DM-Homework-1

March 18, 2023

1.1

39.
e:

p	q	r	$p \leftrightarrow q$	$\neg q \leftrightarrow r$	$(p \leftrightarrow q) \vee (\neg q \leftrightarrow r)$
T	T	T	T	F	T
F	T	T	F	F	F
T	F	T	F	T	T
T	T	F	T	T	T
F	F	T	T	T	T
F	T	F	F	T	T
T	F	F	F	F	F
F	F	F	T	F	T

f:

p	q	r	$\neg p \leftrightarrow \neg q$	$q \leftrightarrow r$	$(\neg p \leftrightarrow \neg q) \leftrightarrow (q \leftrightarrow r)$
T	T	T	T	T	T
T	T	F	T	F	F
T	F	T	F	F	T
F	T	T	F	T	F
T	F	F	F	T	F
F	T	F	F	F	T
F	F	T	T	F	F
F	F	F	T	T	T

1.2

21. Let's assume that someone of them didn't want the coffee, that person would know that the proposition "everyone wants the coffee" is wrong, then the answer of that man would be "No". But after that, the rest would all answer "No" because whether they wanted the coffee or not, they knew that there is someone who didn't want. Fortunately, the first two professors both answered "I don't know". The last professor answered "No". So we can conclude that the first two professors wanted the coffee and the third professors didn't want the coffee.

For 29, 31, 33, 35, there are 6 situations.

A	B	C
knight	knave	spy
knight	spy	knave
knave	knight	spy
knave	spy	knight
spy	knight	knave
spy	knave	knight

29. After analysis we could know that only the second situation is satisfied. So there is a unique solution where A is the knight, B is the spy, C is the knave.
31. There is a unique solution where A is the knight, B is the spy, C is the knave.
33. All these 6 situations are true.
35. All these 6 situations are false because there is always a knave telling the truth.

1.3

21.

p	q	$\neg(p \leftrightarrow q)$	$p \leftrightarrow \neg q$
T	T	F	F
F	T	T	T
T	F	T	T
F	F	F	F

So $\neg(p \leftrightarrow q)$ and $p \leftrightarrow \neg q$ are logically equivalent.

29.
$(p \to r) \vee (q \to r)$
$\neg p \vee r \vee \neg q \vee r$ by theorem
$(\neg p \vee \neg q) \vee (r \vee r)$ by Associative laws
$(\neg p \vee \neg q) \vee r$ by Idempotent laws
$\neg(p \wedge q) \vee r$ by De Morgan's laws
$(p \wedge q) \to r$ by theorem
So $(p \to r) \vee (q \to r)$ and $(p \wedge q) \to r$ are logically equivalent.

33.
$(p \to q) \wedge (q \to r) \to (p \to r)$
$\neg[(\neg p \vee q) \wedge (\neg q \vee r)] \vee (\neg p \vee r)$ by theorem
$(p \wedge \neg q) \vee (q \wedge \neg r) \vee \neg p \vee r$ by De Morgan's law
$[(p \wedge \neg q) \vee \neg p] \vee [(q \wedge \neg r) \vee r]$ by Associative laws
$[(p \vee \neg p) \wedge (\neg q \vee \neg p)] \vee [(q \vee r) \wedge (\neg r \vee r)]$ by Distributige laws
$[T \wedge (\neg q \vee \neg p)] \vee [(q \vee r) \wedge T]$ by Negation laws
$(\neg q \vee \neg p) \vee (q \vee r)$ by Identity laws
$(\neg q \vee q) \vee \neg p \vee r$ by Associative laws
$T \vee \neg p \vee r$ by Negation laws
T by Domination laws
So $(p \to q) \wedge (q \to r) \to (p \to r)$ is a tautology.

51. We can learn from the question that $p \mid q$ means p NAND q, which is true when either p or q, or both, are false; and it is false when both p and q are true. Then we can list the truth tables of $p \mid q$ and $\neg(p \wedge q)$.

p	q	$p \mid q$	$\neg(p \wedge q)$
T	T	F	F
T	F	T	T
F	T	T	T
F	F	T	T

So $p \mid q$ and $\neg(p \wedge q)$ are logically equivalent.

