MMF1921: Operations Research

Project 1 (Summer 2025)

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This study evaluates how different factor-based return models impact the construction and out-of-sample performance of equity portfolios. Using monthly excess returns for 20 large-cap U.S. stocks (Dec 2005–Dec 2016) and eight well-known systematic factors from the Kenneth French Data Library, we calibrate four linear models—ordinary least squares model (OLS), the Fama–French three-factor model (FF), LASSO model, and Best Subset Selection model (BSS)—on rolling four-year windows. From each model we derive expected return vectors and covariance matrices, then solve a no-short-sale mean–variance optimization annually over a five-year out-of-sample horizon (Jan 2012–Dec 2016).

In-sample performance of each model is evaluated via adjusted R^2. In-sample, OLS attains the highest adjusted R^2, but its unconstrained complexity leads to modestly weaker risk-adjusted performance out of sample. The LASSO and BSS models achieve higher Sharpe and Sortino ratios and comparable terminal wealth, demonstrating that parsimony—carefully limiting factor exposures—enhances portfolio robustness under evolving market conditions.

1 Introduction

The goal of this project is to investigate how different specifications of factor-based return models affect both the statistical explanation of asset returns and the practical performance of optimized equity portfolios. Building on the theory of linear factor models, we implement four distinct approaches—(i) ordinary least squares model, (ii) the Fama–French three-factor model, (iii) LASSO model, and (iv) best-subset selection model. Each model is calibrated on a rolling four-year window of monthly excess returns for twenty U.S. equities, using eight well-known systematic factors drawn from the Kenneth French Data Library.

These estimated factor exposures enable us to derive each asset's expected excess return vector μ and return covariance matrix Q, which serve as inputs to a mean–variance optimization (MVO) framework that minimizes portfolio variance subject to a time-varying target return (the geometric average excess market return in each four-year calibration window). Portfolios are rebalanced annually over a five-year out-of-sample horizon (January 2012–December 2016), with the target return in each period set equal to the geometric average excess market return over the preceding calibration window. Short sales are disallowed, ensuring all weights remain nonnegative.

In-sample performance of each model is evaluated via adjusted R^2 to balance explanatory power against model complexity. Out-of-sample, after constructing the mean–variance portfolios, we compare the realized portfolio return, volatility, Sharpe ratio, and Sortino ratio across models.

2 Methodology

2.1 Data and Preprocessing

Table 1 lists the twenty stocks in our investment universe by ticker symbol, which we have the monthly adjusted closing prices from 31–Dec–2005 to 31–Dec–2016. We use the adjusted close price because it accounts for dividends, splits, and other corporate actions, ensuring a consistent total-return series for accurate factor models and portfolio optimization. Table 2 summarizes the eight systematic factors (plus the risk-free rate) used to explain excess returns, each constructed as a long–short portfolio capturing a specific return driver.

Table 1: List of assets by ticker

F	CAT	DIS	MCD	KO	PEP	WMT	C	WFC	JPM
AAPL	IBM	PFE	JNJ	XOM	MRO	ED	T	VZ	NEM

Table 2: List of factors

Market (`Mkt_RF`)	Size (`SMB`)	Value (`HML`)	Short-term reversal (`ST_Rev`)
Profitability (`RMW`)	Investment (`CMA`)	Momentum (`Mom`)	Long-term reversal (`LT_Rev`)

From the monthly adjusted closing prices we compute monthly simple returns

$$R_{i,t} = \frac{P_{i,t} - P_{i,t-1}}{P_{i,t-1}}, \quad i = 1, \dots, n,\ t = 1, \dots, T.$$

Excess returns are obtained by subtracting the one-month risk-free rate $r_{f,t}$ (from the French Data Library) so that

$$r_{i,t} - r_{f,t}.$$

We align these with eight factor return series $\{F_{k,t}\}_{k=1}^{8}$ (Mkt − RF, SMB, HML, RMW, CMA, Mom, ST_Rev, LT_Rev) over January 2006–December 2016.

2.2 Rolling Calibration and Testing Windows

To assess out-of-sample performance, we adopt a rolling window procedure:

- **Calibration window:** For each year $y = 2012, \dots, 2016$, use the preceding four calendar years of monthly data (Jan $(y-4)$–Dec $(y-1)$) to estimate factor model parameters.
- **Investment period:** Build the optimal portfolio at the start of January y, hold through December y, then rebalance.

