$$H_c = \sum_{k\sigma} (\epsilon_{\mathbf{k}} - \mu) c^\dagger_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma}$$

$$H_d + H_U = \sum_{\sigma} (\epsilon_d - \mu) c^\dagger_{d\sigma} c_{d\sigma} + U n_{d\uparrow} n_{d\downarrow}$$

$$H_{hyb} = \sum_{\mathbf{k}\sigma} t_{\mathbf{k}} c^\dagger_{d\sigma} c_{\mathbf{k}\sigma} + \sum_{\mathbf{k}\sigma} t^*_{\mathbf{k}} c^\dagger_{\mathbf{k}\sigma} c_{d\sigma}$$

$$H = H_c + H_d + H_U + H_{hyb}$$

$$\langle n_{d\sigma} \rangle = \int \frac{d\omega}{2\pi} A(d\sigma) n_F(\omega)$$

$$(\omega + i\eta - \epsilon_d + \mu) G^R(d\sigma, \omega) - \sum_k t_{\mathbf{k}} G^R(\mathbf{k}\sigma, d\sigma, \omega) = 1 + U D^R(d\sigma, \omega)$$

$$(\omega + i\eta - \epsilon + \mu) G^R(\mathbf{k}\sigma, d\sigma, \omega) - t^*_{\mathbf{k}} G^R(d\sigma, \omega) = 0$$

$$D^R(d\sigma, t - t') = -i\theta(t - t') \left\langle \left(-[n_{d\uparrow} n_{d\downarrow}, c_{d\sigma}](t), c^\dagger_{d\sigma}(t') \right) \right\rangle$$

$$[n_{d\uparrow} n_{d\downarrow}, c_{d\uparrow}] = n_{d\downarrow} [n_{d\uparrow}, c_{d\uparrow}] = -n_{d\downarrow} c_{d\uparrow}$$

$$H^{MF}_U = U \langle n_{d\uparrow} \rangle n_{d\downarrow} + U \langle n_{d\downarrow} \rangle n_{d\uparrow} - U \langle n_{d\uparrow} \rangle \langle n_{d_\downarrow} \rangle$$

$$\begin{aligned} D^R(d\uparrow, t - t') &= i\theta(t - t') \langle n_{d\downarrow} \rangle \langle \{ c_{d\uparrow}(t), c^\dagger_{d\uparrow}(t') \} \\ &= \langle n_{d_\downarrow} \rangle G^R(d\uparrow, t - t') \end{aligned}$$

$$(\omega + i\eta - \epsilon_d + \mu - U\langle n_{d\downarrow} \rangle) G^R(d\sigma, \omega) - \sum_k \frac{|t_{\mathbf{k}}|^2}{\omega - \epsilon_{\mathbf{k}} + \mu + i\eta} G^R(d\uparrow, \omega) = 1$$

$$G^R(d\uparrow, \omega) = \frac{1}{\omega - \epsilon_d + \mu - U\langle n_{d\downarrow} \rangle - \Sigma^R(\omega)}$$

$$\Sigma^R(\omega) - \sum_{\mathbf{k}} \frac{|t_{\mathbf{k}}|^2}{\omega - \epsilon_{\mathbf{k}} + \mu + i\eta}$$

$$\begin{aligned} \Sigma^R(\omega) &= \int d\epsilon d(\epsilon) \frac{|t(\epsilon)|^2}{\omega - \epsilon + \mu + i\eta} \\ &= P \int d\epsilon d(\epsilon) \frac{|t(\epsilon)|^2}{\omega - \epsilon + \mu} - i\pi d(\omega + \mu) |t(\omega + \mu)|^2 \end{aligned}$$

$$2\pi d(\epsilon) |t(\epsilon)|^2 = \begin{cases} \Gamma & \text{for } -D < \epsilon < D \\ 0 & \text{for } D < |\epsilon| \end{cases}$$

$$\begin{aligned} \Sigma^R(\omega) \; & \frac{\Gamma}{\pi} \int_{-D}^{D} \frac{d\epsilon}{\omega - \epsilon + \mu} - i\Gamma/2 \\ &= -\frac{\Gamma}{\pi} \ln \frac{D + \omega + \mu}{D - \omega - \mu} - i\Gamma/2 \end{aligned}$$

$$\begin{aligned} A(d\uparrow, \omega) &= -2\, \mathrm{Im}\, G^R(d\uparrow, \omega) \\ &= \frac{\Gamma}{(\omega - \tilde{\epsilon} + \mu - U\langle n_{d\downarrow} \rangle)^2 + (\Gamma/2)^2} \end{aligned}$$

$$\begin{aligned} \langle n_{d\uparrow} \rangle &= \int \frac{d\omega}{2\pi} n_F(\omega) A(d\uparrow, \omega) \\ &= \int \frac{d\omega}{2\pi} n_F(\omega) \frac{\Gamma}{(\omega - \tilde{\epsilon} + \mu - U\langle n_{d\downarrow} \rangle)^2 + (\Gamma/2)^2} \end{aligned}$$