EN.625.252.81 Fall 2025
Module 2 Assignment

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Due Sunday September 7th at 10:59 CST

$$\text{In the problems below let } A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

1. Calculate $rref(A)$ by hand showing all the steps of your row reduction.

Ans:

$$\begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$
$$\downarrow$$
$$R_2 \leftarrow R_2 - R_1$$
$$\begin{bmatrix} 1 & 1 & 0 \\ 1-1 & 2-1 & 1-0 \\ 0 & 1 & 1 \end{bmatrix}$$
$$\downarrow$$
$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$
$$\downarrow$$
$$R_1 \leftarrow R_1 - R_2$$
$$\begin{bmatrix} 1-0 & 1-1 & 0-1 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$
$$\downarrow$$
$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$
$$\downarrow$$

$$R_3 \leftarrow R_3 - R_2$$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \\ 0-0 & 1-1 & 1-1 \end{bmatrix}$$

$$\downarrow$$

$$rref(A) = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

2. Using your answer to 1, determine the general solution of $Ax = 0$.

Ans:

$$Ax = 0$$
$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix} * \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$
$$x_1 - x_3 = 0$$
$$x_2 + x_3 = 0$$
$$0 = 0$$
$$\therefore$$
$$x_1 = x_3$$
$$x_2 = -x_3$$
$$x_3 \text{ is a free variable}$$
$$x_3 = t$$
$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_3 \\ -x_3 \\ x_3 \end{bmatrix} \; or \; \begin{bmatrix} t \\ -t \\ t \end{bmatrix} \; or \; t \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$
$$\text{where } t \in \mathbb{R}$$

3. Solve the matrix equation $Ax = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$, or show that no solution exists.

Ans:

$$\begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 2 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix} R_2 \leftarrow R_2 - R_1$$

$$\begin{bmatrix} 1 & 1 & 0 & 1 \\ 1-1 & 2-1 & 1-0 & 0-1 \\ 0 & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & -1 \\ 0 & 1 & 1 & 1 \end{bmatrix}$$

$$R_1 \leftarrow R_1 - R_2$$
$$R_3 \leftarrow R_3 - R_2$$

$$\begin{bmatrix} 1-0 & 1-1 & 0-1 & 1-(-1) \\ 0 & 1 & 1 & -1 \\ 0-0 & 1-1 & 1-1 & 1-(-1) \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 & 2 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

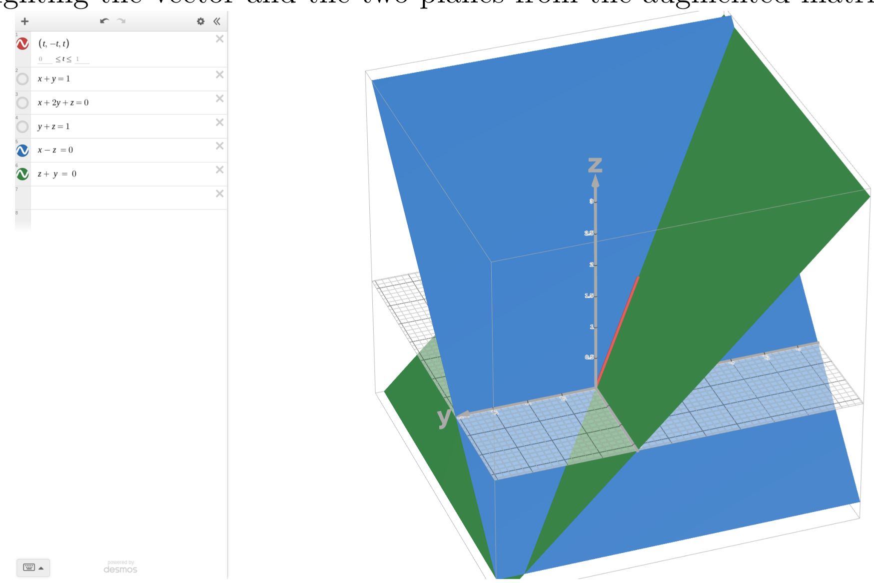
$$x_1 - x_3 = 2$$
$$x_2 + x_3 = -1$$
$$0 = 2$$

This is a contradiction as $0 = 2$ is invalid and this is inconsistent, No Solution

4. Use 3D graphing software (e.g. Geogebra or Desmos) to give a geometric description of your answers to problems 2 and 3. (Note: even if a systems is inconsistent, you can still graph to show no common intersection occurring.)

Ans:

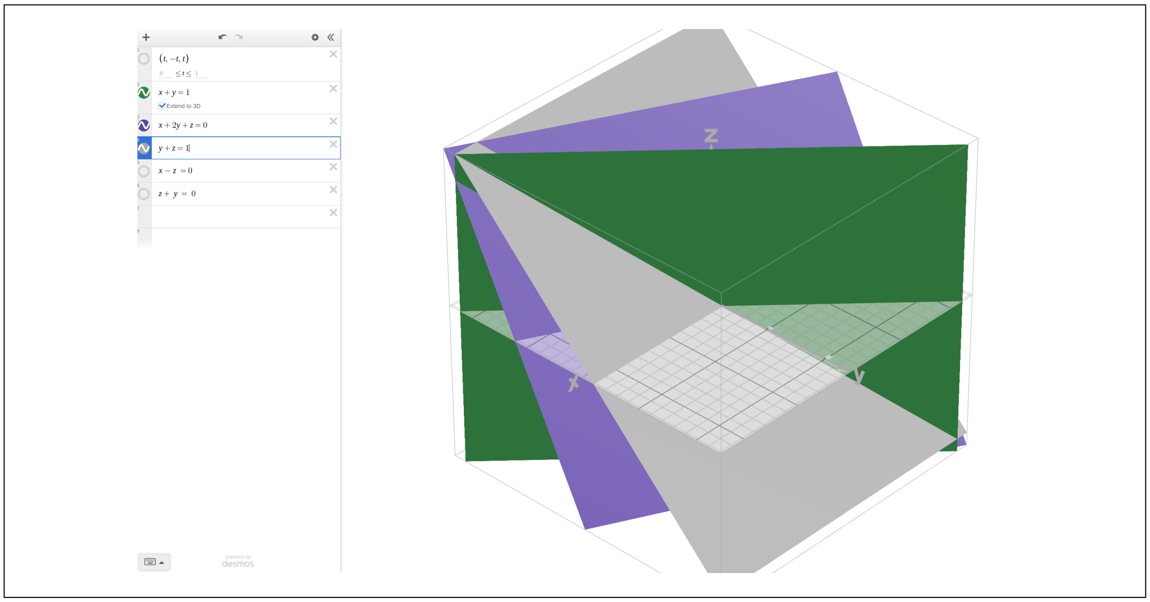
This first graph is representative of the problem 2, it can be seen as vector going from the origin (limited) to $(1, -1, 1)$ as it scales with any given t value. The redline highlighting the vector and the two planes from the augmented matrix



of $Ax = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

This second graph is showing the three equations independently graphed derived from the initial augmented matrix from question 3: $\begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 2 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}$

Following the planes outward, there is no point where all three will converge


(t, −t, t)
0 ≤ t ≤ 1
x + y = 1
Extend to 3D
x + 2y + z = 0
y + z = 1
x − z = 0
z + y = 0
z
x
y
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