

1. Suppose  $f_1, \dots, f_m : \mathbb{R}^d \rightarrow \mathbb{R}$  are convex functions, and  $f(\mathbf{x}) = \max_{i=1, \dots, m} f_i(\mathbf{x})$ . Let  $k$  be any index for which  $f_k(\mathbf{x}) = f(\mathbf{x})$ , and choose  $g \in \partial f_k(\mathbf{x})$  (a convex function on  $\mathbb{R}^d$  has a non-empty subdifferential at all points). Show that  $g \in \partial f(\mathbf{x})$ .