Adaptive Noise Smoothing Filter for Images with Signal-Dependent Noise

DARWIN T. KUAN, MEMBER, IEEE, ALEXANDER A. SAWCHUK, SENIOR MEMBER, IEEE, TIMOTHY C. STRAND, MEMBER, IEEE, AND PIERRE CHAVEL

Abstract—In this paper, we consider the restoration of images with signal-dependent noise. The filter is noise smoothing and adapts to local changes in image statistics based on a nonstationary mean, nonstationary variance (NMNV) image model. For images degraded by a class of uncorrelated, signal-dependent noise without blur, the adaptive noise smoothing filter becomes a point processor and is similar to Lee's local statistics algorithm [16]. The filter is able to adapt itself to the nonstationary local image statistics in the presence of different types of signal-dependent noise. For multiplicative noise, the adaptive noise smoothing filter is a systematic derivation of Lee's algorithm with some extensions that allow different estimators for the local image variance. The advantage of the derivation is its easy extension to deal with various types of signal-dependent noise. Film-grain and Poisson signal-dependent restoration problems are also considered as examples. All the nonstationary image statistical parameters needed for the filter can be estimated from the noisy image and no *a priori* information about the original image is required.

Index Terms—Adaptive noise smoothing, image restoration, nonstationary image model, single-dependent noise.

I. Introduction

VARIOUS image restoration and enhancement methods have been proposed for removing degradations due to blurring and noise. The effectiveness of an image restoration algorithm depends on the validity of the image model, the criterion used to judge the quality of the restored image, and the statistical model for the noise process. Early techniques concentrated on nonrecursive algorithms implemented in the discrete frequency domain [1]. The necessary computations are carried out by fast Fourier transform (FFT) techniques. The use of the FFT makes it possible to do image restoration with reasonable computation time. However, these approaches cannot easily deal with images degraded by space-variant blur and assume a stationary image model. More recent work has centered on two-dimensional recursive filtering techniques extended from the one-dimensional Kalman filtering algorithm [2]-[4]. The main problems of two-dimensional recursive filtering are both the difficulty in establishing a suitable two-dimensional recursive model and the high dimensionality of the resulting state vector. Recent results on the reduced update Kalman filter by Woods *et al.* [5], [6] seem to overcome some of these difficulties and have potential for real applications. The advantages of two-dimensional recursive filters are that they require less computation time than nonrecursive algorithms and can handle space-variant blur easily.

The conventional stationary image model assumes that an image is a wide-sense stationary field. The statistical properties of an image are characterized globally rather than locally by its stationary correlation function. This assumption enables the use of FFT-based algorithms in the restoration procedure and consequently reduces the computation time dramatically. However, the restoration filter designed accordingly is insensitive to abrupt changes of the image intensity, and tends to smooth the edges where stationarity is not justified. To overcome these disadvantages and to have an improved restoration result, Hunt *et al.* [7], [8] proposed a nonstationary mean Gaussian image model. In this nonstationary image model, an image is modeled as stationary fluctuations about a nonstationary ensemble mean which has the gross structure that represents the context of the ensemble. Lebedev and Mirkin [9] suggested a composite image model that assumes an image is composed of many different stationary components. Each component has a distinct stationary correlation structure. Ingle and Woods [10] applied a reduced update Kalman filter to image restoration by using this composite image model. The result is a bank of Kalman filters running in parallel and an identification-estimation approach is used in which each point is assigned a stationary image model and filtered by the specific Kalman filter. Rajala *et al.* [11] derived a two-dimensional recursive filter based on a piecewise stationary image model. They first segment the image into disjoint regions according to the local spatial activity of the region and determine the covariance structure of different segments. In this framework, they also use a visibility adapted observation noise model. Then they use a different Kalman filter for each segment for nonstationary restoration. Compared with Ingle and Woods' approach, this method emphasizes nonstationarity within regions rather than nonstationarity at edges in the scene.

Another nonlinear approach to an improved restoration filter was proposed by Anderson and Netravali [12]. Instead of

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D. T. Kuan is with Central Engineering Laboratories, FMC Corporation, Santa Clara, CA 95052.

A. A. Sawchuk is with the Image Processing Institute, Department of Electrical Engineering, University of Southern California, Los Angeles, CA 90089.

T. C. Strand is with IBM Corporation, San Jose, CA 95193.

P. Chavel is with Institut d'Optique, Universite de Paris-Sud, Orsay, France.

using a nonstationary image model, they used a subjective error criterion based on human visual system models and derived a nonrecursive filter that adapts itself to make a compromise between the loss of resolution and noise smoothing such that the same amount of subjective noise is suppressed throughout the image. Abramatic and Silverman [13] generalized this procedure and related it to the classical Wiener filter. In fact, nonlinear restoration is closely related to nonstationary restoration as we shall see in the following sections.

In contrast to the signal-independent, additive noise model assumed in most image restoration algorithms, many physical noise processes are inherently signal-dependent. Restoration algorithms based on a signal-independent noise model are not expected to be very effective in the signal-dependent noise environment. Naderi and Sawchuk [14] derived a nonstationary discrete Wiener filter for a signal-dependent film-grain noise model. In addition to the generality of the noise model, the filter is able to adapt itself to the local signal statistics given the conditional noise statistics. Lo and Sawchuk [15] developed a nonlinear MAP filter for images degraded by Poisson noise. The algorithm is nonrecursive and requires the solution of a set of nonlinear equations. The sectioning method is used to reduce computation and to do local processing. The MAP filter is optimal, but in some applications, it might be acceptable to trade off theoretical performance for ease of implementation. This becomes more appropriate when we consider the adaptive filtering techniques where the filter parameters have to be estimated from the noisy observations.

In this paper, we introduce a nonstationary mean, nonstationary variance (NMNV) image model. The nonstationary mean describes the gross structure of an image and the nonstationary variance characterizes edge and elementary texture information of the image. A local linear minimum mean square error (LLMMSE) filter for images degraded by blur and a class of signal-dependent, uncorrelated noise is derived based on the NMNV image model. If there is no blur degradation, the LLMMSE filter has a very simple structure and is a point processor. This property is due to the NMNV image model and the uncorrelated noise assumptions. Usually the nonstationary ensemble statistics are not available *a priori* and can only be estimated from the degraded image. With the substitution of the local statistics for the ensemble statistics in the LLMMSE filter, we have an adaptive noise smoothing filter that is able to change characteristics according to the local image statistics and to different types of signal-dependent noise. The methods used to calculate the local statistics from the degraded image are critical to the quality of the restored results. Various ways of estimating these ensemble statistics are discussed and their performance is compared. The adaptive noise smoothing filter has a form similar to Abramatic's filter, and the relationship between the nonstationary image modeling approach and the nonlinear subjective criterion approach becomes clear. Based on this connection, we show that the minimum mean square error criterion is not a bad criterion if it is used locally. The explicit structure of the adaptive noise smoothing filters for multiplicative noise, film-grain noise, and Poisson noise are derived for completeness. The comparison of the adaptive noise smoothing filter with Lee's local statistics algorithm [16] shows that the introduction of the NMNV image model is not only valuable for systematically deriving the optimal estimator structure for different signal-dependent noise models, but is also very useful for the extension of the adaptive noise smoothing filter to image restoration where the images are degraded by both blur and signal-dependent noise. This problem will be discussed in another paper [17].

In this paper we denote a two-dimensional image by $f(i,j) = \boldsymbol{f}$, where (i, j) are spatial coordinates. The nonstationary statistical (ensemble) mean and variance of image $\boldsymbol{f}$ are denoted by $E(f(i, j))$ and $\sigma_f^2(i, j)$, respectively. The nonstationary local spatial mean and variance of $\boldsymbol{f}$ (described in detail later) are denoted by $\bar{f}(i, j)$ and $v_f(i, j)$. Various subscripts on σ^2 and v will refer to different two-dimensional functions.

