The Inverse Trigonometric Functions

The Inverse Sine (or Arcsine) Function

Let us define a function Sin x (note the capital letter) to be $\sin x$, restricted so that its domain is the interval $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$:

The restricted function Sin x

$$\operatorname{Sin} x = \sin x \qquad \text{if } -\frac{\pi}{2} \le x \le \frac{\pi}{2}.$$

$\operatorname{Sin}: \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \to [-1,1]$

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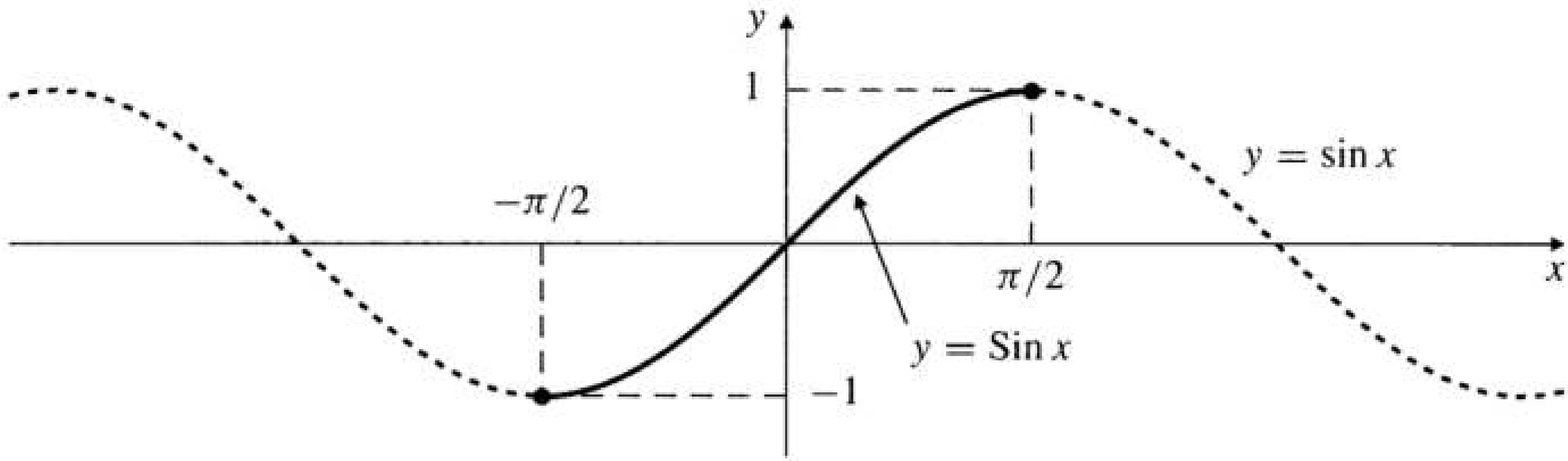

The Inverse Trigonometric Functions

The Inverse Sine (or Arcsine) Function

The inverse sine function $\sin^{-1} x$ or arcsin x

$$y = \sin^{-1} x \quad \Longleftrightarrow \quad x = \operatorname{Sin} y$$

$$\Longleftrightarrow \quad x = \sin y \quad \text{and} \quad -\frac{\pi}{2} \le y \le \frac{\pi}{2}$$

