Ex. 3 p. 4 page 1

8.3.4.

1. $\Delta x_{BD} = L_0$ $\Delta t_{BD} = \frac{L_0}{V}$

$\Delta x''_{BD} = 0$ $\Delta t''_{BD} = t''_B$

$$\Delta s_{BD}^2 = \Delta s''^2_{BD}$$

$$\left(\frac{L_0}{V}\right)^2 - L_0^2 = \left(t''_B\right)^2$$

$$t''_B = L_0 \sqrt{\frac{1}{V^2} - 1} = \frac{L_0}{V\gamma}$$

This is the same as t'_B.

2. We see the same thing when looking at the video; both spaceships reach destiny when their clocks show ~28,5 yrs.

3. $\Delta x_{DB''} = 2L_0 - 0 = 2L_0$

$\Delta t_{DB''} = t_{B''} - 0 = t_{B''}$

$\Delta x''_{DB''} = x''_{B''} - 0 = \dfrac{L_0}{\gamma}$

$\Delta t''_{DB''} = t''_{B''} - 0 = t''_{B} = \dfrac{L_0}{\gamma v}$

4. $\Delta s^2_{DB''} = \Delta s''^2_{DB''}$

$$t_{B''}^2 - (2L_0)^2 = \left(\frac{L_0}{\gamma v}\right)^2 - \left(\frac{L_0}{\gamma}\right)^2$$

$$t_{B''}^2 = L_0^2\left(\frac{1-v^2}{(\gamma v)^2} + 4\right)$$

$$t_{B''}^2 = L_0^2\left(\frac{(1-v^2)^2}{v^2} + 4\right)$$

$$t_{B''}^2 = L_0^2\left(\frac{1}{v^2} + 2 + v^2\right)$$

$$t_{B''}^2 = \frac{L_0^2}{v^2}\left(1 + 2v^2 + v^4\right) = \left(\frac{L_0}{v}\right)^2(1+v^2)^2$$

$$t_{B''} = \frac{L_0}{v}(1+v^2) = \underline{\frac{L_0}{v} + L_0 v}$$

$t_{B''} \approx 400$ yrs. This is logical, since in the incoming-frame they calculate the return-time as measured on Homey to be 4 yrs, giving a total of 404 yrs, as expected. Again it is the case that B and B'' are not simultaneous in the planet frame.

8.3.5

1. $$\Delta S^2_{BB'} = \Delta S'^2_{BB'}$$

$$t_B = \frac{L_0}{v} \quad x_B = L_0 \quad t'_B = \frac{L_0}{\gamma v} \quad x'_B = 0$$

$$t_{B'} = t_{B'} \quad x_{B'} = 0 \quad t'_{B'} = t'_B = \frac{L_0}{\gamma v} \quad x'_{B'} = -\frac{L_0}{\gamma}$$

$$\left(\frac{L_0}{v} - t_{B'}\right)^2 - L_0^2 = \left(\frac{L_0}{\gamma v} - \frac{L_0}{\gamma v}\right)^2 - \left(\frac{L_0}{\gamma}\right)^2$$

$$\left(\frac{L_0}{v}\right)^2 - \frac{2L_0 t_{B'}}{v} + t_{B'}^2 - L_0^2 = -\left(\frac{L_0}{\gamma}\right)^2$$

$$t_{B'}^2 - \frac{2L_0}{v} t_{B'} + \left(\frac{L_0}{v}\right)^2 + \left(\frac{L_0}{\gamma}\right)^2 - L_0^2 = 0$$

$$t_{B'} = \frac{\frac{2L_0}{v} \pm \sqrt{\frac{4L_0^2}{v^2} - 4\left(\frac{L_0^2}{v^2} + \frac{L_0^2}{\gamma^2} - L_0^2\right)}}{2} =$$

$$\frac{L_0}{v} \pm L_0\sqrt{1 - \frac{1}{\gamma^2}} = \frac{L_0}{v} \pm L_0\sqrt{1-(1-v^2)} =$$

$$\underline{\frac{L_0}{v} \pm L_0 v}$$ Must be $\frac{L_0}{v} - L_0 v$.

$$\frac{L_0}{v} - L_0 v = \frac{L_0 - L_0 v^2}{v} = \frac{L_0(1-v^2)}{v} =$$

$$\underline{\underline{\frac{L_0}{v\gamma^2}}}$$

Ex. 3p5 page 2

2. $$v(t) = \begin{cases} v_0 & \text{for } 0 < t < \frac{L_0}{v_0} \\ v_0 + (t - t_B)g & \text{for } t \geq \frac{L_0}{v_0} \end{cases}$$

$v(t) = 0$ gives

$$v_0 + (t - t_B)g = 0$$

$$tg = t_B g - v_0$$

$$\underline{t = t_B - \frac{v_0}{g}}$$

3. If the acceleration is constant, the speed will return to the same value, v_0, but in the opposite direction, so the velocity is $-v_0$.