II. Nonstationary Mean, Nonstationary Variance (NMNV) Image Model

In a conventional stationary image model, an image $\boldsymbol{f}$ is assumed to be a wide-sense stationary random field with constant mean vector and block Toeplitz covariance matrix. The joint probability density function is implicitly assumed to be multivariate Gaussian. All the statistical information of the image is carried by the covariance matrix. For a real world image as in Fig. 1(a), it is apparent from the picture that the image is not a stationary random field and its histogram [Fig. 1(b)] is not Gaussian. Thus the conventional stationary image model is an oversimplified model for its computational purposes. Hunt and Cannon [7] proposed a nonstationary mean and stationary covariance Gaussian image model. They assumed that an image $\boldsymbol{f}$ can be decomposed into a nonstationary statistical mean component $E(\boldsymbol{f})$, and a stationary residual component $\boldsymbol{f}_0 = \boldsymbol{f} - E(\boldsymbol{f})$. The nonstationary statistical mean describes the gross structure of an image and the residual component describes the detail variation of the image. For ease of computation and mathematical tractability, the covariance function C_f is assumed to be stationary and is defined by

$$C_f = E[(\boldsymbol{f} - E(\boldsymbol{f}))(\boldsymbol{f} - E(\boldsymbol{f}))^T]. \tag{1}$$

The joint probability density function (PDF) of $\boldsymbol{f}$ is assumed to be Gaussian, i.e.,

$$P(\boldsymbol{f}) = ((2\pi)^{N^2} \, |C_f|)^{-1/2} \cdot \exp\left[-\tfrac{1}{2}(\boldsymbol{f} - E(\boldsymbol{f}))^T C_f^{-1} (\boldsymbol{f} - E(\boldsymbol{f}))\right] \tag{2}$$

where $|C_f|$ is the determinant of the covariance matrix C_f, and N is the size of the image. If we replace the nonstationary ensemble (statistical) mean $E(\boldsymbol{f})$ by a local (spatial average) mean estimate $\bar{f}$ that is calculated over a 3×3 uniform window, and subtract the local mean from the original image, we have the residual image $\boldsymbol{f}_0$ that is shown for our example in Fig. 1(c). It is obvious from the picture that $\boldsymbol{f}_0$ is still a correlated nonstationary process. However, the shape of the histogram is more Gaussian [Fig. 1(d)]. Because some of the structural information is now carried by the nonstationary mean, it is reasonable to assume that the covariance structure may be simplified compared with the conventional stationary image model. Trussell and Hunt [8] assumed that the covariance matrix of $\boldsymbol{f}_0$ could be approximated by a constant diagonal

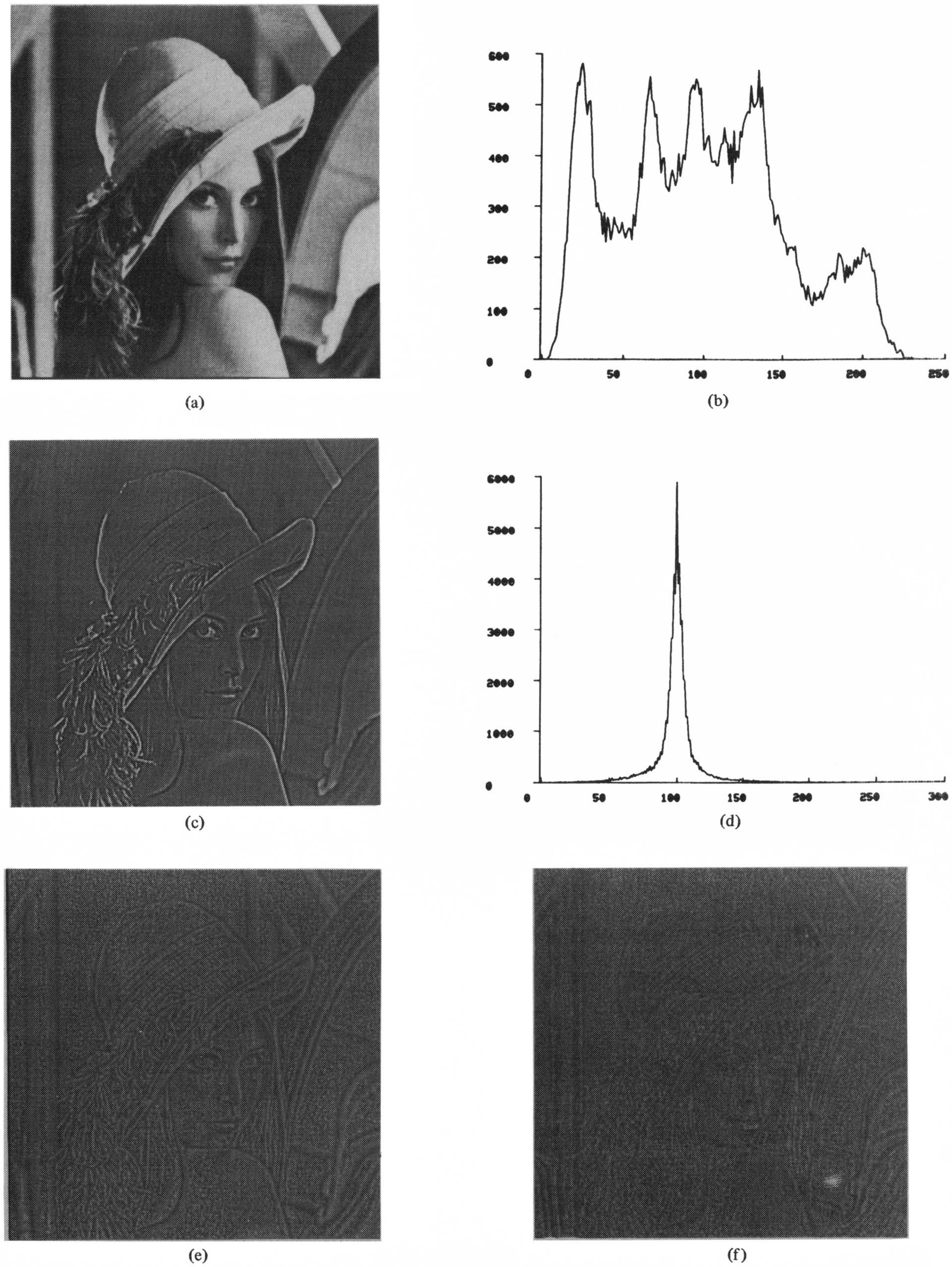


Fig. 1. Nonstationary mean and nonstationary variance image model. (a) Original image. (b) Histogram of (a). (c) Residual image of the original image and the local mean image. (d) Histogram of (c). (e) Normalized residual image with unit variance. (f) Normalized unit variance residual image with an edge detecting intelligent filter to calculate the local statistics.

matrix $\sigma_f^2 I$, where σ_f^2 is a scalar and I is the identity matrix. In this case, all the information is carried by the nonstationary mean. While letting the covariance matrix carry all the statistical information, as in a conventional stationary image model, is not satisfactory, neither is letting the nonstationary mean carry all the information. Therefore, it is heuristically reasonable to include more structure in the covariance matrix of the nonstationary mean Gaussian image model.

In order to consider the nonstationarity of the image and not to complicate the computation too much, we assume that $\boldsymbol{f}_0$ is a nonstationary white process. More specifically, $\boldsymbol{f}_0$ is statistically uncorrelated and is characterized by its nonstationary statistical variance $\sigma_f^2(i, j)$ at spatial position (i, j). This nonstationary mean and nonstationary variance (NMNV) image model has been used implicitly to some extent in the sectioned MAP method. There it was assumed that the nonstationary image can be divided into many sections. Each section has the covariance matrix $\sigma_f^{2(q)} I$, where $\sigma_f^{2(q)}$ is a scalar that varies from section to section. The section size is chosen according to the width of the point spread function and the extent of stationarity assumed. An overlap-save sectioning method is used to suppress the convolution wraparound effect at the section boundary. For the noise smoothing problem (no blur), the section size can be reduced to a single point and this becomes a NMNV image model. However, for the image restoration problem, the minimum section size cannot be reduced to a single point.

