The Absolute Value

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

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absolute value bars

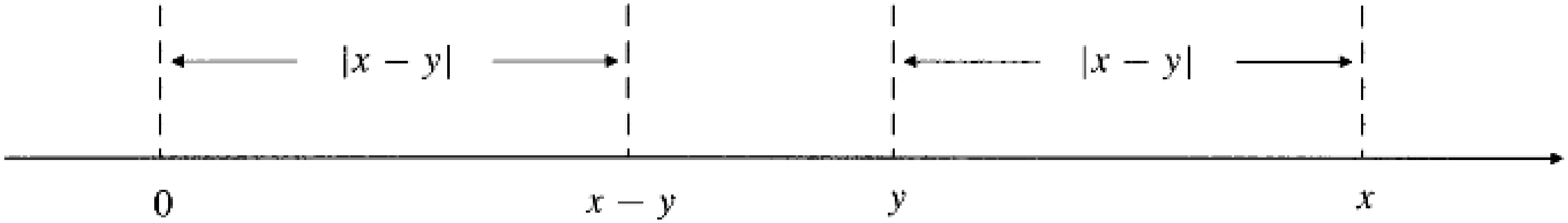
The Absolute Value

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

absolute value bars

- $|x| \geq 0$ for every real number x, and
- $|x| = 0$ only if $x = 0$.

The Absolute Value



absolute value represents the distance between points

The Absolute Value

Properties of absolute values

1. $|-a| = |a|$.

The Absolute Value

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2. $|ab| = |a||b|$ and $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$.

The Absolute Value

Properties of absolute values

1. $|-a| = |a|$.

2. $|ab| = |a||b|$ and $\left|\dfrac{a}{b}\right| = \dfrac{|a|}{|b|}$.

3. $|a \pm b| \leq |a| + |b|$ (the **triangle inequality**).

The Absolute Value

$$|x-a| = D \iff \text{either } x = a - D \text{ or } x = a + D$$

$$|x-a| < D \iff a - D < x < a + D$$

$$|x-a| \le D \iff a - D \le x \le a + D$$

$$|x-a| > D \iff \text{either } x < a - D \text{ or } x > a + D$$

The Absolute Value

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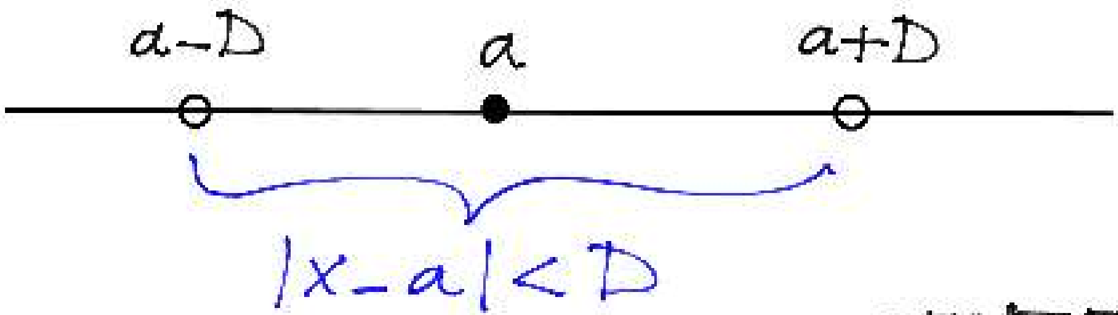


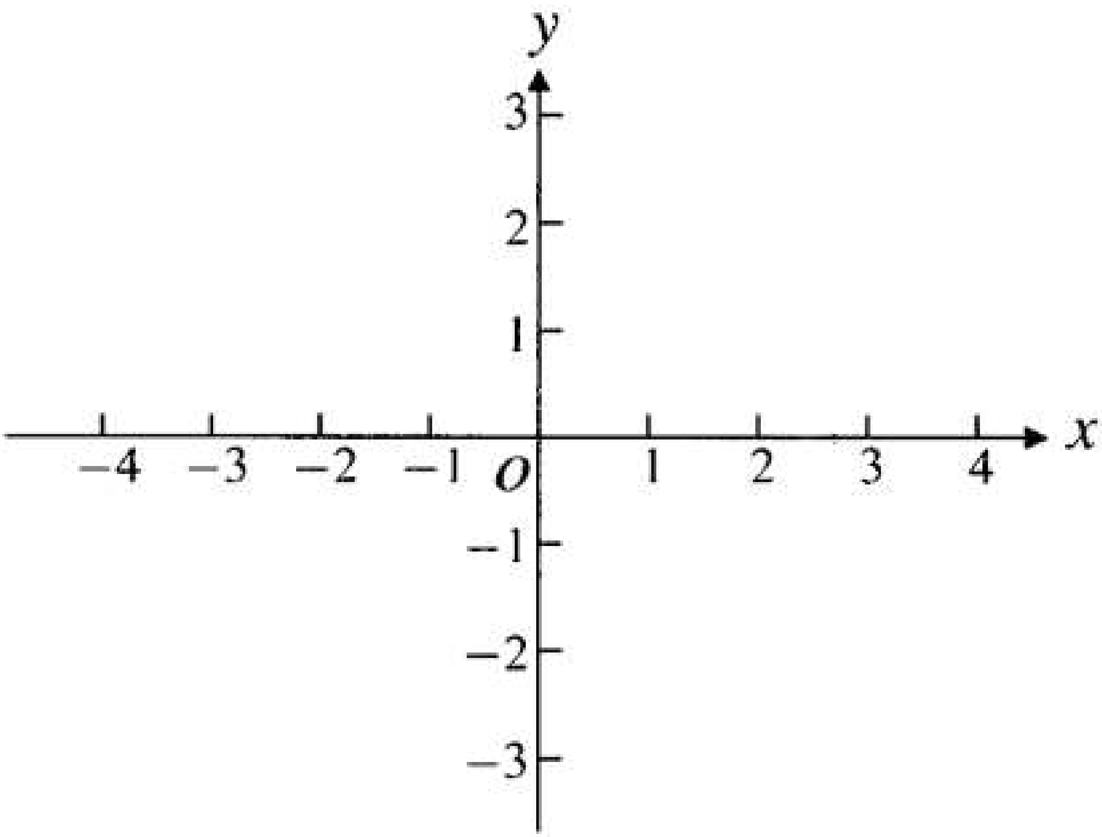
CENTER = a

RADIUS = D

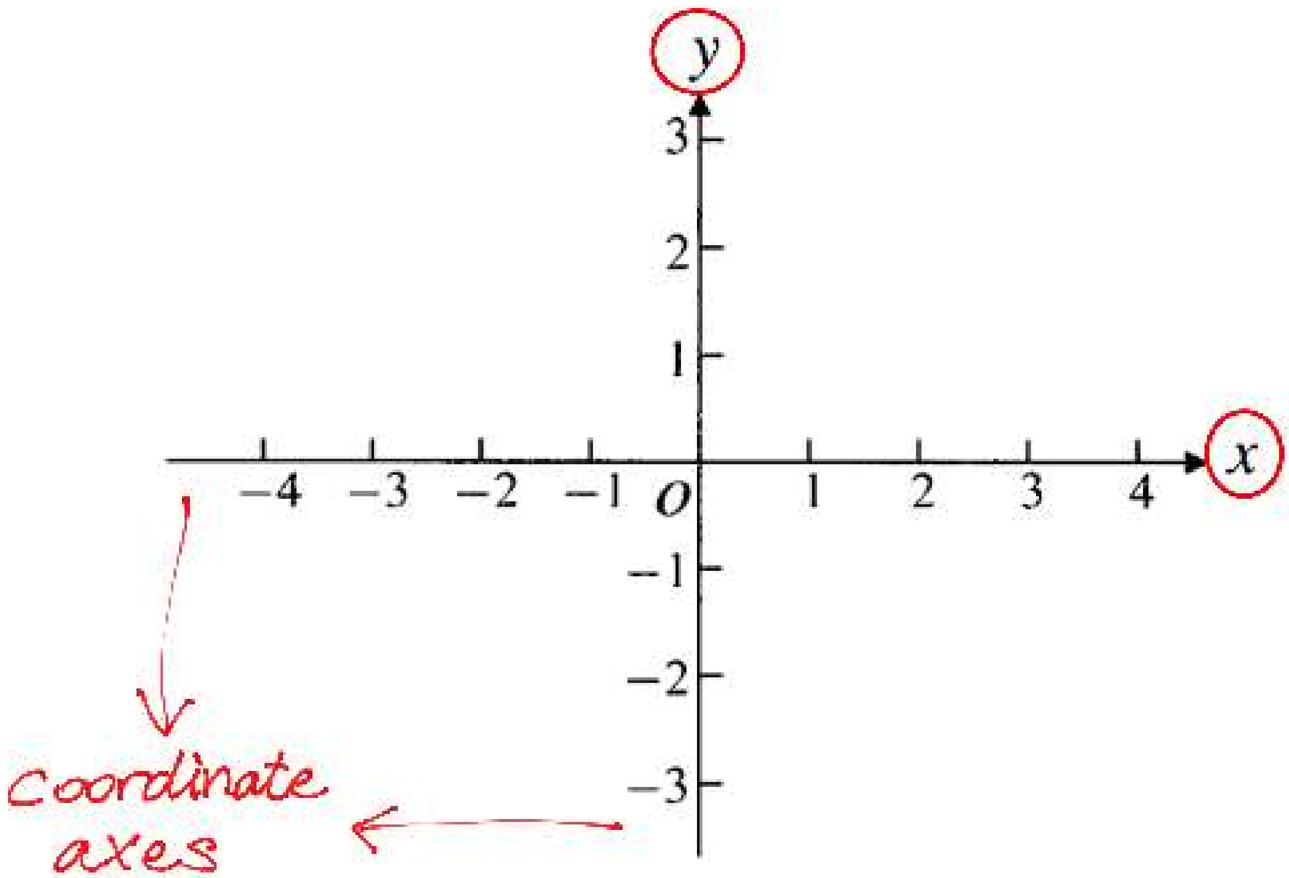
The Absolute Value

EXAMPLE What values of x satisfy the inequality $\left|5 - \frac{2}{x}\right| < 3$?

