

$$E_j(t) = E_0 e^{-i\omega t + i\phi_j(t)} \quad (1)$$

$$E(t) = \sum_{j=1}^N E_j(t) = E_0 e^{-i\omega t} (e^{i\phi_1(t)} + e^{i\phi_2(t)} + e^{i\phi_3(t)} \dots) \quad (2)$$

$$= E_0 e^{i\omega t} r(t) e^{i\phi(t)} \\ I(t) = \epsilon_0 c [Re E(t)]^2 \quad (3)$$

$$\bar{I}(t) = \frac{1}{T} \int_0^T I(t) dt = \frac{1}{2} \epsilon_0 c |E(t)|^2 \quad (4)$$

$$\bar{I}(t) = \frac{1}{2} \epsilon_0 c E_0^2 r(t)^2 \quad (5)$$

$$p(\bar{I}(t)) d\bar{I}(t) = \frac{1}{\langle I \rangle} e^{\frac{\bar{I}(t)}{\langle I \rangle}} d\bar{I}(t) \quad (6)$$

$$\langle I \rangle = \frac{1}{\tau} \int_0^\tau \bar{I}(t) dt = \frac{1}{2} \epsilon_0 c E_0^2 \frac{1}{\tau} \int_0^\tau r^2(t) dt \quad (7)$$

$$\frac{1}{\tau} \int_0^\tau r(t)^2 dt = \frac{1}{\tau} \int_0^\tau \left| e^{i\phi_1(t)} + e^{i\phi_2(t)} + e^{i\phi_3(t)} \dots + e^{i\phi_N(t)} \right|^2 dt \approx N \quad (8)$$

$$\langle I \rangle \approx \frac{1}{2} \epsilon_0 c E_0^2 n$$

$$\langle n \rangle = Tr\{\rho a^\dagger a\} = \sum_n P_n n = \frac{1}{e^{\frac{\hbar\omega}{kT}} - 1} \quad (9)$$

$$P_n = \frac{\langle n \rangle^n}{(1 + \langle n \rangle)^{1+n}} \quad (10)$$

$$\rho = \sum_n P_n |n\rangle \langle n| = \sum_n \frac{\langle n \rangle^n}{(1 + \langle n \rangle)^{1+n}} |n\rangle \langle n| \quad (11)$$

$$(\Delta n)^2 = \langle (n - \langle n \rangle)^2 \rangle = \langle n^2 \rangle - \langle n \rangle^2 = \frac{e^{\frac{\hbar\omega}{kT}}}{(e^{\frac{\hbar\omega}{kT}} - 1)^2} \quad (12)$$

$$(\Delta n)^2 = \langle n \rangle + \langle n \rangle^2 \quad (13)$$

$$\Delta n \approx \langle n \rangle + \frac{1}{2} \approx \langle n \rangle \quad (14)$$

$$0 \quad (15)$$

$$0 \quad (16)$$

$$\langle I(\vec{r}_1, t_1) I(\vec{r}_2, t_2) \rangle = \left\langle I(\vec{r}, t_1 - \frac{|\vec{r}_1 - \vec{r}|}{c}) I(\vec{r}, t_2 - \frac{|\vec{r}_2 - \vec{r}|}{c}) \right\rangle \quad (17) \\ = \langle I(\vec{r}, t) I(\vec{r}, t + \tau) \rangle$$

$$\tau = t_2 - t_1 - \frac{|\vec{r}_2 - \vec{r}| - |\vec{r}_1 - \vec{r}|}{c} \quad (18)$$

$$i_{12}(\tau) \equiv \frac{\langle I(\vec{r}, t_1) I(\vec{r}_2, t_2) \rangle}{\langle I(\vec{r}_1, t_1) \rangle \langle I(\vec{r}_2, t_2) \rangle} = \frac{\langle I(\vec{r}, t) I(\vec{r}, t + \tau) \rangle}{\langle I(\vec{r}, t) \rangle^2} \quad (19)$$

$$i_{12}(\tau|\infty) = 1 \quad (20)$$

$$i_{12}(\tau = 0) = \frac{\langle I^2 \vec{r}, t \rangle}{\langle I(\vec{r}, t) \rangle^2} \quad (21)$$

$$i_{12}(\tau = 0) \geq 1 \quad (22)$$

$$\langle I(\vec{r}_1, t_1) I(\vec{r}_2, t_2) \rangle = (\frac{1}{2}\epsilon_0 c)^2 \langle E^*(\vec{r}_1, t_1) E(\vec{r}_1, t_1) E^*(\vec{r}_2, t_2) E(\vec{r}_2, t_2) \rangle \quad (23)$$

$$\begin{aligned} &= (\frac{1}{2}\epsilon_0 c)^2 \langle E^*(\vec{r}_1, t_1) E^*(\vec{r}_2, t_2) E(\vec{r}_2, t_2) E(\vec{r}_1, t_1) \rangle \\ i_{12}(\tau) &= \frac{\langle E^*(\vec{r}_1, t_1) E^*(\vec{r}_2, t_2) E(\vec{r}_2, t_2) E(\vec{r}_1, t_1) \rangle}{\langle |E(\vec{r}_1, t_1)|^2 \rangle \langle |E(\vec{r}_2, t_2)|^2 \rangle} \\ &= \frac{\langle E^*(\vec{r}, t) E^*(\vec{r}, t + \tau) E(\vec{r}, t + \tau) E(\vec{r}, t) \rangle}{\langle |E(\vec{r}_1, t_1)|^2 \rangle^2} \end{aligned} \quad (24)$$

