01_softmax

December 6, 2025

[ ]:
```
# This mounts your Google Drive to the Colab VM.
from google.colab import drive
drive.mount('/content/drive')

# TODO: Enter the foldername in your Drive where you have saved the unzipped
# assignment folder, e.g. 'dl/assignments/assignment1/'
FOLDERNAME = 'dl/assignments/assignment1/'
assert FOLDERNAME is not None, "[!] Enter the foldername."

# Now that we've mounted your Drive, this ensures that
# the Python interpreter of the Colab VM can load
# python files from within it.
import sys
sys.path.append('/content/drive/My Drive/{}'.format(FOLDERNAME))

# This downloads the CIFAR-10 dataset to your Drive
# if it doesn't already exist.
%cd /content/drive/My\ Drive/$FOLDERNAME/dl/datasets/
!bash get_datasets.sh
%cd /content/drive/My\ Drive/$FOLDERNAME
```

```
Mounted at /content/drive
/content/drive/My Drive/dl/assignments/assignment1/dl/datasets
/content/drive/My Drive/dl/assignments/assignment1
```

1 Softmax Classifier exercise

In this exercise you will:

- implement a fully-vectorized **loss function** for the Softmax classifier.
- implement the fully-vectorized expression for its **analytic gradient**
- **check your implementation** using numerical gradient
- use a validation set to **tune the learning rate and regularization** strength
- **optimize** the loss function with **SGD**
- **visualize** the final learned weights

[ ]:
```
# Run some setup code for this notebook.
import random
```

```
import numpy as np
from dl.data_utils import load_CIFAR10
import matplotlib.pyplot as plt

# This is a bit of magic to make matplotlib figures appear inline in the
# notebook rather than in a new window.
%matplotlib inline
plt.rcParams['figure.figsize'] = (10.0, 8.0) # set default size of plots
plt.rcParams['image.interpolation'] = 'nearest'
plt.rcParams['image.cmap'] = 'gray'
```

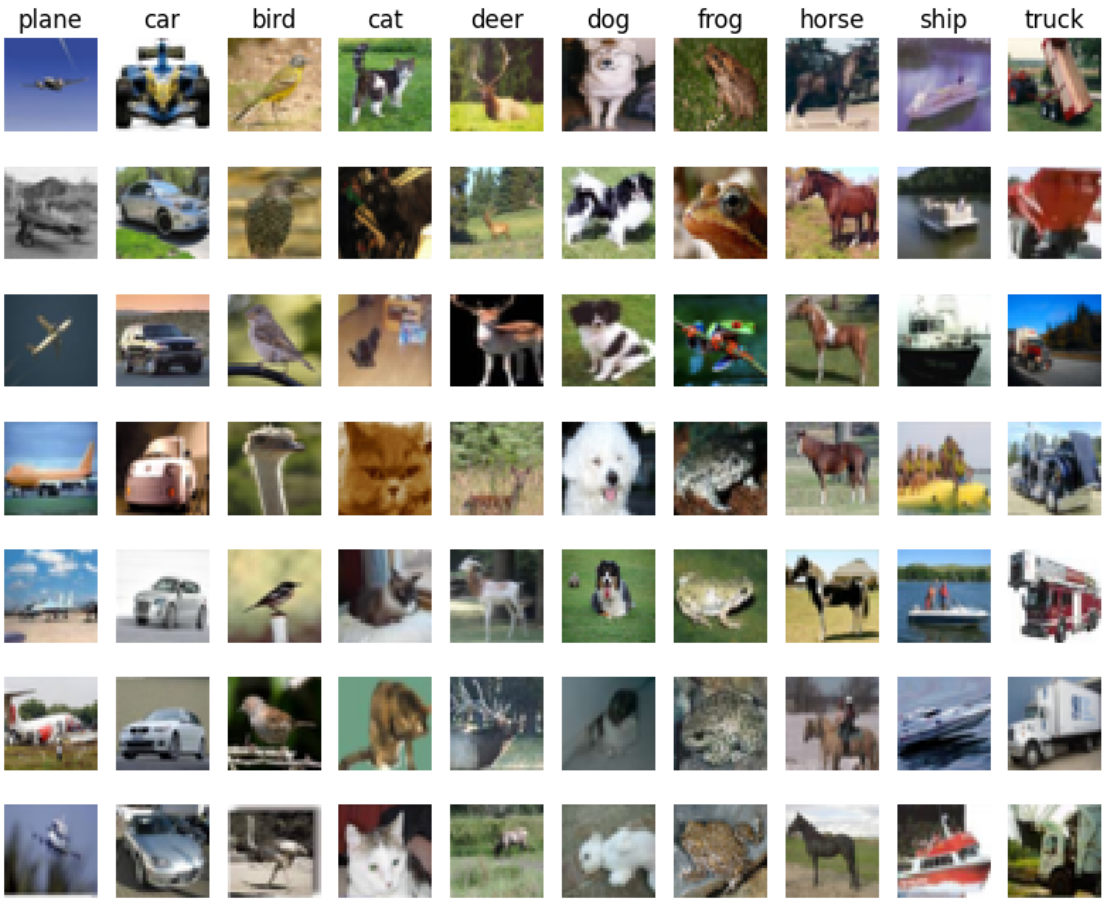
1.1 CIFAR-10 Data Loading and Preprocessing

[ ]:
```
# Load the raw CIFAR-10 data.
cifar10_dir = 'dl/datasets/cifar-10-batches-py'

# Cleaning up variables to prevent loading data multiple times (which may cause␣
 ↪memory issue)
try:
   del X_train, y_train
   del X_test, y_test
   print('Clear previously loaded data.')
except:
   pass

X_train, y_train, X_test, y_test = load_CIFAR10(cifar10_dir)

# As a sanity check, we print out the size of the training and test data.
print('Training data shape: ', X_train.shape)
print('Training labels shape: ', y_train.shape)
print('Test data shape: ', X_test.shape)
print('Test labels shape: ', y_test.shape)
```

```
Training data shape:  (50000, 32, 32, 3)
Training labels shape:  (50000,)
Test data shape:  (10000, 32, 32, 3)
Test labels shape:  (10000,)
```

[ ]:
```
# Visualize some examples from the dataset.
# We show a few examples of training images from each class.
classes = ['plane', 'car', 'bird', 'cat', 'deer', 'dog', 'frog', 'horse',␣
 ↪'ship', 'truck']
num_classes = len(classes)
samples_per_class = 7
for y, cls in enumerate(classes):
    idxs = np.flatnonzero(y_train == y)
    idxs = np.random.choice(idxs, samples_per_class, replace=False)
```

```
    for i, idx in enumerate(idxs):
        plt_idx = i * num_classes + y + 1
        plt.subplot(samples_per_class, num_classes, plt_idx)
        plt.imshow(X_train[idx].astype('uint8'))
        plt.axis('off')
        if i == 0:
            plt.title(cls)
plt.show()
```



[ ]:
```
# Split the data into train, val, and test sets. In addition we will
# create a small development set as a subset of the training data;
# we can use this for development so our code runs faster.
num_training = 49000
num_validation = 1000
num_test = 1000
num_dev = 500

# Our validation set will be num_validation points from the original
```

```
# training set.
mask = range(num_training, num_training + num_validation)
X_val = X_train[mask]
y_val = y_train[mask]

# Our training set will be the first num_train points from the original
# training set.
mask = range(num_training)
X_train = X_train[mask]
y_train = y_train[mask]

# We will also make a development set, which is a small subset of
# the training set.
mask = np.random.choice(num_training, num_dev, replace=False)
X_dev = X_train[mask]
y_dev = y_train[mask]

# We use the first num_test points of the original test set as our
# test set.
mask = range(num_test)
X_test = X_test[mask]
y_test = y_test[mask]

print('Train data shape: ', X_train.shape)
print('Train labels shape: ', y_train.shape)
print('Validation data shape: ', X_val.shape)
print('Validation labels shape: ', y_val.shape)
print('Test data shape: ', X_test.shape)
print('Test labels shape: ', y_test.shape)
```

```
Train data shape:  (49000, 32, 32, 3)
Train labels shape:  (49000,)
Validation data shape:  (1000, 32, 32, 3)
Validation labels shape:  (1000,)
Test data shape:  (1000, 32, 32, 3)
Test labels shape:  (1000,)
```

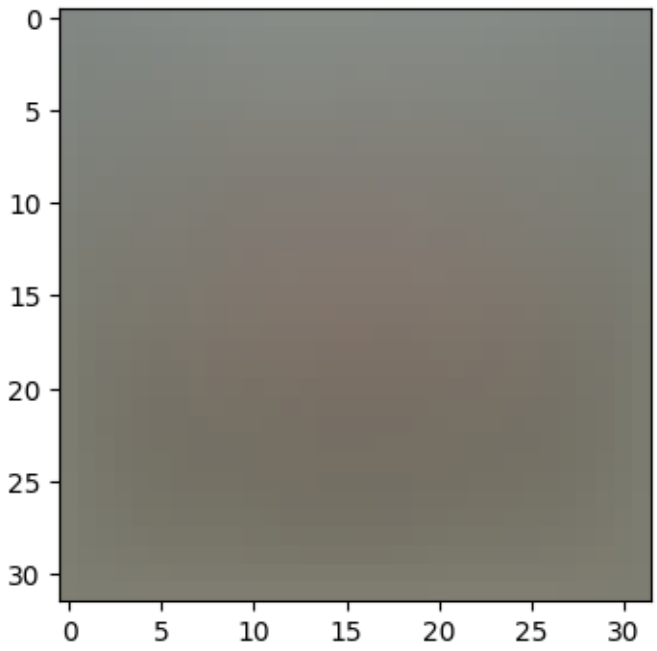
[ ]:
```
# Preprocessing: reshape the image data into rows
X_train = np.reshape(X_train, (X_train.shape[0], -1))
X_val = np.reshape(X_val, (X_val.shape[0], -1))
X_test = np.reshape(X_test, (X_test.shape[0], -1))
X_dev = np.reshape(X_dev, (X_dev.shape[0], -1))

# As a sanity check, print out the shapes of the data
print('Training data shape: ', X_train.shape)
print('Validation data shape: ', X_val.shape)
print('Test data shape: ', X_test.shape)
```

```
print('dev data shape: ', X_dev.shape)
```

```
Training data shape:  (49000, 3072)
Validation data shape:  (1000, 3072)
Test data shape:  (1000, 3072)
dev data shape:  (500, 3072)
```

[ ]:
```
# Preprocessing: subtract the mean image
# first: compute the image mean based on the training data
mean_image = np.mean(X_train, axis=0)
print(mean_image[:10]) # print a few of the elements
plt.figure(figsize=(4,4))
plt.imshow(mean_image.reshape((32,32,3)).astype('uint8')) # visualize the mean␣
 ↪image
plt.show()

# second: subtract the mean image from train and test data
X_train -= mean_image
X_val -= mean_image
X_test -= mean_image
X_dev -= mean_image

# third: append the bias dimension of ones (i.e. bias trick) so that our␣
 ↪classifier
# only has to worry about optimizing a single weight matrix W.
X_train = np.hstack([X_train, np.ones((X_train.shape[0], 1))])
X_val = np.hstack([X_val, np.ones((X_val.shape[0], 1))])
X_test = np.hstack([X_test, np.ones((X_test.shape[0], 1))])
X_dev = np.hstack([X_dev, np.ones((X_dev.shape[0], 1))])

print(X_train.shape, X_val.shape, X_test.shape, X_dev.shape)
```

```
[130.64189796 135.98173469 132.47391837 130.05569388 135.34804082
 131.75402041 130.96055102 136.14328571 132.47636735 131.48467347]
```




```
(49000, 3073) (1000, 3073) (1000, 3073) (500, 3073)
```

1.2 Softmax Classifier

Your code for this section will all be written inside `dl/classifiers/softmax.py`.

As you can see, we have prefilled the function `softmax_loss_naive` which uses for loops to evaluate the softmax loss function.

[ ]:
```python
# Evaluate the naive implementation of the loss we provided for you:
from dl.classifiers.softmax import softmax_loss_naive
import time

# generate a random Softmax classifier weight matrix of small numbers
W = np.random.randn(3073, 10) * 0.0001

loss, grad = softmax_loss_naive(W, X_dev, y_dev, 0.000005)
print('loss: %f' % (loss, ))

# As a rough sanity check, our loss should be something close to -log(0.1).
print('loss: %f' % loss)
print('sanity check: %f' % (-np.log(0.1)))
print(grad.shape)
```

```
loss: 2.361754
loss: 2.361754
sanity check: 2.302585
```

```
(3073, 10)
```

Inline Question 1

Why do we expect our loss to be close to -log(0.1)? Explain briefly.**

YourAnswer : We expect the loss to be close to -log(0.1)because the weights are initialized with very small random values. This makes all the class scores almost the same, and since the softmax outputs probabilities that must sum to 1, each class gets roughly an equal share. With 10 classes, that means each one is about $1/10 = 0.1$, The loss for each sample is -log(score of the correct class), so it becomes apx. log(0.1). This is why the average loss at the start is close to that value—the model is basically guessing equally among all classes.

Also, running the experiment many times shows that the true-class probability usually falls around P 0.1 (typically between [89/1000 ,104/1000]), which is very close to $100/1000 = 0.1$ and matches our intuition for a nearly uniform distribution

The `grad` returned from the function above is right now all zero. Derive and implement the gradient for the softmax loss function and implement it inline inside the function `softmax_loss_naive`. You will find it helpful to interleave your new code inside the existing function.

To check that you have correctly implemented the gradient, you can numerically estimate the gradient of the loss function and compare the numeric estimate to the gradient that you computed. We have provided code that does this for you:

[ ]:
```
# Once you've implemented the gradient, recompute it with the code below
# and gradient check it with the function we provided for you

# Compute the loss and its gradient at W.
loss, grad = softmax_loss_naive(W, X_dev, y_dev, 0.0)

# Numerically compute the gradient along several randomly chosen dimensions, and
# compare them with your analytically computed gradient. The numbers should␣
 ↪match
# almost exactly along all dimensions.
from dl.gradient_check import grad_check_sparse
f = lambda w: softmax_loss_naive(w, X_dev, y_dev, 0.0)[0]
grad_numerical = grad_check_sparse(f, W, grad)

# do the gradient check once again with regularization turned on
# you didn't forget the regularization gradient did you?
loss, grad = softmax_loss_naive(W, X_dev, y_dev, 5e1)
f = lambda w: softmax_loss_naive(w, X_dev, y_dev, 5e1)[0]
grad_numerical = grad_check_sparse(f, W, grad)
```

```
numerical: 3.374718 analytic: 3.374718, relative error: 6.853180e-09
numerical: -0.133647 analytic: -0.133647, relative error: 2.806071e-07
numerical: 0.322214 analytic: 0.322214, relative error: 4.078942e-08
numerical: -0.927757 analytic: -0.927757, relative error: 4.342776e-09
numerical: -2.158621 analytic: -2.158622, relative error: 4.539471e-08
numerical: -4.926574 analytic: -4.926574, relative error: 1.086047e-08
```

```
numerical: -1.087933 analytic: -1.087933, relative error: 4.160728e-08
numerical: 3.167069 analytic: 3.167069, relative error: 2.100865e-08
numerical: -1.861696 analytic: -1.861696, relative error: 7.165298e-09
numerical: 0.333993 analytic: 0.333993, relative error: 8.930398e-08
numerical: 1.668656 analytic: 1.668656, relative error: 1.190839e-08
numerical: -0.895162 analytic: -0.895162, relative error: 4.383323e-09
numerical: 1.381510 analytic: 1.381510, relative error: 1.516064e-08
numerical: -0.969236 analytic: -0.969236, relative error: 1.854407e-08
numerical: 2.857344 analytic: 2.857344, relative error: 2.290417e-08
numerical: -1.228753 analytic: -1.228753, relative error: 8.230108e-09
numerical: -0.859018 analytic: -0.859018, relative error: 1.651656e-08
numerical: 0.240777 analytic: 0.240776, relative error: 1.061114e-07
numerical: 0.655507 analytic: 0.655507, relative error: 5.689034e-08
numerical: -3.002352 analytic: -3.002352, relative error: 1.460566e-08
```

Inline Question 2

Although gradcheck is reliable softmax loss, it is possible that for SVM loss, once in a while, a dimension in the gradcheck will not match exactly.

What could such a discrepancy be caused by? Is it a reason for concern? What is a simple example in one dimension where a svm loss gradient check could fail? How would change the margin affect of the frequency of this happening?

Note that SVM loss for a sample (x_i, y_i) is defined as:

$$L_i = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + \Delta)$$

where j iterates over all classes except the correct class y_i and s_j denotes the classifier score for j^{th} class. Δ is a scalar margin.

Hint: the SVM loss function is not strictly speaking differentiable.

YourAnswer :

A. The discrepancy happens because the SVM hinge loss is not differentiable at the point where $s_j - s_{y_i} + \Delta = 0$. Gradient check uses tiny finite differences, so if we are very close to this kink, the numerical and analytic gradients may not match.

B. It is not a concern because backprop never relies on finite differences. The model almost never lands exactly on the nondifferentiable point, so training is unaffected.

C. A simple 1-D failure case is when $s_j - s_{y_i} + \Delta = 0$

The hinge switches from 0 to positive there, so the slope jumps and finite differences read different values on each side.

D. Changing Δ only moves where the kink is located it does not make the model hit it more often. The nondifferentiable point is still just a single value, so mismatches remain rare.

[ ]:
```
# Next implement the function softmax_loss_vectorized; for now only compute the loss;
# we will implement the gradient in a moment.
```

```
tic = time.time()
loss_naive, grad_naive = softmax_loss_naive(W, X_dev, y_dev, 0.000005)
toc = time.time()
print('Naive loss: %e computed in %fs' % (loss_naive, toc - tic))

from dl.classifiers.softmax import softmax_loss_vectorized
tic = time.time()
loss_vectorized, _ = softmax_loss_vectorized(W, X_dev, y_dev, 0.000005)
toc = time.time()
print('Vectorized loss: %e computed in %fs' % (loss_vectorized, toc - tic))

# The losses should match but your vectorized implementation should be much␣
 ↪faster.
print('difference: %f' % (loss_naive - loss_vectorized))
```

```
Naive loss: 2.361754e+00 computed in 0.074206s
Vectorized loss: 2.361754e+00 computed in 0.012433s
difference: 0.000000
```

[ ]:
```
# Complete the implementation of softmax_loss_vectorized, and compute the␣
 ↪gradient
# of the loss function in a vectorized way.

# The naive implementation and the vectorized implementation should match, but
# the vectorized version should still be much faster.
tic = time.time()
_, grad_naive = softmax_loss_naive(W, X_dev, y_dev, 0.000005)
toc = time.time()
print('Naive loss and gradient: computed in %fs' % (toc - tic))

tic = time.time()
_, grad_vectorized = softmax_loss_vectorized(W, X_dev, y_dev, 0.000005)
toc = time.time()
print('Vectorized loss and gradient: computed in %fs' % (toc - tic))

# The loss is a single number, so it is easy to compare the values computed
# by the two implementations. The gradient on the other hand is a matrix, so
# we use the Frobenius norm to compare them.
difference = np.linalg.norm(grad_naive - grad_vectorized, ord='fro')
print('difference: %f' % difference)
```

```
Naive loss and gradient: computed in 0.109675s
Vectorized loss and gradient: computed in 0.009619s
difference: 0.000000
```

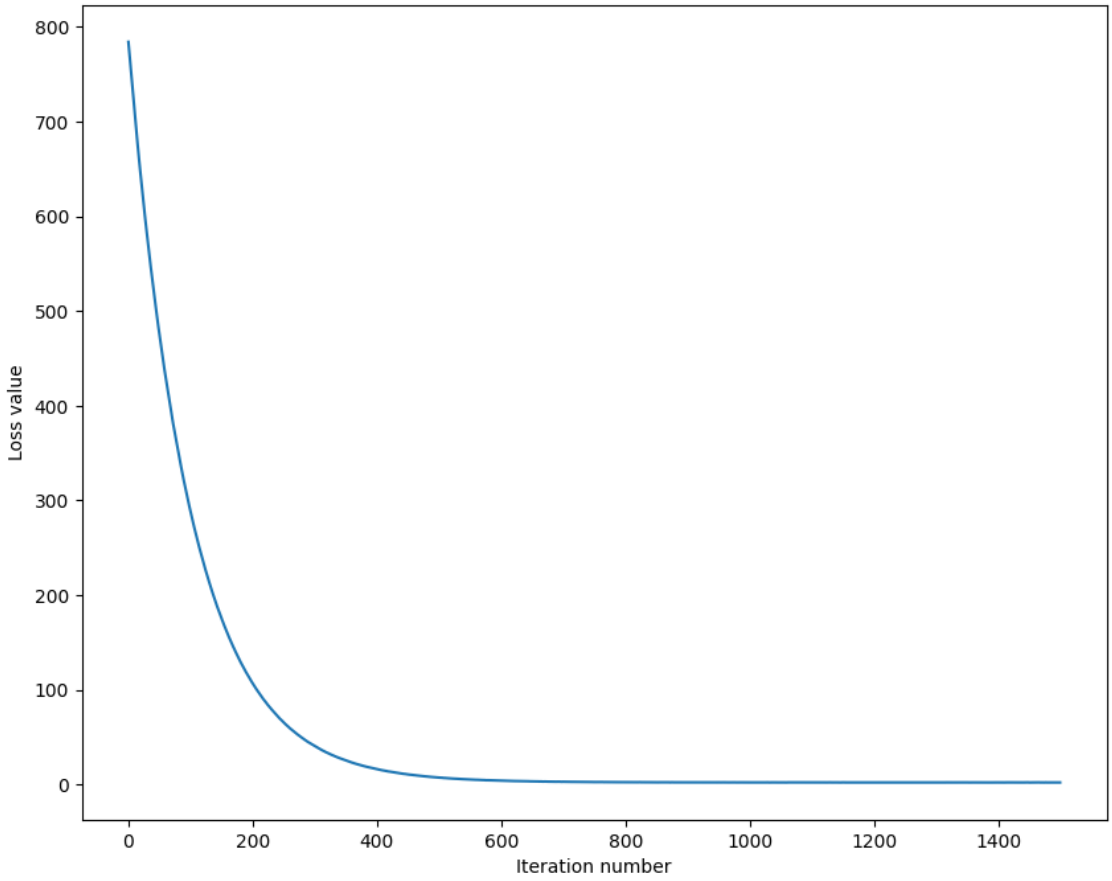
1.2.1 Stochastic Gradient Descent

We now have vectorized and efficient expressions for the loss, the gradient and our gradient matches the numerical gradient. We are therefore ready to do SGD to minimize the loss. Your code for this part will be written inside `dl/classifiers/linear_classifier.py`.

[ ]:
```
# In the file linear_classifier.py, implement SGD in the function
# LinearClassifier.train() and then run it with the code below.
from dl.classifiers import Softmax
softmax = Softmax()
tic = time.time()
loss_hist = softmax.train(X_train, y_train, learning_rate=1e-7, reg=2.5e4,
                      num_iters=1500, verbose=True)
toc = time.time()
print('That took %fs' % (toc - tic))
```

```
iteration 0 / 1500: loss 784.310757
iteration 100 / 1500: loss 288.007781
iteration 200 / 1500: loss 106.742597
iteration 300 / 1500: loss 40.311756
iteration 400 / 1500: loss 16.057462
iteration 500 / 1500: loss 7.198559
iteration 600 / 1500: loss 3.909639
iteration 700 / 1500: loss 2.769399
iteration 800 / 1500: loss 2.342930
iteration 900 / 1500: loss 2.160078
iteration 1000 / 1500: loss 2.110094
iteration 1100 / 1500: loss 2.176059
iteration 1200 / 1500: loss 2.095568
iteration 1300 / 1500: loss 2.123266
iteration 1400 / 1500: loss 2.034464
That took 7.086493s
```

[ ]:
```
# A useful debugging strategy is to plot the loss as a function of
# iteration number:
plt.plot(loss_hist)
plt.xlabel('Iteration number')
plt.ylabel('Loss value')
plt.show()
```



[ ]:
```
# Write the LinearClassifier.predict function and evaluate the performance on
# both the training and validation set
# You should get validation accuracy of about 0.34 (> 0.33).
y_train_pred = softmax.predict(X_train)
print('training accuracy: %f' % (np.mean(y_train == y_train_pred), ))
y_val_pred = softmax.predict(X_val)
print('validation accuracy: %f' % (np.mean(y_val == y_val_pred), ))
```

```
training accuracy: 0.326286
validation accuracy: 0.354000
```

[ ]:
```
# Save the trained model for autograder.
softmax.save("softmax.npy")
```

```
softmax.npy saved.
```

[ ]:
```
# Use the validation set to tune hyperparameters (regularization strength and
# learning rate). You should experiment with different ranges for the learning
# rates and regularization strengths; if you are careful you should be able to
```

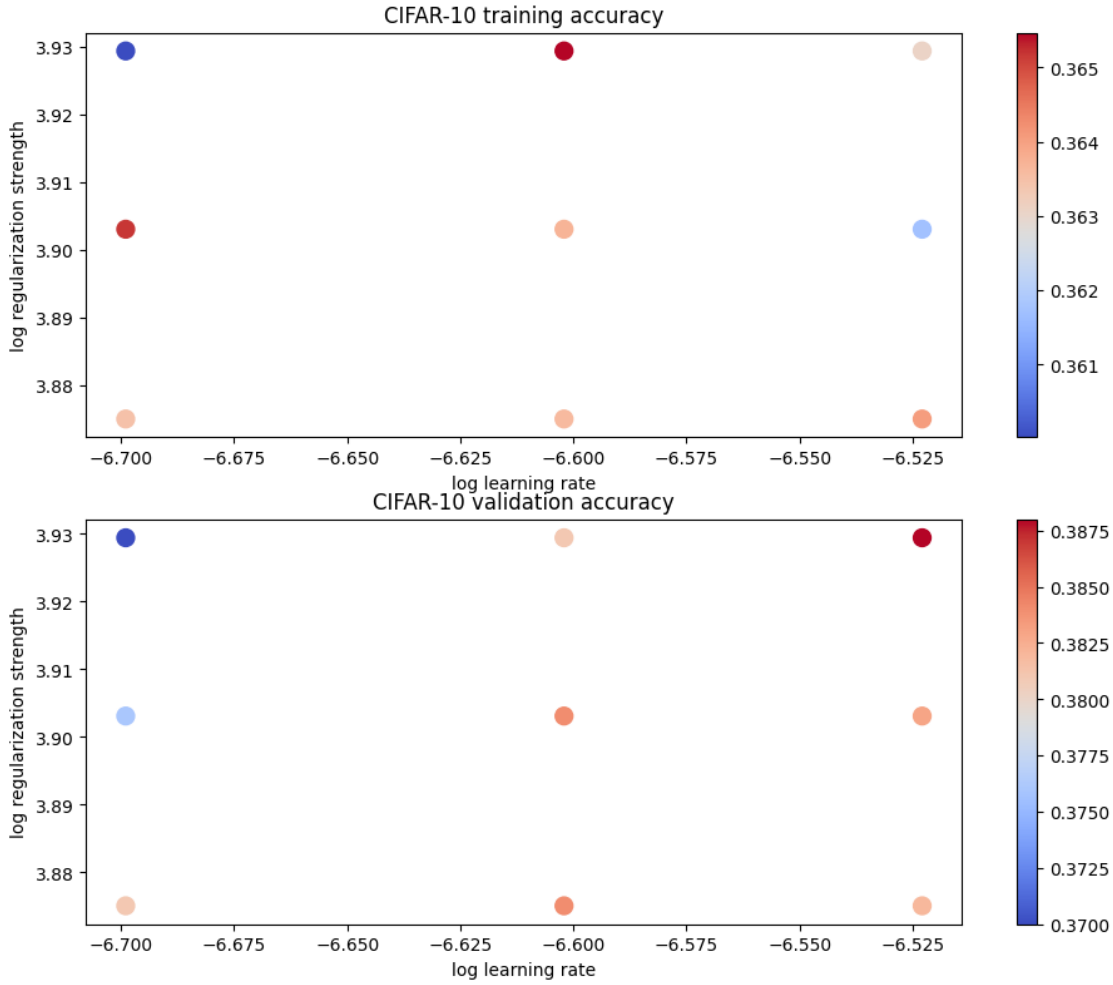
```
# get a classification accuracy of about 0.365 (> 0.36) on the validation set.

# Note: you may see runtime/overflow warnings during hyper-parameter search.
# This may be caused by extreme values, and is not a bug.

# results is dictionary mapping tuples of the form
# (learning_rate, regularization_strength) to tuples of the form
# (training_accuracy, validation_accuracy). The accuracy is simply the fraction
# of data points that are correctly classified.
results = {}
best_val = -1   # The highest validation accuracy that we have seen so far.
best_softmax = None # The Softmax object that achieved the highest validation␣
 ↪rate.
# Provided as a reference. You may or may not want to change these␣
 ↪hyperparameters
learning_rates = [2e-7, 2.5e-7, 3e-7]
regularization_strengths = [0.75e4, 0.8e4, 0.85e4]

# Provided as a reference. You may or may not want to change these␣
 ↪hyperparameters
#learning_rates = [1e-7, 1e-6]
#regularization_strengths = [2.5e4, 1e4]
################################################################################
# TODO:                                                                        #
# Write code that chooses the best hyperparameters by tuning on the validation #
# set. For each combination of hyperparameters, train a Softmax on the         #
# training set, compute its accuracy on the training and validation sets, and  #
# store these numbers in the results dictionary. In addition, store the best   #
# validation accuracy in best_val and the Softmax object that achieves this.   #
# accuracy in best_softmax.                                                    #
#                                                                              #
# Try all combinations of learning rates and regularization strengths
for lr in learning_rates:
    for reg in regularization_strengths:
        # Create a new Softmax classifier for each (lr, reg)
        softmax = Softmax()

        # Train the classifier with these hyperparameters
        loss_history = softmax.train(
            X_train,
            y_train,
            learning_rate=lr,
            reg=reg,
            num_iters=2000,    # you can start with e.g. 200 while debugging
            batch_size=300,
            verbose=False,
        )
```

```
        # Predict on training and validation sets
        y_train_pred = softmax.predict(X_train)
        y_val_pred   = softmax.predict(X_val)

        # Compute accuracies
        train_acc = np.mean(y_train_pred == y_train)
        val_acc   = np.mean(y_val_pred == y_val)

        # Store in results dict
        results[(lr, reg)] = (train_acc, val_acc)

        # Keep track of the best model (highest val accuracy)
        if val_acc > best_val:
            best_val = val_acc
            best_softmax = softmax
# Hint: You should use a small value for num_iters as you develop your         #
# validation code so that the classifiers don't take much time to train; once  #
# you are confident that your validation code works, you should rerun the      #
# code with a larger value for num_iters.                                      #
################################################################################




# Print out results.
for lr, reg in sorted(results):
    train_accuracy, val_accuracy = results[(lr, reg)]
    print('lr %e reg %e train accuracy: %f val accuracy: %f' % (
                lr, reg, train_accuracy, val_accuracy))

print('best validation accuracy achieved during cross-validation: %f' %␣
 ↪best_val)
```

```
lr 2.000000e-07 reg 7.500000e+03 train accuracy: 0.363490 val accuracy: 0.381000
lr 2.000000e-07 reg 8.000000e+03 train accuracy: 0.365184 val accuracy: 0.376000
lr 2.000000e-07 reg 8.500000e+03 train accuracy: 0.360020 val accuracy: 0.370000
lr 2.500000e-07 reg 7.500000e+03 train accuracy: 0.363612 val accuracy: 0.384000
lr 2.500000e-07 reg 8.000000e+03 train accuracy: 0.363714 val accuracy: 0.384000
lr 2.500000e-07 reg 8.500000e+03 train accuracy: 0.365469 val accuracy: 0.381000
lr 3.000000e-07 reg 7.500000e+03 train accuracy: 0.364061 val accuracy: 0.382000
lr 3.000000e-07 reg 8.000000e+03 train accuracy: 0.361714 val accuracy: 0.383000
lr 3.000000e-07 reg 8.500000e+03 train accuracy: 0.363082 val accuracy: 0.388000
best validation accuracy achieved during cross-validation: 0.388000
```