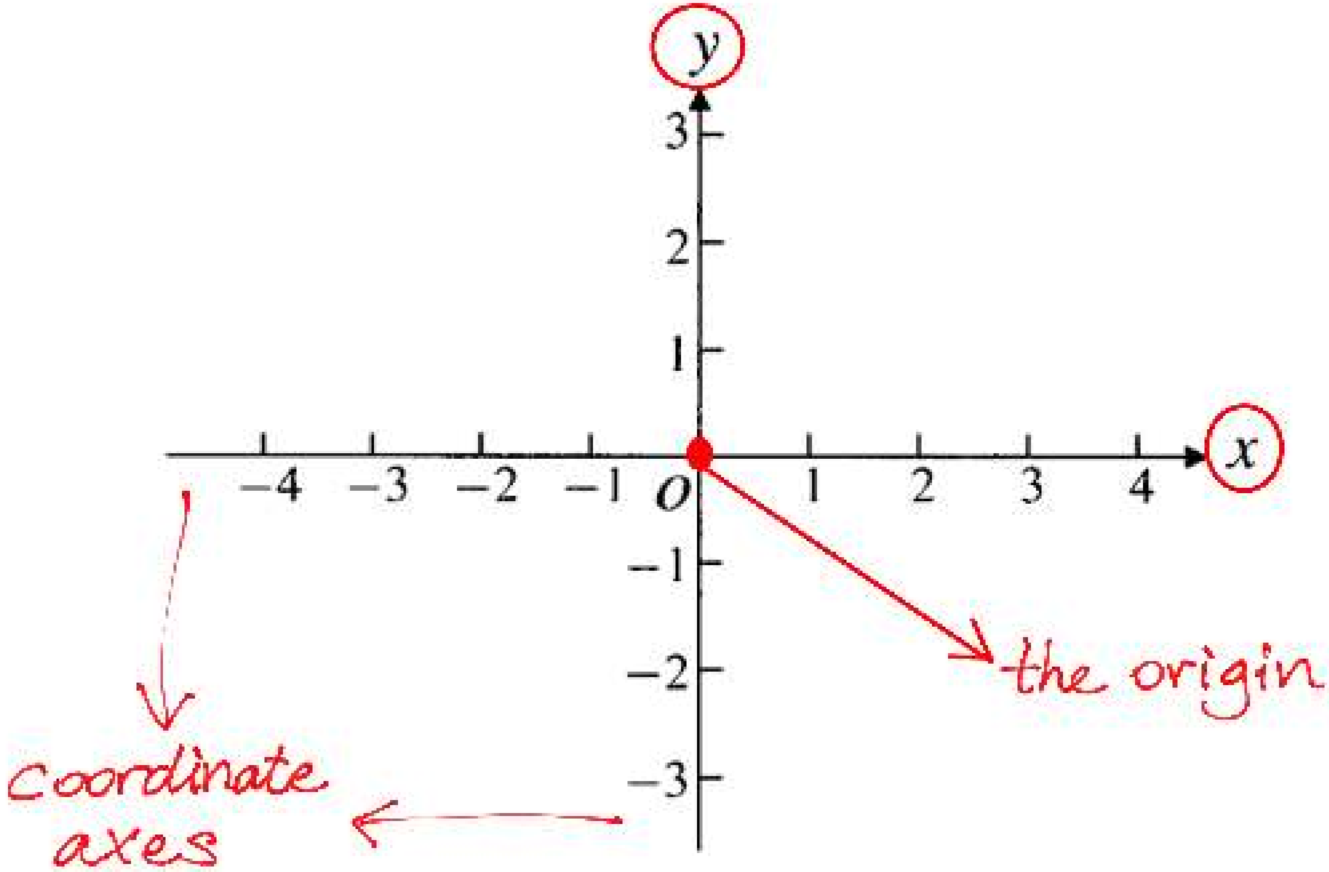
Cartesian Coordinates in the Plane



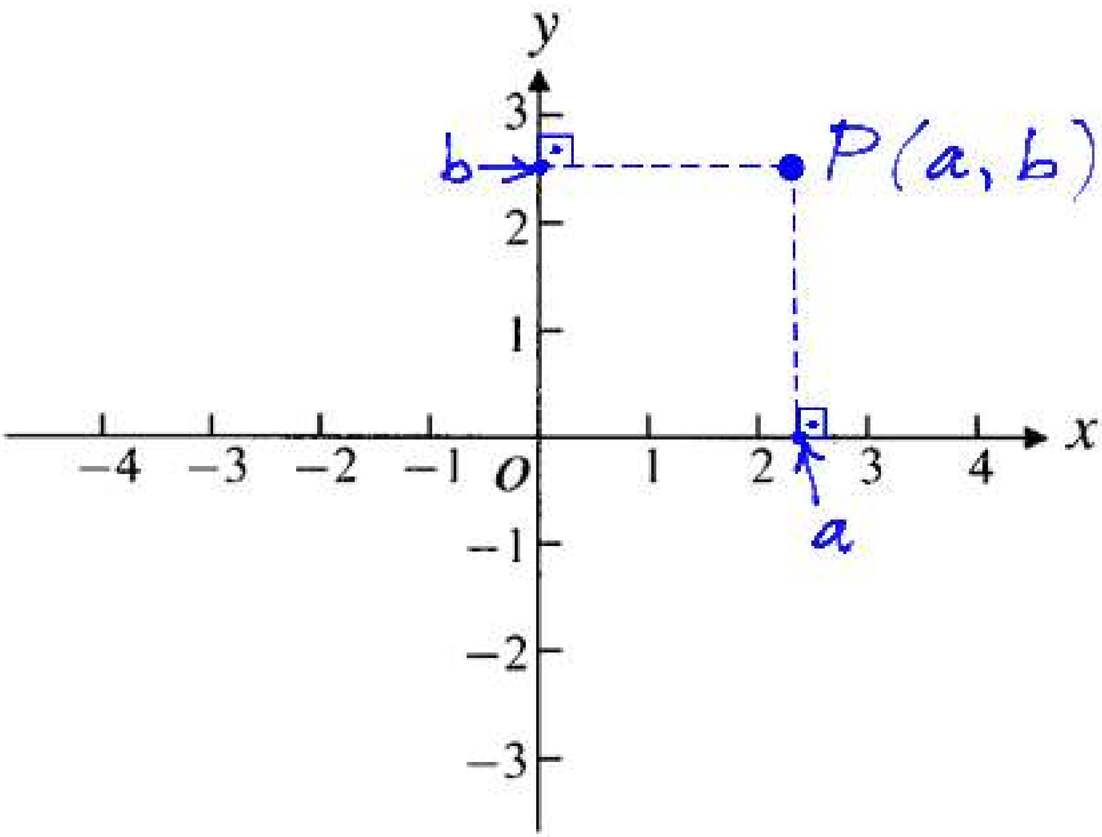
Cartesian Coordinates in the Plane



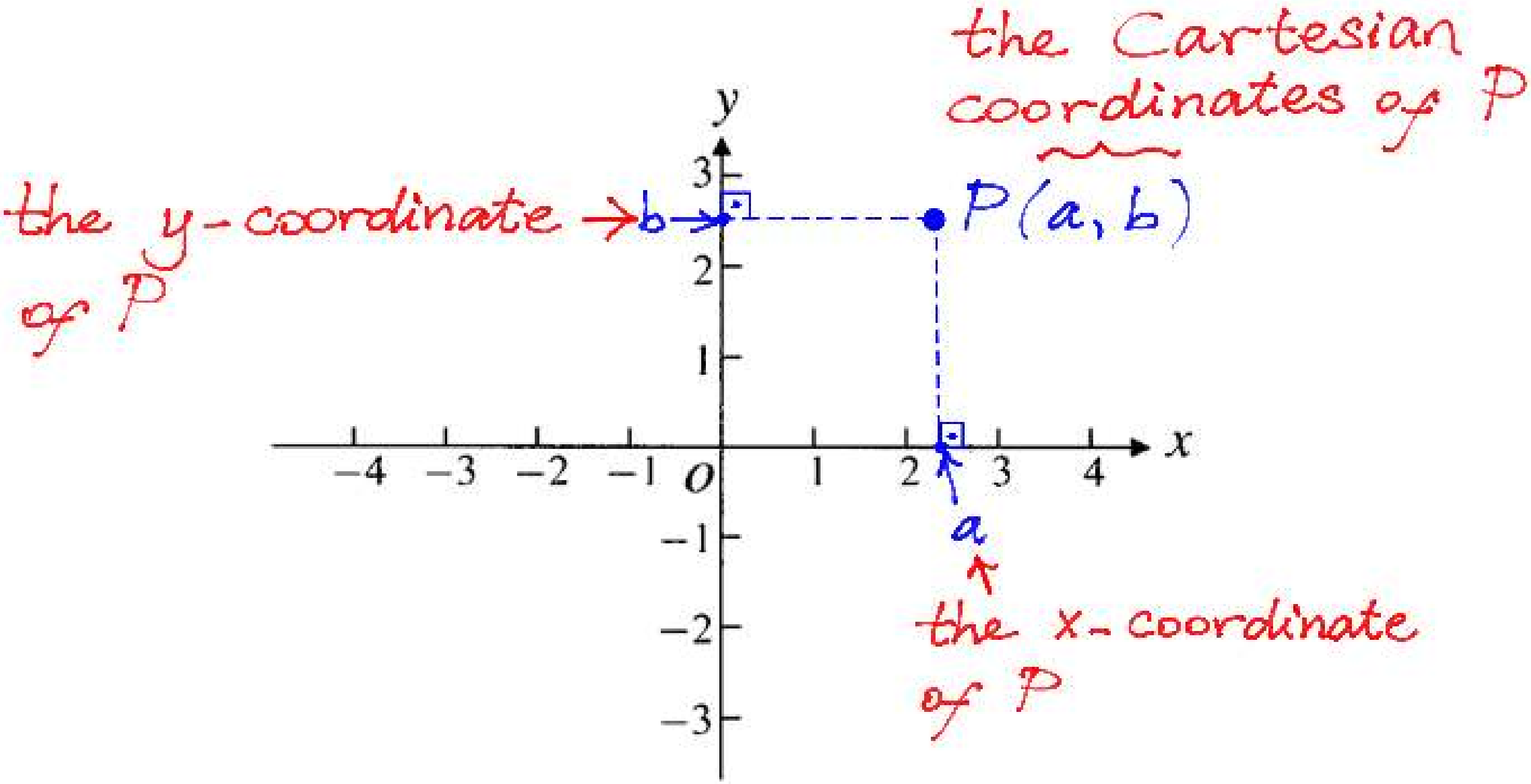
Cartesian Coordinates in the Plane



