11. Active Filters and Tuned Circuits

11.1 Active Low - Pass Filters

Active filters: circuits, containing resistors, capacitors and Op Amps. They are intended for separating of signals having different frequencies.

Requirements for filters:

1. Contain few components.
2. Have a transfer function that is insensitive to component tolerances.
3. Place modest demands on the op amp's gain-bandwidth product, output impedance, slew rate, and other specifications.
4. Be easily adjusted.
5. Require a small spread of component values.
6. Allow a wide of useful transfer functions to be realized.

Butterworth Transfer Function

$$\left|H(f)\right| = \frac{H_0}{\sqrt{1+\left(f/f_b\right)^{2n}}} \qquad (11.1)$$

f_b – **break (cut-off) frequency.**

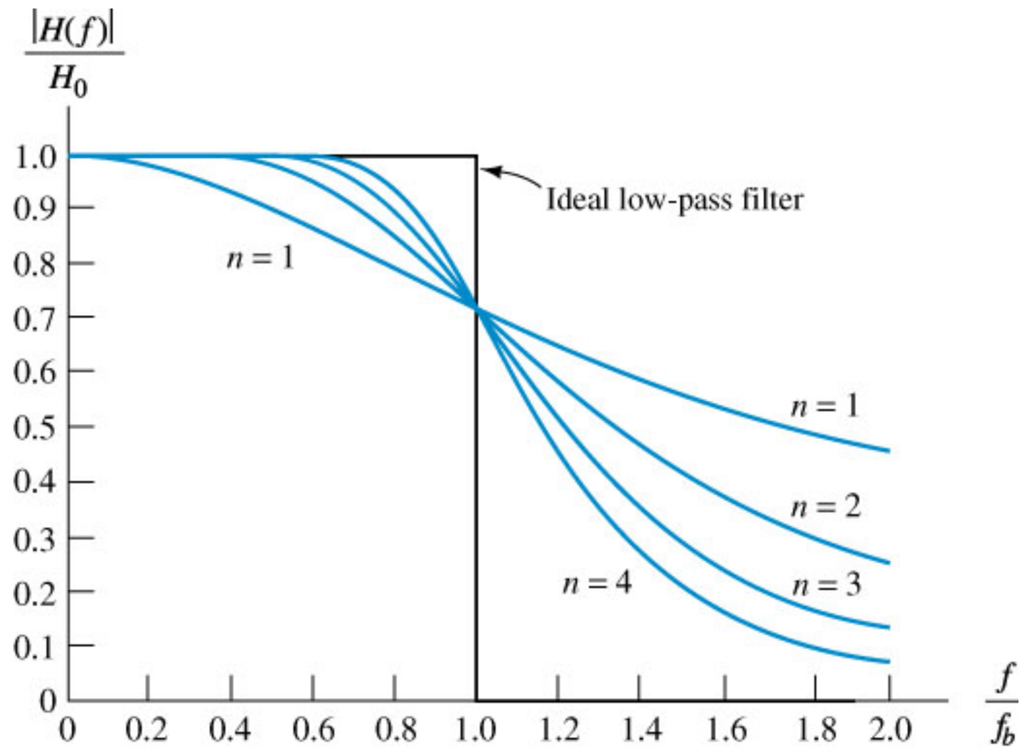


Figure 11.1 Transfer function magnitude versus frequency for low-pass Butterworth filters.

Sallen - Key Circuits

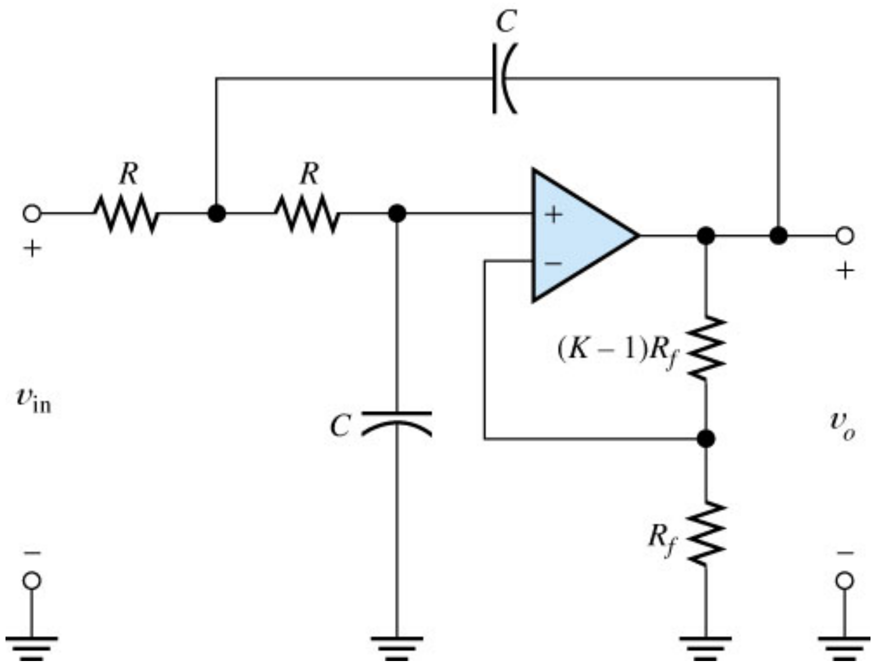


Figure 11.2 Equal-component Sallen-Key low-pass active-filter section.

$$f_b = \frac{1}{2\pi RC} \tag{11.2}$$

Table 11.1. K Values for Low-Pass or High-Pass Butterworth Filters of Various Orders

Order	*K*
2	1.586
4	1.152
	2.235
6	1.068
	1.586
	2.483
8	1.038
	1.337
	1.889
	2.610

Example 11.1 Fourth - Order Low - Pass Butterworth Filter Design

Design a fourth - order low - pass Butterworth filter with a cut - off frequency of 100 Hz. Use the LF411 op amp.

Solution:

We choose $C_1=C_2=C_{11}=C_{12}=0.1\mu F$ and from (11.2)

$$R_1 = R_2 = R_{11} = R_{12} = \frac{1}{2\pi f_b C} = \frac{1}{2\pi \times 100 \times 0.1 \times 10^{-6}} = 15.8\text{k}\Omega$$

From table 11.1 K=1.152; K_1=2.235. We choose $R_4=R_{14}=10\text{k}\Omega$ and

$$R_3 = (K-1)R_4 = (1.152-1)\times 10\times 10^3 = 1.52\text{k}\Omega$$

$$R_{13} = (K-1)R_{14} = (2.235-1)\times 10\times 10^3 = 12.35\text{k}\Omega$$

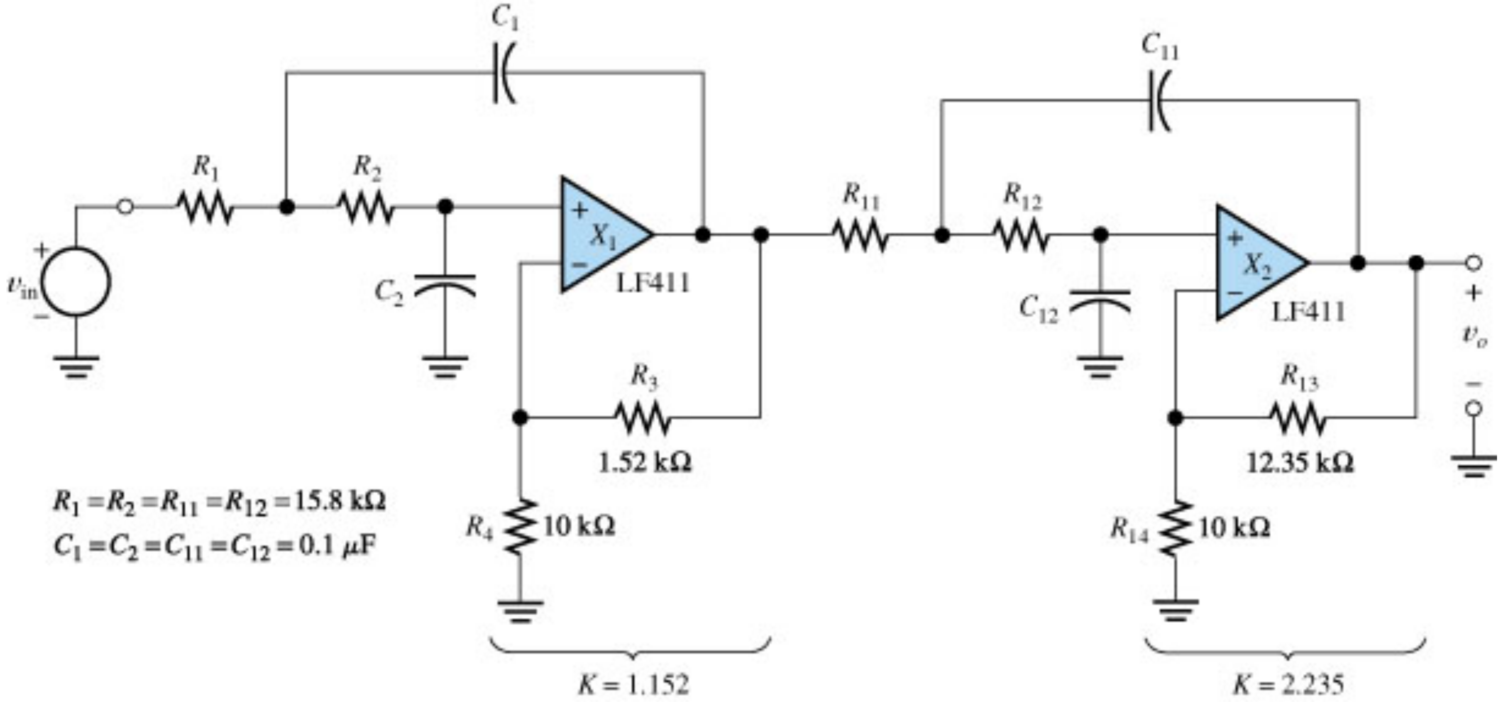


Figure 11.3 Fourth-order Butterworth low-pass filter.

