1 Quantum theory of coherent states

$$\vec{E}(\vec{r},t) = i\sum_k \epsilon_{\vec{k}}\sqrt{\frac{\hbar\omega_k}{2\epsilon_0 V}}(\hat{a}_k e^{-i\omega_k t+i\vec{k}\cdot\vec{r}} + \hat{a}_k^\dagger e^{-i\vec{k}\cdot\vec{r}+i\omega_k t})$$

$$\hat{E}^{(+)}(\vec{r},t) = i\vec{\epsilon}\sqrt{\frac{\hbar\omega}{2\epsilon_0 V}}(\hat{a}e^{-i\omega t+i\vec{k}\cdot\vec{r}})$$

$$\hat{E}^{(-)}(\vec{r},t) = i\vec{\epsilon}\sqrt{\frac{\hbar\omega}{2\epsilon_0 V}}(\hat{a}e^{-i\vec{k}\cdot\vec{r}+i\omega t+})$$

$$\hat{\vec{E}}(\vec{r},t) = \hat{E}^{(+)} + \hat{E}^{(-)}$$

$$\begin{aligned} P \propto \sum_f \left|\langle f|\,\hat{E}^{(+)}(\vec{r},t)\,|i\rangle\right|^2 &= \sum_f \langle i|\,\hat{E}^{(-)}(\vec{r},t)\,|f\rangle\,\langle f|\,\hat{E}^{(+)}(\vec{r},t)\,|i\rangle \\ &= \langle i|\,\hat{E}^{(-)}(\vec{r},t)\hat{E}^{(+)}(\vec{r},t)\,|i\rangle \\ &= Tr\{\rho\hat{E}^{(-)}(\vec{r},t)\hat{E}^{(+)}(\vec{r},t)\} \end{aligned}$$

first-order coherence function $G^{(1)}$:

$$G^{(1)}(x_1,x_2) = \left\langle E^{(-)}(x_1)E^{(+)}(x_1)\right\rangle$$

$$(x_1,x_2) = ((\vec{r}_1,t_1),(\vec{r}_2,t_2))$$