Let $H_0 = -E_c \frac{\partial^2}{\partial \phi^2} - E_J \cos \phi$, $H_0 \psi$ become :

even:
$$-\frac{E_c}{\sqrt{\pi}} \left(\sum_{m=1}^{\infty} (-m^2) a_m \cos m\phi - \frac{a_0}{\sqrt{2}} \gamma \cos \phi - \frac{\gamma}{2} \sum_{m=1}^{\infty} a_m (\cos(m+1)\phi + \cos(m-1)\phi) \right)$$
 (1)

odd:
$$-\frac{E_c}{\sqrt{\pi}} \left(\sum_{m=1}^{\infty} (-m^2) b_m \sin m\phi - \frac{\gamma}{2} \sum_{m=1}^{\infty} a_m (\sin(m+1)\phi + \sin(m-1)\phi) \right)$$
 (2)

And its matrix form is:

$$E_{c} \begin{pmatrix} 0 & -\frac{\gamma}{2} & 0 & 0 & \cdots \\ -\frac{\gamma}{\sqrt{2}}\cos\phi & \cos\phi & -\frac{\gamma}{2}\cos\phi & 0 & \cdots \\ 0 & -\frac{\gamma}{2}\cos2\phi & 4\cos2\phi & -\frac{\gamma}{2}\cos2\phi & \cdots \\ 0 & 0 & -\frac{\gamma}{2}\cos3\phi & 9\cos3\phi & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix} \begin{pmatrix} a_{0} \\ a_{1} \\ a_{2} \\ a_{3} \\ \vdots \end{pmatrix}$$
(3)

$$E_{c} \begin{pmatrix} \sin \phi & -\frac{\gamma}{2} \sin \phi & 0 & 0 & \cdots \\ 0 & -\frac{\gamma}{\sqrt{2}} \sin 2\phi & 4 \sin 2\phi & -\frac{\gamma}{2} \sin 2\phi & \cdots \\ 0 & 0 & -\frac{\gamma}{2} \cos 3\phi & 9 \cos 2\phi & \cdots \\ 0 & 0 & 0 & -\frac{\gamma}{2} \cos 4\phi & \cdots \\ \vdots & \vdots & \vdots & \vdots & \cdots \end{pmatrix} \begin{pmatrix} b_{1} \\ b_{2} \\ b_{3} \\ \vdots \end{pmatrix}$$
(4)

Also, let form of ψ as : $\psi(x) = \sum_{-\infty}^{\infty} \frac{c_m e^{im\phi}}{\sqrt{2\pi}}$, $H_0 \psi$ is :

$$\frac{E_c}{\sqrt{2\pi}} \left(\sum_{-\infty}^{\infty} c_m m^2 e^{im\phi} - \gamma \sum_{-\infty}^{\infty} c_m e^{i(m+1)\phi} - \gamma \sum_{-\infty}^{\infty} c_m e^{i(m-1)\phi} \right)$$
 (5)

for $m \geq 0$,

$$\frac{E_c}{\sqrt{2\pi}} \begin{pmatrix}
0 & -\gamma & 0 & 0 \cdots \\
-\gamma e^{i\phi} & e^{i\phi} & -\gamma e^{i\phi} & 0 \cdots \\
0 & -\gamma e^{i2\phi} & 4e^{i2\phi} & -\gamma e^{i2\phi} \cdots \\
0 & 0 & -\gamma e^{i3\phi} & 9e^{i3\phi} \cdots \\
\vdots & \vdots & \vdots & \cdots
\end{pmatrix} \begin{pmatrix}
c_0 \\
c_1 \\
c_2 \\
c_3 \\
\vdots
\end{pmatrix}$$
(6)