11.2 Active High - Pass Filters

Transformation of low-pass transfer function to high-pass transfer function: $f \rightarrow f_b^2/f$.

$$\left|H_{hp}(f)\right| = \frac{H_0}{\sqrt{1+\left(f_b/f\right)^{2n}}} \qquad (11.3)$$

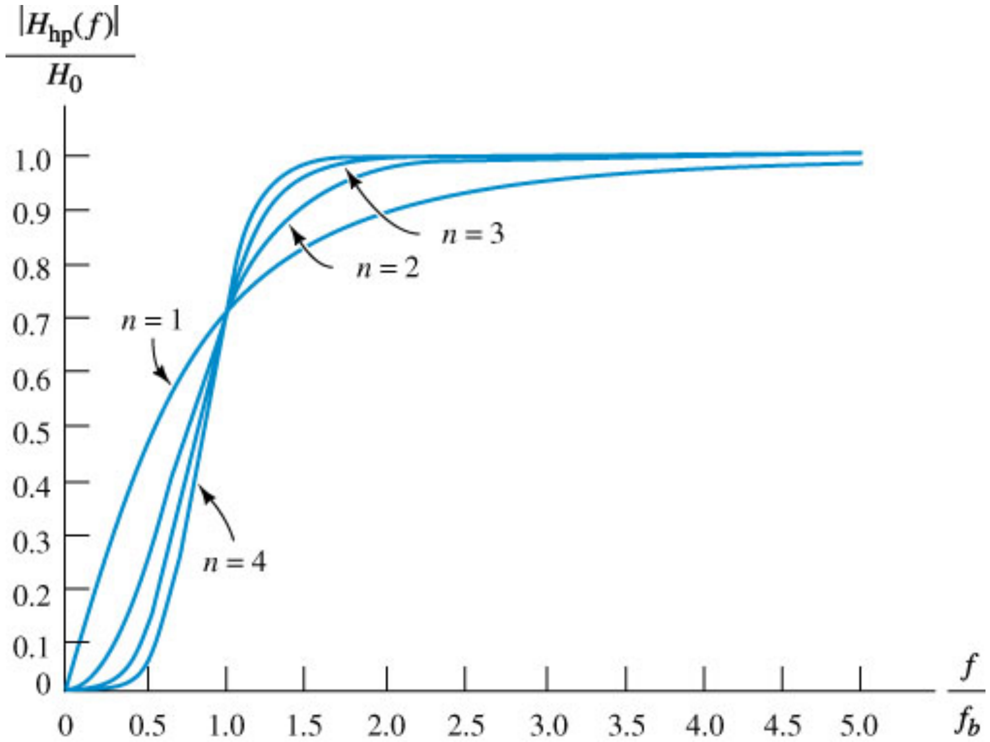


Figure 11.7 Normalized high-pass Butterworth transfer functions.

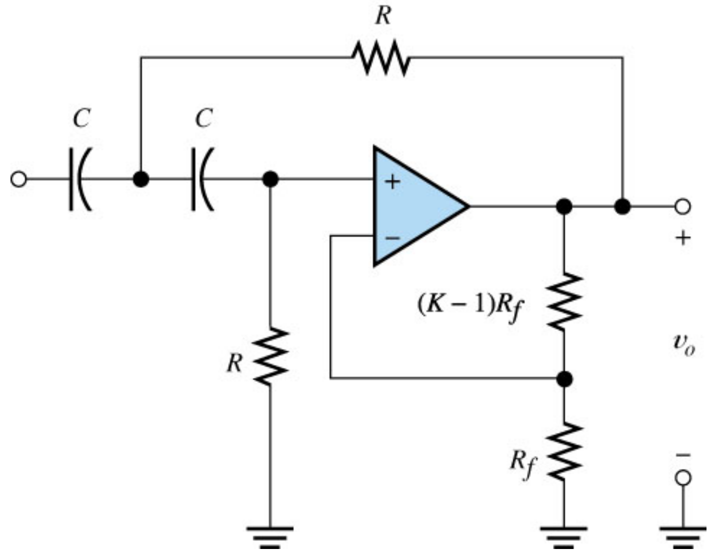


Figure 11.8 Sallen-Key high-pass active-filter section.

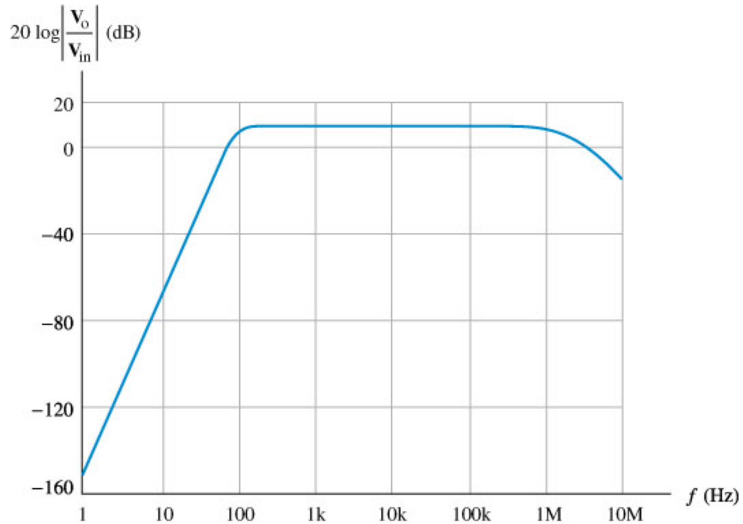


Figure 11.9 Bode magnitude plot for the high-pass filter.

11.3 Active Bandpass Filters

Delyiannis - Friend Bandpass Circuits

$$f_0 = \frac{1}{2\pi C\sqrt{(R_1 \,||\, R_2)R_3}} \qquad (11.6)$$

$$H_0 = \frac{R_3}{2R_1} \qquad (11.7)$$

$$B = \frac{1}{\pi R_3 C} \qquad (11.8)$$

$$Q = \frac{f_0}{B} = \frac{1}{2}\left(\frac{R_3}{R_1 \,||\, R_2}\right)^{1/2} \qquad (11.9)$$

