

Homework 2 - Univariate and Multivariate Function Approximations

Lillian Haas

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1 Exercise 1 - Univariate Approximation

1.1 Taylor series

Along Taylor series approximation, we observe along increasing order approximation better estimates for the approximation of the function that increasing order approximation leads to a larger error term with respect to the true function

$$f(x) = x^0.321$$

1.2 The Ramp Function

As seen in the lecture, the Taylor approximation fails to approximate the ramp at (0,0).

1.3 Exponential, Runge and Ramp Function

The exponential function shows a singularity at $x=0$ which doesn't allow further approximations. a) For spaced interpolation nodes, we observe along increasing order approximation bigger error terms. These oscillations are due to the equal weights of the interpolation nodes. b) Chebyshev approaches the described phenomena (Runge's phenomena, as it becomes highly obvious by interpolating the Runge's Function (1. Function in my Python Script). By using Chebyshev's nodes more weight is given to the edges of the true function. c) By not only using Chebyshev nodes but also Chebyshev function, we observe

The exponential function shows a singularity which implies discontinuity at the zero value. The approximation does not work.

2 Exercise 2 - Multivariate Approximation

2.1 Show elasticity analytically

$$f(k, h) = ((1 - \alpha) * k^{\frac{\sigma-1}{\sigma}} + \alpha * h^{\frac{\sigma-1}{\sigma}})^{\frac{\sigma}{\sigma-1}}$$

1.

$$\sigma = \frac{\delta \ln(h/k)}{\delta \ln(MRS)}$$

$$MRS = \frac{\delta f(k, h)/\delta k}{\delta f(k, h)/\delta h}$$

$$= \frac{1-\alpha}{\alpha} * (k/h)^{\frac{-1}{\sigma}}$$

$$\sigma = \frac{\delta \ln(h/k)}{\delta \ln \frac{1-\alpha}{\alpha} * (k/h)^{\frac{-1}{\sigma}}}$$

2.2 Show labour share analytically

By definition:

$$labour_share = \frac{h * w}{f(k, h)}$$

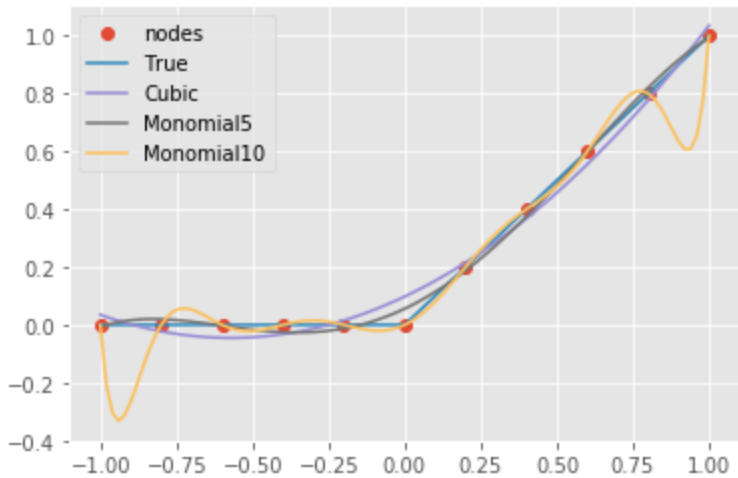
wage in competitive markets is:

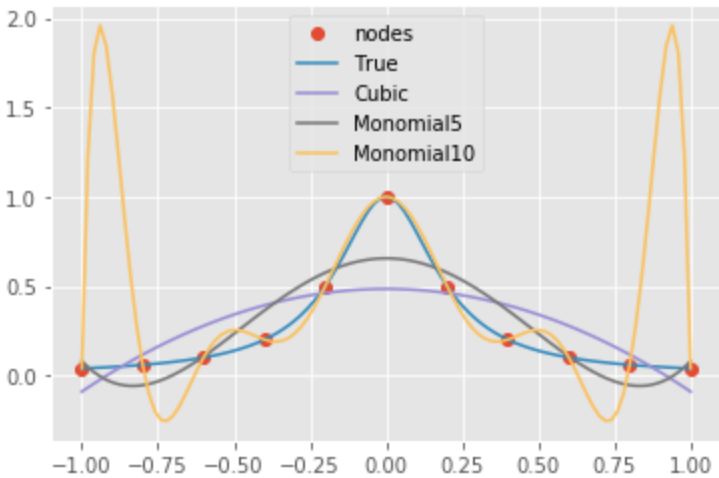
$$w = \alpha * h^{\frac{1}{\sigma}} ((1-\alpha) * k^{\frac{\sigma-1}{\sigma}} + \alpha * h^{\frac{\sigma-1}{\sigma}})^{\frac{1}{\sigma-1}}$$

