9.2.1

$$\Delta\tau_{13} = \Delta\tau_{12} + \Delta\tau_{23}$$

$$\Delta\tau^2 = \Delta s^2 = \left(1-\frac{2M}{r}\right)\Delta t^2 - \frac{\Delta r^2}{1-\frac{2M}{r}} - r^2\Delta\varphi^2$$

$$\Delta\tau_{13} = \sqrt{\left(1-\frac{2M}{r_A}\right)\Delta t_{12}^2 - \frac{\Delta r_{12}^2}{1-\frac{2M}{r_A}} - r_A^2\Delta\varphi_{12}^2} + \sqrt{\left(1-\frac{2M}{r_B}\right)\Delta t_{23}^2 - \frac{\Delta r_{23}^2}{1-\frac{2M}{r_B}} - r_B^2\Delta\varphi_{23}^2}$$

$\blacksquare$

9.2.2

Principle of maximum aging gives that $\Delta\tau^2$ should be at a maximum.

To find this I let

$$\frac{d\Delta\tau}{d\Delta t} = 0$$

$$\frac{d\Delta\tau_{13}}{d\Delta t_{12}} = \frac{d\Delta\tau_{12}}{d\Delta t_{12}} + \frac{d\Delta\tau_{23}}{d\Delta t_{12}}$$

$$\frac{d\,\Delta\tau_{12}}{d\,\Delta t_{12}} = \frac{1}{2}\frac{1}{\Delta\tau_{12}}\left(-2 r_A^2 \Delta\varphi_{12}\right)$$

$$= -\frac{r_A^2 \Delta\varphi_{12}}{\Delta\tau_{12}}$$

$$\frac{d\,\Delta\tau_{23}}{d\,\Delta t_{12}} = \frac{1}{2}\frac{1}{\Delta\tau_{23}}\left(-2 r_B^2 \Delta\varphi_{23}\cdot(-1)\right)$$

$$= \frac{r_B^2 \Delta\varphi_{23}}{\Delta\tau_{23}}.$$

Here I used that $\Delta\varphi_{23} = \Delta\varphi_{13} - \Delta\varphi_{12}$

$$\frac{d\,\Delta\tau_{13}}{d\,\Delta t_{12}} = -\frac{r_A^2 \Delta\varphi_{12}}{\Delta\tau_{12}} + \frac{r_B^2 \Delta\varphi_{23}}{\Delta\tau_{23}} = 0$$

$$\frac{r_A^2 \Delta\varphi_{12}}{\Delta\tau_{12}} = \frac{r_B^2 \Delta\varphi_{23}}{\Delta\tau_{23}} \quad \blacksquare$$

We see that the quantity

$$\frac{r^2 \Delta\varphi}{\Delta\tau}$$

Does not change from interval Δt_{12} to Δt_{23}. We can thus conclude that it is always conserved.

9.2.3

$$\Delta\tau = \Delta\tau_{sn} = \sqrt{\Delta t_{sn}^2 - \Delta x_{sn}^2} = \sqrt{\frac{\Delta t_{sn}^2 \Delta t_{sn}^2}{\Delta t_{sn}^2} - \frac{\Delta x_{sn}^2 \Delta t_{sn}^2}{\Delta t_{sn}^2}}$$

$$= \sqrt{\Delta t_{sn}^2 \left(1 - \frac{\Delta x_{sn}^2}{\Delta t_{sn}^2}\right)}$$

$$= \Delta t_{sn} \sqrt{1 - v_{sn}^2}$$

$$= \frac{\Delta t_{sn}}{\gamma_{sn}}.$$

$$\frac{r^2 \Delta\varphi}{\Delta\tau} = \gamma_{sn} \frac{r^2 \Delta\varphi}{\Delta t_{sn}}$$

$$\frac{r\,\Delta\varphi}{\Delta t_{sn}} = v_{\varphi, sn}$$

$$\frac{r^2 \Delta\varphi}{\Delta\tau} = \gamma_{sn}\, r\, v_{\varphi, sn} \quad \blacksquare$$

9.2.4

For $1 \gg V_{Sh}$, $\gamma_{Sh} = \frac{1}{\sqrt{1 - V_{Sh}^2}}$

goes towards 1 and we get

$$\gamma_{Sh} \, r \, V_{\varphi,Sh} = r \, V_{\varphi,Sh}$$

$$r \, V_{\varphi,Sh} = \underline{r} \times \underline{V}_{Sh}$$

Clasicall spin is defined as

$$L = \underline{r} \times \underline{P} \Rightarrow \frac{L}{m} = \underline{r} \times \underline{V}.$$

We have thus shown that the conserved quantity is spin pr. mass.