$$R_3 = \frac{Q}{\pi f_0 C} \qquad (11.10)$$

$$R_1 = \frac{R_3}{2H_0} \qquad (11.11)$$

$$R_2 = \frac{R_3}{4Q^2 - 2H_0} \qquad (11.12)$$

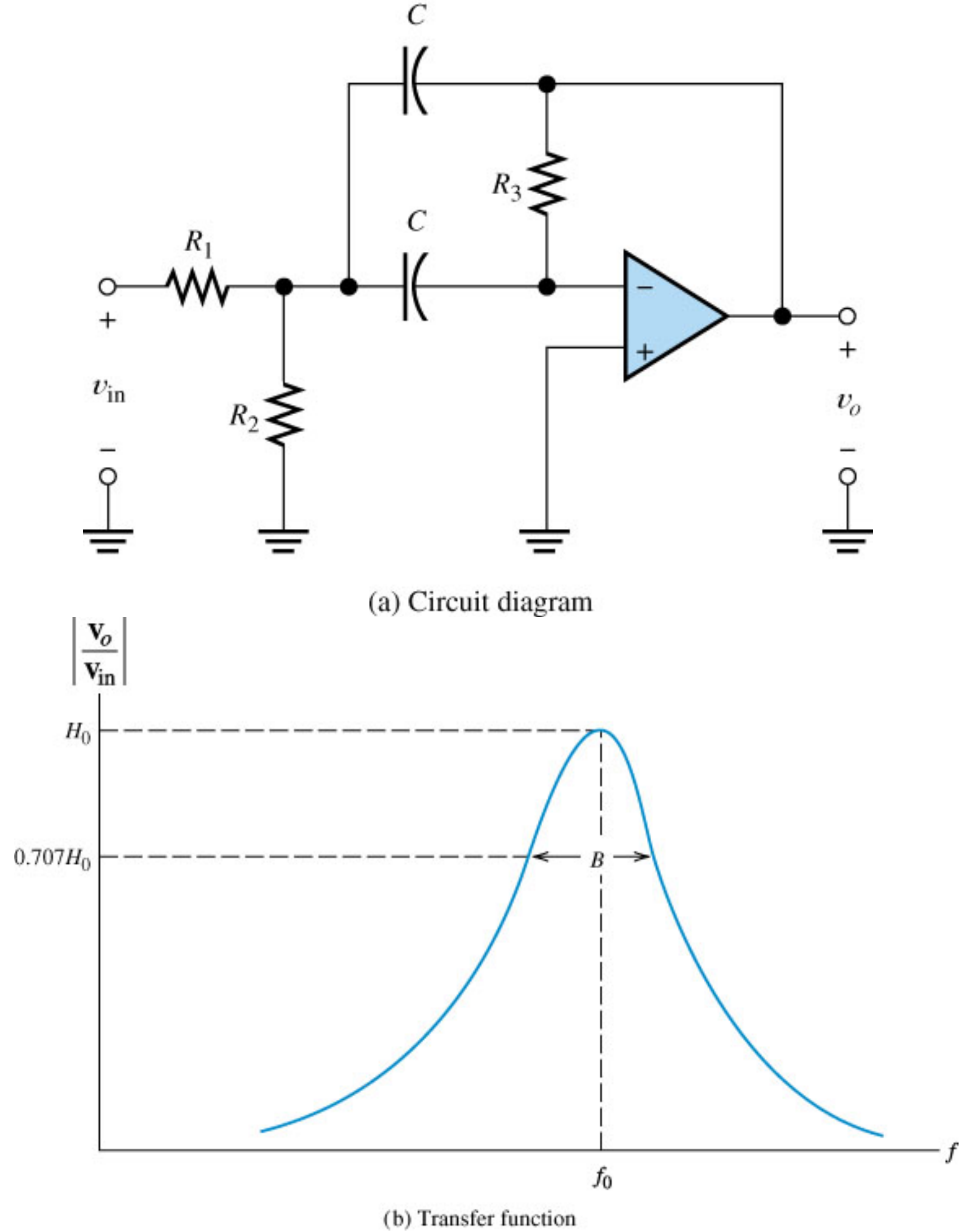


Figure 11.12 Second-order bandpass filter.

Example 11.3 Bandpass Filter Design

Design a bandpass filter with f_0=1 kHz, B=200 Hz, and H_0=10.

Solution:

$$Q = f_0/B = \left(1\times10^3\right)/200 = 5$$

$$R_3 = \frac{Q}{\pi f_0 C} = \frac{1.592\times10^{-3}}{C} \qquad (11.13)$$

$$R_1 = \frac{R_3}{2H_0} = \frac{R_3}{20} \qquad (11.14)$$

$$R_2 = \frac{R_3}{4Q^2 - 2H_0} = \frac{R_3}{80} \qquad (11.15)$$

R_1 must be large to have high input impedance.

From (11.14) $R_3 = 20R_1$ and R_1 must have moderate value to keep the value of R_3 realistic.

The choice $R_1 = 10\text{k}\Omega$ gives

$$R_3 = 20R_1 = 20\times10\times10^3 = 200\text{k}\Omega$$

$$R_2 = \frac{R_3}{80} = \frac{200\times10^3}{80} = 2.5\text{k}\Omega$$

$$C = \frac{1.592\times10^{-3}}{R_3} = \frac{1.592\times10^{-3}}{200\times10^3} = 7.96\text{nF}$$

We select $C = 8.2\text{nF}$ and then calculate again the values of R_1, R_2 and R_3.

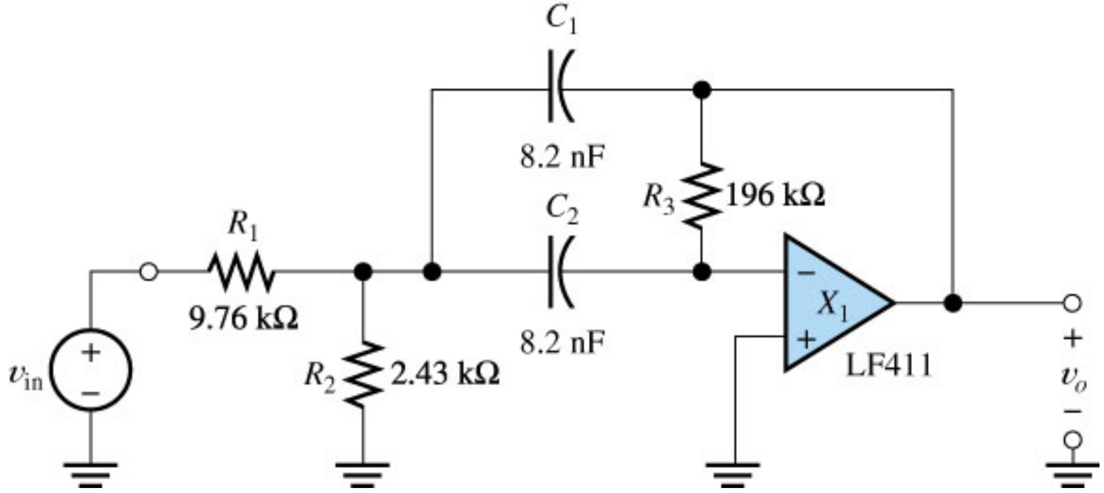


Figure 11.13 Bandpass filter designed in Example 11.3.

11.4 The Series Resonant Circuit

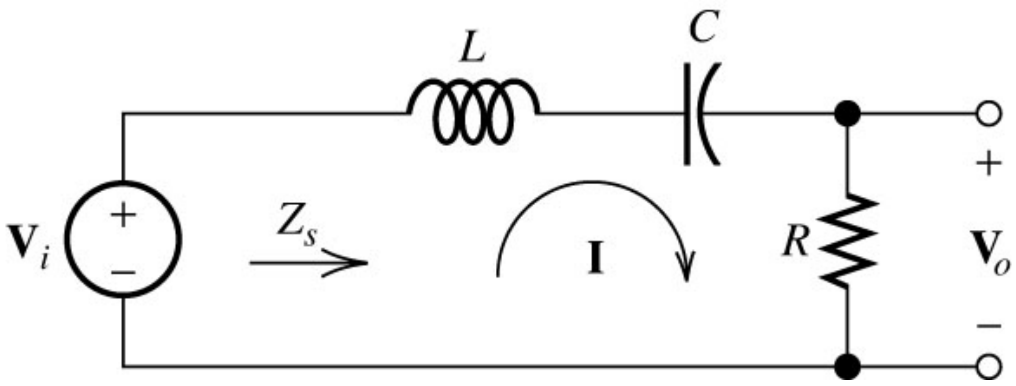


Figure 11.17 Series resonant circuit.

$$I = \frac{V_i}{j\omega L + 1/(j\omega C) + R} \tag{11.16}$$

$$V_o = RI \tag{11.17}$$

$$V_o = \frac{j\omega R V_i}{-\omega^2 L + j\omega R + 1/C} \tag{11.18}$$

$$A_v(j\omega) = \frac{V_o}{V_i} = \frac{j\omega R}{-\omega^2 L + j\omega R + 1/C} \tag{11.19}$$

Resonant Frequency and Quality Factor

Resonant frequency

$$\omega_0 = \frac{1}{\sqrt{LC}} \tag{11.20}$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \tag{11.21}$$

Quality factor

$$Q = \frac{\omega_0 L}{R} \tag{11.22}$$

At the resonance

$$\omega_0 L = \frac{1}{\omega_0 C} \tag{11.23}$$

Another formula for the quality factor

$$Q = \frac{1}{\omega_0 C R} \tag{11.24}$$

Voltage transfer ratio, expressed with Q-factor and resonant frequency

$$A_v(j\omega) = \frac{j(\omega/\omega_0)}{Q\left[1-(\omega/\omega_0)^2\right] + j(\omega/\omega_0)} \qquad (11.25)$$

$|A_v| = 0$ if $\omega = 0$; $|A_v| \to 0$ when $\omega \to \infty$. For frequencies between 0 and ∞ $|A_v| > 0$ and has a maximum at $\omega = \omega_0$.

The circuit is a bandpass filter.