4. To find the position x_y we use the equation of motion with constant acceleration g from time $t = t_B$. Then the position is $x_0 = L_0$. The time from t_B is $\Delta t = (t - t_B)^2$. This gives

$$x_Y = L_0 + v_0(t - t_B) + \frac{1}{2}g(t-t_B)^2.$$

$$t_Y = t_Y$$

$$x'_Y = 0$$

$$t'_Y = t'_Y$$

$$x_{Y'} = 0 \qquad t_{Y'} = t_{Y'}$$

To find $x'_{Y'}$ we use length contraction at time t_Y, which gives

$$\gamma(t_Y) = \frac{1}{\sqrt{1 - v(t_Y)^2}}, \text{ and}$$

$$x'_{Y'} = -\frac{x_Y}{\gamma(t_Y)} \qquad t'_{Y'} = t'_Y, \text{ since}$$

Y' and Y are simultaneous'.

5. $$\Delta s_{YY'}^2 = \Delta s'^2_{YY'}$$

$$(t_Y - t_{Y'})^2 - x_Y^2 = (t'_Y - t'_Y)^2 - \frac{x_Y^2}{\gamma(t_Y)^2}$$

$$(t_Y - t_{Y'})^2 = x_Y^2 - \frac{x_Y^2}{\gamma(t_Y)^2}$$

$$(t_y - t_y')^2 = x_y^2 - x_y^2(1 - v(t_y)^2)$$

$$(t_y - t_y')^2 = x_y^2 v(t_y)^2$$

$$t_y - t_y' = \pm x_y v(t_y)$$

t_y' must be smaller than t_y, so the positive sign is correct.

$$\underline{t_y - t_y' = x_y v(t_y)}$$

$$t_y - t_y' = \left(L_0 + v_0(t_y - t_B) + \tfrac{1}{2}g(t_y - t_B)^2\right)\cdot\left(v_0 + (t_y - t_B)g\right)$$

$$t_y - t_B = \Delta t_y$$

$$t_y - t_y' = \left(L_0 + v_0 \Delta t_y + \tfrac{1}{2}g\,\Delta t_y^2\right)\left(v_0 + \Delta t_y g\right)$$

$$t_y - t_y' = L_0 v_0 + \Delta t_y(gL_0 + v_0^2) + \Delta t_y^2 \tfrac{3}{2} v_0 g + \Delta t_y^3 \tfrac{1}{2} g^2$$

6. $g = -0{,}1\,\frac{m}{s^2} = -\frac{0{,}1}{c}\frac{s}{s^2} = \underline{-\frac{0{,}1}{c}\frac{1}{s}}$

convert to yrs

7. $t_{tp} = t_B - \frac{v_0}{g} = t_B + \frac{c v_0}{0{,}1} = t_B + 10 c v_0 \approx \underline{296_{yrs}}$

8. When $t_y = t_{tp}$, $v(t_y) = 0$. That gives

$$\underline{\underline{t_y = t_{tp} = t_y'}}$$

10. For $t_Y < t_B$, the formula for $t_{Y'}$ is the same as the one for $t_{B'}$, but with $L_0 = v_0 t_Y$. This gives

$$t_{Y'} = \frac{t_Y}{\gamma^2} \quad \text{for} \quad 0 \leq t_Y < t_B$$

For $t_Y \geq t_B$, we use the formula we found in 5.

12. Since we restarted and synchronized the clocks, we can use time dilation. After a time T, Lisa's velocity is $v = Tg$. Time dilation gives

$$\Delta T' = \frac{\Delta T}{\gamma} = \frac{\Delta T}{\frac{1}{\sqrt{1-(Tg)^2}}} =$$

$$\underline{\sqrt{1-g^2T^2}\,\Delta T}$$

13. $T' = \int_0^{T_{dest}} \frac{\Delta T'}{\Delta T}\, dT$. The time T_{dest} when the spaceship reaches Destiny

~~is~~ the same as the time when $v = v_0$. This gives

$T_{dest}\, g = v_0 \Leftrightarrow T_{dest} = \frac{v_0}{g}$. Then

$$T' = \int_0^{\frac{v_0}{g}} \frac{\Delta T'}{\Delta T}\, dT = \underline{\int_0^{\frac{v_0}{g}} \sqrt{1 - g^2 T^2}\, dT}$$

14. $T' = \frac{v_0\sqrt{1-v_0^2} + \arcsin(v_0)}{2g} \approx 74{,}5\,\text{yrs}$

15. To find the position of the space ship we use the equation of motion:

$r = r_0 + v_0 T + \frac{1}{2} g T^2$. Since r_0 and v_0 are both 0, we get

$$\underline{\underline{r = \frac{1}{2} g T^2}}$$

16. We solve the equation for T:

$$r = \frac{1}{2} g T^2 \Rightarrow \underline{\underline{T = \sqrt{\frac{2r}{g}}}}$$

17. Then $\Delta T' = \sqrt{1 - g^2 t^2}\, \Delta T =$

$$\sqrt{1 - g^2\left(\sqrt{\frac{2r}{g}}\right)^2}\, \Delta T = \underline{\underline{\sqrt{1 - 2gr}\, \Delta T}}$$

18. With $g = \frac{GM}{r^2}$ we get

$$\Delta T' = \sqrt{1 - 2\frac{GM}{r^2} r}\, \Delta T = \underline{\underline{\sqrt{1 - \frac{2GM}{r}}\, \Delta T}}$$

This is the same as the formula for time dilation in a gravitational field, showing that having an acceleration g is equivalent to being in a gr. field with

$$g = \frac{GM}{r^2}$$