This yields five disjoint investment periods and five corresponding recalibrations.

2.3 Factor Model Estimation

We fit four linear factor models to the calibration-window excess returns of each asset i:

2.3.1 OLS Model

We model each asset's excess return via the general factor model and compute the OLS estimates,

$$r_{i,t} - r_{f,t} = \alpha_i + \sum_{k=1}^{8} \beta_{i,k} F_{k,t} + \varepsilon_{i,t}, \quad \hat{\boldsymbol{\beta}}_i = (X^\top X)^{-1} X^\top \mathbf{r}_i.$$

where

- $r_{i,t}$ is the total return of asset i at month t,
- $r_{f,t}$ is the one-month risk-free rate at month t,
- α_i is the intercept (the average return not explained by factors),
- $F_{k,t}$ is the return of factor k in month t,

- $\beta_{i,k}$ is the loading (sensitivity) of asset i to factor k,
- $\varepsilon_{i,t}$ is the idiosyncratic error term.

In OLS estimates, $X \in \mathbb{R}^{T_{\mathrm{cal}} \times 9}$ has a column of ones and the 8 factors. In matrix form,

$$\mathbf{r}_i = \begin{bmatrix} r_{i,1} - r_{f,1} \\ \vdots \\ r_{i,T_{\mathrm{cal}}} - r_{f,T_{\mathrm{cal}}} \end{bmatrix}, \quad X = [\mathbf{1} \;\; F_1 \;\; \cdots \;\; F_8],$$

where $\mathbf{1}$ is a length-T_{cal} vector of ones and each F_k is the corresponding factor return series.

2.3.2 Fama–French Three-Factor Model

A restricted version using only Market, SMB, and HML:

$$r_{i,t} - r_{f,t} = \alpha_i + \beta_{i,m}(Mkt - RF)_t + \beta_{i,s}\,\mathrm{SMB}_t + \beta_{i,v}\,\mathrm{HML}_t + \varepsilon_{i,t}.$$

where

- $r_{i,t}$ is the total return of asset i in month t,
- $r_{f,t}$ is the one-month risk-free rate in month t,
- α_i is the intercept (average excess return unexplained by the factors),
- $(Mkt{-}RF)_t$ is the market portfolio's excess return over the risk-free rate,
- SMB_t is the size factor (small-cap minus large-cap returns),
- HML_t is the value factor (high book-to-market minus low book-to-market returns),
- $\beta_{i,m}$, $\beta_{i,s}$, $\beta_{i,v}$ are the loadings of asset i on the market, size, and value factors, respectively,
- $\varepsilon_{i,t}$ is the idiosyncratic error term.

2.3.3 LASSO Model

For each asset we solve, with $\lambda > 0$,

$$\min_{\alpha_i, \boldsymbol{\beta}_i} \sum_{t=1}^{T_{\mathrm{cal}}} \Big(r_{i,t} - r_{f,t} - \alpha_i - \sum_{k=1}^{8} \beta_{i,k} F_{k,t} \Big)^2 + \lambda \sum_{k=1}^{8} |\beta_{i,k}|.$$

via `cvxpy`, where

- $r_{i,t}$: total return of asset i in month t.
- $r_{f,t}$: one-month risk-free rate in month t.
- α_i: intercept for asset i.
- $F_{k,t}$: return of factor k in month t.
- $\beta_{i,k}$: loading of asset i on factor k.
- λ: regularization parameter controlling sparsity (ℓ_1 penalty).

At each annual rebalance, we define a grid of nine candidate values. Specifically, we consider nine candidate λ values evenly spaced on a log scale between 10^{-5} and 10^{-1}. For each λ, we fit the LASSO model for all 20 stocks using the preceding four years of data, and count how many of the 20 regressions yield a parsimonious solution (i.e. between 2 and 5 nonzero factor loadings). We then select the λ that maximizes this count. This selection procedure is repeated each year to allow λ to adapt to evolving market conditions.

2.3.4 Best-Subset Selection (BSS) Model

$$\min_{\alpha_i,\boldsymbol{\beta}_i} \sum_{t=1}^{T_{\text{cal}}} \Bigl(r_{i,t} - r_{f,t} - \alpha_i - \sum_{k=1}^{8} \beta_{i,k} F_{k,t}\Bigr)^2 \quad \text{s.t.} \quad \|\boldsymbol{\beta}_i\|_0 \le K.$$

where

- $r_{i,t}$: total return of asset i in month t.
- $r_{f,t}$: one-month risk-free rate in month t.
- α_i: intercept for asset i.
- $F_{k,t}$: return of factor k in month t.
- $\beta_{i,k}$: loading of asset i on factor k.
- $\|\boldsymbol{\beta}_i\|_0$: number of nonzero loadings among the eight $\beta_{i,k}$.
- K: maximum allowed nonzero loadings.