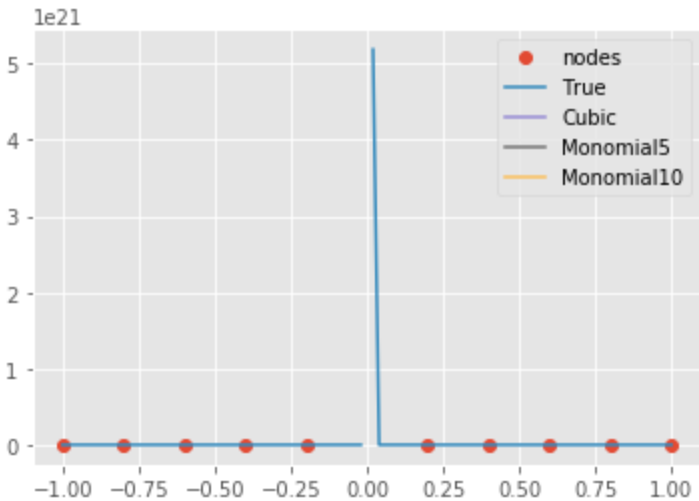
$$\frac{h * w}{f(k, h)} = \frac{\alpha * h^{\frac{\sigma-1}{\sigma}}}{((1-\alpha) * k^{\frac{\sigma-1}{\sigma}} + \alpha * h^{\frac{\sigma-1}{\sigma}})^{\sigma}}$$

2.3 Chebyshev Approximation

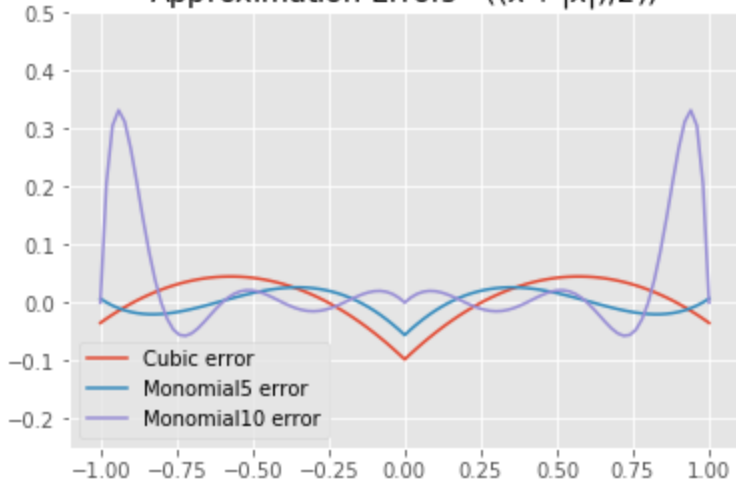
in progress, to be updated by friday...



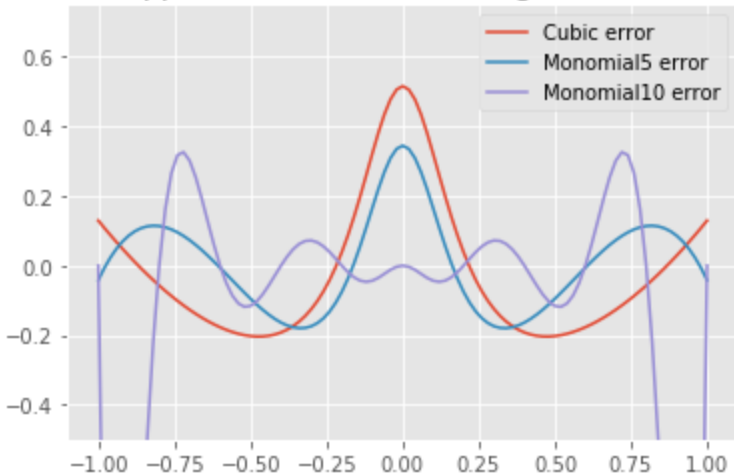




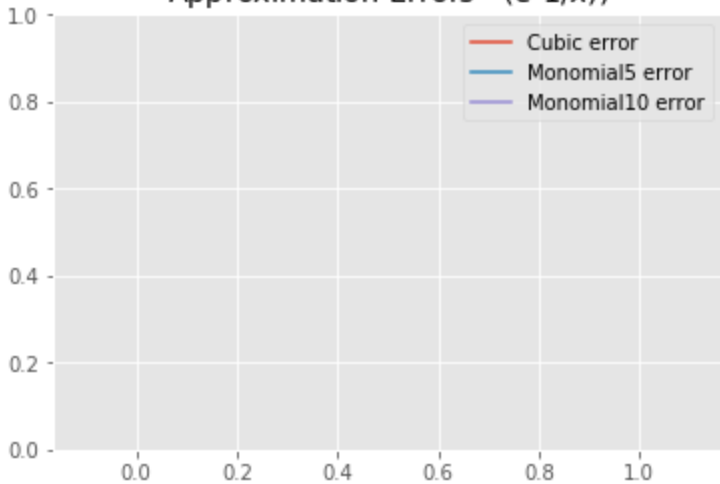
Approximation Errors - $((x + |x|)/2)$



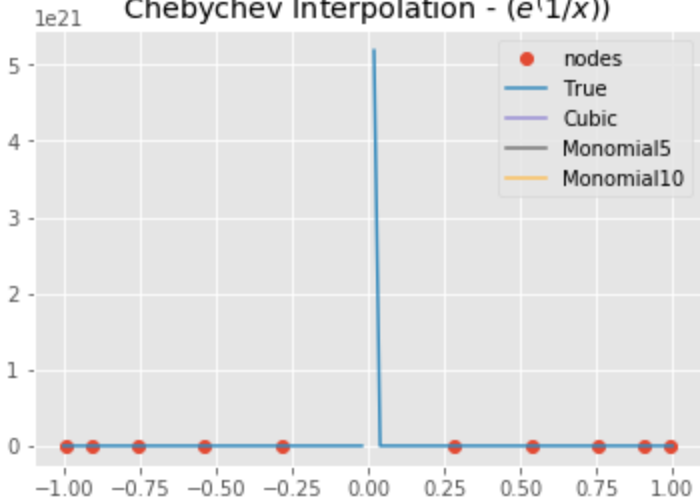
Approximation Errors - Runge Function



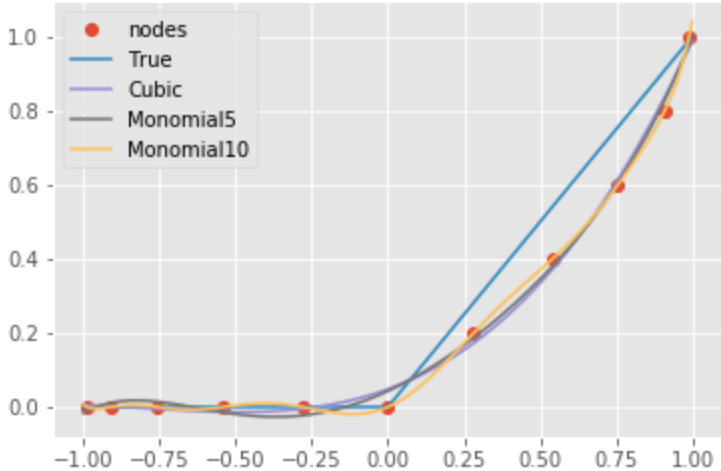
Approximation Errors - ($e^{(1/x)}$)



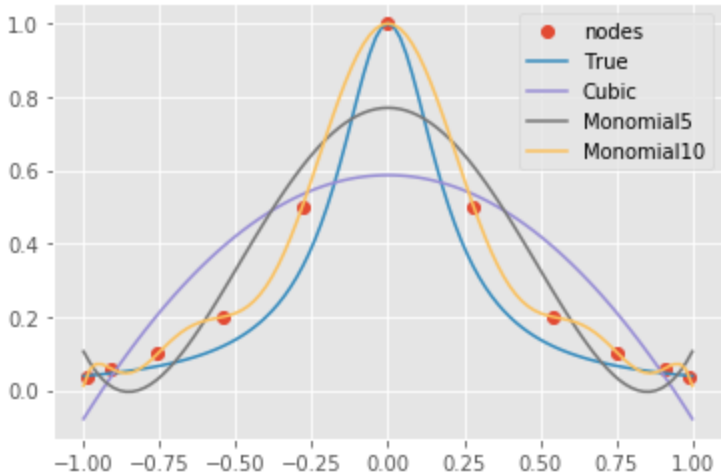
Chebyshev Interpolation - ($e^{1/x}$)



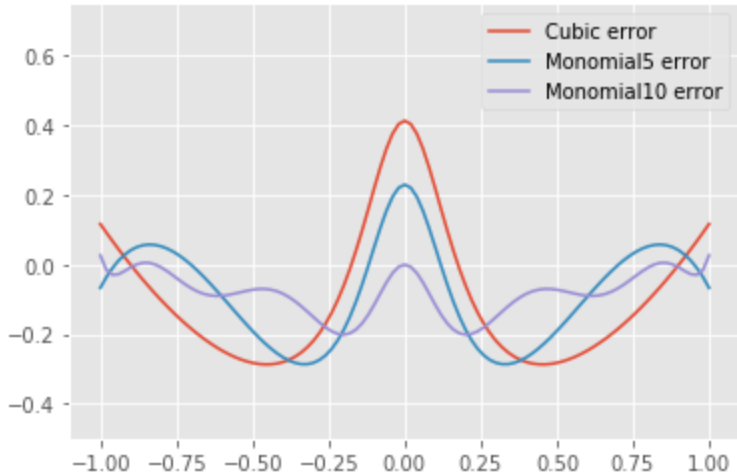
Chebyshev Interpolation - $(x + |x|)/2$



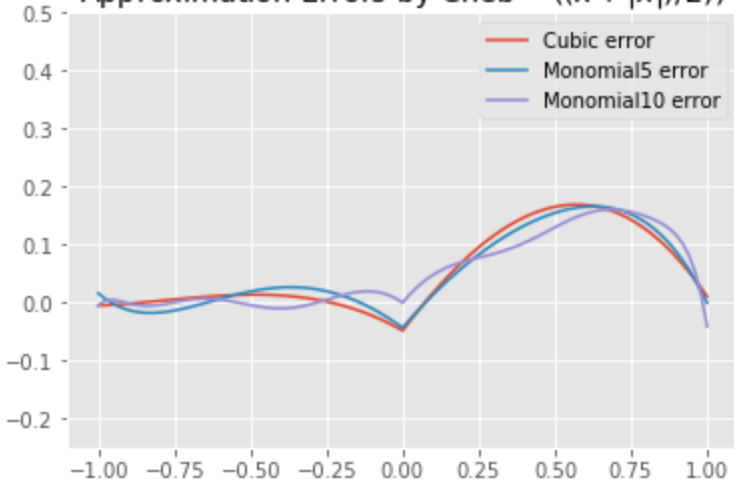
Chebyshev Interpolation - Ramp function



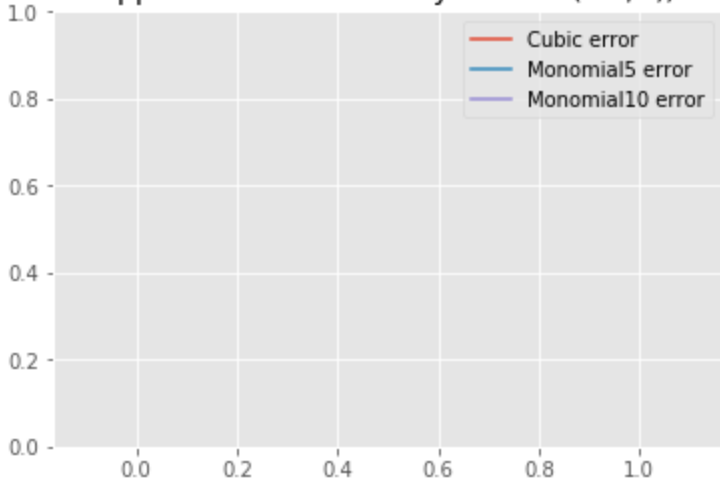
Approximation Errors by Cheb - Runge Function



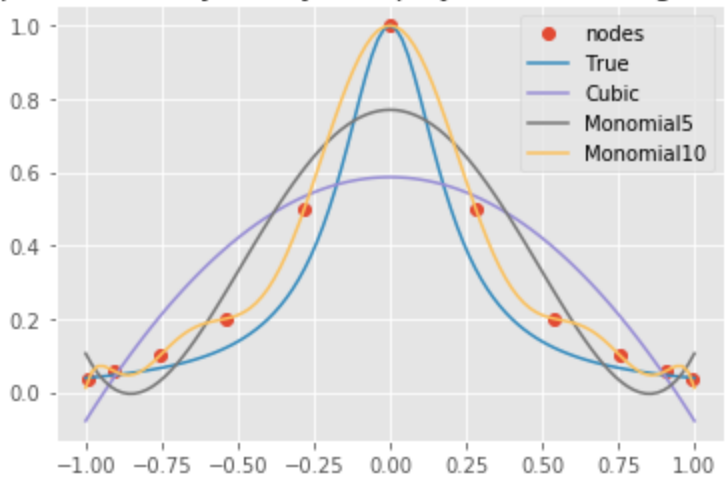
Approximation Errors by Cheb - $((x + |x|)/2)$



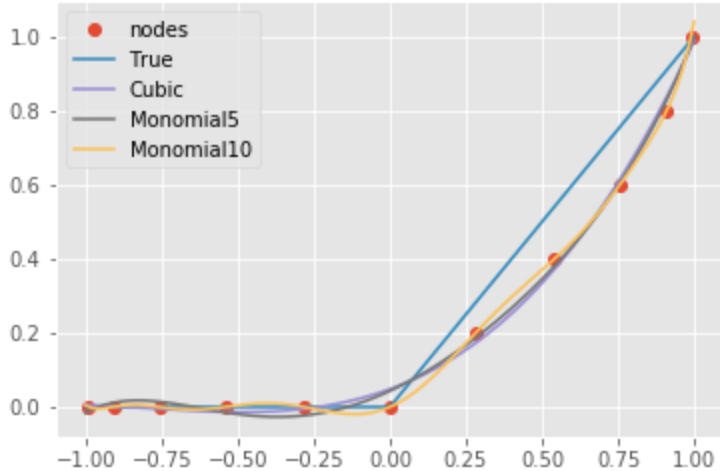
Approximation Errors by Cheb - ($e^{1/x}$)



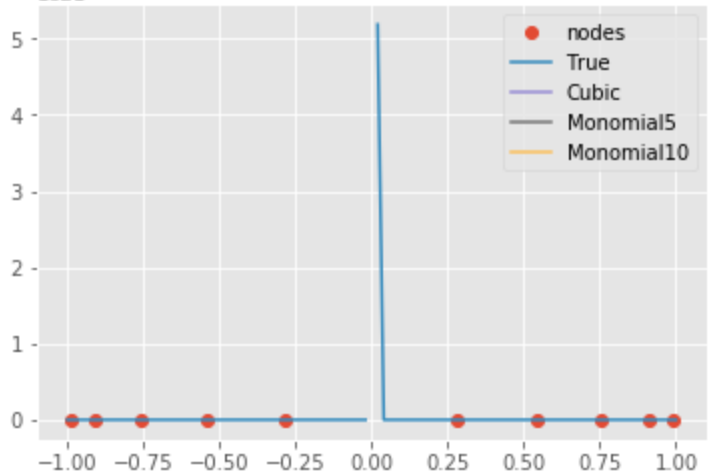
Approximation by Chebychev polynomials - Runge function



Approximation by Chebyshev polynomials - $((x + |x|)/2)$



Approximation by Chebychev polynomials - ($e^{1/x}$)



Approximation Errors by Cheby polyinomial