[ ]:
```
# Visualize the cross-validation results
import math
```

```
import pdb

# pdb.set_trace()

x_scatter = [math.log10(x[0]) for x in results]
y_scatter = [math.log10(x[1]) for x in results]

# plot training accuracy
marker_size = 100
colors = [results[x][0] for x in results]
plt.subplot(2, 1, 1)
plt.tight_layout(pad=3)
plt.scatter(x_scatter, y_scatter, marker_size, c=colors, cmap=plt.cm.coolwarm)
plt.colorbar()
plt.xlabel('log learning rate')
plt.ylabel('log regularization strength')
plt.title('CIFAR-10 training accuracy')

# plot validation accuracy
colors = [results[x][1] for x in results] # default size of markers is 20
plt.subplot(2, 1, 2)
plt.scatter(x_scatter, y_scatter, marker_size, c=colors, cmap=plt.cm.coolwarm)
plt.colorbar()
plt.xlabel('log learning rate')
plt.ylabel('log regularization strength')
plt.title('CIFAR-10 validation accuracy')
plt.show()
```



[ ]:
```
# Evaluate the best softmax on test set
y_test_pred = best_softmax.predict(X_test)
test_accuracy = np.mean(y_test == y_test_pred)
print('Softmax classifier on raw pixels final test set accuracy: %f' %␣
  ↪test_accuracy)
```

```
Softmax classifier on raw pixels final test set accuracy: 0.367000
```

[ ]:
```
# Save best softmax model
best_softmax.save("best_softmax.npy")
```

```
best_softmax.npy saved.
```

[ ]:
```
# Visualize the learned weights for each class.
# Depending on your choice of learning rate and regularization strength, these␣
  ↪may
# or may not be nice to look at.
```

```
w = best_softmax.W[:-1,:] # strip out the bias
w = w.reshape(32, 32, 3, 10)
w_min, w_max = np.min(w), np.max(w)
classes = ['plane', 'car', 'bird', 'cat', 'deer', 'dog', 'frog', 'horse',␣
 ↪'ship', 'truck']
for i in range(10):
    plt.subplot(2, 5, i + 1)

    # Rescale the weights to be between 0 and 255
    wimg = 255.0 * (w[:, :, :, i].squeeze() - w_min) / (w_max - w_min)
    plt.imshow(wimg.astype('uint8'))
    plt.axis('off')
    plt.title(classes[i])
```

Inline question 3

Describe what your visualized Softmax classifier weights look like, and offer a brief explanation for why they look the way they do.

YourAnswer :

The visualized Softmax weights appear as blurry, smoothed "average images" for each class. Since a linear Softmax classifier can only learn one weight vector per class, it captures only broad color and intensity patterns rather than detailed shapes. As a result, each weight map resembles a fuzzy template showing which pixel regions tend to be important for predicting that class (e.g., ships appear bluish, frogs greenish).

Inline Question 4 - *True or False*

Suppose the overall training loss is defined as the sum of the per-datapoint loss over all training examples. It is possible to add a new datapoint to a training set that would change the softmax loss, but leave the SVM loss unchanged.

YourAnswer : True

YourExplanation : The SVM hinge loss becomes zero for datapoints that satisfy the margin (i.e., all $s_j - s_y + \Delta < 0$). Adding such a datapoint does not change the total SVM loss. However, the Softmax loss never becomes exactly zero, since it depends on the negative log of the predicted probability for the correct class. Even a perfectly classified example with high confidence still has a positive Softmax loss. Therefore, adding the datapoint will change the Softmax loss but leave the SVM loss unchanged.

[ ]:

02_two_layer_net

December 6, 2025

[ ]:
```
# This mounts your Google Drive to the Colab VM.
from google.colab import drive
drive.mount('/content/drive')

# TODO: Enter the foldername in your Drive where you have saved the unzipped
# assignment folder, e.g. 'dl/assignments/assignment1/'
FOLDERNAME = 'dl/assignments/assignment1/'
assert FOLDERNAME is not None, "[!] Enter the foldername."

# Now that we've mounted your Drive, this ensures that
# the Python interpreter of the Colab VM can load
# python files from within it.
import sys
sys.path.append('/content/drive/My Drive/{}'.format(FOLDERNAME))

# This downloads the CIFAR-10 dataset to your Drive
# if it doesn't already exist.
%cd /content/drive/My\ Drive/$FOLDERNAME/dl/datasets/
!bash get_datasets.sh
%cd /content/drive/My\ Drive/$FOLDERNAME
```

```
Drive already mounted at /content/drive; to attempt to forcibly remount, call
drive.mount("/content/drive", force_remount=True).
/content/drive/My Drive/dl/assignments/assignment1/dl/datasets
/content/drive/My Drive/dl/assignments/assignment1
```

1 Fully-Connected Neural Nets

In this exercise we will implement fully-connected networks using a modular approach. For each layer we will implement a `forward` and a `backward` function. The `forward` function will receive inputs, weights, and other parameters and will return both an output and a `cache` object storing data needed for the backward pass, like this:

```
def layer_forward(x, w):
  """ Receive inputs x and weights w """
  # Do some computations ...
  z = # ... some intermediate value
  # Do some more computations ...
```

```
  out = # the output

  cache = (x, w, z, out) # Values we need to compute gradients

  return out, cache
```

The backward pass will receive upstream derivatives and the `cache` object, and will return gradients with respect to the inputs and weights, like this:

```
def layer_backward(dout, cache):
  """
  Receive dout (derivative of loss with respect to outputs) and cache,
  and compute derivative with respect to inputs.
  """
  # Unpack cache values
  x, w, z, out = cache

  # Use values in cache to compute derivatives
  dx = # Derivative of loss with respect to x
  dw = # Derivative of loss with respect to w

  return dx, dw
```

After implementing a bunch of layers this way, we will be able to easily combine them to build classifiers with different architectures.

[ ]:
```
# As usual, a bit of setup
from __future__ import print_function
import time
import numpy as np
import matplotlib.pyplot as plt
from dl.classifiers.fc_net import *
from dl.data_utils import get_CIFAR10_data
from dl.gradient_check import eval_numerical_gradient,␣
 ↪eval_numerical_gradient_array
from dl.solver import Solver

%matplotlib inline
plt.rcParams['figure.figsize'] = (10.0, 8.0) # set default size of plots
plt.rcParams['image.interpolation'] = 'nearest'
plt.rcParams['image.cmap'] = 'gray'

def rel_error(x, y):
  """ returns relative error """
  return np.max(np.abs(x - y) / (np.maximum(1e-8, np.abs(x) + np.abs(y))))
```

[ ]:
```
# Load the (preprocessed) CIFAR10 data.

data = get_CIFAR10_data()
```

```
for k, v in list(data.items()):
  print(('%s: ' % k, v.shape))
```

```
('X_train: ', (49000, 3, 32, 32))
('y_train: ', (49000,))
('X_val: ', (1000, 3, 32, 32))
('y_val: ', (1000,))
('X_test: ', (1000, 3, 32, 32))
('y_test: ', (1000,))
```

2 Affine layer: forward

Open the file `dl/layers.py` and implement the `affine_forward` function.

Once you are done you can test your implementaion by running the following:

[ ]:
```
# Test the affine_forward function

num_inputs = 2
input_shape = (4, 5, 6)
output_dim = 3

input_size = num_inputs * np.prod(input_shape)
weight_size = output_dim * np.prod(input_shape)

x = np.linspace(-0.1, 0.5, num=input_size).reshape(num_inputs, *input_shape)
w = np.linspace(-0.2, 0.3, num=weight_size).reshape(np.prod(input_shape),␣
 ↪output_dim)
b = np.linspace(-0.3, 0.1, num=output_dim)

out, _ = affine_forward(x, w, b)
correct_out = np.array([[ 1.49834967,  1.70660132,  1.91485297],
                        [ 3.25553199,  3.5141327,   3.77273342]])

# Compare your output with ours. The error should be around e-9 or less.
print('Testing affine_forward function:')
print('difference: ', rel_error(out, correct_out))
```

```
Testing affine_forward function:
difference:  9.769849468192957e-10
```

3 Affine layer: backward

Now implement the `affine_backward` function and test your implementation using numeric gradient checking.

[ ]:
```
# Test the affine_backward function
np.random.seed(231)
x = np.random.randn(10, 2, 3)
w = np.random.randn(6, 5)
b = np.random.randn(5)
dout = np.random.randn(10, 5)

dx_num = eval_numerical_gradient_array(lambda x: affine_forward(x, w, b)[0], x,␣
 ↪dout)
dw_num = eval_numerical_gradient_array(lambda w: affine_forward(x, w, b)[0], w,␣
 ↪dout)
db_num = eval_numerical_gradient_array(lambda b: affine_forward(x, w, b)[0], b,␣
 ↪dout)

_, cache = affine_forward(x, w, b)
dx, dw, db = affine_backward(dout, cache)

# The error should be around e-10 or less
print('Testing affine_backward function:')
print('dx error: ', rel_error(dx_num, dx))
print('dw error: ', rel_error(dw_num, dw))
print('db error: ', rel_error(db_num, db))
```

```
Testing affine_backward function:
dx error:  5.399100368651805e-11
dw error:  9.904211865398145e-11
db error:  2.4122867568119087e-11
```

4 ReLU activation: forward

Implement the forward pass for the ReLU activation function in the `relu_forward` function and test your implementation using the following:

[ ]:
```
# Test the relu_forward function

x = np.linspace(-0.5, 0.5, num=12).reshape(3, 4)

out, _ = relu_forward(x)
correct_out = np.array([[ 0.,          0.,          0.,          0.,        ],
                        [ 0.,          0.,          0.04545455,  0.13636364,],
                        [ 0.22727273,  0.31818182,  0.40909091,  0.5,       ]])

# Compare your output with ours. The error should be on the order of e-8
print('Testing relu_forward function:')
print('difference: ', rel_error(out, correct_out))
```

```
Testing relu_forward function:
difference:  4.999999798022158e-08
```

5 ReLU activation: backward

Now implement the backward pass for the ReLU activation function in the `relu_backward` function and test your implementation using numeric gradient checking:

[ ]:
```
np.random.seed(231)
x = np.random.randn(10, 10)
dout = np.random.randn(*x.shape)

dx_num = eval_numerical_gradient_array(lambda x: relu_forward(x)[0], x, dout)

_, cache = relu_forward(x)
dx = relu_backward(dout, cache)

# The error should be on the order of e-12
print('Testing relu_backward function:')
print('dx error: ', rel_error(dx_num, dx))
```

```
Testing relu_backward function:
dx error:  3.2756349136310288e-12
```

5.1 Inline Question 1:

We've only asked you to implement ReLU, but there are a number of different activation functions that one could use in neural networks, each with its pros and cons. In particular, an issue commonly seen with activation functions is getting zero (or close to zero) gradient flow during backpropagation. Which of the following activation functions have this problem? If you consider these functions in the one dimensional case, what types of input would lead to this behaviour? 1. Sigmoid 2. ReLU 3. Leaky ReLU

YourAnswer :

Sigmoid :TRUE, zero gradient for very negative/positive inputs .

ReLU :TRUE, zero gradient for all negative inputs.

Leaky ReLU :FALSE, avoids zero gradient because negative slope 0

6 "Sandwich" layers

There are some common patterns of layers that are frequently used in neural nets. For example, affine layers are frequently followed by a ReLU nonlinearity. To make these common patterns easy, we define several convenience layers in the file `dl/layer_utils.py`.

For now take a look at the `affine_relu_forward` and `affine_relu_backward` functions, and run the following to numerically gradient check the backward pass:

[ ]:
```
from dl.layer_utils import affine_relu_forward, affine_relu_backward
np.random.seed(231)
x = np.random.randn(2, 3, 4)
w = np.random.randn(12, 10)
b = np.random.randn(10)
dout = np.random.randn(2, 10)

out, cache = affine_relu_forward(x, w, b)
dx, dw, db = affine_relu_backward(dout, cache)

dx_num = eval_numerical_gradient_array(lambda x: affine_relu_forward(x, w,␣
 ↪b)[0], x, dout)
dw_num = eval_numerical_gradient_array(lambda w: affine_relu_forward(x, w,␣
 ↪b)[0], w, dout)
db_num = eval_numerical_gradient_array(lambda b: affine_relu_forward(x, w,␣
 ↪b)[0], b, dout)

# Relative error should be around e-10 or less
print('Testing affine_relu_forward and affine_relu_backward:')
print('dx error: ', rel_error(dx_num, dx))
print('dw error: ', rel_error(dw_num, dw))
print('db error: ', rel_error(db_num, db))
```

```
Testing affine_relu_forward and affine_relu_backward:
dx error:  2.299579177309368e-11
dw error:  8.162011105764925e-11
db error:  7.826724021458994e-12
```

7 Loss layers: Softmax

Now implement the loss and gradient for softmax in the `softmax_loss` function in `dl/layers.py`. These should be similar to what you implemented in `dl/classifiers/softmax.py`. Other loss functions (e.g. `svm_loss`) can also be implemented in a modular way, however, it is not required for this assignment.

You can make sure that the implementations are correct by running the following:

[ ]:
```
np.random.seed(231)
num_classes, num_inputs = 10, 50
x = 0.001 * np.random.randn(num_inputs, num_classes)
y = np.random.randint(num_classes, size=num_inputs)


dx_num = eval_numerical_gradient(lambda x: softmax_loss(x, y)[0], x,␣
 ↪verbose=False)
loss, dx = softmax_loss(x, y)
```

```
# Test softmax_loss function. Loss should be close to 2.3 and dx error should␣
 ↪be around e-8
print('\nTesting softmax_loss:')
print('loss: ', loss)
print('dx error: ', rel_error(dx_num, dx))
```

```
Testing softmax_loss:
loss:  2.3025458445007376
dx error:  8.234144091578429e-09
```

8 Two-layer network

Open the file `dl/classifiers/fc_net.py` and complete the implementation of the `TwoLayerNet` class. Read through it to make sure you understand the API. You can run the cell below to test your implementation.

[ ]:
```
np.random.seed(231)
N, D, H, C = 3, 5, 50, 7
X = np.random.randn(N, D)
y = np.random.randint(C, size=N)

std = 1e-3
model = TwoLayerNet(input_dim=D, hidden_dim=H, num_classes=C, weight_scale=std)

print('Testing initialization ... ')
W1_std = abs(model.params['W1'].std() - std)
b1 = model.params['b1']
W2_std = abs(model.params['W2'].std() - std)
b2 = model.params['b2']
assert W1_std < std / 10, 'First layer weights do not seem right'
assert np.all(b1 == 0), 'First layer biases do not seem right'
assert W2_std < std / 10, 'Second layer weights do not seem right'
assert np.all(b2 == 0), 'Second layer biases do not seem right'

print('Testing test-time forward pass ... ')
model.params['W1'] = np.linspace(-0.7, 0.3, num=D*H).reshape(D, H)
model.params['b1'] = np.linspace(-0.1, 0.9, num=H)
model.params['W2'] = np.linspace(-0.3, 0.4, num=H*C).reshape(H, C)
model.params['b2'] = np.linspace(-0.9, 0.1, num=C)
X = np.linspace(-5.5, 4.5, num=N*D).reshape(D, N).T
scores = model.loss(X)
correct_scores = np.asarray(
  [[11.53165108,  12.2917344,   13.05181771,  13.81190102,  14.57198434, 15.
 ↪33206765,  16.09215096],
   [12.05769098,  12.74614105,  13.43459113,  14.1230412,   14.81149128, 15.
 ↪49994135,  16.18839143],
```

```
   [12.58373087,  13.20054771,  13.81736455,  14.43418138,  15.05099822, 15.
 ↪66781506,  16.2846319 ]])
scores_diff = np.abs(scores - correct_scores).sum()
assert scores_diff < 1e-6, 'Problem with test-time forward pass'

print('Testing training loss (no regularization)')
y = np.asarray([0, 5, 1])
loss, grads = model.loss(X, y)
correct_loss = 3.4702243556
assert abs(loss - correct_loss) < 1e-10, 'Problem with training-time loss'

model.reg = 1.0
loss, grads = model.loss(X, y)
correct_loss = 26.5948426952
assert abs(loss - correct_loss) < 1e-10, 'Problem with regularization loss'

# Errors should be around e-7 or less
for reg in [0.0, 0.7]:
  print('Running numeric gradient check with reg = ', reg)
  model.reg = reg
  loss, grads = model.loss(X, y)

  for name in sorted(grads):
    f = lambda _: model.loss(X, y)[0]
    grad_num = eval_numerical_gradient(f, model.params[name], verbose=False)
    print('%s relative error: %.2e' % (name, rel_error(grad_num, grads[name])))
```

```
Testing initialization …
Testing test-time forward pass …
Testing training loss (no regularization)
Running numeric gradient check with reg =  0.0
W1 relative error: 1.83e-08
W2 relative error: 3.20e-10
b1 relative error: 9.83e-09
b2 relative error: 4.33e-10
Running numeric gradient check with reg =  0.7
W1 relative error: 2.53e-07
W2 relative error: 2.85e-08
b1 relative error: 1.56e-08
b2 relative error: 9.09e-10
```

9 Solver

Open the file `dl/solver.py` and read through it to familiarize yourself with the API. After doing so, use a `Solver` instance to train a `TwoLayerNet` that achieves about `36%` accuracy on the validation set.

[ ]:
```
input_size = 32 * 32 * 3
hidden_size = 50
num_classes = 10
model = TwoLayerNet(input_size, hidden_size, num_classes)
solver = None

################################################################################
# TODO: Use a Solver instance to train a TwoLayerNet that achieves about 36% #
# accuracy on the validation set.                                              #
################################################################################

solver = Solver(
    model,
    data,
    update_rule='sgd',
    optim_config={'learning_rate': 1e-3},
    lr_decay=0.7,
    num_epochs=10,
    batch_size=40,
    print_every=500
)

solver.train()
################################################################################
#                              END OF YOUR CODE                                #
################################################################################
```

```
(Iteration 1 / 12250) loss: 2.299721
(Epoch 0 / 10) train acc: 0.140000; val_acc: 0.124000
(Iteration 501 / 12250) loss: 1.563805
(Iteration 1001 / 12250) loss: 1.494839
(Epoch 1 / 10) train acc: 0.424000; val_acc: 0.455000
(Iteration 1501 / 12250) loss: 1.244195
(Iteration 2001 / 12250) loss: 1.435301
(Epoch 2 / 10) train acc: 0.495000; val_acc: 0.449000
(Iteration 2501 / 12250) loss: 1.430156
(Iteration 3001 / 12250) loss: 1.738859
(Iteration 3501 / 12250) loss: 1.326043
(Epoch 3 / 10) train acc: 0.493000; val_acc: 0.469000
(Iteration 4001 / 12250) loss: 1.286425
(Iteration 4501 / 12250) loss: 1.270864
(Epoch 4 / 10) train acc: 0.532000; val_acc: 0.501000
(Iteration 5001 / 12250) loss: 1.347897
(Iteration 5501 / 12250) loss: 1.216622
(Iteration 6001 / 12250) loss: 1.372079
(Epoch 5 / 10) train acc: 0.552000; val_acc: 0.505000
(Iteration 6501 / 12250) loss: 1.032235
```

```
(Iteration 7001 / 12250) loss: 1.116262
(Epoch 6 / 10) train acc: 0.577000; val_acc: 0.514000
(Iteration 7501 / 12250) loss: 1.231360
(Iteration 8001 / 12250) loss: 1.083451
(Iteration 8501 / 12250) loss: 1.269069
(Epoch 7 / 10) train acc: 0.577000; val_acc: 0.523000
(Iteration 9001 / 12250) loss: 1.368745
(Iteration 9501 / 12250) loss: 1.174837
(Epoch 8 / 10) train acc: 0.570000; val_acc: 0.516000
(Iteration 10001 / 12250) loss: 1.356955
(Iteration 10501 / 12250) loss: 1.148590
(Iteration 11001 / 12250) loss: 1.059183
(Epoch 9 / 10) train acc: 0.605000; val_acc: 0.509000
(Iteration 11501 / 12250) loss: 1.172649
(Iteration 12001 / 12250) loss: 1.120144
(Epoch 10 / 10) train acc: 0.636000; val_acc: 0.524000
```

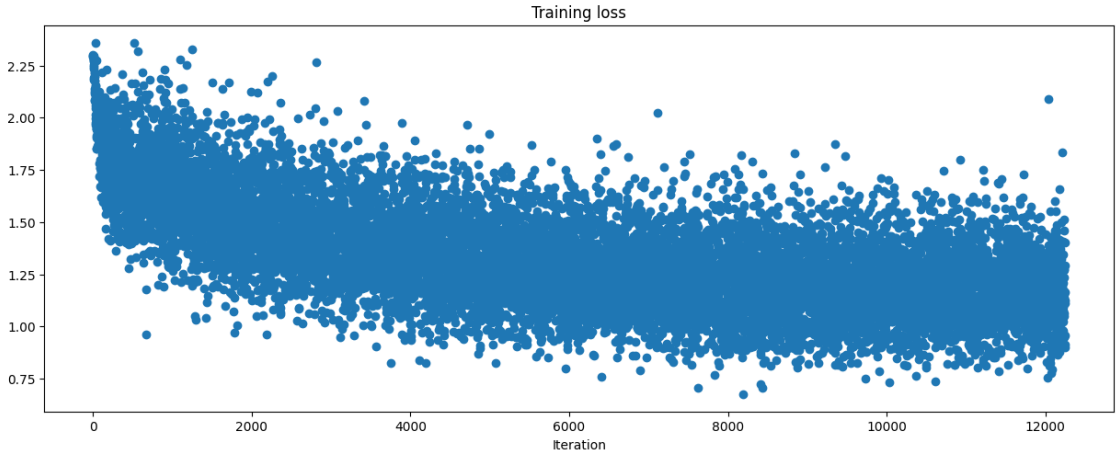
10 Debug the training

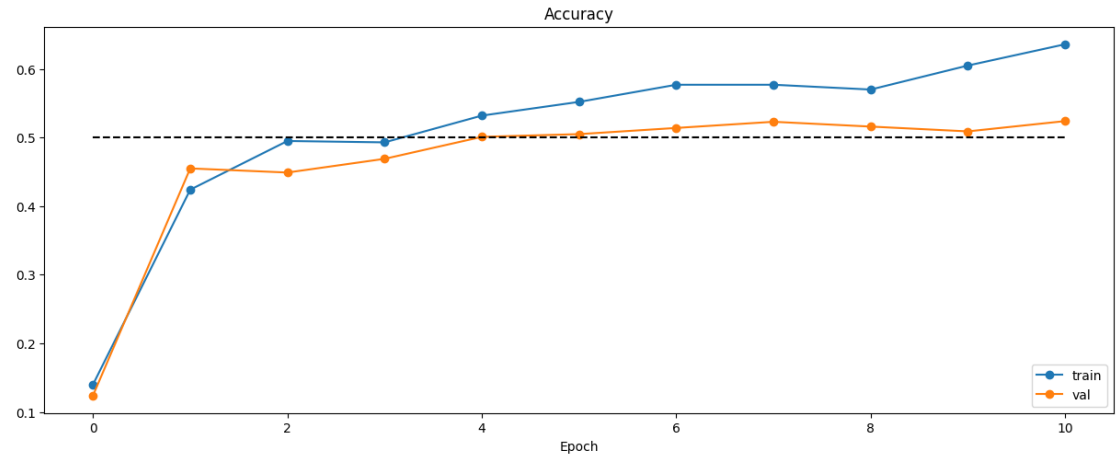
With the default parameters we provided above, you should get a validation accuracy of about 0.36 on the validation set. This isn't very good.

One strategy for getting insight into what's wrong is to plot the loss function and the accuracies on the training and validation sets during optimization.

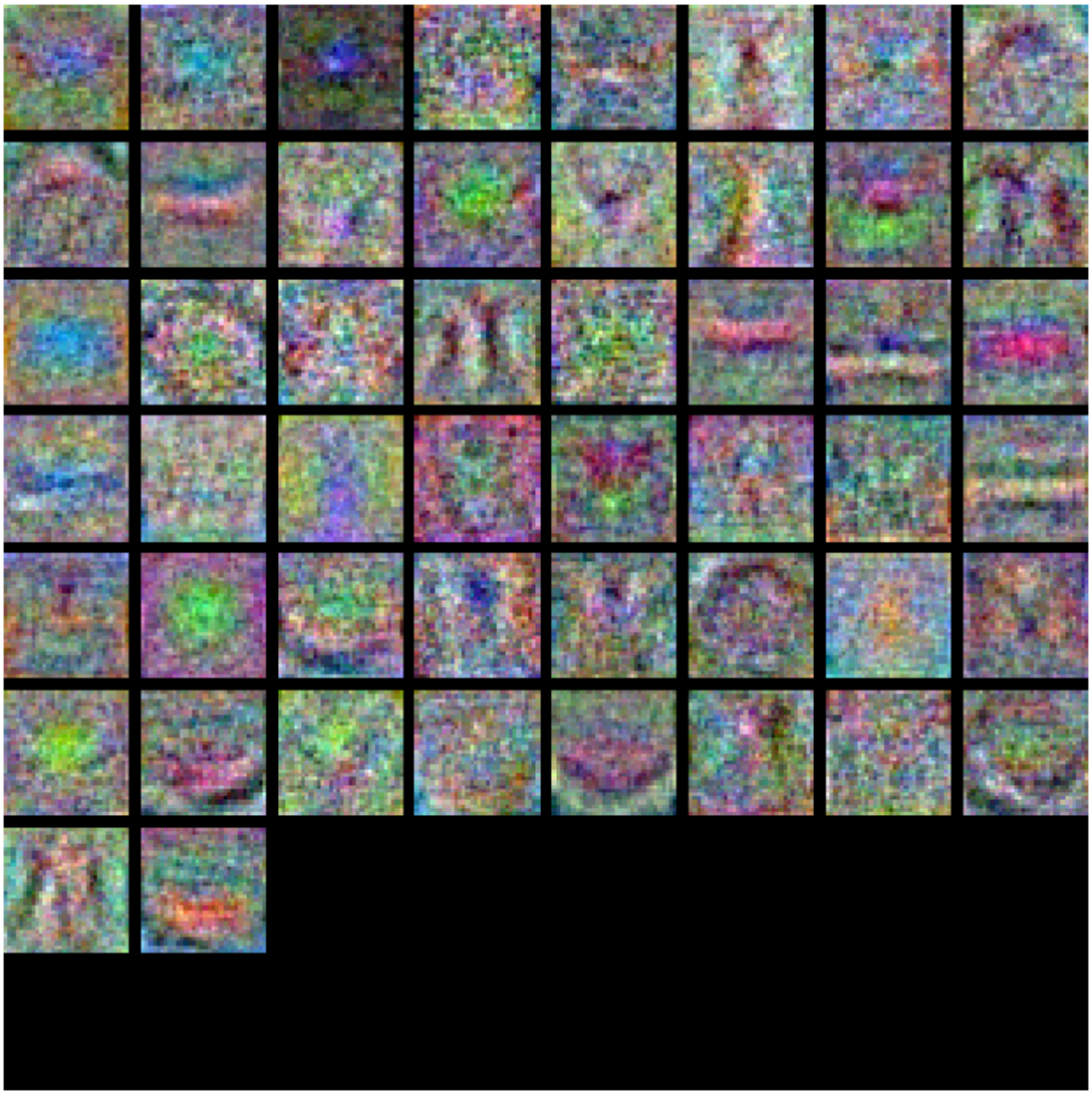
Another strategy is to visualize the weights that were learned in the first layer of the network. In most neural networks trained on visual data, the first layer weights typically show some visible structure when visualized.

