$$E_j(t) = E_0 e^{-i\omega t + i\phi_j(t)} \tag{1}$$

$$E(t) = \sum_{j=1}^{N} E_j(t) = E_0 e^{-i\omega t}(e^{i\phi_1(t)} + e^{i\phi_2(t)} + e^{i\phi_3(t)} \cdots) \tag{2}$$

$$= E_0 e^{i\omega t} r(t) e^{i\phi(t)}$$

$$I(t) = \epsilon_0 c [ReE(t)]^2 \tag{3}$$

$$\overline{I}(t) = \frac{1}{T}\int_0^T I(t)dt = \frac{1}{2}\epsilon_0 c |E(t)|^2 \tag{4}$$

$$\overline{I}(t) = \frac{1}{2}\epsilon_0 c E_0^2 r(t)^2 \tag{5}$$

$$p(\overline{I}(t))d\overline{I}(t) = \frac{1}{\langle I \rangle} e^{\frac{\overline{I}(t)}{\langle I \rangle}} d\overline{I}(t) \tag{6}$$

$$\langle I \rangle = \frac{1}{\tau}\int_0^\tau \overline{I}(t)dt = \frac{1}{2}\epsilon_0 c E_0^2 \frac{1}{\tau}\int_0^\tau r^2(t)dt \tag{7}$$

$$\frac{1}{\tau}\int_0^\tau r(t)^2 dt = \frac{1}{\tau}\int_0^\tau \left| e^{i\phi_1(t)} + e^{i\phi_2(t)} + e^{i\phi_3(t)} \cdots + e^{i\phi_N(t)} \right|^2 dt \approx N \tag{8}$$

$$\langle I \rangle \approx \frac{1}{2}\epsilon_0 c E_0^2 n$$

$$\langle n \rangle = Tr\{\rho a^\dagger a\} = \sum_n P_n n = \frac{1}{e^{\frac{\hbar\omega}{kT}-1}} \tag{9}$$

$$P_n = \frac{\langle n \rangle^n}{(1+\langle n \rangle)^{1+n}} \tag{10}$$

$$\rho = \sum_n P_n |n\rangle \langle n| = \sum_n \frac{\langle n \rangle^n}{(1+\langle n \rangle)^{1+n}} |n\rangle \langle n| \tag{11}$$

$$(\Delta n)^2 = \left\langle (n - \langle n \rangle)^2 \right\rangle = \left\langle n^2 \right\rangle - \langle n \rangle^2 = \frac{e^{\frac{\hbar\omega}{kT}}}{(e^{\frac{\hbar\omega}{kT}} - 1)^2} \tag{12}$$

$$(\Delta n)^2 = \langle n \rangle + \langle n \rangle^2 \tag{13}$$

$$\Delta n \approx \langle n \rangle + \frac{1}{2} \approx \langle n \rangle \tag{14}$$

$$0 \tag{15}$$

$$0 \tag{16}$$

$$\langle I(\vec{r}_1, t_1) I(\vec{r}_2, t_2) \rangle = \left\langle I(\vec{r}, t_1 - \frac{|\vec{r}_1 - \vec{r}|}{c}) I(\vec{r}, t_2 - \frac{|\vec{r}_2 - \vec{r}|}{c}) \right\rangle \tag{17}$$

$$= \langle I(\vec{r}, t) I(\vec{r}, t+\tau) \rangle$$

$$\tau = t_2 - t_1 - \frac{|\vec{r}_2 - \vec{r}| - |\vec{r}_1 - \vec{r}|}{c} \tag{18}$$

$$i_{12}(\tau) \equiv \frac{\langle I(\vec{r}, t_1) I(\vec{r}_2, t_2)\rangle}{\langle I(\vec{r}_1, t_1)\rangle \langle I(\vec{r}_2, t_2)\rangle} = \frac{\langle I(\vec{r}, t) I(\vec{r}, t+\tau)\rangle}{\langle I(\vec{r}, t)\rangle^2} \tag{19}$$

$$i_{12}(\tau|\infty) = 1 \tag{20}$$

$$i_{12}(\tau = 0) = \frac{\left\langle I^2 \vec{r}, t\right\rangle}{\langle I(\vec{r}, t)\rangle^2} \tag{21}$$

$$i_{12}(\tau = 0) \geq 1 \tag{22}$$

$$\langle I(\vec{r}_1, t_1) I(\vec{r}_2, t_2)\rangle = (\frac{1}{2}\epsilon_0 c)^2 \langle E^*(\vec{r}_1, t_1) E(\vec{r}_1, t_1) E^*(\vec{r}_2, t_2) E(\vec{r}_2, t_2)\rangle \tag{23}$$

$$= (\frac{1}{2}\epsilon_0 c)^2 \langle E^*(\vec{r}_1, t_1) E^*(\vec{r}_2, t_2) E(\vec{r}_2, t_2) E(\vec{r}_1, t_1)\rangle$$

$$i_{12}(\tau) = \frac{\langle E^*(\vec{r}_1, t_1) E^*(\vec{r}_2, t_2) E(\vec{r}_2, t_2) E(\vec{r}_1, t_1)\rangle}{\left\langle |E(\vec{r}_1, t_1)|^2\right\rangle \left\langle |E(\vec{r}_2, t_2)|^2\right\rangle} \tag{24}$$

$$= \frac{\langle E^*(\vec{r}, t) E^*(\vec{r}, t+\tau) E(\vec{r}, t+\tau) E(\vec{r}, t)\rangle}{\left\langle |E(\vec{r}_1, t_1)|^2\right\rangle^2}$$

