$$K_c(s) = K_p\left(1 + \frac{1}{T_i s} + T_d \frac{N}{1 + N\frac{1}{s}}\right)$$

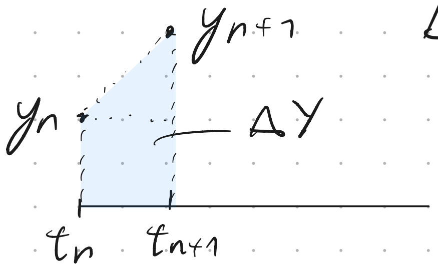
$$\frac{U(s)}{e(s)} = \underbrace{K_p}_{G_p(s)} + \underbrace{\frac{K_p}{T_i s}}_{G_i(s)} + \underbrace{K_p T_d \frac{N}{1 + N\frac{1}{s}}}_{G_d(s)}$$

$\rightarrow$ $U[n] = U_0[n] + U_p[n] + U_i[n] + U_d[n]$

$\boxed{U_p[n] = k_p\, e[n]}$ – Proporsjonaldel

- $U_i[n]$ utledning – Integraldel
  - Ønsker å bruke trapesmetoden for å approksimere integralet
  - kaller $Y[n]$ det diskrete integralet til $y[n]$

$$Y[n+1] = Y[n] + \Delta Y$$



$$\Delta Y = y_n(t_{n+1} - t_n) + \frac{(y_{n+1} - y_n)(t_{n+1} - t_n)}{2}$$

$$= y_n h + \frac{h y_{n+1} - h y_n}{2} \qquad (t_{n+1} - t_n) = h$$

$$= \frac{h}{2} y_{n+1} + \frac{h}{2} y_n$$

$$= \frac{h}{2}(y_n + y_{n+1})$$

notat : Her har jeg undersøkt approksimasjonen av et integral ved bruk av trapesmetoden. Vi ser senere at vi får samme resultat som vi får ved å diskretisere 1/s ved bruk av tusitins metode.

- Sjekke at dette stemmer overens med tustins metode

Utleder transferfunksjonen i z-domenet fra differensligningen vi fikk fra trapesmetoden:

$$y \longrightarrow \boxed{S} \longrightarrow Y$$

$$Y[n+1] = Y[n] + \Delta Y$$

$$= Y[n] + \frac{h}{2}\left(y[n] + y[n+1]\right)$$

$$\Rightarrow Y[n] = Y[n-1] + \frac{h}{2}\left(y[n] + y[n-1]\right)$$

$$\rightarrow Y(z) = z^{-1}Y(z) + \frac{h}{2}y(z) + \frac{h}{2}z^{-1}y(z)$$

$$\rightarrow Y(z)(1 - z^{-1}) = y(z)\frac{h}{2}\left(1 + z^{-1}\right)$$

$$\rightarrow \frac{Y(z)}{y(z)} = \frac{h}{2} \cdot \frac{1+z^{-1}}{1-z^{-1}}$$ ← Trapesmetoden

ekvivalent

tustins metode

$$\frac{Y(s)}{y(s)} = \frac{1}{s} \Rightarrow s = \frac{2}{h} \cdot \frac{1-z^{-1}}{1+z^{-1}}$$ OK

- Utlede $u_i[n]$ vha. tustins metode

$$\frac{U_i(s)}{e(s)} = \frac{K_p}{T_i s}, \quad s = \frac{2}{T_s} \cdot \frac{1-z^{-1}}{1+z^{-1}}$$

– Tar med K_p og T_i

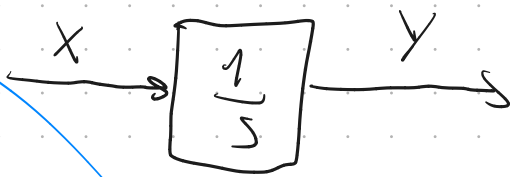
$$\rightarrow \frac{U_i(z)}{e(z)} = \frac{K_p}{T_i} \cdot \frac{T_s}{2} \cdot \frac{1+z^{-1}}{1-z^{-1}}$$

