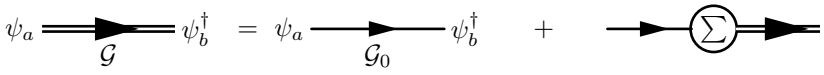


$$Z = \langle \psi_a(\tau)\psi_b^\dagger(\tau')\rangle$$
$$= \mathrm{Tr}[e^{\beta H(\hat{\tau})}\psi_a(\tau)\psi_b^\dagger(\tau')]$$

$$H = H_{loc} + H_{bath} + \mathbf{H_{hyb}} + \mathbf{H_{hyb}^\dagger}$$

$$\mathbf{H_{hyb}} = \sum_i c_\nu \mathbf{V}_i b_i^\dagger$$

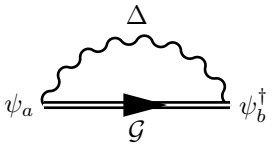
$$\mathcal{G} = Z_{\mathrm{bath}}\mathrm{Tr}_c\Big[\frac{1}{Z_{\mathrm{bath}}}\mathrm{Tr}_{\mathrm{b}}\big[T_\tau e^{-\int_0^\beta d\tau H_{\mathrm{loc}}(\tau)+H_{\mathrm{bath}}(\tau)}\prod_\nu\sum_{k_\nu} Z_{k_\nu}\big]\Big]$$

$$\begin{aligned} Z_{k_\nu} = &\sum_{i_1,\ldots,i_{k_\nu}}\sum_{i'_1,\ldots,i'_{k_\nu}} V^\nu_{i_1}V^{\nu*}_{i'_1}\cdots V^\nu_{i_{k_j}}V^{\nu*}_{i'_{k_j}}\int_0^\beta d\tau \\ &\times\int_{\tau_1}^\beta d\tau_2\cdots\int_{\tau_{k_\nu-1}}^\beta d\tau_{k_\nu}\int_{\tau'_1}^\beta d\tau'_2\cdots\int_{\tau'_{k_\nu-1}}^\beta d\tau'_{k_\nu} \\ &\times c_\nu(\tau_1)b^{\nu^\dagger}_{i_1}(\tau_1)b^\nu_{i'_1}(\tau'_1)c^\dagger_\nu(\tau'_1)c_\nu(\tau_2)b^{\nu^\dagger}_{i_2}(\tau_2)b^\nu_{i'_2}(\tau'_2)c^\dagger_\nu(\tau'_2) \\ &\times\cdots c_\nu(\tau_{k_j})b^{\nu^\dagger}_{i_{k_j}}(\tau_{k_j})b^\nu_{i'_{k_j}}(\tau'_{k_j})c^\dagger_\nu(\tau'_{k_j}) \end{aligned}$$

$$M^{-1} = \begin{pmatrix} \mathcal{G}'_{11} & \mathcal{G}'_{12} & \cdots & \mathcal{G}'_{1j} & \cdots \\ \mathcal{G}'_{21} & \mathcal{G}'_{22} & \cdots & \mathcal{G}'_{2j} & \cdots \\ \vdots & \vdots & \ddots & \vdots & \cdots \\ \mathcal{G}'_{i1} & \mathcal{G}'_{i2} & \cdots & \mathcal{G}'_{ij} & \cdots \\ \vdots & \vdots & \cdots & \cdots & \ddots \end{pmatrix}$$