Cartesian Coordinates in the Plane

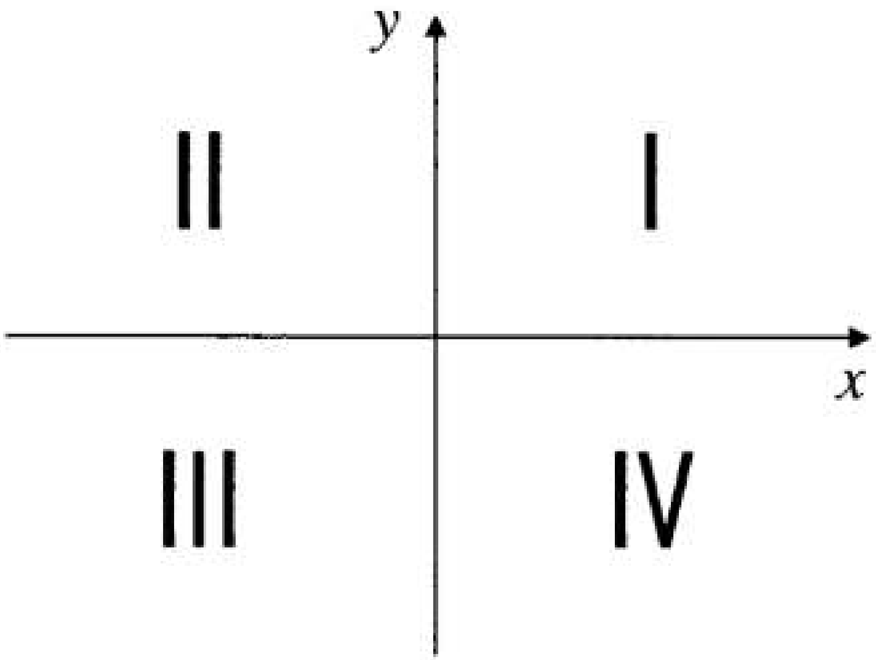


Cartesian Coordinates in the Plane



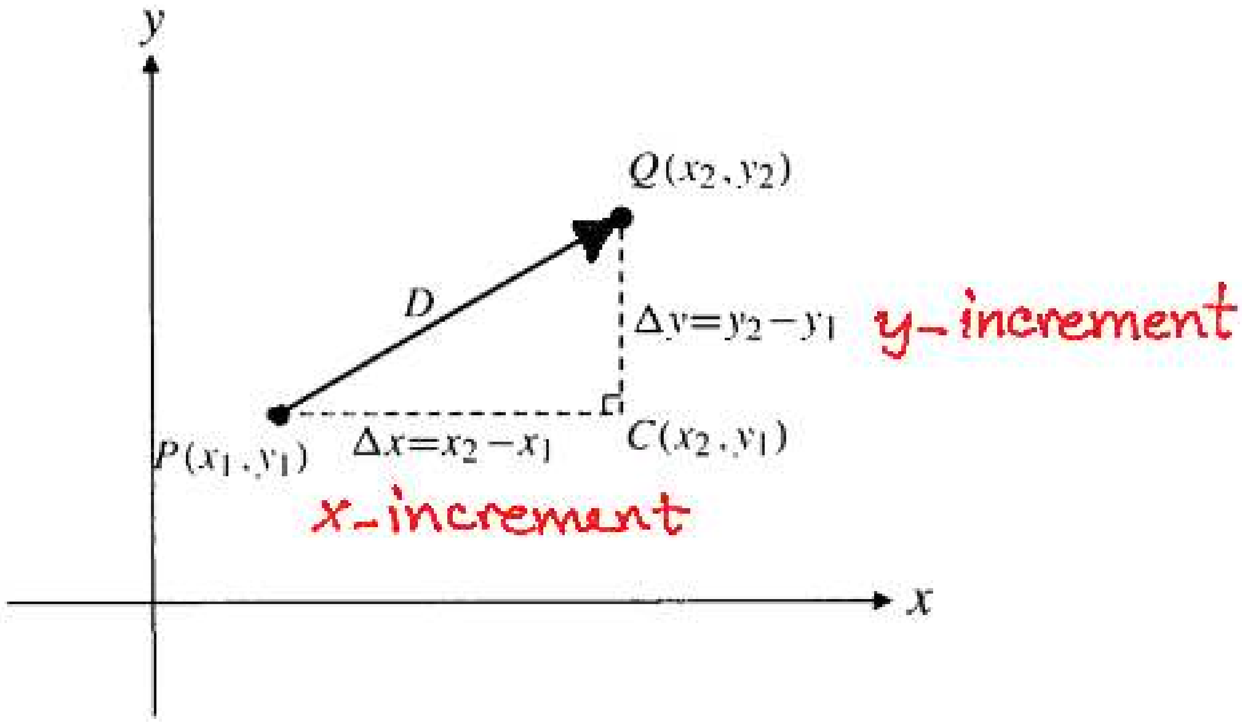
Cartesian coordinate system (after René Descartes)