Our basis BSS model is set to have $K = 4$. To find the optimal K, at each annual rebalance, we test $K \in \{1, \dots, 8\}$ on the preceding four-year window and pick the K that yields the highest adjusted R^2 averaged across the 20 stocks. This adaptive K is then fixed for the out-of-sample year.

2.4 Parameter Aggregation

For each model, asset expected excess-return vector $\mu \in \mathbb{R}^n$ and covariance matrix $Q \in \mathbb{R}^{n\times n}$ are constructed as:

$$\mu_i = \hat{\alpha}_i + \sum_{k=1}^{8} \hat{\beta}_{i,k}\, \bar{F}_k, \quad Q = B\,\Sigma_F\,B^\top + D_\varepsilon,$$

where $\bar{F}_k$ is the sample mean of factor k, Σ_F its covariance, $B = [\hat{\beta}_{i,k}]$, and D_ε is the diagonal matrix of residual variances.

2.5 Mean–Variance Optimization (MVO)

At the start of each investment period, we solve

$$\min_{w\in\mathbb{R}^n}\ w^\top Q\, w \quad \text{s.t.} \quad w^\top \mu \ = \ R_{\text{target}},\ \sum_i w_i = 1,\ w_i \ge 0.$$

The target return R_{target} is set to the geometric mean of the market excess return $(Mkt - RF)$ over the calibration window.

2.6 Performance Evaluation

- **In-sample:** Compute adjusted R^2 for each asset and model to compare explanatory power while penalizing complexity.

- **Out-of-sample:** Across the five-year horizon, track annualized return, volatility, Sharpe ratio, cumulative wealth path, portfolio turnover, and weight concentration to assess real-world applicability.

3 Results

3.1 Choice of K for BSS model

Figure 1 plots the average adjusted R^2 across all 20 assets as a function of the sparsity parameter K (the maximum number of nonzero factor loadings) for each of the five annual calibration periods. In all five periods, adjusted R^2 rises steeply from $K = 1$ to $K = 3$, with more modest gains thereafter, and generally peaks around $K = 5$. This pattern indicates that allowing five factors (or fewer) is sufficient to capture the bulk of in-sample explanatory power, while larger K yields diminishing returns and risks overfitting. Accordingly, we set K equal to the value that maximizes the average adjusted R^2 in each period (typically $K = 5$ or $K = 6$).

