IISER PUNE

INDIAN INSTITUTE OF SCIENCE EDUCATION AND RESEARCH PUNE

MID-Semester Examination, 2020 January

Course code. PHY 1213; Course name: Introductory Electricity and Magnetism

Duration: 2hrs, Date: 25/02/2020

Instructor name: Atikur Rahman Total points: 30

Instructions:

i. This question paper consists of 10 questions (1 to 10) and 4 printed pages. Please verify that the question paper you have received has all the questions in sequential order.

ii. Answer all the questions. Marks are indicated at the lower right of each questions

Plank's constant, h = 6.63×10^{-34} J.s; Charge of electron = 1.6×10^{-19} C; $k_B = 1.38\times10^{-23}$ m^2 kg s^{-2} K^{-1}; dielectric constant in vacuum, $\varepsilon_0 = 8.85\times10^{-12}$ F/m

Some useful expression

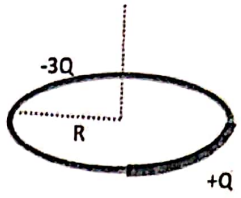
In spherical polar coordinate

$$\vec{\nabla} = \hat{r}\frac{\partial}{\partial r} + \frac{\hat{\theta}}{r}\frac{\partial}{\partial \theta} + \frac{\hat{\phi}}{r\sin\theta}\frac{\partial}{\partial \phi} \qquad \vec{\nabla}\cdot\vec{A} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 A_r\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}\left(A_\theta \sin\theta\right) + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial \phi}$$

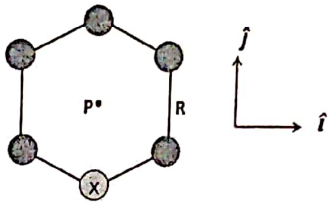
$$\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial \phi^2}$$

$$\vec{\nabla}.\left(\frac{\hat{r}}{r^2}\right) = 4\pi\delta^3(\vec{r})$$

1. (a) A circular plastic rod of radius R has a positive charge +Q uniformly distributed along one-quarter of its circumference and a negative charge of -3Q uniformly distributed along the rest of the circumference as shown in the Figure. What is the electric potential at the center of the circle? (Take V = 0 at infinity as the reference point.)? Marks: 1

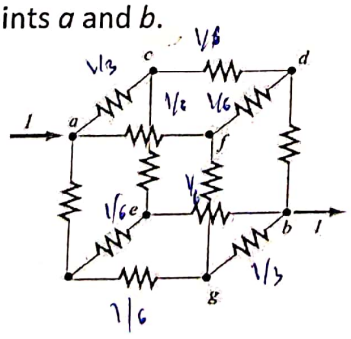


(b) Six equal positive charges q sit at the vertices of a regular hexagon with sides of length R. We remove the bottom charge (marked as "X"). What is the electric field at the center of the hexagon (at point P) now?
Marks: 1



(c) A solid insulating sphere of radius R has a non-uniform volume charge distribution given by $\rho(r) = ar$, where a is a constant. What is the total charge Q of the insulating sphere? Marks: 1

(d) Consider a cube which has identical resistors with resistance R along each edge, as shown in Figure. Find the equivalent resistance between points a and b. Marks: 1



(e) Consider a long straight cylindrical wire with radius R carrying the nonuniform current density, $J(r)=ar^2$, where a is a constant r is the distance from the center axis of the wire. How much total current I passes through the wire? Marks: 1

2. For a given potential V(x,y,z) = ax+by+cz^2, where a = 3 v/m, b = -4 v/m, and c = 5 v/m^2, find is the magnitude of the electric field (in v/m) at the origin, P=(x,y,z)=(0,0,0)? Marks: 2

3. If the electric field in some region is given (in spherical coordinates) by the expression $\vec{E}(\vec{r}) = \frac{(A\hat{r}+B\,Sin\theta Cos\varphi\,\hat{\varphi})}{r}$ where A and B are constants, what is the charge density ρ? Marks: 2

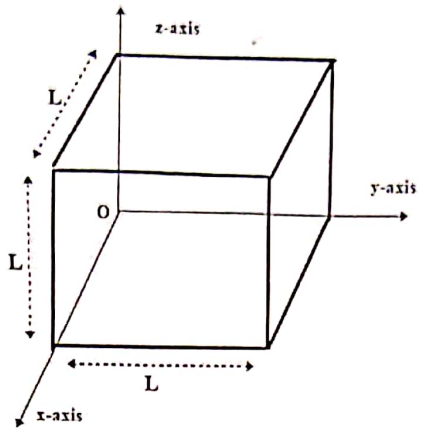
4. A charge Q is divided into two parts such that they repel each other with a maximum force when placed at a certain distance apart. Find the distribution of charge. Marks: 2

5. A capacitor C with an initial charge Q discharges through a resistor R. How many time constants $\tau = RC$ must elapse in order for the capacitor to lose 2/3 of its charge? Marks: 2

6. The resistivity of seawater is about 25 Ω.cm. The charge carriers are chiefly Na^+ and Cl^- ions, and of each there are about 3x10^{20}/cm^3. If we fill a plastic tube 2 meters long with seawater and connect a 12-volt battery to the electrodes at each end, what is the resulting average drift velocity of the ions, in cm/s? Marks: 2

7. In a certain region of space the electric field is given by $\vec{E} = Ay\hat{x}$, where A=100 V/m^2. The electric field points in the x direction and has a constant magnitude given by $E(y)=Ay$. What is the total amount of electric potential energy (in milli Joules) contained in the electric field within the cube with sides of length L=20 meters shown in Figure?

Marks: 3



8. A vertical parallel-plate capacitor is half filled with a dielectric for which the dielectric constant is κ. When this capacitor is positioned horizontally, what **fraction** of it should be filled with the same dielectric so that the two capacitors have equal capacitance? Marks: 3

9. Two point charges *+4Q* and *+Q* are fixed at a distance *r* apart. Where a third point charge *q* should be placed on the line joining the two charges so that it is in equilibrium? Comment on the condition of stable and unstable equilibrium. Marks: 3+1

10. Suppose the electric potential is given by the expression

$$V(\vec{r}) = A\frac{e^{-\lambda r}}{r}$$

for all *r* (A and λ are constants). Find the electric field $\vec{E}(\vec{r})$, the charge density $\rho(\vec{r})$, and the total charge Q. Marks: 1+2+2