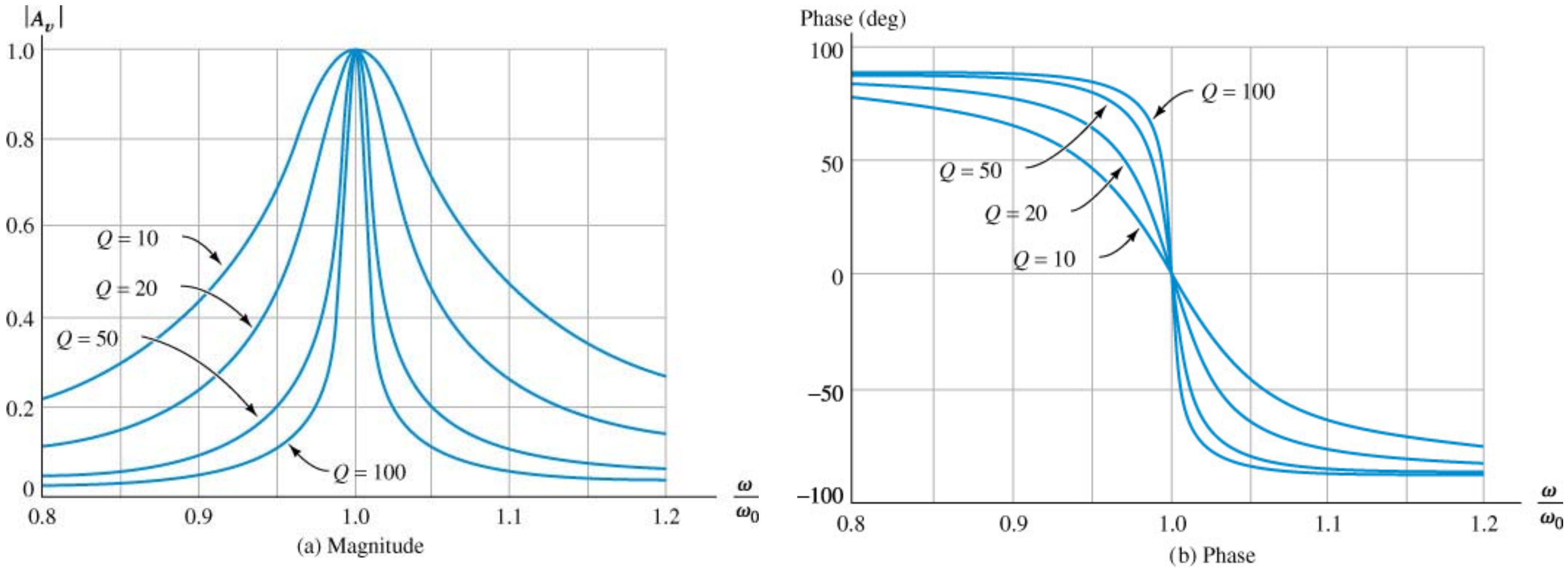


Figure 11.18 Voltage transfer function for the series resonant circuit.

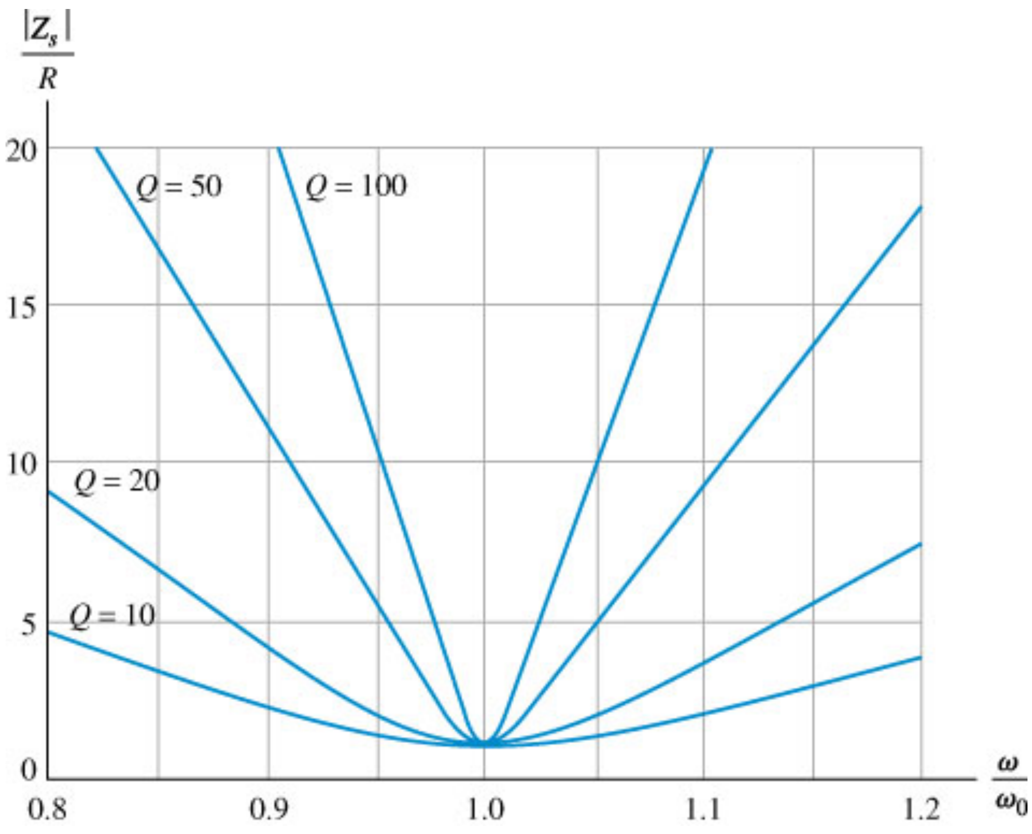


Figure 11.19 Normalized impedance of the series resonant circuit.

Circuit Bandwidth

$$B = f_H - f_L = \frac{f_0}{Q} \tag{11.28}$$

$$f_H f_L = f_0^2 \tag{11.29}$$

If B is relatively small, compared with f_0, then

$$f_L \cong f_0 - B/2 \tag{11.30}$$

$$f_H \cong f_0 + B/2 \tag{11.31}$$

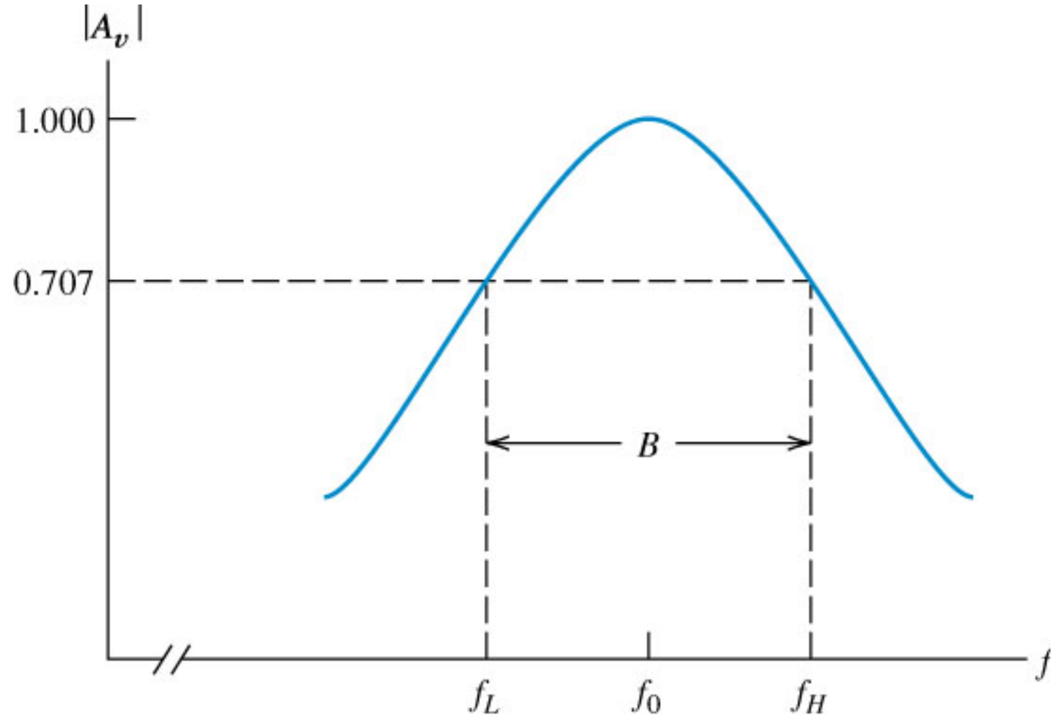


Figure 11.20 Bandwidth and half-power frequencies for the series resonant circuit.

Exercise 11.7

Find resonant frequency, Q, the bandwidth, and half - power frequencies of a series resonance circuit having L=5 μH, C=100 pF, and R=10 Ω.

