Dissipative quantum phase transition in the three-level system coupled with bosonic bath

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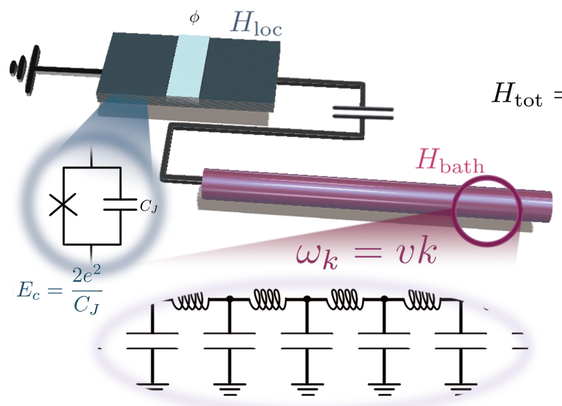
Introduction

As a minimal model of dissipative quantum phase transition in the resistively shunted Josephson junction, we study the three-level system coupled to the bosonic bath. In order to deal with the continuous bath degrees of freedom with linear bosonic dispersions, the strong-coupling approximations such as non-crossing or one-crossing approximations are introduced both in-and-out of the equilibrium setup. The finite-temperature behavior of possible quantum phase transition between the superconducting and insulating phase will be discussed.

Model

▶Resistivily shunted Josephson junction circuit [1]



$$H_{tot} = H_{loc} + H_{int} + H_{bath}$$

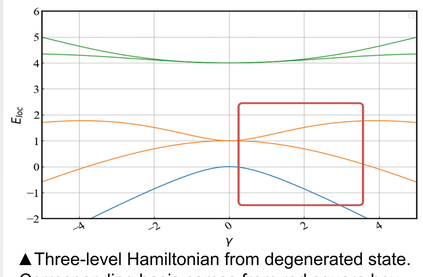
$$H_{loc} = E_C \hat{N} - E_J \cos\phi$$

$$H_{int} = \sum_k g_k \hat{N}(b_k^\dagger + b_k)$$

$$H_{bath} = \sum_k \omega_k b_k^\dagger b_k$$

$$\begin{cases} \gamma = \frac{E_J}{E_c} \\ \alpha = \frac{R_Q}{R} \end{cases}$$

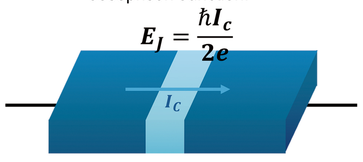
Local System



▲Three-level Hamiltonian from degenerated state. Corresponding basis comes from red square box.

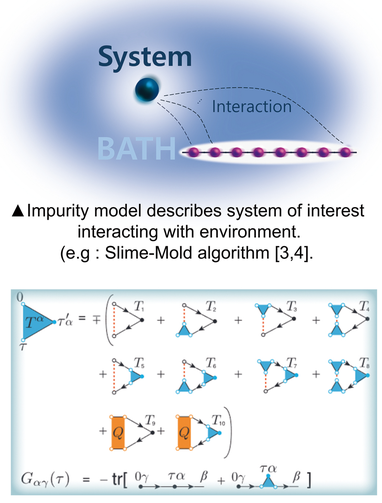
▼ Value of γ comes from critical current of Josephson Junction.

$$E_J = \frac{\hbar I_c}{2e}$$



I_c : Josephson current

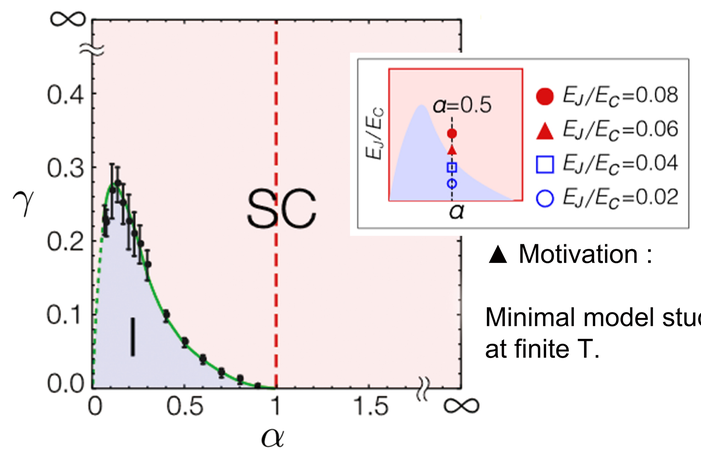
Impurity Model



▲Impurity model describes system of interest interacting with environment. (e.g : Slime-Mold algorithm [3,4].

