

Auction Econometrics

Empirical Industrial Organization

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- Structural approach: we want to estimate primitives of the auction model, i.e. valuations
- From the observation of the bids b_1, \dots, b_n (and the auction rules) we want to recover the valuations v_1, \dots, v_n or equivalently the distribution $F(v)$.
- We can use observations of repeated auctions (assumption of the same bidders)
- When F is estimated, then the following questions can be addressed:
 - Market power of bidders: margin $v - p$
 - Optimal auction format (maximize revenue)
 - Optimal reserve price

Refresher Theory

- Assume there are I bidders
- bidder i has valuation U_i
- U_i s are i.i.d. draws from a distribution F with density f
- in a second price auction, the equilibrium bid of bidder i is $b_i = U_i$
- in a first-price auction, the equilibrium bid of bidder i is given by

$$\beta(U_i) = E[U_{-i} | U_{-i} < U_i]$$

where $U_{-i} = \max_{j \neq i} U_j$ is the highest bid by a competitor

- the bid in a first-price auction can also be written as

$$\beta(U_1) = U_1 - \int_0^{U_1} \left(\frac{F(x)}{F(U_1)} \right)^{I-1} dx$$

Refresher Theory

- There is a revenue equivalence between first-price auctions and second-price auctions: both generate the same expected revenue
- Revenue equivalence also holds when there is a reserve price
- The optimal reserve price is given by

$$r - \frac{1 - F(r)}{f(r)} = c$$

where c are the (opportunity) costs of the seller

Estimation of First-Price auction

Distinct empirical approaches to infer distribution F :

- 1 Laffont, Ossard and Vuong (Econometrica 1995):
“Econometrics of First-Price Auctions”
 - Use revenue equivalence theorem (“elegant”)
- 2 Donald and Paarsch (1993):
 - “Brute-force” approach: computationally intensive
- 3 Guerre, Perrigne and Vuong (Econometrica 2000): Indirect inference approach
 - Very influential methodology, straightforward and relatively simple

Estimation of First-Price auction

Laffont, Ossard and Vuong (1995)

- Observe winning bids only
- Dutch auction (bidders with lower bids never have a chance to bid)
- Idea: revenue equivalence
- By revenue equivalence:

$$E[\textit{Winning Bid}] = E[\textit{2nd Highest Valuation}]$$

Can infer directly the distribution of second highest valuation (second order statistic).

- Need parametric assumption on the distribution of valuation: $f(v|\theta)$, $F(\cdot)$

Estimation of First-Price auction

Laffont, Ossard and Vuong (1995)

- Simulation estimator in practice: for a value of parameter θ and each auction I
 - Prepare S simulations $s = 1 : \dots, S$
 - Draw v_1^s, \dots, v_N^s , vector of simulated valuations for auction I
 - Sort the draws in ascending order
 - Set $b_I = v_{(2)}$ (2nd highest valuation)
 - Approximate $E(b_I; \theta) = \frac{1}{S} \sum b_I^s$
 - Estimate θ by simulated non linear least squares:

$$\min_{\theta} \frac{1}{L} \sum_I (b_I^w - E(b_I^w; \theta))^2$$

- Caveat:
 - Revenue equivalence assumes symmetric bidders (does not work for bidder heterogeneity)
 - Requires a parametric assumption on the distribution function

Estimation of First-Price auction

Direct inference approach

- Donald and Paarsch (1993), and others
- Idea: need to specify the density of observed data (which are bids) to write down likelihood.
- Find inverse bid function:

$$v = b^{-1}(b, \theta)$$

where θ are parameters of density.

- Plug into distribution

$$F(b^{-1}(b, \theta), \theta)$$

Distribution of bids:

$$H(b, \theta) = F(b^{-1}(b, \theta), \theta)$$

with density

$$h(b, \theta) = f(b^{-1}(b, \theta), \theta) \cdot \frac{\partial b^{-1}(b, \theta)}{\partial b}$$

Estimation of First-Price auction

Direct inference approach

- Notice that:

$$\frac{\partial b^{-1}(b, \theta)}{\partial b} = \frac{1}{\frac{\partial b(v)}{\partial v}}$$

which has a simple analytic form for some distributions F .