Differensligning for integraldel:

$$\rightarrow U_i(z)(1-z^{-1}) = \frac{K_p T_s}{2T_i}(1+z^{-1})e(z)$$

$$\rightarrow U_i[n] = U_i[n-1] + \frac{K_p T_s}{2T_i}\left(e[n]+e[n-1]\right)$$

- Generelt sett for en integrator:



$$Y[n] = Y[n-1] + \frac{T_s}{2}\left(X[n] + X[n-1]\right)$$

- Med tracking får vi:

$$X[n] = \frac{K_p}{T_i}e[n] + \frac{1}{T_t}\left(U_R[n] - U[n]\right)$$

$$\Rightarrow Y[n] = Y[n-1] + \frac{T_s}{2}\left[\frac{K_p}{T_i}e[n] + \frac{1}{T_t}\left(U_R[n]-U[n]\right) + \frac{K_p}{T_i}e[n-1] + \frac{1}{T_t}\left(U_R[n-1]-U[n-1]\right)\right]$$

$$= Y[n-1]$$

- Derivatdel

$$\frac{U_d(s)}{e(s)} = K_p T_d \frac{N}{1 + N\frac{1}{s}} = K_p T_d \frac{N s}{s + N}$$

$$= K_p T_d \frac{N}{s+N} s$$

– Merk: Ønsker å ha derivatoren før forsterkningen $K_p T_d$ for å unngå derivatspark ved parameterendring

$$\Rightarrow \quad e \rightarrow \boxed{\text{Filter}} \xrightarrow{e_f} \boxed{s} \xrightarrow{e_f'} \boxed{K_p T_d} \xrightarrow{U_d}$$

1) Filter:

$$H_f(s) = \frac{N}{s+N}$$

- Bruker bakoverdifferanse for å få et 'exponential moving average filter'

NB: Diskretiserer derivatfilteret og derivatdelen hver for seg

$$\Rightarrow s = \frac{1 - z^{-1}}{T_s}$$

$$\Rightarrow H_f(z) = \frac{e_f(z)}{e(z)} = \frac{NT_s}{1 - z^{-1} + NT_s}$$

$$\rightarrow e_f\left(1 + NT_s - z^{-1}\right) = NT_s e$$

$$\rightarrow e_f[n](1 + NT_s) - e_f[n-1] = NT_s e[n]$$

$$\rightarrow e_f[n] = \frac{1}{1 + NT_s}\left[e_f[n-1] + NT_s e[n]\right]$$

$$= \frac{NT_s}{NT_s + 1} e[n] + \frac{1}{NT_s + 1} e_f[n-1]$$

- Sier $\boxed{\beta = \frac{NT_s}{NT_s + 1}} \Rightarrow 1 - \beta = \frac{NT_s + 1}{NT_s + 1} - \frac{NT_s}{NT_s + 1} = \frac{1}{NT_s + 1}$

$$\Rightarrow \boxed{e_f[n] = \beta e[n] + (1 - \beta) e_f[n-1]}$$

Differensligning for derivatfilteret (et «exponential moving average filter»)

