$$\Delta X_1 = \frac{1}{2}e^{-r}, \Delta X_2 = \frac{1}{2}e^{-r}$$

$$\hat{X}_1 \to \hat{X}_1 e^{-r}, \hat{X}_2 \to \hat{X}_2 e^{-r}$$

$$(\hat{a} + \hat{a}^\dagger) \to (\hat{a} + \hat{a}^\dagger)e^{-r}, (\hat{a} - \hat{a}^\dagger) \to (\hat{a} - \hat{a}^\dagger)e^{-r}$$

$$\hat{a} \to \hat{b} = \hat{a}\cosh r - \hat{a}^\dagger \sinh r$$

$$\hat{a} \to \hat{b}^\dagger = \hat{a}^\dagger \cosh r - \hat{a}^\dagger \sinh r$$

If Operator $\hat{S}(r)$ is,

$$\hat{S}^\dagger(r)\hat{a}\hat{S}(r) = \hat{b} = \hat{a}\cosh r - \hat{a}^\dagger \sinh r$$

$$\hat{S}^\dagger(r)\hat{a}^\dagger\hat{S}(r) = \hat{b} = \hat{a}^\dagger \cosh r - \hat{a}\sinh r$$

Or,

$$\hat{S}^\dagger(r)\hat{X}_1\hat{S}(r) = \hat{X}_1 e^{-r}$$

$$\hat{S}^\dagger(r)\hat{X}_2\hat{S}(r) = \hat{X}_2 e^{r}$$

Then Squeezed state given by:

$$|r, \alpha\rangle = \hat{S}(r)\,|\alpha\rangle = \hat{S}(r)\hat{D}(\alpha)\,|0\rangle$$

The Operator which satisfies the above equation is given by:

$$\hat{S}(r) = e^{\frac{r}{2}\hat{a}^2 - \frac{r}{2}\hat{a}^{\dagger 2}}$$

For any complex number $z = re^{i\theta}$,

$$\hat{S}(r) = e^{\frac{z^*}{2}\hat{a}^2 - \frac{z}{2}\hat{a}^{\dagger 2}}$$

Optimize $|F(x, y) - G(x, y, z)|$ where $F = 4x + y$, $G = 0.875(x + y + z)$