State preparation

for quantum tunneling

요약

진행한 일: <K> 가 실제로는 V 값을 넘게 상태가 준비될 수 있으니 |k> 에서 진행하는 것으로 바꿔보라고 하셨습니다.

- 시작 상태에서 특정 |x> 범주에서 Amplitude가 갇혀있을 것.

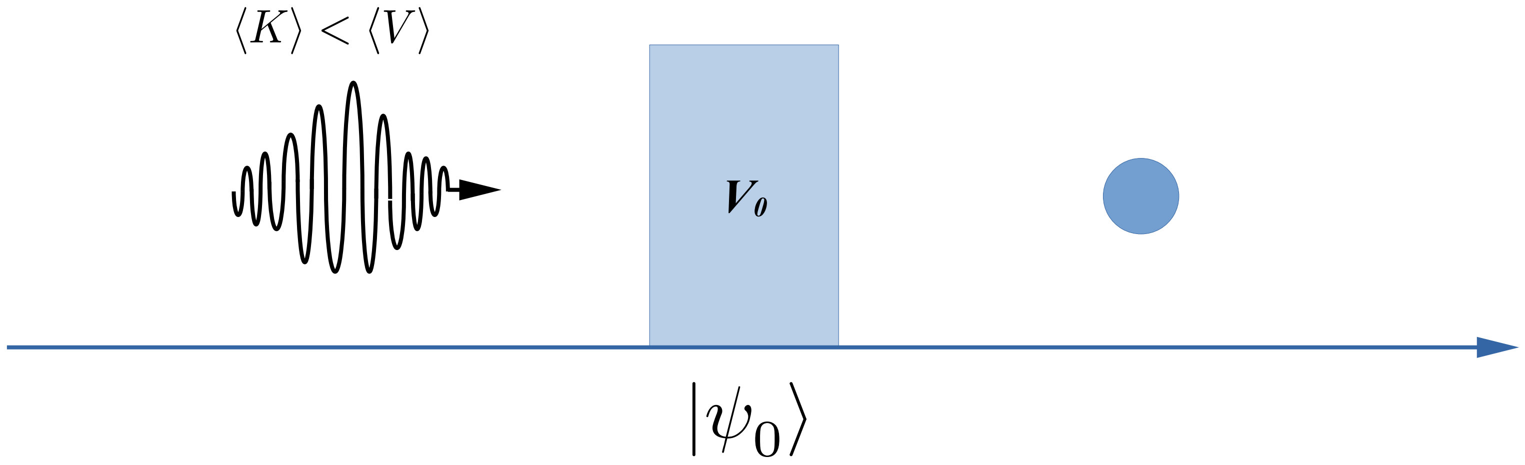
- <K> < V_0 가 성립될 것.

를 구현할 수 있도록 상태를 준비하는 방법을 제시해 보았습니다.

Quantum Tunneling: Initial state

$$|\psi_0\rangle$$

a. It must be **localized** in a specific region, |x>.
b. Its kinetic energy, in |k>, should **not exceed** the maximum value of potential barrier, otherwise it becomes classic or mixed state.



Quantum Tunneling: Initial state

Procedure

a. State preparation to fullfill $\langle K \rangle < V_{max} - \varepsilon$, $(|k\rangle)$.
b. Apply QFT - $(|k\rangle \rightarrow |x\rangle)$.
c. Add a qubit to the register, $(n \rightarrow n+1)$: state is localized in $|x\rangle$.

Practical implementation of a and b are well-explained in the papers.

