Vectors and Scalars

Salars

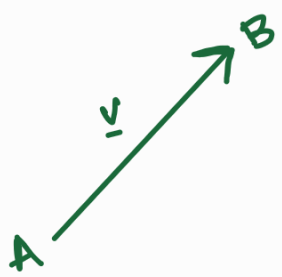
Need only a number and a unit of measurement
Ex: mass, length, speed and time.

Vectors

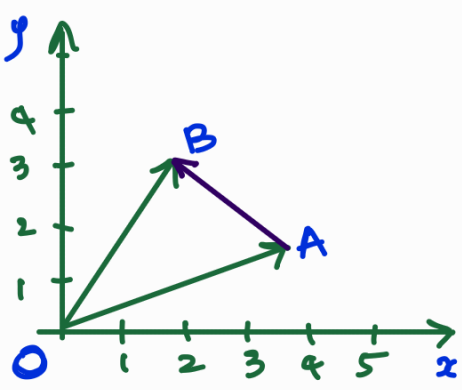
Quantity which need both magnitude and direction.
Ex: Force, displacement, velocity and acceleration.

Vector Notation

- Upper case letter with an arrow head: $\overrightarrow{AB}$
- A single lowercase letter with a tilde below: $\underset{\sim}{v}$
- Bold upper case letters : **AB**
- Bold lower case letter : **v**

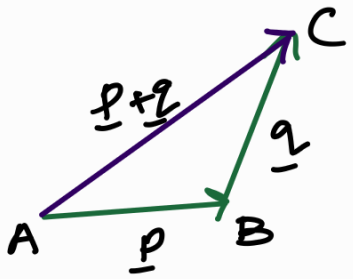


2-dimentional plane / Cartesian plane



- Position of A Relative to Origin (O) $\overrightarrow{OA}$
- Position of B Relative to origin $\overrightarrow{OB}$
- Position of B Relative to Position of A → $\overrightarrow{AB}$

On the 2-dimentional plane above, we see that $\vec{OA} + \vec{AB} = \vec{OB}$

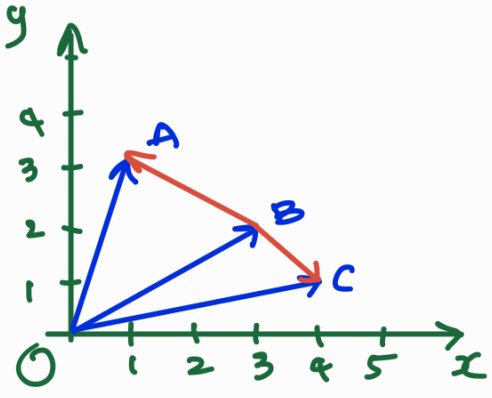


$\therefore \quad \vec{AB} = \vec{OB} + \vec{OA}$

$\underline{p} + \underline{q} = \underline{p} + \underline{2}$

$\vec{BC} = \vec{AC} - \vec{AB}$

Unit Vectors



- $\underline{i}$ is a unit vector in x-axis
- $\underline{j}$ is a unit vector in y-axis

- Position Vector of A is $\vec{OA} = \underline{a} = \underline{i} + 3\underline{j}$
- Position Vector of B is $\vec{OB} = \underline{b} = 3\underline{i} + 2\underline{j}$
- Position Vector of C is $\vec{OC} = \underline{c} = 4\underline{i} + \underline{j}$

