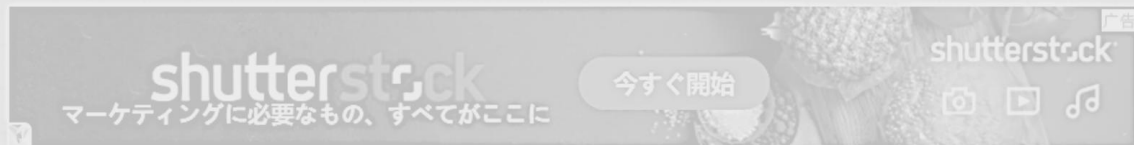
Calculate integrals online – with steps and graphing!

Also check the [Derivative Calculator!](#)

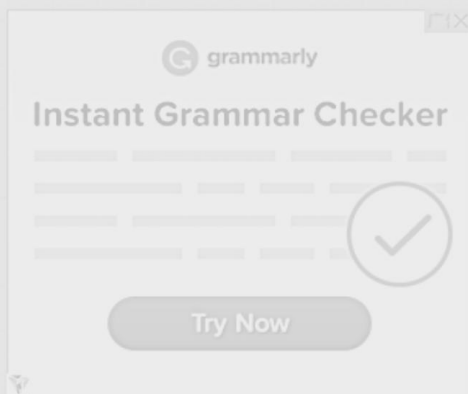
[Calculadora de Integrales](#) en español

[Integralrechner](#) auf Deutsch

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(sin(sqrt(x)+a) * e^sqrt(x)) / sqrt(x) Go!

This will be calculated:

$$\int \frac{\sin(\sqrt{x} + a) e^{\sqrt{x}}}{\sqrt{x}} dx$$

Not what you mean? Use parentheses! Set integration variable and bounds in "Options".

The Integral Calculator lets you calculate integrals and antiderivatives of functions online – for free!

Our calculator allows you to check your solutions to calculus exercises. It helps you practice by showing you the full working (step by step integration). All common integration techniques and even special functions are supported.

The Integral Calculator supports definite and indefinite integrals (antiderivatives) as well as integrating functions with many variables. You can also check your answers! Interactive graphs/plots help visualize and better understand the functions.

For more about how to use the Integral Calculator, go to "Help" or take a look at the examples.

And now: Happy integrating!

Tactic 1 - Integration by Inspection / Recitation

1. Write down the primitives of the following functions.

$e^{\alpha x}$, α is a constant

x^α , $\alpha \in \mathbb{Q} \setminus \{-1\}$ is a constant

$\ln x$

$\sin x$ & $\cos x$

$\frac{1}{1+x^2}$ & $\frac{1}{\sqrt{1-x^2}}$

$\frac{x}{\sqrt{x^2 - a^2}}$

DIY : $\frac{x}{(\sqrt{x^2 + a^2})^n}$

Tactic 2 - Substitution Rule

4.2.4. **Substitution Rule.** Let $f \in \text{Reg}([\alpha, \beta])$ and $g: [a, b] \rightarrow [\alpha, \beta]$ continuously differentiable. Then

$$\int_a^b (f \circ g)(x) g'(x) dx = \int_{g(a)}^{g(b)} f(y) dy.$$

Tactic 2 - Substitution Rule

1, Integrate $\int \frac{1}{\cos x}$

2, Show that $dx = \frac{2}{1+t^2} dt$, if we substitute $t = \tan\left(\frac{x}{2}\right)$ in the integral

3, $\int \left(\frac{1-\cos(2x)}{2}\right)^2 \sin(x) \cos(x) dx$

DIY : $\int \frac{1}{\sqrt{a^2 - x^2}}, a > 0$

$$\int \frac{1}{2 - \sin^2 x} dx$$

Tactic 3 - Integration by Parts

4.2.8. Theorem. Let $f, g \in C^1([a, b], \mathbb{C})$. Then

$$\int_a^b f'(x)g(x) dx = f(x)g(x)\Big|_a^b - \int_a^b f(x)g'(x) dx.$$

Tactic 3 - Integration by Parts

1, Calculate $\int e^{ax} \sin(bx)$ and $\int e^{ax} \cos(bx)$

2, Calculate $\int e^{ax} P_n(x)$ (e.g. $\int e^x \cdot x^4$)

Exercise 11.9

Let $a_n := \int_0^{\pi/2} \sin^n x \, dx$.

