Gap problem

May 26, 2023

1. Using sinh

$$\frac{e^{-\omega_k \tau}}{e^{\omega_k} - 1}$$

$$= \frac{2ie^{-\omega_k \tau}}{2ie^{\frac{\omega_k}{2}}(e^{\frac{\omega_k}{2}} - e^{-\frac{\omega_k}{2}})} = \frac{e^{-\omega_k \tau - \frac{\omega_k}{2}}}{2i \sinh \frac{\omega_k}{2}}$$

$$\frac{\frac{\omega_k}{2}}{\frac{\omega_k}{2} \sinh \frac{\omega_k}{2}} \quad , \quad \lim_{\omega \to 0} \frac{\frac{\omega_k}{2}}{\frac{\omega_k}{2} \sinh \frac{\omega_k}{2}} = \lim_{\omega \to 0} \frac{2}{\omega_k} \to \infty$$

2. Change the Series Expansion form (Wolfram alpha)

$$\frac{e^{-\omega_k \tau}}{e^{\omega_k} - 1}$$

$$\frac{1}{\omega_k} - \tau - \frac{1}{2} + e^{-\omega_k \tau} \quad , \quad \lim_{\omega_k \to 0} \frac{1}{\omega_k} - \tau - \frac{1}{2} + e^{-\omega_k \tau} \to \infty$$