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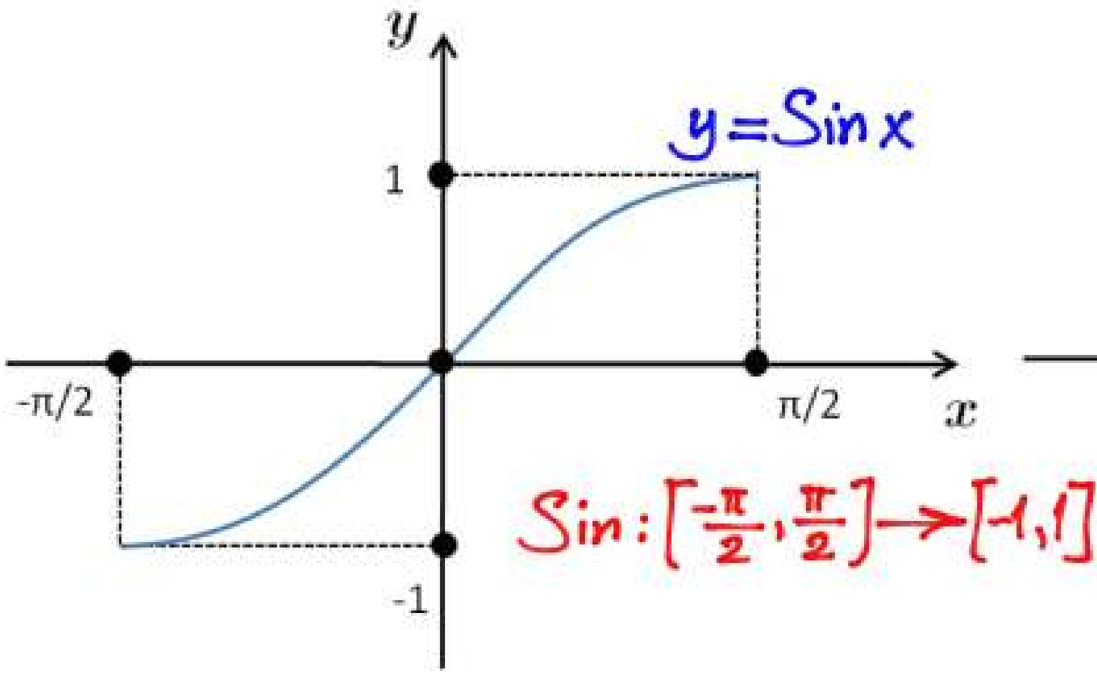

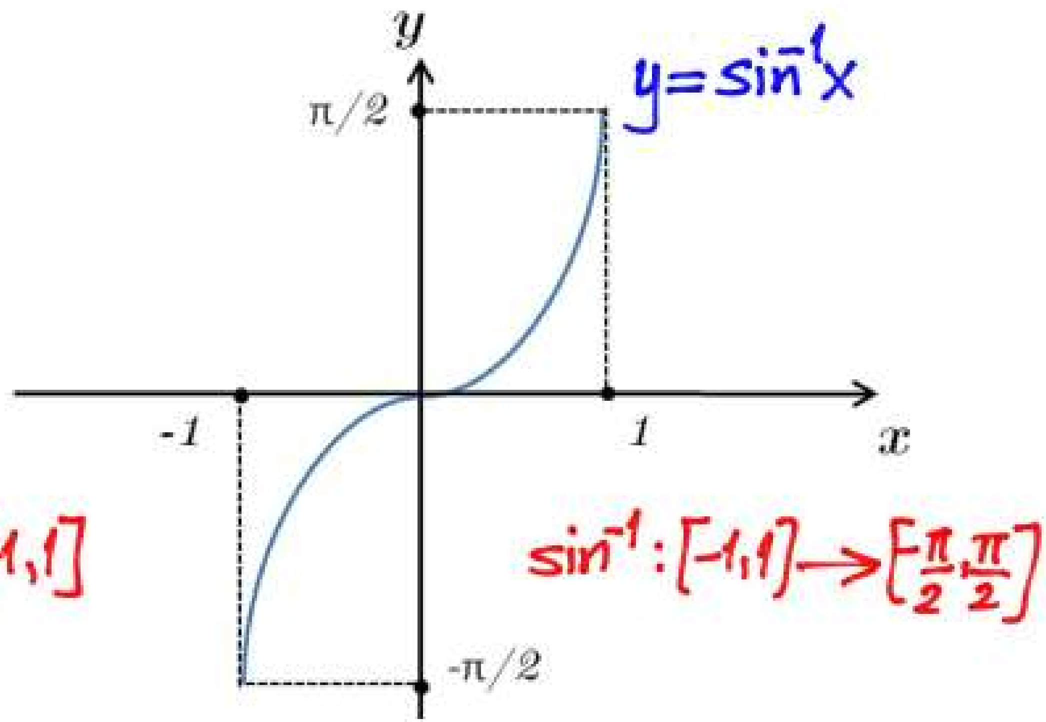

The Inverse Trigonometric Functions

The Inverse Sine (or Arcsine) Function

The cancellation identities for Sin and $\sin^{-1}$

$$\sin^{-1}(\operatorname{Sin} x) = \arcsin(\operatorname{Sin} x) = x \qquad \text{for } -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$$

$$\operatorname{Sin}(\sin^{-1} x) = \operatorname{Sin}(\arcsin x) = x \qquad \text{for } -1 \leq x \leq 1$$

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EXAMPLE

Find (a) $\sin\left(\sin^{-1} 0.7\right)$, (b) $\sin^{-1}(\sin 0.3)$, (c) $\sin^{-1}\left(\sin \frac{4\pi}{5}\right)$, and (d) $\cos\left(\sin^{-1} 0.6\right)$.