2) Derivatdel

- Bruker enkel bakoverdifferanse

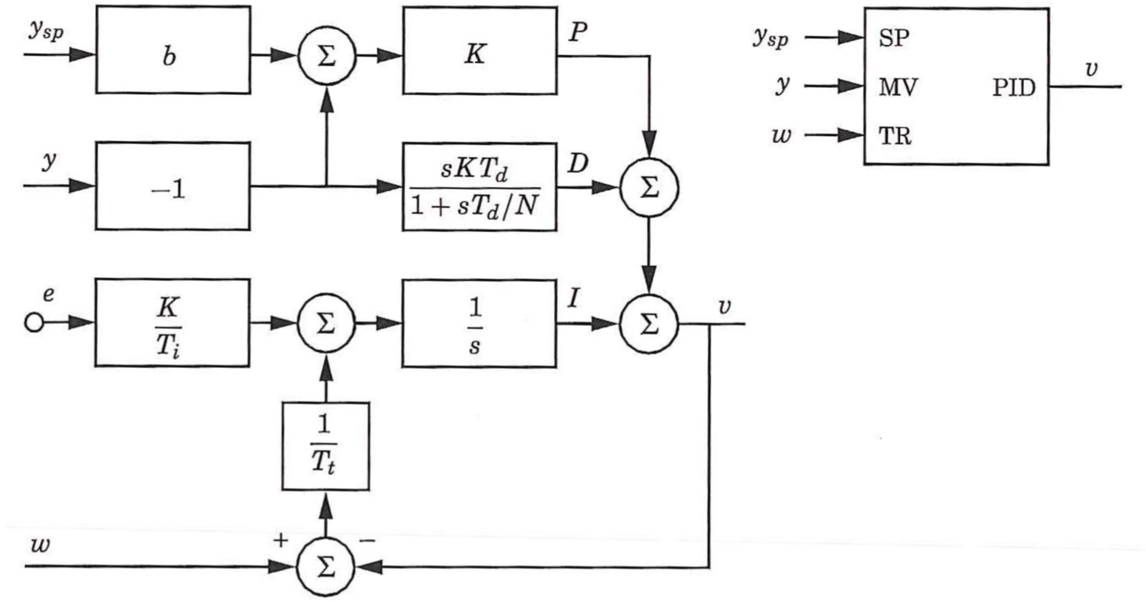
$$\Rightarrow s = \frac{1 - z^{-1}}{T_s}$$

$$\Rightarrow \frac{e_f'(s)}{e_f(s)} = s \Rightarrow e_f'(z) = \frac{1 - z^{-1}}{T_s} e_f(z)$$

$$\Rightarrow e_f'[n] = \frac{1}{T_s}\left(e_f[n] - e_f[n-1]\right)$$

Differensligning for derivatoren. Bruker bakoverdifferanse, som garanterer stabilitet.

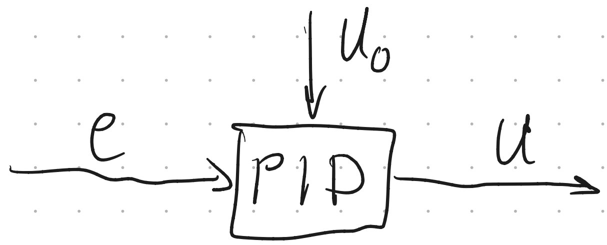
Integrator med tracking



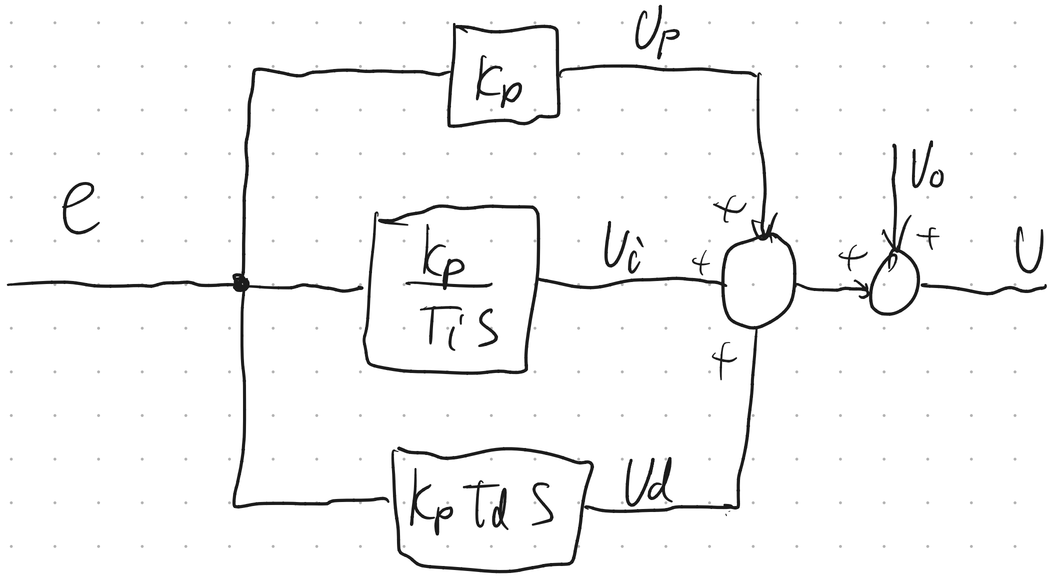
- Bruker Matlab PID-formen for å enkelt kunne deaktivere I og D leddene:

$$P\left(1+I\frac{1}{s}+D\frac{N}{1+N\frac{1}{s}}\right)$$

- Bruker også tracking-koeff. $K_t = \frac{1}{T_t}$ for å enkelt kunne aktivere/deaktivere tracking

