Matsubara Summary(2) : Equations

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Equations for summarizing to derive Green functions

$$S(\nu,\tau) = \frac{1}{\beta}\sum_{ik_n} G(\nu, ik_n)e^{ik\tau} \tag{1}$$

$$S_2(\nu_1,\nu_2,i\omega,\tau) = \frac{1}{\beta}\sum_{ik_n} G_0(\nu, ik_n)G_0\nu_2, ik_n + i\omega_n e^{ik\tau} \tag{2}$$

$$S^B(\tau) = \frac{1}{\beta}\sum_{i\omega_n} g(i\omega_n)e^{i\omega_n\tau} \tag{3}$$

$$n_B(\tau) = \frac{1}{e^{\beta z}-1} \tag{4}$$

$$\begin{aligned} Res_{z=i\omega_n}[n_B(z)] &= \lim_{z\to i\omega_n} \frac{(z-i\omega_n)}{e^{\beta z}-1} \\ &= \lim_{\delta\to 0} \frac{\delta}{e^{\beta i\omega_n}e^{\beta\delta}-1} \\ &= \frac{1}{\beta} \end{aligned}$$

$$\begin{aligned} \oint dz n_B(z)g(z) &= 2\pi i \operatorname*{Res}_{z=i\omega_n}(n_B(z)g_B(i\omega_n)) \\ &= \frac{2\pi i}{\beta}g(i\omega_n) \end{aligned}$$

$$S^B = \int_C \frac{dz}{2\pi i} n_B(z)g(z) \tag{5}$$

$$S_0^F(\tau) = \frac{1}{\beta}\sum_{ik_n} g_0(ik_n)e^{ik_n\tau}, \qquad \tau > 0 \tag{6}$$

$$g_0(z) = \prod_j \frac{1}{z-z_j} \tag{7}$$

$$n_F(z)e^{\tau z} = \frac{e^{\tau z}}{e^{\beta z}+1} \propto \tag{8}$$

$$\begin{aligned} 0 &= \int_{C_\infty} \frac{dz}{2\pi i} n_B(z)g_0(z)e^{\tau z} \\ &= -\frac{1}{\beta}\sum_{ik_n} g_0(ik_n)e^{ik_n\tau} + \sum_j \operatorname*{Res}_{z=z_j}(g_0(z))n_F(z_j)e^{\tau_j\tau} \end{aligned}$$

$$\frac{1}{\beta}\sum_{i\omega_n} e^{o\omega_n\tau} = S_0^B(\tau) = -\sum_j \operatorname*{Res}_{z=z_j}[g_0(z)]n_B(z_j)e^{\tau_j\tau} \tag{9}$$

$$S(\tau) = \frac{1}{\beta}\sum_{ik_n} g(ik_n)e^{ik_n\tau} \tag{10}$$

$$\begin{aligned} S(\tau) &= -\int_{C_1+C_2}\frac{dz}{2\pi}n_F(z)g(z)e^{z\tau} \\ &= -\frac{1}{2\pi}\int_{\infty}^{\infty} d\epsilon n_F(\epsilon)[g(\epsilon+i\eta)-g(\epsilon-i\eta)]e^{\epsilon\tau} \end{aligned}$$

$$\langle a_\nu^\dagger a_\nu\rangle = G(\nu, 0^-) \tag{11}$$

$$\begin{aligned} \leftrightarrow &= \frac{1}{\beta}\sum_{ik_n} G(\nu, ik_n)e^{-ik_n 0^-} \\ &= G_1(\nu, 0^+) \\ &= \int_{-\infty}^{\infty}\frac{d\epsilon}{2\pi}n_F(\epsilon)A(\nu,\epsilon) \end{aligned}$$