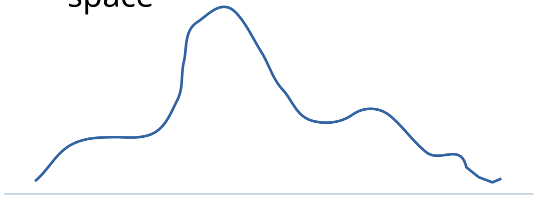
- Sornborger, 10.1038/srep00597
- Feng et al, 10.1038/srep02232

Quantum Tunneling: Localization(|x>)

Additional qubit to the register, (n → n+1)

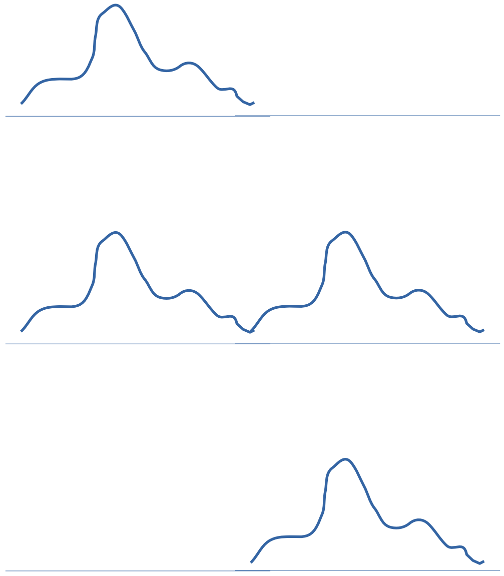
$$|\psi\rangle_{n+1} = |\psi\rangle_n \otimes |x\rangle$$

Arbitrary state on position space



$$\langle \mathbf{x}|\psi\rangle_n$$

$$|\psi\rangle_{n+1}$$



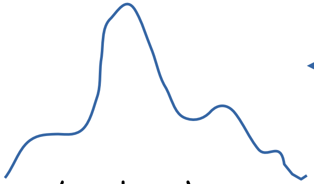
$$|0\rangle$$

$$\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

$$|1\rangle$$

$$\langle \mathbf{x}|\psi\rangle_{n+1}$$

Quantum Tunneling: Localization(|x>)

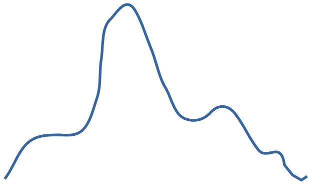


$\langle \mathbf{x} | \psi \rangle_n$

Why arbitrary shape?

A: A preparation directly corresponding to a specific position and <K> requires significant much costs to fullfill the requirements.

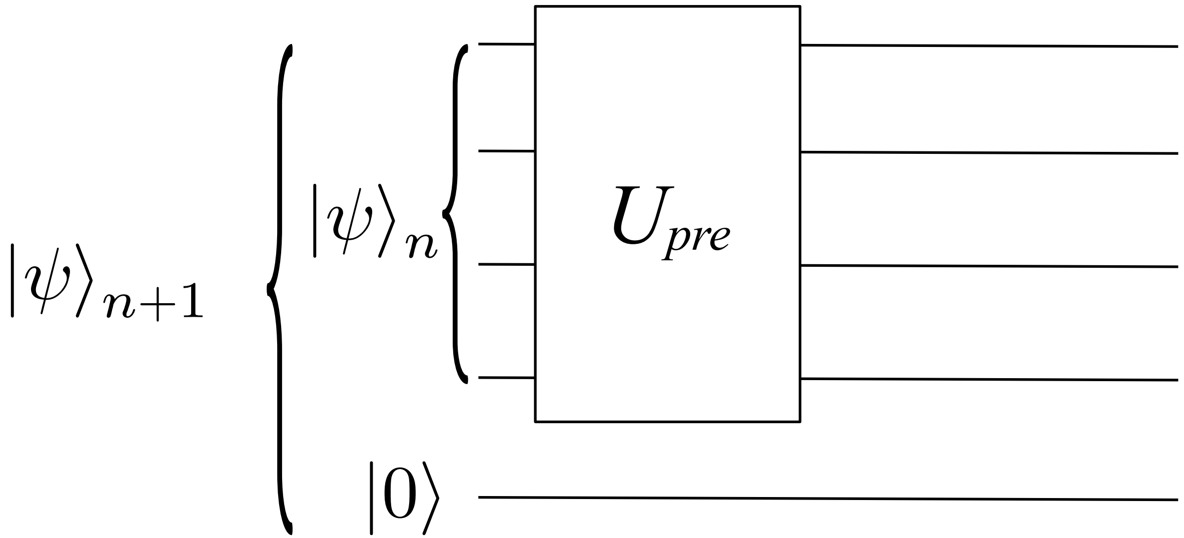
- Require to set all amplitudes and phases considering momentum and position space.



