Computer Algorithms Final Exam **6/15/2021**

1. **(40%) True or False**
   (1) One can give an O (V + E) time algorithm for the single-source shortest paths problem.
   (2) The topological sort of an arbitrary directed acyclic graph G = (V, E) can be computed in linear time.
   (3) Strongly connected components of a graph cannot be found in linear time.
   (4) The Huffman algorithm for constructing an optimal prefix code is a greedy algorithm.
   (5) In a depth-first search of an undirected graph G, every edge of G is either a tree edge or a back edge.
   (6) The running time of Bellman-Ford algorithm is O(|VE|).
   (7) Kruskal's algorithm for finding a minimum spanning tree of a weighted, undirected graph is an example of a dynamic programming algorithm.
   (8) S. Cook had proved that $P \neq NP$.
   (9) If L$\in$ NP-hard and L' α (reduces to) L then L'$\in$ NP-hard.
   (10) One can determine whether a given undirected graph G = (V, E) contains a cycle in O (V) time.

2. **(20%)** The (fractional) Knapsack problem is :

   maximize $\sum_{1\le i\le n} p_i x_i$

   subject to $\sum_{1\le i\le n} w_i x_i \le \mathrm{W}, \quad 0 \le x_i \le 1, \ \forall 1 \le i \le n.$

   Find an optimal solution to the instance $n = 6, \mathrm{W} = 13,$

   $(p_1, p_2, ..., p_6)$ = (20, 8, 15, 5, 6, 18), and $(w_1, w_2, ..., w_6)$ = (5, 4, 3, 2, 3, 4).

3. **(20%)** We are given a directed graph $G = (V, E)$ on which each edge $(u, v) \in E$ has an associated value $r(u, v)$, which is a real number in the range $0 \le r(u, v) \le 1$ that represents the reliability of a communication channel from vertex u to vertex v. We interpret $r(u, v)$ as the probability that the channel from u to v will not fail, and we assume that these probabilities are independent. Give an efficient algorithm to find the most reliable path between two given vertices.

4. **(20%)** Give a simple example of directed graph with negative-weight edges and 3 vertices for which Dijkstra's algorithm produces incorrect answers.

5. **(20%)** Give an example of
   (1) P
   (2) NP
   (3) NP-complete
   (4) NP-hard but not NP.