- i) Show that $\{a_n\}_{n \in \mathbb{N}}$ is a convergent sequence by establishing that it is bounded below and decreasing.
(1 Mark)
- ii) Prove the recursion formula

$$\int_0^{\pi/2} \sin^n x \, dx = \frac{n-1}{n} \int_0^{\pi/2} \sin^{n-2} x \, dx \quad (*)$$

and calculate $\int_0^{\pi/2} \sin^2 x \, dx$, $\int_0^{\pi/2} \sin x \, dx$.

(2 Marks)

- i) Prove (e.g., using integration by parts) that

$$\int_0^{\pi} \cos^{2m} \theta \sin^{2n} \theta \, d\theta = \frac{(2m)!}{2^{2m} m!} \frac{(2n)!}{2^{2n} n!} \frac{\pi}{(n+m)!}, \quad n, m \in \mathbb{N}.$$

(2 Marks)

“ $\int \frac{1}{ax^2+bx+c}, a > 0$ ”: this case should be divided by three subcases

If $\Delta > 0$, use factorization of the denominator.

If $\Delta = 0$, use factorization and substitution rule and the primitive $\frac{1}{x}$.

If $\Delta < 0$, change the denominator into the form $a \left(x + \frac{b}{2a}\right)^2 + E, E > 0$, then use substitution rule and the primitive $\arctan x$

Tactic 4 - Comprehensive Methods

1, $\sin(mx) \cos(nx)$

2, $\int \frac{1}{x^2+4x-5}$

3, $\int \frac{x}{3x^2+6x+10}$

$$\int \frac{1}{x^4-16}, \int \frac{x}{x^2-1}$$

Integrate $\int \frac{1}{\sin(x) \cos^3(x)}$ (SJTU Math textbook, P191)

$$\text{Find: } \int \cos \ln x \cdot dx = \int x \cdot (\sin \ln x)' dx = x \sin \ln x - \int \sin \ln x dx.$$

$$\int \sin \ln x dx = - \int x \cdot (\cos \ln x)' dx = -x \cos \ln x + \int \cos \ln x dx$$

$$\Rightarrow \begin{cases} \int \cos \ln x \cdot dx = \frac{x}{2} (\sin \ln x + \cos \ln x) \\ \int \sin \ln x dx = \frac{x}{2} (\sin \ln x - \cos \ln x) \end{cases}$$

Tactic 5 - Extra!

Multi-variable Method (VV285)

Complex Plane (VV286)

etc...

Improper Integrals - Definition

4.2.10. *Definition.* Assume that $b \leq \infty$ and that $f: [a, b) \rightarrow \mathbb{C}$ is regulated on any closed subinterval $[a, x]$, $x < b$. Then

$$\int_a^b f(t) dt$$

is called an *improper integral* and is said to *converge* or *exist* if

$$\lim_{x \nearrow b} \int_a^x f(t) dt =: I$$

exists.

Improper Integrals - Practise

Whether $\int_1^{\infty} \frac{\ln x}{x^2+1}$ converge or not?

Whether $\int_1^{\infty} \frac{\sin x}{x^{\frac{3}{\sqrt{x}}}}$ converge or not?