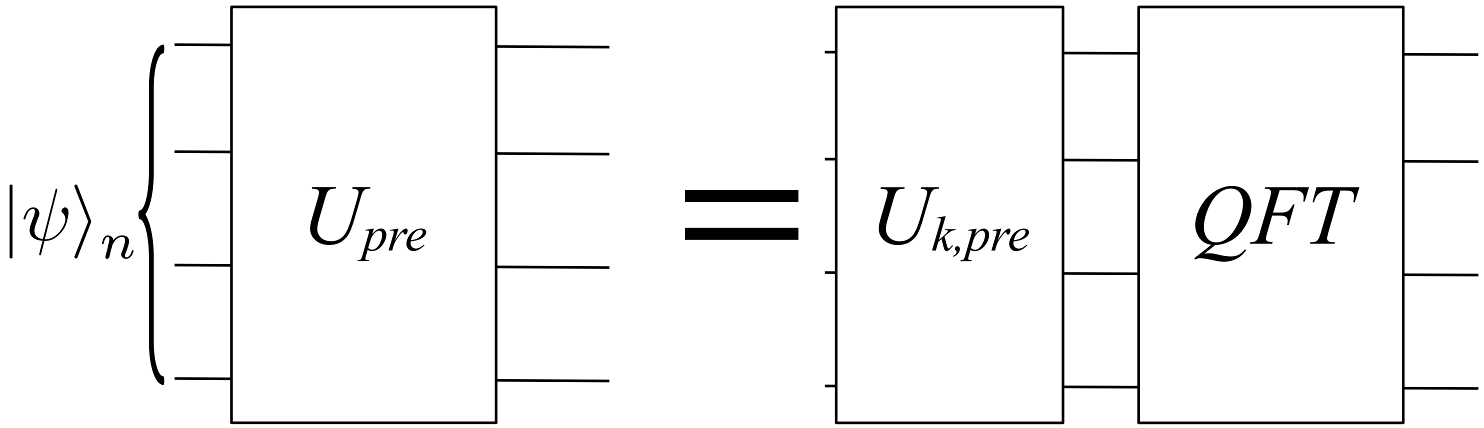
The left is an inverse transformed state of a specific |k> state, which we can prepare easily, comparing to the above.

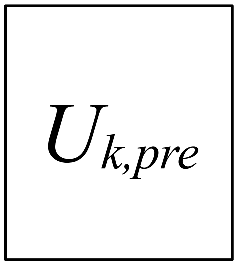
Quantum Tunneling: Localization(|x>)

$$|\psi\rangle_n \rightarrow |\psi\rangle_{n+1}$$



Quantum Tunneling: Localization(|x>)



