Ergu et al.'s 2016 Least Square Method (LSM) with MATLAB'S fmincon

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The LSM could be used to find the optimal values of missing comparisons by minimizing the sum of errors (geometric mean induced bias matrix (GMIBEM) ε) squares formula:

$$\text{Minimize } f(a_{\text{ij}}, x) = \sum_{j=1}^{n} \sum_{i=1}^{n} \left(\varepsilon_{\text{ij}}(x, a_{\text{ij}})\right)^2,$$

subject to the interval constraint $\left[\frac{1}{9}, 9\right]$.

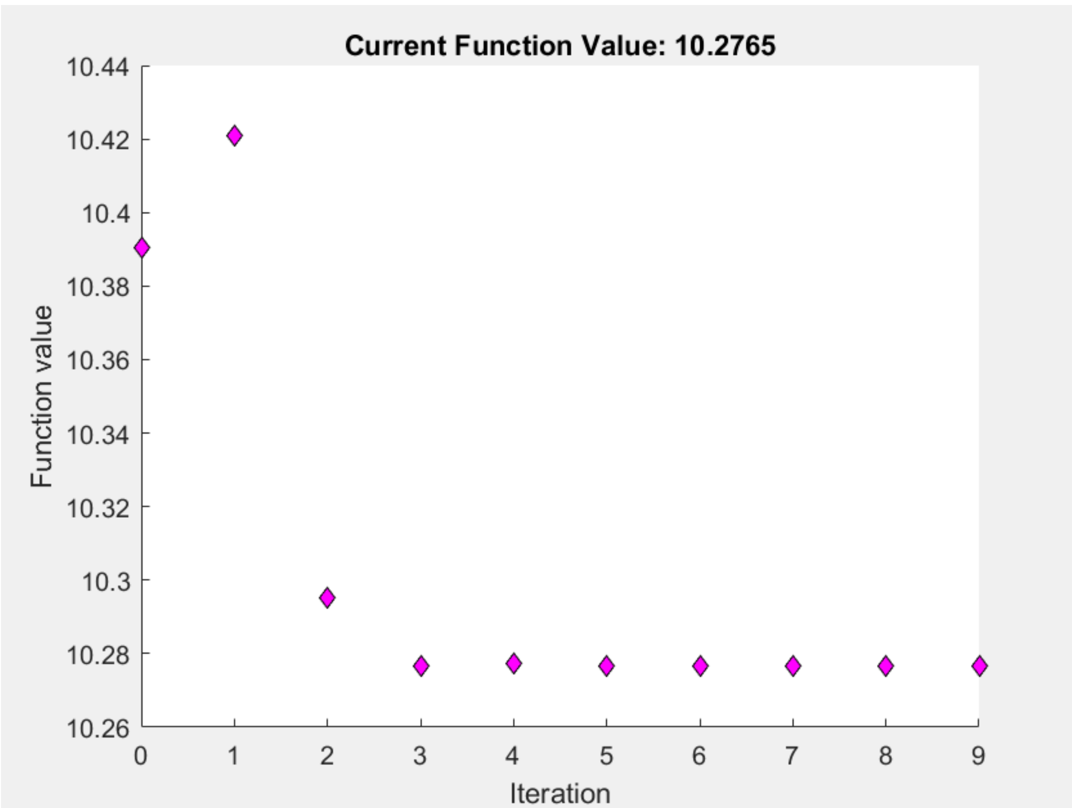
References:

