Questions

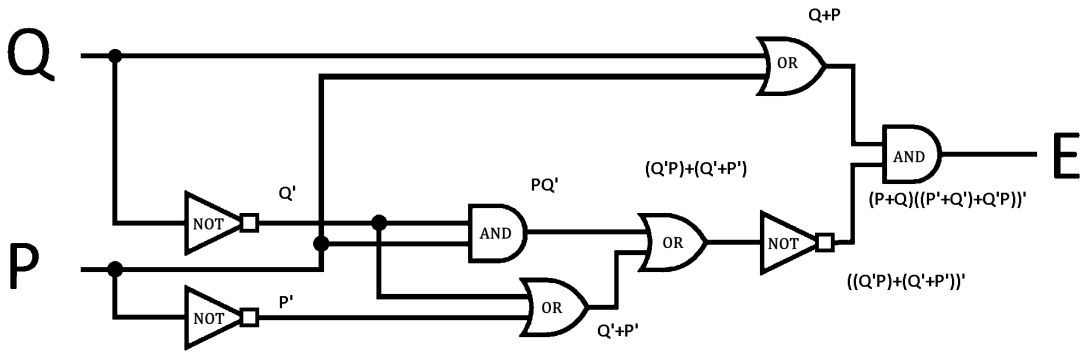
Topic: Boolean algebra and circuits

Question 1 **(2 + 4 + 2 = 8 marks)**

Given the following expression:

$$\boldsymbol{E} = (\boldsymbol{P} + \boldsymbol{Q})((\boldsymbol{P}' + \boldsymbol{Q}') + (\boldsymbol{Q}'\boldsymbol{P}))'$$

a) Draw the circuit that represents this expression as is (do not simplify).



b) Simplify the expression using Boolean algebra rules. State your steps and the rules used.

$$\boldsymbol{E} = (\boldsymbol{P} + \boldsymbol{Q})((\boldsymbol{P}' + \boldsymbol{Q}') + (\boldsymbol{Q}'\boldsymbol{P}))'$$

DeMorgan's Law

$$\boldsymbol{E} = (\boldsymbol{P} + \boldsymbol{Q})(\boldsymbol{P}' + \boldsymbol{Q}')'(\boldsymbol{Q}'\boldsymbol{P})'$$

DeMorgan's Law

$$\boldsymbol{E} = (\boldsymbol{P} + \boldsymbol{Q})(\boldsymbol{P}\boldsymbol{Q})(\boldsymbol{Q} + \boldsymbol{P}')$$

Distributive law

$$\boldsymbol{E} = (\boldsymbol{P}(\mathrm{P}\boldsymbol{Q}) + \boldsymbol{Q}\,(\mathrm{P}\boldsymbol{Q}))\,(\boldsymbol{Q} + \boldsymbol{P}')$$

Idempotent law

$$\boldsymbol{E} = (\boldsymbol{P}\boldsymbol{Q} + \boldsymbol{P}\boldsymbol{Q})(\boldsymbol{Q} + \boldsymbol{P}')$$

$$\boldsymbol{E} = (\boldsymbol{P}\boldsymbol{Q})(\boldsymbol{Q} + \boldsymbol{P}')$$

Distributive law

$$\boldsymbol{E} = (\boldsymbol{P}\boldsymbol{Q}\boldsymbol{Q} + \boldsymbol{P}\boldsymbol{Q}\boldsymbol{P}')$$

Idempotent law

$$\boldsymbol{E} = (\boldsymbol{P}\boldsymbol{Q} + \boldsymbol{P}\boldsymbol{Q}\boldsymbol{P}')$$

Complement law

$$\boldsymbol{E} = (\boldsymbol{P}\boldsymbol{Q} + \boldsymbol{P}\boldsymbol{P}'\boldsymbol{Q})$$

$$\boldsymbol{E} = (\boldsymbol{P}\boldsymbol{Q} + 0\boldsymbol{Q})$$

$$\boldsymbol{E} = \boldsymbol{P}\boldsymbol{Q} + 0$$

Identity law

$$\boldsymbol{E} = \boldsymbol{P}\boldsymbol{Q}$$

c) Given the simplified circuit, state how the depth and size of the circuit have changed compared to the original circuit. The simplified circuit diagram does not need to be included in your submission.

The original circuit has a size of 8 and a depth of 5.

The simplified circuit has a size of 1 and a depth of 1.

The two circuits have a difference in size of 7 and a difference in depth of 4.