$$\begin{aligned} P &\propto \sum_f \left| \langle f | E^{(+)}(\vec{r}_2, t_2) E^{(+)}(\vec{r}_2, t_2) | i \rangle \right|^2 \\ &= \sum_f \langle i | E^{(-)}(\vec{r}_1, t_1) E^{(-)}(\vec{r}_2, t_2) | f \rangle \langle f | E^{(+)}(\vec{r}_2, t_2) E^{(+)}(\vec{r}_1, t_1) | i \rangle \end{aligned} \quad (25)$$

$$\begin{aligned} &= \langle i | E^{(-)}(x_1) E^{(-)}(x_2) E^{(+)}(x_3) E^{(+)}(x_4) | i \rangle \\ G^{(2)} &= (x_1, x_2, x_3, x_4) = \langle E^{(-)}(x_1) E^{(-)}(x_2) E^{(+)}(x_3) E^{(+)}(x_4) \rangle \\ &= Tr\{\rho E^{(-)}(x_1) E^{(-)}(x_2) E^{(+)}(x_3) E^{(+)}(x_4)\} \end{aligned} \quad (26)$$

$$g^{(2)} = \frac{G^{(2)}(x_1, x_2, x_3, x_4)}{\sqrt{G^{(1)}(x_1, x_1) G^{(1)}(x_2, x_2) G^{(3)}(x_3, x_3) G^{(4)}(x_4, x_4)}} \quad (27)$$

$$\begin{aligned} G^{(2)}(x_1, x_2, x_3, x_4) &= \langle \alpha | E^{(-)}(x_1) E^{(-)}(x_2) E^{(+)}(x_2) E^{(+)}(x_1) | \alpha \rangle \\ &= (\frac{\hbar\omega}{2\epsilon_0 V})^2 |\alpha|^4 \end{aligned} \quad (28)$$

$$g^{(2)}(x_1, x_2, x_2, x_1) = g^{(2)}(\tau) = \frac{G^{(2)}(x_1, x_2, x_2, x_1)}{G^{(1)}(x_1, x_1) G^{(1)}(x_2, x_2)} = 1 \quad (29)$$

$$\left| g^{(2)}(x_1, x_2, x_3, x_4) \right| = 1 \quad (30)$$

$$g^{(2)}(x_1, x_2, x_3, x_4) = g^{(2)}(\tau) = 1 \quad (31)$$

$$G^{(2)}(x_1, x_2, x_2, x_1) = G^{(1)}(x_1, x_1) G^{(1)}(x_2, x_2) \quad (32)$$

$$Q = \frac{(\Delta n)^2 - \langle n \rangle}{\langle n \rangle} \quad (33)$$

$$Q = \langle n \rangle (g^{(2)}(\tau) - 1) \quad (34)$$

if  $g^{(2)} < 1$ , anti-bunching

coherent light (Poissonian) :  $[(\Delta n)^2 - \langle n \rangle] = 0$ , super-Poissonian :  $[(\Delta n)^2 - \langle n \rangle] > 0$

sub-Poissonian :  $[(\Delta n)^2 - \langle n \rangle] < 0$

$$g^{(2)}(x_1, x_2, x_2, x_1) = \frac{\langle E^*(x_1)E^*(x_2)E(x_2)E(x_1) \rangle}{\langle E^{(-)}(x_1)E^{(+)}(x_1) \rangle \langle E^{(-)}(x_2)E^{(+)}(x_2) \rangle} \quad (35)$$

$$g^{(2)}(x_1, x_2, x_2, x_1) = \frac{\langle a^\dagger a^\dagger a a \rangle}{\langle a^\dagger a \rangle^2} \quad (36)$$

$$= \frac{\langle a^\dagger a a^\dagger a \rangle - \langle a^\dagger a \rangle^2}{\langle a^\dagger a \rangle^2} = \frac{\langle n^2 \rangle - \langle n \rangle^2}{\langle n \rangle^2}$$

$$g^{(2)}(x_1, x_2, x_2, x_1) = g^{(2)}(\tau) = 1 + \frac{(\Delta n)^2 - \langle n \rangle}{\langle n \rangle^2} \quad (37)$$

$$0 \quad (38)$$

$$0 \quad (39)$$

$$g^{(n)}(x_1, x_2, \dots, x_{2n}) = \frac{G^{(n)}(x_1, x_2, \dots, x_{2n})}{\sqrt{\prod_{j=1}^{2n} G^{(1)}(x_j, x_j)}} \quad (40)$$

$$= \frac{E^{(-)}(x_1) \dots E^{(-)}(x_n) E^{(+)}(x_{n+1}) \dots E^{(+)}(x_{2n})}{\sqrt{\prod_{j=1}^{2n} \langle E^{(-)}(x_j) E^{(+)}(x_j) \rangle}}$$

$$\left| g^{(n)}(x_1, x_2, \dots, x_{2n}) \right| = 1 \quad (41)$$

$$\left| g^{(j)}(x_1, x_2, \dots, x_{2j}) \right| = 1 \quad (42)$$

$$g^{(n)}(x_1, \dots, x_n, x_n, \dots, x_1) = 1 \quad (43)$$

$$G^{(n)}(x_1, \dots, x_n, x_n, \dots, x_1) = \prod_{j=1}^n G^{(1)}(x_j, x_j) \quad (44)$$