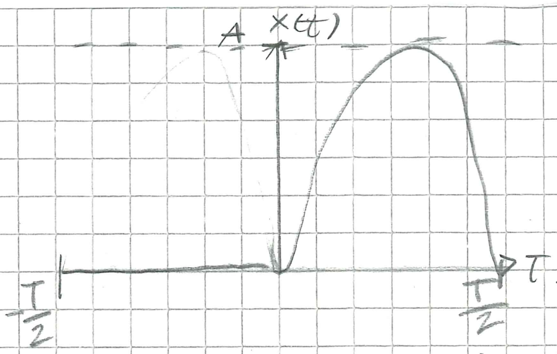
$T = \frac{1}{f} \qquad \omega = 2\pi f \qquad T\omega = 2\pi$

$n \in \mathbb{N}$



$$c_n = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} x(t)\, e^{-jn2\pi ft}\, dt$$

$$x(t) = \begin{cases} A\cdot\sin(2\pi ft), & \sin(2\pi ft) \geq 0 \\ 0, & \sin(2\pi fT) < 0 \end{cases}$$

$$= \frac{A}{T}\int_0^{\frac{T}{2}} \sin(2\pi ft)\, e^{-jn2\pi ft}\, dt = \frac{A}{T}\int_0^{\frac{T}{2}} \sin(\omega t)\cdot e^{-jn\omega t}$$

$\downarrow$ variabelbytte + tidsomforming

$\theta = \omega t \qquad \theta' = \omega \qquad t = 0 \Rightarrow \theta = 0 \qquad t = \frac{T}{2},\ \theta = \pi$

$$= \frac{A}{T\omega}\int_0^{\pi} \sin(\theta)\cdot e^{-jn\theta}\, d\theta = \frac{A}{2\pi}\int_0^{\pi} \frac{e^{j\theta}\cdot e^{-jn\theta} - e^{-j\theta}\cdot e^{-jn\theta}}{2j}\, d\theta$$

$$= \frac{A}{2\pi}\int_0^{\pi} \frac{e^{j\theta(1-n)} - e^{-j\theta(1+n)}}{2j} = \frac{A}{2\pi}\left[\frac{e^{j\theta(1-n)}}{-2(1-n)} + \frac{e^{-j\theta(1+n)}}{-2(1+n)}\right]_{\theta=0}^{\theta=\pi}$$

(ikke definert for $n = \pm 1$)

$$= \frac{A}{2\pi}\left(-\frac{1}{2}\left(\left(\frac{e^{-j\pi n}\cdot e^{j\pi}}{1-n} + \frac{e^{-j\pi n}\cdot e^{-j\pi}}{1+n}\right) - \left(\frac{e^0}{1-n} + \frac{e^0}{1+n}\right)\right)\right)$$

$e^{-j\pi n} = (-1)^n$

$$= \frac{A}{2\pi}\left(\cancel{-}\frac{1}{2}\left(\left(\frac{(-1)^n\cdot(-1)}{1-n} + \frac{(-1)^n\cdot(-1)}{1+n}\right) \cancel{-} \left(\frac{(1+n)}{(1-n)(1+n)} + \frac{(1-n)}{(1-n)(1+n)}\right)\right)\right)$$

$$= \frac{A}{4\pi}\left(\frac{(-1)^n(1+n)}{(1-n)(1+n)} + \frac{(-1)^n(1-n)}{(1+n)(1-n)} + \frac{2}{1-n^2}\right)$$

$$= \frac{A}{4\pi}\left(\frac{((-1)^n+1)\cdot 2}{1-n^2}\right) = A\left(\frac{(-1)^n+1}{2\pi(1-n^2)}\right)$$

for $n = 3, 5 \ldots$ $= 0$ siden $-1+1=0$

for $n = 2, 4, \ldots$ $= A\cdot\left(\frac{2}{2\pi(1-n^2)}\right) = \frac{A}{\pi(1-n^2)}$

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for $n=1$

$$c_1 = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} x(t)\, e^{-j2\cdot\pi f t} = \frac{A}{T}\int_0^{T/2} \sin(\omega t)\, e^{-j\omega t}$$

$$= \frac{A}{2\pi}\int_0^{\pi} \sin(\theta)\, e^{-j\theta} = \frac{A}{2\pi}\int_0^{\pi} \frac{e^{-j\theta}\cdot e^{j\theta} - e^{-j\theta}\cdot e^{-j\theta}}{2j}$$

$$= \frac{A}{4\pi j}\int_0^{\pi} e^{j\theta - j\theta} - e^{-j\theta - j\theta} = \frac{A}{4\pi j}\int_0^{\pi} 1 - e^{-j2\theta}$$

$$= \frac{A}{4\pi j}\left[\theta + \frac{e^{-j2\theta}}{j\cdot 2}\right]_{\theta=0}^{\theta=\pi} = \frac{A}{4\pi j}\left((\pi - 0) + \left(\frac{e^{-j2\pi}}{j\cdot 2} - \frac{e^{0}}{j\cdot 2}\right)\right)$$

$$= \frac{A\cdot\pi}{4\cdot j\,\pi} = \frac{A}{4j}$$

for $n=0$

$$c_0 = \frac{1}{T}\int_{-\frac{T}{2}}^{T/2} x(t)\, e^{0} = \frac{A}{T}\int_0^{T/2}\sin(\omega t) = \frac{A}{2\pi}\int_0^{\pi}\sin(\theta)$$

$$= \frac{A}{2\pi}\left[-\cos(\theta)\right]_{\theta=0}^{\theta=\pi} = \frac{A}{2\pi}\left(\cos(0) - \cos(\pi)\right) = \frac{2A}{2\pi} = \frac{A}{\pi}$$

for $n=-1$

$$c_{-1} = \ldots = \frac{A}{2\pi}\int_0^{\pi}\sin(\theta)\, e^{j\theta} = \frac{A}{2\pi}\int \frac{e^{j\theta}\cdot e^{j\theta} - e^{-j\theta}\cdot e^{j\theta}}{2j}$$

$$= \frac{A}{4\pi j}\int_0^{\pi} e^{j\cdot 2\theta} - 1 = \frac{A}{4\pi j}\left[\cancel{\frac{e^{j\cdot 2\theta}}{j2}} - \theta\right]_{\theta=0}^{\theta=\pi}$$

$$= -\frac{A\pi}{4\pi j} = -\frac{A}{4j}$$

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$$C_n = \begin{cases} \dfrac{A}{\pi(1-n^2)} & \text{for } n = 2, 4, \ldots \\ \pm\dfrac{A}{4j} & \text{for } n = \pm 1 \\ \dfrac{A}{\pi} & \text{for } n = 0 \end{cases}$$

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