1.4

39.

a)

Let the domain of x be all the passengers on the airline.
Let P(x) = "x is an elite flyer", Q(x) = "x flies more than 25,000 miles in a year" , R(x) = "x takes more than 25 flights during that year".
Then the statement can be expressed as

$$\forall x[Q(x) \vee R(x) \rightarrow P(x)]$$

b)

Let the domain of x be people.
Let A(x) = "x is a man", B(x) = "x is a woman", P(x) = "x's best previous time is less than 3 hours", Q(x) = "x's best previous time is less than 3.5 hours" , R(x) = "x qualifies for the marathon".
Then the statement can be expressed as

$$\forall x[(A(x) \wedge P(x)) \vee (B(x) \wedge Q(x))] \rightarrow R(x)$$

c)

Let the domain if x be students.
Let P(x) = "x takes at least 60 course hours", Q(x) = "x takes at least 45 course hours" , R(x) = "x writes a master's thesis", S(x) = "x receives a grade no lower than a B in all required courses", T(x) = "x can receive a master's degree".
Then the statement can be expressed as

$$\forall x\{P(x) \vee [Q(x) \wedge R(x)] \wedge S(x)\} \rightarrow T(x)$$

d)
Let the domain of x be students.
Let P(x) = "x has taken more than 21 credit hours in a semester", Q(x) = "x has received all A's".
Then the statement can be expressed as

$$\exists x P(x) \wedge Q(x)$$

55.
a) $\exists! x P(x) \rightarrow \exists x P(x)$ is true.
b) $\forall x P(x) \rightarrow \exists! x P(x)$ is false because for all x, P(x) is true.So it's not unique.
c) $\exists! x \neg P(x) \rightarrow \neg \forall x P(x)$ is true.

63.
a) $\forall x P(x) \rightarrow \neg Q(x)$
b) $\forall x R(x) \rightarrow \neg S(x)$
c) $\forall x \neg Q(x) \rightarrow S(x)$
d) $\forall x P(x) \rightarrow \neg R(x)$
e) (d) follows (a), (b) and (c).

1.5

9.
a) $\forall x L(x, Jerry)$
b) $\forall x \exists y L(x, y)$
c) $\exists y \forall x L(x, y)$
d) $\neg \exists x \forall y L(x, y)$
e) $\exists y \neg L(Lydia, y)$
f) $\exists y \forall x \neg L(x, y)$
g) $\exists! y \forall x L(x, y)$
h) $\exists a \exists b[(a \neq b) \wedge L(Lynn, a) \wedge L(Lynn, b)] \wedge \forall c\{L(Lynn, c) \rightarrow [(c = a) \vee (c = b)]\}$
i) $\forall x L(x, x)$

j) $\exists x \forall y[(y \neq x) \wedge \neg L(x, y) \wedge L(x, x)]$

11.
a) $S(Lois) \wedge A(Lois, ProfessorMichael)$
b) $\forall x S(x) \rightarrow A(x, ProfessorGross)$
c) $\forall x F(x) \rightarrow (A(x, ProfessorMiller) \vee A(ProfessorMiller, x))$
d) $\exists x\{S(x) \wedge \forall y[F(y) \rightarrow \neg A(x, y)]\}$
e) $\exists! y\{F(y) \wedge \forall x[S(x) \rightarrow \neg A(x, y)]\}$
f) $\exists x\{S(x) \wedge \forall y[F(y) \rightarrow A(x, y)]\}$
g) $\exists! x\{F(x) \wedge \forall y[(y \neq x) \wedge F(y) \rightarrow A(x, y)]\}$
h) $\exists x\{S(x) \wedge \forall y[F(y) \rightarrow \neg A(y, x)]\}$

33.
a) $\exists x \exists y \neg P(x, y)$
b) $\exists y \forall x \neg P(x, y)$
c) $\exists y \exists x(\neg P(x, y) \wedge \neg Q(x, y))$
d) $\forall x \forall y P(x, y) \vee \exists x \exists y \neg Q(x, y)$
e) $\exists x(\forall y \exists z \neg P(x, y, z) \vee \forall z \exists y \neg P(x, y, z))$

