Linear Algebra

Assignment 3

Due Sunday at 11:59 PM

1. Directions Complete the following problems showing all your work. You may use a calculator to check your work, but should write out (or TeX up) all the steps of your solution. Unless otherwise specified, you may skip some steps of row reduction with a calculator, but state that you did so. Please upload your work as a single .pdf file in Blackboard.

2. Problems

   (a) Which of the following are linear transformations? For each problem below, first determine if the transformation is linear or not. Then, for each linear transformation, write the standard matrix of the transformation. When showing a transformation is not linear, provide a specific reason or counterexample.

      1. Reflection about the x-axis in $\mathbb{R}^2$.
      2. Reflection about the line $x = 1$ in $\mathbb{R}^2$.
      3. Translation $T_{\vec{v}} : \mathbb{R}^2 \to \mathbb{R}^2$ by a nonzero vector $\vec{v}$ in $\mathbb{R}^2$. (i.e., $T(\vec{x}) = \vec{x} + \vec{v}$)
      4. $T(x) = x^2$ in $\mathbb{R}$.
      5. For each vector $\vec{v} = \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} \in \mathbb{R}^4$, associate to it the cubic polynomial $f(x) = ax^3 + bx^2 + cx + d$. Then consider the transformation $T(f) = f'$, where $f'(x)$ is the derivative.

   (b) In the (sadly fictitious) suspense movie, *The Linear Combination*, our three main characters Adam, Bob and Carl have each been mailed a mysterious letter directing them to a combination safe and promising that its contents are of extreme value. They were told that there is a linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ and that the combination to the safe is a three-digit number corresponding to the vector $T \begin{pmatrix} 4 \\ 4 \\ 15 \end{pmatrix}$. In their letters, each was given partial information about T. Adam knows $T \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$, Bob knows $T \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$, and Carl knows $T \begin{pmatrix} 0 \\ 0 \\ 7 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$.

1. Explain why, if Adam, Bob and Carl work together, they can determine the combination to the safe. Also, determine the combination to the safe.

2. Explain why, if Bob and Carl work together but they *don't* work with Adam, they won't have enough information to determine the combination.

3. Explain why, if the combination to the safe was the vector $T \begin{pmatrix} 2 \\ 4 \\ -1 \end{pmatrix}$, Bob and Carl could have determined the combination without Adam's help.