

$$\psi_a \text{---} \text{--}\!\! \blacktriangleright \text{--}\!\! = \psi_a \text{---} \blacktriangleright \text{---} \psi_b^\dagger + \text{---} \blacktriangleright (\Sigma) \text{--}\!\! =$$

$$\begin{aligned} Z &= \langle \psi_a(\tau) \psi_b^\dagger(\tau') \rangle \\ &= \text{Tr}[e^{\beta H(\hat{\tau})} \psi_a(\tau) \psi_b^\dagger(\tau')] \end{aligned}$$

$$H = H_{loc} + H_{bath} + \mathbf{H}_{\text{hyb}} + \mathbf{H}_{\text{hyb}}^\dagger$$

$$\mathbf{H}_{\text{hyb}} = \sum_i c_\nu \mathbf{V}_i b_i^\dagger$$

$$\mathcal{G} = Z_{\text{bath}} \text{Tr}_c \left[\frac{1}{Z_{\text{bath}}} \text{Tr}_b \left[T_\tau e^{-\int_0^\beta d\tau H_{\text{loc}}(\tau) + H_{\text{bath}}(\tau)} \prod_\nu \sum_{k_\nu} Z_{k_\nu} \right] \right]$$

$$\begin{aligned}
Z_{k\nu} = & \sum_{i_1, \dots, i_{k\nu}} \sum_{i'_1, \dots, i'_{k\nu}} V_{i_1}^{\nu} V_{i'_1}^{\nu*} \dots V_{i_{k_j}}^{\nu} V_{i'_{k_j}}^{\nu*} \int_0^{\beta} d\tau \\
& \times \int_{\tau_1}^{\beta} d\tau_2 \dots \int_{\tau_{k\nu-1}}^{\beta} d\tau_{k\nu} \int_{\tau'_1}^{\beta} d\tau'_2 \dots \int_{\tau'_{k\nu-1}}^{\beta} d\tau'_{k\nu} \\
& \times c_{\nu}(\tau_1) b_{i_1}^{\nu\dagger}(\tau_1) b_{i'_1}^{\nu}(\tau'_1) c_{\nu}^{\dagger}(\tau'_1) c_{\nu}(\tau_2) b_{i_2}^{\nu\dagger}(\tau_2) b_{i'_2}^{\nu}(\tau'_2) c_{\nu}^{\dagger}(\tau'_2) \\
& \times \dots c_{\nu}(\tau_{k_j}) b_{i_{k_j}}^{\nu\dagger}(\tau_{k_j}) b_{i'_{k_j}}^{\nu}(\tau'_{k_j}) c_{\nu}^{\dagger}(\tau'_{k_j})
\end{aligned}$$

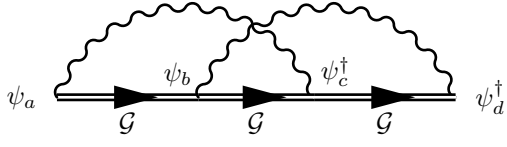
A Feynman diagram representing the decay of a scalar particle into two fermions. A horizontal line with an arrow pointing to the right, labeled \mathcal{G} below it, represents the scalar particle. From this line, two wavy lines branch out upwards and outwards, representing fermions. The left wavy line is labeled ψ_a and the right wavy line is labeled ψ_b^\dagger . Above the wavy lines, a triangle symbol Δ indicates the mass of the fermions.

$$\begin{aligned} \mathcal{G} &= \\ &= \sum_{ab} \left[(\text{sgn}) \psi_a \mathcal{G}(\tau) \psi_b^\dagger \Delta_{ba}(\tau) \right] \end{aligned}$$

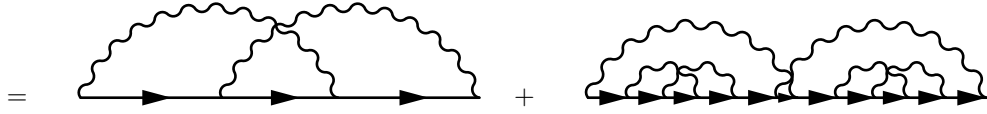
$$= \text{diagram 1} + \text{diagram 2}$$

A diagram of a multi-loop propagator. It consists of a horizontal line with five arrows pointing to the right. Above this line, there are two internal loops, each represented by a cloud-like shape. A large, semi-circular cloud-like shape connects the left and right ends of the horizontal line, passing over the two internal loops. To the left of the diagram is a '+' sign, and to the right is a '+' sign followed by an ellipsis '...'. The entire diagram is enclosed in a rectangular box.

$$\mathcal{G}_{\text{OCA}} =$$



$$= \sum_{\text{abcd}} \int_0^\tau d\tau_1 \int_0^{\tau_1} d\tau_2 \left[\text{sgn}_1 \psi_d^\dagger \mathcal{G}(\tau - \tau_2) \psi_c^\dagger \mathcal{G}(\tau_2 - \tau_1) \psi_b \mathcal{G}(\tau - \tau_1) \psi_a \Delta_{bd}(\tau - \tau_1) \Delta_{ca}(\tau_2) \right]$$



+ ...

$$Z = \text{Tr}[\mathcal{G}(\tau) \lambda_1 \mathcal{G}(0) \lambda_1 \Delta(\tau)]$$

$$Z = Z_{\text{NCA}} + Z_{\text{OCA}}$$

$$= \text{Tr}[\mathcal{G}(\tau) \lambda_1 \mathcal{G}(0) \lambda_1 \Delta(\tau)]$$

$$+ \text{Tr}[\mathcal{G}(\beta - \tau_2) \hat{N} \mathcal{G}(\tau_2 - \tau) \lambda_1 \mathcal{G}(\tau - \tau_1) \hat{N} \mathcal{G}(\tau_1) \lambda_1 \Delta(\tau_2 - \tau_1)]$$