Maximal Matching as ILP

$$
\begin{cases}
\displaystyle\sum_{e \in E} X_e \to \max \\
\displaystyle\forall u \in V \sum_{e \sim u} X_e \leq 1 \\
\forall e \in E \; X_e \in \{0,1\}
\end{cases}
$$

Dual Linear Problem

We can multiply all the non-equations by a constant y_v for each vertex $v \in V$ and get:

$$
\sum_{\substack{e \in E \\ e:\, u \sim v}} X_e (y_u + y_v) \leq \sum_{v \in V} y_v
$$

Now we can constraint:

$$
(\forall \, u, v \in V \mid u \neq v) \; y_u + y_v \geq 1
$$

And:

$$
(\forall \, v \in V) \; y_v \geq 0
$$