Cartesian Coordinates in the Plane



The four quadrants

Increments and Distances

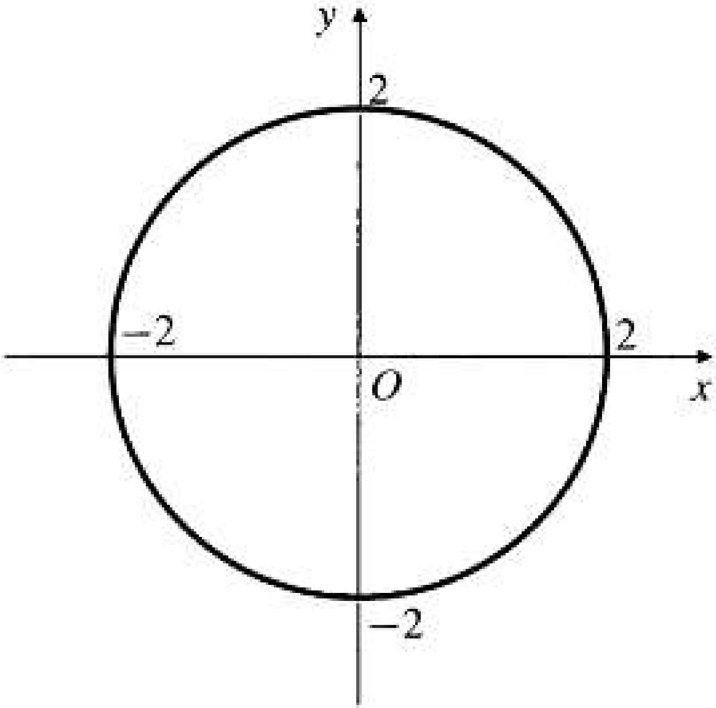


The distance from P to Q is

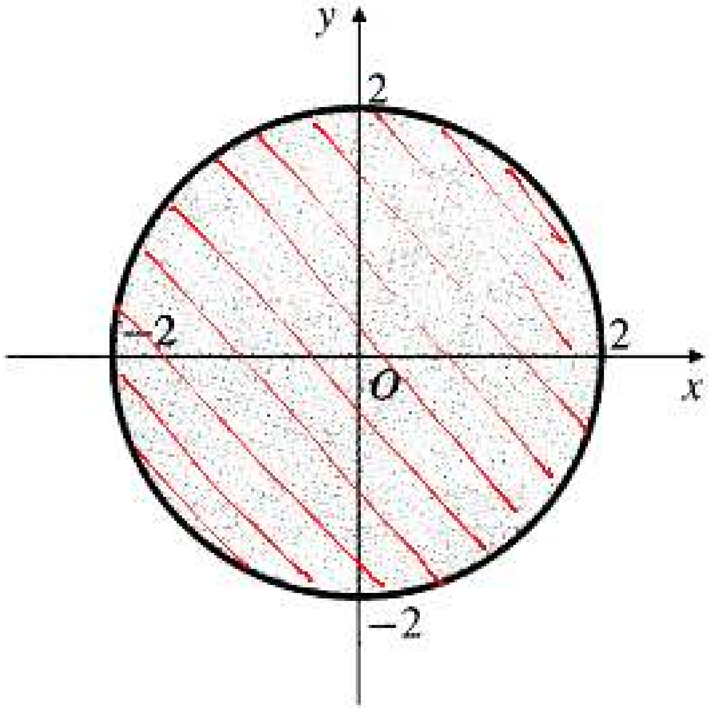
$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Graphs

The **graph** of an equation (or inequality) involving the variables x and y is the set of all points $P(x, y)$ whose coordinates satisfy the equation (or inequality).