Previous Study

▶Previous study at $T = 0$ [1]



▲ Motivation :

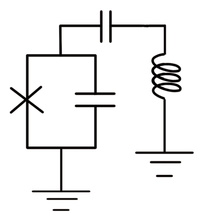
Minimal model study at finite T.

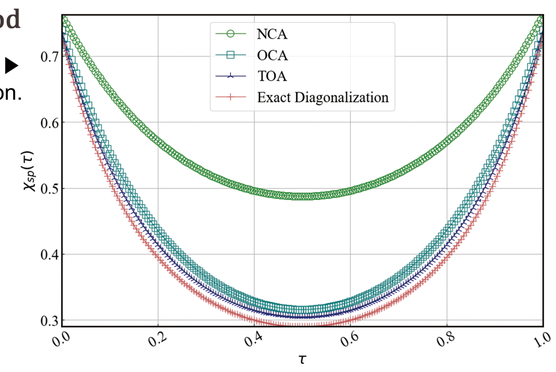
Benchmark Test

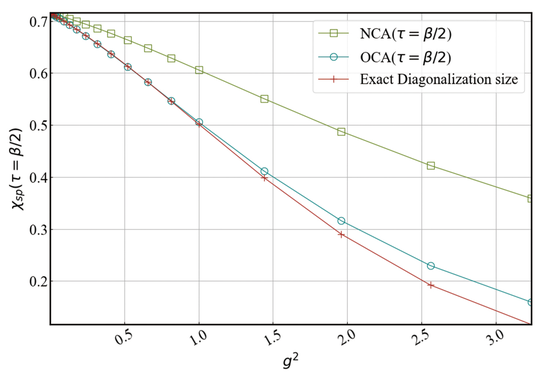
▶Accuracy of the approximation method

Evaluate the accuracy of approximation methods ▶ comparing with the Exact Diagonalization.