[ ]:
```
# Run this cell to visualize training loss and train / val accuracy

plt.subplot(2, 1, 1)
plt.title('Training loss')
plt.plot(solver.loss_history, 'o')
plt.xlabel('Iteration')

plt.subplot(2, 1, 2)
plt.title('Accuracy')
plt.plot(solver.train_acc_history, '-o', label='train')
plt.plot(solver.val_acc_history, '-o', label='val')
plt.plot([0.5] * len(solver.val_acc_history), 'k--')
plt.xlabel('Epoch')
plt.legend(loc='lower right')
plt.gcf().set_size_inches(15, 12)
plt.show()
```





[ ]:
```python
from dl.vis_utils import visualize_grid

# Visualize the weights of the network

def show_net_weights(net):
    W1 = net.params['W1']
    W1 = W1.reshape(3, 32, 32, -1).transpose(3, 1, 2, 0)
    plt.imshow(visualize_grid(W1, padding=3).astype('uint8'))
    plt.gca().axis('off')
    plt.show()

show_net_weights(model)
```



11 Tune your hyperparameters

What's wrong?. Looking at the visualizations above, we see that the loss is decreasing more or less linearly, which seems to suggest that the learning rate may be too low. Moreover, there is no gap between the training and validation accuracy, suggesting that the model we used has low capacity, and that we should increase its size. On the other hand, with a very large model we would expect to see more overfitting, which would manifest itself as a very large gap between the training and validation accuracy.

Tuning. Tuning the hyperparameters and developing intuition for how they affect the final performance is a large part of using Neural Networks, so we want you to get a lot of practice. Below, you should experiment with different values of the various hyperparameters, including hidden layer size, learning rate, numer of training epochs, and regularization strength. You might also consider

tuning the learning rate decay, but you should be able to get good performance using the default value.

Approximate results. You should be aim to achieve a classification accuracy of greater than 48% on the validation set. Our best network gets over 52% on the validation set.

Experiment: You goal in this exercise is to get as good of a result on CIFAR-10 as you can (52% could serve as a reference), with a fully-connected Neural Network. Feel free implement your own techniques (e.g. PCA to reduce dimensionality, or adding dropout, or adding features to the solver, etc.).

[ ]:
```
best_model = None



################################################################################
# TODO: Tune hyperparameters using the validation set. Store your best trained #
# model in best_model.                                                         #
#                                                                              #
# To help debug your network, it may help to use visualizations similar to the #
# ones we used above; these visualizations will have significant qualitative   #
# differences from the ones we saw above for the poorly tuned network.         #
#                                                                              #
# Tweaking hyperparameters by hand can be fun, but you might find it useful to #
# write code to sweep through possible combinations of hyperparameters         #
# automatically like we did on thexs previous exercises.                       #
################################################################################
best_model = None
best_val = -1.0      # best validation accuracy so far
results = {}         # (hidden, lr, reg) -> (train_acc, val_acc)

input_size = 32 * 32 * 3
num_classes = 10

# You can tweak these lists to try more / fewer combos
hidden_sizes = [50, 100]
learning_rates = [7e-4, 1e-3, 1.5e-3]
regs = [0.0, 0.25, 0.5]
```

```
for h in hidden_sizes:
    for lr in learning_rates:
        for reg in regs:
            print('Training model with hidden_size=%d, lr=%e, reg=%e'
                  % (h, lr, reg))

            model = TwoLayerNet(input_dim=input_size,
                                hidden_dim=h,
                                num_classes=num_classes,
                                reg=reg)

            solver = Solver(
                model,
                data,
                update_rule='sgd',
                optim_config={'learning_rate': lr},
                lr_decay=0.95,
                num_epochs=10,
                batch_size=100,
                print_every=0,      # set >0 if you want per-iteration logs
                verbose=False
            )

            solver.train()

            train_acc = solver.train_acc_history[-1]
            val_acc = solver.val_acc_history[-1]
            results[(h, lr, reg)] = (train_acc, val_acc)

            print('  -> train acc: %.4f; val acc: %.4f'
                  % (train_acc, val_acc))

            # Keep track of the best model
            if val_acc > best_val:
                best_val = val_acc
                best_model = model
                best_h, best_lr, best_reg = h, lr, reg

print('========== Search done ==========')
print('Best validation accuracy: %.4f' % best_val)
print('Best hyperparameters: hidden_size=%d, lr=%e, reg=%e'
      % (best_h, best_lr, best_reg))
################################################################################
#                              END OF YOUR CODE                                #
################################################################################
```

```
Training model with hidden_size=50, lr=7.000000e-04, reg=0.000000e+00
  -> train acc: 0.5500; val acc: 0.5070
Training model with hidden_size=50, lr=7.000000e-04, reg=2.500000e-01
  -> train acc: 0.5380; val acc: 0.4800
Training model with hidden_size=50, lr=7.000000e-04, reg=5.000000e-01
  -> train acc: 0.5340; val acc: 0.4970
Training model with hidden_size=50, lr=1.000000e-03, reg=0.000000e+00
  -> train acc: 0.5610; val acc: 0.5050
Training model with hidden_size=50, lr=1.000000e-03, reg=2.500000e-01
  -> train acc: 0.5720; val acc: 0.4970
Training model with hidden_size=50, lr=1.000000e-03, reg=5.000000e-01
  -> train acc: 0.5220; val acc: 0.4780
Training model with hidden_size=50, lr=1.500000e-03, reg=0.000000e+00
  -> train acc: 0.5260; val acc: 0.4660
Training model with hidden_size=50, lr=1.500000e-03, reg=2.500000e-01
  -> train acc: 0.5450; val acc: 0.4820
Training model with hidden_size=50, lr=1.500000e-03, reg=5.000000e-01
  -> train acc: 0.5100; val acc: 0.4910
Training model with hidden_size=100, lr=7.000000e-04, reg=0.000000e+00
  -> train acc: 0.5950; val acc: 0.5230
Training model with hidden_size=100, lr=7.000000e-04, reg=2.500000e-01
  -> train acc: 0.5900; val acc: 0.4970
Training model with hidden_size=100, lr=7.000000e-04, reg=5.000000e-01
  -> train acc: 0.5920; val acc: 0.5200
Training model with hidden_size=100, lr=1.000000e-03, reg=0.000000e+00
  -> train acc: 0.6030; val acc: 0.5180
Training model with hidden_size=100, lr=1.000000e-03, reg=2.500000e-01
  -> train acc: 0.6140; val acc: 0.5070
Training model with hidden_size=100, lr=1.000000e-03, reg=5.000000e-01
  -> train acc: 0.5810; val acc: 0.5200
Training model with hidden_size=100, lr=1.500000e-03, reg=0.000000e+00
  -> train acc: 0.5980; val acc: 0.5140
Training model with hidden_size=100, lr=1.500000e-03, reg=2.500000e-01
  -> train acc: 0.5710; val acc: 0.4970
Training model with hidden_size=100, lr=1.500000e-03, reg=5.000000e-01
  -> train acc: 0.5730; val acc: 0.5240
========== Search done ==========
Best validation accuracy: 0.5240
Best hyperparameters: hidden_size=100, lr=1.500000e-03, reg=5.000000e-01
```

12 Test your model!

Run your best model on the validation and test sets. You should achieve above 48% accuracy on the validation set and the test set.

[ ]:
```
y_val_pred = np.argmax(best_model.loss(data['X_val']), axis=1)
print('Validation set accuracy: ', (y_val_pred == data['y_val']).mean())
```

```
Validation set accuracy:  0.524
```

[ ]:
```
y_test_pred = np.argmax(best_model.loss(data['X_test']), axis=1)
print('Test set accuracy: ', (y_test_pred == data['y_test']).mean())
```

```
Test set accuracy:  0.51
```

[ ]:
```
# Save best model
best_model.save("best_two_layer_net.npy")
```

```
best_two_layer_net.npy saved.
```

12.1 Inline Question 2:

Now that you have trained a Neural Network classifier, you may find that your testing accuracy is much lower than the training accuracy. In what ways can we decrease this gap? Select all that apply.

1. Train on a larger dataset.
2. Add more hidden units.
3. Increase the regularization strength.
4. None of the above.

YourAnswer :

1. Train on a larger dataset.
2. Increase the regularization strength.

YourExplanation :

The gap between the training accuracy and the testing accuracy occurs because the model is overfitting the training data. This means the neural network has learned patterns, noise, and details specific to the training set rather than learning general features that apply to unseen data. As a result, it performs very well on the training data but fails to generalize to new examples, leading to lower test accuracy.

1. **Train on a larger dataset.**
   This helps reduce overfitting because the model is exposed to more diverse examples. With more data, the model cannot simply memorize the training set and must learn more general patterns.

2. **Add more hidden units.**
   Adding hidden units increases the model's capacity, making it more complex. A more complex network is even more likely to memorize the training data, which **increases overfitting** and makes the training-testing gap larger.

3. **Increase the regularization strength.**
   Regularization techniques: limit the complexity and **noise** of the model. By increasing regularization strength, the model is prevented from overfitting, which improves generalization and **reduces the accuracy gap**.

4. **None of the above.**
   This is incorrect because both option (1) and option (3) are valid methods for reducing the gap between training and testing accuracy.

[ ]:

03_FullyConnectedNets

December 6, 2025

[ ]:
```
# This mounts your Google Drive to the Colab VM.
from google.colab import drive
drive.mount('/content/drive')

# TODO: Enter the foldername in your Drive where you have saved the unzipped
# assignment folder, e.g. 'dl/assignments/assignment1/'
FOLDERNAME = 'dl/assignments/assignment1/'
assert FOLDERNAME is not None, "[!] Enter the foldername."

# Now that we've mounted your Drive, this ensures that
# the Python interpreter of the Colab VM can load
# python files from within it.
import sys
sys.path.append('/content/drive/My Drive/{}'.format(FOLDERNAME))

# This downloads the CIFAR-10 dataset to your Drive
# if it doesn't already exist.
%cd /content/drive/My\ Drive/$FOLDERNAME/dl/datasets/
# %cd /content/drive/My\ Drive/$FOLDERNAME
!bash get_datasets.sh
%cd /content/drive/My\ Drive/$FOLDERNAME
```

```
Drive already mounted at /content/drive; to attempt to forcibly remount, call
drive.mount("/content/drive", force_remount=True).
/content/drive/My Drive/dl/assignments/assignment1/dl/datasets
/content/drive/My Drive/dl/assignments/assignment1
```

1 Multi-Layer Fully Connected Network

In this exercise, you will implement a fully connected network with an arbitrary number of hidden layers.

[ ]:
```
# from google.colab import drive
# drive.mount('/content/drive')
```

Read through the `FullyConnectedNet` class in the file `dl/classifiers/fc_net.py`.

Implement the network initialization, forward pass, and backward pass. Throughout this assign-

ment, you will be implementing layers in `dl/layers.py`. You can re-use your implementations for `affine_forward`, `affine_backward`, `relu_forward`, `relu_backward`, and `softmax_loss` from before. For right now, don't worry about implementing dropout or batch/layer normalization yet, as you will add those features later.

[ ]:
```
# Setup cell.
import time
import numpy as np
import matplotlib.pyplot as plt
from dl.classifiers.fc_net import *
from dl.data_utils import get_CIFAR10_data
from dl.gradient_check import eval_numerical_gradient,␣
 ↪eval_numerical_gradient_array
from dl.solver import Solver

%matplotlib inline
plt.rcParams["figure.figsize"] = (10.0, 8.0)  # Set default size of plots.
plt.rcParams["image.interpolation"] = "nearest"
plt.rcParams["image.cmap"] = "gray"

def rel_error(x, y):
    """Returns relative error."""
    return np.max(np.abs(x - y) / (np.maximum(1e-8, np.abs(x) + np.abs(y))))
```

[ ]:
```
# Load the (preprocessed) CIFAR-10 data.
data = get_CIFAR10_data()
for k, v in list(data.items()):
    print(f"{k}: {v.shape}")
```

```
X_train: (49000, 3, 32, 32)
y_train: (49000,)
X_val: (1000, 3, 32, 32)
y_val: (1000,)
X_test: (1000, 3, 32, 32)
y_test: (1000,)
```

1.1 Initial Loss and Gradient Check

As a sanity check, run the following to check the initial loss and to gradient check the network both with and without regularization. This is a good way to see if the initial losses seem reasonable.

For gradient checking, you should expect to see errors around 1e-7 or less.

[ ]:
```
np.random.seed(231)
N, D, H1, H2, C = 2, 15, 20, 30, 10
X = np.random.randn(N, D)
y = np.random.randint(C, size=(N,))

for reg in [0, 3.14]:
```

```
    print("Running check with reg = ", reg)
    model = FullyConnectedNet(
        [H1, H2],
        input_dim=D,
        num_classes=C,
        reg=reg,
        weight_scale=5e-2,
        dtype=np.float64
    )

    loss, grads = model.loss(X, y)
    print("Initial loss: ", loss)

    # Most of the errors should be on the order of e-7 or smaller.
    # NOTE: It is fine however to see an error for W2 on the order of e-5
    # for the check when reg = 0.0
    for name in sorted(grads):
        f = lambda _: model.loss(X, y)[0]
        grad_num = eval_numerical_gradient(f, model.params[name],␣
↪verbose=False, h=1e-5)
        print(f"{name} relative error: {rel_error(grad_num, grads[name])}")
```

```
Running check with reg =  0
Initial loss:  2.300479089768492
W1 relative error: 1.0252674471656573e-07
W2 relative error: 2.2120479295080622e-05
W3 relative error: 4.5623278736665505e-07
b1 relative error: 4.6600944653202505e-09
b2 relative error: 2.085654276112763e-09
b3 relative error: 1.689724888469736e-10
Running check with reg =  3.14
Initial loss:  7.052114776533016
W1 relative error: 1.1358395917166688e-08
W2 relative error: 6.86942277940646e-08
W3 relative error: 3.483989247437803e-08
b1 relative error: 1.4752427965311745e-08
b2 relative error: 1.7223751746766738e-09
b3 relative error: 2.378772438198909e-10
```

As another sanity check, make sure your network can overfit on a small dataset of 50 images. First, we will try a three-layer network with 100 units in each hidden layer. In the following cell, tweak the **learning rate** and **weight initialization scale** to overfit and achieve 100% training accuracy within 20 epochs.

[ ]:
```
# TODO: Use a three-layer Net to overfit 50 training examples by
# tweaking just the learning rate and initialization scale.

num_train = 50
```

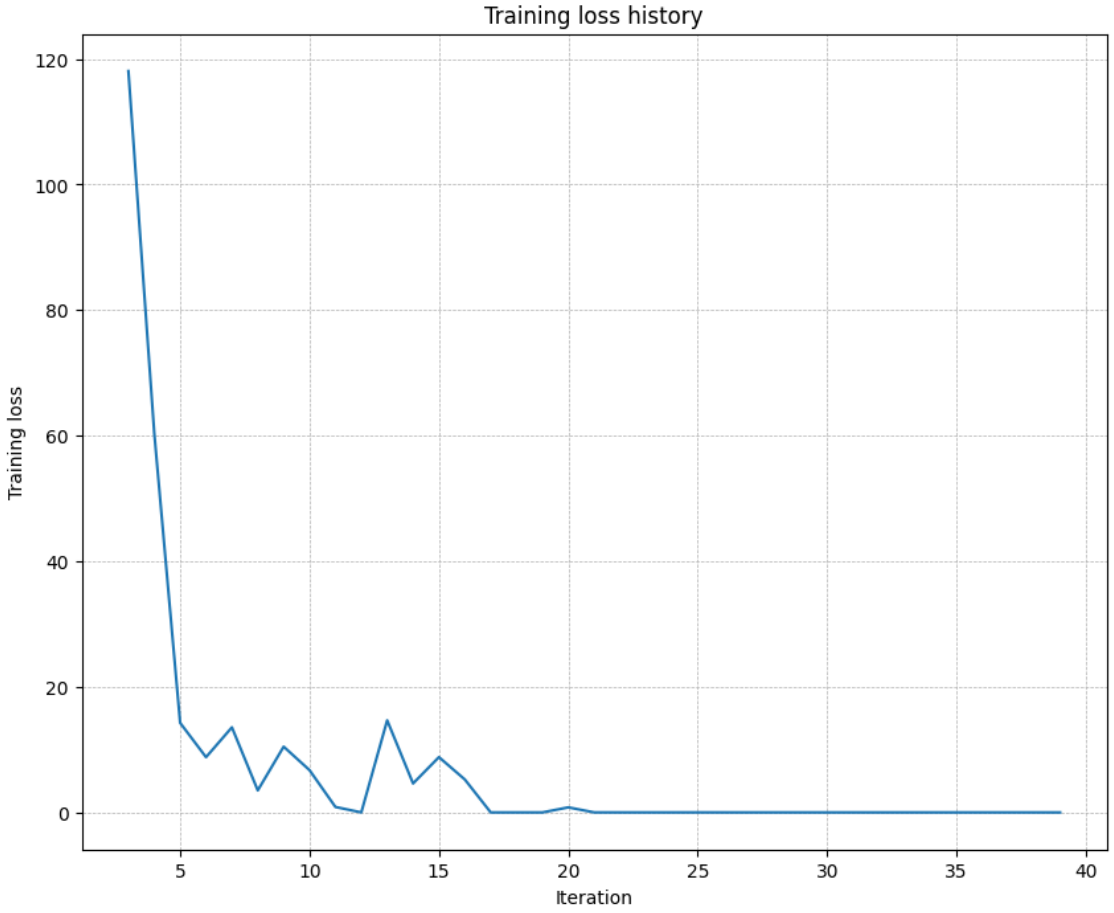
```
small_data = {
  "X_train": data["X_train"][:num_train],
  "y_train": data["y_train"][:num_train],
  "X_val": data["X_val"],
  "y_val": data["y_val"],
}

weight_scale = 1e-1   # Experiment with this!
learning_rate = 1e-3  # Experiment with this!


model = FullyConnectedNet(
    [100, 100],
    weight_scale=weight_scale,
    dtype=np.float64
)
solver = Solver(
    model,
    small_data,
    print_every=10,
    num_epochs=20,
    batch_size=25,
    update_rule="sgd",
    optim_config={"learning_rate": learning_rate},
)
solver.train()

plt.plot(solver.loss_history)
plt.title("Training loss history")
plt.xlabel("Iteration")
plt.ylabel("Training loss")
plt.grid(linestyle='--', linewidth=0.5)
plt.show()
```

```
(Iteration 1 / 40) loss: inf
```

```
/content/drive/My Drive/dl/assignments/assignment1/dl/layers.py:158:
RuntimeWarning: divide by zero encountered in log
  correct_logprobs = -np.log(probs[np.arange(N), y])  # (N,)
```

```
(Epoch 0 / 20) train acc: 0.220000; val_acc: 0.111000
(Epoch 1 / 20) train acc: 0.380000; val_acc: 0.141000
(Epoch 2 / 20) train acc: 0.520000; val_acc: 0.138000
(Epoch 3 / 20) train acc: 0.740000; val_acc: 0.130000
(Epoch 4 / 20) train acc: 0.820000; val_acc: 0.153000
(Epoch 5 / 20) train acc: 0.860000; val_acc: 0.175000
(Iteration 11 / 40) loss: 6.726589
(Epoch 6 / 20) train acc: 0.940000; val_acc: 0.163000
```

```
(Epoch 7 / 20) train acc: 0.960000; val_acc: 0.166000
(Epoch 8 / 20) train acc: 0.960000; val_acc: 0.164000
(Epoch 9 / 20) train acc: 0.980000; val_acc: 0.162000
(Epoch 10 / 20) train acc: 0.980000; val_acc: 0.162000
(Iteration 21 / 40) loss: 0.800243
(Epoch 11 / 20) train acc: 1.000000; val_acc: 0.158000
(Epoch 12 / 20) train acc: 1.000000; val_acc: 0.158000
(Epoch 13 / 20) train acc: 1.000000; val_acc: 0.158000
(Epoch 14 / 20) train acc: 1.000000; val_acc: 0.158000
(Epoch 15 / 20) train acc: 1.000000; val_acc: 0.158000
(Iteration 31 / 40) loss: 0.000000
(Epoch 16 / 20) train acc: 1.000000; val_acc: 0.158000
(Epoch 17 / 20) train acc: 1.000000; val_acc: 0.158000
(Epoch 18 / 20) train acc: 1.000000; val_acc: 0.158000
(Epoch 19 / 20) train acc: 1.000000; val_acc: 0.158000
(Epoch 20 / 20) train acc: 1.000000; val_acc: 0.158000
```



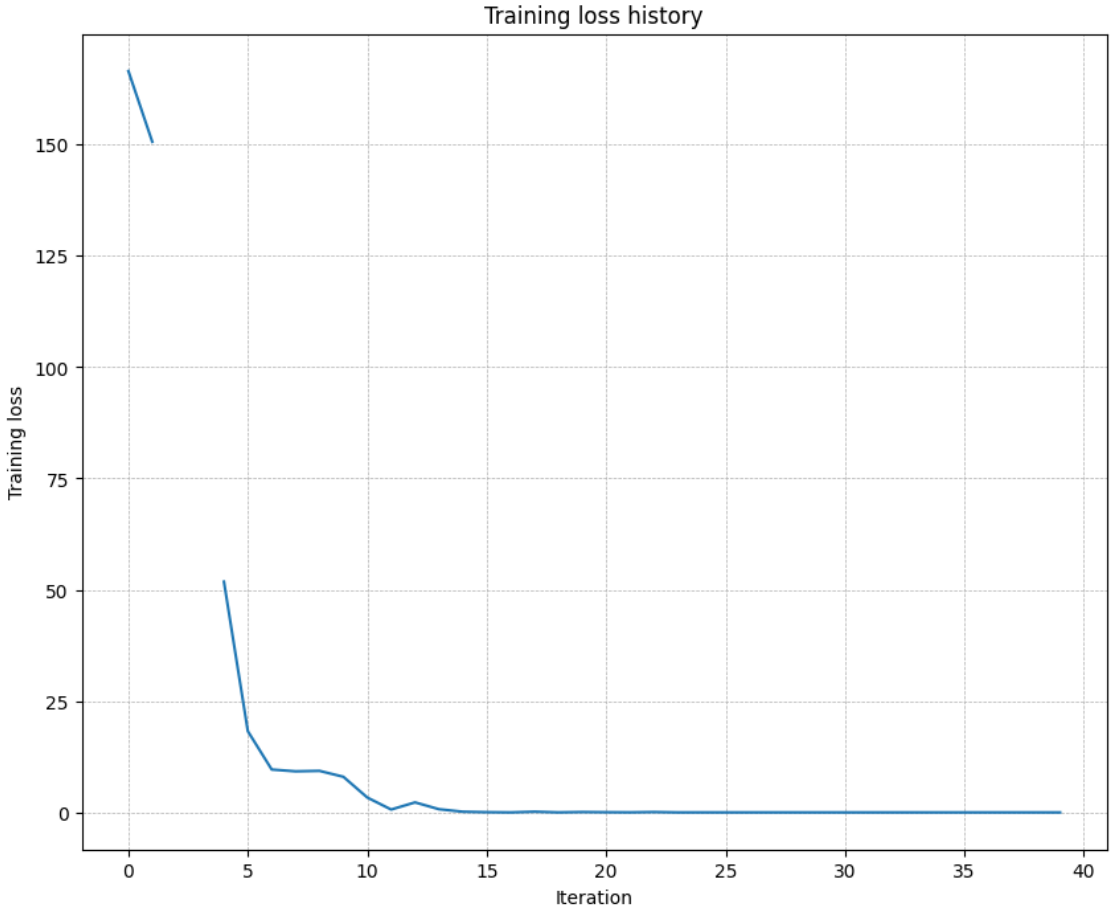
Now, try to use a five-layer network with 100 units on each layer to overfit on 50 training examples. Again, you will have to adjust the learning rate and weight initialization scale, but you should be

able to achieve 100% training accuracy within 20 epochs.

[ ]:
```
# TODO: Use a five-layer Net to overfit 50 training examples by
# tweaking just the learning rate and initialization scale.

num_train = 50
small_data = {
  'X_train': data['X_train'][:num_train],
  'y_train': data['y_train'][:num_train],
  'X_val': data['X_val'],
  'y_val': data['y_val'],
}

learning_rate = 2e-3  # Experiment with this!
weight_scale = 1e-1   # Experiment with this!


model = FullyConnectedNet(
    [100, 100, 100, 100],
    weight_scale=weight_scale,
    dtype=np.float64
)
solver = Solver(
    model,
    small_data,
    print_every=10,
    num_epochs=20,
    batch_size=25,
    update_rule='sgd',
    optim_config={'learning_rate': learning_rate},
)
solver.train()

plt.plot(solver.loss_history)
plt.title('Training loss history')
plt.xlabel('Iteration')
plt.ylabel('Training loss')
plt.grid(linestyle='--', linewidth=0.5)
plt.show()
```

```
(Iteration 1 / 40) loss: 166.501707
(Epoch 0 / 20) train acc: 0.100000; val_acc: 0.107000
(Epoch 1 / 20) train acc: 0.320000; val_acc: 0.101000
(Epoch 2 / 20) train acc: 0.160000; val_acc: 0.122000
(Epoch 3 / 20) train acc: 0.380000; val_acc: 0.106000
(Epoch 4 / 20) train acc: 0.520000; val_acc: 0.111000
(Epoch 5 / 20) train acc: 0.760000; val_acc: 0.113000
(Iteration 11 / 40) loss: 3.343141
```

```
(Epoch 6 / 20) train acc: 0.840000; val_acc: 0.122000
(Epoch 7 / 20) train acc: 0.920000; val_acc: 0.113000
(Epoch 8 / 20) train acc: 0.940000; val_acc: 0.125000
(Epoch 9 / 20) train acc: 0.960000; val_acc: 0.125000
(Epoch 10 / 20) train acc: 0.980000; val_acc: 0.121000
(Iteration 21 / 40) loss: 0.039138
(Epoch 11 / 20) train acc: 0.980000; val_acc: 0.123000
(Epoch 12 / 20) train acc: 1.000000; val_acc: 0.121000
(Epoch 13 / 20) train acc: 1.000000; val_acc: 0.121000
(Epoch 14 / 20) train acc: 1.000000; val_acc: 0.121000
(Epoch 15 / 20) train acc: 1.000000; val_acc: 0.121000
(Iteration 31 / 40) loss: 0.000644
(Epoch 16 / 20) train acc: 1.000000; val_acc: 0.121000
(Epoch 17 / 20) train acc: 1.000000; val_acc: 0.121000
(Epoch 18 / 20) train acc: 1.000000; val_acc: 0.121000
(Epoch 19 / 20) train acc: 1.000000; val_acc: 0.121000
(Epoch 20 / 20) train acc: 1.000000; val_acc: 0.121000
```



1.2 Inline Question 1:

Did you notice anything about the comparative difficulty of training the three-layer network vs. training the five-layer network? In particular, based on your experience, which network seemed more sensitive to the initialization scale? Why do you think that is the case?

1.3 Answer:

When training these networks on a small dataset to achieve 100% training accuracy, I observed that the **five-layer network was more sensitive to the initialization scale** compared to the three-layer network.

Here's why:

1. **Increased Depth and Complexity**: The five-layer network has more layers and thus more parameters, giving it a higher capacity. This complexity means that propagating signals (both forward and backward during training) across these layers becomes more challenging.
2. **Vanishing/Exploding Gradients**: With a larger number of layers, the issue of vanishing or exploding gradients becomes more pronounced. If the initial weights are too small, gradients can shrink rapidly to zero (vanishing gradients), making it difficult for earlier layers to learn. If the weights are too large, gradients can grow uncontrollably (exploding gradients), leading to unstable training. Therefore, a very precise `weight_scale` is often required for deeper networks to ensure stable gradient flow.
3. **Signal Propagation**: A good initialization scale helps maintain the variance of activations and gradients across layers. For a shallower network (3 layers), there are fewer transformations for the signal to pass through, making it somewhat more forgiving to the `weight_scale`. For a deeper network (5 layers), the cumulative effect of many layers means that even slight deviations in initial weight magnitudes can significantly impact signal propagation, requiring more careful tuning of the `weight_scale` to get the network to learn effectively and avoid issues like underfitting (if the scale is too small and leads to vanishing gradients) or struggling to converge (if too large).