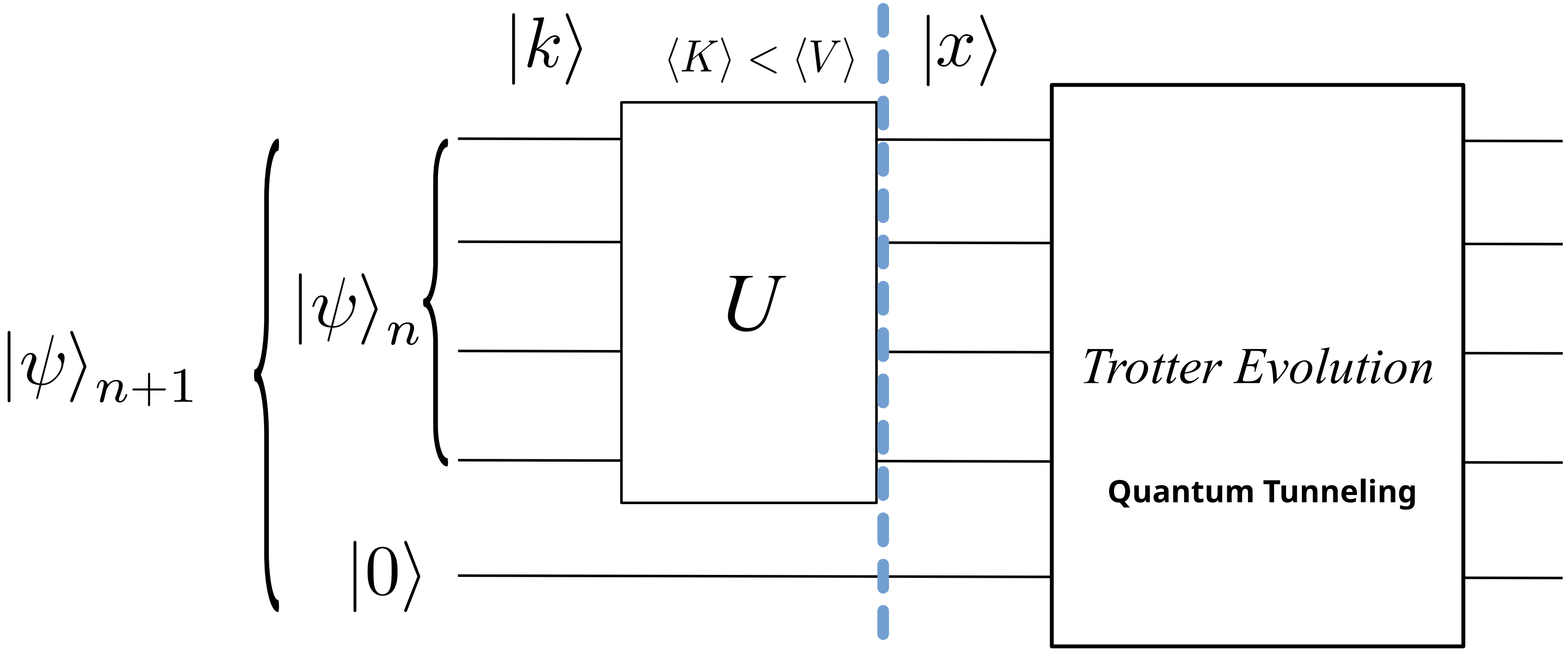


See further references for a general distribution implementation on the amplitude of states.
However, some state can be encoded with simple **X**, **H** gates.

- Dasgupta, Paine, arXiv:2208.13372

Quantum Tunneling: Localization(|x>)