1. Tekile, H. A., Brunelli, M., & Fedrizzi, M. (2023). A numerical comparative study of completion methods for pairwise comparison matrices. *Operations Research Perspectives*, 100272.
2. Ergu, D.; Kou, G.; Peng, Y.; Zhang, M. Estimating the missing values for the incomplete decision matrix and consistency optimization in emergency management. Appl. Math. Model. 2016, 40, 254–267


```
clear; close; clc

% global variables
global A;
global NumberOfMissingEntries;
global n;
global xPosition;


% Incomplete PCM
A=[...
 1   0 3 1;...
 0 1  1/2 4;...
 1/3 2  1 5;...
 1 1/4 1/5 1];

%size of A
n = size(A,2);

% number of missing entries in  the upper triangular  matrix A
NumberOfMissingEntries = 1;

x0 = ones(1,NumberOfMissingEntries); % initial value
lb = 1/9*ones(1,NumberOfMissingEntries); % lower bound
```

```
ub = 9*ones(1,NumberOfMissingEntries); % upper bound

%% MATLAB fmincon - applying the algorithm 'interior-point'
options = optimoptions('fmincon','Display','iter','Algorithm',...
    'interior-point','PlotFcns',@optimplotfval);
x= fmincon(@objectiveFunction,x0, [],[],[],[],lb,ub,[],options)
```

```
                                            First-order      Norm of
 Iter F-count            f(x)  Feasibility   optimality         step
    0       2    1.039049e+01    0.000e+00    1.119e+00
    1       5    1.042097e+01    0.000e+00    8.330e-01    5.386e-01
    2       7    1.029509e+01    0.000e+00    2.749e-01    2.092e-01
    3       9    1.027665e+01    0.000e+00    1.047e-01    1.148e-01
    4      11    1.027751e+01    0.000e+00    7.834e-02    3.409e-02
    5      13    1.027651e+01    0.000e+00    1.906e-02    1.914e-02
    6      15    1.027646e+01    0.000e+00    5.193e-04    4.920e-03
    7      17    1.027646e+01    0.000e+00    1.796e-04    1.184e-04
    8      19    1.027646e+01    0.000e+00    1.803e-06    4.198e-05
    9      21    1.027646e+01    0.000e+00    1.796e-08    3.643e-07
```




```
Local minimum found that satisfies the constraints.

Optimization completed because the objective function is non-decreasing in
feasible directions, to within the value of the optimality tolerance,
and constraints are satisfied to within the value of the constraint tolerance.

<stopping criteria details>
x = 1.2247
```

```
% to write the reconstructed complete PCM
for r = 1:NumberOfMissingEntries % r is another index
    A(xPosition(r,1),xPosition(r,2)) = x(r); % put x(r) in the position (i,j);
    A(xPosition(r,2),xPosition(r,1)) = 1/x(r); % put 1/x(r) in the position (j,i)
```

```
end

% The Completed matrix
B = A;
disp(B)
```

```
    1.0000    1.2247    3.0000    1.0000
    0.8165    1.0000    0.5000    4.0000
    0.3333    2.0000    1.0000    5.0000
    1.0000    0.2500    0.2000    1.0000
```

```
 function LSM =objectiveFunction(x) % objective function
%
global A;
global xPosition;
global NumberOfMissingEntries;
global n;
%xPosition=ones(NumberMissingEntries,2); % preallocate memory for a variable
                                  % to speed up the algorithm
index = 1; % an index that verifies the position of i and j
for j = 2:n % for i < j
    for i = 1:j-1
        if A(i,j) == 0
            xPosition(index,1) = i; % position of t in the ith row
            xPosition(index,2) = j; % position of t in the jth column
            index = index+1; % update index until (A(i,j) == 0) ends
        end
    end
end

%t0 = log(ones(1,numberOfMissing)); % initial value: t0 = log([1,1]) or x0 = (1,1).
% using exponential function x = exp(t)
for s = 1:NumberOfMissingEntries % s is another index
    A(xPosition(s,1),xPosition(s,2)) = x(s); % put x in the position (i,j);
    A(xPosition(s,2),xPosition(s,1)) = 1/x(s); % put 1/x in the position (j,i)
end

% Least square Method - Ergu et al. 2016
AT = A'; % transpose of A
L = ones(n,1);
R = ones(1,n);
for i=1:n
    for j=1:n
        L(i,:)=nthroot(prod(A(i,:)),n); % row geometric mean
        R(:,j)=nthroot(prod(A(:,j)),n); % column geometric mean
    end
end

L; %row geometric mean
R;  % column geometric mean
```

```
P = L*R; % matrix product
ErrorMatrix = P.*AT - 1; % GMIBEM error matrix

% Error square
LSM = 0;
for i = 1:n
    for j = 1:n
  LSM = LSM + ErrorMatrix(i,j).^2; % LSM
 %ErrorSquare = ErrorSquare + abs(ErrorMatrix(i,j)); % LAE

    end
end
LSM;

end
```