We can substitute the local (spatial average) variance $v_f(i, j)$ of $\boldsymbol{f}_0$ at position (i, j) for the nonstationary variance $\sigma^2(i, j)$ and normalize $\boldsymbol{f}_0$ in Fig. 1(c) by computing the normalized unit variance residual image

$$f_0'(i, j) = f_0(i, j)/[v_f(i, j)]^{1/2}. \tag{3}$$

This residual image $\boldsymbol{f}_0'$ is shown in Fig. 1(e). Note that in the uniform intensity region, there is no correlation and $\boldsymbol{f}_0'$ looks like white noise. This verifies that the NMNV image model is valid in those regions. However, in the neighborhood of an edge, visible correlations still exist in the normalized residual image. If we want to include these correlations into the image model, we need to specify a nonstationary mean and nonstationary covariance image model. While this is the direction to go, the possible performance improvement of using this model is hindered by the complexity of the restoration filter and the model identification procedure.

There is another factor that contributes to the correlation of the residual image in Fig. 1(e). Note that we used the local statistics to substitute for the ensemble statistics in the procedure. When there is a sharp edge in the image, the averaging window tends to blur the local mean estimate by using pixels on both sides of the edge. To avoid this we can use an intelligent filter that recognizes an edge and calculates the local mean only using those pixels on the one side of the edge. Fig. 1(f) is the normalized residual image where the local mean and local variance are calculated by such a filter. Note that the correlations around the edges are reduced, and the residual image is more like a white noise process.

From the discussion above, we know that the validity of the NMNV image model depends on the methods we use to estimate the local mean and variance. Therefore, the process of estimating local statistics can be thought of as a way to find an operation that transforms an image into a white noise process in order to facilitate the restoration procedure. This transformation is usually a nonlinear one, and the inherent correlation between adjacent pixels in an image is implicitly imbedded in the nonstationary mean.

III. Local Linear Minimum Mean Square Error Filter for a Class of Signal-Dependent Noise

Consider the observation equation

$$\boldsymbol{g} = H\boldsymbol{f} + \boldsymbol{n} \tag{4}$$

where $\boldsymbol{g}$ is the degraded observation, $\boldsymbol{f}$ is the original signal, $\boldsymbol{n}$ is a zero mean noise that can be signal-independent or signal-dependent, and H is the blurring matrix. The minimum mean square error (MMSE) estimate of $\boldsymbol{f}$ given observation $\boldsymbol{g}$ is the conditional mean estimate

$$\hat{\boldsymbol{f}} = E(\boldsymbol{f}|\boldsymbol{g}). \tag{5}$$

In general, the MMSE estimate is nonlinear and depends on the probability density functions of $\boldsymbol{f}$ and $\boldsymbol{n}$. The explicit form of the MMSE estimator is difficult to obtain for the general case. If we impose a linear constraint on the estimator structure, we have the linear minimum mean square error (LMMSE) estimator [18]

$$\hat{\boldsymbol{f}}_{\text{LMMSE}} = E(\boldsymbol{f}) + C_{fg} C_g^{-1}(\boldsymbol{g} - E(\boldsymbol{g})) \tag{6}$$

where C_{fg} is the cross-covariance matrix of $\boldsymbol{f}$ and $\boldsymbol{g}$. Also C_g and $E(\boldsymbol{g})$ are the covariance matrix and ensemble mean of $\boldsymbol{g}$, respectively. Unlike the MMSE filter, the LMMSE filter only requires the second order statistics of the signal and noise. Nonstationary ensemble mean and covariance statistics can be used in (6) if they are available. Because of the local nature of these nonstationary statistics, we refer to it as a local (or nonstationary) linear minimum mean square error (LLMMSE) filter. In this way, we distinguish it from the conventional stationary Wiener filter. Generally speaking, C_g is a nonstationary covariance matrix and matrix inversion has to be carried out without the help of FFT techniques. In a later section we replace the ensemble statistics with spatial averages obtained from the images themselves.

To see the structure of the LLMMSE filter more explicitly, we need to calculate the covariance matrices in (6). The cross-covariance matrix C_{fg} is given by

$$\begin{aligned} C_{fg} &= E[(\boldsymbol{f} - E(\boldsymbol{f}))(\boldsymbol{g} - E(\boldsymbol{g}))^T] \\ &= E\{(\boldsymbol{f} - E(\boldsymbol{f}))[H(\boldsymbol{f} - E(\boldsymbol{f})) + \boldsymbol{n}]^T\} \\ &= C_f H^T + E[(\boldsymbol{f} - E(\boldsymbol{f}))\boldsymbol{n}^T]. \end{aligned} \tag{7}$$

Similarly, the covariance matrix C_g can be calculated as

$$\begin{aligned} C_g &= E[(\boldsymbol{g} - E(\boldsymbol{g}))(\boldsymbol{g} - E(\boldsymbol{g}))^T] \\ &= E\{[H(\boldsymbol{f} - E(\boldsymbol{f})) + \boldsymbol{n}][H(\boldsymbol{f} - E(\boldsymbol{f})) + \boldsymbol{n}]^T \\ &= HC_f H^T + C_n + HE[(\boldsymbol{f} - E(\boldsymbol{f}))\boldsymbol{n}^T] \\ &\quad + E[\boldsymbol{n}(\boldsymbol{f} - E(\boldsymbol{f}))^T] H^T \end{aligned} \tag{8}$$

where C_n is the covariance matrix of $\boldsymbol{n}$.

The results of (7) and (8) simplify considerably if we can assume that the conditional mean of $\boldsymbol{n}$ given $\boldsymbol{f}$ is 0, i.e.,

$$E(\boldsymbol{n}|\boldsymbol{f}) = \boldsymbol{0}. \tag{9}$$

We now look at the term $E[(\boldsymbol{f} - E(\boldsymbol{f}))\,\boldsymbol{n}^T]$ in (7) and (8) and rewrite it as

$$E[(\boldsymbol{f} - E(\boldsymbol{f}))\,\boldsymbol{n}^T] = E_f[E_n[(\boldsymbol{f} - E(\boldsymbol{f}))\,\boldsymbol{n}^T|\boldsymbol{f}]]. \tag{10}$$

Now the inner expectation on the right side of (10) can be written

$$E_n[(\boldsymbol{f} - E(\boldsymbol{f}))\,\boldsymbol{n}^T|\boldsymbol{f}] = (\boldsymbol{f} - E(\boldsymbol{f}))\,E[\boldsymbol{n}^T|\boldsymbol{f}] = [0] \tag{11}$$

from the condition of (9). Substituting this result back in (10) gives

$$E[(\boldsymbol{f} - E(\boldsymbol{f}))\,\boldsymbol{n}^T] = [0] \tag{12}$$

and using this result in (7) and (8) gives the simplified result

$$\begin{aligned} C_{fg} &= C_f H^T \\ C_g &= HC_f H^T + C_n. \end{aligned} \tag{13}$$

Note that although (4) has the form of signal plus signal-dependent noise, the noise need not be strictly additive in the usual sense. Any noise degradation can always be expressed in terms of signal plus signal-dependent noise, although this may not be the most convenient way to represent it. If we examine

$$\begin{aligned} E(\boldsymbol{g}|\boldsymbol{f}) &= E[(H\boldsymbol{f} + \boldsymbol{n})|\boldsymbol{f}] \\ &= E[H\boldsymbol{f}|\boldsymbol{f}] + E(\boldsymbol{n}|\boldsymbol{f}) \\ &= HE(\boldsymbol{f}|\boldsymbol{f}) = H\boldsymbol{f} \end{aligned} \tag{14}$$

we see that the condition of (9) implies that the signal-dependent noise $\boldsymbol{n}$ has no bias. This condition is generally satisfied by many physical noise models such as multiplicative noise, film-grain noise, and Poisson noise. Instead of deriving a different filter for each individual case, we give a unified approach to design a noise smoothing filter for this class of signal-dependent observations.

Conditions for a Scalar (Point) Processor

If we consider the special case of no degradation by blurring, i.e., $H = I$, then the degraded image $g(i, j)$ can be expressed as the scalar equation

$$g(i,j) = f(i,j) + n(i,j) \tag{15}$$

where $n(i, j)$ is the (possibly) signal-dependent noise satisfying (9).