- Example: Uniform distribution with N bidders: $v \sim U[0, \theta]$

$$b(v) = \frac{n-1}{n} v$$

$$b'(v) = \frac{n-1}{n}$$

- Likelihood:

$$L(\theta) = \prod_{t=1}^T \prod_{i=1}^N f(b^{-1}(b_i^t, \theta), \theta) \cdot \frac{\partial b^{-1}(b_i^t, \theta)}{\partial b}$$

Estimation of First-Price auction

Direct inference approach

- Caveat: regularity condition of Maximum Likelihood is violated, support of bids depends on θ
- Donald and Paarsch (1993) derive asymptotic distribution of ML estimator.
- Asymmetric bidders, bidder heterogeneity \implies Need to numerically solve for the equilibrium (no analytic expression is known).
- Computationally very intensive.

Estimation of First-Price auction

Indirect Inference

- Guerre, Perrigne and Vuong (Econometrica, 2000)
- Idea: Use best response vis-a-vis the empirical distribution of opponents' bids.
- Observe bids, and thus observe density and distribution of bids.
- Bidder i 's problem: win if $b_i \geq b_j$ for all $j \neq i$ Probability of winning is $H(b)^{N-1}$
- A risk neutral bidder will choose a bid that solves

$$\max_b [v - b] H(b)^{N-1}$$

Estimation of First-Price auction

Indirect Inference

- FOC:

$$-H(b)^{N-1} + [v - b](N - 1)H(b)^{N-2}H'(b) = 0$$

or

$$v = b + \frac{H(b)}{(N - 1)H'(b)}$$

- We need first to estimate the distribution (and density) of bids $H(b)$

Estimation of First-Price auction

Indirect Inference

- Can estimate \hat{H} consistently using the empirical distribution function

$$\hat{H}(b) = \frac{1}{TN} \sum_t \sum_i \mathbf{1}(b_i^t \leq b)$$

and Kernel estimator for $\hat{H}'(b)$

$$\hat{H}'(b) = \frac{1}{TN} \sum_t \sum_i \frac{1}{h_g} \kappa\left(\frac{b - b_i^t}{h_g}\right)$$

where $\kappa(\cdot)$ is a kernel function (e.g. normal pdf).

Estimation of First-Price auction

Indirect Inference

- h_g is bandwidth parameter (goes to zero as T goes to infinity)
- We can find optimal bandwidth
- Rule of thumb: $h = \text{std}(\text{bids}) \times (\# \text{observations})^{-1/5}$
- If h_g is zero we get empirical cdf
- Kernel:
 - Epanechnikov: $\kappa(u) = 0.01(1 - u^2)(|u| \leq 1)$
 - Normal: $\kappa(u) = \frac{1}{\sqrt{2\pi}} \exp(-\frac{u^2}{2})$
 - Uniform: $\kappa(u) = \frac{1}{2}|u| \leq 1$
 - Triangular: $\kappa(u) = (1 - |u|)(|u| \leq 1)$

Estimation of First-Price auction

Indirect Inference

- To estimate distribution of v , generate *pseudo-values* \hat{v} :

$$\hat{v}_{it} = b_{it} + \frac{\hat{H}(b_{it})}{(N-1)\hat{H}'(b_{it})}$$

and then estimate distribution function

$$\hat{F}(v) = \frac{1}{TN} \sum_t \sum_i \mathbf{1}(\hat{v}_{it} \leq v)$$

and pdf

$$\hat{f}(v) = \frac{1}{TN} \sum_t \sum_i \frac{1}{h_f} \kappa \left(\frac{\hat{v}_{it} - v}{h_f} \right)$$

Estimation of First-Price auction

Indirect Inference

- With \hat{F} in hand we can:
 - design optimal auction,
 - find optimal reserve price,
 - market design.
- Indirect inference approach extends to:
 - asymmetric bidders,
 - common values,
 - other auction rules.