Solution:

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{5\times10^{-6}\times100\times10^{-12}}} = 7.12\text{MHz}$$

$$Q = \frac{2\pi f_0 L}{R} = \frac{2\pi\times7.12\times10^{6}\times5\times10^{-6}}{10} = 22.4$$

$$B = \frac{f_0}{Q} = \frac{7.12\times10^{6}}{22.4} = 318\text{kHz}$$

$$f_L \approx f_0 - \frac{B}{2} = 7.12\times10^{6} - \frac{318\times10^{3}}{2} = 6.96\text{MHz}$$

$$f_H \approx f_0 + \frac{B}{2} = 7.12\times10^{6} + \frac{318\times10^{3}}{2} = 7.28\text{MHz}$$

11.5 The Parallel Resonant Circuit

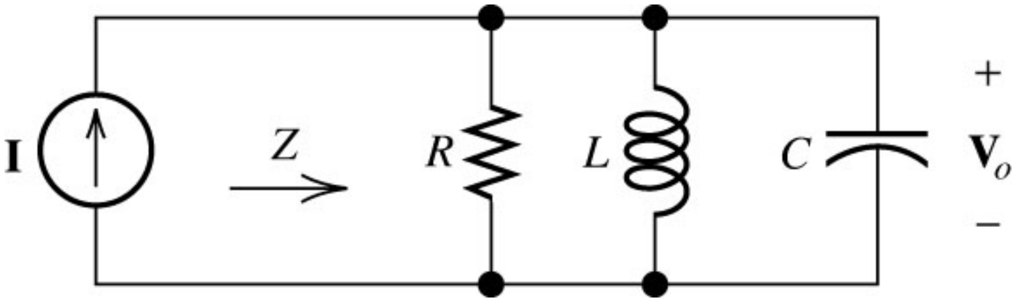


Figure 11.25 Parallel resonant circuit.

$$Z(j\omega) = \frac{1}{1/R + 1/(j\omega L) + j\omega C} \qquad (11.32)$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \qquad (11.33)$$

$$Q = \frac{R}{\omega_0 L} \qquad (11.34)$$

$$\omega_0 L = \frac{1}{\omega_0 C}$$

$$Q = \omega_0 RC \qquad (11.35)$$

$$Z(j\omega) = R\frac{j(\omega/\omega_0)}{Q\left[1-(\omega/\omega_0)^2\right] + j(\omega/\omega_0)} \qquad (11.36)$$

$$B = f_H - f_L = \frac{f_0}{Q}$$

$$f_H f_L = f_0^2$$

$$f_L \cong f_0 - B/2$$

$$f_H \cong f_0 + B/2$$

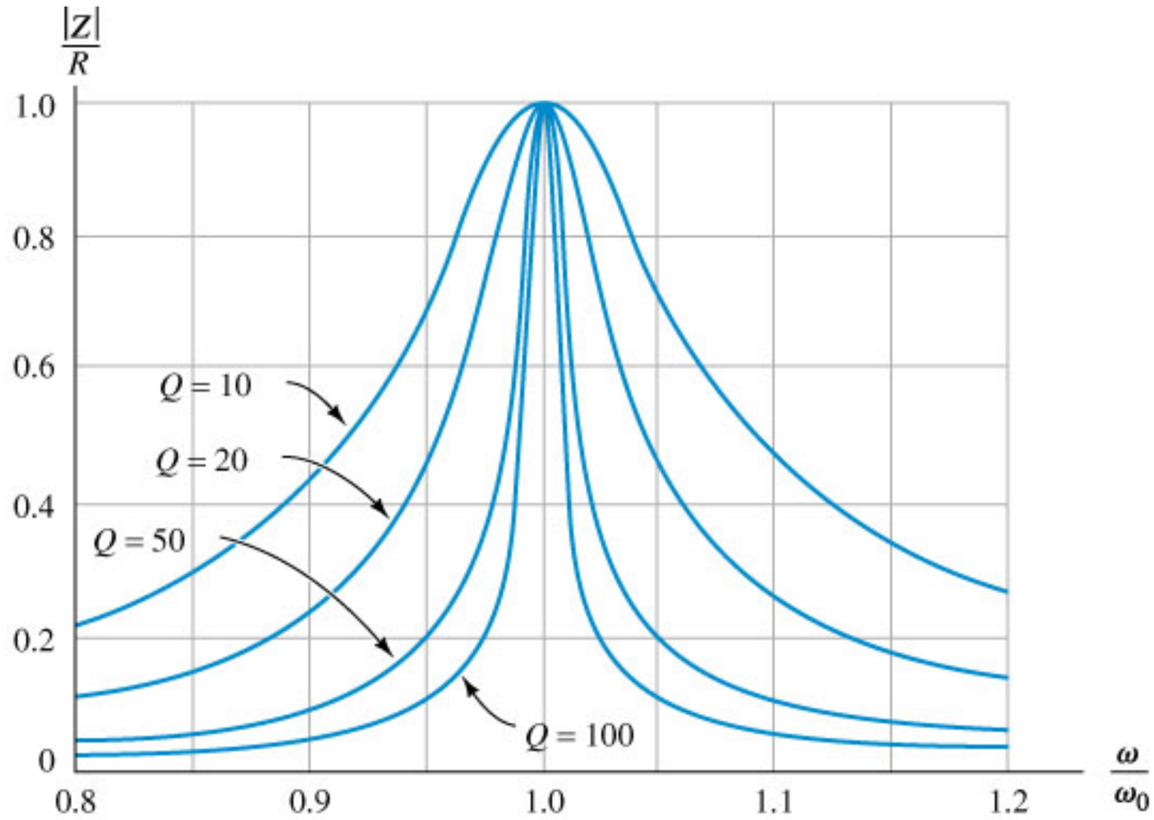


Figure 11.26 Normalized impedance of the parallel resonant circuit.

11.8 Tuned Amplifiers

Tuned amplifiers have resonant circuits at their inputs or outputs or both and they amplify in a narrow bandwidth.

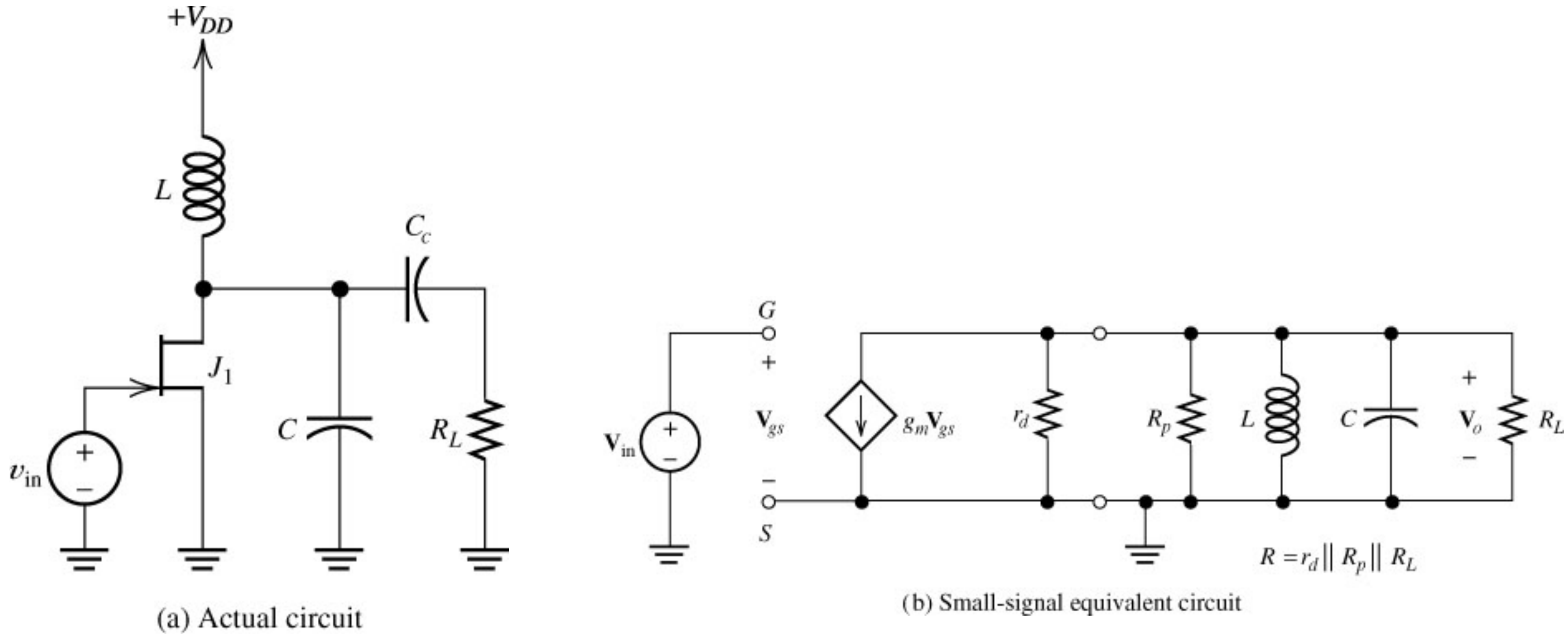


Figure 11.43 Tuned amplifier.