The Exact calculation ▶ of correlation function available in the single bath

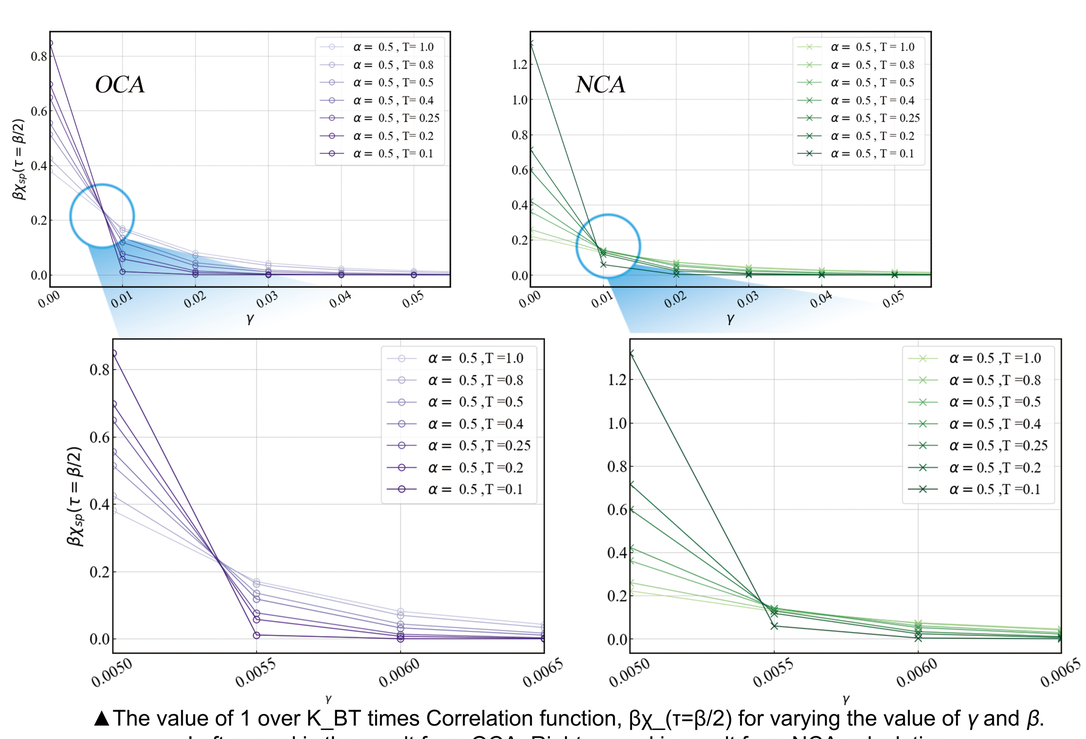




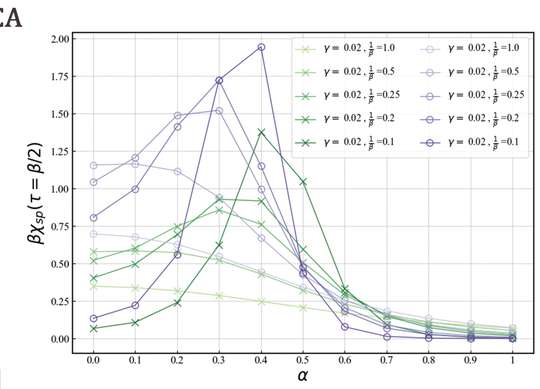


RESULT - Correlation function

▶Phase transition from localized state to delocalized excited states in both NCA & OCA

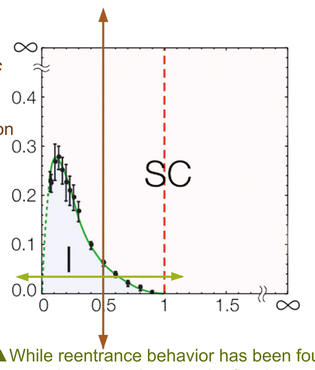


▲The value of 1 over K_BT times Correlation function, βχ_(τ=β/2) for varying the value of γ and β. Left pannel is the result from OCA, Right pannel is result from NCA calculation.



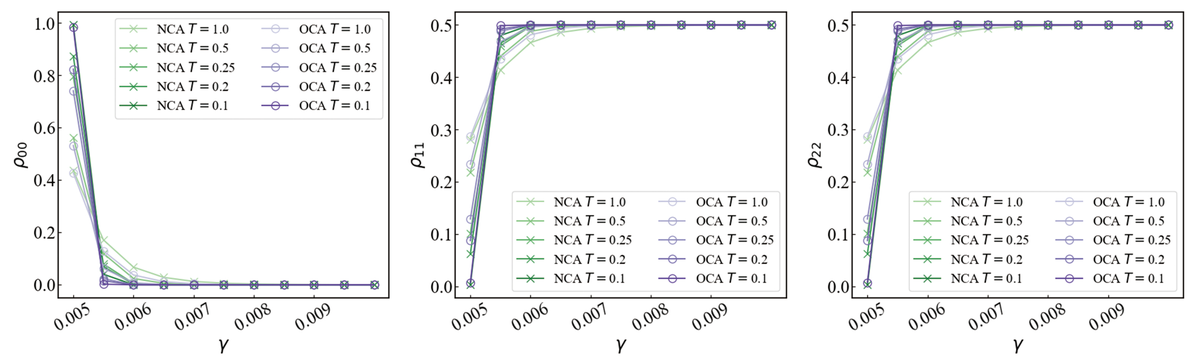
▲Reentrant Transition by varying the value of α.

▶ Changing c under fixed α, A point where phase transition occurs can be found due to the response (resistivity) of the system.

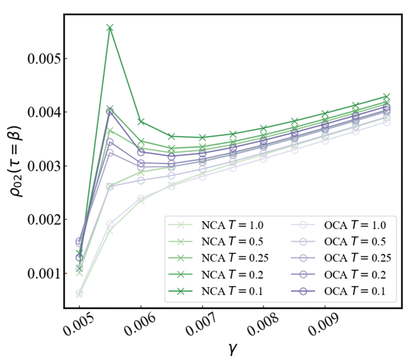


▲While reentrance behavior has been found when changing α under fixed γ

RESULT - Density matrix



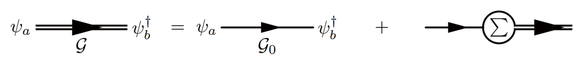
▲Elements of density matrix, ρ , by varying the value of the value of γ and T. As γ increases system evolves from local ground state to equal mixture of 1st & 2nd excited states .