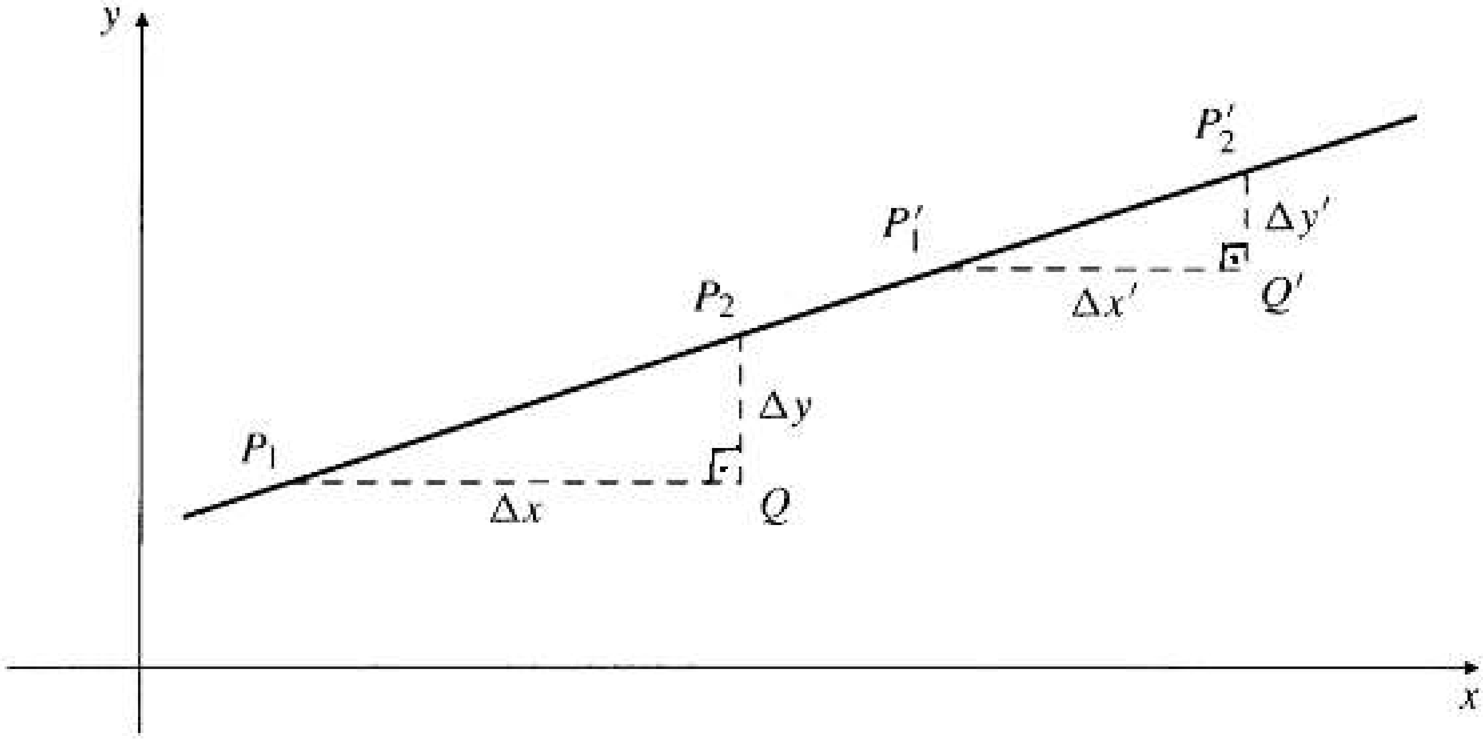


$x^2+y^2=4$



$x^2+y^2 \leq 4$

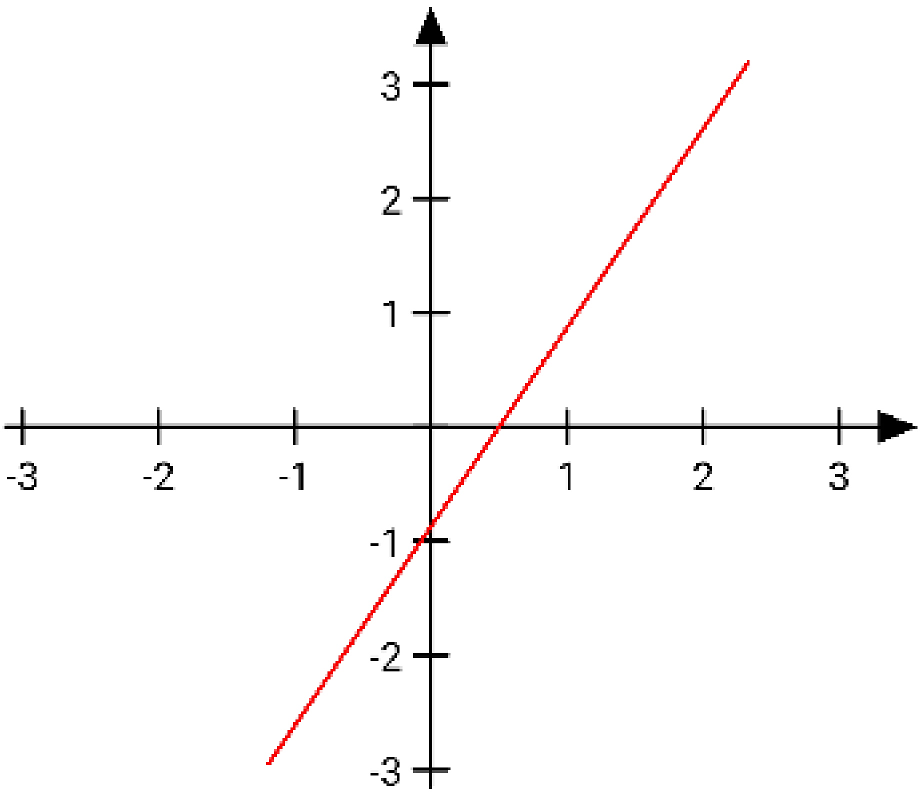
Straight Lines



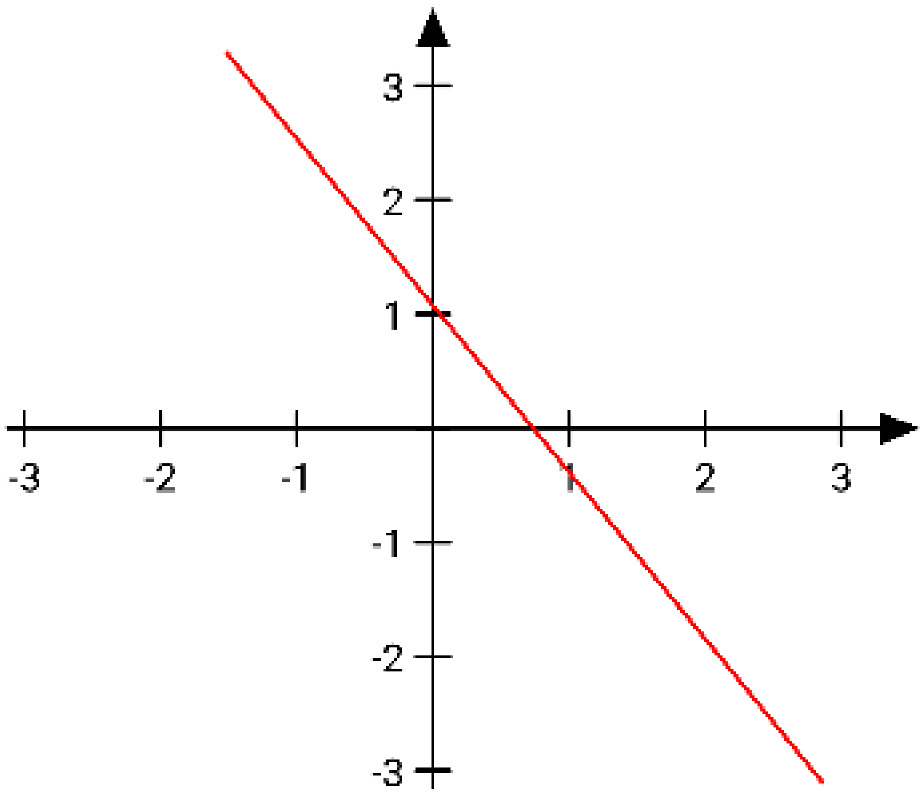
$$m = \frac{\Delta y}{\Delta x} = \frac{\Delta y'}{\Delta x'}$$

slope

Straight Lines

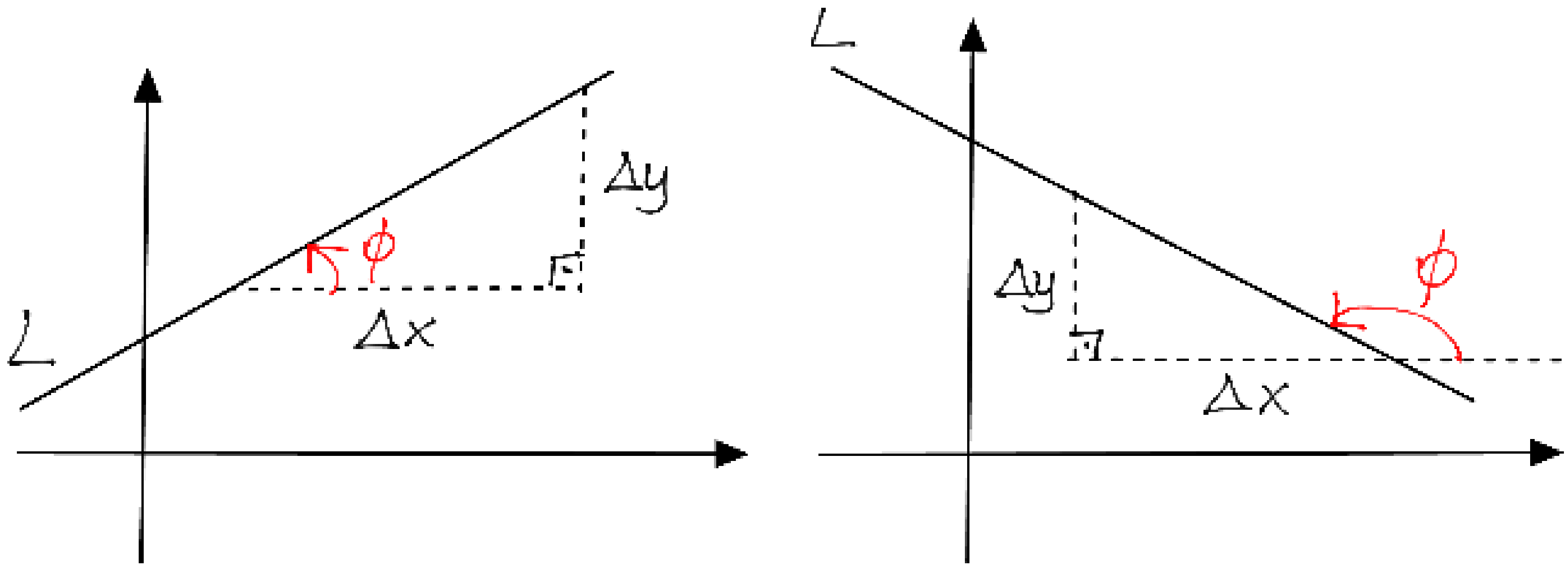


slope > 0



slope < 0

Straight Lines



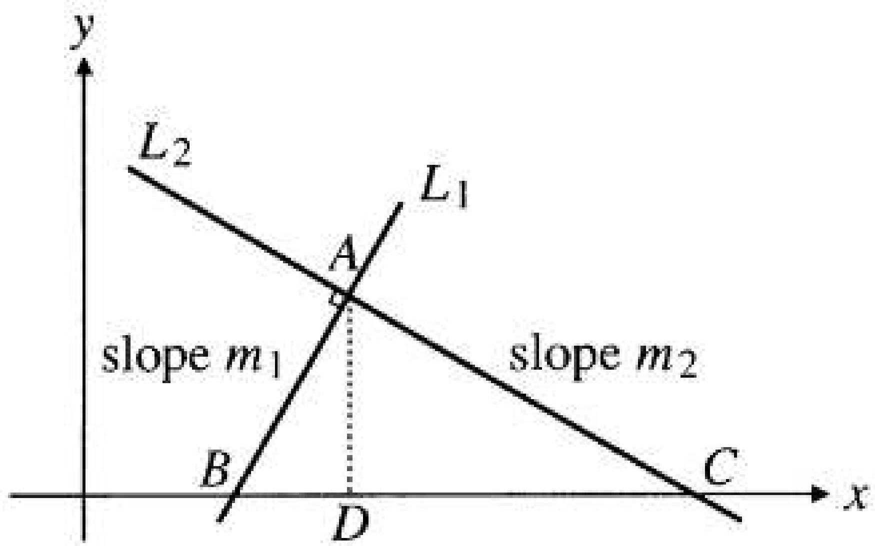
The line L has inclination φ

$$m = \frac{\Delta y}{\Delta x} = \tan\phi$$

Straight Lines

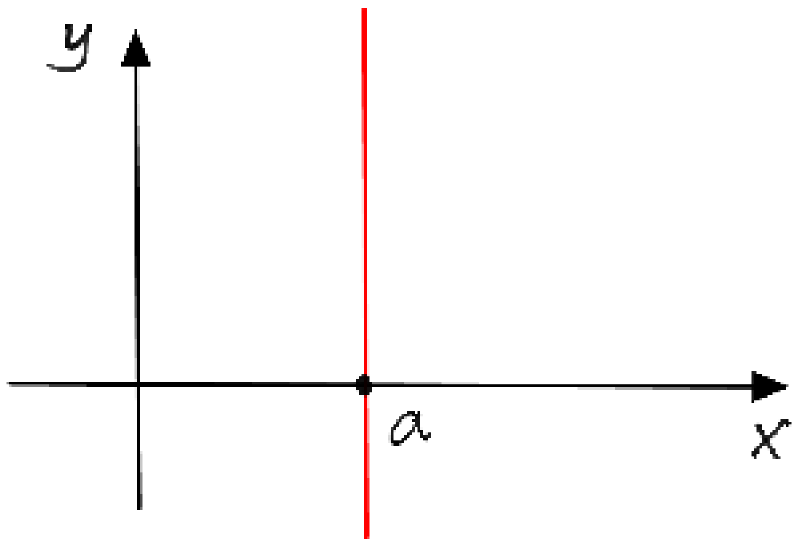
If two nonvertical lines, L_1 and L_2, are perpendicular, their slopes m_1 and m_2 satisfy $m_1 m_2 = -1$, so each slope is the *negative reciprocal* of the other:

