Maximal Matching as ILP

$$\begin{cases} \sum_{e \in E} X_e \to \max \\ \forall u \in V \sum_{e \sim u} X_e \leq 1 \\ \forall e \in E \; X_e \in \{0,1\} \end{cases}$$

Dual Linear Problem

We can multiply all the non-equations by a constant y_v for each vertex $v \in V$ and get:

$$\sum_{\substack{e \in E \\ e:\, u \sim v}} X_e (y_u + y_v) \leq \sum_{v \in V} y_v$$

Now we can constraint:

$$(\forall\, u, v \in V \mid u \neq v)\; y_u + y_v \geq 1$$

And:

$$(\forall\, v \in V)\; y_v \geq 0$$

Finally, we need to minimize:

$$\sum_{v \in V} y_v \to \min$$