The effective Hamiltonian describing cross-Kerr interaction between modes a and b can be given as:

$$\hat{H_{kerr}} = \hbar\chi^(3)\hat{n_a}\hat{n_b}$$

where $\chi^(3)$ is the (rescaled) third-order susceptibility of the nonlinear medium, and $\hat{n_a} = \hat{a}^\dagger\hat{a}$ and $\hat{n}_b = \hat{b}^\dagger\hat{b}$ are the photon number operators given in terms of the annihilation ($\hat{a}$ and $\hat{b}$)and creation($\hat{a}^\dagger$ and $\hat{b}^\dagger$)operators.

We analyze photon-number qubits as superpositions of vacuum and single-photon Fock states. Using an appropriately strong cross-Kerr interaction,
it is possible to perform the CPHASE gate on two qubits so that the states $|00\rangle, |01\rangle$, and $|10\rangle$ are unchanged,

but the two single-photon states gain some additional phase, δ.(i.e., $|11\rangle \rightarrow e^{i\delta}|11\rangle$). In particular, for $\delta = \pi$, the CPHASE gate becomes the controlled-sign (CSIGN) gate, which is equivalent up to a unitary transformation, to the controlled-NOT gate.