$$|\psi\rangle_n \rightarrow |\psi\rangle_{n+1}$$



Quantum Tunneling: Energy expectation value

Energy expectation value

Q

$$|\psi\rangle_n$$

$$|\psi\rangle_{n+1} = |\psi\rangle_n \otimes |x\rangle$$

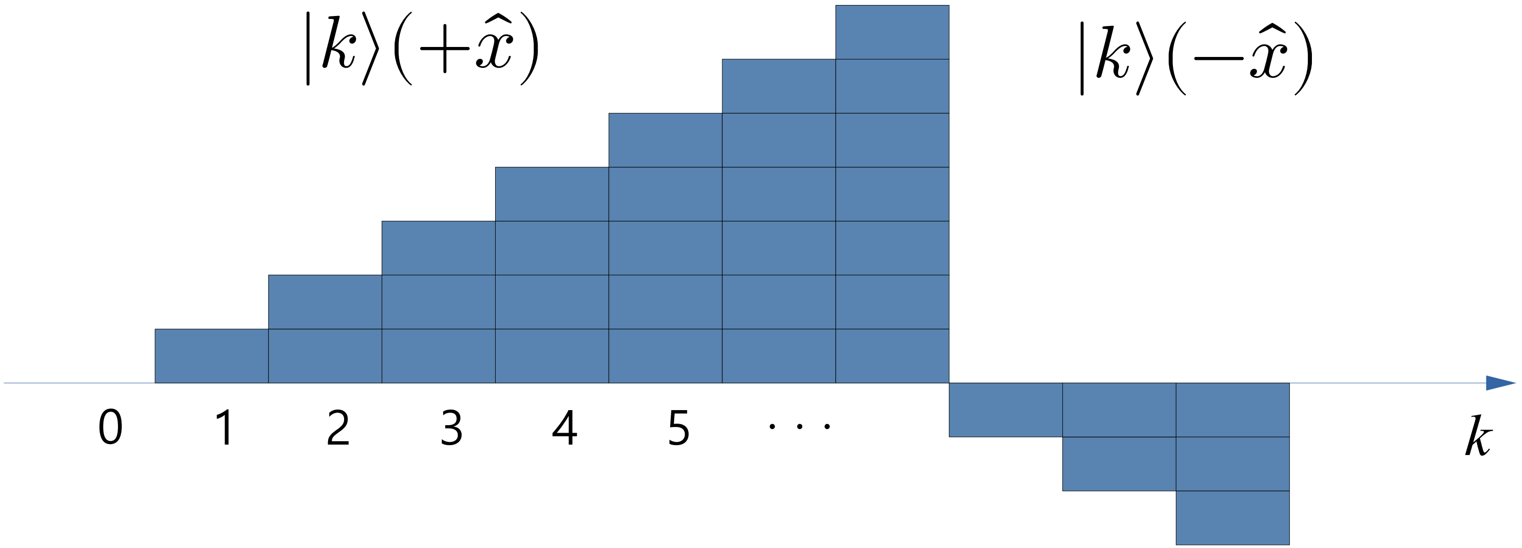
$$\langle K\rangle_n < V_0 \longrightarrow \langle K\rangle_{n+1} < V_0 \ ?$$

A: No, but we can calculate amount of change.

Quantum Tunneling

Momentum Operator n+1

$$\hat{\mathbf{P}}|_p = \frac{2\pi}{2^{n+1}} \left(\sum_{j'=0}^{2^n} j' |j'\rangle\langle j'| + \sum_{j'=2^n+1}^{2^{n+1}-1} (2^n - j')|j'\rangle\langle j'| \right)$$

