MAT1856/APM466 Assignment 1

Your Name, Student #: Junlong Zhang, 1006751711

February, 2024

Fundamental Questions - 25 points

1.

   (a) The main reason is that printing more money means increase of money supply hence putting more money in circulation, leading to inflation.

   (b) One example can be when people anticipates stable inflation, and economic growth; in this case the market for long-term bonds is at equilibrium: the current long term yields is acceptable for both parties.

   (c) Quantitative easing refers to the central bank purchasing bonds to increase money supply, push down interest rates hence yields; during the COVID-19 the Fed aimed to lower long-term interest rates, purchased a lot of long-term bonds, which lowers the long term yield hence making borrowing cheaper for investors and consumers, encouraging investment and spending.

2. Our close price starts from Jan 8th, 2024 to Jan 22nd, 2024, and the bonds all have semi-annual coupon. In an ideal situation, we want the Bonds to have : similar coupon rate, maturity date separated by 6 months. However, none of the bonds in our data follows this pattern so we will use the following strategy: we pick bonds that have 6 months gap in maturity date (between Mar 1st to Sept 1), if there are duplicates, choose similar coupon rate. The bonds chosen are: CAN 1.5 Sep 1st 2024, CAN 1.25 Mar 1st 2025, CAN 0.5 Sep 1st 2025, CAN 0.25 Mar 1st 2026, CAN 1.00 Sep 1st 2026, CAN 1.25 Mar 1st 2027, CAN 2.75 Sep 1 2027, CAN 3.5 Mar 1st 2028, CAN 3.25 Sep 1st 2028, CAN 4.00 Mar 1st 2029, CAN 2.25 Dec 1 2029.
   Note: The last bond matures on Dec 1 because we can't find one that matures in September. I had 11 bonds to make it having approximately 5 years time horizon.

3. Eigenvalues tell us about how the movement of the curve is captured by each corresponding eigenvector (principle component), greater magnitude means the corresponding eigenvector captures more variance. Whereas the eigenvectors provide a direction along which the stochastic curve vary the most (common trend).

Empirical Questions - 75 points

4.

   (a) In this question, I first converted all the prices to clean price then I use the python library called ”bond-pricing” to calculate YTM.

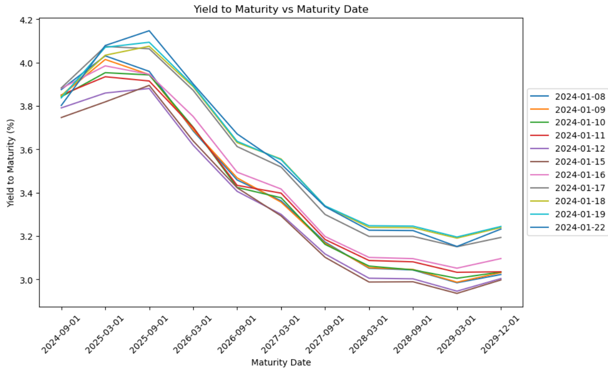

Figure 1: YTM

(b) When deriving the spot curve, I use the bootstrapping method. First I create an empty array to store the spot rates, in order of time periods length(from shortest to longest). Next, given all the 10 (11) bonds, I start from the one with the closest date of maturity, for each bond, calculate:

   i. number of coupon payments it has
   ii. the time period from the previous payment for each payment (in fraction of a year)
   iii. its discounted cash flow (continuous compounding) until but not including the last payment (coupon + notional), discounted by the spot rates calculated from each time

   Then, take its dirty price and the discounted cash flow, calculate the spot rate(T) so that price = discounted cash flow so far + final payment discounted on spot rate (T).
   Since I start from the bond with the closest time and each bond's maturity date is separated by 6 months, I will have a approximate spot rate for each time of payment.