▲Mixing between ground state & 2nd-excited state via 1st excited state

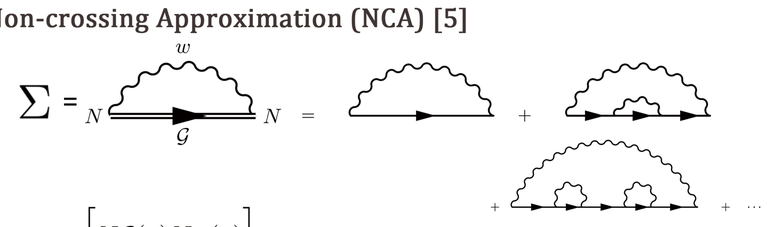
Method - Impurity solver

▶Dyson's equation with diagrammatic scheme



$$\mathcal{G} = \langle \psi_a(\tau)\psi_b^\dagger(\tau')\rangle = \mathrm{Tr}[e^{\beta H(\tau)}\psi_a(\tau)\psi_b^\dagger(\tau')]$$

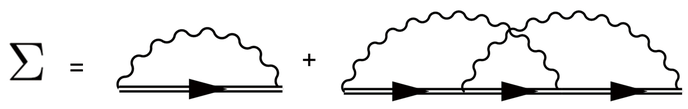
▶Non-crossing Approximation (NCA) [5]



$$= \left[N\mathcal{G}(\tau)N\omega(\tau)\right]$$

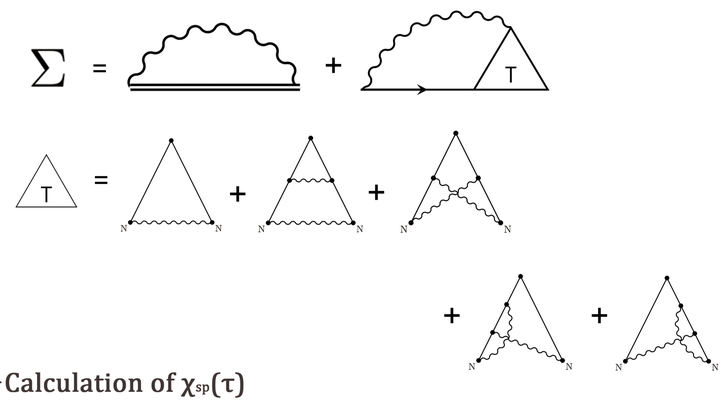
▲ ω(τ) , which is used for describing Hybridization between the target system and bath, (wiggly line in the diagramm)

▶One-crossing Approximation (OCA)

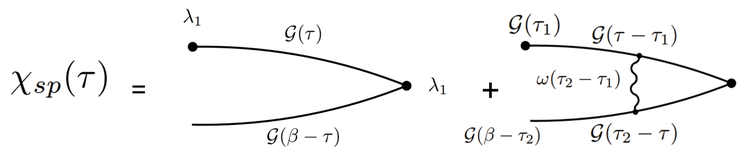


$$= \int_0^\tau d\tau_1 \int_0^{\tau_1} d\tau_2 \left[N\mathcal{G}(\tau-\tau_2)N\mathcal{G}(\tau_2-\tau_1)N\mathcal{G}(\tau-\tau_1)N\omega(\tau-\tau_1)\omega(\tau_2)\right]$$

▶Third-Order approximation (TOA)



▶Calculation of $\chi_{sp}(\tau)$



$$\chi_{sp} = \mathrm{Tr}[\mathcal{G}(\beta-\tau)\lambda_1\mathcal{G}(\tau)\lambda_1] + \mathrm{Tr}[\omega\mathcal{G}(\beta-\tau_1)\hat{N}\mathcal{G}(\tau_1-\tau_2)\lambda_1\mathcal{G}(\tau_2-\tau_3)N\mathcal{G}(\tau_3)\lambda_1]$$

▶Definition of Gellmann opertaor, λ_1

$$\hat{\lambda}_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

◀ Gellmann operator as a form of Matrix

◀ First Gellman matrix was used for measuring the effect of mixing between local ground state and excited state.

CONCLUSION

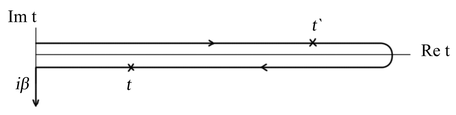
▶ Using strong-coupling expasions, we obsereve the phase transitions localized ground state to delocalized excited state, in the minimal model of resistive-shunted Josephson Junction.

▶ Both NCA and OCA, Phase transition appear.

▶ As a function of R, there appears re-entrance behavior of the localized phase.

FUTURE WORK

▶ Non-equilllibrium Green's function approach (Keldysh time contour).



▶ Higher order Impurity solver under various parameter-condition.

▶The another form of entanglement via different type of Gellmann operator.

REFERENCE

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[3] A J. Kim, K. Lenk, J. Li, P. Werner, M. Eckstein, Phys. Rev. Lett. **130**, 036901 (2023)

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[5] H. Aoki, N. Tsuji, M. Eckstein, M. Kollar, T. Oka, P. Werner, Rev. Mod. Phys. **86**, 779 (2014)