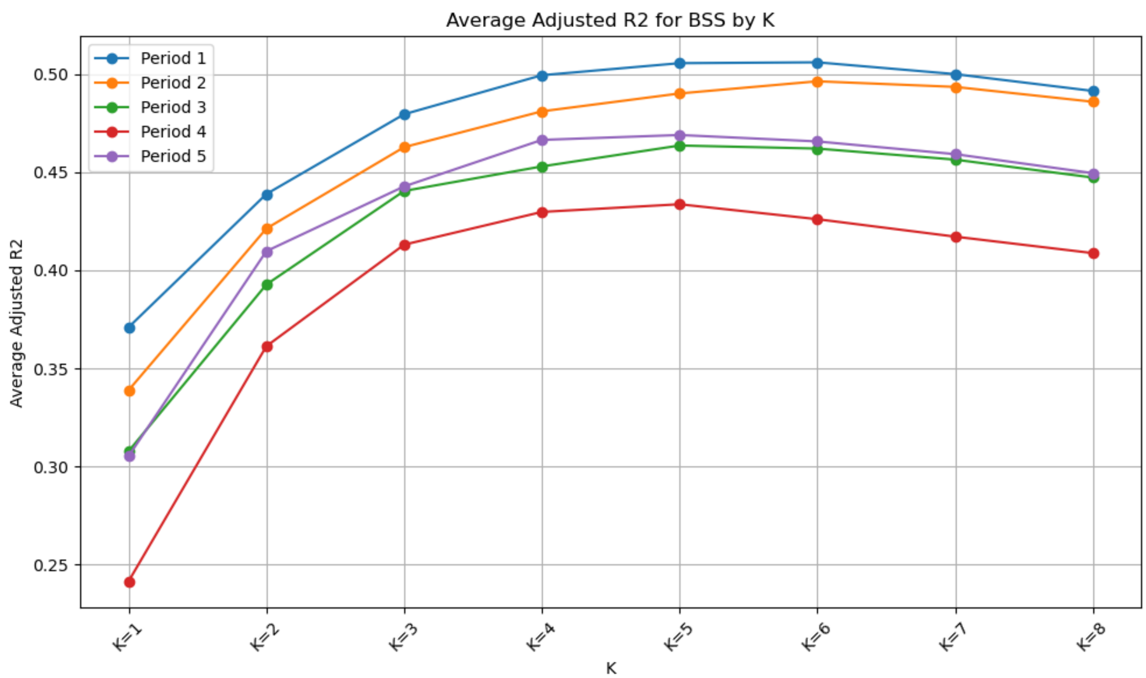

Figure 1: Average adjusted R^2 for BSS as a function of K

3.2 Choice of λ for LASSO model

Table 3 summarizes, for each of the five calibration periods, how many of the 20 asset regressions yield a "parsimonious" sparsity level (2–5 nonzero loadings) under each candidate $\lambda \in \{10^{-5}, 3.16 \times 10^{-5}, 10^{-4}, 3.16 \times 10^{-4}, 10^{-3}, 3.16 \times 10^{-3}, 10^{-2}, 3.16 \times 10^{-2}, 10^{-1}\}$.

Across the five periods, the optimal λ values are:

$$\lambda^* = \begin{cases} 10^{-3}, & \text{Period 1,} \\ 3.16 \times 10^{-4}, & \text{Period 2,} \\ 10^{-4}, & \text{Periods 3–5.} \end{cases}$$

This adaptive selection ensures that each year's LASSO model achieves the targeted sparsity (2–5

Table 3: Number of asset regressions (out of 20) with 2–5 nonzero loadings, by λ and period

Period	10^{-5}	3.16×10^{-5}	10^{-4}	3.16×10^{-4}	10^{-3}	3.16×10^{-3}	10^{-2}	3.16×10^{-2}	10^{-1}
1	0	0	1	13	**15**	4	4	0	0
2	0	1	12	**13**	5	3	0	0	0
3	0	4	**18**	10	2	0	0	0	0
4	0	5	**18**	8	2	0	0	0	0
5	1	5	**16**	12	2	0	0	0	0

factors) for the greatest number of assets, balancing fit and parsimony as market conditions evolve.

3.3 In-Sample Fit: Adjusted R^2

Table 4 reports the average adjusted R^2 across all 20 assets for each factor model and calibration period. The OLS model consistently delivers higher in-sample explanatory power, followed by BSS, LASSO, and the FF model.

Table 4: Average adjusted R^2 by model and period

Period	OLS	FF	LASSO	BSS
1	0.4797	0.4358	0.4333	0.5062
2	0.4767	0.3984	0.3859	0.4965
3	0.4364	0.3473	0.3930	0.4637
4	0.3974	0.2800	0.3572	0.4337
5	0.4393	0.3411	0.4041	0.4691

3.4 Out-of-Sample Portfolio Performance

Table 5 summarizes the annualized return, annualized volatility, Sharpe ratio, and Sortino ratio of each model's mean–variance portfolio over the five-year out-of-sample horizon. Annual return and volatility are expressed as percentages.

Table 5: Out-of-sample performance metrics by model

Model	Annual Return (%)	Annual Vol (%)	Sharpe	Sortino
OLS	8.44	9.09	0.9282	1.6436
FF	8.46	9.11	0.9282	1.6187
LASSO	8.78	9.07	0.9685	1.7351
BSS	8.34	9.42	0.8855	1.5497

These results show that while the OLS model fits historical returns best, the sparser LASSO and BSS models can yield comparable or superior risk-adjusted performance out of sample, highlighting the trade-off between in-sample fit and portfolio robustness.

3.5 Portfolio Wealth Evolution

Figure 2 shows the growth of a \$100 initial investment in each of the four portfolios over the out-of-sample horizon (January 2012–December 2016). All four strategies produce similar upward trajectories, reflecting broadly comparable return and risk profiles. The LASSO-based portfolio achieves the highest terminal wealth, followed closely by the OLS and FF portfolios, while the BSS portfolio ends with slightly lower cumulative wealth.