$$m_1 = -\frac{1}{m_2} \qquad \text{and} \qquad m_2 = -\frac{1}{m_1}.$$



Straight Lines

Equations of Lines



the line $x=a$

the line $y=b$

Straight Lines

Equations of Lines

The equation

$$y = m(x - x_1) + y_1$$

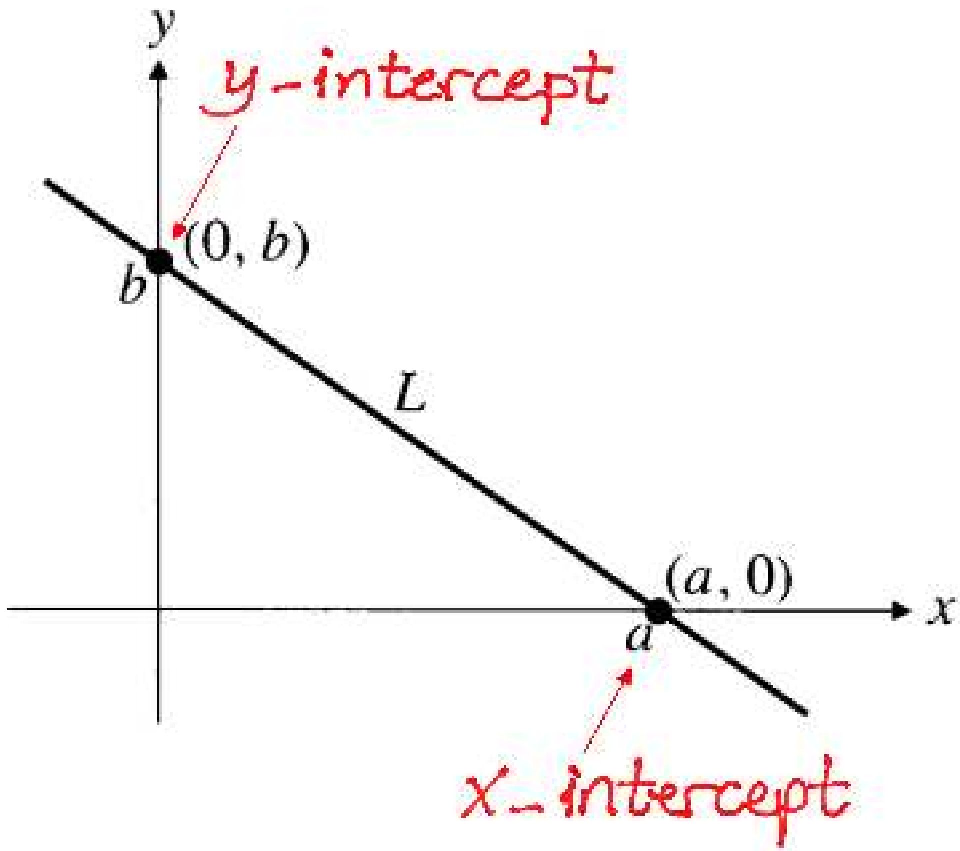
is the **point-slope equation** of the line that passes through the point (x_1, y_1) and has slope m.

EXAMPLE

Find an equation of the line through the points $(1, -1)$ and $(3, 5)$.

Straight Lines

Equations of Lines



$$y = mx + b$$

the **slope–y-intercept equation**

$$y = m(x - a)$$

the **slope–x-intercept equation**

The equation $Ax + By = C$ (where A and B are not both zero) is called the **general linear equation** in x and y.

Graphs of Quadratic Equations

Circles and Disks

Standard equation of a circle

The circle with centre (h, k) and radius $a > 0$ has equation

$$(x - h)^2 + (y - k)^2 = a^2.$$

In particular, the circle with centre at the origin $(0, 0)$ and radius a has equation

$$x^2 + y^2 = a^2.$$

Graphs of Quadratic Equations

Circles and Disks

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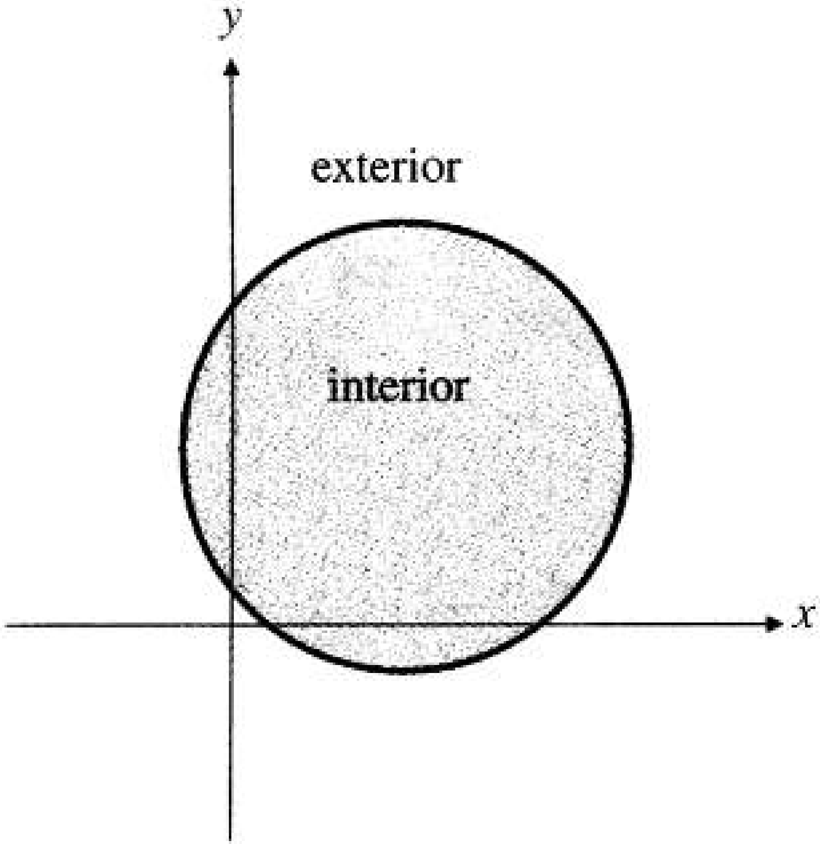
In particular, the circle with centre at the origin $(0, 0)$ and radius a has equation

$$x^2 + y^2 = a^2.$$

$$x^2 + y^2 + 2ax + 2by = c \longrightarrow \begin{cases} \text{circle when } c+a^2+b^2 > 0 \\ \text{single point when } c+a^2+b^2 = 0 \\ \text{no points when } c+a^2+b^2 < 0 \end{cases}$$

Graphs of Quadratic Equations

Circles and Disks



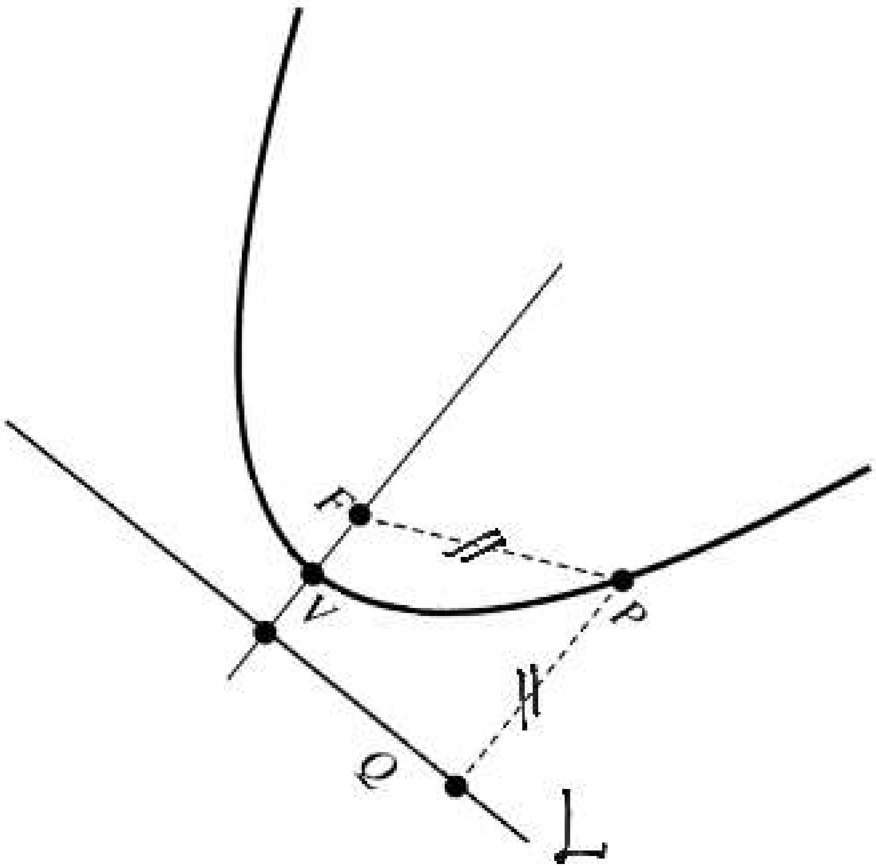
A disk of radius a with center (h,k)

$$(x-h)^2 + (y-k)^2 \le a^2$$

Graphs of Quadratic Equations

Equations of Parabolas

A **parabola** is a plane curve whose points are equidistant from a fixed point F and a fixed straight line L that does not pass through F.