Another special case arises if the noise term $n(i, j)$ is uncorrelated, i.e.,

$$E[n(i,j)\,n(r,s)] = 0 \quad \text{for} \quad (i,j) \neq (r,s). \tag{16}$$

Assuming the condition of (15) and the NMNV assumption that C_f is diagonal, the cross-covariance matrix C_{fg} becomes diagonal. These assumptions together with the assumption of (16) imply that the covariance matrix C_g is diagonal. Under these conditions, the LLMMSE filter (6) becomes a scalar processor of the form

$$\hat{f}_{\text{LLMMSE}}(i,j) = E(f(i,j)) + \frac{\sigma_f^2(i,j)}{\sigma_f^2(i,j) + \sigma_n^2(i,j)} \cdot [g(i,j) - E(g(i,j))] \tag{17}$$

where $\sigma_n^2(i, j)$ is the nonstationary noise ensemble variance.

Interpretation of the Filter

If all the *a priori* image and noise statistics on the right side of (17) are known, then $\hat{f}_{\text{LLMMSE}}(i, j)$ is a function of measurements $g(i, j)$ only, and each estimated pixel can be produced in parallel, perhaps in real-time. Using the fact that the noise $\boldsymbol{n}$ is zero mean, we have

$$E(g(i,j)) = E(f(i,j)) \tag{18}$$

and we can rearrange (17) as

$$\hat{f}_{\text{LLMMSE}}(i,j) = (1 - w(i,j))\,E(f(i,j)) + w(i,j)\,g(i,j) \tag{19}$$

$$w(i,j) = \frac{\sigma_f^2(i,j)}{\sigma_f^2(i,j) + \sigma_n^2(i,j)}. \tag{20}$$

Thus, the LLMMSE estimate is a weighted sum of the ensemble mean $E(f(i, j))$ and the normalized observation $g(i, j)$, where the weight is determined by the ratio of the signal variance to the noise variance. For a low signal-to-noise (SNR) ratio, the LLMMSE filter puts more weight on the *a priori* mean $E(f(i, j))$ because the observation is too noisy to make an accurate estimate of the original image. Conversely, for high SNR, the LLMMSE estimate puts more weight on the noisy observation and the result is to preserve the edge sharpness.

It is interesting to compare these properties of the LLMMSE filter with the results obtained by Anderson and Netravali [12], in which they derived a nonlinear restoration filter based on a subjective visibility function to make a balance between noise smoothing and resolution. The nonlinear filter tends to average out the random noise in the flat areas and preserve edge sharpness so that the same amount of subjective noise is suppressed in the whole image. Thus, the filter response of their nonlinear approach is similar to that of the LLMMSE filter. The nonlinear approach uses a "masking function" to measure the spatial activity or the nonstationarity of the image and uses this measure to suppress the same amount of subjective noise over the image according to an exponentially decreased visibility function, whereas the LLMMSE filter directly uses a nonstationary image model and tries to minimize the local mean square error. The net effect of these two approaches are similar because one is varying the subjective noise variance according to the contextual information of the image, while the other imbeds the contextual information directly into a nonstationary image model. Therefore, the local image variance not only has its statistical meaning but also serves as a spatial "masking function," and the LLMMSE filter is similar to a subjective smoothing filter with a linearly decreasing visibility function. The two seemingly different approaches are now related and support the idea that the minimum mean square error criterion is a reasonable measure for image restoration if it is used locally with a nonstationary image model.

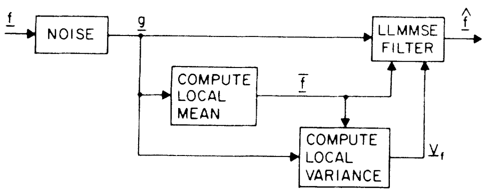


Fig. 2. Adaptive noise smoothing filter structure.

IV. Adaptive Noise Smoothing Filter

The LLMMSE filter shown in (17) requires the ensemble mean and variance of $f(i, j)$. Usually these statistics are not available *a priori* and can only be estimated from the degraded image. If we assume as before that the ensemble statistics can be replaced by local spatial statistics that are estimated from the degraded image, we have the adaptive noise smoothing filter

$$\hat{f}(i,j) = \bar{f}(i,j) + \frac{v_f(i,j)}{v_f(i,j) + \sigma_n^2(i,j)} (g(i,j) - \bar{g}(i,j)) \tag{21}$$

where $\bar{f}(i, j)$ and $\bar{g}(i, j)$ are the local spatial means of $f(i, j)$ and $g(i, j)$, respectively, and $v_f(i,j)$ is the local spatial variance of $f(i,j)$. Computation of these quantities is discussed in the following section. It is interesting to see that the adaptive noise smoothing filter is effectively a nonlinear filter even though it has the same linear form as the LLMMSE filter. The nonlinearity is introduced by the ratio of the local spatial variances in (21) and their estimation by nonlinear functions of the noisy observation $g(i,j)$. The performance of the adaptive noise smoothing filter depends heavily on the method used to calculate the local statistics. A block diagram of the adaptive noise smoothing filter is illustrated in Fig. 2.

Computation of the Local Statistics

The underlying assumption of the adaptive noise smoothing filter is that an image is locally ergodic such that the ensemble statistics can be replaced by the local spatial statistics. Therefore, the choice of method used for estimating local statistics is critical. One way to obtain the local mean and local variance is to calculate over a uniform moving average window of size $(2r + 1) \times (2s + 1)$. We then have

$$\bar{g}(i,j) = \frac{1}{(2r+1)(2s+1)} \sum_{p=i-r}^{i+r} \sum_{q=j-s}^{j+s} g(p,q) \tag{22}$$

and

$$v_g(i,j) = \frac{1}{(2r+1)(2s+1)} \sum_{p=i-r}^{i+r} \sum_{q=j-s}^{j+s} (g(p,q) - \bar{g}(i,j))^2 \tag{23}$$

where $g(i, j)$ and $v_g(i, j)$ are the local mean and local variance of $g(i, j)$, respectively. These statistics are commonly known as the sample mean and sample variance. They are widely used in statistical analysis and can be shown to be the maximum likelihood estimates of the unknown mean and variance of a Gaussian probability density function [19] by assuming that the samples in the summation are from the same ensemble.

The local statistics of $f(i, j)$ can be calculated from the local statistics of $g(i, j)$ by assuming the relationship between their ensemble statistics also holds for their local statistics. Therefore, the functional forms of these transformations depend on the particular noise structure. The local statistics of $f(i, j)$ then feed into the filter to adapt the filter to the nonstationary content of the image and signal-dependent characteristics of the noise.

As an example of the computation of local statistics of $f(i, j)$, consider the signal-independent, additive statistically stationary noise model with no blurring. The local mean of $f(i, j)$ is equal to the local mean of $g(i, j)$ from (18). It is straightforward to show that the local variance of $f(i, j)$ is given by

$$v_f(i,j) = v_g(i,j) - \sigma_n^2(i,j) \tag{24}$$

where $\sigma_n^2(i, j)$ is the nonstationary noise variance. The function $\sigma_n^2(i, j)$ is assumed known from *a priori* measurements on the imaging system [1]. Substituting $v_f(i, j)$ into (21), we have the adaptive noise smoothing filter for the signal-independent, additive noise model.

The adaptive estimate is a balance between the local mean estimate $\bar{f}(i, j)$ and the noisy observation $g(i, j)$. The local variance $v_f(i, j)$ is an indication of our confidence in the local mean estimate. For the signal-independent, additive noise model, the adaptive noise smoothing filter with local statistics defined as in (22) and (23) is the same as Lee's local statistics algorithm [16], which is derived using a heuristic argument. Using the NMNV image model gives a better understanding of the problem and simplifies the extension of the filter to signal-dependent noise. For the multiplicative noise model and signal-dependent noise in general, we will show that Lee's approach is not optimal. Abramatic and Silverman's [13] nonlinear "signal equivalent" filter also has a similar structure to the adaptive noise smoothing filter. It can be treated as a special case of the adaptive noise smoothing filter where the local mean and local variance are estimated by a Wiener filter and a "masking function," respectively.