Exercise: Find $\vec{BA}$, $\vec{BC}$, $\vec{AC}$

(i) $\vec{BA} = \vec{OA} - \vec{OB}$

$\vec{BA} = \underline{i} + 3\underline{j} - (3\underline{i} + 2\underline{j})$

$\vec{BA} = -2\underline{i} + \underline{j}$

(ii) $\vec{BC} = \vec{OC} - \vec{OB}$

$= 4\underline{i} + \underline{j} - (3\underline{i} + 2\underline{j})$

$= \underline{i} - \underline{j}$

(iii) $\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA}$

$= 4\underline{i} + \underline{j} - (\underline{i} + 3\underline{j})$

$= 3\underline{i} - 2\underline{j}$

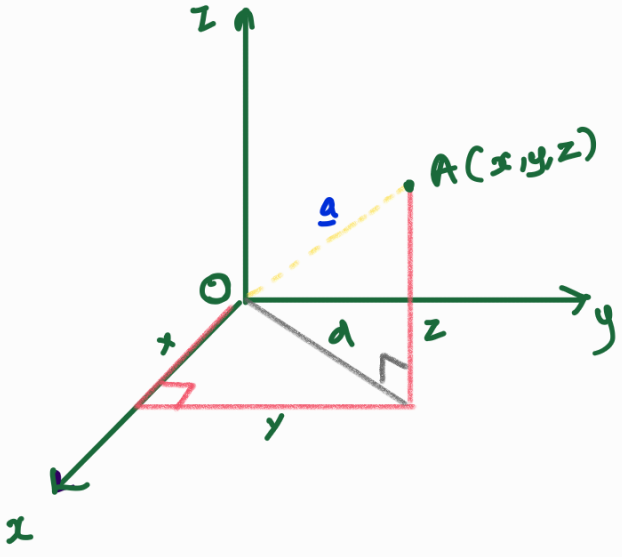
Magnitude of 2-dimensional vector

Magnitude of $\underline{a} = |\underline{a}| = \sqrt{x^2 + y^2}$

where $x \Rightarrow$ Coefficient of i

$y =$ Coeficiant of j

Magnitude of 3-dimensional vector



- $d^2 = x^2 + y^2$
- $|\underline{a}|^2 = d^2 + z^2$
  $= x^2 + y^2 + z^2$
- $|\underline{a}| = \sqrt{x^2 + y^2 + z^2}$

Exercise : 1

For points $D(2, 3, -1)$ and $E(-1, 2, -4)$; find $(|\underline{d}|, |e|, |\overrightarrow{DE}|$

$\overrightarrow{OD} = 2\underline{i} + 3\underline{j} - \underline{k}$, $\overrightarrow{OE} = -\underline{i} + 2\underline{j} - 4k$

$|\overrightarrow{OD}| = \sqrt{4+9+1} = \sqrt{14}$, $|\overrightarrow{OE}| = \sqrt{1+4+16} = \sqrt{21}$

$$\vec{DE} = \vec{OE} - \vec{OD} = -\underline{i} + 2\underline{j} - 4\underline{k} - 2\underline{i} - 3\underline{j} + \underline{k}$$