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EXAMPLE Simplify the expression $\tan(\sin^{-1} x)$.

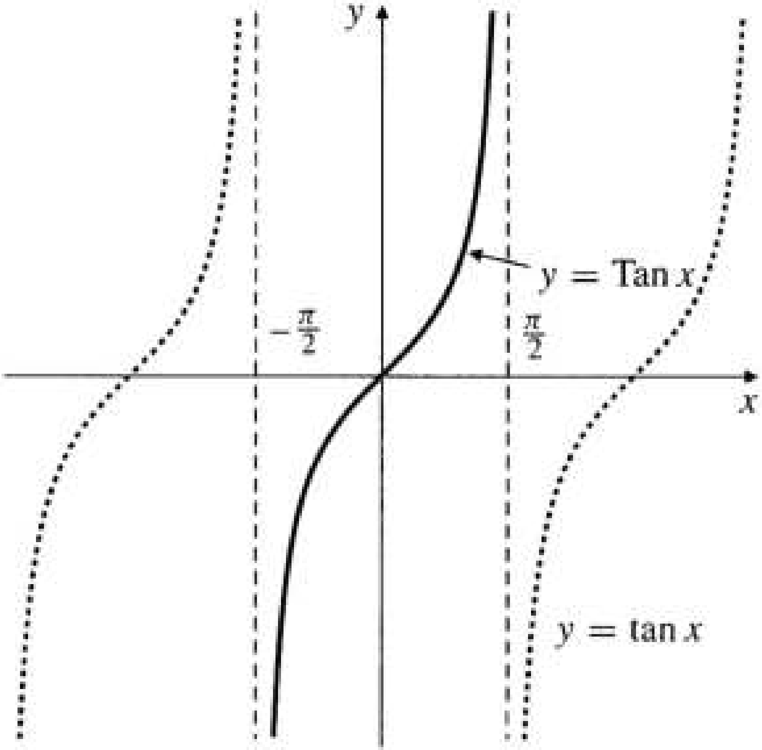
The Inverse Trigonometric Functions

The Inverse Tangent (or Arctangent) Function

The restricted function Tan x

$$\operatorname{Tan} x = \tan x \qquad \text{if } -\frac{\pi}{2} < x < \frac{\pi}{2}.$$

$\operatorname{Tan} : \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \longrightarrow \mathbb{R}$



The Inverse Trigonometric Functions

The Inverse Tangent (or Arctangent) Function

The inverse tangent function $\tan^{-1} x$ or $\arctan x$

$$y = \tan^{-1} x \iff x = \text{Tan}\, y$$

$$\iff x = \tan y \quad \text{and} \quad -\frac{\pi}{2} < y < \frac{\pi}{2}$$

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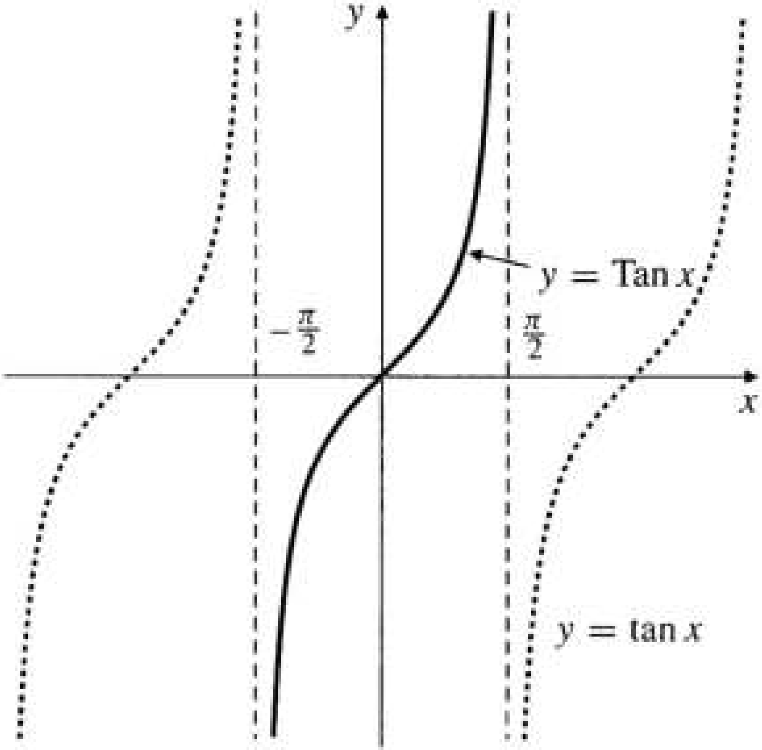