Topic: Counting

Question **2** **(3 + 2 + 2 = 7 marks)**

A theme park offers 6 attractions that visitors can experience in one day. These include:

- Roller Coaster
- Ferris Wheel
- Water Slide
- Carousel
- Bumper Cars
- Balloon Darts

Assume that each attraction can be visited at most once a day (e.g., once you have visited the Ferris Wheel, you won't visit it again), and it is not compulsory to visit all attractions. The sequence of visiting the attractions matters. For example, visiting the Bumper Cars before the Water Slide is different from visiting the Water Slide before the Bumper Cars. We're interested in finding out the different ways that visitors can visit these attractions.

a) How many different visiting patterns can be formed from these 6 attractions? Assume that you must visit at least 2 attractions, and it is not compulsory to visit all attractions.

_ _ + _ _ _ + _ _ _ _ + _ _ _ _ _ + _ _ _ _ _ _ ?

6 x 5 + 6 x 5 x 4 + 6 x 5 x 4 x 3 + 6 x 5 x 4 x 3 x 2 + 6 x 5 x 4 x 3 x 2 x 1=1950

$$\text{P(n,k)} = \frac{n!}{(n-k)!}\ , n = 6$$

P(6,2) + P(6,3) + P(6,4) + P(6,5) + P(6,6)

$$= \frac{6!}{(6-2)!} + \frac{6!}{(6-3)!} + \frac{6!}{(6-4)!} + \frac{6!}{(6-5)!} + \frac{6!}{(6-6)!}$$

$$= 30 + 120 + 360 + 720 + 720$$

$$= 1950$$

Answer: 1950

b) How many visiting patterns contain exactly 5 attractions?

P(6,5) $\frac{6!}{(6-5)!}$ = 720

Answer: 720

c) How many visiting patterns with 3 attractions that start with visiting the Carousel?

_ _ _

Carousel _ _ so you only have 2 slots left to fill

Since you already used up 1 attraction, n = 6 – 1 = 5.

Carousel x 5 x 4

5 x 4 = 20

P(5,2) = $\frac{5!}{(5-2)!}$ = 20

Answer: 20

Note: *Include combinatoric/factorial notation as your working steps. An answer consisting of solely an integer will be awarded 0 marks.*

Question 3 (2 + 2 + 2 + 2 + 2 = 10 marks)

You are organizing activities for 11 friends during the upcoming semester break.

a) If you have 7 entry tickets for an E-sports tournament, how many ways are there to choose your friends to join you?

$$\boldsymbol{C(n,r) = \frac{n!}{(n-r)!\ r!}}$$

$$\boldsymbol{C(11,7) = \frac{11!}{(11-7)!\ 7!}}$$

= 330

b) You invite all 11 friends to join you for a movie at cinema. How many possible outcomes are there from this invitation? (Hint: each friend can accept or decline.)

_ _ _ _ _ _ _ _ _ _ _

2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 x 2 = 2^{11}

2 choices, so 2 to the power of friends

2^{11} = 2048

c) You and 10 friends are going for dinner at a Korean steamboat restaurant. You have reserved 3 tables, each with a maximum seating capacity of 4 people. Using the concept of partitioning, how many ways can you organize the seating such that two tables are full and one table is not full?

You and 10 friends means n = 11

$$\boldsymbol{\frac{11!}{4! \times 4! \times 3!} = 11550}$$

d) Show an alternative way to solve the problem in Question 3(c).

C(11,4) x C(7,4) x C(3,3) = 11550

e) A coding bootcamp offers tutorial exercises in 5 programming languages (Python, C++, C#, JavaScript, PHP). How many random tutorial exercises must you complete to guarantee you've done at least 4 tutorial exercises in the same programming language?

$n = km + 1$

$m = 5,$ $k+1 = 4,$

$k + 1 = 4$

$k = 3$

$n = (5)(3) + 1$

$n = 16$

Note: *Include combinatoric/factorial notation as your working steps. An answer consisting of solely an integer will be awarded 0 marks.*

Topic: Graphs and Trees

Question 4 **(2 + 8 + 5 = 15 marks)**

Using the following representation of weighted undirected graph $G(V,E)$:

