$$\sum_{\nu"}[\delta_{\nu\nu"}(\omega+i\eta)-t_{\nu\nu"}]G^R(\nu"\nu';\omega)=\delta_{\nu\nu'}$$

$$(G_0^R)^{-1}(\nu\nu';\omega)=\delta_{\nu\nu'}(\omega+i\eta)-t_{\nu\nu'}\equiv(\mathbf{G}_0^R)^{-1}_{\nu\nu'}$$

$$(\mathbf{G}_0^R)^{-1}(\mathbf{G}_0^R)=\mathbf{1}$$

$$(\mathbf{G}_0^R)_{\nu\nu'}=G_0^R(\nu,\omega)\delta_{\nu\nu'}=\frac{1}{\omega-\epsilon_\nu+i\eta}\delta_{\nu\nu'}$$

$$H=H_0+H_{hyb}+H_l$$

$$H_0=\sum_\nu \xi_\nu c_\nu^\dagger c_\nu$$

$$H_l=\xi_0 c_l^\dagger c_l$$

$$H_{hyb}=\sum_\nu(t*_\nu c_\nu^\dagger c_l+t_\nu c_l^\dagger c_\nu)$$

$$G^R(l,l,t-t')=-i\theta(t-t')\langle\{c_l(t),c_l^\dagger(t')\}\rangle$$
$$G^R(\nu,l,t-t')=-i\theta(t-t')\langle 1\{c_\nu(t),c_{\cdot}^\dagger(t')\}\rangle$$

$$(\omega+i\eta-\xi_0)G^R(l,l,\omega)-\sum_\nu t_\nu G^R(\nu,l,\omega)=1$$
$$(\omega+i\eta-\xi_\nu)G^R(\nu,l,\omega)-t_\nu^* G^R(l,l,\omega)=0$$

$$H=H+V_{int} \tag{1}$$

$$G^R(l,l,\omega)=\frac{1}{\omega-\xi_0-\Sigma^R(\omega)} \tag{2}$$

$$\Sigma^R(\omega)=\sum_\nu\frac{|t_\nu|^2}{\omega-\xi_\nu+i\eta} \tag{3}$$

Lehmann representation

$$G^>(\nu;t,t')=-i(c_\nu(t)c_\nu^\dagger(t'))=i\frac{1}{Z}\sum_n\langle n|\,e^{\beta H}c_\nu(t)c_\nu^\dagger(t')\,|n\rangle \tag{4}$$
$$=-i\frac{1}{Z}\sum_{nn'}e^{-\beta E_n}\langle n|\,c_\nu\,|n'\rangle\langle n'|\,c_\nu^\dagger\,|n\rangle\, e^{i-(E_n-E_{n'})(t-t')} \tag{5}$$

$$G^>(\nu;\omega)=\frac{-2\pi i}{Z}\sum_{nn'}e^{-\beta E_n}\langle n|\,c_\nu\,|n'\rangle\langle n'|\,c_\nu^\dagger\,|n\rangle\,\delta(E_n-E_{n'}+\omega) \tag{6}$$

$$G^<(\nu;\omega)=\frac{-2\pi i}{Z}\sum_{nn'}e^{(-\beta E_n)}\langle n|\,c_\nu\,|n'\rangle\langle n'|\,c_\nu^\dagger\,|n\rangle\,\delta(E_n-E_{n'}+\omega) \tag{7}$$
$$=\frac{-2\pi i}{Z}\sum_{nn'}e^{(-\beta E_n)}\langle n|\,c_\nu^\dagger\,|n'\rangle\langle n'|\,c_n u\,|n\rangle\,\delta(E_n-E_{n'}+\omega) \tag{8}$$
$$=\frac{-2\pi i}{Z}\sum_{nn'}e^{(-\beta E_n+\omega)}\langle n|\,c_\nu^\dagger\,|n'\rangle\langle n'|\,c_n u\,|n\rangle\,\delta(E_n-E_{n'}+\omega) \tag{9}$$
$$=-G^>(\nu;\omega)e^{-\beta\omega} \tag{10}$$

$$G^R(\nu,\omega) = -i\int_0^\infty dt e^{i(\omega+i\eta)t}\frac{1}{Z}\sum_{nn'} e^{\beta E_n}\left(\langle n| c_\nu |n'\rangle \langle n'| c_\nu^\dagger |n\rangle e^{i(E_n - E_{n'})t} + \langle n| c_\nu^\dagger |n'\rangle \langle n'| c_n u |n\rangle e^{-i(E_n - E_{n'})t}\right) \tag{11}$$

