

FAST MATRIX COMPOSITION OF PAULI POLYNOMIAL *

DIANNE DOE[†], PAUL T. FRANK[‡], AND JANE E. SMITH[‡]

Abstract. This is an example SIAM L^AT_EX article. This can be used as a template for new articles. Abstracts must be able to stand alone and so cannot contain citations to the paper's references, equations, etc. An abstract must consist of a single paragraph and be concise. Because of online formatting, abstracts must appear as plain as possible. Any equations should be inline.

Key words. example, L^AT_EX

MSC codes. 68Q25, 68R10, 68U05

1. Introduction. The introduction introduces the context and summarizes the manuscript. It is important to clearly state the contributions of this piece of work. The next two paragraphs are text filler, generated by the `lipsum` package.

Nam dui ligula, fringilla a, euismod sodales, sollicitudin vel, wisi. Morbi auctor lorem non justo. Nam lacus libero, pretium at, lobortis vitae, ultricies et, tellus. Donec aliquet, tortor sed accumsan bibendum, erat ligula aliquet magna, vitae ornare odio metus a mi. Morbi ac orci et nisl hendrerit mollis. Suspendisse ut massa. Cras nec ante. Pellentesque a nulla. Cum sociis natoque penatibus et magnis dis parturient montes, nascetur ridiculus mus. Aliquam tincidunt urna. Nulla ullamcorper vestibulum turpis. Pellentesque cursus luctus mauris.

Nulla malesuada porttitor diam. Donec felis erat, congue non, volutpat at, tincidunt tristique, libero. Vivamus viverra fermentum felis. Donec nonummy pellentesque ante. Phasellus adipiscing semper elit. Proin fermentum massa ac quam. Sed diam turpis, molestie vitae, placerat a, molestie nec, leo. Maecenas lacinia. Nam ipsum ligula, eleifend at, accumsan nec, suscipit a, ipsum. Morbi blandit ligula feugiat magna. Nunc eleifend consequat lorem. Sed lacinia nulla vitae enim. Pellentesque tincidunt purus vel magna. Integer non enim. Praesent euismod nunc eu purus. Donec bibendum quam in tellus. Nullam cursus pulvinar lectus. Donec et mi. Nam vulputate metus eu enim. Vestibulum pellentesque felis eu massa.

The paper is organized as follows. Our main results are in section 2, our new algorithm is in section 3, experimental results are in section 4, and the conclusions follow in section 6.

2. Main results. We interleave text filler with some example theorems and theorem-like items.

Quisque ullamcorper placerat ipsum. Cras nibh. Morbi vel justo vitae lacus tincidunt ultrices. Lorem ipsum dolor sit amet, consectetur adipiscing elit. In hac habitasse platea dictumst. Integer tempus convallis augue. Etiam facilisis. Nunc elementum fermentum wisi. Aenean placerat. Ut imperdiet, enim sed gravida sollicitudin, felis odio placerat quam, ac pulvinar elit purus eget enim. Nunc vitae tortor. Proin tempus nibh sit amet nisl. Vivamus quis tortor vitae risus porta vehicula.

Here we state our main result as Theorem 2.1; the proof is deferred to ??.

THEOREM 2.1 (*LDL^T Factorization [1]*). *If $A \in \mathbb{R}^{n \times n}$ is symmetric and the*

*Submitted to the editors DATE.

Funding: This work was funded by the Fog Research Institute under contract no. FRI-454.

[†]Imagination Corp., Chicago, IL (ddoe@imag.com, <http://www.imag.com/~ddoe/>).

[‡]Department of Applied Mathematics, Fictional University, Boise, ID (ptfrank@fictional.edu, jesmith@fictional.edu).

principal submatrix $A(1 : k, 1 : k)$ is nonsingular for $k = 1 : n - 1$, then there exists a unit lower triangular matrix L and a diagonal matrix

$$D = \text{diag}(d_1, \dots, d_n)$$

such that $A = LDL^T$. The factorization is unique.

Suspendisse vel felis. Ut lorem lorem, interdum eu, tincidunt sit amet, laoreet vitae, arcu. Aenean faucibus pede eu ante. Praesent enim elit, rutrum at, molestie non, nonummy vel, nisl. Ut lectus eros, malesuada sit amet, fermentum eu, sodales cursus, magna. Donec eu purus. Quisque vehicula, urna sed ultricies auctor, pede lorem egestas dui, et convallis elit erat sed nulla. Donec luctus. Curabitur et nunc. Aliquam dolor odio, commodo pretium, ultricies non, pharetra in, velit. Integer arcu est, nonummy in, fermentum faucibus, egestas vel, odio.