The local statistics calculated according to (22) and (23) assume that the samples within the averaging window are from the same ensemble. This is not true if there is a sharp edge within the window. The value of the sample variance near the edge will be larger than the ensemble variance because we use samples in two entirely different ensembles to calculate the local variance. The sample mean will tend to smear out compared with the ensemble mean. To avoid these effects, we could use an intelligent filter that can find edges and use the correct neighborhood of a pixel for calculating the local statistics. Various approaches for using edge detectors have been proposed for designing an anisotropic restoration filter and calculating the local statistics [12], [20], [24], and they do a reasonable job. Here, we introduce a simple function form for estimating the local variance by including the nonstationarity of the image in the function.

The inaccurate sample mean seems to have less effect on the filter output than the inaccurate sample variance because the NMNV image model only uses the nonstationary mean to describe the gross structure of the image, while the nonstationary

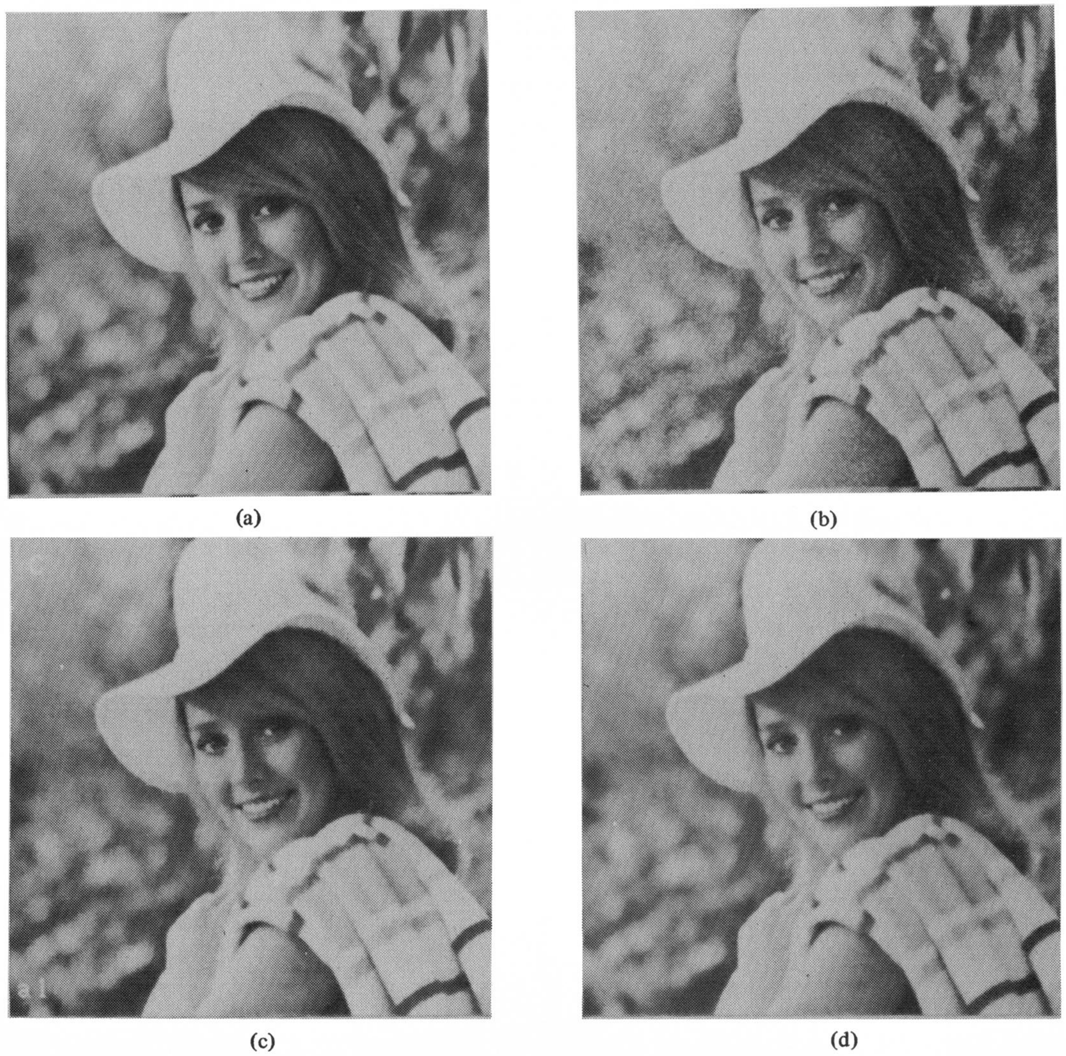


Fig. 3. Adaptive noise smoothing filter for signal-dependent additive noise. (a) Original image. (b) Original degraded by additive noise with $\sigma_n^2 = 100$. (c) Adaptive noise smoothing estimate with local variance of (23). (The same as Lee's local statistics algorithm.) (d) Adaptive noise smoothing estimate with local variance of (25).

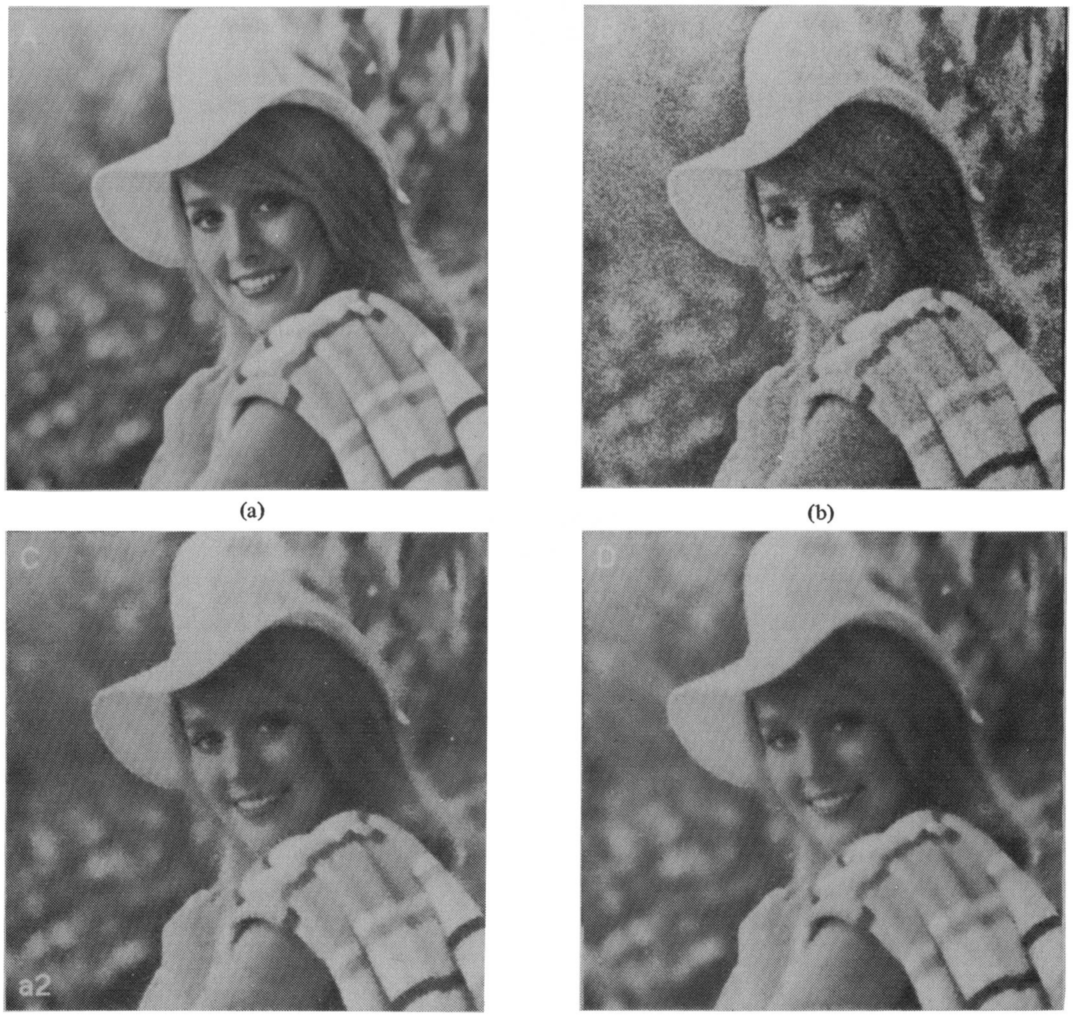
variance is used to characterize the edge information. In order to preserve the noise smoothing ability of the local mean, we still use the sample mean as our local mean estimate. The new local variance is defined as