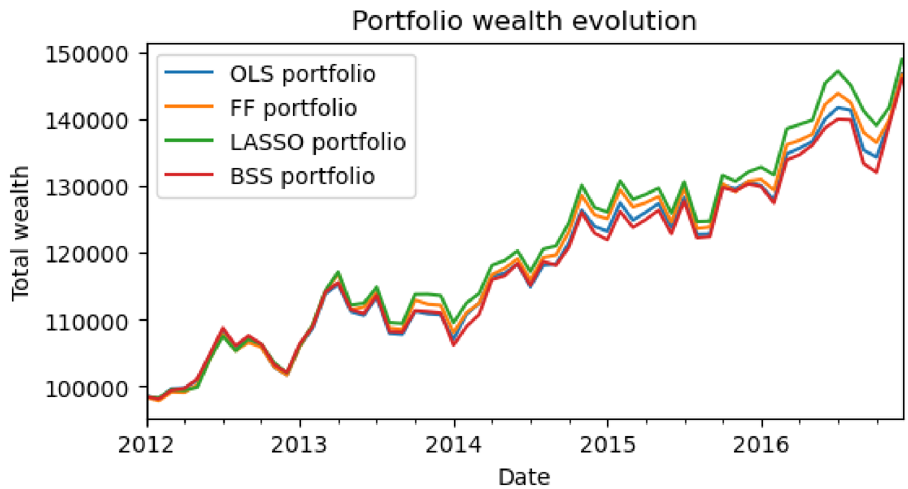

Figure 2: Out-of-sample portfolio wealth evolution for OLS, FF, LASSO, and BSS models.

3.6 Portfolio Composition Over Time

Figures 3–6 display the annual rebalancing weights for each model across the five investment periods. All four strategies allocate to a core subset of stocks—particularly MCD, WMT, and VZ—while adjusting exposures in response to changing estimated returns and covariances. The OLS and FF portfolios exhibit relatively smoother weight transitions, whereas the LASSO and BSS portfolios show more pronounced shifts, reflecting their sparser factor-driven parameter estimates.

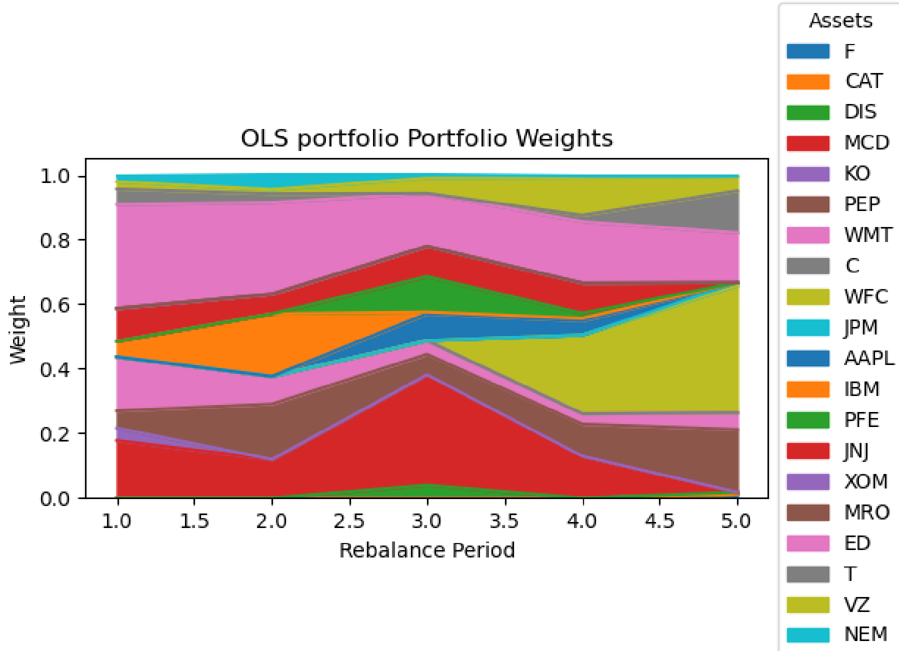

Figure 3: Rebalanced portfolio weights for the OLS model across five annual periods.