In essence, the deeper network requires a more finely tuned initialization scale to overcome the challenges associated with signal propagation through many layers, making it appear more sensitive to this hyperparameter.

2 Update rules

So far we have used vanilla stochastic gradient descent (SGD) as our update rule. More sophisticated update rules can make it easier to train deep networks. We will implement a few of the most commonly used update rules and compare them to vanilla SGD.

2.1 SGD+Momentum

Stochastic gradient descent with momentum is a widely used update rule that tends to make deep networks converge faster than vanilla stochastic gradient descent. See the Momentum Update section at http://cs231n.github.io/neural-networks-3/#sgd for more information.

Open the file `dl/optim.py` and read the documentation at the top of the file to make sure you understand the API. Implement the SGD+momentum update rule in the function `sgd_momentum` and run the following to check your implementation. You should see errors less than e-8.

[ ]:
```
from dl.optim import sgd_momentum

N, D = 4, 5
w = np.linspace(-0.4, 0.6, num=N*D).reshape(N, D)
dw = np.linspace(-0.6, 0.4, num=N*D).reshape(N, D)
v = np.linspace(0.6, 0.9, num=N*D).reshape(N, D)

config = {"learning_rate": 1e-3, "velocity": v}
next_w, _ = sgd_momentum(w, dw, config=config)

expected_next_w = np.asarray([
  [ 0.1406,      0.20738947,  0.27417895,  0.34096842,  0.40775789],
  [ 0.47454737,  0.54133684,  0.60812632,  0.67491579,  0.74170526],
  [ 0.80849474,  0.87528421,  0.94207368,  1.00886316,  1.07565263],
  [ 1.14244211,  1.20923158,  1.27602105,  1.34281053,  1.4096    ]])
expected_velocity = np.asarray([
  [ 0.5406,      0.55475789,  0.56891579, 0.58307368,  0.59723158],
  [ 0.61138947,  0.62554737,  0.63970526,  0.65386316,  0.66802105],
  [ 0.68217895,  0.69633684,  0.71049474,  0.72465263,  0.73881053],
  [ 0.75296842,  0.76712632,  0.78128421,  0.79544211,  0.8096    ]])

# Should see relative errors around e-8 or less
print(next_w)
print("next_w error: ", rel_error(next_w, expected_next_w))
print("velocity error: ", rel_error(expected_velocity, config["velocity"]))
```

```
[[0.1406     0.20738947 0.27417895 0.34096842 0.40775789]
 [0.47454737 0.54133684 0.60812632 0.67491579 0.74170526]
 [0.80849474 0.87528421 0.94207368 1.00886316 1.07565263]
 [1.14244211 1.20923158 1.27602105 1.34281053 1.4096    ]]
next_w error:  8.882347033505819e-09
velocity error:  4.269287743278663e-09
```

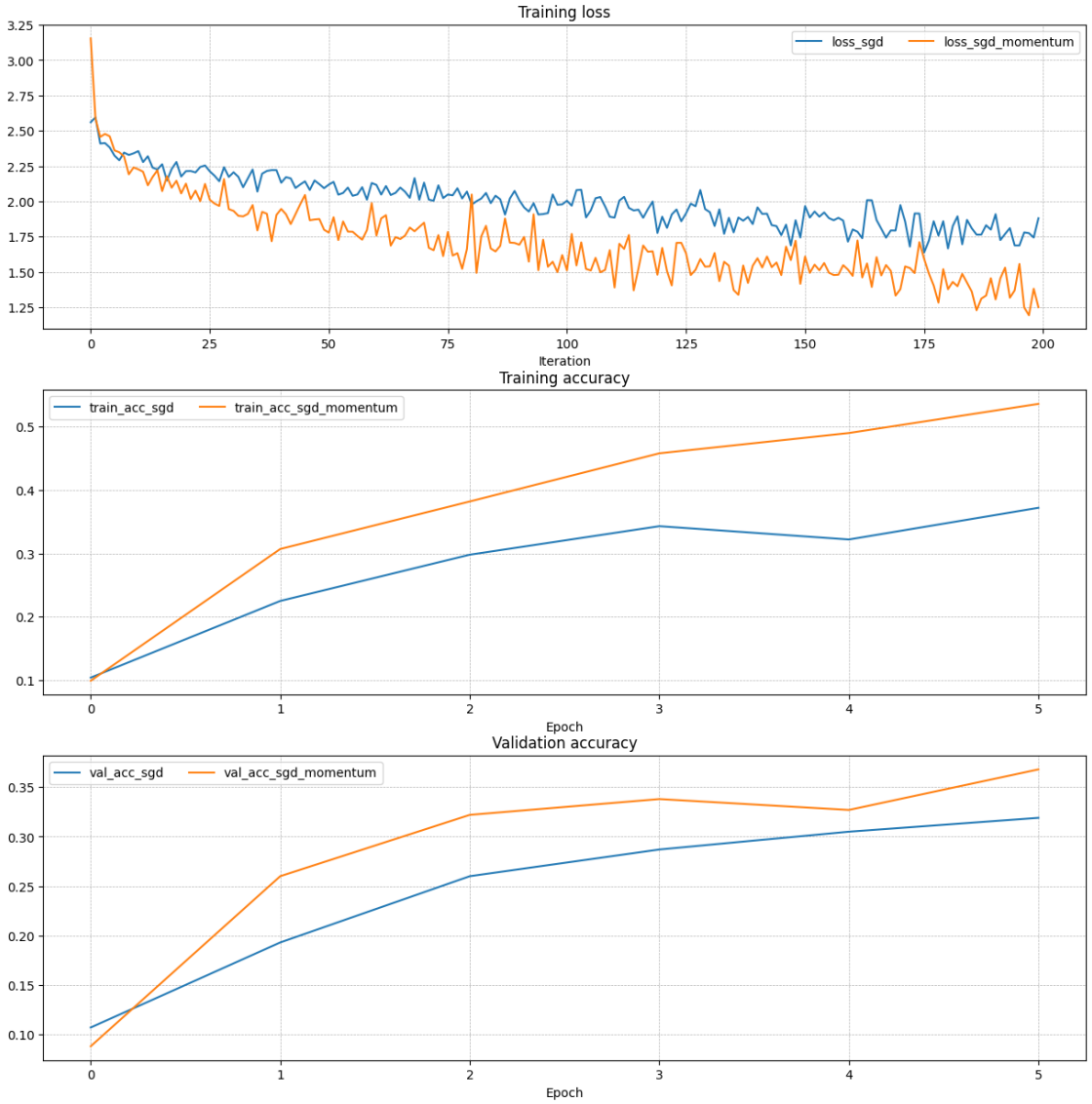
Once you have done so, run the following to train a six-layer network with both SGD and SGD+momentum. You should see the SGD+momentum update rule converge faster.

[ ]:
```
num_train = 4000
small_data = {
  'X_train': data['X_train'][:num_train],
  'y_train': data['y_train'][:num_train],
  'X_val': data['X_val'],
  'y_val': data['y_val'],
}

solvers = {}

for update_rule in ['sgd', 'sgd_momentum']:
    print('Running with ', update_rule)
```

```
    model = FullyConnectedNet(
        [100, 100, 100, 100, 100],
        weight_scale=5e-2
    )

    solver = Solver(
        model,
        small_data,
        num_epochs=5,
        batch_size=100,
        update_rule=update_rule,
        optim_config={'learning_rate': 5e-3},
        verbose=True,
    )
    solvers[update_rule] = solver
    solver.train()

fig, axes = plt.subplots(3, 1, figsize=(15, 15))

axes[0].set_title('Training loss')
axes[0].set_xlabel('Iteration')
axes[1].set_title('Training accuracy')
axes[1].set_xlabel('Epoch')
axes[2].set_title('Validation accuracy')
axes[2].set_xlabel('Epoch')

for update_rule, solver in solvers.items():
    axes[0].plot(solver.loss_history, label=f"loss_{update_rule}")
    axes[1].plot(solver.train_acc_history, label=f"train_acc_{update_rule}")
    axes[2].plot(solver.val_acc_history, label=f"val_acc_{update_rule}")

for ax in axes:
    ax.legend(loc="best", ncol=4)
    ax.grid(linestyle='--', linewidth=0.5)

plt.show()
```

```
Running with  sgd
(Iteration 1 / 200) loss: 2.559978
(Epoch 0 / 5) train acc: 0.104000; val_acc: 0.107000
(Iteration 11 / 200) loss: 2.356070
(Iteration 21 / 200) loss: 2.214091
(Iteration 31 / 200) loss: 2.205928
(Epoch 1 / 5) train acc: 0.225000; val_acc: 0.193000
(Iteration 41 / 200) loss: 2.132095
(Iteration 51 / 200) loss: 2.118950
(Iteration 61 / 200) loss: 2.116443
```

```
(Iteration 71 / 200) loss: 2.132549
(Epoch 2 / 5) train acc: 0.298000; val_acc: 0.260000
(Iteration 81 / 200) loss: 1.977227
(Iteration 91 / 200) loss: 2.007528
(Iteration 101 / 200) loss: 2.004762
(Iteration 111 / 200) loss: 1.885342
(Epoch 3 / 5) train acc: 0.343000; val_acc: 0.287000
(Iteration 121 / 200) loss: 1.891516
(Iteration 131 / 200) loss: 1.923677
(Iteration 141 / 200) loss: 1.957743
(Iteration 151 / 200) loss: 1.966736
(Epoch 4 / 5) train acc: 0.322000; val_acc: 0.305000
(Iteration 161 / 200) loss: 1.801483
(Iteration 171 / 200) loss: 1.973780
(Iteration 181 / 200) loss: 1.666573
(Iteration 191 / 200) loss: 1.909494
(Epoch 5 / 5) train acc: 0.372000; val_acc: 0.319000
Running with  sgd_momentum
(Iteration 1 / 200) loss: 3.153778
(Epoch 0 / 5) train acc: 0.099000; val_acc: 0.088000
(Iteration 11 / 200) loss: 2.227203
(Iteration 21 / 200) loss: 2.125706
(Iteration 31 / 200) loss: 1.932695
(Epoch 1 / 5) train acc: 0.307000; val_acc: 0.260000
(Iteration 41 / 200) loss: 1.946488
(Iteration 51 / 200) loss: 1.778583
(Iteration 61 / 200) loss: 1.758119
(Iteration 71 / 200) loss: 1.849137
(Epoch 2 / 5) train acc: 0.382000; val_acc: 0.322000
(Iteration 81 / 200) loss: 2.048671
(Iteration 91 / 200) loss: 1.693223
(Iteration 101 / 200) loss: 1.511693
(Iteration 111 / 200) loss: 1.390754
(Epoch 3 / 5) train acc: 0.458000; val_acc: 0.338000
(Iteration 121 / 200) loss: 1.670614
(Iteration 131 / 200) loss: 1.540271
(Iteration 141 / 200) loss: 1.597365
(Iteration 151 / 200) loss: 1.609851
(Epoch 4 / 5) train acc: 0.490000; val_acc: 0.327000
(Iteration 161 / 200) loss: 1.472687
(Iteration 171 / 200) loss: 1.378620
(Iteration 181 / 200) loss: 1.378175
(Iteration 191 / 200) loss: 1.305934
(Epoch 5 / 5) train acc: 0.536000; val_acc: 0.368000
```



2.2 RMSProp and Adam

RMSProp [1] and Adam [2] are update rules that set per-parameter learning rates by using a running average of the second moments of gradients.

In the file `dl/optim.py`, implement the RMSProp update rule in the `rmsprop` function and implement the Adam update rule in the `adam` function, and check your implementations using the tests below.

NOTE: Please implement the *complete* Adam update rule (with the bias correction mechanism), not the first simplified version mentioned in the course notes.

[1] Tijmen Tieleman and Geoffrey Hinton. "Lecture 6.5-rmsprop: Divide the gradient by a running average of its recent magnitude." COURSERA: Neural Networks for Machine Learning 4 (2012).

[2] Diederik Kingma and Jimmy Ba, "Adam: A Method for Stochastic Optimization", ICLR 2015.

[ ]:
```
# Test RMSProp implementation
from dl.optim import rmsprop

N, D = 4, 5
w = np.linspace(-0.4, 0.6, num=N*D).reshape(N, D)
dw = np.linspace(-0.6, 0.4, num=N*D).reshape(N, D)
cache = np.linspace(0.6, 0.9, num=N*D).reshape(N, D)

config = {'learning_rate': 1e-2, 'cache': cache}
next_w, _ = rmsprop(w, dw, config=config)

expected_next_w = np.asarray([
  [-0.39223849, -0.34037513, -0.28849239, -0.23659121, -0.18467247],
  [-0.132737,   -0.08078555, -0.02881884,  0.02316247,  0.07515774],
  [ 0.12716641,  0.17918792,  0.23122175,  0.28326742,  0.33532447],
  [ 0.38739248,  0.43947102,  0.49155973,  0.54365823,  0.59576619]])
expected_cache = np.asarray([
  [ 0.5976,      0.6126277,   0.6277108,   0.64284931,  0.65804321],
  [ 0.67329252,  0.68859723,  0.70395734,  0.71937285,  0.73484377],
  [ 0.75037008,  0.7659518,   0.78158892,  0.79728144,  0.81302936],
  [ 0.82883269,  0.84469141,  0.86060554,  0.87657507,  0.8926    ]])

# You should see relative errors around e-7 or less
print('next_w error: ', rel_error(expected_next_w, next_w))
print('cache error: ', rel_error(expected_cache, config['cache']))
```

```
next_w error:  9.524687511038133e-08
cache error:  2.6477955807156126e-09
```

[ ]:
```
# Test Adam implementation
from dl.optim import adam

N, D = 4, 5
w = np.linspace(-0.4, 0.6, num=N*D).reshape(N, D)
dw = np.linspace(-0.6, 0.4, num=N*D).reshape(N, D)
m = np.linspace(0.6, 0.9, num=N*D).reshape(N, D)
v = np.linspace(0.7, 0.5, num=N*D).reshape(N, D)

config = {'learning_rate': 1e-2, 'm': m, 'v': v, 't': 5}
next_w, _ = adam(w, dw, config=config)

expected_next_w = np.asarray([
  [-0.40094747, -0.34836187, -0.29577703, -0.24319299, -0.19060977],
  [-0.1380274,  -0.08544591, -0.03286534,  0.01971428,  0.0722929],
  [ 0.1248705,   0.17744702,  0.23002243,  0.28259667,  0.33516969],
  [ 0.38774145,  0.44031188,  0.49288093,  0.54544852,  0.59801459]])
```

```
expected_v = np.asarray([
  [ 0.69966,     0.68908382,  0.67851319,  0.66794809,  0.65738853,],
  [ 0.64683452,  0.63628604,  0.6257431,   0.61520571,  0.60467385,],
  [ 0.59414753,  0.58362676,  0.57311152,  0.56260183,  0.55209767,],
  [ 0.54159906,  0.53110598,  0.52061845,  0.51013645,  0.49966,   ]])
expected_m = np.asarray([
  [ 0.48,        0.49947368,  0.51894737,  0.53842105,  0.55789474],
  [ 0.57736842,  0.59684211,  0.61631579,  0.63578947,  0.65526316],
  [ 0.67473684,  0.69421053,  0.71368421,  0.73315789,  0.75263158],
  [ 0.77210526,  0.79157895,  0.81105263,  0.83052632,  0.85      ]])

# You should see relative errors around e-7 or less
print('next_w error: ', rel_error(expected_next_w, next_w))
print('v error: ', rel_error(expected_v, config['v']))
print('m error: ', rel_error(expected_m, config['m']))
```

```
next_w error:  1.1395691798535431e-07
v error:  4.208314038113071e-09
m error:  4.214963193114416e-09
```

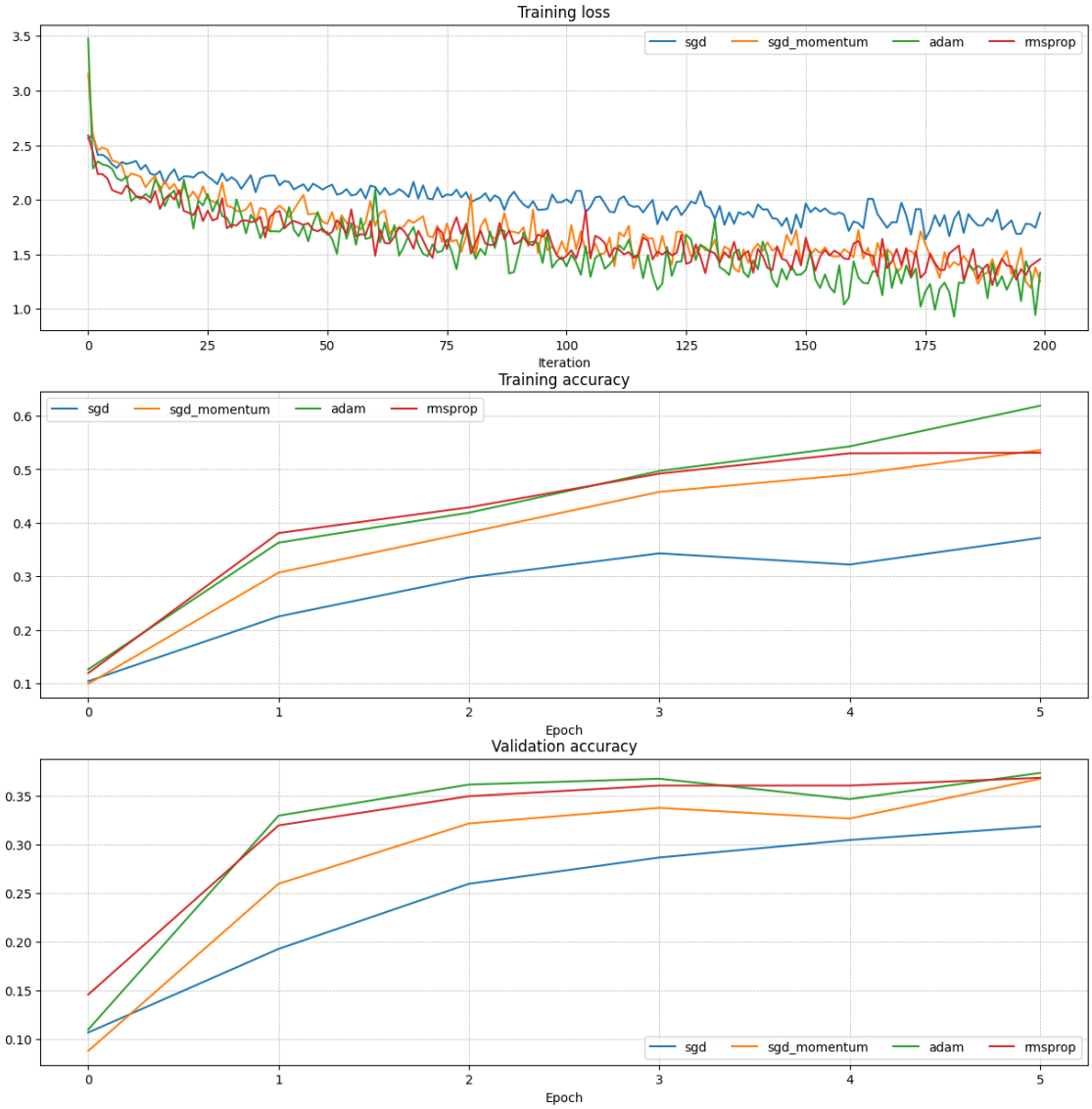
Once you have debugged your RMSProp and Adam implementations, run the following to train a pair of deep networks using these new update rules:

[ ]:
```
learning_rates = {'rmsprop': 1e-4, 'adam': 1e-3}
for update_rule in ['adam', 'rmsprop']:
    print('Running with ', update_rule)
    model = FullyConnectedNet(
        [100, 100, 100, 100, 100],
        weight_scale=5e-2
    )
    solver = Solver(
        model,
        small_data,
        num_epochs=5,
        batch_size=100,
        update_rule=update_rule,
        optim_config={'learning_rate': learning_rates[update_rule]},
        verbose=True
    )
    solvers[update_rule] = solver
    solver.train()
    print()

fig, axes = plt.subplots(3, 1, figsize=(15, 15))

axes[0].set_title('Training loss')
axes[0].set_xlabel('Iteration')
axes[1].set_title('Training accuracy')
```

```
axes[1].set_xlabel('Epoch')
axes[2].set_title('Validation accuracy')
axes[2].set_xlabel('Epoch')

for update_rule, solver in solvers.items():
    axes[0].plot(solver.loss_history, label=f"{update_rule}")
    axes[1].plot(solver.train_acc_history, label=f"{update_rule}")
    axes[2].plot(solver.val_acc_history, label=f"{update_rule}")

for ax in axes:
    ax.legend(loc='best', ncol=4)
    ax.grid(linestyle='--', linewidth=0.5)

plt.show()
```

```
Running with  adam
(Iteration 1 / 200) loss: 3.476928
(Epoch 0 / 5) train acc: 0.126000; val_acc: 0.110000
(Iteration 11 / 200) loss: 2.027712
(Iteration 21 / 200) loss: 2.183357
(Iteration 31 / 200) loss: 1.744257
(Epoch 1 / 5) train acc: 0.363000; val_acc: 0.330000
(Iteration 41 / 200) loss: 1.707951
(Iteration 51 / 200) loss: 1.703835
(Iteration 61 / 200) loss: 2.094758
(Iteration 71 / 200) loss: 1.505558
(Epoch 2 / 5) train acc: 0.419000; val_acc: 0.362000
(Iteration 81 / 200) loss: 1.594431
(Iteration 91 / 200) loss: 1.511452
(Iteration 101 / 200) loss: 1.389237
(Iteration 111 / 200) loss: 1.463575
(Epoch 3 / 5) train acc: 0.497000; val_acc: 0.368000
(Iteration 121 / 200) loss: 1.231313
(Iteration 131 / 200) loss: 1.520198
(Iteration 141 / 200) loss: 1.363221
(Iteration 151 / 200) loss: 1.355143
(Epoch 4 / 5) train acc: 0.543000; val_acc: 0.347000
(Iteration 161 / 200) loss: 1.436402
(Iteration 171 / 200) loss: 1.231426
(Iteration 181 / 200) loss: 1.153575
(Iteration 191 / 200) loss: 1.209479
(Epoch 5 / 5) train acc: 0.619000; val_acc: 0.374000

Running with  rmsprop
(Iteration 1 / 200) loss: 2.589166
(Epoch 0 / 5) train acc: 0.119000; val_acc: 0.146000
(Iteration 11 / 200) loss: 2.032921
```

```
(Iteration 21 / 200) loss: 1.897277
(Iteration 31 / 200) loss: 1.770793
(Epoch 1 / 5) train acc: 0.381000; val_acc: 0.320000
(Iteration 41 / 200) loss: 1.895731
(Iteration 51 / 200) loss: 1.681091
(Iteration 61 / 200) loss: 1.487204
(Iteration 71 / 200) loss: 1.629973
(Epoch 2 / 5) train acc: 0.429000; val_acc: 0.350000
(Iteration 81 / 200) loss: 1.506686
(Iteration 91 / 200) loss: 1.610742
(Iteration 101 / 200) loss: 1.486124
(Iteration 111 / 200) loss: 1.559454
(Epoch 3 / 5) train acc: 0.492000; val_acc: 0.361000
(Iteration 121 / 200) loss: 1.497406
(Iteration 131 / 200) loss: 1.530736
(Iteration 141 / 200) loss: 1.550957
(Iteration 151 / 200) loss: 1.652046
(Epoch 4 / 5) train acc: 0.530000; val_acc: 0.361000
(Iteration 161 / 200) loss: 1.599574
(Iteration 171 / 200) loss: 1.401073
(Iteration 181 / 200) loss: 1.509365
(Iteration 191 / 200) loss: 1.365772
(Epoch 5 / 5) train acc: 0.531000; val_acc: 0.369000
```



2.3 Inline Question 2:

AdaGrad, like Adam, is a per-parameter optimization method that uses the following update rule:

```
cache += dw**2
w += - learning_rate * dw / (np.sqrt(cache) + eps)
```

John notices that when he was training a network with AdaGrad that the updates became very small, and that his network was learning slowly. Using your knowledge of the AdaGrad update rule, why do you think the updates would become very small? Would Adam have the same issue?

2.4 Answer:

AdaGrad accumulates the squares of all past gradients for each parameter into its cache variable (cache += dw**2). Since dw**2 is always non-negative, the cache will continuously grow (or stay the same) as training progresses.

The learning rate for each parameter is effectively learning_rate / (np.sqrt(cache) + eps). As cache grows, np.sqrt(cache) also grows, causing the effective learning rate to continuously decrease. This means that with each iteration, the steps taken in the parameter space become smaller and smaller.

Eventually, the effective learning rate can become so tiny that the model makes negligible updates to its weights, leading to very slow learning or even premature stalling.

No, Adam would not have the same issue. Adam addresses this problem by using an exponentially decaying average of past squared gradients, rather than a simple sum.

3 Train a Good Model!

Train the best fully connected model that you can on CIFAR-10, storing your best model in the `best_model` variable. We require you to get at least 50% accuracy on the validation set using a fully connected network.

If you are careful it should be possible to get accuracies above 55%, but we don't require it for this part and won't assign extra credit for doing so. Later in the next assignment, we will ask you to train the best convolutional network that you can on CIFAR-10, and we would prefer that you spend your effort working on convolutional networks rather than fully connected networks.

Note: Later in the assignment, you will implement techniques like BatchNormalization and Dropout which can help you train powerful models.

[ ]:
```
best_model = None

################################################################################
# TODO: Train the best FullyConnectedNet that you can on CIFAR-10. You might   #
# find batch/layer normalization and dropout useful. Store your best model in  #
# the best_model variable.                                                     #
################################################################################
# *****START OF YOUR CODE (DO NOT DELETE/MODIFY THIS LINE)*****

learning_rate = 1e-3
weight_scale = 2e-2 # Adjusted for better initial performance on full data
reg = 1e-3 # L2 regularization strength
num_epochs = 10
batch_size = 256

hidden_dims = [200, 200, 200, 200] # A 5-layer network

model = FullyConnectedNet(
    hidden_dims,
    weight_scale=weight_scale,
```

```
    reg=reg,
    dtype=np.float64
)

solver = Solver(
    model,
    data, # Use the full CIFAR-10 data
    print_every=100,
    num_epochs=num_epochs,
    batch_size=batch_size,
    update_rule='adam',
    optim_config={'learning_rate': learning_rate},
    verbose=True,
    lr_decay=0.95 # Add learning rate decay
)
solver.train()

best_model = model

plt.plot(solver.loss_history)
plt.title('Training loss history for best_model')
plt.xlabel('Iteration')
plt.ylabel('Training loss')
plt.grid(linestyle='--', linewidth=0.5)
plt.show()

# *****END OF YOUR CODE (DO NOT DELETE/MODIFY THIS LINE)*****
################################################################################
#                                END OF YOUR CODE                              #
################################################################################
```

```
(Iteration 1 / 1910) loss: 2.448006
(Epoch 0 / 10) train acc: 0.164000; val_acc: 0.160000
(Iteration 101 / 1910) loss: 1.824769
(Epoch 1 / 10) train acc: 0.444000; val_acc: 0.465000
(Iteration 201 / 1910) loss: 1.653619
(Iteration 301 / 1910) loss: 1.584860
(Epoch 2 / 10) train acc: 0.487000; val_acc: 0.492000
(Iteration 401 / 1910) loss: 1.572211
(Iteration 501 / 1910) loss: 1.505669
(Epoch 3 / 10) train acc: 0.540000; val_acc: 0.491000
(Iteration 601 / 1910) loss: 1.387680
(Iteration 701 / 1910) loss: 1.416892
(Epoch 4 / 10) train acc: 0.579000; val_acc: 0.513000
(Iteration 801 / 1910) loss: 1.313968
(Iteration 901 / 1910) loss: 1.304877
(Epoch 5 / 10) train acc: 0.613000; val_acc: 0.524000
```

```
(Iteration 1001 / 1910) loss: 1.245060
(Iteration 1101 / 1910) loss: 1.321057
(Epoch 6 / 10) train acc: 0.622000; val_acc: 0.515000
(Iteration 1201 / 1910) loss: 1.297997
(Iteration 1301 / 1910) loss: 1.348924
(Epoch 7 / 10) train acc: 0.621000; val_acc: 0.536000
(Iteration 1401 / 1910) loss: 1.185974
(Iteration 1501 / 1910) loss: 1.116548
(Epoch 8 / 10) train acc: 0.647000; val_acc: 0.524000
(Iteration 1601 / 1910) loss: 1.296468
(Iteration 1701 / 1910) loss: 1.078236
(Epoch 9 / 10) train acc: 0.657000; val_acc: 0.537000
(Iteration 1801 / 1910) loss: 1.238451
(Iteration 1901 / 1910) loss: 1.080319
(Epoch 10 / 10) train acc: 0.651000; val_acc: 0.513000
```

4 Test Your Model!

Run your best model on the validation and test sets. You should achieve at least 50% accuracy on the validation set and the test set.

