APPLICATION TO COUPLED CAVITY ARRAY

Bound states, symmetry-protected BIC, quasi-BIC

$$\hat{H}_light = -\frac{J}{2}\sum_x(\hat{a}^\dagger_{x+1}\hat{a}_x + H.c.) + \hbar\omega_c\sum_x \hat{a}^\dagger_x\hat{a}_x$$

ω_c
$\hat{a}_x \equiv \frac{1}{\sqrt{L}}\sum_k \hat{a}_k e^{-ikx}$

$$\hbar\omega_k = \hbar\omega_c - Jcosk$$

$$\hat{A} = A(\hat{a}_{x=0} + \hat{a}^\dagger_{x=0})$$

$f_k = \frac{A}{\sqrt{L}}$
$g = qA\sqrt{\frac{\omega_c}{m\hbar}}$
$V(Q) = v\big(1 - \frac{Q^2}{d^2}\big)^2$
$\Omega_0\ \frac{(E_2-E_1)}{\hbar} \simeq \omega_c$

$$\delta E_{sca} \in [\hbar\omega_c - J, \hbar\omega_c + J]$$

$$\delta E_{BS} \notin [\hbar\omega_c - J, \hbar\omega_c + J]$$

$$\delta E_{(q)BIC} \in [\hbar\omega_c - J, \hbar\omega_c + J]$$

$$\Gamma_{qBIC}\ \frac{(J/\hbar)^2}{g\sqrt{m\omega_c^3 d^3}}\left(\frac{v^3}{m_{eff}}\right)^{1/4} \propto g^{-3/2}$$

Dressed potential

$$V_{eff}(Q) = v_{eff}\left(1 - \frac{Q^2}{d_{eff}}\right)^2$$

$$v_{eff} = \begin{cases} v\big(1 - \frac{3\xi^2}{d^2}\big)^2, & \xi \leq \frac{d}{\sqrt{3}}, \\ 0, & \frac{d}{\sqrt{3}} \end{cases}$$

Two-level effective model and its breakdown in the Coulomb gauge

$$\hat{H}_U^{JC} = \frac{\hbar\Delta_g}{2}\hat{\sigma}^z + \left(\hat{\sigma}^-\sum_n \hbar\tilde{g}_n\hat{b}^\dagger_n + H.c.\right) + \sum_n \hbar\Omega_n\hat{b}^\dagger_n\hat{b}_n$$

$$\Delta_g \equiv \frac{E_2 - E_1}{\hbar} > 0,$$

$$\tilde{g}_n\frac{\xi}{\hbar}\langle\psi_1|\frac{dV}{dQ}|\psi_2\rangle$$

$$E_{ex} - \Delta_g = \sum_n \frac{\tilde{g}_n^2}{E_{ex} - \Omega_n}$$

$$\hat{H}_U^{JC} = \frac{\hbar\Delta}{2}\hat{\sigma}^z + \left(\hat{\sigma}^- \sum_k \hbar\tilde{g}_k \hat{a}_k^\dagger + H.c.\right) + \sum_k \hbar\Omega_k \hat{a}_k^\dagger \hat{a}_k$$

$\Delta \equiv \Delta_{g=0}$

$$\tilde{g}_k \equiv \frac{g}{\sqrt{L}}\kappa_{\omega_c} \left\langle \psi_1^{g=0} \middle| \partial_Q \middle| \psi_2^{g=0} \right\rangle$$

$$\hat{H}_U^{Rabi} = \frac{\hbar\Delta_g}{2}\hat{\sigma}^z + \hat{\sigma}^x \left(\sum_n \hbar\tilde{g}_n \hat{b}_n^\dagger + H.c.\right) + \sum_n \hbar\Omega_n \hat{b}_n^\dagger \hat{b}_n$$

$$|\Psi(t)\rangle_U = e^{-i\hat{H}_U t} |\Psi(0)\rangle_U = \sum_i c_i e^{-iE_i t} |\Psi_i\rangle_U$$

$$\langle \hat{O} \rangle_C = \langle \hat{U}^\dagger \hat{O} \hat{U} \rangle_U$$

$$\omega_{osc} = \sqrt{\frac{8v}{d_f^2 m_{eff}}\left(1 - \frac{3\xi^2}{d_f^2}\right)} \propto g^{-1}$$