EN.625.252.81 Fall 2025
Module 1 Assignment

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Due Sunday August 31st at 10:59 CST

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1. Find the general solution of the system below two ways:
   1) Row reduction by hand.
   2) Verify your answer using any program or calculator of your choice. Clearly state which program you used and include a screen shot of your code and output

$$\begin{bmatrix} 1 & 3 & 4 & 7 \\ 3 & 9 & 7 & 6 \end{bmatrix}$$

Ans: Multiplying the bottom row by '-3(Row1)'

$$\begin{bmatrix} 1 & 3 & 4 & 7 \\ 3-3(1) & 9-3(3) & 7-3(4) & 6-3(7) \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & 4 & 7 \\ 0 & 0 & -5 & -15 \end{bmatrix}$$

Which can simply to

$$\begin{bmatrix} 1 & 3 & 4 & 7 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

This can be turned into Reduced Row Echelon Form by multiplying the top row by '-4(Row2)'

$$\begin{bmatrix} 1-4(0) & 3-4(0) & 4-4(1) & 7-4(3) \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

Which simplifies to

$$\begin{bmatrix} 1 & 3 & 0 & -5 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

$$x_1 + 3x_2 = -5$$
$$x_3 = 3$$

x_2 is a free variable which results in the following:

$$x_1 = -5 - 3t$$
$$x_2 = t$$
$$x_3 = 3$$

To verify, I plugged the original values into Matlab

```
>> a = [1  3  4  7; 3  9  7  6];
b = rref(a);
disp(b);
     1     3     0    -5
     0     0     1     3

>> 
```

which aligns with my solution

2. What would you have to know about the pivot columns in an augmented matrix in order to know that the linear system is consistent and has a unique solution? Explain in complete sentences and provide examples to justify your reasoning.

Ans: For a linear system to be consistent, the last column pf the augmented matrix must not be a pivot. In addition to that, if it is unique, that means every column (variable) has a single pivot. The following is an example of such.

$$\begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 7 \end{bmatrix}$$

An example that would not be consistent nor have pivots for every column would be as follows.

$$\begin{bmatrix} 1 & 0 & 3 & 5 \\ 0 & 1 & 7 & 2 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$

3. Consider the linear system

$$x_1 + hx_2 = 2$$
$$4x_1 + 8x_2 = k$$

(a) Find values of h and k such that the system has no solution

(b) Find values of h and k such that the system has a unique solution.

(c) Find values of h and k such that the system has infinitely many solutions.

Ans:

$$\begin{bmatrix} 1 & h & 2 \\ 4 & 8 & k \end{bmatrix}$$

$R_2 \leftarrow R_2 - 4(R_1)$

$$\begin{bmatrix} 1 & h & 2 \\ 4-4(1) & 8-4(h) & k-4(2) \end{bmatrix}$$

$$\begin{bmatrix} 1 & h & 2 \\ 0 & 8-4h & k-8 \end{bmatrix}$$

(a) $(8-4h)x_2 = k-8$
So $8-4h = 0$ and $k-8 \neq 0$
For no solution, $h = 2$ and $k = 8$

(b) As long as the coefficient of x_2 is nonzero $k-8$ can be any value, therefore:
$h \neq 2$ and k can be any real number

(c) To have infinite solutions, it must be consistent and one free variable.
Using $h = 2 \rightarrow \begin{bmatrix} 1 & 2 & 2 \\ 0 & 0 & k-8 \end{bmatrix}$ to be consistent, $k-8$ must also equal 0, thus
$k = 8 \rightarrow \begin{bmatrix} 1 & 2 & 2 \\ 0 & 0 & 0 \end{bmatrix}$
Resulting in $x_1 + 2x_2 = 2 \rightarrow x_1 = 2 - 2x_2$ with infinite solutions for $h = 2$ and $k = 8$