$$V_o = -g_m V_{gs} Z(j\omega) \qquad (11.57)$$

$$A_v = \frac{V_o}{V_{in}} = -g_m Z(j\omega) \qquad (11.58)$$

Input Impedance

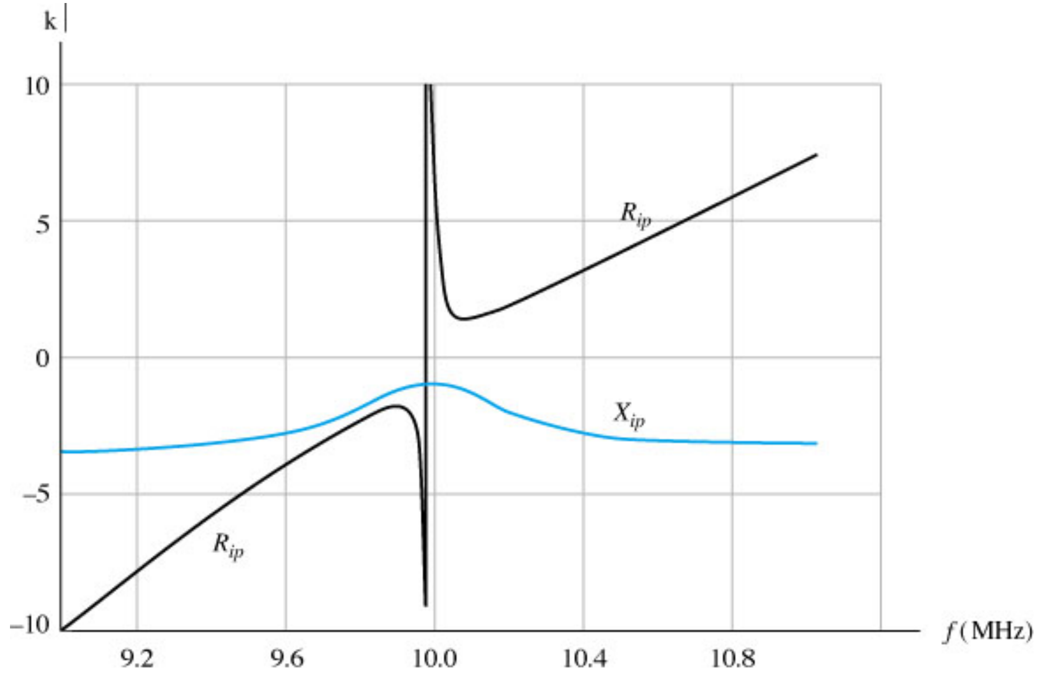


Figure 11.47 Parallel input resistance and reactance for the tuned amplifier of Figure 11.44. Pay attention to the negative real part at frequencies below the resonant frequency.

Neutralization

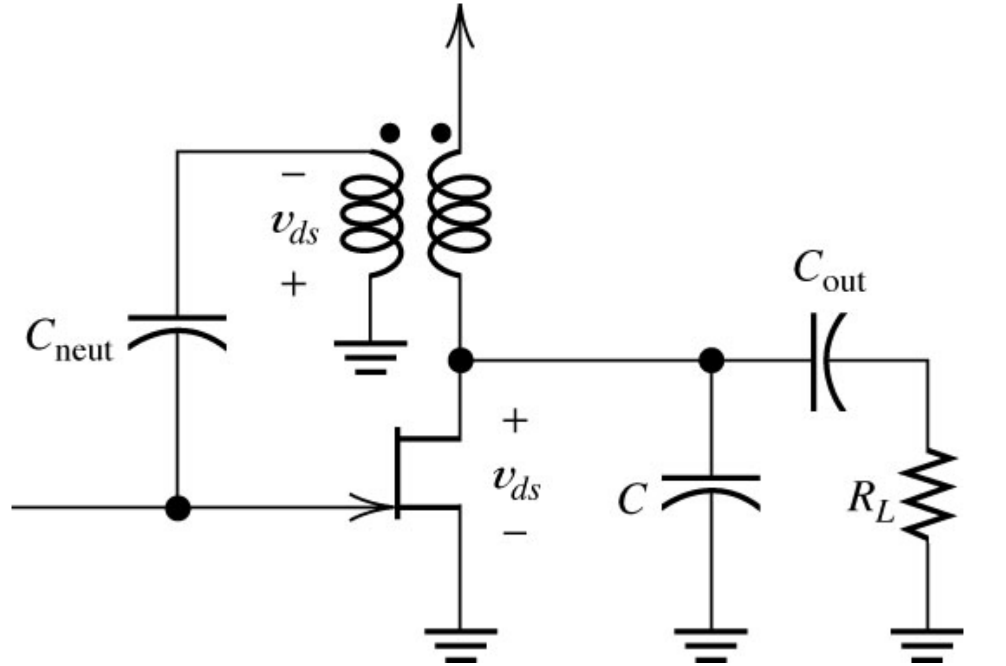


Figure 11.48 C_{neut} is used to cancel the feedback through C_{gd}.

11.9 *LC* Oscillators

The Hartley Oscillators

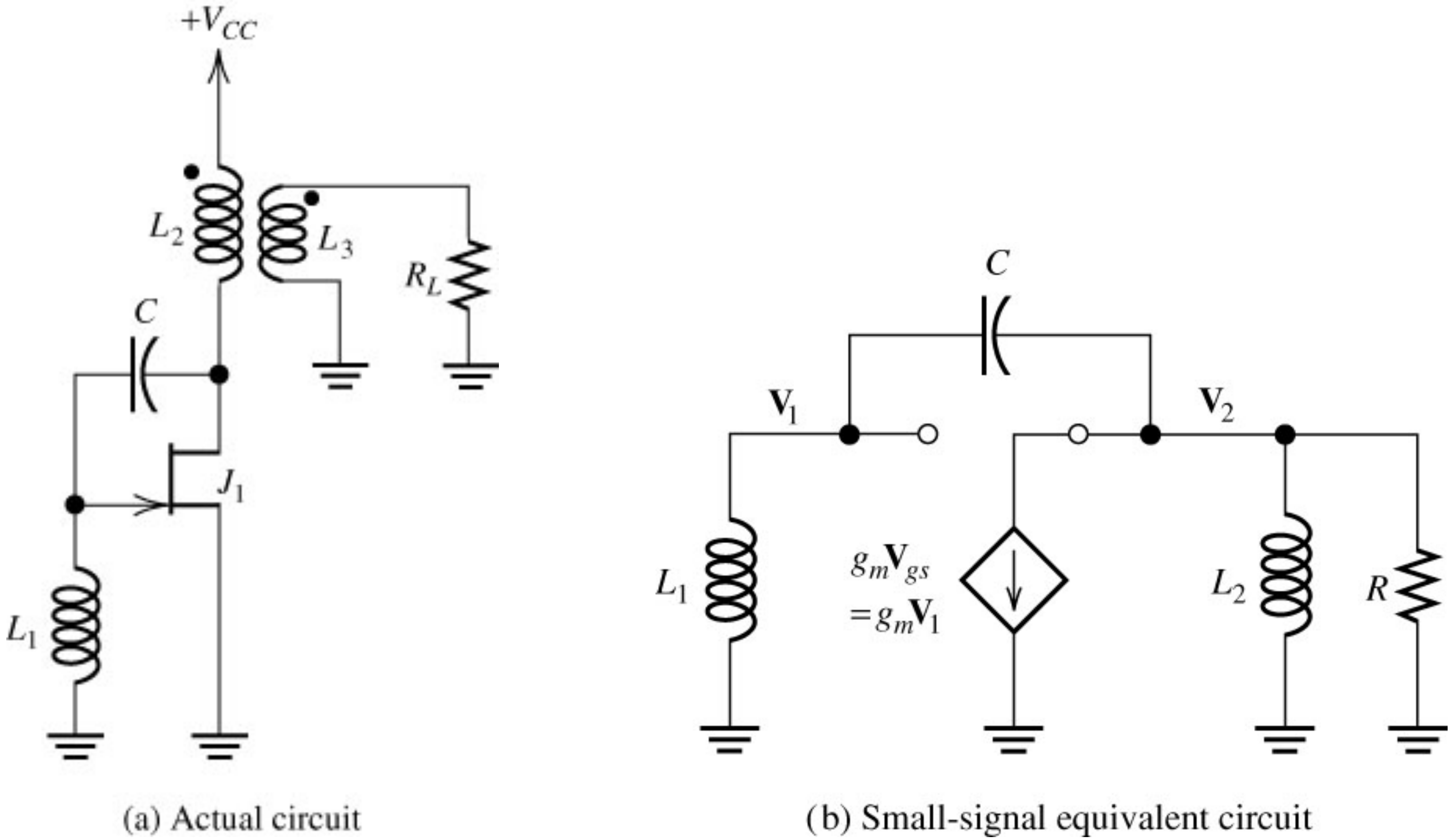
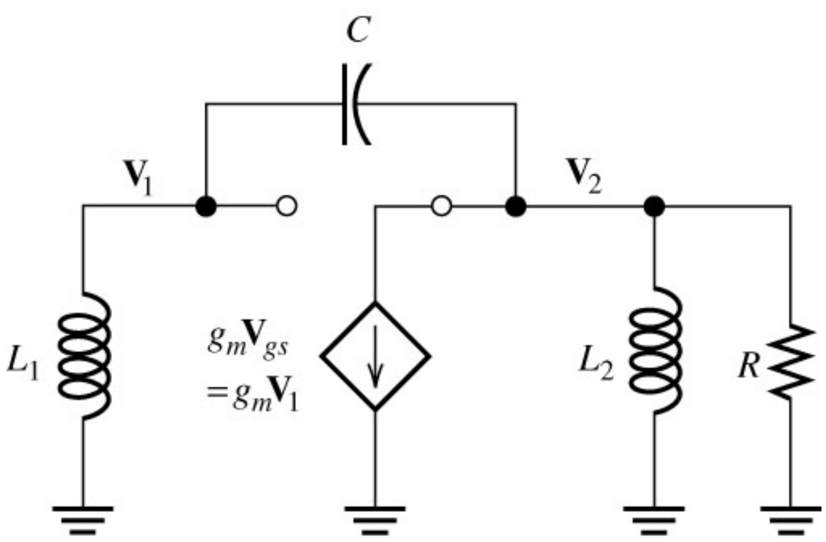