Graphs of Quadratic Equations

Equations of Parabolas

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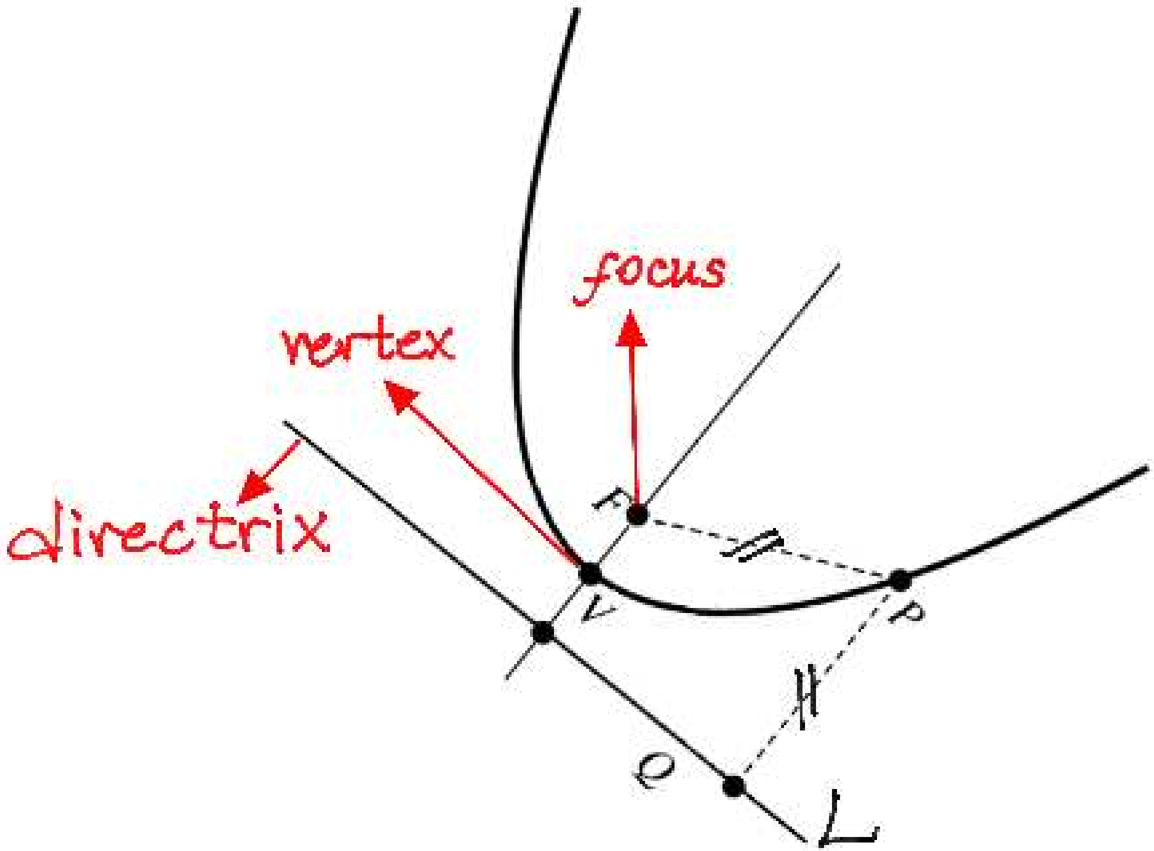
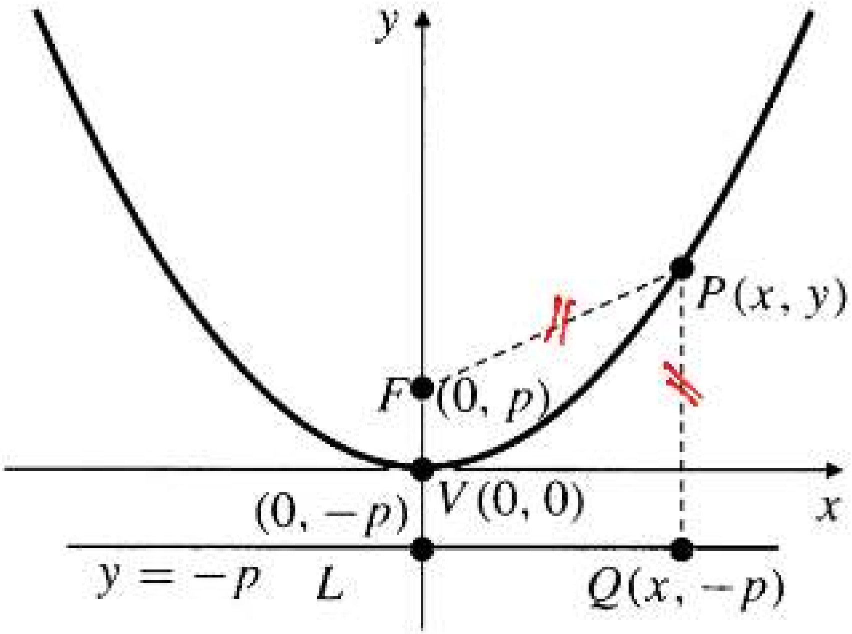


Graphs of Quadratic Equations

Equations of Parabolas

EXAMPLE Find an equation of the parabola having the point $F(0, p)$ as focus and the line L with equation $y = -p$ as directrix.



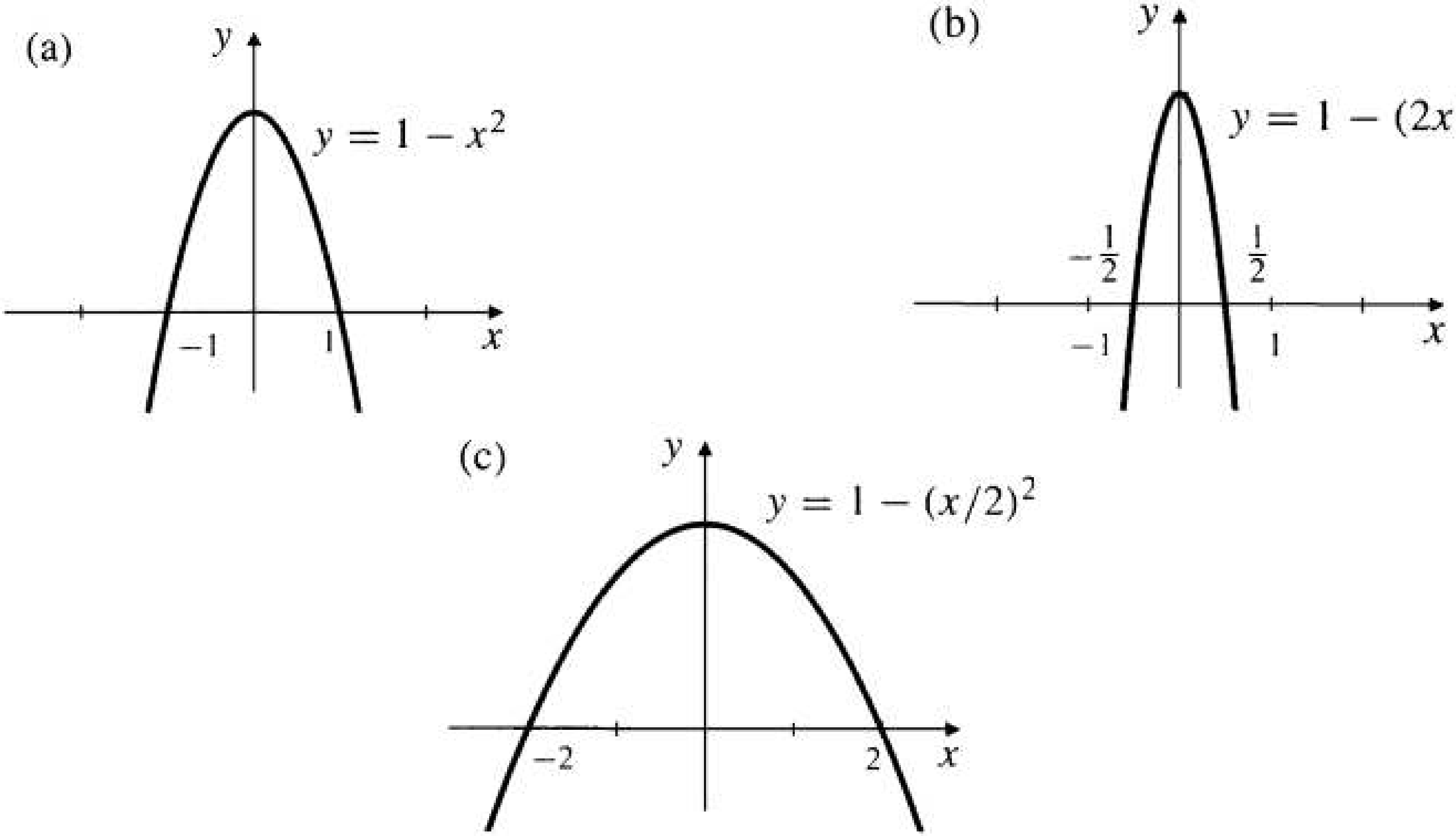
$|FP| = |PQ|$

$\downarrow$

$y = \dfrac{x^2}{4p}$

Graphs of Quadratic Equations

Scaling a Graph



Graphs of Quadratic Equations

Shifting a Graph

SHIFTING A GRAPH HORIZONTALLY

$c > 0, \quad x \longleftrightarrow x - c$	shifting c units to the right
$c < 0, \quad x \longleftrightarrow x - c$	shifting c units to the left

SHIFTING A GRAPH VERTICALLY

$c > 0, \quad y \longleftrightarrow y - c$	shifting c units to downward
$c < 0, \quad y \longleftrightarrow y - c$	shifting c units to upward

Graphs of Quadratic Equations

EXAMPLE Describe the graph of $y = x^2 - 4x + 3$.