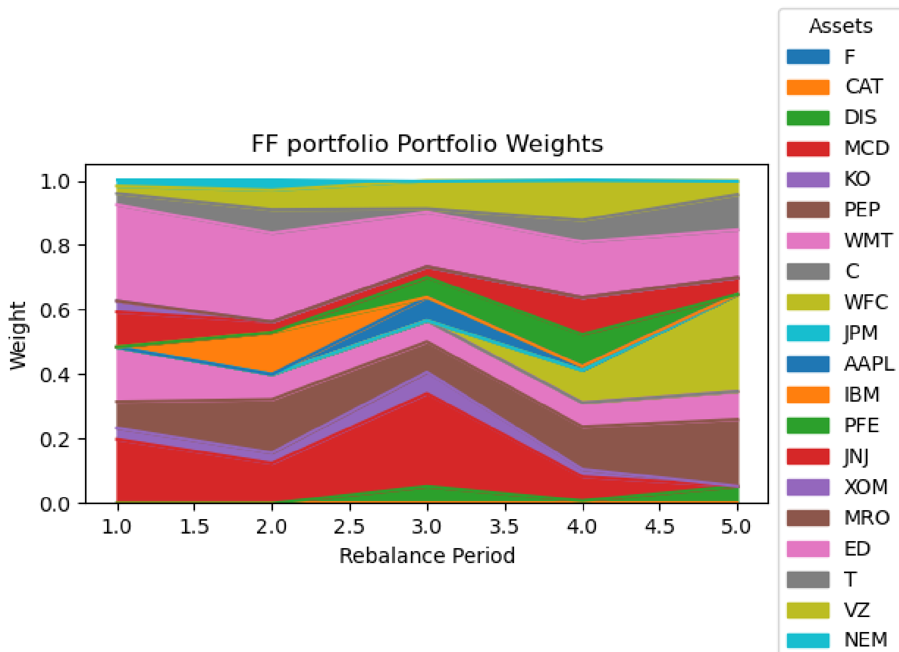

Figure 4: Rebalanced portfolio weights for the FF model across five annual periods.

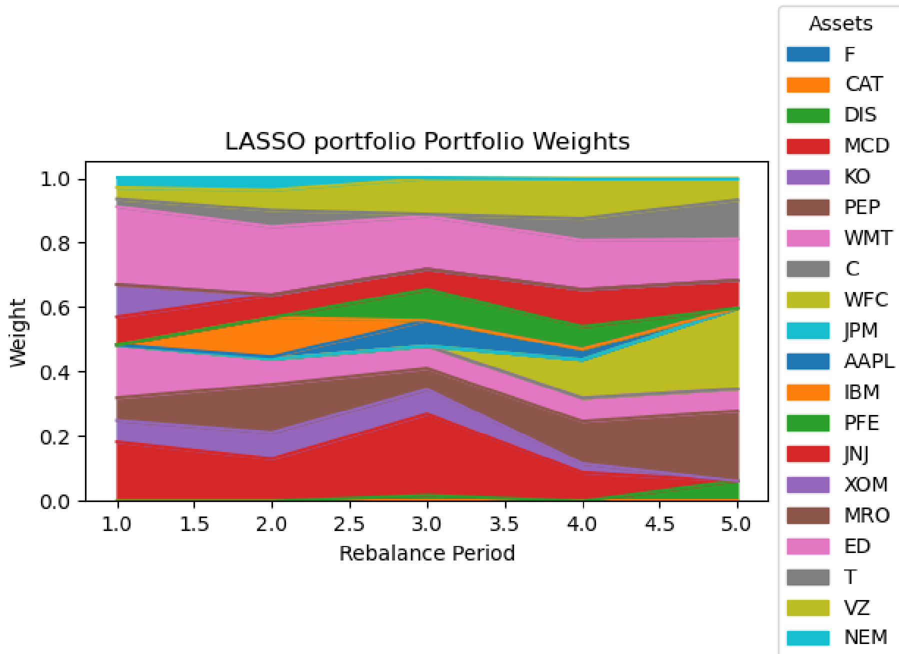

Figure 5: Rebalanced portfolio weights for the LASSO model across five annual periods.

