Equation of motion in Heisenberg picture with Quadratic Hamiltonian.
let $\hbar = 1$.

$$\frac{\partial}{\partial t} a_\nu = -i[H, a_\nu]$$

Where H represents the Hamiltonian

$$\begin{aligned}
[\sum_{\nu\nu'} \epsilon_{\nu\nu'} a_\nu a^\dagger_{\nu'}, a_\nu] &= \sum_{\nu\nu'} \epsilon_{\nu\nu'} a_\nu a^\dagger_{\nu'} a_\nu - a_\nu \sum_{\nu\nu'} \epsilon_{\nu\nu'} a_\nu a^\dagger_{\nu'} \\
&= \sum_{\nu\nu'} \epsilon_{\nu\nu'} a_\nu (1 - a_\nu a^\dagger_{\nu'}) - a_\nu \sum_{\nu\nu'} \epsilon_{\nu\nu'} a_\nu a^\dagger_{\nu'} \\
&= \sum_{\nu\nu'} \epsilon_{\nu\nu'} a_\nu
\end{aligned}$$

Thus

$$\frac{\partial}{\partial t} a_\nu = \sum_{\nu\nu'} \epsilon_{\nu\nu'} a_\nu$$

$$a_\nu = e^{-i\epsilon t} \tag{1}$$