The spin-boson model

studies the dynamics of a two-level system interacting with a bosonic environment
→ Assumes a linear coupling between a two-level system and a collective coordinate of the bosonic bath.
→ A qunatum simulation can be done by connecting a superconducting qubit to an open transmission line
→ To achieve strong coupling, desing system characteristc impedance comaprable to the resistance quantum.
→ coupling strngth effectively increased by creating a Hamiltonian in the rotating frame.

Notations

spin-boson hamiltonian is :

$$\hat{H}_{SB} = -\frac{\hbar\Delta}{2}\hat{\sigma}_x + \frac{\epsilon}{2}\hat{\sigma}_z + \frac{q_0}{2}\hat{\sigma}\sum_i c_i\hat{x}_i + \underbrace{\sum_i\left[\frac{1}{2}m_i\omega_i^2\hat{x}_i^2 + \frac{1}{2m_i}\hat{p}_i^2\right]}_{\text{bath}}$$

And its spectral density is :

$$J(\omega) = \frac{\pi}{2}\sum_i \frac{c_i^2}{m_i\omega_i}\delta(\omega - \omega_i)$$

In this paper, using the operator $\hat{b} = \sqrt{\frac{m_i\omega_i}{2\hbar}}\left(x_i + i\frac{p_i}{m_i\omega_i}\right)$, total hamiltonian become:

$$\hat{H} = \frac{\hbar\Delta}{2}\hat{\sigma}_z + \frac{q_0}{2}\hat{\sigma}_x\sum_i g_i(\hat{b}_i + \hat{b}_i^\dagger) + \sum_i \hbar\omega_i\hat{b}_i\hat{b}_i^\dagger$$

And spectral density is :

$$J(\omega) = \frac{\pi}{\hbar}\sum_i g_i^2\delta(\omega - \omega_i)$$

relaxation dynamics

In the spin-boson model, widely studied effect is the hopping dynamics between the two trapped positions of the fictitious particle. Particle inserted initially in the left-hand well, the probability to found from this well again is :

$$P(t) = \langle\hat{\sigma}_x(t)\rangle$$

and if there is no interaction with environment,

$$P(t) = \cos(\Delta t)$$

Relaxation dynamics for different environments

At the Ohmic environment,

$$J(\omega) = \eta\omega F_c(\omega)$$

where $F_c(\omega)$ is an environment cutoff function.
In ohmic system, important parameter describing the coupling between the system and the environment is Kondo parameter, $\alpha = \eta\frac{a_0^2}{2\pi\hbar}$.

		$\alpha = \eta\frac{q_0^2}{2\pi\hbar}$
$\alpha < 1/2$		damped osillations
$\alpha > 1/2$		all dynamics incoherent
$\alpha \geqslant 1$	T=0	total supperssion of hopping
	$T \gtrsim 0$	very slow thermal relaxation

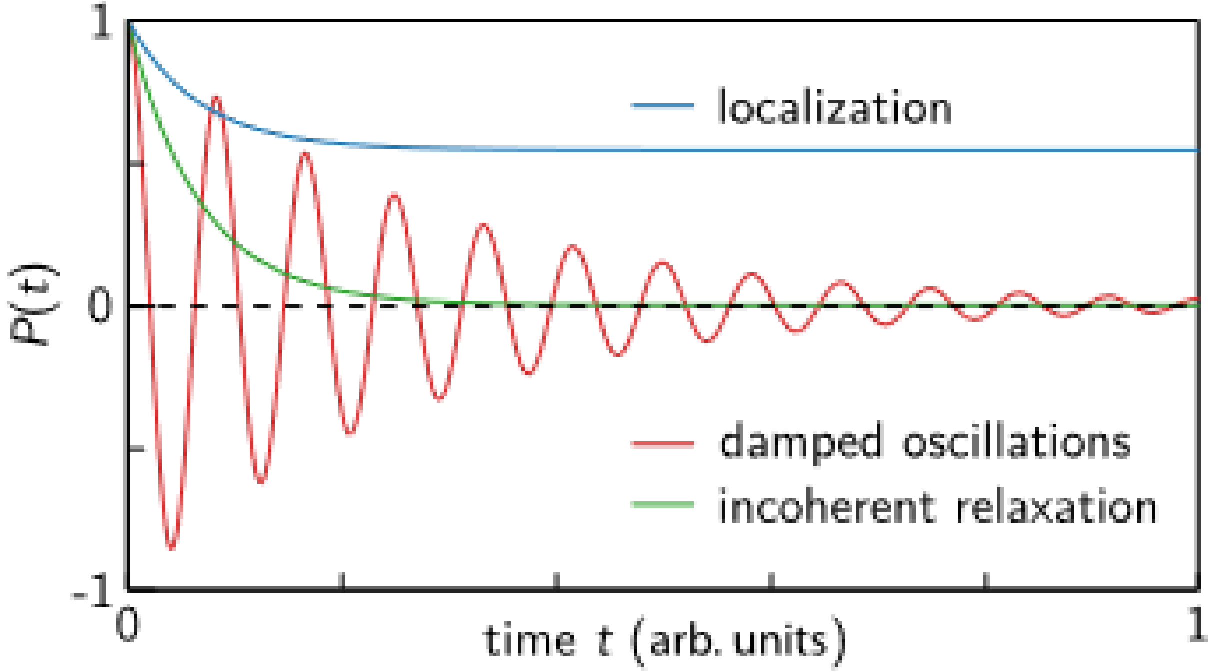
1
0
-1
$P(t)$
localization
damped oscillations
incoherent relaxation
0
1
time t (arb. units)

Figure 1: