Homework 2 - Univariate and Multivariate Function Approximations

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1 Exercise 1 - Univariate Approximation

1.1 Taylor series

Along Taylor series approximation, we observe along increasing order approximation better estimates for the approximation of the functionthat increasing order approximation leads to a larger error term with respect to the true function

$$f(x) = x^0.321$$

1.2 The Ramp Function

As seen in the lecture, the Taylor approximation fails to approximate the ramp at (0,0).

1.3 Exponential, Runge and Ramp Function

The exponential function shows a singularity at x=0 which doesn't allow further approximations. a)For spaced interpolation nodes, we observe along increasing order approximation bigger error terms. These oscillation are due to the equal weights of the interpolation nodes. b) Chebyshev approaches the described phenomena (Runge's phenomena, as it becomes highly obvious by interpolating the Runge's Function (1. Function in my Python Script). By using Chebychevs nodes more weight is given to the edges of the true function. c) By not only using Chebyshev nodes but also Chebyshev function, we observe

The exponential function shows a singularity which implies discontinuity at the zero value. The approximation does not work.

2 Exercise 2 - Multivariate Approximation

2.1 Show elasticity analytically

$$f(k,h) = ((1-\alpha) * k^{\frac{\sigma-1}{\sigma}} + \alpha * h^{\frac{\sigma-1}{\sigma}})^{\frac{\sigma}{\sigma-1}})$$

1.
$$\sigma = \frac{\delta ln(h/k)}{\delta ln(MRS)}$$

$$MRS = \frac{\delta f(k,h)/\delta k}{\delta f(k,h)/\delta h}$$

$$= \frac{1-\alpha}{\alpha} * (k/h)^{\frac{-1}{\sigma}}$$

$$\sigma = \frac{\delta ln(h/k)}{\delta ln\frac{1-\alpha}{\alpha} * (k/h)^{\frac{-1}{\sigma}}}$$

2.2 Show labour share analytically

By definition:

$$labour_share = \frac{h * w}{f(k,h)}$$

wage in competitive markets is:

$$w = \alpha * h^{\frac{1}{\sigma}})((1-\alpha) * k^{\frac{\sigma-1}{\sigma}} + \alpha * h^{\frac{\sigma-1}{\sigma}})^{\frac{1}{\sigma-1}})$$

$$\frac{h * w}{f(k,h)} = \frac{\alpha * h^{\frac{\sigma-1}{\sigma}}}{((1-\alpha) * k^{\frac{\sigma-1}{\sigma}} + \alpha * h^{\frac{\sigma-1}{\sigma}})^{\sigma}}$$

2.3 Chebyshev Approximation

in progress, to be updated by friday...