Technical Documentation: Computing Descending Factorial Moments from Raw Moments

The procedure for computing descending factorial moments (also called factorial moments) from raw moments involves a transformation that expresses factorial moments as a linear combination of raw moments using Stirling numbers of the second kind. Given the first four raw moments m_1, m_2, m_3, m_4, the corresponding factorial moments f_1, f_2, f_3, f_4 are obtained using the recurrence relation:

$$f_r = \sum_{k=1}^{r} S(r,k) m_k$$

where $S(r,k)$ are Stirling numbers of the second kind, which count the number of ways to partition a set of r elements into k non-empty subsets. Explicitly, the first four factorial moments are computed as:

$$\begin{aligned} f_1 &= m_1 \\ f_2 &= m_2 - m_1 \\ f_3 &= m_3 - 3m_2 + 2m_1 \\ f_4 &= m_4 - 6m_3 + 11m_2 - 6m_1 \end{aligned}$$

This transformation ensures that factorial moments are always less than or equal to their corresponding raw moments, reflecting the nature of the descending factorial operation, which progressively reduces contributions from larger values in the dataset. This approach is particularly useful in combinatorial probability, discrete distributions, and moment-based statistical inference.

Technical Documentation: Computing Raw Moments from Variance, Skewness, and Kurtosis

Given a dataset with mean m_1, variance (second central moment) σ_2, skewness γ_3, and kurtosis γ_4, the second, third, and fourth raw moments m_2, m_3, m_4 can be derived as follows.

1. Second Raw Moment The second raw moment m_2 is computed directly from the variance:

$$m_2 = \sigma_2 + m_1^2$$

2. Third Raw Moment The third raw moment m_3 is computed using the relationship between the third central moment σ_3 and the standardized skewness:

$$\sigma_3 = \gamma_3 \cdot \sigma_2^{3/2}$$

Using the transformation between raw and central moments:

$$m_3 = \sigma_3 + 3m_1\sigma_2 + m_1^3$$

3. Fourth Raw Moment The fourth raw moment m_4 is computed using the relationship between the fourth central moment σ_4 and the standardized kurtosis:

$$\sigma_4 = \gamma_4 \cdot \sigma_2^2$$

Using the transformation between raw and central moments:

$$m_4 = \sigma_4 + 4m_1\sigma_3 + 6m_1^2\sigma_2 + m_1^4$$

These transformations allow for the computation of raw moments from commonly used statistical measures such as variance, skewness, and kurtosis, ensuring consistency in moment-based statistical analysis.