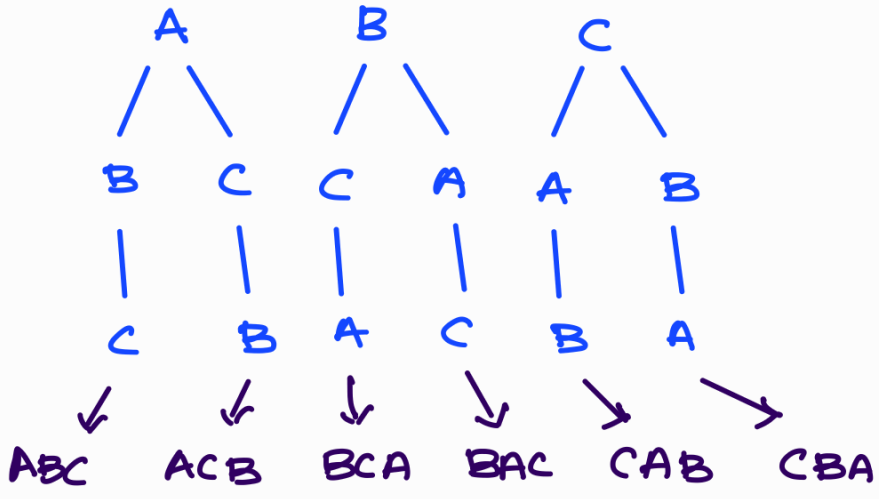
Permutation and Combinatorics

Permutation

A permutation is an arrangment of objects in a definite order.

There are 6 permutations of the letter A, B, C



Exercise: write the permutations for 4 letters a, b, c, d

a b c d	b a c d	c a b d	d a b c
a b d c	b a d c	c a d b	d a c b
a c b d	b c a d	c b a d	d b a c
a c d b	b c d a	c b d a	d b c a
a d c b	b d a c	c d a b	d c a b
a d b c	b d c a	c d b a	d c b a

Permutation of n elements

* There are $n!$ permutations $\Rightarrow n \times (n-1) \times (n-2) \ldots 2 \times 1$

k permutations of n elements

* when we have n elements and we can only use k elements only to build permutations.

$$P(n,k) = \frac{n!}{(n-p)!}$$

Example : $n = 4$, a, b, c, d and $p = 3$

$$P(4,3) = \frac{4!}{(4-3)!} = 24 \text{ //}$$

bcd	acd	abd	abc
bdc	adc	adb	acb
cbd	cad	bad	bac
cdb	cda	bda	bca
dcb	dac	dab	cab
dbc	dca	dba	cba

Example :

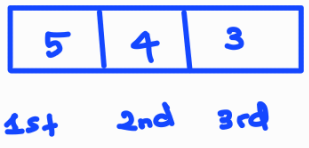
(a) In how many ways 6 people can be arranged.

$$6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1$$
$$= 720$$

Acording to the multiplication rule if one event can ocure in m ways and a second event ocure in n ways after the first event has ocured then the two events can ocure in $m \times n$ ways.

Example: Assume that 5 cars are in a race in how many ways 3 cars finish in first, second and third place

C_1, C_2, C_3, C_4, C_5



$\Rightarrow 5 \times 4 \times 3 = 60$ ways

Repetition of an event

when some event ocures repeatedly and the numberf chances for outcomes does not change.

Example: tossing a coin

(a) 2 times $\Rightarrow 6^2$

(b) 3 times $\Rightarrow 6^3$

(c) r times $\Rightarrow 6^r$

Example : 2

(a) How many different car number plates are possible with 3 letters followed by 3 digits?

$26^3 \times 10^3$ ← This answer is enough we don't have to simplify.

$= 17576 \times 1000$

$= 17576000$

(b) How many of these number plates begins with ABC?

$10^3 = 1000$

(c) If a plate is chosen at random, what is the Probality that it begins with ABC?

$$\frac{1000}{17576000} = 0.0000569$$

Permutations with restrictions

Example: In how many ways 5 boys and 4 Girls be arranged on a bench if

(a) there are no restrictions?

$9!$

(b) boys and girls alternate?

$5 \times 4 \times 4 \times 3 \times 3 \times 2 \times 2 \times 1 \times 1 = 5! \times 4!$

(c) boys and girls are in seperate groups?

$(5! \times 4!)\ 2!$

(d) Anne and Jim wish to stay together?

$8! \times 2!$

From 2, 3, 4, 5, 6

(a) how many numbers greater than 4000 can be found.

$3 \times 4! + 5! = 192$

(b) how many 4 digit numbers would be even

$3 \times 4 \times 3 \times 2 = 3 \times 4!$

Arrangements with Repetition

If we have n elements of which X are alike of one kind, Y are alike of another kind, Z are alike another kind, then the number of ordered selections or permutations is given by:

$$\frac{n!}{x!\,y!\,z!}$$

Example: K, C, C How many Permutations?

total 3 letters → $\frac{3!}{2!} = 3$ ← 2 times C

KCC
CKC
CCK

Example: How many different arrangements of the word PARRAMATTA are posible?