1.6

9.
a)
Conclusions: I didn't take Tuesday off, I took Thursday off, It rained on Thursday
Let p = "I took the day off", q = "The day rains", r = "The day snows".
According to the question, we can know that $p \rightarrow (q \vee r)$
If I took the Tuesday off, then we have

Step	Reason
$\neg(q \vee r)$	Premise(1)
$p \rightarrow (q \vee r)$	Premise(2)
$\neg p$	Modus tollens

So I didn't take Tuesday off.
Let s="I took Tuesday off", t="I took Thursday off". Then we have

Step	Reason
$s \vee t$	Premise(3)
$\neg s$	Premise(4)
t	Disjunctive syllogism

So I took Thursday off.
What's more, on Thursday

Step	Reason
p	Premise(5)
$p \to (q \vee r)$	Premise(6)
$q \vee r$	Modus ponens
$\neg r$	Premise(7)
q	Disjunctive syllogism

So it rained on Thursday.

b)

Conclusions: I didn't eat spicy foods and there was no thunder while slept. Let p = "I eat spicy foods", q = "I have strange dreams", r = "There is thunder while I sleep".

Step	Reason
$\neg q$	Premise(1)
$p \to q$	Premise(2)
$\neg p$	Modus tollens

So I didn't eat spicy foods.

Step	Reason
$\neg q$	Premise(3)
$r \to q$	Premise(4)
$\neg r$	Modus tollens

So there was no thunder while slept.

c)

Conclusion: I am clever.
Let p = "I am clever", q = "I am lucky", r = "I will win the lottery".

Step	Reason
$p \vee q$	Premise(1)
$\neg q$	Premise(2)
p	Disjunctive syllogism

So I am clever.

d)
Conclusion: Ralph doesn't major in computer science.
Let M(x) = "x majors in computer science", C(x) = "x has a personal computer".

Step	Reason
$\forall x(M(x) \rightarrow C(x))$	Premise
$M(x) \rightarrow C(x)$	Universal instantiation
$\neg C(Ralph)$	Premise
$\neg M(Ralph)$	Modus tollens

So Ralph doesn't major in computer science.

e)
Conclusions: Buying lots of stuff is good for you.
Let P(x) = "x is good for corporations", Q(x) = "x is good for the United States", R(x) = "x is good for you".

Step	Reason
$P(x) \rightarrow Q(x)$	Premise
$Q(x) \rightarrow R(x)$	Premise
$P(x) \rightarrow R(x)$	Hypothetical syllogism
$P(YouBuyLotsofStuff)$	Premise
$R(YouBuyLotsofStuff)$	Modus ponens

So You buy lots of stuff is good for you.

f)

Conclusions: Mice gnaw their food and rabbits are not rodents. Let R(x) = "x are rodents", G(x) = "x gnaw their food".

Step	Reason
$\forall x(R(x) \to G(x)))$	Premise
$R(x) \to G(x)$	Universal instantiation
$R(Mice)$	Premise
$G(Mice)$	Modus ponens
$\neg G(Rabbits)$	Premise
$\neg R(Rabbits)$	Modus tollens

So mice gnaw their food and rabbits are not rodents.

19. Let the domain of n be real numbers.

a) $p = n > 1$, $q = n^2 > 1$, $p \to q$, but $((p \to q) \wedge q) \to (q \to p)$ is not a tautology because it's false when p is false and q is true. This is called the **fallacy of affirming the conclusion**.

b) $p = n > 3$, $q = n^2 > 9$,$p \to q$, assume that $\neg q$ is true

Step	Reason
$p \to q$	Premise
$\neg q$	Assume
$\neg p$	Modus tollens

So if $n^2 \leq 9$, then $n \leq 3$.

c) $p = n > 2$, $q = n^2 > 4$,$p \to q$, but $((p \to q) \wedge \neg p) \to \neg\neg q$ is not a tautology, because it is false when p is false and q is true. This is called the **fallacy of denying the hypothesis**.

23. The 6th step is wrong when using Conjunction because the "c" is not definitely the same in P(c) and Q(c).