[ ]:
```
y_test_pred = np.argmax(best_model.loss(data['X_test']), axis=1)
y_val_pred = np.argmax(best_model.loss(data['X_val']), axis=1)
print('Validation set accuracy: ', (y_val_pred == data['y_val']).mean())
print('Test set accuracy: ', (y_test_pred == data['y_test']).mean())
```

```
Validation set accuracy:  0.537
Test set accuracy:  0.526
```

04_Dropout

December 6, 2025

[1]:
```
# This mounts your Google Drive to the Colab VM.
from google.colab import drive
drive.mount('/content/drive')

# TODO: Enter the foldername in your Drive where you have saved the unzipped
# assignment folder, e.g. 'dl/assignments/assignment1/'
FOLDERNAME = 'dl/assignments/assignment1/'
assert FOLDERNAME is not None, "[!] Enter the foldername."

# Now that we've mounted your Drive, this ensures that
# the Python interpreter of the Colab VM can load
# python files from within it.
import sys
sys.path.append('/content/drive/My Drive/{}'.format(FOLDERNAME))

# This downloads the CIFAR-10 dataset to your Drive
# if it doesn't already exist.
%cd /content/drive/My\ Drive/$FOLDERNAME/dl/datasets/
!bash get_datasets.sh
%cd /content/drive/My\ Drive/$FOLDERNAME
```

```
Drive already mounted at /content/drive; to attempt to forcibly remount, call
drive.mount("/content/drive", force_remount=True).
/content/drive/My Drive/dl/assignments/assignment1/dl/datasets
/content/drive/My Drive/dl/assignments/assignment1
```

1 Dropout

Dropout [1] is a technique for regularizing neural networks by randomly setting some output activations to zero during the forward pass. In this exercise, you will implement a dropout layer and modify your fully connected network to optionally use dropout.

[1] Geoffrey E. Hinton et al, "Improving neural networks by preventing co-adaptation of feature detectors", arXiv 2012

[2]:
```
# Setup cell.
import time
import numpy as np
```

```
import matplotlib.pyplot as plt
from dl.classifiers.fc_net import *
from dl.data_utils import get_CIFAR10_data
from dl.gradient_check import eval_numerical_gradient,␣
 ↪eval_numerical_gradient_array
from dl.solver import Solver

%matplotlib inline
plt.rcParams["figure.figsize"] = (10.0, 8.0)  # Set default size of plots.
plt.rcParams["image.interpolation"] = "nearest"
plt.rcParams["image.cmap"] = "gray"

def rel_error(x, y):
    """Returns relative error."""
    return np.max(np.abs(x - y) / (np.maximum(1e-8, np.abs(x) + np.abs(y))))
```

[3]:
```
# Load the (preprocessed) CIFAR-10 data.
data = get_CIFAR10_data()
for k, v in list(data.items()):
    print(f"{k}: {v.shape}")
```

```
X_train: (49000, 3, 32, 32)
y_train: (49000,)
X_val: (1000, 3, 32, 32)
y_val: (1000,)
X_test: (1000, 3, 32, 32)
y_test: (1000,)
```

2 Dropout: Forward Pass

In the file `dl/layers.py`, implement the forward pass for dropout. Since dropout behaves differently during training and testing, make sure to implement the operation for both modes.

Once you have done so, run the cell below to test your implementation.

[4]:
```
np.random.seed(231)
x = np.random.randn(500, 500) + 10

for p in [0.25, 0.4, 0.7]:
    out, _ = dropout_forward(x, {'mode': 'train', 'p': p})
    out_test, _ = dropout_forward(x, {'mode': 'test', 'p': p})

    print('Running tests with p = ', p)
    print('Mean of input: ', x.mean())
    print('Mean of train-time output: ', out.mean())
    print('Mean of test-time output: ', out_test.mean())
    print('Fraction of train-time output set to zero: ', (out == 0).mean())
```

```
    print('Fraction of test-time output set to zero: ', (out_test == 0).mean())
    print()
```

```
Running tests with p =  0.25
Mean of input:  10.000207878477502
Mean of train-time output:  10.014059116977283
Mean of test-time output:  10.000207878477502
Fraction of train-time output set to zero:  0.749784
Fraction of test-time output set to zero:  0.0

Running tests with p =  0.4
Mean of input:  10.000207878477502
Mean of train-time output:  9.977917658761159
Mean of test-time output:  10.000207878477502
Fraction of train-time output set to zero:  0.600796
Fraction of test-time output set to zero:  0.0

Running tests with p =  0.7
Mean of input:  10.000207878477502
Mean of train-time output:  9.987811912159426
Mean of test-time output:  10.000207878477502
Fraction of train-time output set to zero:  0.30074
Fraction of test-time output set to zero:  0.0
```

3 Dropout: Backward Pass

In the file `dl/layers.py`, implement the backward pass for dropout. After doing so, run the following cell to numerically gradient-check your implementation.

[5]:
```
np.random.seed(231)
x = np.random.randn(10, 10) + 10
dout = np.random.randn(*x.shape)

dropout_param = {'mode': 'train', 'p': 0.2, 'seed': 123}
out, cache = dropout_forward(x, dropout_param)
dx = dropout_backward(dout, cache)
dx_num = eval_numerical_gradient_array(lambda xx: dropout_forward(xx,␣
 ↪dropout_param)[0], x, dout)

# Error should be around e-10 or less.
print('dx relative error: ', rel_error(dx, dx_num))
```

```
dx relative error:  5.44560814873387e-11
```

3.1 Inline Question 1:

What happens if we do not divide the values being passed through inverse dropout by `p` in the dropout layer? Why does that happen?

3.2 Answer:

The key is that expected output of a layer must be the same during tarining and inference, it is crucial fo consistent network behavior. Without scaling by 1/p we can say that the expected value of the layer would be E[output] = p * x, while x is the expected output without droping neurons which equals to the expected value of the output of layer during test. For making them equlas the most effiecnt and simple way is scaling each activation by 1/p so we get that expected value is p * x * (1/p) which is x

4 Fully Connected Networks with Dropout

In the file `dl/classifiers/fc_net.py`, modify your implementation to use dropout. Specifically, if the constructor of the network receives a value that is not 1 for the `dropout_keep_ratio` parameter, then the net should add a dropout layer immediately after every ReLU nonlinearity. After doing so, run the following to numerically gradient-check your implementation.

[6]:
```
np.random.seed(231)
N, D, H1, H2, C = 2, 15, 20, 30, 10
X = np.random.randn(N, D)
y = np.random.randint(C, size=(N,))

for dropout_keep_ratio in [1, 0.75, 0.5]:
    print('Running check with dropout = ', dropout_keep_ratio)
    model = FullyConnectedNet(
        [H1, H2],
        input_dim=D,
        num_classes=C,
        weight_scale=5e-2,
        dtype=np.float64,
        dropout_keep_ratio=dropout_keep_ratio,
        seed=123
    )

    loss, grads = model.loss(X, y)
    print('Initial loss: ', loss)

    # Relative errors should be around e-6 or less.
    # Note that it's fine if for dropout_keep_ratio=1 you have W2 error be on␣
 ↪the order of e-5.
    for name in sorted(grads):
        f = lambda _: model.loss(X, y)[0]
        grad_num = eval_numerical_gradient(f, model.params[name],␣
 ↪verbose=False, h=1e-5)
```

```
        print('%s relative error: %.2e' % (name, rel_error(grad_num,␣
↪grads[name])))
    print()
```

```
Running check with dropout =  1
Initial loss:  2.300479089768492
W1 relative error: 1.03e-07
W2 relative error: 2.21e-05
W3 relative error: 4.56e-07
b1 relative error: 4.66e-09
b2 relative error: 2.09e-09
b3 relative error: 1.69e-10

Running check with dropout =  0.75
Initial loss:  2.302371489704412
W1 relative error: 1.85e-07
W2 relative error: 2.15e-06
W3 relative error: 4.56e-08
b1 relative error: 1.16e-08
b2 relative error: 1.82e-09
b3 relative error: 1.48e-10

Running check with dropout =  0.5
Initial loss:  2.30427592207859
W1 relative error: 3.11e-07
W2 relative error: 2.48e-08
W3 relative error: 6.43e-08
b1 relative error: 5.37e-09
b2 relative error: 1.91e-09
b3 relative error: 1.85e-10
```

5 Regularization Experiment

As an experiment, we will train a pair of two-layer networks on 500 training examples: one will use no dropout, and one will use a keep probability of 0.25. We will then visualize the training and validation accuracies of the two networks over time.

[7]:
```
# Train two identical nets, one with dropout and one without.
np.random.seed(231)
num_train = 500
small_data = {
    'X_train': data['X_train'][:num_train],
    'y_train': data['y_train'][:num_train],
    'X_val': data['X_val'],
    'y_val': data['y_val'],
}
```

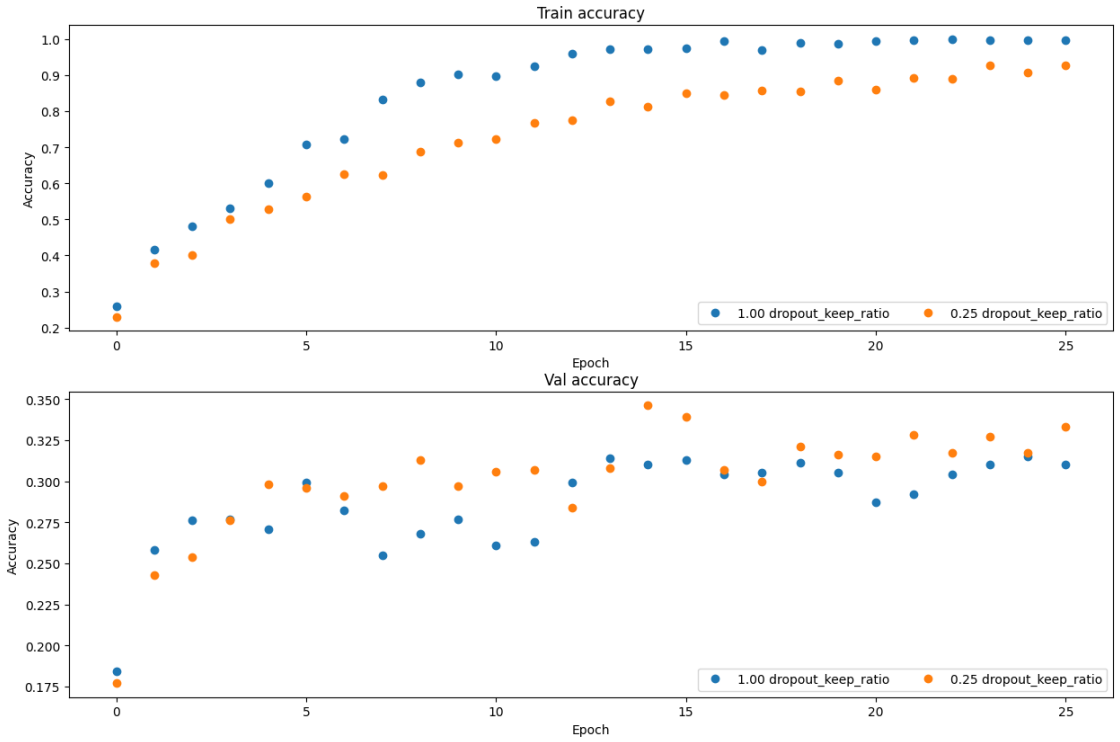
```
solvers = {}
dropout_choices = [1, 0.25]
for dropout_keep_ratio in dropout_choices:
    model = FullyConnectedNet(
        [500],
        dropout_keep_ratio=dropout_keep_ratio
    )
    print(dropout_keep_ratio)

    solver = Solver(
        model,
        small_data,
        num_epochs=25,
        batch_size=100,
        update_rule='adam',
        optim_config={'learning_rate': 5e-4,},
        verbose=True,
        print_every=100
    )
    solver.train()
    solvers[dropout_keep_ratio] = solver
    print()
```

```
1
(Iteration 1 / 125) loss: 7.856644
(Epoch 0 / 25) train acc: 0.260000; val_acc: 0.184000
(Epoch 1 / 25) train acc: 0.416000; val_acc: 0.258000
(Epoch 2 / 25) train acc: 0.482000; val_acc: 0.276000
(Epoch 3 / 25) train acc: 0.532000; val_acc: 0.277000
(Epoch 4 / 25) train acc: 0.600000; val_acc: 0.271000
(Epoch 5 / 25) train acc: 0.708000; val_acc: 0.299000
(Epoch 6 / 25) train acc: 0.722000; val_acc: 0.282000
(Epoch 7 / 25) train acc: 0.832000; val_acc: 0.255000
(Epoch 8 / 25) train acc: 0.880000; val_acc: 0.268000
(Epoch 9 / 25) train acc: 0.902000; val_acc: 0.277000
(Epoch 10 / 25) train acc: 0.898000; val_acc: 0.261000
(Epoch 11 / 25) train acc: 0.924000; val_acc: 0.263000
(Epoch 12 / 25) train acc: 0.960000; val_acc: 0.299000
(Epoch 13 / 25) train acc: 0.972000; val_acc: 0.314000
(Epoch 14 / 25) train acc: 0.972000; val_acc: 0.310000
(Epoch 15 / 25) train acc: 0.974000; val_acc: 0.313000
(Epoch 16 / 25) train acc: 0.994000; val_acc: 0.304000
(Epoch 17 / 25) train acc: 0.970000; val_acc: 0.305000
(Epoch 18 / 25) train acc: 0.990000; val_acc: 0.311000
(Epoch 19 / 25) train acc: 0.986000; val_acc: 0.305000
(Epoch 20 / 25) train acc: 0.994000; val_acc: 0.287000
```

```
(Iteration 101 / 125) loss: 0.001869
(Epoch 21 / 25) train acc: 0.996000; val_acc: 0.292000
(Epoch 22 / 25) train acc: 1.000000; val_acc: 0.304000
(Epoch 23 / 25) train acc: 0.996000; val_acc: 0.310000
(Epoch 24 / 25) train acc: 0.998000; val_acc: 0.315000
(Epoch 25 / 25) train acc: 0.998000; val_acc: 0.310000

0.25
(Iteration 1 / 125) loss: 17.318478
(Epoch 0 / 25) train acc: 0.230000; val_acc: 0.177000

/content/drive/My Drive/dl/assignments/assignment1/dl/layers.py:158:
RuntimeWarning: divide by zero encountered in log
  correct_logprobs = -np.log(probs[np.arange(N), y])  # (N,)

(Epoch 1 / 25) train acc: 0.378000; val_acc: 0.243000
(Epoch 2 / 25) train acc: 0.402000; val_acc: 0.254000
(Epoch 3 / 25) train acc: 0.502000; val_acc: 0.276000
(Epoch 4 / 25) train acc: 0.528000; val_acc: 0.298000
(Epoch 5 / 25) train acc: 0.562000; val_acc: 0.296000
(Epoch 6 / 25) train acc: 0.626000; val_acc: 0.291000
(Epoch 7 / 25) train acc: 0.622000; val_acc: 0.297000
(Epoch 8 / 25) train acc: 0.688000; val_acc: 0.313000
(Epoch 9 / 25) train acc: 0.712000; val_acc: 0.297000
(Epoch 10 / 25) train acc: 0.724000; val_acc: 0.306000
(Epoch 11 / 25) train acc: 0.768000; val_acc: 0.307000
(Epoch 12 / 25) train acc: 0.774000; val_acc: 0.284000
(Epoch 13 / 25) train acc: 0.828000; val_acc: 0.308000
(Epoch 14 / 25) train acc: 0.812000; val_acc: 0.346000
(Epoch 15 / 25) train acc: 0.850000; val_acc: 0.339000
(Epoch 16 / 25) train acc: 0.844000; val_acc: 0.307000
(Epoch 17 / 25) train acc: 0.858000; val_acc: 0.300000
(Epoch 18 / 25) train acc: 0.856000; val_acc: 0.321000
(Epoch 19 / 25) train acc: 0.884000; val_acc: 0.316000
(Epoch 20 / 25) train acc: 0.860000; val_acc: 0.315000
(Iteration 101 / 125) loss: 4.393245
(Epoch 21 / 25) train acc: 0.892000; val_acc: 0.328000
(Epoch 22 / 25) train acc: 0.890000; val_acc: 0.317000
(Epoch 23 / 25) train acc: 0.928000; val_acc: 0.327000
(Epoch 24 / 25) train acc: 0.908000; val_acc: 0.317000
(Epoch 25 / 25) train acc: 0.928000; val_acc: 0.333000
```

[8]:
```python
# Plot train and validation accuracies of the two models.
train_accs = []
val_accs = []
for dropout_keep_ratio in dropout_choices:
    solver = solvers[dropout_keep_ratio]
```

```
    train_accs.append(solver.train_acc_history[-1])
    val_accs.append(solver.val_acc_history[-1])

plt.subplot(3, 1, 1)
for dropout_keep_ratio in dropout_choices:
    plt.plot(
        solvers[dropout_keep_ratio].train_acc_history, 'o', label='%.2f␣
 ↪dropout_keep_ratio' % dropout_keep_ratio)
plt.title('Train accuracy')
plt.xlabel('Epoch')
plt.ylabel('Accuracy')
plt.legend(ncol=2, loc='lower right')

plt.subplot(3, 1, 2)
for dropout_keep_ratio in dropout_choices:
    plt.plot(
        solvers[dropout_keep_ratio].val_acc_history, 'o', label='%.2f␣
 ↪dropout_keep_ratio' % dropout_keep_ratio)
plt.title('Val accuracy')
plt.xlabel('Epoch')
plt.ylabel('Accuracy')
plt.legend(ncol=2, loc='lower right')

plt.gcf().set_size_inches(15, 15)
plt.show()
```



5.1 Inline Question 2:

Compare the validation and training accuracies with and without dropout – what do your results suggest about dropout as a regularizer?

5.2 Answer:

We can see that during train, model using dropout layer reached less accuracy compraing to the model without droped layers. On the other hand, in validation, the model using dropout manage equaly and even better then the undropout model. Basicly that tells us that adding droput layers helps the model to be more robust to noise so generallization is better, meaning that dropout is a powerful regularizer.

[8]:

[ ]:

05_BatchNormalization

December 6, 2025

[ ]:
```
# This mounts your Google Drive to the Colab VM.
from google.colab import drive
drive.mount('/content/drive')

# TODO: Enter the foldername in your Drive where you have saved the unzipped
# assignment folder, e.g. 'dl/assignments/assignment1/'
FOLDERNAME = 'dl/assignments/assignment1/'
assert FOLDERNAME is not None, "[!] Enter the foldername."

# Now that we've mounted your Drive, this ensures that
# the Python interpreter of the Colab VM can load
# python files from within it.
import sys
sys.path.append('/content/drive/My Drive/{}'.format(FOLDERNAME))

# This downloads the CIFAR-10 dataset to your Drive
# if it doesn't already exist.
%cd /content/drive/My\ Drive/$FOLDERNAME/dl/datasets/
!bash get_datasets.sh
%cd /content/drive/My\ Drive/$FOLDERNAME
```

```
Drive already mounted at /content/drive; to attempt to forcibly remount, call
drive.mount("/content/drive", force_remount=True).
/content/drive/My Drive/dl/assignments/assignment1/dl/datasets
/content/drive/My Drive/dl/assignments/assignment1
```

1 Batch Normalization

One way to make deep networks easier to train is to use more sophisticated optimization procedures such as SGD+momentum, RMSProp, or Adam. Another strategy is to change the architecture of the network to make it easier to train. One idea along these lines is batch normalization, proposed by [1] in 2015.

To understand the goal of batch normalization, it is important to first recognize that machine learning methods tend to perform better with input data consisting of uncorrelated features with zero mean and unit variance. When training a neural network, we can preprocess the data before feeding it to the network to explicitly decorrelate its features. This will ensure that the first layer of the network sees data that follows a nice distribution. However, even if we preprocess the input

data, the activations at deeper layers of the network will likely no longer be decorrelated and will no longer have zero mean or unit variance, since they are output from earlier layers in the network. Even worse, during the training process the distribution of features at each layer of the network will shift as the weights of each layer are updated.

The authors of [1] hypothesize that the shifting distribution of features inside deep neural networks may make training deep networks more difficult. To overcome this problem, they propose to insert into the network layers that normalize batches. At training time, such a layer uses a minibatch of data to estimate the mean and standard deviation of each feature. These estimated means and standard deviations are then used to center and normalize the features of the minibatch. A running average of these means and standard deviations is kept during training, and at test time these running averages are used to center and normalize features.

It is possible that this normalization strategy could reduce the representational power of the network, since it may sometimes be optimal for certain layers to have features that are not zero-mean or unit variance. To this end, the batch normalization layer includes learnable shift and scale parameters for each feature dimension.

[1] Sergey Ioffe and Christian Szegedy, "Batch Normalization: Accelerating Deep Network Training by Reducing Internal Covariate Shift", ICML 2015.

[ ]:
```
# Setup cell.
import time
import numpy as np
import matplotlib.pyplot as plt
from dl.data_utils import get_CIFAR10_data
from dl.gradient_check import eval_numerical_gradient,␣
 ↪eval_numerical_gradient_array
from dl.solver import Solver

%matplotlib inline
plt.rcParams["figure.figsize"] = (10.0, 8.0)  # Set default size of plots.
plt.rcParams["image.interpolation"] = "nearest"
plt.rcParams["image.cmap"] = "gray"


def rel_error(x, y):
    """Returns relative error."""
    return np.max(np.abs(x - y) / (np.maximum(1e-8, np.abs(x) + np.abs(y))))

def print_mean_std(x,axis=0):
    print(f"  means: {x.mean(axis=axis)}")
    print(f"  stds:  {x.std(axis=axis)}\n")
```

[ ]:
```
# Load the (preprocessed) CIFAR-10 data.
data = get_CIFAR10_data()
for k, v in list(data.items()):
    print(f"{k}: {v.shape}")
```

```
X_train: (49000, 3, 32, 32)
y_train: (49000,)
X_val: (1000, 3, 32, 32)
y_val: (1000,)
X_test: (1000, 3, 32, 32)
y_test: (1000,)
```

2 Batch Normalization: Forward Pass

In the file `dl/layers.py`, implement the batch normalization forward pass in the function `batchnorm_forward`. Once you have done so, run the following to test your implementation.

Referencing the paper linked to above in [1] may be helpful!

[ ]:
```python
from dl.layers import *

# Check the training-time forward pass by checking means and variances
# of features both before and after batch normalization

# Simulate the forward pass for a two-layer network.
np.random.seed(231)
N, D1, D2, D3 = 200, 50, 60, 3
X = np.random.randn(N, D1)
W1 = np.random.randn(D1, D2)
W2 = np.random.randn(D2, D3)
a = np.maximum(0, X.dot(W1)).dot(W2)

print('Before batch normalization:')
print_mean_std(a,axis=0)

gamma = np.ones((D3,))
beta = np.zeros((D3,))

# Means should be close to zero and stds close to one.
print('After batch normalization (gamma=1, beta=0)')
a_norm, _ = batchnorm_forward(a, gamma, beta, {'mode': 'train'})
print_mean_std(a_norm,axis=0)

gamma = np.asarray([1.0, 2.0, 3.0])
beta = np.asarray([11.0, 12.0, 13.0])

# Now means should be close to beta and stds close to gamma.
print('After batch normalization (gamma=', gamma, ', beta=', beta, ')')
a_norm, _ = batchnorm_forward(a, gamma, beta, {'mode': 'train'})
print_mean_std(a_norm,axis=0)
```

```
Before batch normalization:
  means: [ -2.3814598  -13.18038246   1.91780462]
```

```
  stds:  [27.18502186 34.21455511 37.68611762]

After batch normalization (gamma=1, beta=0)
  means: [5.32907052e-17 7.04991621e-17 1.85962357e-17]
  stds:  [0.99999999 1.         1.        ]

After batch normalization (gamma= [1. 2. 3.] , beta= [11. 12. 13.] )
  means: [11. 12. 13.]
  stds:  [0.99999999 1.99999999 2.99999999]
```

[ ]:
```python
# Check the test-time forward pass by running the training-time
# forward pass many times to warm up the running averages, and then
# checking the means and variances of activations after a test-time
# forward pass.

np.random.seed(231)
N, D1, D2, D3 = 200, 50, 60, 3
W1 = np.random.randn(D1, D2)
W2 = np.random.randn(D2, D3)

bn_param = {'mode': 'train'}
gamma = np.ones(D3)
beta = np.zeros(D3)

for t in range(50):
  X = np.random.randn(N, D1)
  a = np.maximum(0, X.dot(W1)).dot(W2)
  batchnorm_forward(a, gamma, beta, bn_param)

bn_param['mode'] = 'test'
X = np.random.randn(N, D1)
a = np.maximum(0, X.dot(W1)).dot(W2)
a_norm, _ = batchnorm_forward(a, gamma, beta, bn_param)

# Means should be close to zero and stds close to one, but will be
# noisier than training-time forward passes.
print('After batch normalization (test-time):')
print_mean_std(a_norm,axis=0)
```

```
After batch normalization (test-time):
  means: [-0.03927354 -0.04349152 -0.10452688]
  stds:  [1.01531428 1.01238373 0.97819988]
```

3 Batch Normalization: Backward Pass

Now implement the backward pass for batch normalization in the function `batchnorm_backward`.

To derive the backward pass you should write out the computation graph for batch normalization and backprop through each of the intermediate nodes. Some intermediates may have multiple outgoing branches; make sure to sum gradients across these branches in the backward pass. Referencing the paper linked to above in [1] may be helpful!

Once you have finished, run the following to numerically check your backward pass.

[ ]:
```
# Gradient check batchnorm backward pass.
np.random.seed(231)
N, D = 4, 5
x = 5 * np.random.randn(N, D) + 12
gamma = np.random.randn(D)
beta = np.random.randn(D)
dout = np.random.randn(N, D)

bn_param = {'mode': 'train'}
fx = lambda x: batchnorm_forward(x, gamma, beta, bn_param)[0]
fg = lambda a: batchnorm_forward(x, a, beta, bn_param)[0]
fb = lambda b: batchnorm_forward(x, gamma, b, bn_param)[0]

dx_num = eval_numerical_gradient_array(fx, x, dout)
da_num = eval_numerical_gradient_array(fg, gamma.copy(), dout)
db_num = eval_numerical_gradient_array(fb, beta.copy(), dout)

_, cache = batchnorm_forward(x, gamma, beta, bn_param)
dx, dgamma, dbeta = batchnorm_backward(dout, cache)

# You should expect to see relative errors between 1e-13 and 1e-8.
print('dx error: ', rel_error(dx_num, dx))
print('dgamma error: ', rel_error(da_num, dgamma))
print('dbeta error: ', rel_error(db_num, dbeta))
```

```
dx error:  1.7029235612572515e-09
dgamma error:  7.420414216247087e-13
dbeta error:  2.8795057655839487e-12
```

4 Batch Normalization: Alternative Backward Pass

In class we talked about two different implementations for the sigmoid backward pass. One strategy is to write out a computation graph composed of simple operations and backprop through all intermediate values. Another strategy is to work out the derivatives on paper. For example, you can derive a very simple formula for the sigmoid function's backward pass by simplifying gradients on paper.

Surprisingly, it turns out that you can do a similar simplification for the batch normalization

backward pass too!

In the forward pass, given a set of inputs $X = \begin{bmatrix} x_1 \\ x_2 \\ ... \\ x_N \end{bmatrix}$,

we first calculate the mean μ and variance v. With μ and v calculated, we can calculate the standard deviation σ and normalized data Y. The equations and graph illustration below describe the computation (y_i is the i-th element of the vector Y).

$$\mu = \frac{1}{N}\sum_{k=1}^{N} x_k \qquad v = \frac{1}{N}\sum_{k=1}^{N}(x_k - \mu)^2 \tag{1}$$

$$\sigma = \sqrt{v + \epsilon} \qquad y_i = \frac{x_i - \mu}{\sigma} \tag{2}$$

The meat of our problem during backpropagation is to compute $\frac{\partial L}{\partial X}$, given the upstream gradient we receive, $\frac{\partial L}{\partial Y}$. To do this, recall the chain rule in calculus gives us $\frac{\partial L}{\partial X} = \frac{\partial L}{\partial Y} \cdot \frac{\partial Y}{\partial X}$.

The unknown/hard part is $\frac{\partial Y}{\partial X}$. We can find this by first deriving step-by-step our local gradients at $\frac{\partial v}{\partial X}$, $\frac{\partial \mu}{\partial X}$, $\frac{\partial \sigma}{\partial v}$, $\frac{\partial Y}{\partial \sigma}$, and $\frac{\partial Y}{\partial \mu}$, and then use the chain rule to compose these gradients (which appear in the form of vectors!) appropriately to compute $\frac{\partial Y}{\partial X}$.

If it's challenging to directly reason about the gradients over X and Y which require matrix multiplication, try reasoning about the gradients in terms of individual elements x_i and y_i first: in that case, you will need to come up with the derivations for $\frac{\partial L}{\partial x_i}$, by relying on the Chain Rule to first calculate the intermediate $\frac{\partial \mu}{\partial x_i}$, $\frac{\partial v}{\partial x_i}$, $\frac{\partial \sigma}{\partial x_i}$, then assemble these pieces to calculate $\frac{\partial y_i}{\partial x_i}$.