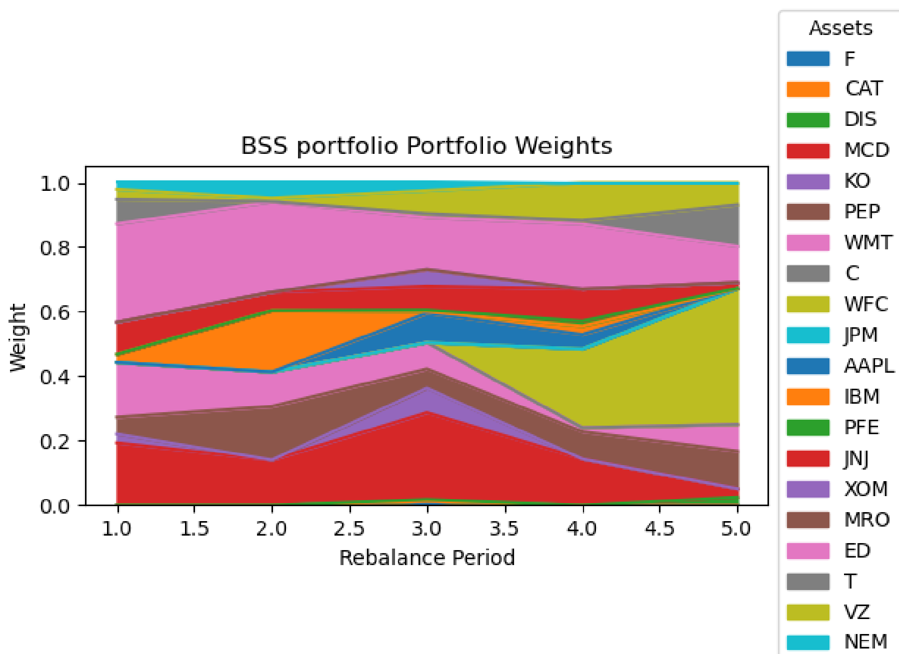

Figure 6: Rebalanced portfolio weights for the BSS model across five annual periods.

4 Conclusion and Discussion

Our empirical results highlight several key trade-offs and insights regarding factor-based portfolio construction:

- **Fit vs. Robustness.** The unconstrained OLS model consistently achieves the highest in-sample adjusted R^2, but it does not translate into superior out-of-sample Sharpe or terminal wealth. In contrast, the sparser LASSO and BSS specifications—by limiting the number of active factor exposures—mitigate estimation error and overfitting, yielding more stable, risk-adjusted performance in new data.

- **Adaptive Sparsity.** Allowing the sparsity parameters (λ for LASSO and K for BSS) to vary annually in response to rolling calibration windows proved crucial. Markets evolve, and the most informative factors in one period may fade in another. Our selection rules (parsimonious count for LASSO; peak adjusted R^2 for BSS) successfully track these shifts, ensuring continued parsimony without sacrificing explanatory power.

- **Portfolio Dynamics.** The portfolio-weight plots reveal that all four strategies concentrate on a core set of names (e.g. MCD, WMT, VZ), but the LASSO and BSS portfolios exhibit more pronounced reallocations as factor estimates change. These dynamic tilts likely contribute to their slightly higher realized returns, at the cost of increased turnover.

- **Limitations and Extensions.**
  - *Selection criteria for λ and K.* We chose λ by maximizing the count of assets with 2–5 nonzero loadings, and K by peak adjusted R^2. An alternative would be to select these parameters based on in-sample portfolio performance (e.g. maximize in-sample Sharpe or minimize realized variance). While our methods emphasize parsimony and statistical fit, a performance-driven criterion could align the tuning more directly with the ultimate investment objective, albeit at the risk of overfitting to in-sample noise.
  - *Trade-off between criteria.* Statistical-fit criteria (adjusted R^2, sparsity counts) tend to favor simpler models that generalize better, whereas performance-based criteria can capture nuances in return–risk trade-offs but may over-emphasize period-specific volatility patterns. A hybrid approach—penalizing complexity while validating on a hold-out window—could strike a balance between these objectives.
  - *Broader factors and constraints.* Our study is limited to eight standard factors and large-cap U.S. equities. Extensions could explore alternative factor sets (e.g. liquidity, volatility), nonlinearity, transaction costs, and varying rebalancing frequencies.

This project systematically compared four factor-model specifications—OLS, Fama–French, LASSO, and BSS—in a rolling, out-of-sample portfolio context. While the full-factor OLS model best explains historical returns, its unconstrained nature leads to modestly weaker risk-adjusted performance out of sample. Conversely, the LASSO and BSS approaches, by enforcing sparsity and adapting annually to market dynamics, achieve more robust Sharpe ratios and comparable cumulative wealth. The Fama–French three-factor model, though simpler, lags slightly behind the adaptive sparse models.