The DEMATEL-based completion method for iPCMs in AHP (Zahou et al. 2018)

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It includes the warning in the case I-N is singular. Finally, in the completed matrix A, the existing/known entries of the incomplete PCM remain unchanged.

```
clear; clc
```

Step 1: Replace the missing entries with zeros and write the known entries as it is. And then normalize the direct-relation D using N=D/maxSum.

```
% Example
D = [...
   1    0   0.1428   5        0.1111;...
   0    1   0        0.1111   0.1250;...
   7    0   1        6.0000   4;...
   0.2  9   0.1667   1        0;...
   9    8    0.2500  0        1];


% Compute the sum of each row sum(dij) and column sum(dij) of DRM D
n = size(D,1); % matrix order
I = eye(n);  % n × n identity matrix.

% Preallocate memory to speed up the algorithm
SumOfRows=zeros(n,1);
SumOfColumns=zeros(n,1);
    for i=1:n
        SumOfRows(i,1) =sum(D(i,:)); % Calculate the sum of each row of D
        SumOfColumns(i,1) = sum(D(:,i)); % Calculate the sum of each column of D
    end

% Normalize the direct-relation D
maxSumOfRows = max(SumOfRows); %maximum of row sum
maxSumOfColumns = max(SumOfColumns); %maximum of column sum
maxSum = max(maxSumOfRows,maxSumOfColumns); % maximum of the maximum row sum and maxmum
                                            % column sum
disp('The normalized matrix N is:')
```

```
The normalized matrix N is:
```

```
N = D/maxSum % Normalize the DRM D
```

```
N = 5×5
    0.0548         0    0.0078    0.2740    0.0061
         0    0.0548         0    0.0061    0.0068
```

```
    0.3836         0    0.0548    0.3288    0.2192
    0.0110    0.4932    0.0091    0.0548         0
    0.4932    0.4384    0.0137         0    0.0548
```

Step 2: Convert the direct-relation matrix D into total relation matrix where T = N(I − N)^{-1}.

```
disp ('The total-relation matrix T is: ')
```

```
The total-relation matrix T is:
```

```
T = (I - N)\N
```

```
T = 5×5
    0.0723    0.1700    0.0121    0.3161    0.0109
    0.0043    0.0659    0.0002    0.0082    0.0078
    0.5743    0.4010    0.0717    0.5418    0.2551
    0.0202    0.5619    0.0106    0.0711    0.0067
    0.5697    0.5888    0.0219    0.1766    0.0710
```

Step 3: Transform the total-relation matrix T into the complete matrix C

```
disp ('The complete matrix C using DEMATEL: ')
```

```
The complete matrix C using DEMATEL:
```

```
% C = sqrt(T ./ T') % alternative way;
% Fill in the upper triangular first and then lower triangular by reciprocal condition
C = zeros(n,n);
 for i=1:n-1
     for j=i+1:n
    C(i,j) = sqrt(T(i,j)/T(j,i));
    C(j,i) = 1/C(i,j);
     end
 end %for i<j

 % all diagonals of C are 1
 for i=1:n
     C(i,i)=1;
 end
 C % completed matrix, but some of the existing entries are changed
```

```
C = 5×5
    1.0000    6.3180    0.1451    3.9555    0.1386
    0.1583    1.0000    0.0238    0.1206    0.1151
    6.8920   41.9934    1.0000    7.1442    3.4096
    0.2528    8.2893    0.1400    1.0000    0.1943
    7.2161    8.6864    0.2933    5.1475    1.0000
```

```
%  C.*C' % to check reciprocal condition (if all ones)

%
% To write the original matrix with the missing values obtained without
% changing the existing entry values. Because in C, the original known
% entries were changed.  I complete the matrix D_completed with the elements
% taken from D
```

```
D_completed = zeros(n,n); %preallocate memory
for i = 1:n
    for j=1:n
        if D(i,j)==0  % Missing entries position
       D_completed(i,j) = C(i,j); % new matrix C from DEMATEL
        else
       D_completed(i,j)  = D(i,j); % the kownn entries in the given
                                   % incomplete PCM
        end
    end
end
D_completed; % The comleted matrix - full matrix
```

Step 4: Restrict the missing comparisons to be in the interval [1/9, 9]: if cij>9, then aij=9, and if cij<1/9, then aij=1/9. Because it may happen that the estmated values are out of [1/9,9]. For instance, C(3,2)=41.9934 in the given example.

```
completedMatrix = D_completed;
for i = 1:n
    for j = 1:n
        if completedMatrix(i,j)<1/9
            completedMatrix(i,j)=1/9;
        elseif completedMatrix(i,j)>9
             completedMatrix(i,j)=9;
        else
            completedMatrix(i,j)=completedMatrix(i,j);
        end
    end
end
disp('The completed matrix A with a restriction is: ')
```

```
The completed matrix A with a restriction is:
```

```
A = completedMatrix % full matrix
```

```
A = 5×5
    1.0000    6.3180    0.1428    5.0000    0.1111
    0.1583    1.0000    0.1111    0.1111    0.1250
    7.0000    9.0000    1.0000    6.0000    4.0000
    0.2000    9.0000    0.1667    1.0000    0.1943
    9.0000    8.0000    0.2500    5.1475    1.0000
```