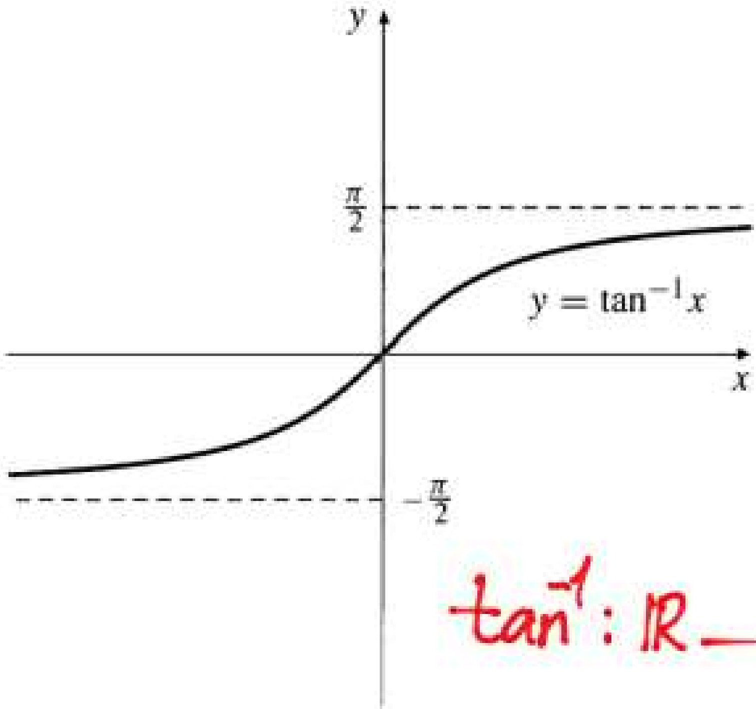

$\tan^{-1} : \mathbb{R} \longrightarrow \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

The Inverse Tangent (or Arctangent) Function

The cancellation identities for Tan and $\tan^{-1}$

$$\tan^{-1}(\operatorname{Tan} x) = \arctan(\operatorname{Tan} x) = x \qquad \text{for } -\frac{\pi}{2} < x < \frac{\pi}{2}$$

$$\operatorname{Tan}(\tan^{-1} x) = \operatorname{Tan}(\arctan x) = x \qquad \text{for } -\infty < x < \infty$$

Other Inverse Trigonometric Functions

The function $\cos x$ is one-to-one on the interval $[0, \pi]$.

$$\boxed{y = \cos^{-1} x \quad \Longleftrightarrow \quad x = \cos y \quad \text{and} \quad 0 \le y \le \pi.}$$

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$$\boxed{y = \cos^{-1} x \quad \Longleftrightarrow \quad x = \cos y \quad \text{and} \quad 0 \le y \le \pi.}$$

$$y = \cos^{-1} x \iff x = \sin\left(\frac{\pi}{2} - y\right) \iff \sin^{-1} x = \frac{\pi}{2} - y = \frac{\pi}{2} - \cos^{-1} x.$$

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$$\boxed{\cos^{-1} x = \frac{\pi}{2} - \sin^{-1} x \qquad \text{for} \quad -1 \le x \le 1.}$$

The Inverse Trigonometric Functions

Other Inverse Trigonometric Functions

The cancellation identities for $\cos^{-1}$

$$\cos^{-1}(\cos x) = \arccos(\cos x) = x \qquad \text{for } 0 \le x \le \pi$$

$$\cos(\cos^{-1} x) = \cos(\arccos x) = x \qquad \text{for } -1 \le x \le 1$$

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The cancellation identities for $\cos^{-1}$