$$v_g(i,j) = \frac{1}{(2r+1)(2s+1)} \sum_{p=i-r}^{i+r} \sum_{q=j-s}^{j+s} \cdot c(i-p, j-q)(g(p,q) - \bar{g}(p,q))^2 \tag{25}$$

where $c(i, j)$ is a weighting function. The weights $c(i, j)$ are chosen such that $\Sigma_i \Sigma_j c(i, j) = 1$, and $c(i, j)$ is a monotonically decreasing function (e.g., Gaussian function) to put more confidence on the center variance estimates. The sample variance estimate in (23) implicitly assumes that $g(i, j)$ is locally stationary such that $\bar{g}(i, j)$ can be used as the local mean for all $g(p, q)$ within the averaging window. In our new local variance estimate, the locally stationary assumption is removed and the local mean $\bar{g}(p, q)$ is allowed to vary for each $g(p, q)$ within the window. Therefore, this new local variance should have good performance near the edges.

Simulation Results

The original girl image is shown in Fig. 3(a). Fig. 3(b) is the original image degraded by a signal-independent, additive, white noise with variance 100. The degraded image is processed by the adaptive noise smoothing filter with the sample mean and sample variance calculated over a 5×5 window as the local statistics, and the smoothed image is shown in Fig. 3(c). In the uniform region, the estimate is close to the local mean estimate and the noise is smoothed by a large amount. Conversely, in the edge area, the restored image is close to the noisy observation and the edge sharpness is preserved. The squared error (SE) between the smoothed image and the original image is 36.7 per pixel. This value is very small compared with the SE of 94.1 per pixel for the local mean estimate that is essentially a spatially invariant low-pass filter. Fig. 3(d) shows the adaptive noise smoothing estimate of the original



(a) (b) (c) (d)

Fig. 4. Adaptive noise smoothing filter for signal-dependent additive noise. (a) Original image. (b) Original degraded by additive noise with $\sigma_n^2 = 400$. (c) Adaptive noise smoothing estimate with local variance of (23). (The same as Lee's local statistics algorithm.) (d) Adaptive noise smoothing estimate with local variance of (25).

image using (25) to calculate the local variance. This estimate seems to have better visual quality than Fig. 3(c). It looks smoother both at the step edges and ramp edges. The SE of Fig. 3(d) is 34.6 per pixel. This is slightly smaller than for the sample variance case indicating the performance improvement of using the new local variance estimate. The same sequence of the simulation results for a low SNR case where the noise variance is equal to 400 is shown in Fig. 4.

V. Adaptive Noise Smoothing Filter for Various Signal-Dependent Noise Models

The general form of the adaptive noise smoothing filter for a class of uncorrelated, signal-dependent noise is shown in (21). It is useful to examine the explicit structure of this filter for some physical noise models that are frequently encountered in practical imaging systems. In this section, we derive the explicit structure of the adaptive noise smoothing filter for multiplicative noise, film-grain noise, and Poisson noise.

Multiplicative Noise

The degradation model for the multiplicative noise model can be written as

$$g'(i,j) = u(i,j)\, f(i,j) \tag{26}$$

where $u(i,j)$ is independent of $f(i,j)$. The multiplicative noise is assumed to have a stationary mean and variance given by

$$E(u(i,j)) = E(\boldsymbol{u}) \tag{27}$$

and

$$E[(u(i,j) - E(\boldsymbol{u}))^2] = \sigma_u^2, \tag{28}$$

respectively. We now define a normalized observation

$$g(i,j) = g'(i,j)/E(\boldsymbol{u}) \tag{29}$$

such that

$$E[g(i,j)] = E[f(i,j)]. \tag{30}$$

If we represent (29) in terms of signal plus signal-dependent

(a)

(b)

(c)

(d)

Fig. 5. Adaptive noise smoothing filter for multiplicative noise. (a) Original image. (b) Original degraded by multiplicative noise with $\sigma_u^2 = 0.007$. (c) Adaptive noise smoothing estimate with local variance of (23). (Similar to Lee's local statistics algorithm.) (d) Adaptive noise smoothing estimate with local variance of (25).

additive noise, we have

$$g(i,j) = f(i,j) + \left[\frac{(u(i,j) - E(\boldsymbol{u}))}{E(\boldsymbol{u})}\right] \cdot f(i,j) \tag{31}$$

and we can identify

$$n(i,j) = (u(i,j)/E(\boldsymbol{u}) - 1)\, f(i,j) \tag{32}$$

from (15). It is straightforward to compute the variance

$$\sigma_n^2(i,j) = \frac{\sigma_u^2\,[(E(f(i,j)))^2 + \sigma_f^2(i,j)]}{[E(\boldsymbol{u})]^2} \tag{33}$$

and to verify that $n(i, j)$ satisfies (9). From these facts and (30) we can solve for the statistical variance

$$\sigma_f^2(i,j) = \frac{\sigma_g^2(i,j) - \sigma_u^2\,[E(g(i,j))^2]/[E(\boldsymbol{u})]^2}{1 + \sigma_u^2/[E(\boldsymbol{u})]^2} \tag{34}$$

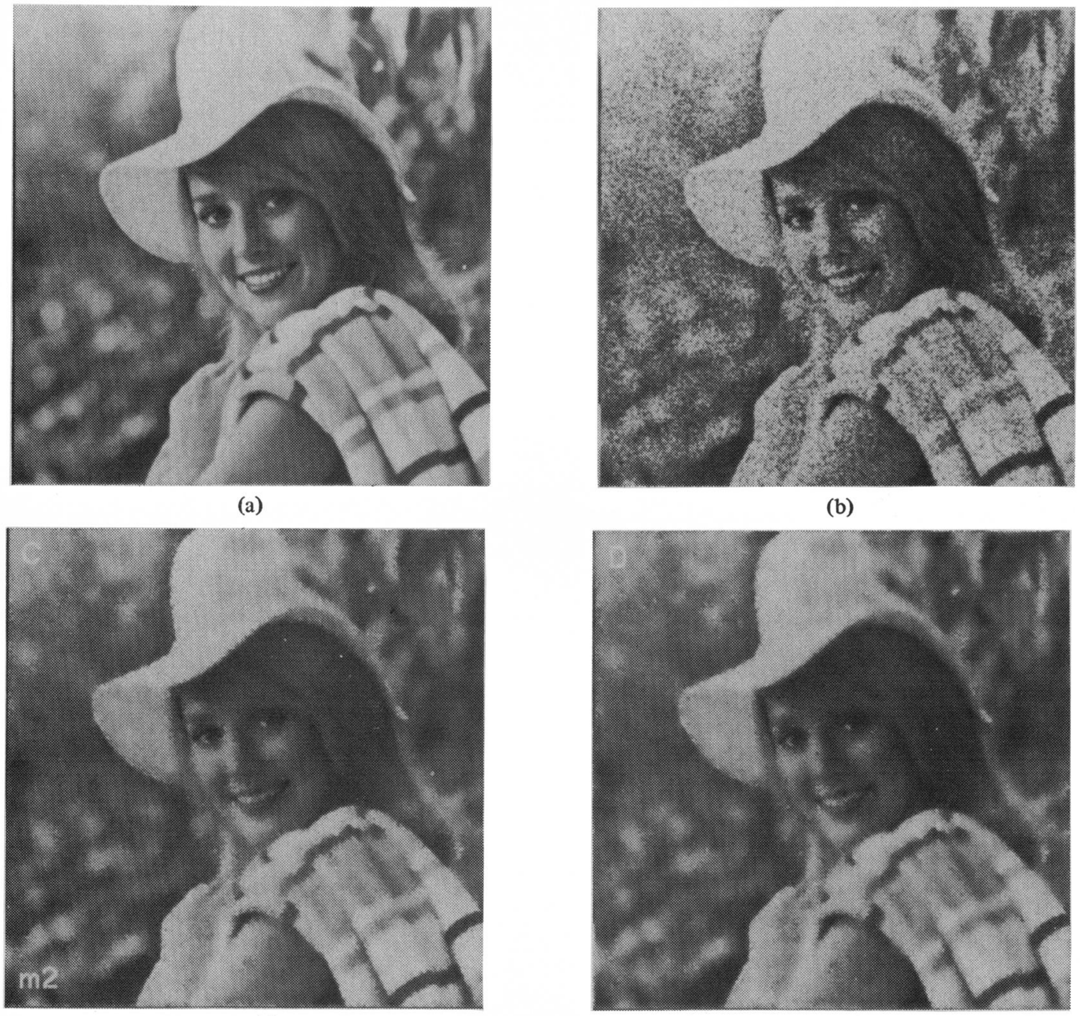
in terms of statistics of $g(i,j)$.