Figure 11.50 Hartley oscillator.

Frequency of Oscillation and the Minimum Transconductance Requirement



(b) Small-signal equivalent circuit

Figure 11.50 Hartley oscillator.

$$\frac{V_1}{j\omega L_1}+j\omega C(V_1-V_2)=0 \qquad (11.60)$$

$$g_m V_1+j\omega C(V_2-V_1)+\frac{V_2}{j\omega L_2}+\frac{V_2}{R}=0 \qquad (11.61)$$

$$\left(j\omega C-j\frac{1}{\omega L_1}\right)V_1-j\omega C V_2=0 \qquad (11.62)$$

$$(g_m-j\omega C)V_1+\left(j\omega C-j\frac{1}{\omega L_2}+\frac{1}{R}\right)V_2=0 \qquad (11.63)$$

(11.62) and (11.63) form a homogeneous set of linear equations concerning V_1 and V_2. The solution is nonzero if the system determinant is zero:

$$\begin{vmatrix} \left(j\omega C-j\frac{1}{\omega L_1}\right) & (-j\omega C) \\ (g_m-j\omega C) & \left(j\omega C-j\frac{1}{\omega L_2}+\frac{1}{R}\right) \end{vmatrix}=0 \qquad (11.64)$$

$$\left(\frac{C}{L_1}+\frac{C}{L_2}-\frac{1}{\omega^2 L_1 L_2}\right)+j\left(\frac{\omega C}{R}-\frac{1}{\omega R L_1}+\omega C g_m\right)=0$$

(11.66)

$$\left(\frac{C}{L_1}+\frac{C}{L_2}-\frac{1}{\omega^2 L_1 L_2}\right)+j\left(\frac{\omega C}{R}-\frac{1}{\omega R L_1}+\omega C g_m\right)=0 \tag{11.66}$$

From zeroing of the real part of (11.66):

$$\omega=\frac{1}{\sqrt{C(L_1+L_2)}} \tag{11.68}$$

From zeroing of the imaginary part of (11.66):

$$g_m=\frac{L_2}{RL_1} \tag{11.71}$$

11.10 Crystal - Controlled Oscillators

The Piezoelectric Effect

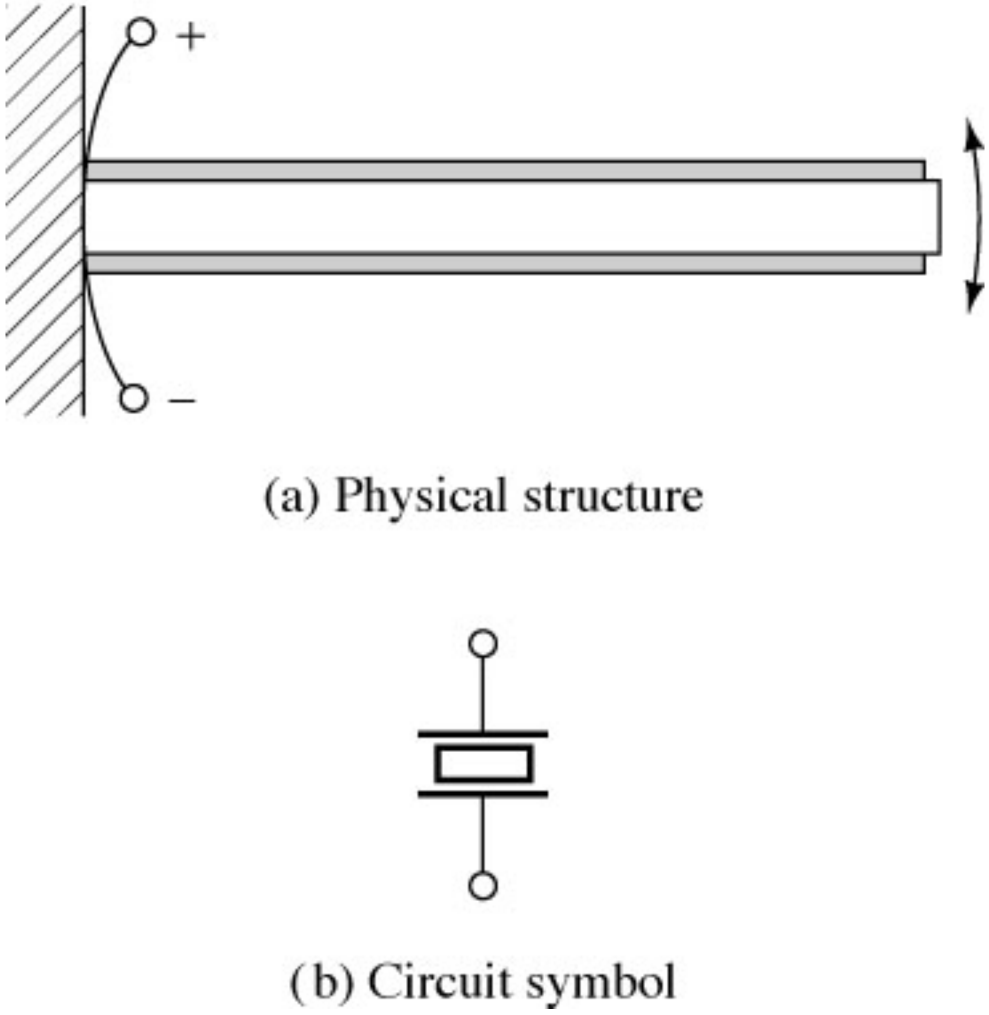


Figure 11.54 Crystal.

Piezoelectric effect: if we apply an electric field to the plates, forces on the ions in the lattice deform the material.

Piezoelectric effect is **reciprocal**: if deform the crystal, a voltage appears between the plates.

When used as frequency-determining element, quartz crystal vibrates freely at the desired frequency. The mechanical vibrations result in an ac current in the external circuit.

The Equivalent Circuit of the Crystal

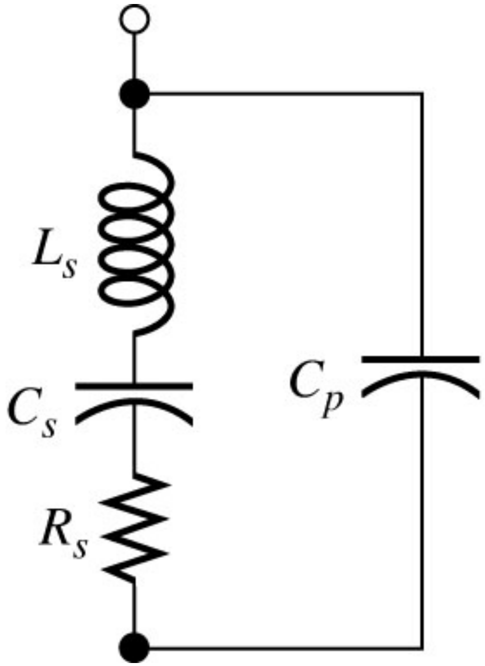


Figure 11.56 Equivalent circuit for a crystal.

Table 11.2 Parameters of a typical 10-MHz Crystal.

