8.3.1.

1 $s = vt$

$$t = \frac{s}{v} = \frac{200\,ly}{0{,}99c} = 202.(02)\ yr$$

$$t' = \frac{t}{\gamma} = t \cdot \sqrt{1-0{,}99^2} = 28.5\,yr$$

2. She travels the same distance at the same speed, so the times are the same.

8.3.2.

1 $t = 28{,}5\,yr$

$$t' = \frac{t}{\gamma} = 28{,}5 \cdot \sqrt{1-0{,}99^2} = 4{,}0\,yr$$

2. Again using symetry, the time elapsed is the same.

8.3.3

1. $L_0 = v t_B \Rightarrow \underline{t_B = \frac{L_0}{v}}$

2. $t'_B = \frac{t_B}{\gamma} = \underline{\frac{L_0\sqrt{1-v^2}}{v}}$

$L_0 = 200 \quad v = 0{,}99$

$t'_B = \frac{200\sqrt{1-0{,}99^2}}{0{,}99} = \underline{\underline{28{,}5\,\text{yr}}}$

3. Since B' and B are simultaneous in the outgoing frame, we have $t'_{B'} = t'_B$.

The distance from Lisa to B' in her frame $(x'_{B'})$ is the same as the distance to Homey, which is

$-L'_0 = -\frac{L_0}{\gamma} = -L_0\sqrt{1-v^2}$

We can now find expressions for the two spacetime intervals

$\Delta S'^2_{BB'}$ and $\Delta S^2_{BB'}$ between events B and B'. We have

$$\Delta S'^2_{BB'} = \Delta t'^2_{BB'} - \Delta x'^2_{BB'} = (t'_B - t'_{B'})^2$$

$$- (x'_B - x'_{B'})^2 = (t'_B - t'_B)^2 - \left(L_0\sqrt{1-v^2}\right)^2 =$$

$$0 - L_0^2(1-v^2) = \underline{-L_0^2(1-v^2)}$$

This must be the same as $\Delta S^2_{BB'}$.

$$\Delta S^2_{BB'} = \Delta t^2_{BB'} - \Delta x^2_{BB'} = (t_B - t_{B'})^2$$

$$- (x_B - x_{B'})^2 = \left(\frac{L_0}{v} - t_{B'}\right)^2 - (L_0)^2 =$$

$$\underline{\left(\frac{L_0}{v}\right)^2 - \frac{2L_0 t_{B'}}{v} + t_{B'}^2 - L_0^2}$$

$$-L_0^2(\cancel{1} - v^2) = \left(\frac{L_0}{v}\right)^2 - \frac{2L_0 t_{B'}}{v} + t_{B'}^2 - \cancel{L_0^2}$$

$$t_{B'}^2 - \frac{2L_0 t_{B'}}{v} + \left(\frac{L_0}{v}\right)^2 - (L_0 v)^2 = 0$$

$$t_{B'} = \frac{\frac{2L_0}{v} \pm \sqrt{4\left(\frac{L_0}{v}\right)^2 + 4(L_0 v)^2 - 4\left(\frac{L_0}{v}\right)^2}}{2} =$$

$$= \frac{\frac{2L_0}{v} \pm \sqrt{4(L_0 v)^2}}{2} = \frac{L_0}{v} \pm L_0 v$$

We have two options for $t_{B'}$:

$t_{B'} = \frac{L_0}{v} + vL_0$ or $t_{B'} = \frac{L_0}{v} - vL_0$

$\frac{L_0}{v}$ is t_B. We now have a similar situation to the one in exercise 1, where two events, in this case B and B', are simultaneous in a marked frame, but not an unmarked. By the same reasoning we used there, we find that B' must happen before B in the planet frame.

Therefore $t_{B'} < t_B$, so

$$\underline{\underline{t_{B'} = \frac{L_0}{v} - vL_0}} \blacksquare$$

With numbers this gives $t_{B'} = 202 - 198 = \underline{4}$