You should make sure each of the intermediary gradient derivations are all as simplified as possible, for ease of implementation.

After doing so, implement the simplified batch normalization backward pass in the function `batchnorm_backward_alt` and compare the two implementations by running the following. Your two implementations should compute nearly identical results, but the alternative implementation should be a bit faster.

[ ]:
```
np.random.seed(231)
N, D = 100, 500
x = 5 * np.random.randn(N, D) + 12
gamma = np.random.randn(D)
beta = np.random.randn(D)
dout = np.random.randn(N, D)

bn_param = {'mode': 'train'}
out, cache = batchnorm_forward(x, gamma, beta, bn_param)

t1 = time.time()
dx1, dgamma1, dbeta1 = batchnorm_backward(dout, cache)
t2 = time.time()
```

```
dx2, dgamma2, dbeta2 = batchnorm_backward_alt(dout, cache)
t3 = time.time()

print('dx difference: ', rel_error(dx1, dx2))
print('dgamma difference: ', rel_error(dgamma1, dgamma2))
print('dbeta difference: ', rel_error(dbeta1, dbeta2))
print('speedup: %.2fx' % ((t2 - t1) / (t3 - t2)))
```

```
dx difference:  1.6697540272642257e-12
dgamma difference:  0.0
dbeta difference:  0.0
speedup: 1.30x
```

5 Fully Connected Networks with Batch Normalization

Now that you have a working implementation for batch normalization, go back to your `FullyConnectedNet` in the file `dl/classifiers/fc_net.py`. Recall that you implemented the network initialization, forward pass, and backward pass in Assignment 1. Copy that implementation here, and modify it to incorporate batch normalization.

Concretely, when the `normalization` flag is set to `"batchnorm"` in the constructor, you should insert a batch normalization layer before each ReLU nonlinearity. The outputs from the last layer of the network should not be normalized. Once you are done, run the following to gradient-check your implementation.

Hint: You might find it useful to define an additional helper layer similar to those in the file `dl/layer_utils.py`.

[ ]:
```
from dl.classifiers.fc_net import *
from dl.gradient_check import *

np.random.seed(231)
N, D, H1, H2, C = 2, 15, 20, 30, 10
X = np.random.randn(N, D)
y = np.random.randint(C, size=(N,))

# You should expect losses between 1e-4~1e-10 for W,
# losses between 1e-08~1e-10 for b,
# and losses between 1e-08~1e-09 for beta and gammas.
for reg in [0, 3.14]:
  print('Running check with reg = ', reg)
  model = FullyConnectedNet([H1, H2], input_dim=D, num_classes=C,
                            reg=reg, weight_scale=5e-2, dtype=np.float64,
                            normalization='batchnorm')

  loss, grads = model.loss(X, y)
  print('Initial loss: ', loss)
```

```
  for name in sorted(grads):
    f = lambda _: model.loss(X, y)[0]
    grad_num = eval_numerical_gradient(f, model.params[name], verbose=False,␣
↪h=1e-5)
    print('%s relative error: %.2e' % (name, rel_error(grad_num, grads[name])))
  if reg == 0: print()
```

```
Running check with reg =  0
Initial loss:  2.2611955101340957
W1 relative error: 1.10e-04
W2 relative error: 5.65e-06
W3 relative error: 4.14e-10
b1 relative error: 2.22e-08
b2 relative error: 5.55e-09
b3 relative error: 1.02e-10
beta1 relative error: 7.33e-09
beta2 relative error: 1.17e-09
gamma1 relative error: 7.47e-09
gamma2 relative error: 3.35e-09

Running check with reg =  3.14
Initial loss:  6.996533220108303
W1 relative error: 1.98e-06
W2 relative error: 2.29e-06
W3 relative error: 1.11e-08
b1 relative error: 5.55e-09
b2 relative error: 2.22e-08
b3 relative error: 2.10e-10
beta1 relative error: 6.32e-09
beta2 relative error: 3.48e-09
gamma1 relative error: 6.27e-09
gamma2 relative error: 4.14e-09
```

6 Batch Normalization for Deep Networks

Run the following to train a six-layer network on a subset of 1000 training examples both with and without batch normalization.

[ ]:
```
np.random.seed(231)

# Try training a very deep net with batchnorm.
hidden_dims = [100, 100, 100, 100, 100]

num_train = 1000
small_data = {
  'X_train': data['X_train'][:num_train],
  'y_train': data['y_train'][:num_train],
```

```
  'X_val': data['X_val'],
  'y_val': data['y_val'],
}

weight_scale = 2e-2
bn_model = FullyConnectedNet(hidden_dims, weight_scale=weight_scale,␣
 ↪normalization='batchnorm')
model = FullyConnectedNet(hidden_dims, weight_scale=weight_scale,␣
 ↪normalization=None)

print('Solver with batch norm:')
bn_solver = Solver(bn_model, small_data,
                num_epochs=10, batch_size=50,
                update_rule='adam',
                optim_config={
                  'learning_rate': 1e-3,
                },
                verbose=True,print_every=20)
bn_solver.train()

print('\nSolver without batch norm:')
solver = Solver(model, small_data,
                num_epochs=10, batch_size=50,
                update_rule='adam',
                optim_config={
                  'learning_rate': 1e-3,
                },
                verbose=True, print_every=20)
solver.train()
```

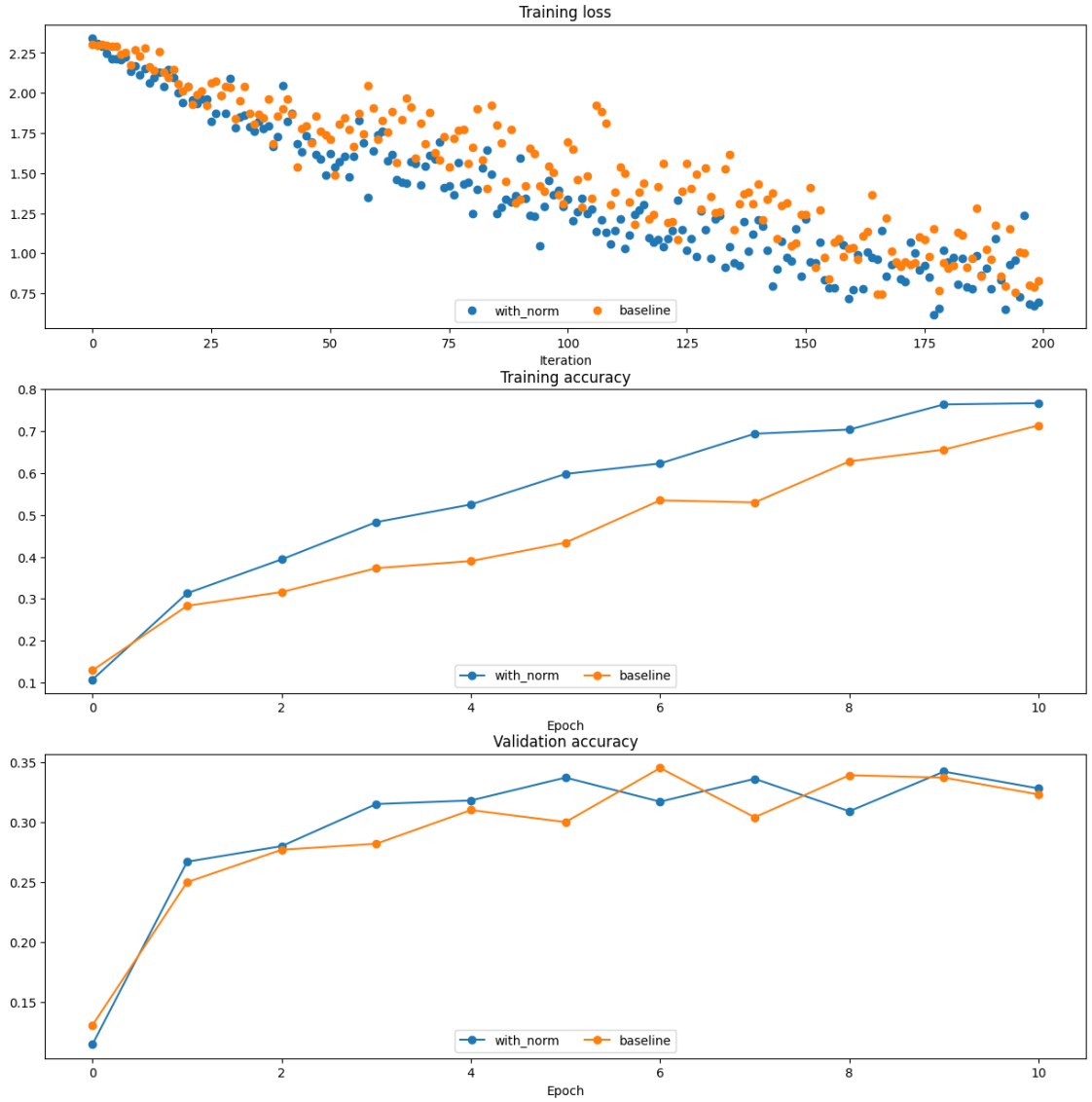
```
Solver with batch norm:
(Iteration 1 / 200) loss: 2.340974
(Epoch 0 / 10) train acc: 0.107000; val_acc: 0.115000
(Epoch 1 / 10) train acc: 0.313000; val_acc: 0.267000
(Iteration 21 / 200) loss: 2.039345
(Epoch 2 / 10) train acc: 0.394000; val_acc: 0.280000
(Iteration 41 / 200) loss: 2.047471
(Epoch 3 / 10) train acc: 0.483000; val_acc: 0.315000
(Iteration 61 / 200) loss: 1.739554
(Epoch 4 / 10) train acc: 0.525000; val_acc: 0.318000
(Iteration 81 / 200) loss: 1.247064
(Epoch 5 / 10) train acc: 0.598000; val_acc: 0.337000
(Iteration 101 / 200) loss: 1.335661
(Epoch 6 / 10) train acc: 0.623000; val_acc: 0.317000
(Iteration 121 / 200) loss: 1.040249
(Epoch 7 / 10) train acc: 0.694000; val_acc: 0.336000
(Iteration 141 / 200) loss: 1.208633
```

```
(Epoch 8 / 10) train acc: 0.704000; val_acc: 0.309000
(Iteration 161 / 200) loss: 0.771428
(Epoch 9 / 10) train acc: 0.764000; val_acc: 0.342000
(Iteration 181 / 200) loss: 0.947944
(Epoch 10 / 10) train acc: 0.767000; val_acc: 0.328000

Solver without batch norm:
(Iteration 1 / 200) loss: 2.302331
(Epoch 0 / 10) train acc: 0.129000; val_acc: 0.131000
(Epoch 1 / 10) train acc: 0.283000; val_acc: 0.250000
(Iteration 21 / 200) loss: 2.041970
(Epoch 2 / 10) train acc: 0.316000; val_acc: 0.277000
(Iteration 41 / 200) loss: 1.900473
(Epoch 3 / 10) train acc: 0.373000; val_acc: 0.282000
(Iteration 61 / 200) loss: 1.713156
(Epoch 4 / 10) train acc: 0.390000; val_acc: 0.310000
(Iteration 81 / 200) loss: 1.662209
(Epoch 5 / 10) train acc: 0.434000; val_acc: 0.300000
(Iteration 101 / 200) loss: 1.696059
(Epoch 6 / 10) train acc: 0.535000; val_acc: 0.345000
(Iteration 121 / 200) loss: 1.557987
(Epoch 7 / 10) train acc: 0.530000; val_acc: 0.304000
(Iteration 141 / 200) loss: 1.432189
(Epoch 8 / 10) train acc: 0.628000; val_acc: 0.339000
(Iteration 161 / 200) loss: 1.033931
(Epoch 9 / 10) train acc: 0.656000; val_acc: 0.337000
(Iteration 181 / 200) loss: 0.908564
(Epoch 10 / 10) train acc: 0.714000; val_acc: 0.323000
```

Run the following to visualize the results from two networks trained above. You should find that using batch normalization helps the network to converge much faster.

[ ]:
```
def plot_training_history(title, label, baseline, bn_solvers, plot_fn,␣
↪bl_marker='.', bn_marker='.', labels=None):
    """utility function for plotting training history"""
    plt.title(title)
    plt.xlabel(label)
    bn_plots = [plot_fn(bn_solver) for bn_solver in bn_solvers]
    bl_plot = plot_fn(baseline)
    num_bn = len(bn_plots)
    for i in range(num_bn):
        label='with_norm'
        if labels is not None:
            label += str(labels[i])
        plt.plot(bn_plots[i], bn_marker, label=label)
    label='baseline'
    if labels is not None:
        label += str(labels[0])
```

```
    plt.plot(bl_plot, bl_marker, label=label)
    plt.legend(loc='lower center', ncol=num_bn+1)


plt.subplot(3, 1, 1)
plot_training_history('Training loss','Iteration', solver, [bn_solver], \
                      lambda x: x.loss_history, bl_marker='o', bn_marker='o')
plt.subplot(3, 1, 2)
plot_training_history('Training accuracy','Epoch', solver, [bn_solver], \
                      lambda x: x.train_acc_history, bl_marker='-o',␣
 ↪bn_marker='-o')
plt.subplot(3, 1, 3)
plot_training_history('Validation accuracy','Epoch', solver, [bn_solver], \
                      lambda x: x.val_acc_history, bl_marker='-o',␣
 ↪bn_marker='-o')

plt.gcf().set_size_inches(15, 15)
plt.show()
```



7 Batch Normalization and Initialization

We will now run a small experiment to study the interaction of batch normalization and weight initialization.

The first cell will train eight-layer networks both with and without batch normalization using different scales for weight initialization. The second layer will plot training accuracy, validation set accuracy, and training loss as a function of the weight initialization scale.

[ ]:
```
np.random.seed(231)

# Try training a very deep net with batchnorm.
```

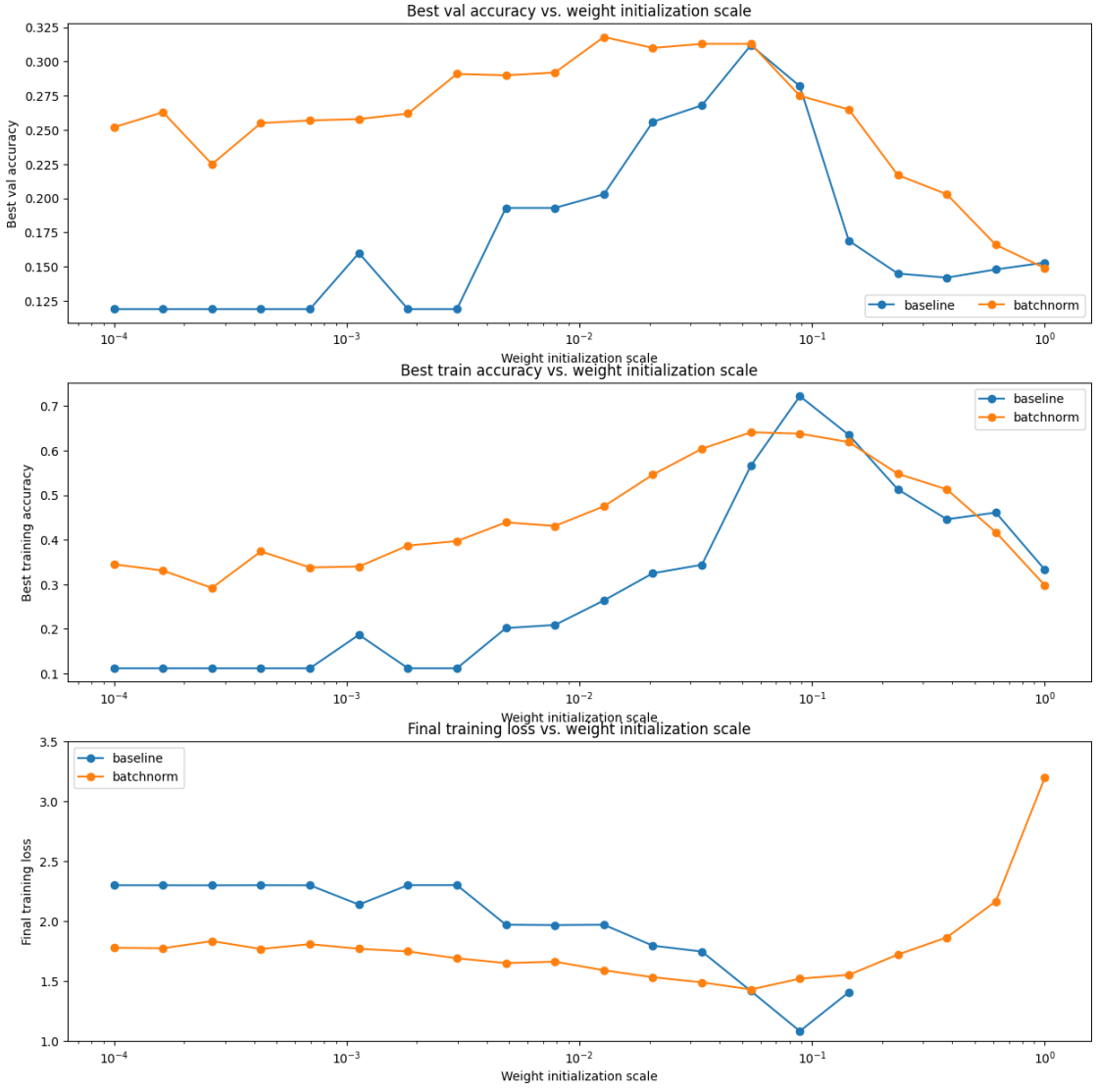
```
hidden_dims = [50, 50, 50, 50, 50, 50, 50]
num_train = 1000
small_data = {
  'X_train': data['X_train'][:num_train],
  'y_train': data['y_train'][:num_train],
  'X_val': data['X_val'],
  'y_val': data['y_val'],
}

bn_solvers_ws = {}
solvers_ws = {}
weight_scales = np.logspace(-4, 0, num=20)
for i, weight_scale in enumerate(weight_scales):
    print('Running weight scale %d / %d' % (i + 1, len(weight_scales)))
    bn_model = FullyConnectedNet(hidden_dims, weight_scale=weight_scale,␣
 ↪normalization='batchnorm')
    model = FullyConnectedNet(hidden_dims, weight_scale=weight_scale,␣
 ↪normalization=None)

    bn_solver = Solver(bn_model, small_data,
                    num_epochs=10, batch_size=50,
                    update_rule='adam',
                    optim_config={
                      'learning_rate': 1e-3,
                    },
                    verbose=False, print_every=200)
    bn_solver.train()
    bn_solvers_ws[weight_scale] = bn_solver

    solver = Solver(model, small_data,
                    num_epochs=10, batch_size=50,
                    update_rule='adam',
                    optim_config={
                      'learning_rate': 1e-3,
                    },
                    verbose=False, print_every=200)
    solver.train()
    solvers_ws[weight_scale] = solver
```

```
Running weight scale 1 / 20
Running weight scale 2 / 20
Running weight scale 3 / 20
Running weight scale 4 / 20
Running weight scale 5 / 20
Running weight scale 6 / 20
Running weight scale 7 / 20
Running weight scale 8 / 20
```

```
Running weight scale 9 / 20
Running weight scale 10 / 20
Running weight scale 11 / 20
Running weight scale 12 / 20
Running weight scale 13 / 20
Running weight scale 14 / 20
Running weight scale 15 / 20
Running weight scale 16 / 20

/content/drive/My Drive/dl/assignments/assignment1/dl/layers.py:158:
RuntimeWarning: divide by zero encountered in log
  correct_logprobs = -np.log(probs[np.arange(N), y])  # (N,)

Running weight scale 17 / 20
Running weight scale 18 / 20
Running weight scale 19 / 20
Running weight scale 20 / 20
```

[ ]:
```python
# Plot results of weight scale experiment.
best_train_accs, bn_best_train_accs = [], []
best_val_accs, bn_best_val_accs = [], []
final_train_loss, bn_final_train_loss = [], []

for ws in weight_scales:
  best_train_accs.append(max(solvers_ws[ws].train_acc_history))
  bn_best_train_accs.append(max(bn_solvers_ws[ws].train_acc_history))

  best_val_accs.append(max(solvers_ws[ws].val_acc_history))
  bn_best_val_accs.append(max(bn_solvers_ws[ws].val_acc_history))

  final_train_loss.append(np.mean(solvers_ws[ws].loss_history[-100:]))
  bn_final_train_loss.append(np.mean(bn_solvers_ws[ws].loss_history[-100:]))

plt.subplot(3, 1, 1)
plt.title('Best val accuracy vs. weight initialization scale')
plt.xlabel('Weight initialization scale')
plt.ylabel('Best val accuracy')
plt.semilogx(weight_scales, best_val_accs, '-o', label='baseline')
plt.semilogx(weight_scales, bn_best_val_accs, '-o', label='batchnorm')
plt.legend(ncol=2, loc='lower right')

plt.subplot(3, 1, 2)
plt.title('Best train accuracy vs. weight initialization scale')
plt.xlabel('Weight initialization scale')
plt.ylabel('Best training accuracy')
plt.semilogx(weight_scales, best_train_accs, '-o', label='baseline')
plt.semilogx(weight_scales, bn_best_train_accs, '-o', label='batchnorm')
plt.legend()
```

```
plt.subplot(3, 1, 3)
plt.title('Final training loss vs. weight initialization scale')
plt.xlabel('Weight initialization scale')
plt.ylabel('Final training loss')
plt.semilogx(weight_scales, final_train_loss, '-o', label='baseline')
plt.semilogx(weight_scales, bn_final_train_loss, '-o', label='batchnorm')
plt.legend()
plt.gca().set_ylim(1.0, 3.5)

plt.gcf().set_size_inches(15, 15)
plt.show()
```



7.1 Inline Question 1:

Describe the results of this experiment. How does the weight initialization scale affect models with/without batch normalization differently, and why?

7.2 Answer:

The first that we can see is without batch_norm layer the model is more sensitive to changes in the weight scaling initialliztion. With Too Small Weights: If weights are initialized too small, the activations in deeper layers can shrink towards zero. This leads to very small gradients during backpropagation. With Too Large Weights, the activations can grow uncontrollably, leading to very large gradients (the 'exploding gradient' problem). This can cause training to become unstable. The models with batch normalization show much more consistent performance across a broad range of weight initialization scales. They achieve good accuracy and lower loss even with weight scales that would cause the baseline model to fail.

8 Batch Normalization and Batch Size

We will now run a small experiment to study the interaction of batch normalization and batch size.

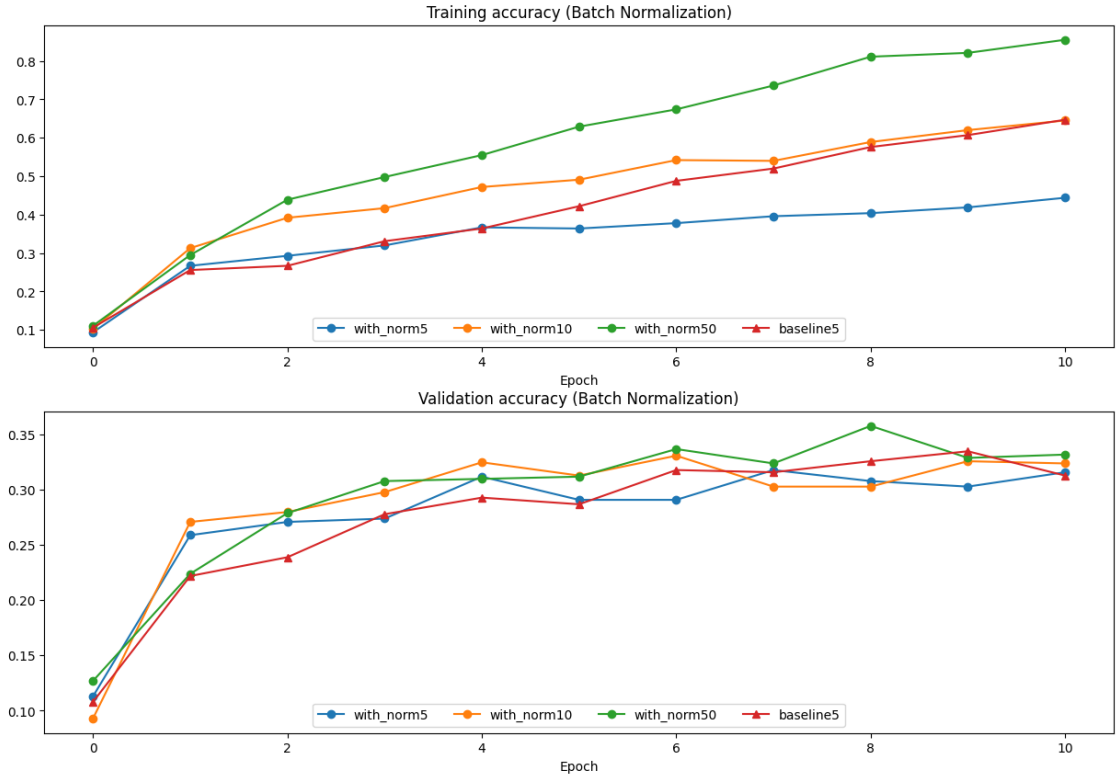
The first cell will train 6-layer networks both with and without batch normalization using different batch sizes. The second layer will plot training accuracy and validation set accuracy over time.

[ ]:
```
def run_batchsize_experiments(normalization_mode):
    np.random.seed(231)

    # Try training a very deep net with batchnorm.
    hidden_dims = [100, 100, 100, 100, 100]
    num_train = 1000
    small_data = {
      'X_train': data['X_train'][:num_train],
      'y_train': data['y_train'][:num_train],
      'X_val': data['X_val'],
      'y_val': data['y_val'],
    }
    n_epochs=10
    weight_scale = 2e-2
    batch_sizes = [5,10,50]
    lr = 10**(-3.5)
    solver_bsize = batch_sizes[0]

    print('No normalization: batch size = ',solver_bsize)
    model = FullyConnectedNet(hidden_dims, weight_scale=weight_scale,␣
↪normalization=None)
    solver = Solver(model, small_data,
                    num_epochs=n_epochs, batch_size=solver_bsize,
                    update_rule='adam',
                    optim_config={
```

```
                    'learning_rate': lr,
                  },
                  verbose=False)
    solver.train()

    bn_solvers = []
    for i in range(len(batch_sizes)):
        b_size=batch_sizes[i]
        print('Normalization: batch size = ',b_size)
        bn_model = FullyConnectedNet(hidden_dims, weight_scale=weight_scale,␣
 ↪normalization=normalization_mode)
        bn_solver = Solver(bn_model, small_data,
                        num_epochs=n_epochs, batch_size=b_size,
                        update_rule='adam',
                        optim_config={
                          'learning_rate': lr,
                        },
                        verbose=False)
        bn_solver.train()
        bn_solvers.append(bn_solver)

    return bn_solvers, solver, batch_sizes

batch_sizes = [5,10,50]
bn_solvers_bsize, solver_bsize, batch_sizes =␣
 ↪run_batchsize_experiments('batchnorm')
```

```
No normalization: batch size =  5
Normalization: batch size =  5
Normalization: batch size =  10
Normalization: batch size =  50
```

[ ]:
```
plt.subplot(2, 1, 1)
plot_training_history('Training accuracy (Batch Normalization)','Epoch',␣
 ↪solver_bsize, bn_solvers_bsize, \
                      lambda x: x.train_acc_history, bl_marker='-^',␣
 ↪bn_marker='-o', labels=batch_sizes)
plt.subplot(2, 1, 2)
plot_training_history('Validation accuracy (Batch Normalization)','Epoch',␣
 ↪solver_bsize, bn_solvers_bsize, \
                      lambda x: x.val_acc_history, bl_marker='-^',␣
 ↪bn_marker='-o', labels=batch_sizes)

plt.gcf().set_size_inches(15, 10)
plt.show()
```



8.1 Inline Question 2:

Describe the results of this experiment. What does this imply about the relationship between batch normalization and batch size? Why is this relationship observed?

8.2 Answer:

This experiment implies a direct and important relationship: the performance and stability of Batch Normalization are influenced by the batch size. Batch Normalization is most effective when the batch statistics (mean and variance) computed from the mini-batch are a good approximation of the statistics of the entire training dataset. This approximation tends to be more accurate with larger batch sizes.

9 Layer Normalization

Batch normalization has proved to be effective in making networks easier to train, but the dependency on batch size makes it less useful in complex networks which have a cap on the input batch size due to hardware limitations.

Several alternatives to batch normalization have been proposed to mitigate this problem; one such technique is Layer Normalization [2]. Instead of normalizing over the batch, we normalize over the features. In other words, when using Layer Normalization, each feature vector corresponding to a single datapoint is normalized based on the sum of all terms within that feature vector.

[2] Ba, Jimmy Lei, Jamie Ryan Kiros, and Geoffrey E. Hinton. "Layer Normalization." stat 1050 (2016): 21.

9.1 Inline Question 3:

Which of these data preprocessing steps is analogous to batch normalization, and which is analogous to layer normalization?