THEOREM 2.2 (Mean Value Theorem). *Suppose f is a function that is continuous on the closed interval $[a, b]$. and differentiable on the open interval (a, b) . Then there exists a number c such that $a < c < b$ and*

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

In other words,

$$f(b) - f(a) = f'(c)(b - a).$$

Observe that Theorems 2.1 and 2.2 and Corollary 2.3 correctly mix references to multiple labels.

COROLLARY 2.3. *Let $f(x)$ be continuous and differentiable everywhere. If $f(x)$ has at least two roots, then $f'(x)$ must have at least one root.*

Proof. Let a and b be two distinct roots of f . By Theorem 2.2, there exists a number c such that

$$f'(c) = \frac{f(b) - f(a)}{b - a} = \frac{0 - 0}{b - a} = 0. \quad \square$$

Note that it may require two L^AT_EX compilations for the proof marks to show.

Display matrices can be rendered using environments from `amsmath`:

$$(2.1) \quad S = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \quad \text{and} \quad C = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

Equation (2.1) shows some example matrices.

We calculate the Fréchet derivative of F as follows:

$$(2.2a) \quad F'(U, V)(H, K) = \langle R(U, V), H\Sigma V^T + U\Sigma K^T - P(H\Sigma V^T + U\Sigma K^T) \rangle \\ = \langle R(U, V), H\Sigma V^T + U\Sigma K^T \rangle$$

$$(2.2b) \quad = \langle R(U, V)V\Sigma^T, H \rangle + \langle \Sigma^T U^T R(U, V), K^T \rangle.$$

Equation (2.2a) is the first line, and (2.2b) is the last line.

3. Algorithm. Sed gravida lectus ut purus. Morbi laoreet magna. Pellentesque eu wisi. Proin turpis. Integer sollicitudin augue nec dui. Fusce lectus. Vivamus faucibus nulla nec lacus. Integer diam. Pellentesque sodales, enim feugiat cursus volutpat, sem mauris dignissim mauris, quis consequat sem est fermentum ligula. Nullam justo lectus, condimentum sit amet, posuere a, fringilla mollis, felis. Morbi nulla nibh, pellentesque at, nonummy eu, sollicitudin nec, ipsum. Cras neque. Nunc augue. Nullam vitae quam id quam pulvinar blandit. Nunc sit amet orci. Aliquam erat elit, pharetra nec, aliquet a, gravida in, mi. Quisque urna enim, viverra quis, suscipit quis, tincidunt ut, sapien. Cras placerat consequat sem. Curabitur ac diam. Curabitur diam tortor, mollis et, viverra ac, tempus vel, metus.

Our analysis leads to the algorithm in Algorithm 3.1.

Algorithm 3.1 Build tree

```

Define  $P := T := \{\{1\}, \dots, \{d\}\}$ 
while  $\#P > 1$  do
  Choose  $C' \in \mathcal{C}_p(P)$  with  $C' := \operatorname{argmin}_{C \in \mathcal{C}_p(P)} \varrho(C)$ 
  Find an optimal partition tree  $T_{C'}$ 
  Update  $P := (P \setminus C') \cup \{\bigcup_{t \in C'} t\}$ 
  Update  $T := T \cup \{\bigcup_{t \in \tau} t : \tau \in T_{C'} \setminus \mathcal{L}(T_{C'})\}$ 
end while
return  $T$ 

```

Curabitur ac lorem. Vivamus non justo in dui mattis posuere. Etiam accumsan ligula id pede. Maecenas tincidunt diam nec velit. Praesent convallis sapien ac est. Aliquam ullamcorper euismod nulla. Integer mollis enim vel tortor. Nulla sodales placerat nunc. Sed tempus rutrum wisi. Duis accumsan gravida purus. Nunc nunc. Etiam facilisis dui eu sem. Vestibulum semper. Praesent eu eros. Vestibulum tellus nisl, dapibus id, vestibulum sit amet, placerat ac, mauris. Maecenas et elit ut erat placerat dictum. Nam feugiat, turpis et sodales volutpat, wisi quam rhoncus neque, vitae aliquam ipsum sapien vel enim. Maecenas suscipit cursus mi.

4. Experimental results. Quisque facilisis auctor sapien. Pellentesque gravida hendrerit lectus. Mauris rutrum sodales sapien. Fusce hendrerit sem vel lorem. Integer pellentesque massa vel augue. Integer elit tortor, feugiat quis, sagittis et, ornare non, lacus. Vestibulum posuere pellentesque eros. Quisque venenatis ipsum dictum nulla. Aliquam quis quam non metus eleifend interdum. Nam eget sapien ac mauris malesuada adipiscing. Etiam eleifend neque sed quam. Nulla facilisi. Proin a ligula. Sed id dui eu nibh egestas tincidunt. Suspendisse arcu.