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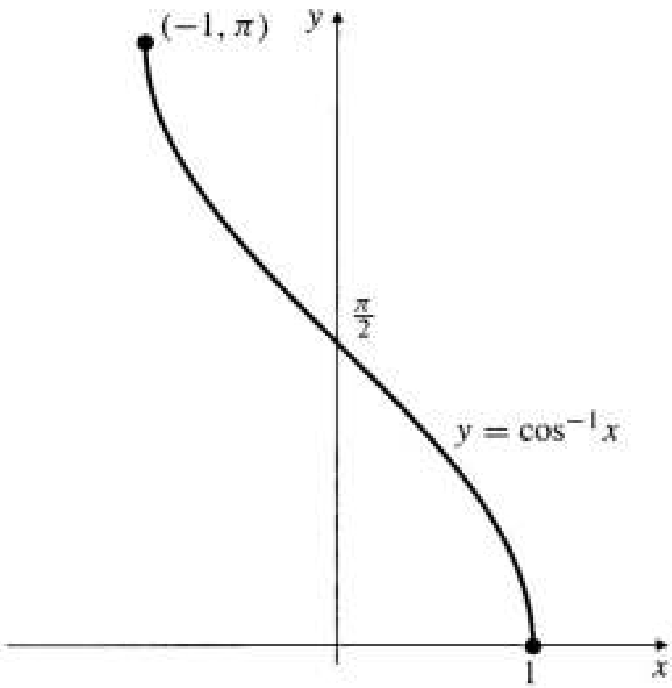


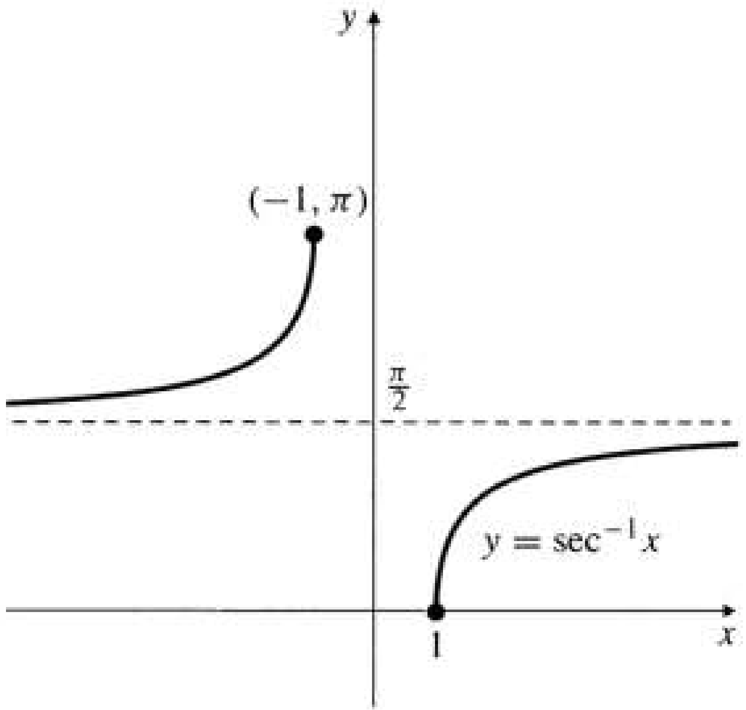
The Inverse Trigonometric Functions

Other Inverse Trigonometric Functions

The inverse secant function $\sec^{-1} x$ (or arcsec x)

$$\sec^{-1} x = \cos^{-1}\left(\frac{1}{x}\right) \qquad \text{for} \quad |x| \geq 1.$$

$\sec^{-1} : (-\infty, -1] \cup [1, \infty) \longrightarrow [0, \pi/2) \cup (\pi/2, \pi]$



$\sec^{-1}(\sec x) = x$

$\sec(\sec^{-1} x) = x$

Other Inverse Trigonometric Functions

The inverse cosecant and inverse cotangent functions

$$\csc^{-1} x = \sin^{-1}\left(\frac{1}{x}\right), \quad (|x| \geq 1); \qquad \cot^{-1} x = \tan^{-1}\left(\frac{1}{x}\right), \quad (x \neq 0)$$

