

$$\psi_a \text{---} \underbrace{\hspace{0.8cm}}_{\mathcal{G}} \text{---} \psi_b^\dagger = \psi_a \text{---} \underbrace{\hspace{0.8cm}}_{\mathcal{G}_0} \text{---} \psi_b^\dagger + \text{---} \underbrace{\hspace{0.6cm}}_{\circlearrowleft \Sigma \circlearrowright} \text{---}$$

$$\begin{aligned} Z &= \langle \psi_a(\tau) \psi_b^\dagger(\tau') \rangle \\ &= \text{Tr}[e^{\beta H(\tau)} \psi_a(\tau) \psi_b^\dagger(\tau')] \end{aligned}$$

$$H = H_{loc} + H_{bath} + \mathbf{H}_{\text{hyb}} + \mathbf{H}_{\text{hyb}}^\dagger$$

$$\mathbf{H}_{\text{hyb}} = \sum_i c_\nu \mathbf{V}_i b_i^\dagger$$

$$\mathcal{G} = Z_{\text{bath}} \text{Tr}_{\text{c}} \left[\frac{1}{Z_{\text{bath}}} \text{Tr}_{\text{b}} [T_{\tau} e^{-\int_0^{\beta} d\tau H_{\text{loc}}(\tau) + H_{\text{bath}}(\tau)} \prod_{\nu} \sum_{k_{\nu}} Z_{k_{\nu}}] \right]$$

$$\begin{aligned}
Z_{k_\nu} = & \sum_{i_1, \dots, i_{k_\nu}} \sum_{i'_1, \dots, i'_{k_\nu}} V_{i_1}^\nu V_{i'_1}^{\nu*} \dots V_{i_{k_j}}^\nu V_{i'_{k_j}}^{\nu*} \int_0^\beta d\tau \\
& \times \int_{\tau_1}^\beta d\tau_2 \dots \int_{\tau_{k_\nu-1}}^\beta d\tau_{k_\nu} \int_{\tau'_1}^\beta d\tau'_2 \dots \int_{\tau'_{k_\nu-1}}^\beta d\tau'_{k_\nu} \\
& \times c_\nu(\tau_1) b_{i_1}^{\nu\dagger}(\tau_1) b_{i'_1}^{\nu}(\tau'_1) c_\nu^\dagger(\tau'_1) c_\nu(\tau_2) b_{i_2}^{\nu\dagger}(\tau_2) b_{i'_2}^{\nu}(\tau'_2) c_\nu^\dagger(\tau'_2) \\
& \times \dots c_\nu(\tau_{k_j}) b_{i_{k_j}}^{\nu\dagger}(\tau_{k_j}) b_{i'_{k_j}}^{\nu}(\tau'_{k_j}) c_\nu^\dagger(\tau'_{k_j})
\end{aligned}$$

$$M^{-1} = \begin{pmatrix} \mathcal{G}'_{11} & \mathcal{G}'_{12} & \cdots & \mathcal{G}'_{1j} & \cdots \\ \mathcal{G}'_{21} & \mathcal{G}'_{22} & \cdots & \mathcal{G}'_{2j} & \cdots \\ \vdots & \vdots & \ddots & \vdots & \cdots \\ \mathcal{G}'_{i1} & \mathcal{G}'_{i2} & \cdots & \mathcal{G}'_{ij} & \cdots \\ \vdots & \vdots & \cdots & \cdots & \ddots \end{pmatrix}$$

$$\det M^{-1} = \Delta$$

$$\begin{aligned} \mathcal{G}' &= \langle |V^\nu| b^{\dagger\nu}(\tau) b^\nu(\tau') \rangle \\ &= \begin{cases} |V^a|^2 e^{-\epsilon(\tau-\tau')} \frac{1}{(1+e^{-\beta\epsilon p})} & (\text{if } \tau > \tau') \\ |V^a|^2 e^{-\epsilon(\tau'-\tau)} \frac{e^{-\beta\epsilon}}{(1+e^{-\beta\epsilon})} & (\text{if } \tau < \tau') \end{cases} \end{aligned}$$

A Feynman diagram showing a scalar particle (represented by a wavy line) decaying into two fermions (represented by straight lines). The scalar particle is labeled Δ at the top. The fermions are labeled ψ_a on the left and ψ_b^\dagger on the right. The interaction is mediated by a vertex labeled \mathcal{G} .

$$\begin{aligned} \mathcal{G} &= \\ &= \sum_{ab} \left[(\text{sgn}) \psi_a \mathcal{G}(\tau) \psi_b^\dagger \Delta_{ba}(\tau) \right] \end{aligned}$$

$$\begin{aligned}
&= \text{diagram 1} + \text{diagram 2} \\
&+ \text{diagram 3} + \dots
\end{aligned}$$

$$\mathcal{G}_{\text{OCA}} =$$

$$\begin{array}{c}
\psi_a \text{---} \text{diagram} \text{---} \psi_d^\dagger \\
\text{diagram labels: } \mathcal{G}, \psi_b, \mathcal{G}, \psi_c^\dagger, \mathcal{G}
\end{array}$$

$$= \sum_{\text{abcd}} \int_0^\tau d\tau_1 \int_0^{\tau_1} d\tau_2 \left[\text{sgn}_1 \psi_d^\dagger \mathcal{G}(\tau - \tau_2) \psi_c^\dagger \mathcal{G}(\tau_2 - \tau_1) \psi_b \mathcal{G}(\tau - \tau_1) \psi_a \Delta_{bd}(\tau - \tau_1) \Delta_{ca}(\tau_2) \right]$$

$$= \text{diagram 1} + \text{diagram 2}$$

$$+ \dots$$

$$Z = \text{Tr}[\mathcal{G}(\tau) \lambda_1 \mathcal{G}(0) \lambda_1 \Delta(\tau)]$$

$$\begin{aligned}
Z &= Z_{\text{NCA}} + Z_{\text{OCA}} \\
&= \text{Tr}[\mathcal{G}(\tau) \lambda_1 \mathcal{G}(0) \lambda_1 \Delta(\tau)] \\
&\quad + \text{Tr}[\mathcal{G}(\beta - \tau_2) \hat{N} \mathcal{G}(\tau_2 - \tau) \lambda_1 \mathcal{G}(\tau - \tau_1) \hat{N} \mathcal{G}(\tau_1) \lambda_1 \Delta(\tau_2 - \tau_1)]
\end{aligned}$$

$$H_{\text{tot}} = H_{\text{loc}} + H_{\text{int}} + H_{\text{bath}}$$

$$H_{\text{loc}} = E_c \hat{N} - E_J \cos \phi$$

$$H_{\text{int}} = \sum_k g_k \hat{N} (b_k^\dagger + b_k)$$

$$H_{\text{bath}} = \sum_k \omega_k b_k^\dagger b_k$$