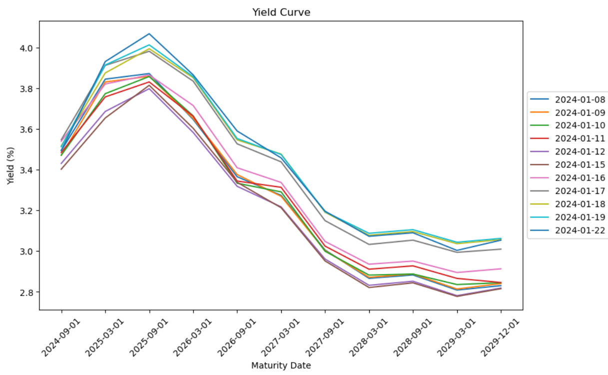

Figure 2: Spot rate

(c) Here I picked the 5 years spot rate as all the dates of March 1st. (2025-03-01, 2026-03-01...). As I calculated the spot rates in continuous compounding I used the formula.

$$F_{t,t+n} = \frac{S_{t+n} \cdot (t+n) - S_t \cdot t}{n} \tag{1}$$

   where t is 1 in our case and $n = 1, 2, 3, 4$

5. I created a list of 11 elements (each corresponds to 1 date) for each year and calculated the covariance table.

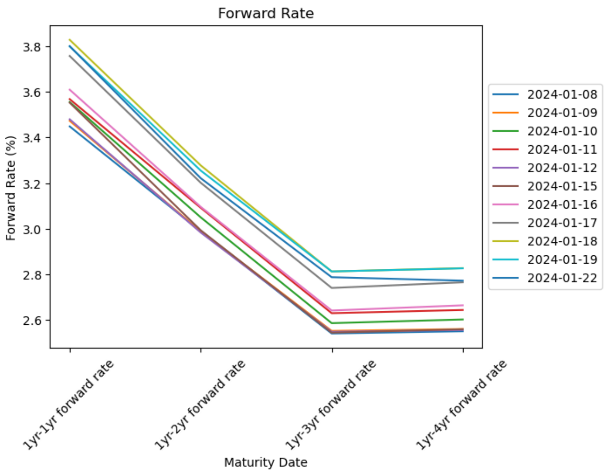

Figure 3: Spot rate

0.00035951	0.00024732	0.00027948	0.00028671	0.00029107
0.00024732	0.00024082	0.00027277	0.00026745	0.00028398
0.00027948	0.00027277	0.00033968	0.00033722	0.00036186
0.00028671	0.00026745	0.00033722	0.00034102	0.00036365
0.00029107	0.00028398	0.00036186	0.00036365	0.00039538

Table 1: Covariance Matrix for yield

0.00033336	0.00034824	0.00031274	0.00034176
0.00034824	0.00047741	0.00045827	0.00049594
0.00031274	0.00045827	0.00045832	0.00049067
0.00034176	0.00049594	0.00049067	0.00053889

Table 2: Covariance Matrix for forward rate

6. The first eigenvalue and its eigenvector represent the percent of variance captured and the direction along which the data varies the most.

0.001544	0.000106	0.000022	0.000002	0.000003
-0.423162	-0.380712	-0.463487	-0.465135	-0.494795
-0.878901	-0.025700	0.245199	0.194136	0.359251
-0.196087	0.878776	0.082698	-0.281806	-0.321014
-0.089863	0.217893	-0.657988	0.702387	-0.134731
0.044069	0.186208	-0.534108	-0.416172	0.710573

(a)

0.001702	0.000094	0.000005	0.000007
0.389800	0.527291	0.511524	0.555304
0.889616	-0.004130	-0.320830	-0.325014
0.188552	-0.641478	0.719878	-0.186361
0.145215	-0.557183	-0.342325	0.742477

(b)

Figure 4: Eigenvalue and eigenvectors

Each column is the eigenvalue and row is the eigenvector. Left is for YTM and right is for forward rate.

References and GitHub Link to Code

https://github.com/HZjl829/APM466_A1.git