$$G^R(\mathbf{r}t,\mathbf{r}'t') = -i\theta(t-t')\langle[\Psi(\mathbf{r}t),\Psi^\dagger(\mathbf{r}'t')_{B,F}]\rangle$$

$$\begin{aligned} i\frac{\partial}{\partial t}G^R(\mathbf{r}t,\mathbf{r}'t') &= (-i)(i\partial_t\theta(t-t'))\langle[\Psi(\mathbf{r}t),\Psi^\dagger(\mathbf{r}'t')_{B,F}]\rangle + (-i)\theta(t-t')\langle[i\partial\Psi(\mathbf{r}t),\Psi^\dagger(\mathbf{r}'t')_{B,F}]\rangle \\ &= \nabla(t-t')\nabla(\mathbf{r}-\mathbf{r}') + (-i)\theta(t-t')\langle[i\partial\Psi(\mathbf{r}t),\Psi^\dagger(\mathbf{r}'t')_{B,F}]\rangle \end{aligned}$$

$$i\partial_t\Psi(\mathbf{r}t) = -[H,\Psi(\mathbf{r})](t) = -[H_0,\Psi(\mathbf{r})](t) - [V_{int},\Psi(\mathbf{r})](t)$$

$$\begin{aligned} -[H_0,\Psi(r)] &= \frac{1}{2m}\int d\mathbf{r}'[\Psi^\dagger(\mathbf{r}')\nabla^2_{\mathbf{r}'}\Psi^\dagger(\mathbf{r}'),\Psi(\mathbf{r})] \\ &= \frac{1}{2m}\nabla^2_{\mathbf{r}}\Psi(\mathbf{r}) \end{aligned}$$

$$\left(i\partial_t + \frac{1}{2m}\nabla^2_{\mathbf{r}}\right)G^R(\mathbf{r}t,\mathbf{r}'t') = \nabla(t-t')\nabla(\mathbf{r}-\mathbf{r}') + D^R(\mathbf{r}t,\mathbf{r}'t'),\quad D^R(\mathbf{r}t,\mathbf{r}'t') = -i\theta(t-t')\langle[-V_{int},\psi(\mathbf{r})(t),\psi^\dagger(\mathbf{r}'t')]_{B,F}\rangle$$

$$H_0 = \sum_{\nu\nu'} t_{\nu'\nu} a^\dagger_{\nu'} a_{\nu'}$$

$$G^R(\nu t,\nu' t') = -i\theta(t-t')\langle[a_\nu(t)a^\dagger_{\nu'}(t')]_{B,F}\rangle$$

$$-[H_0,a_\nu] = \sum_{\nu"} t_{\nu\nu"} a_{\nu"}$$

$$\sum_{\nu"}(i\delta_{\nu\nu"}\partial_t - t_{\nu\nu"})G^R(\nu" t,\nu' t') = \delta(t-t')\delta_{\nu\nu'} + D^R(\nu t,\nu' t')$$

$$D^R(\nu t,\nu' t') = -i\theta(t-t')\langle\big[-[V_{int},a_\nu](t),a_n u^\dagger]_{B,F}\rangle$$

$$\sum_{\nu"}[\delta_{\nu\nu"}(\omega+i\eta) - t_{\nu\nu"}]G^R(\nu"\nu';\omega) = \delta_{\nu\nu'} + D^R(\nu\nu';\omega)$$

$$D^R(\nu,\nu';\omega) = -i\int_{-\infty}^{\infty} dt e^{i(\omega+i\eta)(t-t')}\theta(t-t')\langle\Big[-[V_{int},a_\nu](t),a^\dagger_{\nu'}(t')]_{B,F}\rangle$$

$$\sum_{\nu"}[\delta_{\nu\nu"}(\omega+i\eta) - t_{\nu\nu"}]G^R(\nu"\nu';\omega) = \delta_{\nu\nu'}$$

$$(G^R_0)^{-1}(\nu\nu';\omega) = \delta_{\nu\nu'}(\omega+i\eta) - t_{\nu\nu'} \equiv (\mathbf{G}^R_0)^{-1}_{\nu\nu'}$$

$$(\mathbf{G}^R_0)^{-1}(\mathbf{G}^R_0) = \mathbf{1}$$

$$(\mathbf{G}^R_0)_{\nu\nu'} = G^R_0(\nu,\omega)\delta_{\nu\nu'} = \frac{1}{\omega-\epsilon_\nu+i\eta}\delta_{\nu\nu'}$$

$$H = H_0 + H_{hyb} + H_l$$

$$H_0 = \sum_\nu \xi_\nu c^\dagger_\nu c_\nu$$

$$H_l = \xi_0 c^\dagger_l c_l$$

$$H_{hyb} = \sum_{\nu} (t *_{\nu} c_{\nu}^{\dagger} c_l + t_{\nu} c_l^{\dagger} c_{\nu})$$

$$G^R(l, l, t - t') = -i\theta(t - t')\langle\{c_l(t), c_l^{\dagger}(t')\}\rangle$$
$$G^R(\nu, l, t - t') = -i\theta(t - t')\langle 1\{c_{\nu}(t), c_{.}^{\dagger}(t')\}\rangle$$

$$(\omega + i\eta - \xi_0)G^R(l, l, \omega) - \sum_{\nu} t_{\nu} G^R(\nu, l, \omega) = 1$$
$$(\omega + i\eta - \xi_{\nu})G^R(\nu, l, \omega) - t_{\nu}^* G^R(l, l, \omega) = 0$$

$$H = H + V_{int} \tag{1}$$

$$G^R(l, l, \omega) = \frac{1}{\omega - \xi_0 - \Sigma^R(\omega)} \tag{2}$$

$$\Sigma^R(\omega) = \sum_{\nu} \frac{|t_{\nu}|^2}{\omega - \xi_{\nu} + i\eta} \tag{3}$$