535.641 Mathematical Methods Assignment 2

Ben Minnick   **Name**_____

1	/20
2	/20
3	/20
4	/20
TOTAL	/80

1. In the following problems check to see if the set S is a subspace of the corresponding vector space. If it is not, explain why not. If it is, then find a basis and the dimension.

(a)
$$S = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \ 2x_1 + x_2 = 1 \right\} \subset \mathbb{R}^3$$

(b)
$$S = \left\{ \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \ x_1 \leq x_2 \right\} \subset \mathbb{R}^2$$

(c)
$$S = \left\{ \begin{bmatrix} a & 0 \\ 0 & -a \end{bmatrix}, \ \forall a \in \mathbb{R} \right\} \subset \mathbb{R}^{2 \times 2}$$

(d)
$$S = \left\{ f(x), \ \frac{df}{dx} = A\sin(2x), \forall A \in \mathbb{R} \right\} \subset \mathbb{R}$$

2. Use Cramer's rule to compute the solutions of the system

$$2x_1 + x_2 = 7$$
$$-3x_1 + x_3 = -8$$
$$x_2 + 2x_3 = -3$$

3. *Read section 1.15 of the instructor's notes.* You are tasked with: 1. developing a model for an elastic beam supported on the edges, and 2. applying this model for a specific application, i.e. find the loading given a displacement. You decide to discretize the beam so that there are three nodes (P_1, P_2, P_3). Assume Hooke's law applies.

   (a) You have tested the beam under unit forces applied to each node and respectfully found the following displacements:

$$\mathbf{y}_1 = \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix}, \quad \mathbf{y}_2 = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}, \quad \mathbf{y}_3 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

where $\mathbf{y}_1$ is the displacement that results from loading $\mathbf{F}_1 = [1, \ 0, \ 0]^{\mathrm{T}}$, $\mathbf{y}_2$ is the displacement from loading $\mathbf{F}_2 = [0, \ 1, \ 0]^{\mathrm{T}}$, and $\mathbf{y}_3$ is the displacement from loading $\mathbf{F}_3 = [0, \ 0, \ 1]^{\mathrm{T}}$.

Given this information, build the model and construct the flexibility matrix.

   (b) Using the model you have just derived, determine the loading that produces the displacement

$$\mathbf{y} = \begin{bmatrix} 4 \\ 5 \\ 3 \end{bmatrix}$$

4. Consider the (x, y) vertices of an equilateral triangle in $\mathbb{R}^2$ given by,

$$(0,0) \ , \ (1,0) \ , \ (.5, \sqrt{3}/2)$$

Perform the following actions in order using matrix transformations:

(a) Reflect the triangle over the x-axis,

(b) Compress the triangle along the y-axis by a factor of 2,

(c) Rotate the triangle counterclockwise by $\pi/6$ radians about the origin, and

(d) Shear the triangle along the x-axis by a factor of 0.5.

Clearly show the matrix transformations used and compute the final set of vertices of the triangle.