Replacing ensemble statistics in (17), (30), and (34) by local spatial statistics gives the adaptive noise smoothing filter for the multiplicative noise model as

$$\hat{f}(i,j) = \bar{f}(i,j) + \frac{v_f(i,j)(g(i,j) - \bar{f}(i,j))}{v_f(i,j) + \sigma_u^2\,[(\bar{f}(i,j))^2 + v_f(i,j)]/[E(\boldsymbol{u})]^2} \tag{35}$$

where $g(i, j)$ is the normalized observation and $\sigma_u^2/[E(\boldsymbol{u})]^2$ is a parameter characterizing the multiplicative noise level. The term $(\bar{f}(i, j))^2 + v_f(i, j)$ in (35) shows the signal-dependent properties of multiplicative noise, and makes the adaptive noise smoothing filter change its characteristics adaptively to smooth the signal-dependent noise. Note that in the case of very low noise, $\sigma_u^2 \ll [E(\boldsymbol{u})]^2$, and the best estimate is the measurement $g(i,j)$ itself.

The result of (35) is a generalization of Lee's local statistics algorithm for multiplicative noise [20] in that an additional term $\sigma_u^2 v_f(i, j)/[E(\boldsymbol{u})]^2$ appears in the denominator. The term does not appear in Lee's derivation due to the linear approxi-



(a) (b) (c) (d)

Fig. 6. Adaptive noise smoothing filter for multiplicative noise. (a) Original image. (b) Original degraded by multiplicative noise with $\sigma_u^2 = 0.04$. (c) Adaptive noise smoothing estimate with local variance of (23). (Similar to Lee's local statistics algorithm.) (d) Adaptive noise smoothing estimate with local variance of (25).

mation made there for the nonlinear multiplicative noise model. In the approach presented in this paper, we derive the optimal linear filter for the given classes of signal-dependent noise by first representing the noise model in signal plus additive noise form, and then obtaining the variance of the noise term. Thus, the linear constraint is applied to the estimator structure rather than to the nonlinear observation model. Of course, the significance of this additional term in actual application depends on the relative magnitude of σ_u^2, $E(\boldsymbol{u})$, $v_f(i, j)$ and $\bar{f}(i, j)$. Lee [21] has examined a multiplicative noise model of the form of (26) for synthetic aperture radar (SAR) imagery degraded by coherent speckle noise and has found the effects of the additional term to be small.

The simulation results are shown in Fig. 5. The original image is in Fig. 5(a). The image degraded by a multiplicative noise with unit mean and variance equal to 0.007 is shown in Fig. 5(b). Fig. 5(c) is the adaptive noise smoothing estimate of the original image by using the sample mean and sample variance as local statistics. Fig. 5(d) is the adaptive estimate using the new local variance estimate as in Fig. 3(d). These results are comparable with those of Fig. 3. Another set of simulation results for $\sigma_u^2 = 0.04$ are shown in Fig. 6. In these examples the effects of the additional term in the denominator of (35) are small.

Film-Grain Noise

Film-grain noise inherently exists in the process of photographic recording and reproduction. If we process the film in the linear region of the D-log E curve and ignore the blurring effect of the model [22], we have

$$g(i, j) = f(i, j) + \alpha f^{1/3}(i, j)\, u(i, j) \tag{36}$$

where $u(i, j)$ is a signal-independent noise and α is a proportionality constant. This model is very similar to the additive form of the multiplicative noise model except for the nonlinear effect of $f^{1/3}(i, j)$. Therefore, the derivation of the adaptive noise smoothing filter is similar to that for multiplicative noise.

(a) (b) (c) (d)

Fig. 7. Adaptive noise smoothing filter for Poisson noise. (a) Original image. (b) Original degraded by Poisson noise with $\lambda = 1$. (c) Adaptive noise smoothing estimate with local variance of (23). (d) Adaptive noise smoothing estimate with local variance of (25).

Poisson Noise

Photon noise is a fundamental limitation of images detected at low light levels [23]. The degradation model of Poisson noise is given by

$$g'(i,j) = \text{Poisson}_\lambda(f(i,j)) \tag{37}$$

where $\text{Poisson}_\lambda(\cdot)$ is a Poisson random number generator, and λ is a proportionality factor. The probabilistic description of a Poisson process is given by

$$P(g'(i,j)|f(i,j)) = \frac{(\lambda f(i,j))^{g(i,j)}\, e^{-\lambda f(i,j)}}{g(i,j)!}. \tag{38}$$

The conditional ensemble mean and variance of $g'(i,j)$ given $f(i,j)$ are

$$E[g'(i,j)|f(i,j)] = \lambda f(i,j) \tag{39}$$

$$\text{Var}\,[g'(i,j)|f(i,j)] = \lambda f(i,j). \tag{40}$$

We define the normalized observation as

$$g(i,j) = g'(i,j)/\lambda = \frac{\text{Poisson}_\lambda(f(i,j))}{\lambda}. \tag{41}$$

If we represent the normalized Poisson observation in terms of signal plus signal-dependent additive noise, we have

$$g(i,j) = f(i,j) + (g(i,j) - f(i,j)). \tag{42}$$

Therefore, the noise part has the form

$$n(i,j) = g(i,j) - f(i,j) = \frac{\text{Poisson}_\lambda(f(i,j))}{\lambda} - f(i,j) \tag{43}$$