Estimation of First-Price auction

Identification

When is the distribution of private information identified?

- Distribution is identified if for any F_1, F_2 consistent with data it must be that $F_1 = F_2$.
- Problem illustration: Suppose we have many data points (bids) Question: When is true distribution F uniquely determined from the data?
- Note that non-identification may arise in pooling equilibria \implies need a separating equilibrium (single crossing).
- Ideally: Examine identification problem prior to estimation.

Estimation of First-Price auction

Identification

- From earlier result we know that the inverse bid function

$$v = b + \frac{H(b)}{(N-1)H'(b)}$$

is strictly monotone.

- Guerre, Perrigne and Vuong (Econometrica, 2000):
Proposition: Distribution F is identified if and only if $\frac{H(b)}{H'(b)}$ is strictly monotone.
- Does identification result extend to bidder asymmetry in first-price auctions?
- Yes, because the bid function remains strict monotone.
- FOC characterizes v as a residual vis-a-vis the distribution of opponents' bids.

Estimation of First-Price auction

Second-price auction

- Consider the dominant strategy equilibrium $b(v) = v$
- Distribution of bids is identical to the distribution of private values.
- Hence: Distribution F is identified.

Estimation of a Second-Price Auction

- Assumption: Data, $\left((b_i^t)_{i=1}^N \right)_{t=1}^T$ on a cross section of auctions, $t = 1, \dots, T$, is available, each auction with
 - 1 an identical object
 - 2 fixed number of bidders N
 - 3 independent observations
- Assumption: bids are generated from the dominant strategy equilibrium in which

$$b_i(v_i) = v_i \quad \text{for all } i$$

- Estimate distribution function F using frequency estimator

$$\hat{F}(v) = \frac{1}{TN} \sum_t \sum_i \mathbf{1}(b_i^t \leq v)$$

- What if bids are not generated from dominant strategy equilibrium?

Estimation of a Second-Price Auction

- When strategic equivalence applies then the earlier results from second-price auction extend.
- Note: English auctions used in practice may not share this strategic equivalence.
- English auction may feature:
 - 1 Discrete price increases sometimes step-size in the increment is chosen by bidder
 - 2 Open access: bidders may re-enter later-on, the number of remaining bidders may not be known
 - 3 Bidding costs bid preparation costs, costs to participating in the auction, etc.
- Ebay: late bidding (e.g. Bajari and Hortacsu, Rand (2003)).
- Haile and Tamer (2003) Incomplete model of an English auction.

- Auction is effective price discovery mechanism **under competition**
- Auction very sensitive to collusion
 - Small number of bidders, easily identifiable
 - Often repeated auctions
 - Collusion among bidders can benefit participants at the expense of the seller
- Collusion here is explicit, involves direct communication and side payments

Collusion

Collusive mechanisms

- Second price auction with IPV: ex post efficient collusion by any subset of bidders is possible
 - Implementation with a pre-sale knockout auction
- First price auction with IPV: efficient collusion possible if the ring includes all the bidders
 - E.g. territorial division of bidding privileges
 - Submits many identical bids at the reserve price

- Main difficulty: bidders can submit artificially low bids to hide collusion
- Test for collusion (Porter & Zona 1993):
 - New-York state highway paving jobs
 - They know which bidders were part of the ring
 - Compare the distribution of bids within the two groups
 - Specifically the order of the bids (not the value/magnitude)
 - Exploit the theoretical relation between the order of the bid and cost measures
 - One should expect high cost bidders to place higher bids
 - Cost variables: firm capacity, distance, utilization rate
 - Bids from the cartel do not pass the test while bids outside the cartel pass the test

- Test for collusion (Porter & Zona 1999):
 - School milk procurement process
 - They know which bidders were part of the ring
 - Compare the magnitude of bids near the firm's plant and beyond their local territories.
 - Find that bids further away were not higher than bids for local territory
 - Suggests that bids in the local territory not competitive
 - Consistent with territory allocation
 - Key element here: distance as a cost shifter