



INDIAN INSTITUTE OF SCIENCE EDUCATION AND RESEARCH PUNE  
End-semester Exam, 2019 August Semester

Course Name: Calculus I  
Date: 29-Nov-2019  
Instructor: Baskar Balasubramanyam

Course Code: MT1113  
Duration: 2 hrs  
Total Marks: 30

**Instructions:**

- This exam contains 1 page(s) and 9 problems. Check to see if any pages are missing.
- You may **not** use your calculators, books or notes.
- You are required to show your work for each problem on this test. A correct answer unsupported by explanation will receive no credit.
- If you use any theorem covered in class, you must indicate this and explain why the theorem may be applied.
- Justify any other theorem/result that you use.

1. (3 points) Find the radius and interval of convergence of the power series

$$\sum_{n=0}^{\infty} \frac{2^{-n}}{n+1} (x-1)^n.$$

2. (3 points) Find the arc length of the curve given by the graph of the function  $\cosh(x) = \frac{e^x + e^{-x}}{2}$  on the interval  $[0, \ln 2]$ .

3. (3 points) Determine if the series

$$\sum_{n=0}^{\infty} \frac{1}{1+n^2}$$

converges or diverges.

4. (3 points) Compute  $\int_0^{\frac{\pi}{2}} e^{2x} \cos(x) dx$ .

5. (3 points) Let  $f$  be a continuous function on  $[a, b]$  and a partition  $P$  of  $[a, b]$  such that  $L(f, P) = U(f, P)$ . Show that  $f$  is a constant function.

6. (3 points) Compute the integral  $\int_0^{\pi/4} \sin(2x) dx$ .

7. (4 points) Using the power series method, find the first 6 terms (i.e., up to  $x^5$  term) of the solution of the differential equation  $y'' = x^2 y$  with initial values  $y(0) = 1$  and  $y'(0) = 1$ .

8. (4 points) Compute the integral  $\int_0^3 \frac{1}{(x-1)^{2/3}} dx$ .

9. (4 points) Determine if the series

$$\sum_{n=2}^{\infty} (-1)^{n+1} \frac{\ln(n)}{n}$$

converges or diverges.

$$\frac{(-1)^{n+2}}{(-1)^{n+1}} \cdot \frac{\ln(n+1)}{\ln(n)} \cdot \frac{n}{n+1}$$

$$\frac{\ln(n)}{n}$$

$$\ln(1+n)$$

$$C_0 x^0 + C_1 x^1 + \dots$$