4.2.13. **Cauchy Criterion.** Let $a \in \mathbb{R}$ and $f: [a, \infty) \rightarrow \mathbb{R}$ be integrable on every interval $[a, x]$, $x \in \mathbb{R}$. The improper integral

$$\int_a^{\infty} f(x) dx$$

converges if and only if

$$\forall \varepsilon > 0 \exists R > 0 \forall x, y > R \quad \left| \int_x^y f(t) dt \right| < \varepsilon.$$

4.2.16. **Comparison Test.** Let $I \subset \mathbb{R}$ and $f: I \rightarrow \mathbb{C}$, $g: I \rightarrow [0, \infty)$. Suppose that $|f(t)| \leq g(t)$ for $t \in I$ and $\int_I g(t) dt$ converges. Then $\int_I f(t) dt$ also converges.

i) Show that $\int_1^{\infty} \frac{\ln x}{x^2+1}$ is integrable

ii) Show that $\int_0^1 \frac{\ln x}{x^2+1} = - \int_1^{\infty} \frac{\ln x}{x^2+1}$

iii) Calculate the value of the integral $\int_0^{\infty} \frac{\ln x}{x^2+1}$

What's left after integration.....

- Integral test for series.
- Function series.
- Taylor's theorem.
- Riemann Zeta Function
- Stirling's Formula

Important Detail 1: Taylor's Theorem

Taylor's Theorem

4.3.5. Taylor's Theorem. Let $I \subset \mathbb{R}$ an open interval and $f \in C^k(I)$. Let $x \in I$ and $y \in \mathbb{R}$ such that $x + y \in I$. Then for all $p \leq k$,

$$f(x + y) = f(x) + \frac{1}{1!} f'(x)y + \cdots + \frac{1}{(p-1)!} f^{(p-1)}(x)y^{p-1} + R_p \quad (4.3.5)$$

with the remainder term

$$R_p := \int_0^1 \frac{(1-t)^{p-1}}{(p-1)!} f^{(p)}(x + ty)y^p dt.$$

Very important to remember! 6 points in last years' final!

But how to use?

Just remember the formula and
 Use it in approximation and inequality!

Important Detail 2: Stirling's Formula

4.3.15. Stirling's Formula. $n! \sim \sqrt{2\pi n} \left(\frac{n}{e}\right)^n$ as $n \rightarrow \infty$.

The proof is not important! Remember the conclusion!

When to use? Use it in approximation when n goes to infinity!

Important Detail 2: Stirling's Formula

Exercise 1. Comprehensive Exercise

Prove the following:

- $\frac{e^n \cdot n!}{n^n} = \infty$ as n goes to infinity
- $\frac{n!}{n^n} = 0$ as n goes to infinity
- $\frac{n!}{(n/3)^n} = \infty$ as n goes to infinity
- $\frac{n!}{(n/2)^n} = 0$ as n goes to infinity
- Find a positive number $\alpha \in \mathbb{R}$, such that when $x > \alpha$, $\frac{n!}{(n/x)^n} = 0$

when $0 < x < \alpha$, $\frac{n!}{(n/x)^n} = \infty$

Note: remind yourself of Stirling's formula whenever you see n factorial ! ! !

See you!

Thanks for your coming!

I am very glad to be your taa. Thanks for your support and companion!

Perhaps I am not going to apply for TA of any courses in the future...

But wish you the best and find your own beauty in your life!

VV186 is a start, which opens the huge door of Math for you...

Honors Mathematics: **VV186** - VV285 - VV286

Discrete Mathematics: **VE203**

Linear Algebra: VV214 / VV417

Probability: VE401

Partial Differential Equations VV557

Want more?

Further explore in Analysis, Algebra, Geometry, Topology, Number Theory...

Reflection

Find the true beauty in your life.

Find your passion, and flourish.

Be brave, try something new in your youth.

Reference

2021 & 2022 VV186 Lecture Slides Horst Hohberger

2019 VV186 TA Zhang Leyang

2021 VV186 TA Ni Yincheng

2021 VV186 TA Huang Yue

2022 VV186 TA Sun Meng

2022 VV186 TA Ding zizhao

2022 VV186 TA Ma tianyi

Thanks and Have Fun!

This is not the end... See you in big RC!