$$P \propto \sum_f \left|\langle f| E^{(+)}(\vec{r}_2, t_2) E^{(+)}(\vec{r}_2, t_2) |i\rangle\right|^2 \tag{25}$$

$$= \sum_f \langle i| E^{(-)}(\vec{r}_1, t_1) E^{(-)}(\vec{r}_2, t_2) |f\rangle \langle f| E^{(+)}(\vec{r}_2, t_2) E^{(+)}(\vec{r}_1, t_1) |i\rangle$$

$$= \langle i| E^{(-)}(x_1) E^{(-)}(x_2) E^{(+)}(x_3) E^{(+)}(x_4) |i\rangle$$

$$G^{(2)} = (x_1, x_2, x_3, x_4) = \left\langle E^{(-)}(x_1) E^{(-)}(x_2) E^{(+)}(x_3) E^{(+)}(x_4)\right\rangle \tag{26}$$

$$= Tr\{\rho E^{(-)}(x_1) E^{(-)}(x_2) E^{(+)}(x_3) E^{(+)}(x_4)\}$$

$$g^{(2)} = \frac{G^{(2)}(x_1, x_2, x_3, x_4)}{\sqrt{G^{(1)}(x_1, x_1) G^{(1)}(x_2, x_2) G^{(3)}(x_3, x_3) G^{(4)}(x_4, x_4)}} \tag{27}$$

$$G^{(2)}(x_1, x_2, x_2, x_1) = \langle\alpha| E^{(-)}(x_1) E^{(-)}(x_2) E^{(+)}(x_2) E^{(+)}(x_1) |\alpha\rangle \tag{28}$$

$$= (\frac{\hbar\omega}{2\epsilon_0 V})^2 |\alpha|^4$$

$$g^{(2)}(x_1, x_2, x_2, x_1) = g^{(2)}(\tau) = \frac{G^{(2}(x_1, x_2, x_2, x_1)}{G^{(1)}(x_1, x_1) G^{(1)}(x_2, x_2)} = 1 \tag{29}$$

$$\left|g^{(2)}(x_1, x_2, x_3, x_4)\right| = 1 \tag{30}$$

$$g^{(2)}(x_1, x_2, x_3, x_4) = g^{(2)}(\tau) = 1 \tag{31}$$

$$G^{(2)}(x_1, x_2, x_2, x_1) = G^{(1)}(x_1, x_1) G^{(1)}(x_2, x_2) \tag{32}$$

$$Q = \frac{()\Delta n)^2 - \langle n \rangle}{\langle n \rangle} \tag{33}$$

$$Q = \langle n \rangle \left(g^{(2)}(\tau) - 1\right) \tag{34}$$

if $g^{(2)} < 1$, anti-bunching
coherent light (Poissonian) : $[(\Delta n)^2 - \langle n \rangle] = 0$, super-Poissonian : $[(\Delta n)^2 - \langle n \rangle] > 0$
sub-Poissonian : $[(\Delta n)^2 - \langle n \rangle] < 0$

$$g^{(2)}(x_1, x_2, x_2, x_1) = \frac{\langle E^*(x_1)E^*(x_2)E(x_2)E(x_1)\rangle}{\left\langle E^{(-)}(x_1)E^{(+)}(x_1)\right\rangle \left\langle E^{(-)}(x_2)E^{(+)}(x_2)\right\rangle} \tag{35}$$

$$g^{(2)}(x_1, x_2, x_2, x_1) = \frac{\left\langle a^\dagger a^\dagger a a\right\rangle}{\left\langle a^\dagger a\right\rangle^2} \tag{36}$$

$$= \frac{\left\langle a^\dagger a a^\dagger a\right\rangle - \left\langle a^\dagger a\right\rangle}{\left\langle a^\dagger a\right\rangle^2} = \frac{\left\langle n^2\right\rangle - \langle n \rangle}{\langle n \rangle^2}$$

$$g^{(2)}(x_1, x_2, x_2, x_1) = g^{(2)}(\tau) = 1 + \frac{(\Delta n)^2 - \langle n \rangle}{\langle n \rangle^2} \tag{37}$$

$$0 \tag{38}$$

$$0 \tag{39}$$

$$g^{(n)}(x_1, x_2, \cdots, x_{2n}) = \frac{G^{(n)}(x_1, x_2, \cdots, x_{2n})}{\sqrt{\prod_{j=1}^{2n} G^{(1)}(x_j, x_j)}} \tag{40}$$

$$= \frac{E^{(-)}(x_1) \cdots E^{(-)}(x_n)E^{(+)}(x_{n+1}) \cdots E^{(+)}(x_{2n})}{\sqrt{\prod_{j=1}^{2n} \left\langle E^{(-)}(x_j)E^{(+)}(x_j)\right\rangle}}$$

$$\left| g^{(n)}(x_1, x_2, \cdots, x_{2n}) \right| = 1 \tag{41}$$

$$\left| g^{(j)}(x_1, x_2, \cdots, x_{2j}) \right| = 1 \tag{42}$$

for all $n \leq j$

$$g^{(n)}(x_1, \cdots, x_n, x_n, \cdots, x_1) = 1 \tag{43}$$

$$G^{(n)}(x_1, \cdots, x_n, x_n, \cdots, x_1) = \prod_{j=1}^{n} G^{(1)}(x_j, x_j) \tag{44}$$

$$G^{(n)}(x_1, \cdots, x_n, x_n, \cdots, x_1)G^{(n)}(x_{n+1}, \cdots, x_{2n}, x_{2n}, \cdots, x_{n+1}) \geq \left| G^{(n)}(x_1, \cdots, x_n, x_{n+1}, \cdots, x_{2n}) \right|^2$$