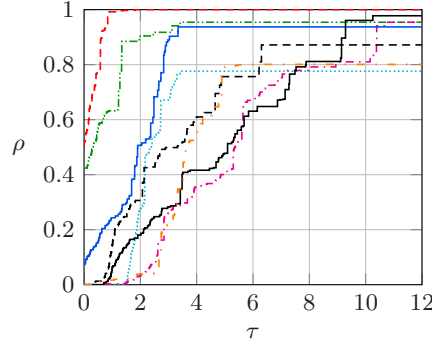
Figure 1 shows some example results. Additional results are available in the supplement in Table 1.

Table 1 shows additional supporting evidence.

TABLE 1
Example table.

Species	Mean	Std. Dev.
1	3.4	1.2
2	5.4	0.6
3	7.4	2.4
4	9.4	1.8

Maecenas dui. Aliquam volutpat auctor lorem. Cras placerat est vitae lectus.

FIG. 1. *Example figure using external image files.*

104 Curabitur massa lectus, rutrum euismod, dignissim ut, dapibus a, odio. Ut eros erat,
 105 vulputate ut, interdum non, porta eu, erat. Cras fermentum, felis in porta congue,
 106 velit leo facilisis odio, vitae consectetur lorem quam vitae orci. Sed ultrices, pede eu
 107 placerat auctor, ante ligula rutrum tellus, vel posuere nibh lacus nec nibh. Maecenas
 108 laoreet dolor at enim. Donec molestie dolor nec metus. Vestibulum libero. Sed quis
 109 erat. Sed tristique. Duis pede leo, fermentum quis, consectetur eget, vulputate sit
 110 amet, erat.

111 **5. Discussion of $Z = X \cup Y$.** Curabitur nunc magna, posuere eget, venenatis
 112 eu, vehicula ac, velit. Aenean ornare, massa a accumsan pulvinar, quam lorem laoreet
 113 purus, eu sodales magna risus molestie lorem. Nunc erat velit, hendrerit quis,
 114 malesuada ut, aliquam vitae, wisi. Sed posuere. Suspendisse ipsum arcu, scelerisque
 115 nec, aliquam eu, molestie tincidunt, justo. Phasellus iaculis. Sed posuere lorem non
 116 ipsum. Pellentesque dapibus. Suspendisse quam libero, laoreet a, tincidunt eget, con-
 117 sequat at, est. Nullam ut lectus non enim consequat facilisis. Mauris leo. Quisque
 118 pede ligula, auctor vel, pellentesque vel, posuere id, turpis. Cras ipsum sem, cursus
 119 et, facilisis ut, tempus euismod, quam. Suspendisse tristique dolor eu orci. Mauris
 120 mattis. Aenean semper. Vivamus tortor magna, facilisis id, varius mattis, hendrerit
 121 in, justo. Integer purus.

122 **6. Conclusions.** Some conclusions here.

123 **Appendix A. An example appendix.** Aenean tincidunt laoreet dui. Vestibu-
 124 lum ante ipsum primis in faucibus orci luctus et ultrices posuere cubilia Curae; Integer
 125 ipsum lectus, fermentum ac, malesuada in, eleifend ut, lorem. Vivamus ipsum turpis,
 126 elementum vel, hendrerit ut, semper at, metus. Vivamus sapien tortor, eleifend id,
 127 dapibus in, egestas et, pede. Pellentesque faucibus. Praesent lorem neque, dignissim
 128 in, facilisis nec, hendrerit vel, odio. Nam at diam ac neque aliquet viverra. Morbi da-
 129 pibus ligula sagittis magna. In lobortis. Donec aliquet ultricies libero. Nunc dictum
 130 vulputate purus. Morbi varius. Lorem ipsum dolor sit amet, consectetur adipiscing
 131 elit. In tempor. Phasellus commodo porttitor magna. Curabitur vehicula odio vel
 132 dolor.

133 **LEMMA A.1. Test Lemma.**

134 **Acknowledgments.** We would like to acknowledge the assistance of volunteers
 135 in putting together this example manuscript and supplement.

136

REFERENCES

- 137 [1] G. H. GOLUB AND C. F. VAN LOAN, *Matrix Computations*, The Johns Hopkins University Press,
138 Baltimore, 4th ed., 2013.