1. Scaling each image in the dataset, so that the RGB channels for each row of pixels within an image sums up to 1.
2. Scaling each image in the dataset, so that the RGB channels for all pixels within an image sums up to 1.
3. Subtracting the mean image of the dataset from each image in the dataset.
4. Setting all RGB values to either 0 or 1 depending on a given threshold.

9.2 Answer:

Batch Normalization is analogous to 3. Subtracting the mean image of the dataset from each image in the dataset.

This is because Batch Normalization computes statistics (mean and variance) across the batch for each feature, just as subtracting the 'mean image of the dataset' involves a statistic computed across the dataset and applied feature-wise (pixel-wise).

Layer Normalization is analogous to 2. Scaling each image in the dataset, so that the RGB channels for all pixels within an image sums up to 1.

This is because Layer Normalization computes statistics (mean and variance) per individual sample (in this case, per image) across all its features. Option 2 describes normalizing each image independently based on its own pixel values.

10 Layer Normalization: Implementation

Now you'll implement layer normalization. This step should be relatively straightforward, as conceptually the implementation is almost identical to that of batch normalization. One significant difference though is that for layer normalization, we do not keep track of the moving moments, and the testing phase is identical to the training phase, where the mean and variance are directly calculated per datapoint.

Here's what you need to do:

- In `dl/layers.py`, implement the forward pass for layer normalization in the function `layernorm_forward`.

Run the cell below to check your results. * In `dl/layers.py`, implement the backward pass for layer normalization in the function `layernorm_backward`.

Run the second cell below to check your results. * Modify `dl/classifiers/fc_net.py` to add layer normalization to the `FullyConnectedNet`. When the `normalization` flag is set to `"layernorm"` in the constructor, you should insert a layer normalization layer before each ReLU nonlinearity.

Run the third cell below to run the batch size experiment on layer normalization.

[ ]:
```
# Check the training-time forward pass by checking means and variances
# of features both before and after layer normalization.

# Simulate the forward pass for a two-layer network.
np.random.seed(231)
N, D1, D2, D3 =4, 50, 60, 3
X = np.random.randn(N, D1)
W1 = np.random.randn(D1, D2)
W2 = np.random.randn(D2, D3)
a = np.maximum(0, X.dot(W1)).dot(W2)

print('Before layer normalization:')
print_mean_std(a,axis=1)

gamma = np.ones(D3)
beta = np.zeros(D3)

# Means should be close to zero and stds close to one.
print('After layer normalization (gamma=1, beta=0)')
a_norm, _ = layernorm_forward(a, gamma, beta, {'mode': 'train'})
print_mean_std(a_norm,axis=1)

gamma = np.asarray([3.0,3.0,3.0])
beta = np.asarray([5.0,5.0,5.0])

# Now means should be close to beta and stds close to gamma.
print('After layer normalization (gamma=', gamma, ', beta=', beta, ')')
a_norm, _ = layernorm_forward(a, gamma, beta, {'mode': 'train'})
print_mean_std(a_norm,axis=1)
```

```
Before layer normalization:
  means: [-59.06673243 -47.60782686 -43.31137368 -26.40991744]
  stds:  [10.07429373 28.39478981 35.28360729  4.01831507]

After layer normalization (gamma=1, beta=0)
  means: [ 4.81096644e-16 -7.40148683e-17  2.22044605e-16 -5.92118946e-16]
  stds:  [0.99999995 0.99999999 1.         0.99999969]

After layer normalization (gamma= [3. 3. 3.] , beta= [5. 5. 5.] )
  means: [5. 5. 5. 5.]
  stds:  [2.99999985 2.99999998 2.99999999 2.99999907]
```

[ ]:
```
# Gradient check batchnorm backward pass.
np.random.seed(231)
N, D = 4, 5
```

```
x = 5 * np.random.randn(N, D) + 12
gamma = np.random.randn(D)
beta = np.random.randn(D)
dout = np.random.randn(N, D)

ln_param = {}
fx = lambda x: layernorm_forward(x, gamma, beta, ln_param)[0]
fg = lambda a: layernorm_forward(x, a, beta, ln_param)[0]
fb = lambda b: layernorm_forward(x, gamma, b, ln_param)[0]

dx_num = eval_numerical_gradient_array(fx, x, dout)
da_num = eval_numerical_gradient_array(fg, gamma.copy(), dout)
db_num = eval_numerical_gradient_array(fb, beta.copy(), dout)

_, cache = layernorm_forward(x, gamma, beta, ln_param)
dx, dgamma, dbeta = layernorm_backward(dout, cache)

# You should expect to see relative errors between 1e-12 and 1e-8.
print('dx error: ', rel_error(dx_num, dx))
print('dgamma error: ', rel_error(da_num, dgamma))
print('dbeta error: ', rel_error(db_num, dbeta))
```

```
dx error:  1.4336161049967258e-09
dgamma error:  4.519489546032799e-12
dbeta error:  2.276445013433725e-12
```

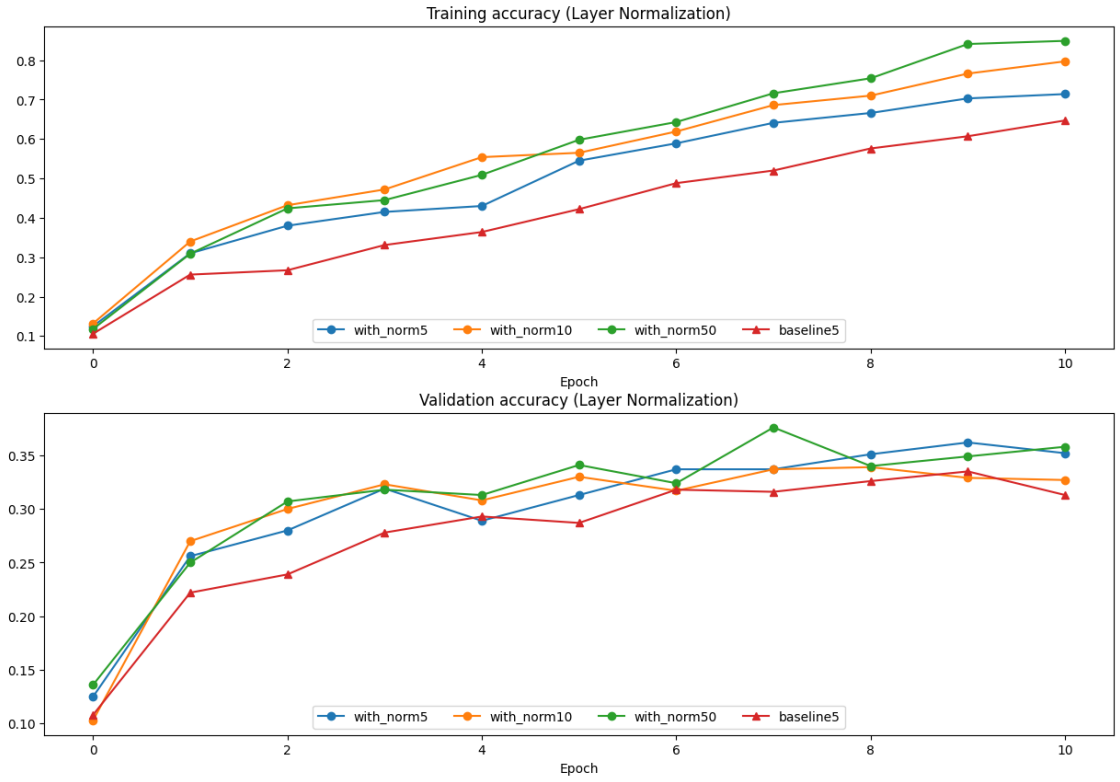
11 Layer Normalization and Batch Size

We will now run the previous batch size experiment with layer normalization instead of batch normalization. Compared to the previous experiment, you should see a markedly smaller influence of batch size on the training history!

[ ]:
```
ln_solvers_bsize, solver_bsize, batch_sizes = run_batchsize_experiments('layernorm')

plt.subplot(2, 1, 1)
plot_training_history('Training accuracy (Layer Normalization)','Epoch', solver_bsize, ln_solvers_bsize, \
                      lambda x: x.train_acc_history, bl_marker='-^', bn_marker='-o', labels=batch_sizes)
plt.subplot(2, 1, 2)
plot_training_history('Validation accuracy (Layer Normalization)','Epoch', solver_bsize, ln_solvers_bsize, \
                      lambda x: x.val_acc_history, bl_marker='-^', bn_marker='-o', labels=batch_sizes)

plt.gcf().set_size_inches(15, 10)
```

```
plt.show()
```

```
No normalization: batch size =  5
Normalization: batch size =  5
Normalization: batch size =  10
Normalization: batch size =  50
```



11.1 Inline Question 4:

When is layer normalization likely to not work well, and why?

1. Using it in a very deep network
2. Having a very small dimension of features
3. Having a high regularization term

11.2 Answer:

Layer Normalization is most likely to not work well when 2. Having a very small dimension of features.

Layer Normalization operates by calculating the mean and variance across all feature dimensions for each individual sample. When the number of features (the feature dimension) is very small, the statistics (mean and variance) computed from such a limited set of values become less reliable and

more susceptible to noise. This means the normalization applied might not effectively standardize the activations, potentially leading to unstable training or reduced performance.

06_ConvolutionalNetworks

December 6, 2025

[1]:
```
# This mounts your Google Drive to the Colab VM.
from google.colab import drive
drive.mount('/content/drive')

# TODO: Enter the foldername in your Drive where you have saved the unzipped
# assignment folder, e.g. 'dl/assignments/assignment1/'
FOLDERNAME = 'dl/assignments/assignment1/'
assert FOLDERNAME is not None, "[!] Enter the foldername."

# Now that we've mounted your Drive, this ensures that
# the Python interpreter of the Colab VM can load
# python files from within it.
import sys
sys.path.append('/content/drive/My Drive/{}'.format(FOLDERNAME))

# This downloads the CIFAR-10 dataset to your Drive
# if it doesn't already exist.
%cd /content/drive/My\ Drive/$FOLDERNAME/dl/datasets/
!bash get_datasets.sh
%cd /content/drive/My\ Drive/$FOLDERNAME
```

```
Drive already mounted at /content/drive; to attempt to forcibly remount, call
drive.mount("/content/drive", force_remount=True).
/content/drive/My Drive/dl/assignments/assignment1/dl/datasets
/content/drive/My Drive/dl/assignments/assignment1
```

1 Convolutional Networks

So far we have worked with deep fully connected networks, using them to explore different optimization strategies and network architectures. Fully connected networks are a good testbed for experimentation because they are very computationally efficient, but in practice all state-of-the-art results use convolutional networks instead.

First you will implement several layer types that are used in convolutional networks. You will then use these layers to train a convolutional network on the CIFAR-10 dataset.

[2]:
```
# Setup cell.
import numpy as np
```

```
import matplotlib.pyplot as plt
from dl.classifiers.cnn import *
from dl.data_utils import get_CIFAR10_data
from dl.gradient_check import eval_numerical_gradient_array,␣
 ↪eval_numerical_gradient
from dl.layers import *
from dl.fast_layers import *
from dl.solver import Solver

%matplotlib inline
plt.rcParams['figure.figsize'] = (10.0, 8.0) # set default size of plots
plt.rcParams['image.interpolation'] = 'nearest'
plt.rcParams['image.cmap'] = 'gray'


def rel_error(x, y):
  """ returns relative error """
  return np.max(np.abs(x - y) / (np.maximum(1e-8, np.abs(x) + np.abs(y))))
```

[3]:
```
# Load the (preprocessed) CIFAR-10 data.
data = get_CIFAR10_data()
for k, v in list(data.items()):
    print(f"{k}: {v.shape}")
```

```
X_train: (49000, 3, 32, 32)
y_train: (49000,)
X_val: (1000, 3, 32, 32)
y_val: (1000,)
X_test: (1000, 3, 32, 32)
y_test: (1000,)
```

2 Convolution: Naive Forward Pass

The core of a convolutional network is the convolution operation. In the file `dl/layers.py`, implement the forward pass for the convolution layer in the function `conv_forward_naive`.

You don't have to worry too much about efficiency at this point; just write the code in whatever way you find most clear.

You can test your implementation by running the following:

[4]:
```
x_shape = (2, 3, 4, 4)
w_shape = (3, 3, 4, 4)
x = np.linspace(-0.1, 0.5, num=np.prod(x_shape)).reshape(x_shape)
w = np.linspace(-0.2, 0.3, num=np.prod(w_shape)).reshape(w_shape)
b = np.linspace(-0.1, 0.2, num=3)

conv_param = {'stride': 2, 'pad': 1}
```

```
out, _ = conv_forward_naive(x, w, b, conv_param)
correct_out = np.array([[[[-0.08759809, -0.10987781],
                          [-0.18387192, -0.2109216 ]],
                         [[ 0.21027089,  0.21661097],
                          [ 0.22847626,  0.23004637]],
                         [[ 0.50813986,  0.54309974],
                          [ 0.64082444,  0.67101435]]],
                        [[[-0.98053589, -1.03143541],
                          [-1.19128892, -1.24695841]],
                         [[ 0.69108355,  0.66880383],
                          [ 0.59480972,  0.56776003]],
                         [[ 2.36270298,  2.36904306],
                          [ 2.38090835,  2.38247847]]]])

# Compare your output to ours; difference should be around e-8
print('Testing conv_forward_naive')
print('difference: ', rel_error(out, correct_out))
```

```
Testing conv_forward_naive
difference:  2.2121476417505994e-08
```

2.1 Aside: Image Processing via Convolutions

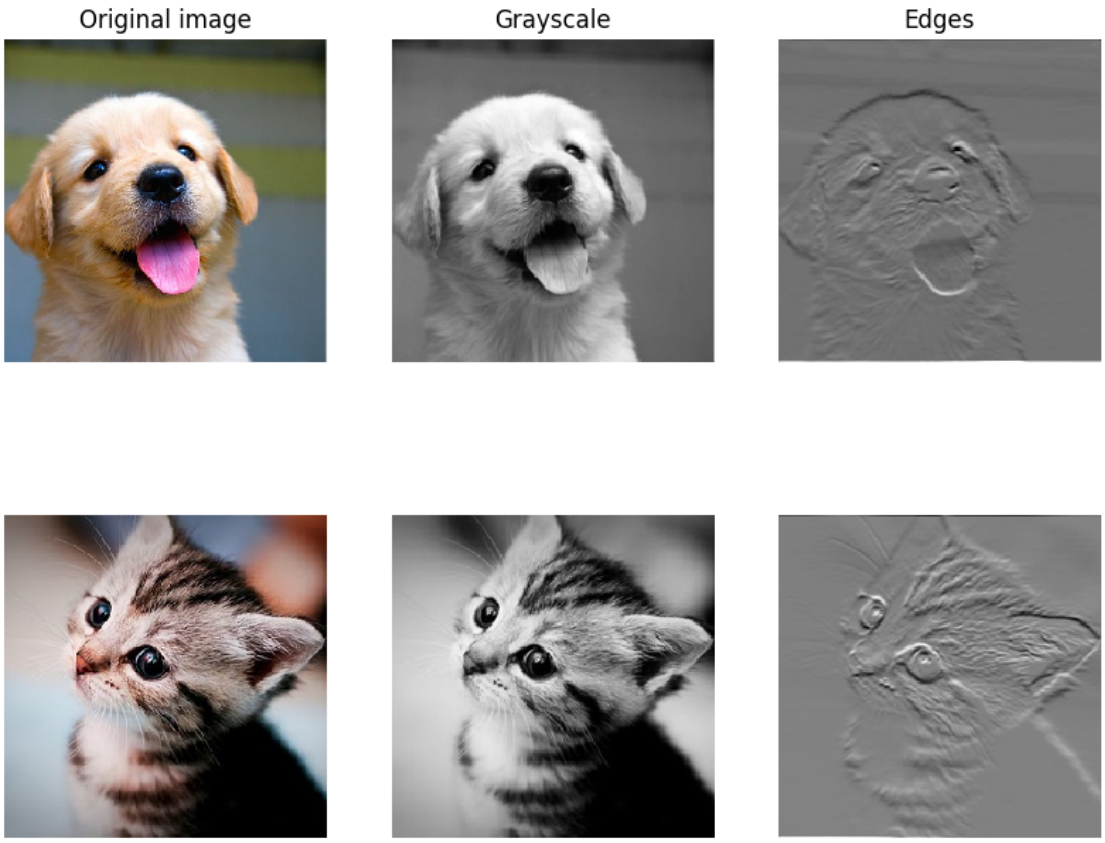
As fun way to both check your implementation and gain a better understanding of the type of operation that convolutional layers can perform, we will set up an input containing two images and manually set up filters that perform common image processing operations (grayscale conversion and edge detection). The convolution forward pass will apply these operations to each of the input images. We can then visualize the results as a sanity check.

[5]:
```
from imageio import imread
from PIL import Image

kitten = imread('dl/notebook_images/kitten.jpg')
puppy = imread('dl/notebook_images/puppy.jpg')
# kitten is wide, and puppy is already square
d = kitten.shape[1] - kitten.shape[0]
kitten_cropped = kitten[:, d//2:-d//2, :]

img_size = 200   # Make this smaller if it runs too slow
resized_puppy = np.array(Image.fromarray(puppy).resize((img_size, img_size)))
resized_kitten = np.array(Image.fromarray(kitten_cropped).resize((img_size,␣
 ↪img_size)))
x = np.zeros((2, 3, img_size, img_size))
x[0, :, :, :] = resized_puppy.transpose((2, 0, 1))
x[1, :, :, :] = resized_kitten.transpose((2, 0, 1))

# Set up a convolutional weights holding 2 filters, each 3x3
w = np.zeros((2, 3, 3, 3))
```

```
# The first filter converts the image to grayscale.
# Set up the red, green, and blue channels of the filter.
w[0, 0, :, :] = [[0, 0, 0], [0, 0.3, 0], [0, 0, 0]]
w[0, 1, :, :] = [[0, 0, 0], [0, 0.6, 0], [0, 0, 0]]
w[0, 2, :, :] = [[0, 0, 0], [0, 0.1, 0], [0, 0, 0]]

# Second filter detects horizontal edges in the blue channel.
w[1, 2, :, :] = [[1, 2, 1], [0, 0, 0], [-1, -2, -1]]

# Vector of biases. We don't need any bias for the grayscale
# filter, but for the edge detection filter we want to add 128
# to each output so that nothing is negative.
b = np.array([0, 128])

# Compute the result of convolving each input in x with each filter in w,
# offsetting by b, and storing the results in out.
out, _ = conv_forward_naive(x, w, b, {'stride': 1, 'pad': 1})

def imshow_no_ax(img, normalize=True):
    """ Tiny helper to show images as uint8 and remove axis labels """
    if normalize:
        img_max, img_min = np.max(img), np.min(img)
        img = 255.0 * (img - img_min) / (img_max - img_min)
    plt.imshow(img.astype('uint8'))
    plt.gca().axis('off')

# Show the original images and the results of the conv operation
plt.subplot(2, 3, 1)
imshow_no_ax(puppy, normalize=False)
plt.title('Original image')
plt.subplot(2, 3, 2)
imshow_no_ax(out[0, 0])
plt.title('Grayscale')
plt.subplot(2, 3, 3)
imshow_no_ax(out[0, 1])
plt.title('Edges')
plt.subplot(2, 3, 4)
imshow_no_ax(kitten_cropped, normalize=False)
plt.subplot(2, 3, 5)
imshow_no_ax(out[1, 0])
plt.subplot(2, 3, 6)
imshow_no_ax(out[1, 1])
plt.show()
```

```
/tmp/ipython-input-3754794099.py:4: DeprecationWarning: Starting with ImageIO v3
the behavior of this function will switch to that of iio.v3.imread. To keep the
```

```
current behavior (and make this warning disappear) use `import imageio.v2 as
imageio` or call `imageio.v2.imread` directly.
  kitten = imread('dl/notebook_images/kitten.jpg')
/tmp/ipython-input-3754794099.py:5: DeprecationWarning: Starting with ImageIO v3
the behavior of this function will switch to that of iio.v3.imread. To keep the
current behavior (and make this warning disappear) use `import imageio.v2 as
imageio` or call `imageio.v2.imread` directly.
  puppy = imread('dl/notebook_images/puppy.jpg')
```



3 Convolution: Naive Backward Pass

Implement the backward pass for the convolution operation in the function `conv_backward_naive` in the file `dl/layers.py`. Again, you don't need to worry too much about computational efficiency.

When you are done, run the following to check your backward pass with a numeric gradient check.

[6]:
```
np.random.seed(231)
x = np.random.randn(4, 3, 5, 5)
w = np.random.randn(2, 3, 3, 3)
b = np.random.randn(2,)
dout = np.random.randn(4, 2, 5, 5)
```

```
conv_param = {'stride': 1, 'pad': 1}

dx_num = eval_numerical_gradient_array(lambda x: conv_forward_naive(x, w, b,␣
 ↪conv_param)[0], x, dout)
dw_num = eval_numerical_gradient_array(lambda w: conv_forward_naive(x, w, b,␣
 ↪conv_param)[0], w, dout)
db_num = eval_numerical_gradient_array(lambda b: conv_forward_naive(x, w, b,␣
 ↪conv_param)[0], b, dout)

out, cache = conv_forward_naive(x, w, b, conv_param)
dx, dw, db = conv_backward_naive(dout, cache)

# Your errors should be around e-8 or less.
print('Testing conv_backward_naive function')
print('dx error: ', rel_error(dx, dx_num))
print('dw error: ', rel_error(dw, dw_num))
print('db error: ', rel_error(db, db_num))
```

```
Testing conv_backward_naive function
dx error:  1.159803161159293e-08
dw error:  2.2471264748452487e-10
db error:  3.37264006649648e-11
```

4 Max-Pooling: Naive Forward Pass

Implement the forward pass for the max-pooling operation in the function `max_pool_forward_naive` in the file `dl/layers.py`. Again, don't worry too much about computational efficiency.

Check your implementation by running the following:

[7]:
```
x_shape = (2, 3, 4, 4)
x = np.linspace(-0.3, 0.4, num=np.prod(x_shape)).reshape(x_shape)
pool_param = {'pool_width': 2, 'pool_height': 2, 'stride': 2}

out, _ = max_pool_forward_naive(x, pool_param)

correct_out = np.array([[[[-0.26315789, -0.24842105],
                          [-0.20421053, -0.18947368]],
                         [[-0.14526316, -0.13052632],
                          [-0.08631579, -0.07157895]],
                         [[-0.02736842, -0.01263158],
                          [ 0.03157895,  0.04631579]]],
                        [[[ 0.09052632,  0.10526316],
                          [ 0.14947368,  0.16421053]],
                         [[ 0.20842105,  0.22315789],
                          [ 0.26736842,  0.28210526]],
```

```
                        [[ 0.32631579,  0.34105263],
                         [ 0.38526316,  0.4       ]]]])

# Compare your output with ours. Difference should be on the order of e-8.
print('Testing max_pool_forward_naive function:')
print('difference: ', rel_error(out, correct_out))
```

```
Testing max_pool_forward_naive function:
difference:  4.1666665157267834e-08
```

5 Max-Pooling: Naive Backward

Implement the backward pass for the max-pooling operation in the function `max_pool_backward_naive` in the file `dl/layers.py`. You don't need to worry about computational efficiency.

Check your implementation with numeric gradient checking by running the following:

[8]:
```
np.random.seed(231)
x = np.random.randn(3, 2, 8, 8)
dout = np.random.randn(3, 2, 4, 4)
pool_param = {'pool_height': 2, 'pool_width': 2, 'stride': 2}

dx_num = eval_numerical_gradient_array(lambda x: max_pool_forward_naive(x,␣
 ↪pool_param)[0], x, dout)

out, cache = max_pool_forward_naive(x, pool_param)
dx = max_pool_backward_naive(dout, cache)

# Your error should be on the order of e-12
print('Testing max_pool_backward_naive function:')
print('dx error: ', rel_error(dx, dx_num))
```

```
Testing max_pool_backward_naive function:
dx error:  3.27562514223145e-12
```

6 Fast Layers

Making convolution and pooling layers fast can be challenging. To spare you the pain, we've provided fast implementations of the forward and backward passes for convolution and pooling layers in the file `dl/fast_layers.py`.

6.0.1 Execute the below cell, save the notebook, and restart the runtime

The fast convolution implementation depends on a Cython extension; to compile it, run the cell below. Next, save the Colab notebook (`File > Save`) and **restart the runtime** (`Runtime > Restart runtime`). You can then re-execute the preceeding cells from top to bottom and skip the cell below as you only need to run it once for the compilation step.

[9]:
```
# Remember to restart the runtime after executing this cell!
%cd /content/drive/My\ Drive/$FOLDERNAME/dl/
!python setup.py build_ext --inplace
%cd /content/drive/My\ Drive/$FOLDERNAME/
```

```
/content/drive/My Drive/dl/assignments/assignment1/dl
/content/drive/My Drive/dl/assignments/assignment1
```

The API for the fast versions of the convolution and pooling layers is exactly the same as the naive versions that you implemented above: the forward pass receives data, weights, and parameters and produces outputs and a cache object; the backward pass recieves upstream derivatives and the cache object and produces gradients with respect to the data and weights.

Note: The fast implementation for pooling will only perform optimally if the pooling regions are non-overlapping and tile the input. If these conditions are not met then the fast pooling implementation will not be much faster than the naive implementation.

You can compare the performance of the naive and fast versions of these layers by running the following:

[10]:
```
# Rel errors should be around e-9 or less.
from dl.fast_layers import conv_forward_fast, conv_backward_fast
from time import time
np.random.seed(231)
x = np.random.randn(100, 3, 31, 31)
w = np.random.randn(25, 3, 3, 3)
b = np.random.randn(25,)
dout = np.random.randn(100, 25, 16, 16)
conv_param = {'stride': 2, 'pad': 1}

t0 = time()
out_naive, cache_naive = conv_forward_naive(x, w, b, conv_param)
t1 = time()
out_fast, cache_fast = conv_forward_fast(x, w, b, conv_param)
t2 = time()

print('Testing conv_forward_fast:')
print('Naive: %fs' % (t1 - t0))
print('Fast: %fs' % (t2 - t1))
print('Speedup: %fx' % ((t1 - t0) / (t2 - t1)))
print('Difference: ', rel_error(out_naive, out_fast))

t0 = time()
dx_naive, dw_naive, db_naive = conv_backward_naive(dout, cache_naive)
t1 = time()
dx_fast, dw_fast, db_fast = conv_backward_fast(dout, cache_fast)
t2 = time()

print('\nTesting conv_backward_fast:')
```

```
print('Naive: %fs' % (t1 - t0))
print('Fast: %fs' % (t2 - t1))
print('Speedup: %fx' % ((t1 - t0) / (t2 - t1)))
print('dx difference: ', rel_error(dx_naive, dx_fast))
print('dw difference: ', rel_error(dw_naive, dw_fast))
print('db difference: ', rel_error(db_naive, db_fast))
```

```
Testing conv_forward_fast:
Naive: 4.457484s
Fast: 0.016818s
Speedup: 265.037964x
Difference:  4.926407851494105e-11

Testing conv_backward_fast:
Naive: 8.462239s
Fast: 0.015270s
Speedup: 554.157036x
dx difference:  1.949764775345631e-11
dw difference:  3.681156828004736e-13
db difference:  3.481354613192702e-14
```

[11]:
```
# Relative errors should be close to 0.0.
from dl.fast_layers import max_pool_forward_fast, max_pool_backward_fast
np.random.seed(231)
x = np.random.randn(100, 3, 32, 32)
dout = np.random.randn(100, 3, 16, 16)
pool_param = {'pool_height': 2, 'pool_width': 2, 'stride': 2}

t0 = time()
out_naive, cache_naive = max_pool_forward_naive(x, pool_param)
t1 = time()
out_fast, cache_fast = max_pool_forward_fast(x, pool_param)
t2 = time()

print('Testing pool_forward_fast:')
print('Naive: %fs' % (t1 - t0))
print('fast: %fs' % (t2 - t1))
print('speedup: %fx' % ((t1 - t0) / (t2 - t1)))
print('difference: ', rel_error(out_naive, out_fast))

t0 = time()
dx_naive = max_pool_backward_naive(dout, cache_naive)
t1 = time()
dx_fast = max_pool_backward_fast(dout, cache_fast)
t2 = time()

print('\nTesting pool_backward_fast:')
```

```
print('Naive: %fs' % (t1 - t0))
print('fast: %fs' % (t2 - t1))
print('speedup: %fx' % ((t1 - t0) / (t2 - t1)))
print('dx difference: ', rel_error(dx_naive, dx_fast))
```

```
Testing pool_forward_fast:
Naive: 0.365863s
fast: 0.005093s
speedup: 71.841713x
difference:  0.0

Testing pool_backward_fast:
Naive: 0.987610s
fast: 0.014558s
speedup: 67.841547x
dx difference:  0.0
```

7 Convolutional "Sandwich" Layers

In the previous assignment, we introduced the concept of "sandwich" layers that combine multiple operations into commonly used patterns. In the file `dl/layer_utils.py` you will find sandwich layers that implement a few commonly used patterns for convolutional networks. Run the cells below to sanity check their usage.