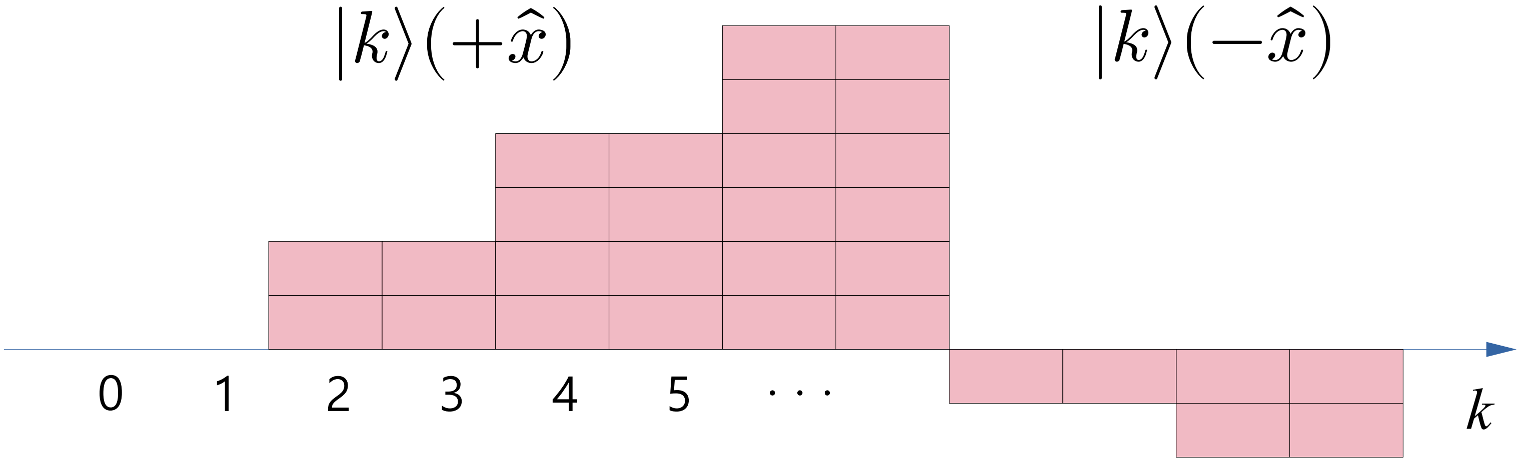


Quantum Tunneling

Momentum Operator n-extended

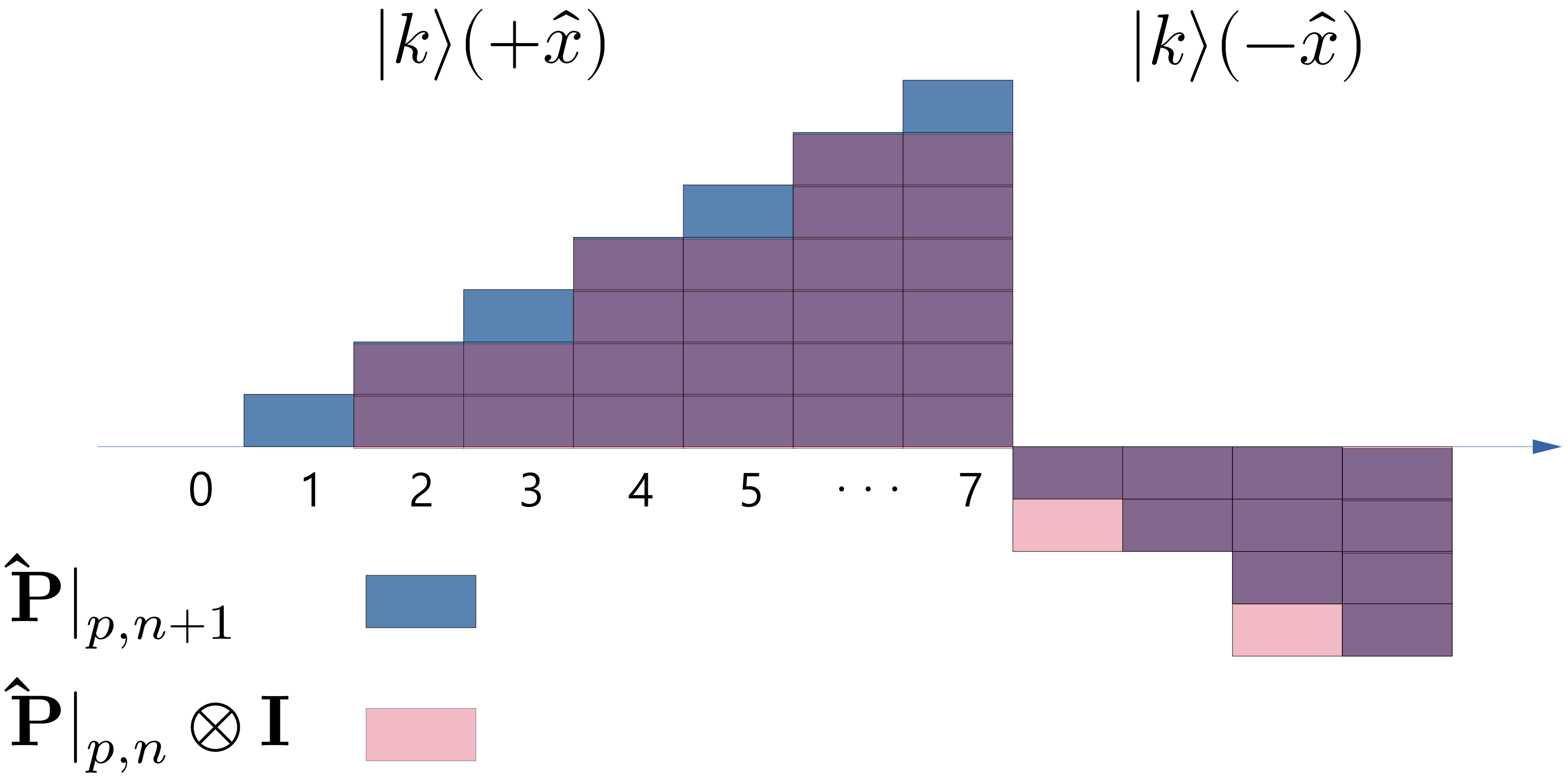
$$\hat{\mathbf{P}}\big|_{p,n} \otimes \mathbf{I}$$

