MFE5209 ARCH/GARCH

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## Problem 1

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This is a AR(1)/ARCH(1) model, the conditional mean and variance of will be **non-constant**.

Since and , we can rewrite the equation of as follows:

**a)**Conditional Expectation

The conditional mean of is

According to P479 of *SDAFE*, for a ARCH(1) process , the conditional mean =0. Thus we have

So,

**b)**Conditional Variance

The conditional variance of is

Thus, we have

And we also know

Therefore,

## Problem 2

Assume , we can rewrite the model

**a)**Conditional Expectation

**b)**Conditional Variance

**c)**Is the conditional distribution of given and normal? Why or why not?

Yes, it is normal. Reason: There is only one stochastic item in the formula which is normal. And we know the linear combination of a normal distribution is normal.

**d)**Is the marginal distribution of normal? Why or why not?

No, it isn't. Reason: In this case, is a ARCH(1) process, which is of a non-Gaussian distribution with a heavy tail. So is also not Gaussian/normal.

## Problem 3

This problem uses monthly observations of the two-month yield, that is, with T equal to two months, in the data set Irates in the Ecdat package. The rates are log-transformed to stabilize the variance. To fit a GARCH model to the changes in the log rates, run the following R code.

library(fGarch)  
library(Ecdat)  
data(Irates)  
r = as.numeric(log(Irates[,2]))  
n = length(r)  
lagr = r[1:(n-1)]  
diffr = r[2:n] - lagr  
g=garchFit(~arma(1,0)+garch(1,1),data=diffr, cond.dist = "std")

**a)** What model is being fit to the changes in ? Describe the model in detail.

The model is a AR(1)/GARCH(1,1) model, assuming a t-distributed errors. We are trying to use AR(1) to model the return conditional mean and GARCH(1,1) to model its conditional variance. We can write is mathematically as follows:

$$\begin{align\*}
\Delta r\_t=Y\_t=\mu+ar\_1 Y\_{t-1}+a\_t\\
\\
a\_t=\varepsilon\_t \sqrt{\omega + \alpha\_1 a\_{t-1}^2+ \beta\_1 \sigma\_{t-1}^2},
\varepsilon\_t \thicksim t(.)
\end{align\*}$$

In this case, the fitted model is:

$$\begin{align}
\Delta r\_t=Y\_t=0.00910873+0.09508060 Y\_{t-1}+a\_t\\
\\
a\_t=\varepsilon\_t \sqrt{0.00054864 + 0.32444019 a\_{t-1}^2+ 0.74026870 \sigma\_{t-1}^2}\\
\\
\varepsilon\_t \thicksim t(2.83060758)
\end{align}$$

The details of the fitted model are as follows:

summary(g)

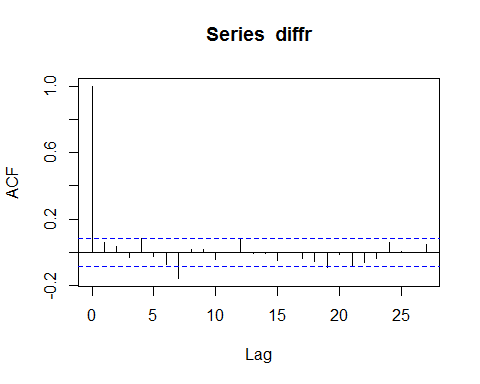
##   
## Title:  
## GARCH Modelling   
##   
## Call:  
## garchFit(formula = ~arma(1, 0) + garch(1, 1), data = diffr, cond.dist = "std")   
##   
## Mean and Variance Equation:  
## data ~ arma(1, 0) + garch(1, 1)  
## <environment: 0x000000000b840608>  
## [data = diffr]  
##   
## Conditional Distribution:  
## std   
##   
## Coefficient(s):  
## mu ar1 omega alpha1 beta1 shape   
## 0.00910873 0.09508060 0.00054864 0.32444019 0.74026870 2.83060758   
##   
## Std. Errors:  
## based on Hessian   
##   
## Error Analysis:  
## Estimate Std. Error t value Pr(>|t|)   
## mu 0.0091087 0.0023895 3.812 0.000138 \*\*\*  
## ar1 0.0950806 0.0432758 2.197 0.028014 \*   
## omega 0.0005486 0.0002968 1.848 0.064564 .   
## alpha1 0.3244402 0.1368876 2.370 0.017782 \*   
## beta1 0.7402687 0.0656629 11.274 < 2e-16 \*\*\*  
## shape 2.8306076 0.4243716 6.670 2.56e-11 \*\*\*  
## ---  
## Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1  
##   
## Log Likelihood:  
## 630.9 normalized: 1.19   
##   
## Description:  
## Sat Oct 11 00:35:58 2014 by user: wufuh\_000   
##   
##   
## Standardised Residuals Tests:  
## Statistic p-Value  
## Jarque-Bera Test R Chi^2 1048 0   
## Shapiro-Wilk Test R W 0.9114 0   
## Ljung-Box Test R Q(10) 7.962 0.6326   
## Ljung-Box Test R Q(15) 11.93 0.6846   
## Ljung-Box Test R Q(20) 12.76 0.8874   
## Ljung-Box Test R^2 Q(10) 13.12 0.2173   
## Ljung-Box Test R^2 Q(15) 14.7 0.4736   
## Ljung-Box Test R^2 Q(20) 17.79 0.6014   
## LM Arch Test R TR^2 14.28 0.2829   
##   
## Information Criterion Statistics:  
## AIC BIC SIC HQIC   
## -2.358 -2.310 -2.358 -2.339

**b)** What are the estimates of the parameters of the model?

The estimates of the model parameters are as follows

**c)** What is the estimated ACF of ?

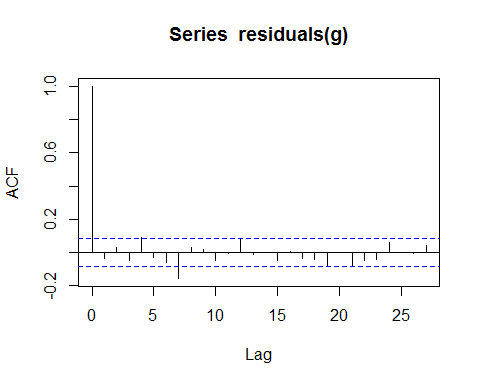
acf(diffr)



**d)** What is the estimated ACF of ?

is uncorrelated, aka if .

acf(residuals(g))



**e)** What is the estimated ACF of ?

acf(residuals(g)^2)