$$= \frac{1}{Z}\sum_{nn'} e^{\beta E_n}\left(\frac{\langle n| c_\nu |n'\rangle \langle n'| c_\nu^\dagger |n\rangle}{\omega + E_n + E_n' + i\eta} + \frac{\langle n| c_\nu^\dagger |n'\rangle \langle n'| c_\nu |n\rangle}{\omega - E_n + E_n' + i\eta}\right) \tag{12}$$

$$= \frac{1}{Z}\sum_{nn'} \langle n| c_\nu |n'\rangle \langle n'| c_\nu^\dagger |n\rangle \left(e^{\beta E_n} + e^{\beta E_{n'}}\right) \tag{13}$$

$$2\mathrm{Im}G^R(\nu,\omega) = -\frac{2\pi}{Z}\sum_{nn'} \langle n| c_\nu |n'\rangle \langle n'| c_\nu^\dagger |n\rangle \left(e^{\beta E_n} + e^{\beta E_{n'}}\right)\delta(\omega + E_n - E_{n'}) \tag{14}$$

$$= -\frac{2\pi}{Z}\sum_{nn'} \langle n| c_\nu |n'\rangle \langle n'| c_\nu^\dagger |n\rangle e^{\beta - E_n}(1 + e^{\beta\omega})\delta(\omega + E_n - E_{n'}) \tag{15}$$

$$= -i(1 + e^{\beta\omega})G^>(\nu,\omega) \tag{16}$$

$$A(\nu,\omega) = -2\mathrm{Im}G^R(\nu,\omega) \tag{17}$$

$$iG^>(\nu,\omega) = A(\nu,\omega)[1 - n_F(\omega)] \tag{18}$$

$$iG^>(\nu,\omega) = A(\nu,\omega)[1 - n_F(\omega)] \tag{19}$$

$$-iG^<(\nu,\omega) = A(\nu,\omega)n_F(\omega) \tag{20}$$

$$G^R(\nu,\omega) = \int \frac{d\omega}{2\pi}\frac{A(\nu,\omega)}{\omega - \omega' + i\eta} \tag{21}$$

$$G^A(\nu,\omega) = \int \frac{d\omega}{2\pi}\frac{A(\nu,\omega)}{\omega - \omega' - i\eta} \tag{22}$$

$$G^R(\nu,\omega) = [G^A(\nu,\omega)]* \tag{23}$$

$$G^R(\nu,\nu',\omega) = [G^A(\nu,\nu,\omega)]^* \tag{24}$$

The spectral function

$$G_0^R(\mathbf{k}\sigma,\omega) = -i\int_\infty^\infty dt\theta(t - t')e^{i\omega(t-t')\eta(t-t')} \tag{25}$$

$$= \frac{1}{\omega - \xi_k + i\eta} \tag{26}$$

$$A_0(k\sigma,\omega) = -2\mathrm{Im}G_0^R(\mathbf{k}\sigma,\omega) = 2\pi\delta(\omega - \xi_k) \tag{27}$$

$$\int_\infty^\infty \frac{d\omega}{2\pi}A(\nu,\omega) = 1 \tag{28}$$

$$\int_\infty^\infty \frac{d\omega}{2\pi}A(\nu,\omega) = -\int_\infty^\infty \frac{d\omega}{2\pi}2ImG^R(\nu,\omega) \tag{29}$$

$$= \int_\infty^\infty d\omega\frac{1}{Z}\sum_{nn'} \langle n| c_\nu |n'\rangle \langle n'| c_\nu^\dagger |n\rangle \left(e^{\beta E_n} + e^{\beta E_{n'}}\right)\delta(\omega + E_n - E_{n'}) \tag{30}$$

$$= \frac{1}{Z}\sum_{nn'} \langle n| c_\nu |n'\rangle \langle n'| c_\nu^\dagger |n\rangle \left(e^{\beta E_n} + e^{\beta E_{n'}}\right) = \langle c_\nu c_\nu^\dagger\rangle + \langle c_\nu^\dagger c_\nu\rangle = \langle c_\nu c_\nu^\dagger + c_\nu^\dagger c_\nu\rangle \tag{31}$$

$$\int_\infty^\infty \frac{d\omega}{2\pi}G^R(\nu,\omega) = G^R(\nu,t=0) = -i\theta(0)\langle\{c_\nu, c_\nu^\dagger\}\rangle = -i\frac{1}{2} \tag{32}$$

Since for fermions the occupation n_ν of a given state ν is given by:

$$\bar{n}_\nu = \langle c_\nu^\dagger c_\nu\rangle = -iG^<(\nu,t=0) \tag{33}$$

$$= -i\int_\infty^\infty \frac{d\omega}{2\pi}G^<(\nu,\omega) \tag{34}$$

$$= \int_\infty^\infty A(\nu,\omega)n_F(\omega) \tag{35}$$