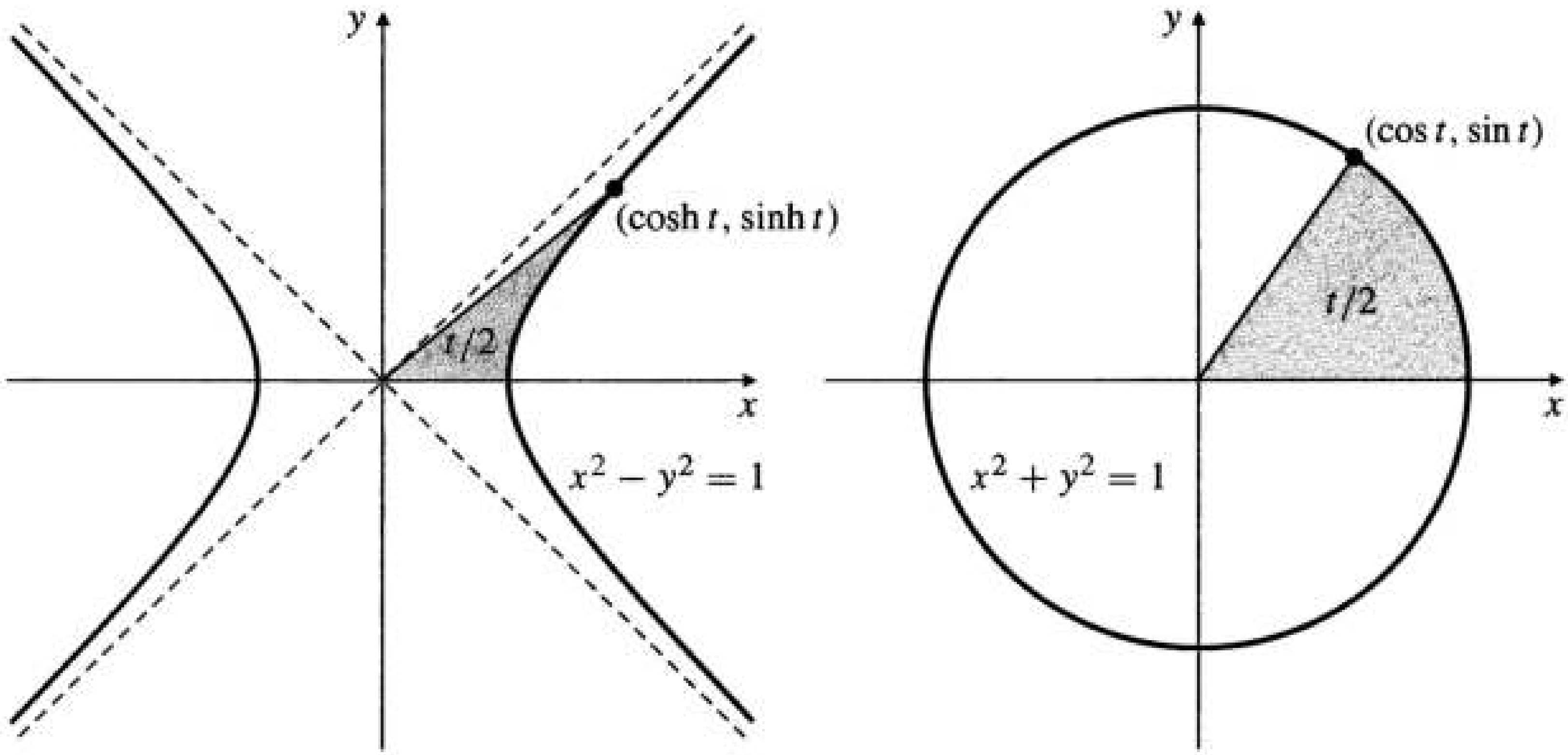
Hyperbolic Functions

The hyperbolic cosine and hyperbolic sine functions

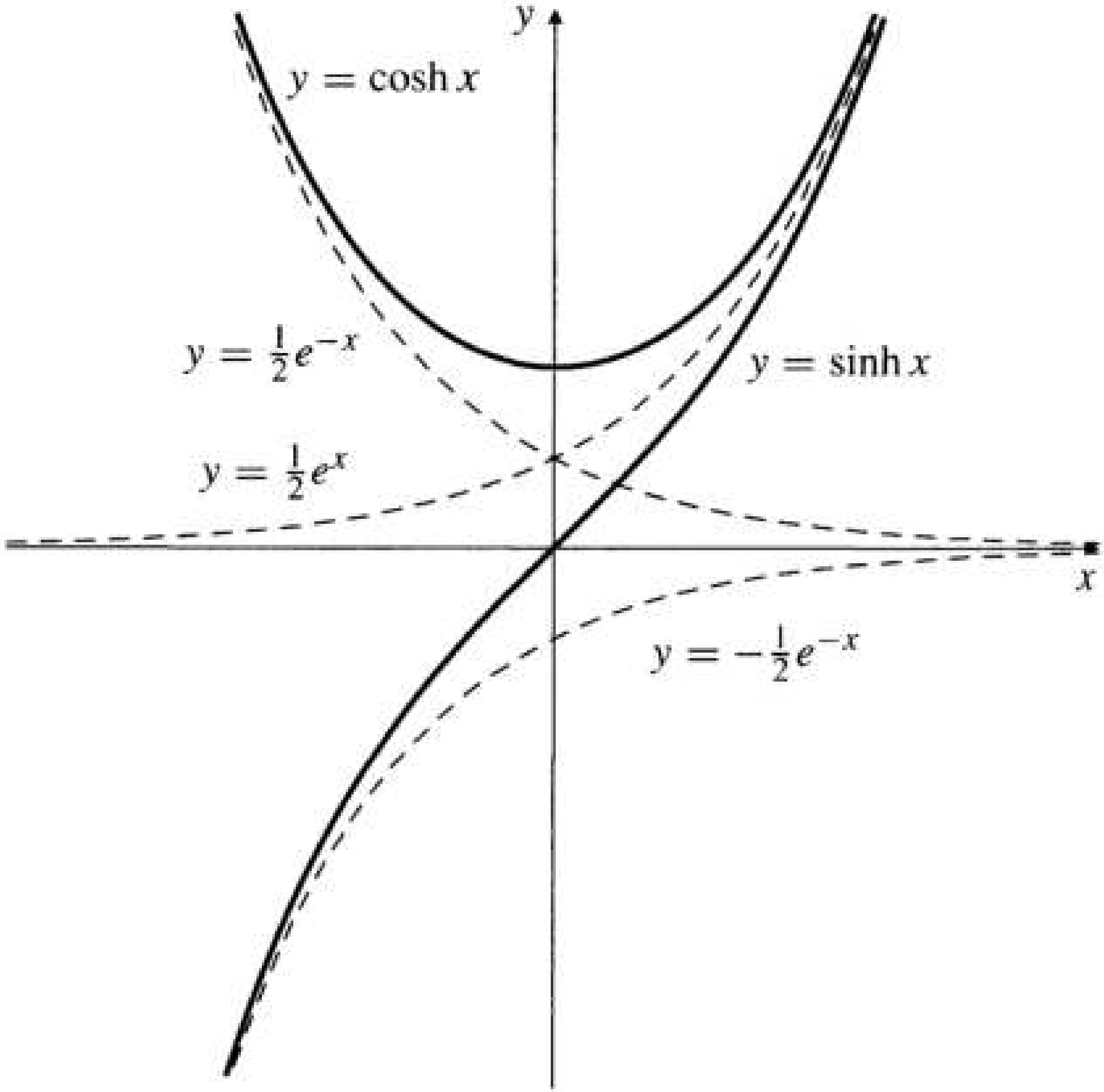
For any real x the **hyperbolic cosine**, $\cosh x$, and the **hyperbolic sine**, $\sinh x$, are defined by

$$\cosh x = \frac{e^x + e^{-x}}{2}, \qquad \sinh x = \frac{e^x - e^{-x}}{2}.$$

Hyperbolic Functions



Hyperbolic Functions



Hyperbolic Functions

Other hyperbolic functions

$$\tanh x = \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}} \qquad \operatorname{sech} x = \frac{1}{\cosh x} = \frac{2}{e^x + e^{-x}}$$

$$\coth x = \frac{\cosh x}{\sinh x} = \frac{e^x + e^{-x}}{e^x - e^{-x}} \qquad \operatorname{csch} x = \frac{1}{\sinh x} = \frac{2}{e^x - e^{-x}}$$