$$\vec{DE} = -3\underline{i} - \underline{j} - 3\underline{k}$$

$$|\vec{DE}| = \sqrt{9 + 1 + 9}$$

$$|\vec{DE}| = \sqrt{19} = 4.3588$$

Unit Vector $\hat{a}$

$$\underline{a} = \underline{i} + 3\underline{j}$$

$$|\underline{a}| = \sqrt{1^2 + 3^2} = \sqrt{10}$$

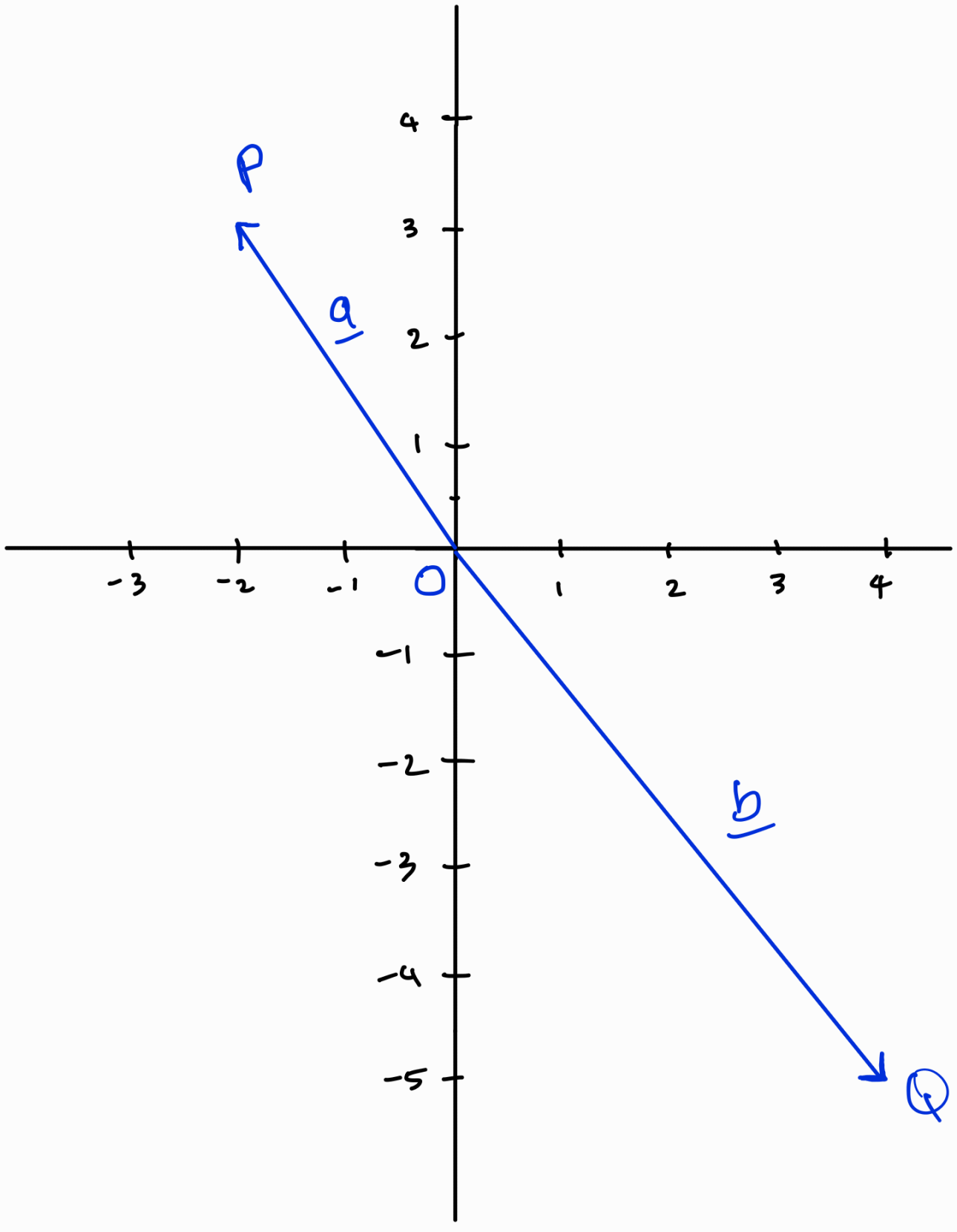
$$\therefore \hat{a} = \frac{\underline{a}}{|\underline{a}|} = \frac{(\underline{i} + 3\underline{j})}{\sqrt{10}}$$

Exercise : 2

For vector $\underline{a}$ and $\underline{b}$ where $\underline{a} = -2\underline{i} + 3\underline{j}$ and $\underline{b} = 4\underline{i} - 5\underline{j}$

plot the points on graph paper and draw and label position vector $\vec{OP} = \underline{a}$, $\vec{OQ} = \underline{b}$

(1) $|\underline{a}|$ (2) $|\underline{b}|$ (3) $\hat{a}$ (4) $\hat{b}$



(1) $|\underline{a}| = \sqrt{4+9} = \sqrt{15}$

(2) $|\underline{b}| = \sqrt{16+25} = \sqrt{41}$

(3) $\hat{\underline{a}} = \dfrac{-2\underline{i}+3\underline{j}}{\sqrt{15}}$ (4) $\dfrac{4\underline{i}-5\underline{j}}{\sqrt{41}}$