and its variance can be shown to be

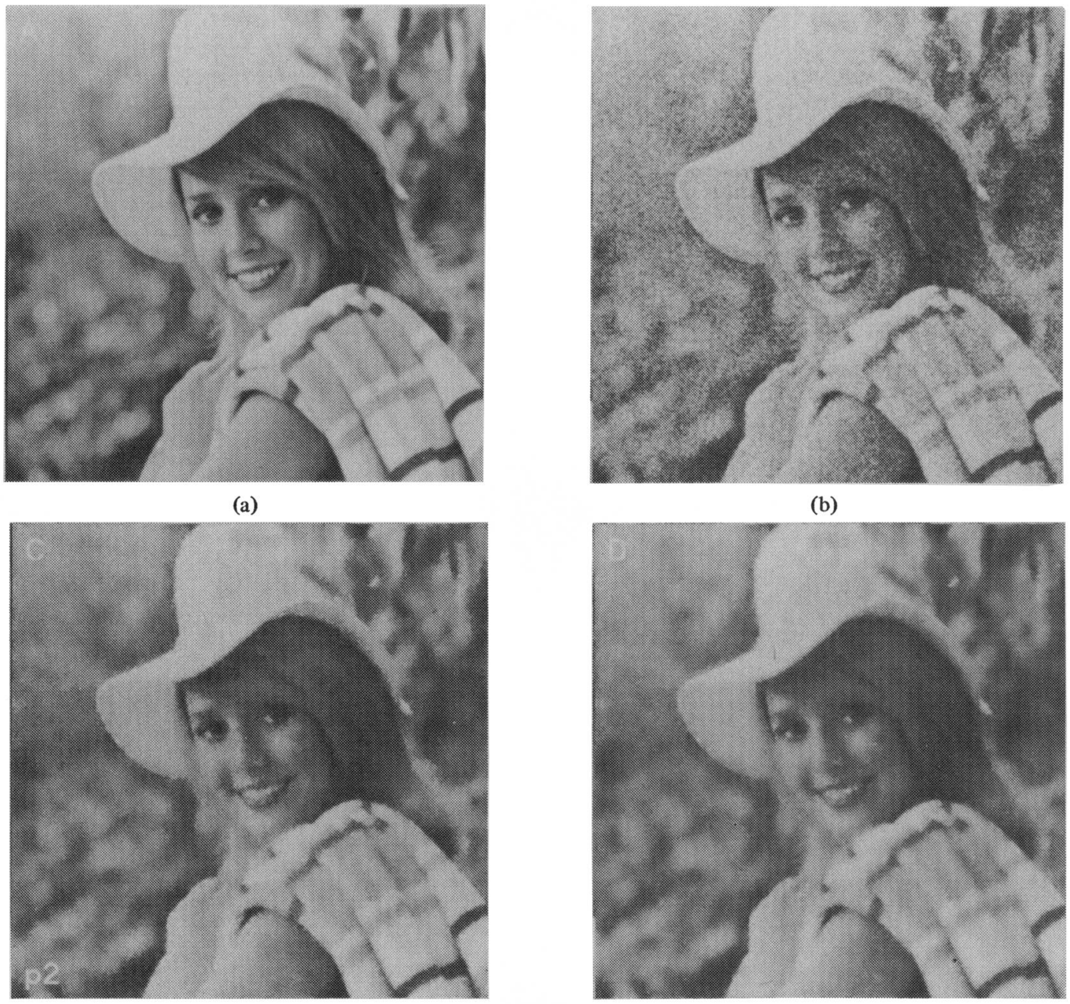
$$\sigma_n^2(i,j) = E(f(i,j))/\lambda. \tag{44}$$

From these equations, we solve for the ensemble variance of $f(i,j)$ and replace all ensemble statistics by local spatial statistics to obtain

$$v_f(i,j) = v_g(i,j) - (\bar{f}(i,j)/\lambda). \tag{45}$$

Thus, the adaptive noise smoothing filter for images with Poisson noise can be expressed as

$$\hat{f}(i,j) = \bar{f}(i,j) + \frac{v_f(i,j)(g(i,j) - \bar{f}(i,j))}{v_f(i,j) + (\bar{f}(i,j)/\lambda)} \tag{46}$$



(a) (b) (c) (d)

Fig. 8. Adaptive noise smoothing filter for Poisson noise. (a) Original image. (b) Original degraded by Poisson noise with $\lambda = 0.25$. (c) Adaptive noise smoothing estimate with local variance of (23). (d) Adaptive noise smoothing estimate with local variance of (25).

where $g(i, j)$ is the normalized observation and $\bar{f}(i, j)/\lambda$ is an indication of the Poisson noise level at point (i,j).

The simulation results are shown in Fig. 7. Fig. 7(a) is the original image. Fig. 7(b) is the original image degraded by a Poisson noise with $\lambda = 1$. Fig. 7(c) is the adaptive noise smoothing estimate of the original image obtained by using the sample mean and sample variance as local statistics. Fig. 7(d) is the result of using the new local variance of (25). The same sequence of simulation results for $\lambda = 0.25$ is shown in Fig. 8.

VI. Conclusions

The adaptive noise smoothing filter has a very simple structure and does not require any *a priori* information of the original image. All the parameters needed are estimated from the noisy image. The filter adopts a nonstationary image model and can be applied to different types of signal-dependent noise. The minimum mean square error criterion is used locally rather than globally and has some of the desirable properties of a subjective error criterion. The calculation of the local image statistics is critical to the quality of the restored image. The adaptive noise smoothing filter has the advantage of separating the estimation of local statistics from the image restoration filter. Thus, we can use various sophisticated methods to estimate the local image statistics while keeping the restoration filter structure fixed. Furthermore, the local statistics can be estimated by using a recursive filter to reduce the computation. The extension of adaptive noise smoothing filter to recursive image restoration is discussed in another paper [17].

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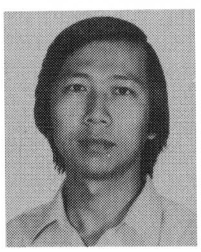


Darwin T. Kuan (S'79–M'82) received the B.S. degree from National Taiwan University in 1978 and the M.S. and Ph.D. degrees from the University of Southern California, Los Angeles, in 1979 and 1982, respectively, all in electrical engineering.

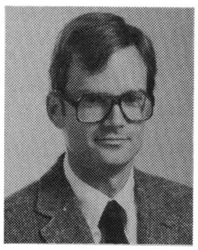
He was employed by Shell Development Company, Bellaire Research Center, Houston, TX, during the summer of 1980 working on seismic data processing. He spent three months of 1982 at the Institute d'Optique, University of Paris-Sud, Orsay, France, as a Visiting Scientist working on speckle noise reduction techniques. From 1982–1983 he was with Advanced Information & Decision Systems, Mountain View, CA. He is currently with FMC Corporation, Central Engineering Laboratories, Santa Clara, CA. His research interests include computer vision, robotics, artificial intelligence, and image and signal processing.

Dr. Kuan is a member of the American Association for Artificial Intelligence.

Alexander A. Sawchuk (S'65–M'71–SM'79) was born in Washington, DC, in 1945. He received the S.B. degree from the Massachusetts Institute of Technology, Cambridge, in 1966 and the M.S. and Ph.D. degrees from Stanford University, Stanford, CA, in 1968 and 1972, respectively, all in electrical engineering.

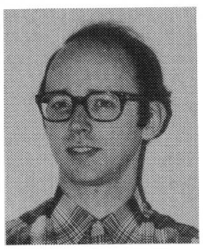
He has been employed in instrumentation and communications systems analysis by the National Bureau of Standards, the NASA Goddard Space Flight Center, and the Communications Satellite Corporation. In 1971, he joined the University of Southern California, Los Angeles, where he is presently Director of the Image Processing Institute and Professor in the Department of Electrical Engineering. His research interests include optical information processing, digital image processing, multidimensional signal processing, machine vision, and image analysis.

Dr. Sawchuk is a Fellow of the Optical Society of America and past Chairman of the Information Processing and Holography Technical Group. He is a Fellow of the Society of Photo-Optical Instrumentation Engineers and is a member of the Optical Society of Southern California, the Society of Information Display, and Sigma Xi. In 1980, he received the Halliburton Award for Exceptional Service from the University of Southern California.



Timothy C. Strand (M'80) received the B.A. degree in physics from the University of Iowa and the M.S. and Ph.D. degrees in applied physics from the University of California, San Diego.

From 1976 until 1983, he was associated with the Image Processing Institute at the University of Southern California where he was involved in teaching and research in the areas of optical information processing, image processing, and holography. He is currently with IBM Corporation at the San Jose Research Laboratory where he is working in the areas of optical sensing and metrology.



Pierre Chavel was born in Strasbourg, France, in 1950. He received the agrégation de physique degree from the Ecole Normale Supérieure de Saint-Cloud, Saint-Cloud, France, in 1972 and the doctorat és Sciences from the University of Paris-Sud, Orsay, in 1979.

Since 1972 he has been a Research Scientist at CNRS (Centre National de la Recherche Scientifique), working at the Institut d'Optique at Orsay. He spent the academic year 1979–1980 as a Visiting Scientist at the Image Processing Institute, University of Southern California, Los Angeles. His research interests include optical-digital image processing, computer generated holography, and applications of optical coherence.

Dr. Chavel is the Secretary of the Société Française d'Optique and is a member of the Optical Society of America, the Society of Photo-Optical Instrumentation Engineers, Société Française de Physique, and Union des Physiciens.