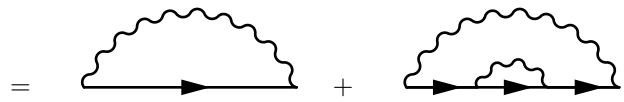
$$\det M^{-1} = \Delta$$

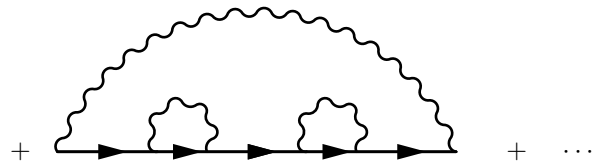
$$\mathcal{G}' = \langle |V^\nu| b^{\dagger\nu}(\tau)b^\nu(\tau')\rangle$$
$$= \begin{cases} |V^a|^2 e^{-\epsilon(\tau-\tau')}\frac{1}{(1+e^{-\beta\epsilon_p})} & (\text{if} \quad \tau > \tau') \\ |V^a|^2 e^{-\epsilon(\tau'-\tau)}\frac{e^{-\beta\epsilon}}{(1+e^{-\beta\epsilon})} & (\text{if} \quad \tau < \tau') \end{cases}$$



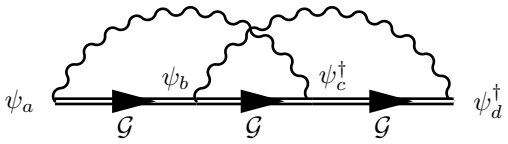
$$\mathcal{G} =$$

$$= \sum_{ab}\Big[(\mathrm{sgn})\psi_a\mathcal{G}(\tau)\psi_b^\dagger\Delta_{ba}(\tau)\Big]$$

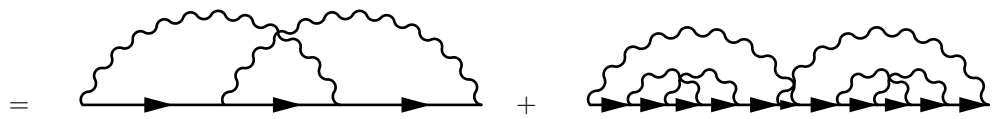




$\mathcal{G}_{\mathrm{OCA}} =$



$$= \sum_{\mathrm{abcd}} \int_0^{\tau} d\tau_1 \int_0^{\tau_1} d\tau_2 \Big[\mathrm{sgn}_1 \psi_d^\dagger \mathcal{G}(\tau - \tau_2) \psi_c^\dagger \mathcal{G}(\tau_2 - \tau_1) \psi_b \mathcal{G}(\tau - \tau_1) \psi_a \Delta_{bd}(\tau - \tau_1) \Delta_{ca}(\tau_2) \Big]$$



$+ \quad \cdots$

$$Z = \mathrm{Tr}[\mathcal{G}(\tau)\lambda_1 \mathcal{G}(0)\lambda_1 \Delta(\tau)]$$

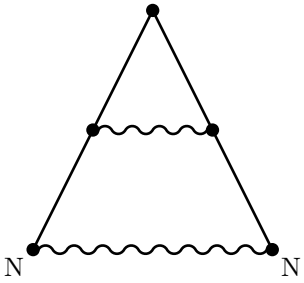
$$\begin{aligned} Z &= Z_{\mathrm{NCA}} + Z_{\mathrm{OCA}} \\ &= \mathrm{Tr}[\mathcal{G}(\tau)\lambda_1 \mathcal{G}(0)\lambda_1 \Delta(\tau)] \\ &\quad + \mathrm{Tr}[\mathcal{G}(\beta - \tau_2)\hat{N}\mathcal{G}(\tau_2 - \tau)\lambda_1 \mathcal{G}(\tau - \tau_1)\hat{N}\mathcal{G}(\tau_1)\lambda_1 \Delta(\tau_2 - \tau_1)] \end{aligned}$$

