8.6.

2. $P'_\mu(p) = m_p V'_\mu(p) = m_p \gamma'_p (1, v'_p) =$

$$\frac{m_p}{\sqrt{1-(v'_p)^2}} (1, v'_p)$$

4. Conservation of momenergy gives

$P'_\mu(n) = P'_\mu(p) + P'_\mu(e)$. Inserting

our expressions we get

$$(m_n, 0) = m_p \gamma'_p (1, v'_p) + m_e \gamma'_e (1, v'_e)$$

This gives

I $\quad m_n = m_p \gamma'_p + m_e \gamma'_e$ and

II $\quad m_p \gamma'_p v'_p + m_e \gamma'_e v'_e = 0$

using $v'_p = \sqrt{1 - \frac{1}{(\gamma'_p)^2}}$ and

$V'_e = \sqrt{1 - \frac{1}{(\gamma'_e)^2}}$ we get

$$m_p \gamma'_p \sqrt{1-\frac{1}{(\gamma'_p)^2}} + m_e \gamma'_e \sqrt{1-\frac{1}{(\gamma'_e)^2}} = 0$$

We get an expession for γ'_p from I:

$$\gamma'_p = \frac{m_n - m_e \gamma'_e}{m_p}$$

Iserting into II:

$$(m_n - m_e \gamma'_e)\sqrt{1-\frac{m_p^2}{(m_n - m_e\gamma'_e)^2}} + m_e \gamma'_e \sqrt{1-\frac{1}{(\gamma'_e)^2}} = 0$$

$$\sqrt{(m_n - m_e\gamma'_e)^2 - m_p^2} = -m_e\gamma'_e\sqrt{1-\frac{1}{(\gamma'_e)^2}}$$

$$(m_n - m_e\gamma'_e)^2 - m_p^2 = m_e^2(\gamma'_e)^2\left(1-\frac{1}{(\gamma'_e)^2}\right)$$

$$(m_n - m_e\gamma'_e)^2 - m_p^2 = m_e^2(\gamma'_e)^2 - m_e^2$$

$$m_n^2 - 2m_n m_e \gamma'_e + \cancel{m_e^2(\gamma'_e)^2} - m_p^2 = \cancel{m_e^2(\gamma'_e)^2} - m_e^2$$

$$m_n^2 + m_e^2 - m_p^2 = 2 m_n m_e \gamma'_e$$

$$\underline{\gamma'_e = \frac{m_n^2 + m_e^2 - m_p^2}{2 m_n m_e}}$$

$$\gamma'_p = \frac{m_n - m_e\gamma'_e}{m_p} = \frac{m_n}{m_p} - \frac{m_n^2 + m_e^2 - m_p^2}{2 m_n m_p} =$$

$$\frac{2m_n^2 - m_n^2 - m_e^2 + m_p^2}{2 m_n m_p} = \underline{\frac{m_n^2 + m_p^2 - m_e^2}{2 m_n m_p}}$$

Iserting the numerical values for the masses, we get

$$\gamma'_e = \frac{m_n^2 + m_e^2 - m_p^2}{2 m_n m_e} \approx 2{,}529865$$

This gives

$$v'_e = \pm\sqrt{1 - \frac{1}{2{,}529865^2}} \approx \pm 0{,}91856c$$

$$\gamma'_p = \frac{m_n^2 + m_p^2 - m_e^2}{2 m_n m_p} \approx 1{,}0000008$$

$$v'_p = \pm\sqrt{1 - \frac{1}{1{,}0000008^2}} = \pm\, 0{,}00127c$$

We see that v'_e and v'_p have two possible signs. For conservation of momentum to hold, these need to be opposite. We will choose v'_e to be positive and v'_p to be negative.

Ex 6 page 4

5. $p'_\mu = c_{\mu\nu} p_\nu$

$$\begin{pmatrix} E' \\ p'_x \\ p'_y \\ p'_z \end{pmatrix} = \begin{pmatrix} \gamma_{rel} & -v_{rel}\gamma_{rel} & 0 & 0 \\ -v_{rel}\gamma_{rel} & \gamma_{rel} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} E \\ p_x \\ p_y \\ p_z \end{pmatrix}$$

$$\boxed{\begin{aligned} E' &= \gamma_{rel} E - v_{rel}\gamma_{rel} p_x \\ p'_x &= \gamma_{rel} p_x - v_{rel}\gamma_{rel} E \end{aligned}}$$

For the electron we have

$$E'(e) = \gamma_{rel} E(e) - v_{rel}\gamma_{rel} p_x(e)$$

$$p'_x(e) = \gamma_{rel} p_x(e) - v_{rel}\gamma_{rel} E(e)$$

$$p_x(e) = \frac{\gamma_{rel} E(e) - E'(e)}{v_{rel}\gamma_{rel}}$$

$$p'_x(e) = \frac{\gamma_{rel} E(e) - E'(e)}{v_{rel}} - v_{rel}\gamma_{rel} E(e)$$

$$v_{rel} p'_x(e) + E'(e) = E(e)\gamma_{rel}(1 - v_{rel}^2)$$

$$\underline{E(e) = \gamma_{rel} v_{rel} p'_x(e) + \gamma_{rel} E'(e)}$$

$$p_x(e) = \frac{\gamma_{rel}^2 (v_{rel}\, p'_x(e) + E'(e)) - E'(e)}{v_{rel}\, \gamma_{rel}}$$

$$p'_x(e) = \gamma'_e\, m_e\, v'_e$$

$$E'(e) = \gamma'_e\, m_e$$

Inserting all values, we get

$$E(e) \approx 7{,}70804 \cdot 10^{-30} \text{ kg}$$

$$\underline{p_x(e) \approx 7{,}65403 \cdot 10^{-30} \text{ kg}}$$

For the proton, we also get

$$E(p) = \gamma_{rel}\, v_{rel}\, p'_x(p) + \gamma_{rel}\, E'(p)$$

$$p_x(p) = \frac{\gamma_{rel}^2 (v_{rel}\, p'_x(p) + E'(p)) - E'(p)}{v_{rel}\, \gamma_{rel}}$$

$$E(p) \approx 3{,}14306 \cdot 10^{-27} \text{ kg}$$

$$\underline{p_x(p) \approx 2{,}66105 \cdot 10^{-27} \text{ kg}}$$

Ex6 page6

6. $E(e) = \gamma_e m_e$

$$\frac{1}{\sqrt{1-v_e^2}} = \frac{E(e)}{m_e}$$

$$\frac{m_e^2}{E(e)^2} = 1 - v_e^2$$

$$v_e = \sqrt{1-\left(\frac{m_e}{E(e)}\right)^2} \approx \underline{\underline{0{,}9930\,c}}$$

$E(p) = \gamma_p m_p$

$$v_p = \sqrt{1-\left(\frac{m_p}{E(p)}\right)^2} \approx \underline{\underline{0{,}8466\,c}}$$

7. Assume $m_n = m_e + m_p$.

$$\gamma_p' = \frac{(m_e+m_p)^2 + m_p^2 - m_e^2}{2m_p(m_e+m_p)} = \frac{2m_e m_p + 2m_p^2}{2m_p(m_e+m_p)} =$$

$$\frac{m_e+m_p}{m_e+m_p} = \underline{1}$$ When $\gamma_p' = 1$,

v_p' must be 0. By conservation of momentum, v_e' must also be 0. Then $\gamma_e' = \gamma_p' = 1$, so energy must also be conserved

$\left(E'(n) = \gamma_n' m_n = m_n = m_e + m_p = \gamma_e' m_e + \gamma_p' m_p = E'(e) + E'(p) \right)$. Since energy is released in the reaction, this cannot be true.

8. $$v_x' = \frac{v_x - v_{rel}}{1 - v_{rel} v_x}$$

$$v_x' - v_{rel} v_x v_x' = v_x - v_{rel}$$

$$v_x' + v_{rel} = v_x (1 + v_{rel} v_x')$$

$$v_x = \frac{v_x' + v_{rel}}{1 + v_{rel} v_x'}$$

$$v_x(e) = \frac{v_x'(e) + v_{rel}}{1 + v_{rel} v_x'(e)} \approx \frac{0{,}91856 + 0{,}847}{1 + 0{,}91856 \cdot 0{,}847}$$

$$\approx \underline{\underline{0{,}9930\,c}}$$

$$v_x(p) = \frac{v_x'(p) + v_{rel}}{1 + v_{rel} v_x'(p)} \approx \underline{\underline{0{,}8466\,c}}$$

<u>This is the same as we got earlier</u>