$$\hat{\mathbf{P}}\big|_{p} = \frac{2\pi}{2^n}\left(\sum_{j=0}^{2^{n-1}} j|j\rangle\langle j| + \sum_{j=2^{n-1}+1}^{2^n-1} (2^{n-1} - j)|j\rangle\langle j|\right)$$



Quantum Tunneling

Momentum Operators



Quantum Tunneling

Expectation Values

$$\langle\psi|\hat{\mathbf{P}}|_{\mathbf{p,n}}|\psi\rangle_n = \langle\psi|\hat{\mathbf{P}}|_{\mathbf{p,n}} \otimes \mathbf{I}|\psi\rangle_{n+1}$$

$$\langle\psi|\hat{\mathbf{P}}|_{\mathbf{p,n}}|\psi\rangle_n = \langle\psi|\hat{\mathbf{P}}|_{\mathbf{p,n}} \otimes \mathbf{I}|\psi\rangle_{n+1}$$

$$|\psi\rangle_n = \sum_{k=0}^{2^n-1} \lambda_k|k\rangle$$

$$\langle\psi|\hat{\mathbf{P}}|_{\mathbf{p,n}} \otimes \mathbf{I}|\psi\rangle_{n+1} \leq \langle\psi|\hat{\mathbf{P}}|_{p,n+1}|\psi\rangle_{n+1}$$

when $\lambda_k = 0,\ k > 2^{n-1}$

The above inequality becomes reversed when $\lambda_k = 0, k < 2^{n-1}$

Quantum Tunneling

Expectation Values

$$\lambda_k = 0,\ k > 2^{n-1}$$

$$\langle\psi|\hat{\mathbf{P}}|_{p,n+1}|\psi\rangle_{n+1} = \sum_{k=0}[\lambda_{2k}^2 2k + \lambda_{2k+1}^2(2k+1)]$$

$$\langle\psi|\hat{\mathbf{P}}|_{p,n} \otimes \mathbf{I}|\psi\rangle_{n+1} = \sum_{k=0}[\lambda_{2k}^2 + \lambda_{2k+1}^2]2k$$

$$\Delta\langle K\rangle_{n\to n+1} = C\sum_{k=0}\lambda_{2k+1}^2(4k+1)$$

The above term becomes “ε” in the kinetic energy threshold.

Quantum Tunneling: Summary

Procedure

a. State preparation to fullfill $\langle K \rangle < V_{max}$, $(|k\rangle)$.
b. Apply QFT - ($|k\rangle \rightarrow |x\rangle$).
c. Add a qubit to the register, (n → n+1): state is localized in $|x\rangle$.
d. Set $V'_{max} = V_{max} + \Delta\langle K \rangle$, where $\Delta\langle K \rangle_{n \rightarrow n+1} = C \sum_{k=0} \lambda^2_{2k+1}(4k+1)$
e. Time evolution.