R_s	15 Ω
C_s	25×10^{-15} F
L_s	10.132118 mH
C_p	6×10^{-12} F
f_s	10.00000 MHz
f_p	10.02100 MHz
Q	42440

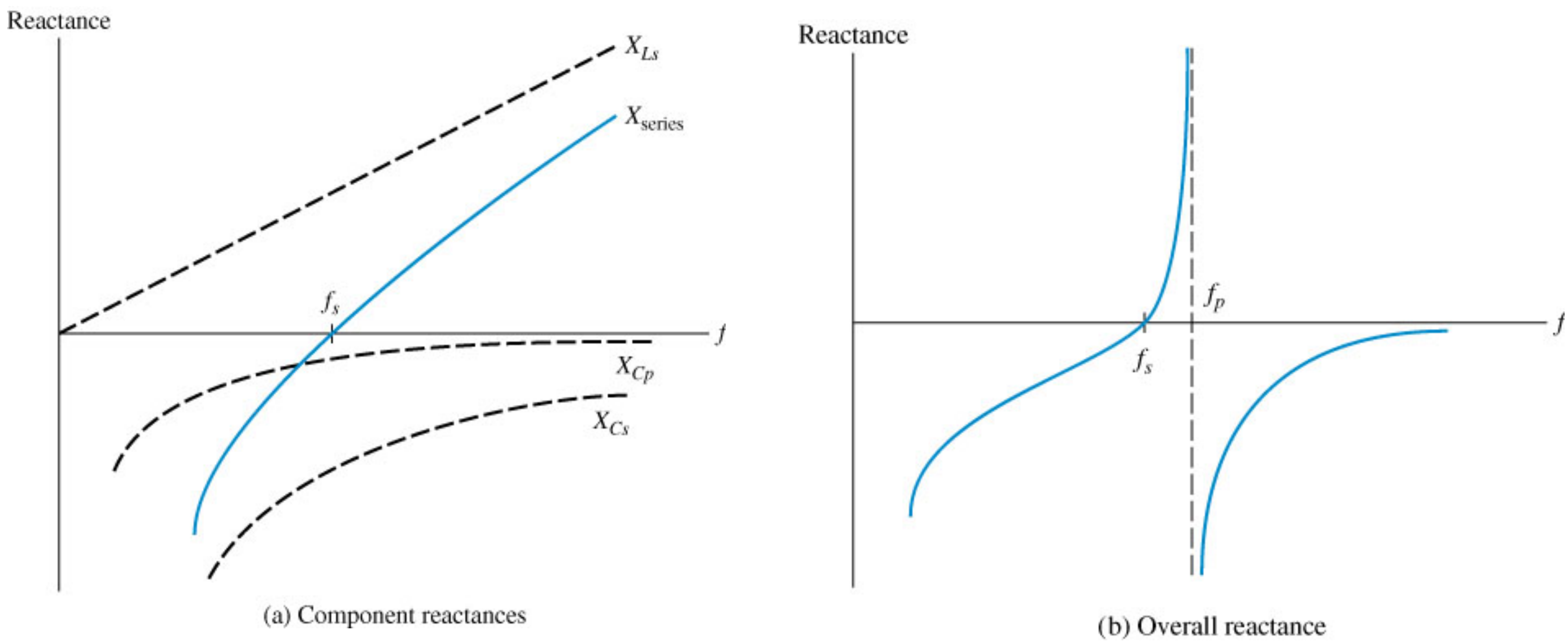


Figure 11.57 Crystal reactance versus frequency.

Crystal Oscillator Circuits

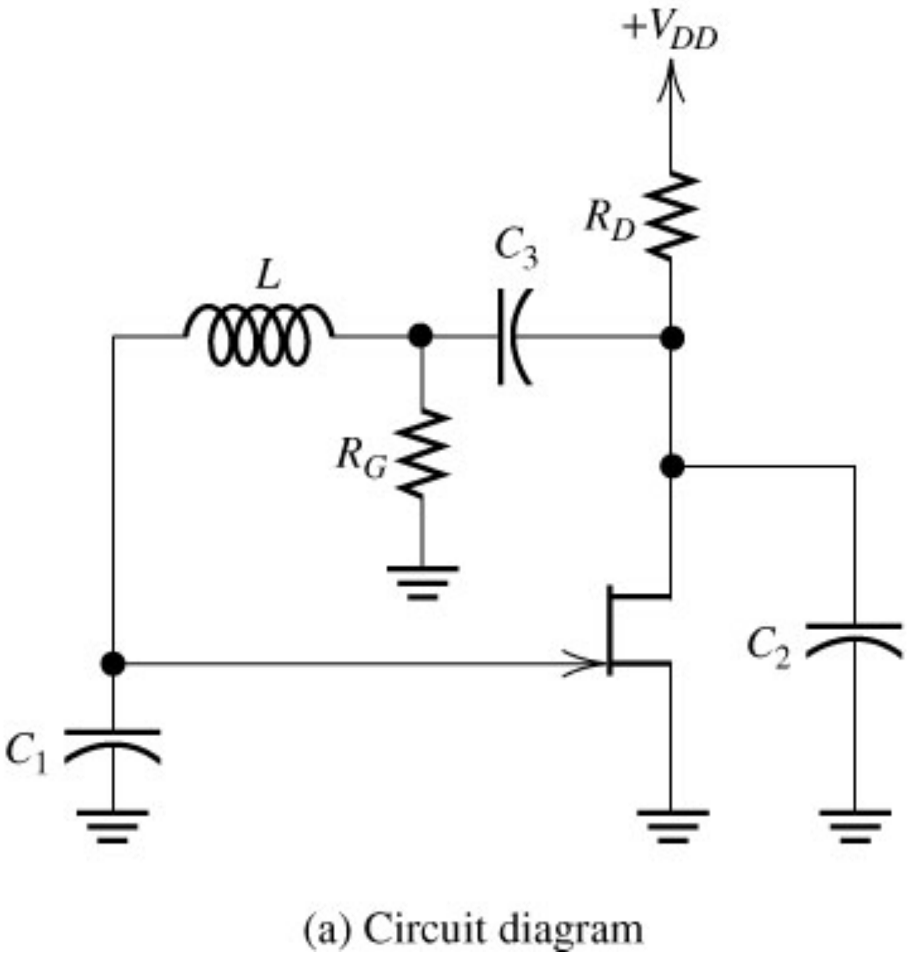


Figure 11.53a Colpitts oscillator.

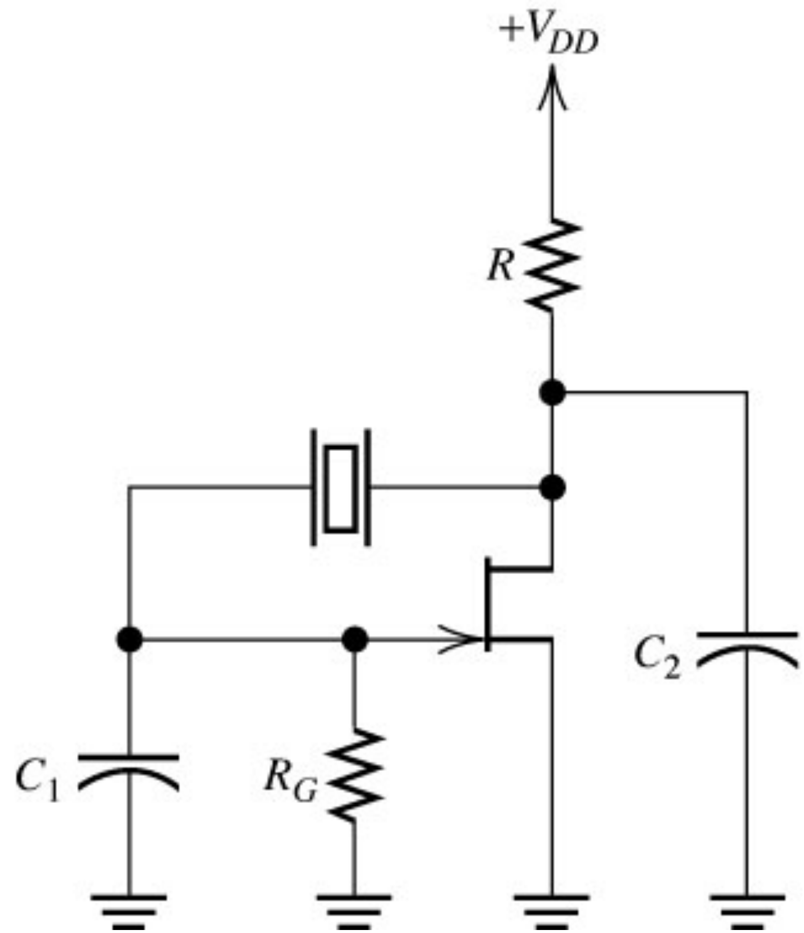


Figure 11.58 Pierce oscillator results from the Colpitts oscillator (Figure 11.53) if the inductor is replaced by a crystal. The dc blocking capacitor C_3 shown in Figure 11.53 can be omitted because the crystal is an open circuit for dc.