EN.625.252.81 Fall 2025
Module 1 Assignment

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Due Sunday August 31st at 10:59 CST

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1. Find the general solution of the system below two ways:
   1) Row reduction by hand.
   2) Verify your answer using any program or calculator of your choice. Clearly state which program you used and include a screen shot of your code and output

$$\begin{bmatrix} 1 & 3 & 4 & 7 \\ 3 & 9 & 7 & 6 \end{bmatrix}$$

Ans: Multiplying the bottom row by '-3(Row1)'

$$\begin{bmatrix} 1 & 3 & 4 & 7 \\ 3-3(1) & 9-3(3) & 7-3(4) & 6-3(7) \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & 4 & 7 \\ 0 & 0 & -5 & -15 \end{bmatrix}$$

Which can simply to

$$\begin{bmatrix} 1 & 3 & 4 & 7 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

This can be turned into Reduced Row Echelon Form by multiplying the top row by '-4(Row2)'

$$\begin{bmatrix} 1-4(0) & 3-4(0) & 4-4(1) & 7-4(3) \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

Which simplifies to

$$\begin{bmatrix} 1 & 3 & 0 & -5 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

$$x_1 + 3x_2 = -5$$
$$x_3 = 3$$

x_2 is a free variable which results in the following:

$$x_1 = -5 - 3t$$
$$x_2 = t$$
$$x_3 = 3$$

Using the coding language C, I created this program to verify the findings

```c
#include <stdio.h>
#include <stdlib.h>

#define rows 2
#define cols 4

void printMatrix(double matrix[rows][cols])
{
    for (int i = 0; i < rows; i++)
    {
        printf("[ ");
        for (int j = 0; j < cols; j++)
        {
            printf("%7.2f", matrix[i][j]);
        }
        printf(" ]\n");
    }
}

int main()
{
    double a[rows][cols] = {
        {1.0, 3.0, 4.0, 7.0},
        {3.0, 9.0, 7.0, 6.0}
    };
    printf("Initial state\n");
    printMatrix(a);

    for (int j = 0; j < cols; j++)
    {
        a[1][j] = a[1][j] - 3.0 * a[0][j];
    }
    printf("\nAfter R2 = R2 - 3*R1:\n");
    printMatrix(a);

    double stepConst = -1.0/5.0;
```

```c
    for (int j = 0; j < cols; j++)
    {
        a[1][j] = stepConst * a[1][j];
    }
    printf("\nAfter R2 = (-1/5)*R2:\n");
    printMatrix(a);

    for (int j = 0; j < cols; j++)
    {
        a[0][j] = a[0][j] - 4.0 * a[1][j];
    }
    printf("\nAfter R1 = R1 - 4*R2 (Reduced Row Echelon Form):\n");
    printMatrix(a);

}
```

And this is the output when compiled and executed

```
hmajeed@haadiOS  ~/repos/jhu/linearAlg/module1  master  make
gcc -Wall -Wextra -std=c17 -O2 verify.c -o verify
hmajeed@haadiOS  ~/repos/jhu/linearAlg/module1  master  ./verify
Initial state
[    1.00    3.00    4.00    7.00 ]
[    3.00    9.00    7.00    6.00 ]

After R2 = R2 - 3*R1:
[    1.00    3.00    4.00    7.00 ]
[    0.00    0.00   -5.00  -15.00 ]

After R2 = (-1/5)*R2:
[    1.00    3.00    4.00    7.00 ]
[   -0.00   -0.00    1.00    3.00 ]

After R1 = R1 - 4*R2 (Reduced Row Echelon Form):
[    1.00    3.00    0.00   -5.00 ]
[   -0.00   -0.00    1.00    3.00 ]
```

2. What would you have to know about the pivot columns in an augmented matrix in order to know that the linear system is consistent and has a unique solution? Explain in complete sentences and provide examples to justify your reasoning.

Ans: For a linear system to be consistent, the last column pf the augmented matrix must not be a pivot. In addition to that, if it is unique, that means every column (variable) has a single pivot. The following is an example of such.

$$\begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 7 \end{bmatrix}$$

An example that would not be consistent nor have pivots for every column would be as follows.

$$\begin{bmatrix} 1 & 0 & 3 & 5 \\ 0 & 1 & 7 & 2 \\ 0 & 0 & 0 & 3 \end{bmatrix}$$

3. Consider the linear system

$$\begin{aligned} x_1 + hx_2 &= 2 \\ 4x_1 + 8x_2 &= k \end{aligned}$$

(a) Find values of h and k such that the system has no solution

(b) Find values of h and k such that the system has a unique solution.

(c) Find values of h and k such that the system has infinitely many solutions.

Ans:

$$\begin{bmatrix} 1 & h & 2 \\ 4 & 8 & k \end{bmatrix}$$

$R_2 \leftarrow R_2 - 4(R_1)$

$$\begin{bmatrix} 1 & h & 2 \\ 4-4(1) & 8-4(h) & k-4(2) \end{bmatrix}$$

$$\begin{bmatrix} 1 & h & 2 \\ 0 & 8-4h & k-8 \end{bmatrix}$$

(a) $(8-4h)x_2 = k-8$
So $8-4h=0$ and $k-8 \neq 0$
For no solution, $h=2$ and $k=8$

(b) As long as the coefficient of x_2 is nonzero $k-8$ can be any value, therefore: $h \neq 2$ and k can be any real number

(c) To have infinite solutions, it must be consistent and one free variable.
Using $h = 2 \rightarrow \begin{bmatrix} 1 & 2 & 2 \\ 0 & 0 & k-8 \end{bmatrix}$ to be consistent, $k - 8$ must also equal 0, thus $k = 8 \rightarrow \begin{bmatrix} 1 & 2 & 2 \\ 0 & 0 & 0 \end{bmatrix}$
Resulting in $x_1 + 2x_2 = 2 \rightarrow x_1 = 2 - 2x_2$ with infinite solutions for $h = 2$ and $k = 8$