1 Quantum Coherence

1.1 Quantum theory of coherent states

$$\vec{E}(\vec{r},t) = i\sum_k \vec{\epsilon_k}\sqrt{\frac{\hbar\omega_k}{2\epsilon_0 V}}(\hat{a}_k e^{-i\omega_k t + i\vec{k}\cdot\vec{r}} + \hat{a}_k^\dagger e^{-i\vec{k}\cdot\vec{r}+i\omega_k t})$$

$$\hat{E}^{(+)}(\vec{r},t) = i\vec{\epsilon}\sqrt{\frac{\hbar\omega}{2\epsilon_0 V}}(\hat{a}e^{-i\omega t + i\vec{k}\cdot\vec{r}})$$

$$\hat{E}^{(-)}(\vec{r},t) = i\vec{\epsilon}\sqrt{\frac{\hbar\omega}{2\epsilon_0 V}}(\hat{a}e^{-i\vec{k}\cdot\vec{r}+i\omega t+})$$

$$\hat{\vec{E}}(\vec{r},t) = \hat{E}^{(+)} + \hat{E}^{(-)}$$

$$\begin{aligned} P \propto \sum_f \left|\langle f|\,\hat{E}^{(+)}(\vec{r},t)\,|i\rangle\right|^2 &= \sum_f \langle i|\,\hat{E}^{(-)}(\vec{r},t)\,|f\rangle\,\langle f|\,\hat{E}^{(+)}(\vec{r},t)\,|i\rangle \\ &= \langle i|\,\hat{E}^{(-)}(\vec{r},t)\hat{E}^{(+)}(\vec{r},t)\,|i\rangle \\ &= Tr\{\rho\hat{E}^{(-)}(\vec{r},t)\hat{E}^{(+)}(\vec{r},t)\} \end{aligned}$$

first-order coherence function $G^{(1)}$:

$$G^{(1)}(x_1,x_2) = \left\langle E^{(-)}(x_1)E^{(+)}(x_1)\right\rangle$$

$$(x_1,x_2) = ((\vec{r}_1,t_1),(\vec{r}_2,t_2))$$

normalized first-order coherence function:

$$g^{(1)} = \frac{G^{(1)}(x_1,x_2)}{\sqrt{G^{(1)}(x_1,x_1)G^{(2)}(x_2,x_2)}}$$

$g^{(1)} \approx 1$: 1st order coherence
$g^{(1)} \leq 1$: Visibility low

1.2 Coherent state

The coherent state is a Eigenstate of the photon annihilation operator , $\hat{E}^{(+)}$ for single mode,

$$\hat{a}\,|\alpha\rangle = \alpha\,|\alpha\rangle$$

$$|\alpha\rangle = \sum_n |n\rangle\,\langle n|\alpha\rangle = \sum_n c_n\,|n\rangle$$

Using $\langle n|\,\hat{a} = \sqrt{n+1}n \hat{+} 1$,

$$\langle n|\,\hat{a}\,|\alpha\rangle = \sqrt{n+1}\,\langle n+1|\alpha\rangle = \alpha\,\langle n|n\rangle$$

$$\therefore \langle n+1|\alpha\rangle = \frac{\alpha}{\sqrt{n+1}}\,\langle n|\alpha\rangle\,, \langle n|\alpha\rangle = \frac{\alpha^n}{n!}\,\langle 0|\alpha\rangle$$

$$|\alpha\rangle = \langle 0|\alpha\rangle \sum_n \frac{\alpha^n}{\sqrt{n!}} |n\rangle$$

$$1 = \langle\alpha|\alpha\rangle = |\langle 0|\alpha\rangle|^2 \sum_n \frac{\alpha^{2n}}{\sqrt{n!}} |n\rangle = |\langle 0|\alpha\rangle|^2 e^{|\alpha|^2}$$

neglecting phase affection,

$$\langle 0|\alpha\rangle = e^{-\frac{|\alpha|^2}{2}}$$

The definition of coherent state is:

$$\therefore |\alpha\rangle = e^{-\frac{|\alpha|^2}{2}} \sum_n \frac{\alpha^{2n}}{\sqrt{n!}} |n\rangle$$

$$P_n = |\langle n|\alpha\rangle|^2 = |c_n|^2 = e^{-\frac{|\alpha|^2}{2}} \frac{\alpha^{2n}}{\sqrt{n!}} |n\rangle$$

This is an Poissonian distribution (for independent particles)