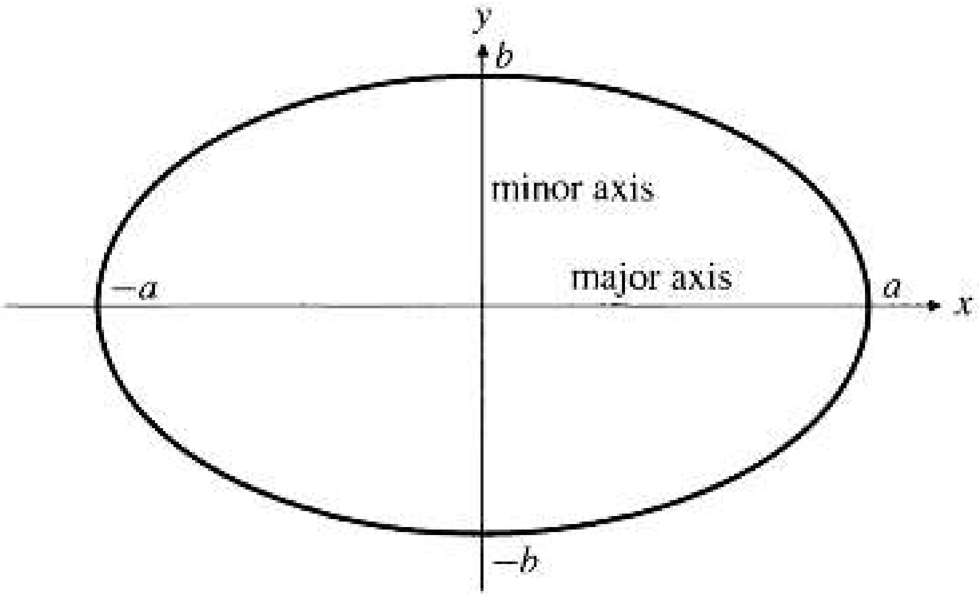
Graphs of Quadratic Equations

Ellipses and Hyperbolas

If a and b are positive numbers, the equation

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

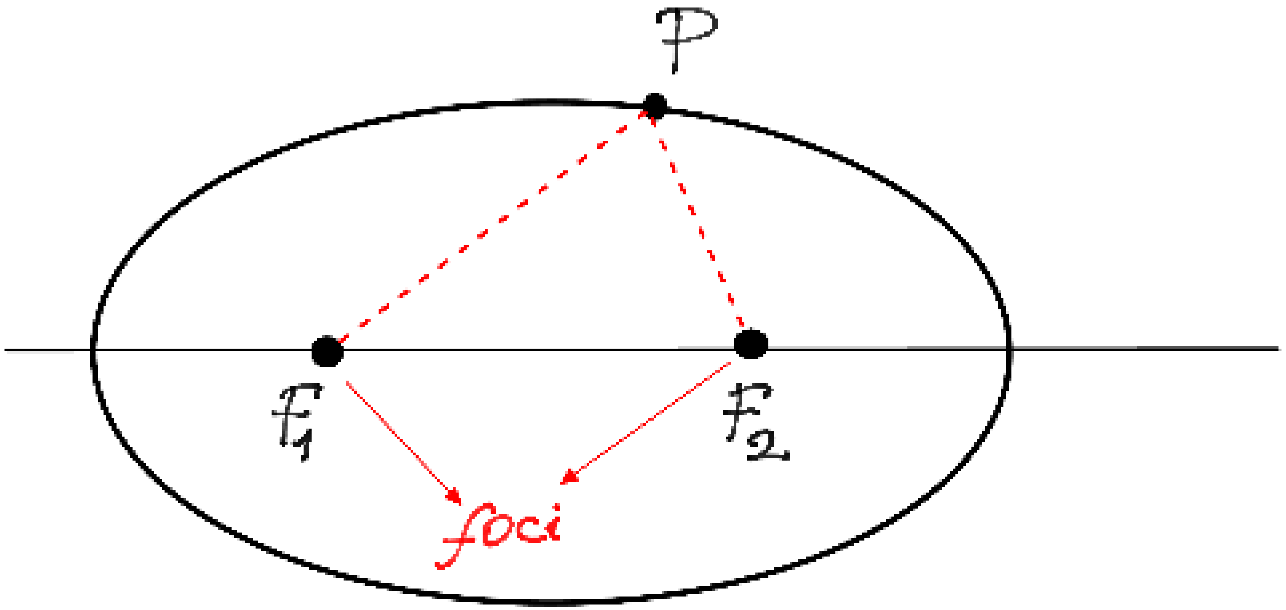
represents a curve called an **ellipse** that lies wholly within the rectangle $-a \le x \le a$, $-b \le y \le b$.



The ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

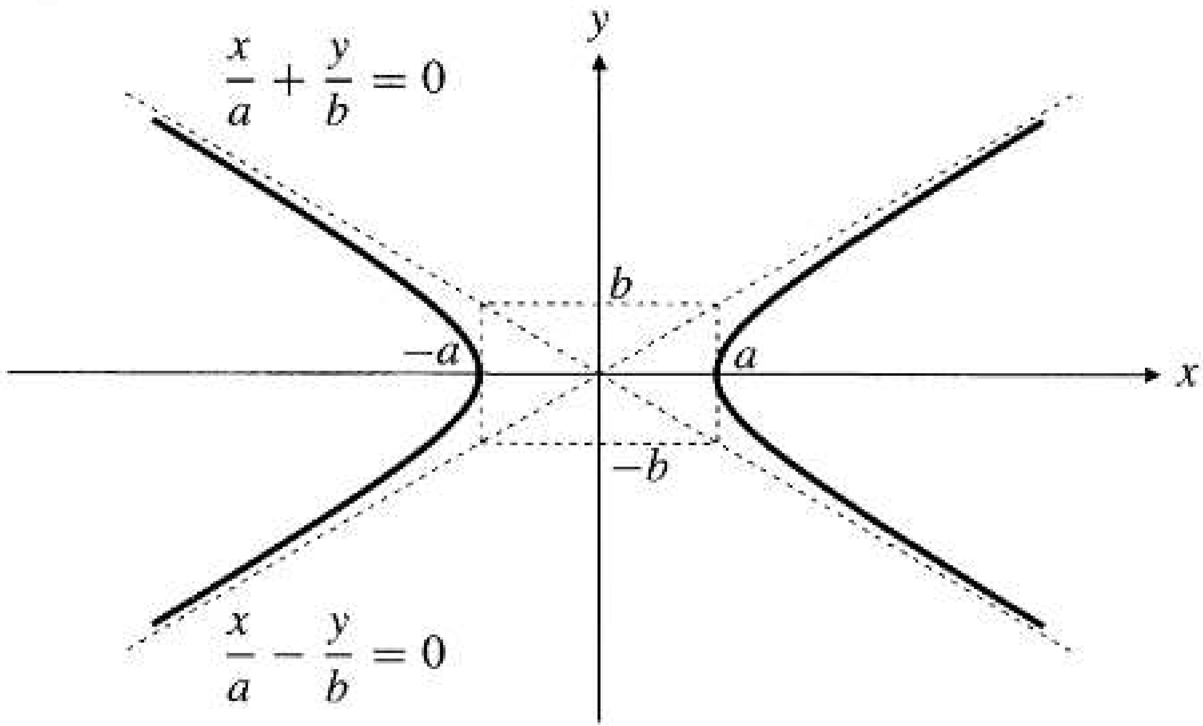
Graphs of Quadratic Equations

General definition of an ellipse



$$|PF_1| + |PF_2| = \text{constant}$$

Graphs of Quadratic Equations



The hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and its asymptotes