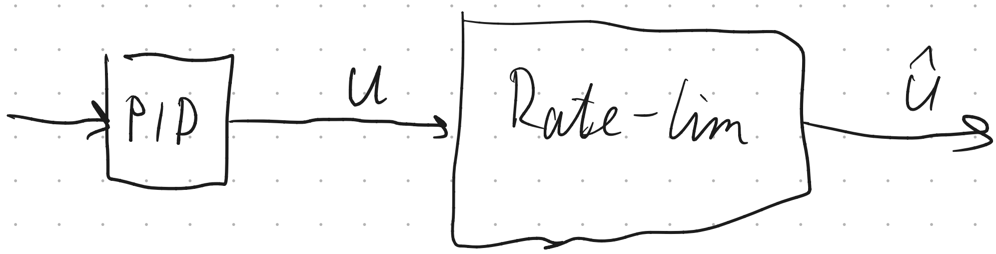


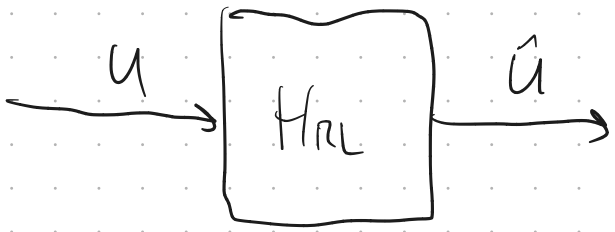
e
PID
u
u_0



e
k_p
u_p
$\frac{k_p}{T_i s}$
u_i
$k_p T_d s$
u_d
+
+
+
+
+
u_0
u

Ratebegrensning :



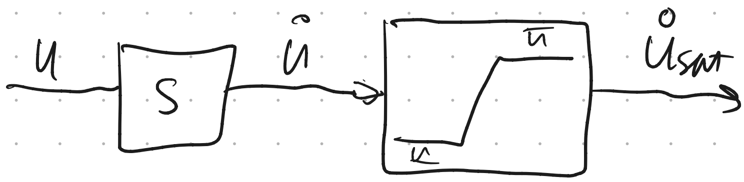


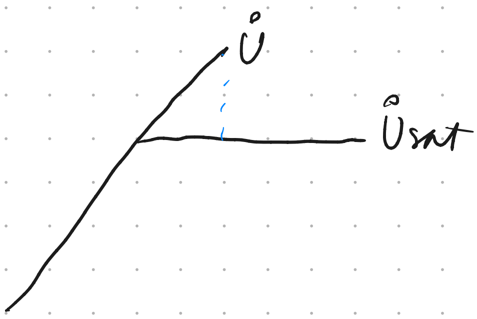
Dersom $\mathring{u}_{min} < \mathring{u} < \mathring{u}_{max} \Rightarrow \hat{u} = u$

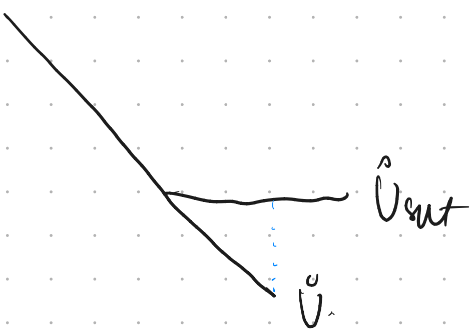
Dersom $\mathring{u} > \mathring{u}_{max}$

$$u_t = u_{t-1} + \mathring{u}_{max} h$$

$$\mathring{u} < \mathring{u}_{min} \Rightarrow u_t = u_{t-1} + \mathring{u}_{min} h$$







Bumpless transfer