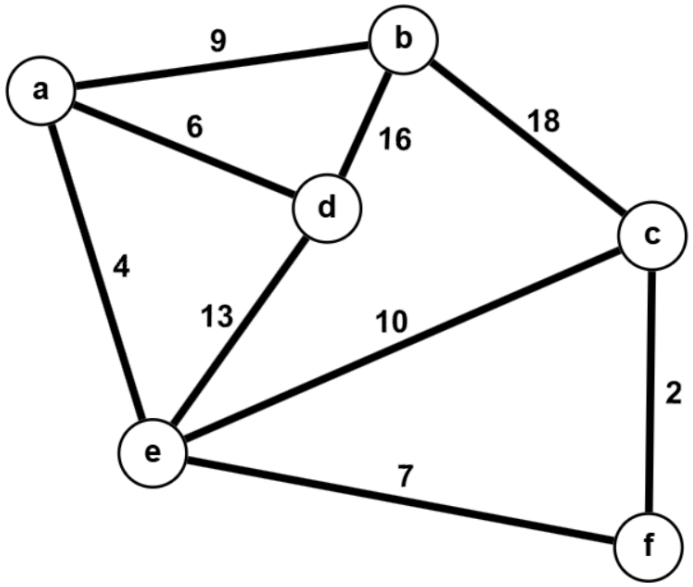
$V = \{a,b,c,d,e,f\}$

Weight of edges E:

$(a,b) = 9$, $(a,d) = 6$, $(a,e) = 4$, $(b,d) = 16$, $(d,e) = 13$,

$(b,c) = 18$, $(c,e) = 10$, $(c,f) = 2$, $(e,f) = 7$

a) Draw the graph G including weights.



b) Using Dijkstra's algorithm, find the shortest distance between vertex d and vertex c in graph G. Use vertex d as the starting point. State the path that should be travelled in order to achieve that shortest distance to vertex c.

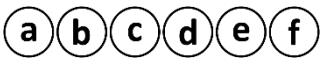
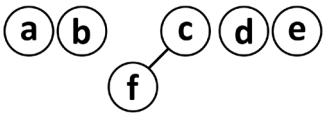
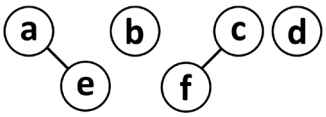
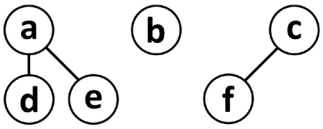
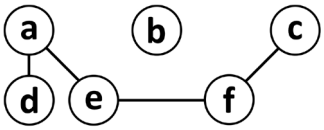
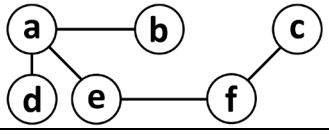
w	**a**	**b**	**c**	**d**	**e**	**f**
d	6_d	16_d	∞	0_d	13_d	∞
a	6_d	$6+9=15_a$	∞	X	$6+4=10_a$	∞
e	X	15_a	$10+10=20_e$	X	10_a	$10+7=17_e$
b	X	15_a	20_e	X	X	17_e
f	X	x	$17+2=19_f$	X	X	17_e
c	x	x	19_f	X	X	x

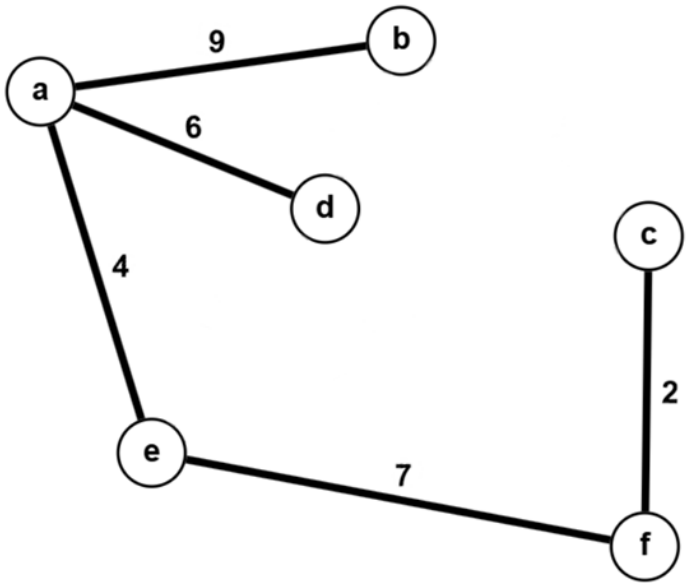
d -> a -> e -> f -> c

Answer: c – f – e – a – d

Note: *Use the method taught in lecture to compose your answer. Any answer consisting of solely sentences to describe the whole process will be awarded 0 marks.*

c) Using Kruskal's algorithm, find a minimum spanning tree for the graph G.

Edge	Cost	Spanning Forest
		
(c,f)	2	
(a,e)	4	
(a,d)	6	
(e,f)	7	
(a,b)	9	



Total weight of minimum spanning tree: 28

Note: *Use the method taught in lecture to compose your answer. Any answer consisting of solely sentences to describe the whole process will be awarded 0 marks.*