Solution 10 letters
- But 2 - R
4 - A
2 - A

$$\therefore \frac{10!}{2! \times 4! \times 2!} = 37800$$

Question : How many different arrangements of words

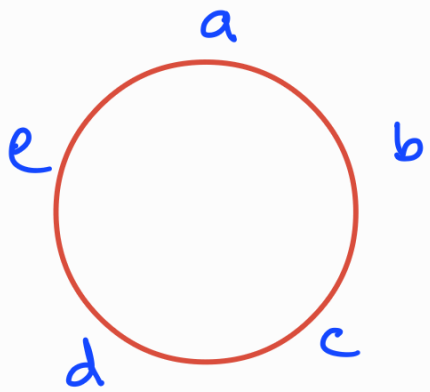
- MATHEMATICS $\frac{11!}{2! \times 2! \times 2!}$

- MATARA $\frac{6!}{3!}$

Circular Arrangements

Consider Arranging 5 objects (a, b, c, d, e) around a circular table

Arrangements are:



abcde
bcdea
cdeab
deabc
eabcd

- these are different in a line, But identical around a circle

So the number of arrangements in a circle $\Rightarrow (n-1)!$

At a dinner party 6 men and 6 women sit at a round table. In how many ways can they sit if:

(a) there are no restrictions.

$$(12-1)! = 11!$$

(b) men and women altered.

$$6! \times 5!$$

Combinations

Example: How many ways can a basket Ball team of 5 players be chosen from 8 players?

Solution: $${}^{8}C_{5} = \frac{8!}{5! \times 3!}$$

$$= 8 \times 7 = 56$$

Example: 2

A committee of 5 people is to be chosen from a group of 6 men and 4 women. How many Commitees are posible:

(a) there are no restrictions:

$$^{10}C_5 = \frac{10!}{5!5!}$$

(b) One perticular person must be chosen on the committee?

$$^{9}C_4 = \frac{9!}{4! \times 5!}$$

(c) One women should be excluded.

$$^{9}C_5 = \frac{9!}{5!4!}$$

(d) One perticular man and one perticular women must be chosen on the committee But ~~total~~ number of men should be 3 and total number of women should be 2 in the committee.

$$^{5}C_2 \times {}^{3}C_1$$

$$= \frac{5!}{2! \times 3!} \times \frac{3!}{2! \times 1!} = 5 \times 2 \times 3 = 30 //$$

Practice questions

Part - 1

1 - GIFT

$4! = 4 \times 3 \times 2 \times 1$

$= 24$ words

Answer - B

2 -

3	2	1

$3! = 3 \times 2 \times 1$

$= 6$

Answer - C

3 - $5! = 5 \times 4 \times 3 \times 2 \times 1$

$= 120$ ways

Answer - A

4 - MAGIC

$5! = 5 \times 4 \times 3 \times 2 \times 1$

$= 120$ ways

Answer - B

5 – AI M G C

$4! = 4 \times 3 \times 2 \times 1$

$= 24$

IA M G C

$4! = 24$

$\therefore$ 48 ways ✓

Answer – C

6 – BEAUTY – 6 Letters

$6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1$ ✓

$= 720$ ways.

Answer – C

7 – [EAU] [B] [T] [Y]

$3!$

$4!$

$\therefore 3! \times 4!$ ✓

Answer – A

8 – $^4C_4 + {}^4C_3 + {}^4C_2 + {}^4C_1$

$\frac{4!}{4! \times 0!} + \frac{4!}{3! \times 1!} + \frac{4!}{2! \times 2!} + \frac{4!}{3! \times 1!}$

$= 1 + 4 + 6 + 4$

$= 15$ ways

Answer – B

Part – 2

1 –

5	4	3

$\therefore 5 \times 4 \times 3 = 60$

Answer – A

2 – SINGAPORE – 9 letters.

9	8	7	6

$\rightarrow 9 \times 8 \times 7 \times 6 = 3024$

$$
\begin{array}{r}
1 \\
72 \\
\times \quad 7 \\
\hline
504 \\
\times \quad 6 \\
\hline
3024
\end{array}
$$

Answer – C

3 -

$7+6+5+4+3+2+1$

2 - 7
3 - 6
4 - 5
5 - 4
6 - 3
7 - 2
8 - 1

$6+5+4+3+2+1$

$5+4+3+2+1$

$4+3+2+1$

$3+2+1$

$2+1$

1

$(7\times1)+(2\times6)+(3\times5)+(4\times4)+(5\times3)+(6\times2)+(7\times1)$

$7+12+15+16+15+12+7$

1
14
24
30
16

84

84

Answer - D

✓

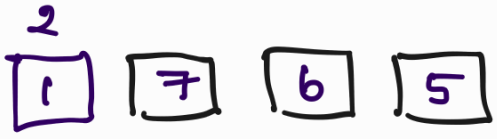
4 - 

1, 3, 4, 5

$4! = 24$ ways ✓

Answer - A

5 - 2000 to 3000



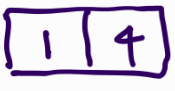
0 - 7 ⇒ 7

(2 not allowed Because Repetition not Allowed)

$7 \times 6 \times 5$
$= 210$ ways ✓

Answer - C

6 - ALHBD ⇒ 1 Vowel
4 Consonant



$= 4$ ✓

Answer - A

7 - GOOD - 4 letters
O - Repeated 2 times

In the Answer they are not Considering 2 - O ↳ repeated

$\therefore \frac{4!}{2!} = \frac{4 \times 3 \times \cancel{2 \times 1}}{\cancel{2 \times 1}} = 12$

Answer B 24

8 – SCISSORS – 8 Letters

4-S

$\therefore \frac{8!}{4!} = 8 \times 7 \times 6 \times 5$

$= 1680$ //

Answer – B

9 – MINIMUM – 7 Letters

3 - M

2 – I

$\therefore \frac{7!}{3! \times 2!} = \frac{7 \times 6 \times 5 \times \cancel{4^2}}{\cancel{2}}$

$= 210 \times 2 = 420$

Answer - A

Part - 3

1 – 13 diamond

including k of ◇

3 kings ♡♧♤

∴ total 16 ways

Answer – A

2 - while One Chair is identical

First Person has 6 different options.

So, $6! = 720$

Answer - B