The Constrained Task Assigment Problem

Given

- a set of N tasks,
- a set of M processors,
- the execution cost e_{ik} of task i if it is assigned to processor k,
- the communication cost c_{ij} between tasks i and j if they are assigned to different processors,
- the total memory n_k of processor k, and
- s_i the memory requirement of module i the sum of memory requirements of the modules assigned to a processor k must not exceed n_k. These kind of constraints are called *memory constraints.*

The Constrained Task Assigment Problem consists in finding an assignment of the tasks to the processors such that the memory constraints are satisfied, and such that the total execution and communication cost is minimized.

A natural mathematical formulation of CMAP can be considered by taking the variables vector $x = (x_{ik})$ $(i = 1, \ldots, N; k = 1, \ldots, M)$ where x_{ik} is equal to 1 if module i is allocated to processor k and is equal to 0 otherwise.
Let $c_0 = \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} c_{ij}$, the CMAP can be formulated by the following quadratic 0-1 problem:

$$(\text{Q01}) : \min F(x) = c_0 + \sum_{i=1}^{n} \sum_{k=1}^{M} e_{ik} x_{ik} - \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \sum_{k=1}^{M} c_{ij} x_{ik} x_{jk} \tag{1}$$

Subject to:

$$\sum_{k=1}^{M} x_{ik} = 1 \qquad i = 1, \ldots, N \tag{2}$$

$$\sum_{i=1}^{N} s_i x_{ik} \leq n_k \qquad k = 1, \ldots, M \tag{3}$$

$$x \in \{0,1\}^{N \times M} \tag{4}$$

Instances generation

The instances we use have been introduced in [1]. In these instances, 4 configurations are considered. For each configuration, two classes of instances are generated: a class with a complete communication graph, and a second class where the density of the communication graph is of 50%. This gives a total of 8 types of instances. For each type, 5 instances of size 10 modules and 3 processors and 5 instances of 20 modules and 5 processors are generated.

	Config A	Config B	Config C	Config D
int_e	[0,100]	[0,10]	[0,100]	[0,0]
int_c	[0,100]	[0,100]	[0,10]	[0,100]

Table 1: Different configurations of the instances

Table 1 describes the different configurations. The execution costs e_{ik} are generated in the interval int_e, and the communication costs c_{ij} are generated in the interval int_c. For all the configurations, the sizes of the modules s_i are generated in the interval [1, 10], and the capacities of the processors n_k in the interval $[S/M,\ 2 * S/M]$ where $S = \sum_{i=1}^{N} s_i$ is the sum of all the module sizes. In this way, we are sure that problem (Q01) has at least one fractional solution $\tilde{x}$ where $\forall (i,k),\ \tilde{x}_{ik} = \frac{1}{M}$.

References

[1] S. Elloumi. *Contribution à la résolution des programmes non linéaires en variables 0-1, application aux problèmes de placement de tâches dans les systèmes distribués.* Thèse de doctorat en informatique, Conservatoire National des Arts et Métiers, Paris, 1991.