$$H_{\mathrm{tot}} = H_{\mathrm{loc}} + H_{\mathrm{int}} + H_{\mathrm{bath}}$$

$$H_{\mathrm{loc}} = E_C \hat{N} - E_J \cos\phi$$

$$H_{\mathrm{int}} = \sum_k g_k \hat{N}(b_k^\dagger + b_k)$$

$$H_{\mathrm{bath}} = \sum_k \omega_k b_k^\dagger b_k$$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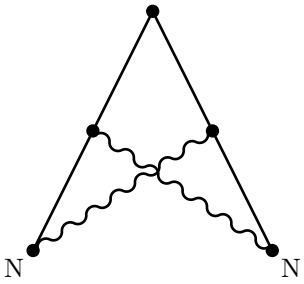
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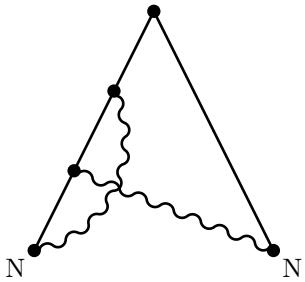
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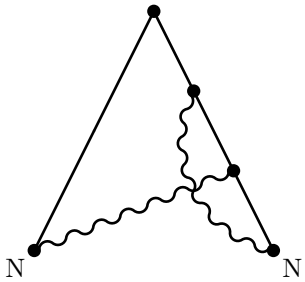
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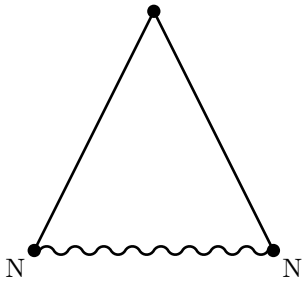
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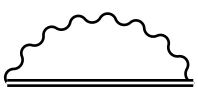
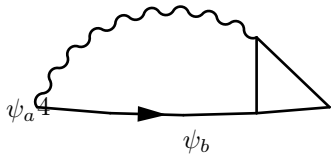


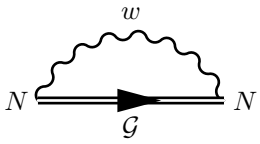


Σ Σ_{OCA} Σ_{TOA}

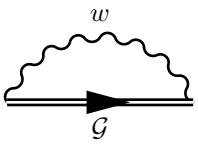
$$\begin{cases} \gamma = \frac{E_J}{E_c} \\ \alpha = \frac{R_Q}{R} \end{cases}$$

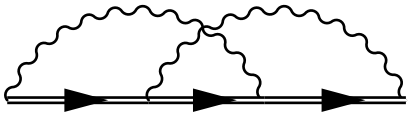
$\omega_k = vk$ α γ

$\mathbf{H}_{\text{hyb}}$ $\mathbf{H}_{\text{hyb}}^{\dagger}$

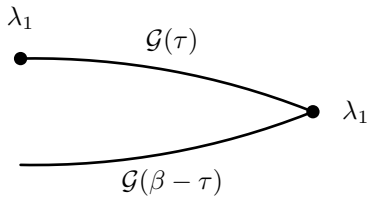


$$= \left[N\mathcal{G}(\tau) N\omega(\tau) \right]$$





$$= \int_0^{\tau} d\tau_1 \int_0^{\tau_1} d\tau_2 \left[N\mathcal{G}(\tau - \tau_2) N\mathcal{G}(\tau_2 - \tau_1) N\mathcal{G}(\tau - \tau_1) N\omega(\tau - \tau_1) \omega(\tau_2) \right]$$



$\chi_{sp}(\tau)$

$$\hat{\lambda}_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$E_c = \frac{2e^2}{C_J}$$

I_c : Josephson current

$\chi_{sp} = \mathrm{Tr}[\mathcal{G}(\beta-\tau)\lambda_1\mathcal{G}(\tau)\lambda_1] + \mathrm{Tr}[\omega\mathcal{G}(\beta-\tau_1)\hat{N}\mathcal{G}(\tau_1-\tau_2)\lambda_1\mathcal{G}(\tau_2-\tau_3)\hat{N}\mathcal{G}(\tau_3)\lambda_1]$

$\mathcal{G}(\tau-\tau_1)$ $\mathcal{G}(\tau_2-\tau)$ $\omega(\tau_2-\tau_1)$ $\mathcal{G}(\beta-\tau_2)$ $\mathcal{G}(\tau_1)$ λ_1

$\psi_a \cdot \mathcal{G}_0$ $\psi_b^\dagger$