[12]:
```
from dl.layer_utils import conv_relu_pool_forward, conv_relu_pool_backward
np.random.seed(231)
x = np.random.randn(2, 3, 16, 16)
w = np.random.randn(3, 3, 3, 3)
b = np.random.randn(3,)
dout = np.random.randn(2, 3, 8, 8)
conv_param = {'stride': 1, 'pad': 1}
pool_param = {'pool_height': 2, 'pool_width': 2, 'stride': 2}

out, cache = conv_relu_pool_forward(x, w, b, conv_param, pool_param)
dx, dw, db = conv_relu_pool_backward(dout, cache)

dx_num = eval_numerical_gradient_array(lambda x: conv_relu_pool_forward(x, w,␣
 ↪b, conv_param, pool_param)[0], x, dout)
dw_num = eval_numerical_gradient_array(lambda w: conv_relu_pool_forward(x, w,␣
 ↪b, conv_param, pool_param)[0], w, dout)
db_num = eval_numerical_gradient_array(lambda b: conv_relu_pool_forward(x, w,␣
 ↪b, conv_param, pool_param)[0], b, dout)

# Relative errors should be around e-8 or less
print('Testing conv_relu_pool')
print('dx error: ', rel_error(dx_num, dx))
print('dw error: ', rel_error(dw_num, dw))
```

```
print('db error: ', rel_error(db_num, db))
```

```
Testing conv_relu_pool
dx error:  9.591132621921372e-09
dw error:  5.802391137330214e-09
db error:  1.0146343411762047e-09
```

[13]:
```
from dl.layer_utils import conv_relu_forward, conv_relu_backward
np.random.seed(231)
x = np.random.randn(2, 3, 8, 8)
w = np.random.randn(3, 3, 3, 3)
b = np.random.randn(3,)
dout = np.random.randn(2, 3, 8, 8)
conv_param = {'stride': 1, 'pad': 1}

out, cache = conv_relu_forward(x, w, b, conv_param)
dx, dw, db = conv_relu_backward(dout, cache)

dx_num = eval_numerical_gradient_array(lambda x: conv_relu_forward(x, w, b,␣
 ↪conv_param)[0], x, dout)
dw_num = eval_numerical_gradient_array(lambda w: conv_relu_forward(x, w, b,␣
 ↪conv_param)[0], w, dout)
db_num = eval_numerical_gradient_array(lambda b: conv_relu_forward(x, w, b,␣
 ↪conv_param)[0], b, dout)

# Relative errors should be around e-8 or less
print('Testing conv_relu:')
print('dx error: ', rel_error(dx_num, dx))
print('dw error: ', rel_error(dw_num, dw))
print('db error: ', rel_error(db_num, db))
```

```
Testing conv_relu:
dx error:  1.5218619980349303e-09
dw error:  2.702022646099404e-10
db error:  1.451272393591721e-10
```

8 Three-Layer Convolutional Network

Now that you have implemented all the necessary layers, we can put them together into a simple convolutional network.

Open the file `dl/classifiers/cnn.py` and complete the implementation of the `ThreeLayerConvNet` class. Remember you can use the fast/sandwich layers (already imported for you) in your implementation. Run the following cells to help you debug:

8.1 Sanity Check Loss

After you build a new network, one of the first things you should do is sanity check the loss. When we use the softmax loss, we expect the loss for random weights (and no regularization) to be about `log(C)` for `C` classes. When we add regularization the loss should go up slightly.

[14]:
```
model = ThreeLayerConvNet()

N = 50
X = np.random.randn(N, 3, 32, 32)
y = np.random.randint(10, size=N)

loss, grads = model.loss(X, y)
print('Initial loss (no regularization): ', loss)

model.reg = 0.5
loss, grads = model.loss(X, y)
print('Initial loss (with regularization): ', loss)
```

```
Initial loss (no regularization):  2.302586071243987
Initial loss (with regularization):  2.508255638232932
```

8.2 Gradient Check

After the loss looks reasonable, use numeric gradient checking to make sure that your backward pass is correct. When you use numeric gradient checking you should use a small amount of artifical data and a small number of neurons at each layer. Note: correct implementations may still have relative errors up to the order of e-2.

[15]:
```
num_inputs = 2
input_dim = (3, 16, 16)
reg = 0.0
num_classes = 10
np.random.seed(231)
X = np.random.randn(num_inputs, *input_dim)
y = np.random.randint(num_classes, size=num_inputs)

model = ThreeLayerConvNet(
    num_filters=3,
    filter_size=3,
    input_dim=input_dim,
    hidden_dim=7,
    dtype=np.float64
)
loss, grads = model.loss(X, y)
# Errors should be small, but correct implementations may have
# relative errors up to the order of e-2
for param_name in sorted(grads):
    f = lambda _: model.loss(X, y)[0]
```

```
    param_grad_num = eval_numerical_gradient(f, model.params[param_name],␣
 ↪verbose=False, h=1e-6)
    e = rel_error(param_grad_num, grads[param_name])
    print('%s max relative error: %e' % (param_name, rel_error(param_grad_num,␣
 ↪grads[param_name])))
```

```
W1 max relative error: 3.053965e-04
W2 max relative error: 1.822723e-02
W3 max relative error: 3.422399e-04
b1 max relative error: 3.397321e-06
b2 max relative error: 2.517459e-03
b3 max relative error: 9.711800e-10
```

8.3 Overfit Small Data

A nice trick is to train your model with just a few training samples. You should be able to overfit small datasets, which will result in very high training accuracy and comparatively low validation accuracy.

[16]:
```
np.random.seed(231)

num_train = 100
small_data = {
  'X_train': data['X_train'][:num_train],
  'y_train': data['y_train'][:num_train],
  'X_val': data['X_val'],
  'y_val': data['y_val'],
}

model = ThreeLayerConvNet(weight_scale=1e-2)

solver = Solver(
    model,
    small_data,
    num_epochs=15,
    batch_size=50,
    update_rule='adam',
    optim_config={'learning_rate': 1e-3,},
    verbose=True,
    print_every=1
)
solver.train()
```

```
(Iteration 1 / 30) loss: 2.414060
(Epoch 0 / 15) train acc: 0.200000; val_acc: 0.137000
(Iteration 2 / 30) loss: 3.102925
(Epoch 1 / 15) train acc: 0.140000; val_acc: 0.087000
(Iteration 3 / 30) loss: 2.270331
```

```
(Iteration 4 / 30) loss: 2.096705
(Epoch 2 / 15) train acc: 0.240000; val_acc: 0.094000
(Iteration 5 / 30) loss: 1.838880
(Iteration 6 / 30) loss: 1.934188
(Epoch 3 / 15) train acc: 0.510000; val_acc: 0.173000
(Iteration 7 / 30) loss: 1.827912
(Iteration 8 / 30) loss: 1.639574
(Epoch 4 / 15) train acc: 0.520000; val_acc: 0.188000
(Iteration 9 / 30) loss: 1.330082
(Iteration 10 / 30) loss: 1.756115
(Epoch 5 / 15) train acc: 0.630000; val_acc: 0.167000
(Iteration 11 / 30) loss: 1.024162
(Iteration 12 / 30) loss: 1.041826
(Epoch 6 / 15) train acc: 0.750000; val_acc: 0.229000
(Iteration 13 / 30) loss: 1.142777
(Iteration 14 / 30) loss: 0.835706
(Epoch 7 / 15) train acc: 0.790000; val_acc: 0.247000
(Iteration 15 / 30) loss: 0.587786
(Iteration 16 / 30) loss: 0.645509
(Epoch 8 / 15) train acc: 0.820000; val_acc: 0.252000
(Iteration 17 / 30) loss: 0.786844
(Iteration 18 / 30) loss: 0.467054
(Epoch 9 / 15) train acc: 0.820000; val_acc: 0.178000
(Iteration 19 / 30) loss: 0.429880
(Iteration 20 / 30) loss: 0.635498
(Epoch 10 / 15) train acc: 0.900000; val_acc: 0.206000
(Iteration 21 / 30) loss: 0.365807
(Iteration 22 / 30) loss: 0.284220
(Epoch 11 / 15) train acc: 0.820000; val_acc: 0.201000
(Iteration 23 / 30) loss: 0.469343
(Iteration 24 / 30) loss: 0.509369
(Epoch 12 / 15) train acc: 0.920000; val_acc: 0.211000
(Iteration 25 / 30) loss: 0.111638
(Iteration 26 / 30) loss: 0.145389
(Epoch 13 / 15) train acc: 0.930000; val_acc: 0.213000
(Iteration 27 / 30) loss: 0.155576
(Iteration 28 / 30) loss: 0.143400
(Epoch 14 / 15) train acc: 0.960000; val_acc: 0.212000
(Iteration 29 / 30) loss: 0.158156
(Iteration 30 / 30) loss: 0.118937
(Epoch 15 / 15) train acc: 0.990000; val_acc: 0.220000
```

[17]:
```python
# Print final training accuracy.
print(
    "Small data training accuracy:",
    solver.check_accuracy(small_data['X_train'], small_data['y_train'])
)
```

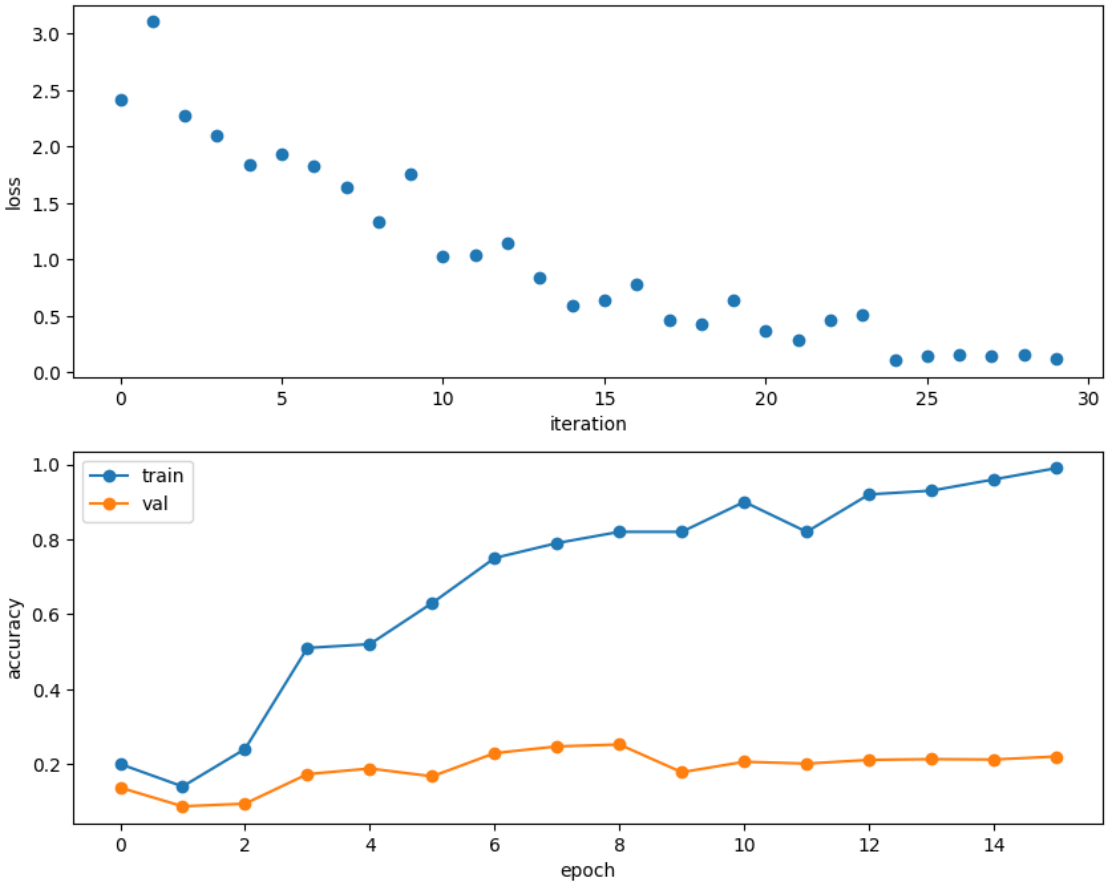
```
Small data training accuracy: 0.82
```

[18]:
```
# Print final validation accuracy.
print(
    "Small data validation accuracy:",
    solver.check_accuracy(small_data['X_val'], small_data['y_val'])
)
```

```
Small data validation accuracy: 0.252
```

Plotting the loss, training accuracy, and validation accuracy should show clear overfitting:

[19]:
```
plt.subplot(2, 1, 1)
plt.plot(solver.loss_history, 'o')
plt.xlabel('iteration')
plt.ylabel('loss')

plt.subplot(2, 1, 2)
plt.plot(solver.train_acc_history, '-o')
plt.plot(solver.val_acc_history, '-o')
plt.legend(['train', 'val'], loc='upper left')
plt.xlabel('epoch')
plt.ylabel('accuracy')
plt.show()
```



8.4 Train the Network

By training the three-layer convolutional network for one epoch, you should achieve greater than 40% accuracy on the training set:

[20]:
```
model = ThreeLayerConvNet(weight_scale=0.001, hidden_dim=500, reg=0.001)

solver = Solver(
    model,
    data,
    num_epochs=1,
    batch_size=50,
    update_rule='adam',
    optim_config={'learning_rate': 1e-3,},
    verbose=True,
    print_every=20
)
solver.train()
```

```
(Iteration 1 / 980) loss: 2.304740
(Epoch 0 / 1) train acc: 0.103000; val_acc: 0.107000
(Iteration 21 / 980) loss: 2.098229
(Iteration 41 / 980) loss: 1.949740
(Iteration 61 / 980) loss: 1.824802
(Iteration 81 / 980) loss: 1.879293
(Iteration 101 / 980) loss: 1.923165
(Iteration 121 / 980) loss: 1.725399
(Iteration 141 / 980) loss: 1.884197
(Iteration 161 / 980) loss: 1.935079
(Iteration 181 / 980) loss: 1.784737
(Iteration 201 / 980) loss: 1.908147
(Iteration 221 / 980) loss: 1.885975
(Iteration 241 / 980) loss: 1.573188
(Iteration 261 / 980) loss: 1.732478
(Iteration 281 / 980) loss: 1.817697
(Iteration 301 / 980) loss: 1.752375
(Iteration 321 / 980) loss: 1.832898
(Iteration 341 / 980) loss: 1.564610
(Iteration 361 / 980) loss: 1.866280
(Iteration 381 / 980) loss: 1.356685
(Iteration 401 / 980) loss: 1.876740
(Iteration 421 / 980) loss: 1.553664
(Iteration 441 / 980) loss: 1.646373
(Iteration 461 / 980) loss: 1.794048
(Iteration 481 / 980) loss: 1.652758
(Iteration 501 / 980) loss: 1.687621
(Iteration 521 / 980) loss: 1.722508
(Iteration 541 / 980) loss: 1.745398
(Iteration 561 / 980) loss: 1.624082
(Iteration 581 / 980) loss: 1.203774
(Iteration 601 / 980) loss: 1.654945
(Iteration 621 / 980) loss: 1.525178
(Iteration 641 / 980) loss: 1.579597
(Iteration 661 / 980) loss: 1.760286
(Iteration 681 / 980) loss: 1.653154
(Iteration 701 / 980) loss: 1.520100
(Iteration 721 / 980) loss: 1.524231
(Iteration 741 / 980) loss: 1.609275
(Iteration 761 / 980) loss: 1.685576
(Iteration 781 / 980) loss: 1.866236
(Iteration 801 / 980) loss: 1.682262
(Iteration 821 / 980) loss: 1.857055
(Iteration 841 / 980) loss: 1.556042
(Iteration 861 / 980) loss: 1.646650
(Iteration 881 / 980) loss: 1.657959
(Iteration 901 / 980) loss: 1.423653
(Iteration 921 / 980) loss: 1.588974
```

```
(Iteration 941 / 980) loss: 1.613119
(Iteration 961 / 980) loss: 1.616299
(Epoch 1 / 1) train acc: 0.496000; val_acc: 0.489000
```

[22]:
```
# Print final training accuracy.
print(
    "Full data training accuracy:",
    solver.check_accuracy(data['X_train'], data['y_train'])
)
```

```
Full data training accuracy: 0.4820204081632653
```

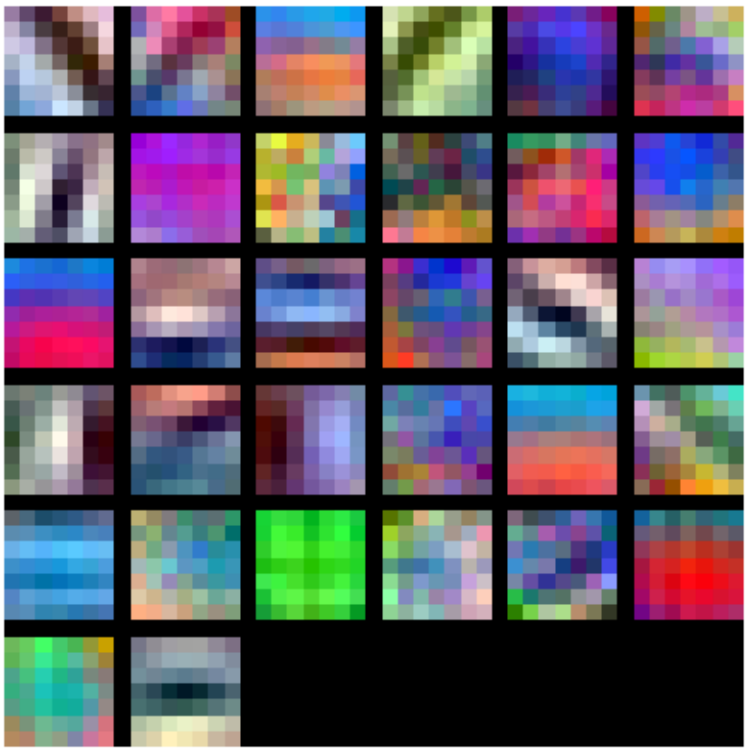
[24]:
```
# Print final validation accuracy.
print(
    "Full data validation accuracy:",
    solver.check_accuracy(data['X_val'], data['y_val'])
)
```

```
Full data validation accuracy: 0.489
```

8.5 Visualize Filters

You can visualize the first-layer convolutional filters from the trained network by running the following:

[23]:
```
from dl.vis_utils import visualize_grid

grid = visualize_grid(model.params['W1'].transpose(0, 2, 3, 1))
plt.imshow(grid.astype('uint8'))
plt.axis('off')
plt.gcf().set_size_inches(5, 5)
plt.show()
```



9 Spatial Batch Normalization

We already saw that batch normalization is a very useful technique for training deep fully connected networks. As proposed in the original paper (link in `BatchNormalization.ipynb`), batch normalization can also be used for convolutional networks, but we need to tweak it a bit; the modification will be called "spatial batch normalization."

Normally, batch-normalization accepts inputs of shape `(N, D)` and produces outputs of shape `(N, D)`, where we normalize across the minibatch dimension `N`. For data coming from convolutional layers, batch normalization needs to accept inputs of shape `(N, C, H, W)` and produce outputs of shape `(N, C, H, W)` where the `N` dimension gives the minibatch size and the `(H, W)` dimensions give the spatial size of the feature map.

If the feature map was produced using convolutions, then we expect every feature channel's statistics e.g. mean, variance to be relatively consistent both between different images, and different locations within the same image – after all, every feature channel is produced by the same convolutional filter! Therefore, spatial batch normalization computes a mean and variance for each of the `C` feature channels by computing statistics over the minibatch dimension `N` as well the spatial dimensions `H` and `W`.

[1] Sergey Ioffe and Christian Szegedy, "Batch Normalization: Accelerating Deep Network Training by Reducing Internal Covariate Shift", ICML 2015.

10 Spatial Batch Normalization: Forward Pass

In the file `dl/layers.py`, implement the forward pass for spatial batch normalization in the function `spatial_batchnorm_forward`. Check your implementation by running the following:

[9]:
```
np.random.seed(231)

# Check the training-time forward pass by checking means and variances
# of features both before and after spatial batch normalization.
N, C, H, W = 2, 3, 4, 5
x = 4 * np.random.randn(N, C, H, W) + 10

print('Before spatial batch normalization:')
print('  shape: ', x.shape)
print('  means: ', x.mean(axis=(0, 2, 3)))
print('  stds: ', x.std(axis=(0, 2, 3)))

# Means should be close to zero and stds close to one
gamma, beta = np.ones(C), np.zeros(C)
bn_param = {'mode': 'train'}
out, _ = spatial_batchnorm_forward(x, gamma, beta, bn_param)
print('After spatial batch normalization:')
print('  shape: ', out.shape)
print('  means: ', out.mean(axis=(0, 2, 3)))
print('  stds: ', out.std(axis=(0, 2, 3)))

# Means should be close to beta and stds close to gamma
gamma, beta = np.asarray([3, 4, 5]), np.asarray([6, 7, 8])
out, _ = spatial_batchnorm_forward(x, gamma, beta, bn_param)
print('After spatial batch normalization (nontrivial gamma, beta):')
print('  shape: ', out.shape)
print('  means: ', out.mean(axis=(0, 2, 3)))
print('  stds: ', out.std(axis=(0, 2, 3)))
```

```
Before spatial batch normalization:
  shape:  (2, 3, 4, 5)
  means:  [9.33463814 8.90909116 9.11056338]
  stds:  [3.61447857 3.19347686 3.5168142 ]
After spatial batch normalization:
  shape:  (2, 3, 4, 5)
  means:  [ 6.18949336e-16  5.99520433e-16 -1.22124533e-16]
  stds:  [0.99999962 0.99999951 0.9999996 ]
After spatial batch normalization (nontrivial gamma, beta):
  shape:  (2, 3, 4, 5)
  means:  [6. 7. 8.]
  stds:  [2.99999885 3.99999804 4.99999798]
```

[10]:
```
np.random.seed(231)

# Check the test-time forward pass by running the training-time
# forward pass many times to warm up the running averages, and then
# checking the means and variances of activations after a test-time
# forward pass.
N, C, H, W = 10, 4, 11, 12

bn_param = {'mode': 'train'}
gamma = np.ones(C)
beta = np.zeros(C)
for t in range(50):
  x = 2.3 * np.random.randn(N, C, H, W) + 13
  spatial_batchnorm_forward(x, gamma, beta, bn_param)
bn_param['mode'] = 'test'
x = 2.3 * np.random.randn(N, C, H, W) + 13
a_norm, _ = spatial_batchnorm_forward(x, gamma, beta, bn_param)

# Means should be close to zero and stds close to one, but will be
# noisier than training-time forward passes.
print('After spatial batch normalization (test-time):')
print('  means: ', a_norm.mean(axis=(0, 2, 3)))
print('  stds: ', a_norm.std(axis=(0, 2, 3)))
```

```
After spatial batch normalization (test-time):
  means:  [-0.08034406  0.07562881  0.05716371  0.04378383]
  stds:  [0.96718744 1.0299714  1.02887624 1.00585577]
```

11 Spatial Batch Normalization: Backward Pass

In the file `dl/layers.py`, implement the backward pass for spatial batch normalization in the function `spatial_batchnorm_backward`. Run the following to check your implementation using a numeric gradient check:

[4]:
```
np.random.seed(231)
N, C, H, W = 2, 3, 4, 5
x = 5 * np.random.randn(N, C, H, W) + 12
gamma = np.random.randn(C)
beta = np.random.randn(C)
dout = np.random.randn(N, C, H, W)

bn_param = {'mode': 'train'}
fx = lambda x: spatial_batchnorm_forward(x, gamma, beta, bn_param)[0]
fg = lambda a: spatial_batchnorm_forward(x, gamma, beta, bn_param)[0]
fb = lambda b: spatial_batchnorm_forward(x, gamma, beta, bn_param)[0]

dx_num = eval_numerical_gradient_array(fx, x, dout)
```

```
da_num = eval_numerical_gradient_array(fg, gamma, dout)
db_num = eval_numerical_gradient_array(fb, beta, dout)

#You should expect errors of magnitudes between 1e-12~1e-06
_, cache = spatial_batchnorm_forward(x, gamma, beta, bn_param)
dx, dgamma, dbeta = spatial_batchnorm_backward(dout, cache)
print('dx error: ', rel_error(dx_num, dx))
print('dgamma error: ', rel_error(da_num, dgamma))
print('dbeta error: ', rel_error(db_num, dbeta))
```

```
dx error:  2.786648193872555e-07
dgamma error:  7.0974817113608705e-12
dbeta error:  3.275608725278405e-12
```

12 Spatial Group Normalization

In the previous notebook, we mentioned that Layer Normalization is an alternative normalization technique that mitigates the batch size limitations of Batch Normalization. However, as the authors of [2] observed, Layer Normalization does not perform as well as Batch Normalization when used with Convolutional Layers:

> With fully connected layers, all the hidden units in a layer tend to make similar contributions to the final prediction, and re-centering and rescaling the summed inputs to a layer works well. However, the assumption of similar contributions is no longer true for convolutional neural networks. The large number of the hidden units whose receptive fields lie near the boundary of the image are rarely turned on and thus have very different statistics from the rest of the hidden units within the same layer.

The authors of [3] propose an intermediary technique. In contrast to Layer Normalization, where you normalize over the entire feature per-datapoint, they suggest a consistent splitting of each per-datapoint feature into G groups and a per-group per-datapoint normalization instead.

Visual comparison of the normalization techniques discussed so far (image edited from [3])

Even though an assumption of equal contribution is still being made within each group, the authors hypothesize that this is not as problematic, as innate grouping arises within features for visual recognition. One example they use to illustrate this is that many high-performance handcrafted features in traditional computer vision have terms that are explicitly grouped together. Take for example Histogram of Oriented Gradients [4] – after computing histograms per spatially local block, each per-block histogram is normalized before being concatenated together to form the final feature vector.

You will now implement Group Normalization.

[2] Ba, Jimmy Lei, Jamie Ryan Kiros, and Geoffrey E. Hinton. "Layer Normalization." stat 1050 (2016): 21.

[3] Wu, Yuxin, and Kaiming He. "Group Normalization." arXiv preprint arXiv:1803.08494 (2018).

[4] N. Dalal and B. Triggs. Histograms of oriented gradients for human detection. In Computer Vision and Pattern Recognition (CVPR), 2005.

13 Spatial Group Normalization: Forward Pass

In the file `dl/layers.py`, implement the forward pass for group normalization in the function `spatial_groupnorm_forward`. Check your implementation by running the following:

[5]:
```
np.random.seed(231)

# Check the training-time forward pass by checking means and variances
# of features both before and after spatial batch normalization.
N, C, H, W = 2, 6, 4, 5
G = 2
x = 4 * np.random.randn(N, C, H, W) + 10
x_g = x.reshape((N*G,-1))
print('Before spatial group normalization:')
print('  shape: ', x.shape)
print('  means: ', x_g.mean(axis=1))
print('  stds: ', x_g.std(axis=1))

# Means should be close to zero and stds close to one
gamma, beta = np.ones((1,C,1,1)), np.zeros((1,C,1,1))
bn_param = {'mode': 'train'}

out, _ = spatial_groupnorm_forward(x, gamma, beta, G, bn_param)
out_g = out.reshape((N*G,-1))
print('After spatial group normalization:')
print('  shape: ', out.shape)
print('  means: ', out_g.mean(axis=1))
print('  stds: ', out_g.std(axis=1))
```

```
Before spatial group normalization:
  shape:  (2, 6, 4, 5)
  means:  [9.72505327 8.51114185 8.9147544  9.43448077]
  stds:  [3.67070958 3.09892597 4.27043622 3.97521327]
After spatial group normalization:
  shape:  (2, 6, 4, 5)
  means:  [-2.14643118e-16  5.25505565e-16  2.65528340e-16 -3.38618023e-16]
  stds:  [0.99999963 0.99999948 0.99999973 0.99999968]
```

14 Spatial Group Normalization: Backward Pass

In the file `dl/layers.py`, implement the backward pass for spatial batch normalization in the function `spatial_groupnorm_backward`. Run the following to check your implementation using a numeric gradient check:

[6]:
```
np.random.seed(231)
N, C, H, W = 2, 6, 4, 5
G = 2
x = 5 * np.random.randn(N, C, H, W) + 12
```

```
gamma = np.random.randn(1,C,1,1)
beta = np.random.randn(1,C,1,1)
dout = np.random.randn(N, C, H, W)

gn_param = {}
fx = lambda x: spatial_groupnorm_forward(x, gamma, beta, G, gn_param)[0]
fg = lambda a: spatial_groupnorm_forward(x, gamma, beta, G, gn_param)[0]
fb = lambda b: spatial_groupnorm_forward(x, gamma, beta, G, gn_param)[0]

dx_num = eval_numerical_gradient_array(fx, x, dout)
da_num = eval_numerical_gradient_array(fg, gamma, dout)
db_num = eval_numerical_gradient_array(fb, beta, dout)

_, cache = spatial_groupnorm_forward(x, gamma, beta, G, gn_param)
dx, dgamma, dbeta = spatial_groupnorm_backward(dout, cache)

# You should expect errors of magnitudes between 1e-12 and 1e-07.
print('dx error: ', rel_error(dx_num, dx))
print('dgamma error: ', rel_error(da_num, dgamma))
print('dbeta error: ', rel_error(db_num, dbeta))
```

```
dx error:  7.413109332145332e-08
dgamma error:  9.468195772749234e-12
dbeta error:  3.354494437653335e-12
```

[ ]: