Derivation of the equations

1 Ch.1

2 Ch.2

2.1 2.2 Quantization of field

$$\vec{\nabla} \cdot \vec{A} = 0 \tag{1}$$

$$\nabla^2 \vec{A} - \frac{1}{c^2} \frac{\partial^2 \vec{A}}{\partial t^2} \tag{2}$$

$$\vec{E} = -\frac{\partial \vec{A}}{\partial t}, \vec{B} = \vec{\nabla} \times \vec{A} \tag{3}$$

fourier series expansion of A:

$$\vec{A}(\vec{r}, t) = \sum_k (\vec{A_k}(t) \exp^{i\vec{k}\cdot\vec{r}} + \vec{A_k}^*(t) \exp^{-i\vec{k}\cdot\vec{r}}) \tag{4}$$

$$k_x = \frac{2\pi n_x}{L}, k_y = \frac{2\pi n_y}{L}, k_z = \frac{2\pi n_z}{L} \tag{5}$$

$$\vec{k} \cdot \vec{A_k} = \vec{k} \cdot \vec{A_k}^* = 0 \tag{6}$$

substitute eq(2.4) with (2.2)

$$\nabla^2 = (\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2})$$

$$\sum_k (\vec{A_k}(t) (\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}) \exp^{i(k_x x + k_y y + k_z z)} + c.c)$$

$$\sum_k (-k^2 \vec{A_k}(t) \exp^{i\vec{k}\cdot\vec{r}} + - k^2 \vec{A_k}^*(t) \exp^{-i\vec{k}\cdot\vec{r}})$$

for single k,

$$k^2 \vec{A_k}(t) + \frac{1}{c^2} \frac{\partial^2 \vec{A_k}(t)}{\partial t^2} = 0 \tag{7}$$

$$\vec{A_k} = \vec{A_k} \exp^{-i\omega_k t}, \omega_k = ck \tag{8}$$

$$\vec{A}(\vec{r}, t) = \sum_k (\vec{A_k} \exp^{-i\omega_k t} \exp^{i\vec{k}\cdot\vec{r}} + \vec{A_k}^* \exp^{-i\vec{k}\cdot\vec{r}} \exp^{i\omega_k t}) \tag{9}$$