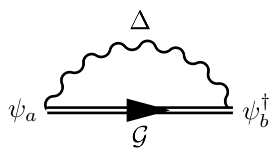
$$
\begin{aligned}
Z &= \langle \psi_a(\tau)\psi_b^\dagger(\tau')\rangle \\
&= \mathrm{Tr}[e^{\beta H(\hat{\tau})}\psi_a(\tau)\psi_b^\dagger(\tau')]
\end{aligned}
$$

$$H = H_{loc} + H_{bath} + \mathbf{H_{hyb}} + \mathbf{H}^\dagger_{\mathbf{hyb}}$$

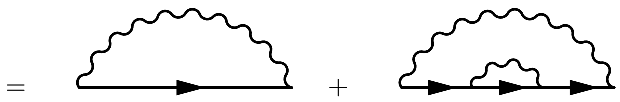
$$\mathbf{H_{hyb}} = \sum_i c_\nu \mathbf{V}_i b_i^\dagger$$

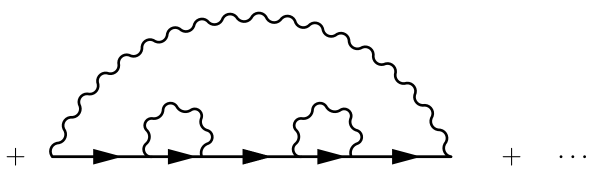
$$\mathcal{G} = Z_{\mathrm{bath}}\mathrm{Tr}_c\Big[\frac{1}{Z_{\mathrm{bath}}}\mathrm{Tr}_{\mathrm{b}}\big[T_\tau e^{-\int_0^\beta d\tau H_{\mathrm{loc}}(\tau)+H_{\mathrm{bath}}(\tau)}\prod_\nu\sum_{k_\nu} Z_{k_\nu}\big]\Big]$$

$$
\begin{aligned}
Z_{k_\nu} = &\sum_{i_1,\ldots,i_{k_\nu}}\sum_{i'_1,\ldots,i'_{k_\nu}} V^\nu_{i_1}V^{\nu *}_{i'_1}\cdots V^\nu_{i_{k_j}}V^{\nu *}_{i'_{k_j}}\int_0^\beta d\tau \\
&\times \int_{\tau_1}^\beta d\tau_2\cdots\int_{\tau_{k_\nu-1}}^\beta d\tau_{k_\nu}\int_{\tau_1^{'}}^\beta d\tau_2^{'}\cdots\int_{\tau^{'}_{k_\nu-1}}^\beta d\tau^{'}_{k_\nu} \\
&\times c_\nu(\tau_1)b^{\nu^\dagger}_{i_1}(\tau_1)b^\nu_{i'_1}(\tau_1^{'})c_\nu^\dagger(\tau_1^{'})c_\nu(\tau_2)b^{\nu^\dagger}_{i_2}(\tau_2)b^\nu_{i'_2}(\tau_2^{'})c_\nu^\dagger(\tau_2^{'}) \\
&\times\cdots c_\nu(\tau_{k_j})b^{\nu^\dagger}_{i_{k_j}}(\tau_{k_j})b^\nu_{i'_{k_j}}(\tau^{'}_{k_j})c^\dagger_\nu(\tau^{'}_{k_j})
\end{aligned}
$$

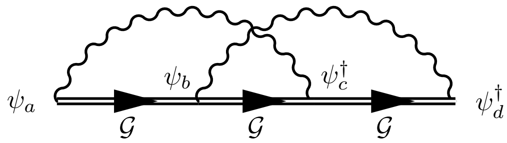


$$
\begin{aligned}
\mathcal{G} &= \\
&= \sum_{ab}\Big[(\mathrm{sgn})\psi_a\mathcal{G}(\tau)\psi_b^\dagger\Delta_{ba}(\tau)\Big]
\end{aligned}
$$

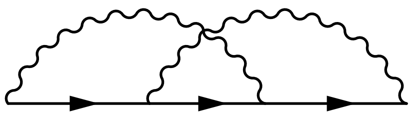
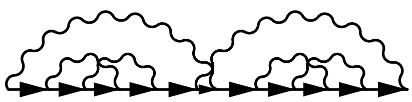




$\mathcal{G}_{\mathrm{OCA}} =$



$$= \sum_{\mathrm{abcd}} \int_0^{\tau} d\tau_1 \int_0^{\tau_1} d\tau_2 \left[\mathrm{sgn}_1 \psi_d^\dagger \mathcal{G}(\tau - \tau_2) \psi_c^\dagger \mathcal{G}(\tau_2 - \tau_1) \psi_b \mathcal{G}(\tau - \tau_1) \psi_a \Delta_{bd}(\tau - \tau_1) \Delta_{ca}(\tau_2) \right]$$

$=$  $+$ 

$+ \quad \cdots$

$$Z = \mathrm{Tr}[\mathcal{G}(\tau)\lambda_1 \mathcal{G}(0) \lambda_1 \Delta(\tau)]$$

$$\begin{aligned} Z &= Z_{\mathrm{NCA}} + Z_{\mathrm{OCA}} \\ &= \mathrm{Tr}[\mathcal{G}(\tau)\lambda_1 \mathcal{G}(0)\lambda_1 \Delta(\tau)] \\ &\quad + \mathrm{Tr}[\mathcal{G}(\beta - \tau_2)\hat{N}\mathcal{G}(\tau_2 - \tau)\lambda_1 \mathcal{G}(\tau - \tau_1)\hat{N}\mathcal{G}(\tau_1)\lambda_1 \Delta(\tau_2 - \tau_1)] \end{aligned}$$