535.641 Mathematical Methods Assignment 2

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TOTAL	/80

1. In the following problems check to see if the set S is a subspace of the corresponding vector space. If it is not, explain why not. If it is, then find a basis and the dimension.
(a)

$$S = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \ , \ 2x_1 + x_2 = 1 \right\} \subset \mathbb{R}^3$$

Testing Relevant Axioms:
A4:

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \text{ (the zero vector) must be in the set}$$

$$2(0) + 0 = 0 \therefore 0 = 1 \text{ which is a contradiction, and thus not in } S$$

(b)

$$S = \left\{ \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \ , \ x_1 \leq x_2 \right\} \subset \mathbb{R}^2$$

(c)

$$S = \left\{ \begin{bmatrix} a & 0 \\ 0 & -a \end{bmatrix} \ , \ \forall a \in \mathbb{R} \right\} \subset \mathbb{R}^{2\times 2}$$

(d)

$$S = \left\{ f(x) \ , \ \frac{df}{dx} = A\sin(2x), \forall A \in \mathbb{R} \right\} \subset \mathbb{R}$$

2. Use Cramer's rule to compute the solutions of the system

$$\begin{aligned} 2x_1 + x_2 &= 7 \\ -3x_1 + x_3 &= -8 \\ x_2 + 2x_3 &= -3 \end{aligned}$$

3. Read section 1.15 of the instructor's notes. You are tasked with: 1. developing a model for an elastic beam supported on the edges, and 2. applying this model for a specific application, i.e. find the loading given a displacement. You decide to discretize the beam so that there are three nodes (P_1, P_2, P_3). Assume Hooke's law applies.

(a) You have tested the beam under unit forces applied to each node and respectfully found the following displacements:

$$\mathbf{y}_1 = \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix} , \ \mathbf{y}_2 = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} , \ \mathbf{y}_3 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

where $\mathbf{y}_1$ is the displacement that results from loading $\mathbf{F}_1 = [1, 0, 0]^T$, $\mathbf{y}_2$ is the displacement from loading $\mathbf{F}_2 = [0, 1, 0]^T$, and $\mathbf{y}_3$ is the displacement from loading $\mathbf{F}_3 = [0, 0, 1]^T$. Given this information, build the model and construct the flexibility matrix.

(b) Using the model you have just derived, determine the loading that produces the displacement

$$\mathbf{y} = \begin{bmatrix} 4 \\ 5 \\ 3 \end{bmatrix}$$

4. Consider the (x, y) vertices of an equilateral triangle in $\mathbb{R}^2$ given by,

$$(0,0),(1,0),(.5,\sqrt{3}/2)$$

Perform the following actions in order using matrix transformations:
(a) Reflect the triangle over the x-axis,

(b) Compress the triangle along the y-axis by a factor of 2,

(c) Rotate the triangle counterclockwise by $\pi/6$ radians about the origin, and

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(d) Shear the triangle along the x-axis by a factor of 0.5.
Clearly show the matrix transformations used and compute the final set of vertices of the triangle.