Homework3 - Macroeconomics

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1 Exercise

Given: $\theta = 0.67$, $h_t = 0.31$, $n = 0$

1. $E_o \Big\{ \beta^t u(c_t) \Big\}$ s.t.
2. $c_t + i_t = y_t$
3. $y_t = k_t^{1-\theta} * (z * h_t)^\theta$
4. $i_t = k_{t+1} - (1-\delta) * k_t$

1.1 Steady state $\mathbf{k}_{ss1} = 4$

1. By assuming $y_{ss} = 1$: By $y_t/k_t = 4$ $k_{ss} = 4$
2. $i_t = k_{t+1} - (1-\delta)k_t = 0.25$: In steady state $k_t = k_{t+1}$: $i_t = \delta * k_t$. So $\delta = 0.25/4 = 0.0625$
3. From (3) we can solve for z: $z = (y_t/k_t)^(\frac{1}{\theta}) * (k_t/h_t)$ $z = 1.62967$
4. From the Utility equation, we can solve for the Euler equation:
   max β^t $ln(k_t^{1-\theta}(zh_t)^\theta - k_{t+1} + (1-\delta)k_t)$
   (a) $\frac{\delta L}{\delta k_{t+1}} = -\beta^t \frac{1}{ct}$
   (b) $\frac{\delta L}{\delta k_t} = \beta^{t+1} \frac{1}{c_{t+1}}((1-\theta)k_t^{-1*\theta}(zh)^{theta} + (1-\delta))$
   (c) Results in **Euler Equation**: $\frac{1}{ct} = \beta^t \frac{1}{c_{t+1}}((1-\theta)k_t^{-1*\theta}(zh)^{theta} + (1-\delta))$
   $\beta = 0.98039$

1.2 Steady state for doubled z

Based on the Euler equation, we observe for a doubled $z_{new} = 2 * z = 3.25936$ a doubled $k_{ss} = 2 * 4 = 8$. Values in the new steady state ($ss2$):

1. $k_{ss2} = 8$
2. $i_{ss2} = \delta * k_{ss2} = \delta * 8 = 0.5$
3. $c_{ss2} = k_t^{1-\theta} * (z * h_t)^\theta - k_{t+1} + (1 + \delta)k_t = 1.5$
4. $y_{ss2} = c_{ss2} + i_{ss2} = 2$

1.3 Transition path

It takes 17 time periods to converge to the new steady state after the shock occurred.

please see attachment(below) for graphs

1.4 Transition for second shock by z at t=10

For the time period from t=1 to t=10, we assume a transition path shown above. A guess is made for the agents behaviour to react to the occurring shock by it's consumption behaviour. As soon as the shock occurs, we assume a reaction in consumption by the agent. This guess/reaction defines the transition path to the old steady state. As soon as the shock occurs, I guess the agents reacts with a reduction in consumption of -0.33 takes the agent t=5 time periods to reach the initial steady state after the shock occurred.

please see attachment(below) for graphs

2 Exercise

Based on $parameters_h w3.pdf$ for countries (A,B) and skill levels

1. $max\left\{c_l^(\frac{1-\omega}{1-signa} - \kappa\frac{h_l^(1+\frac{1}{nu})}{1+\frac{1}{nu}})\right\}$
2. s.t. $c = \lambda(w_l h_l *)^{1-\phi_l} + r_l * k_l + r_{-l}(\overline{k}_l - k_l)$
3. max $Z(K_l^d)^{1-\theta}(H_l^d)^\theta - w_l H_l^d - r_l K_l^d$

2.1 Solving General Equilibrium in closed economies

Each country reaches its own general equilibria, because we consider closed economies. Hence, they don't affect each other. GE is reached if all markets clear. By Walras law, we know that n markets are cleared if $n - 1$ markets are cleared. Market clearing is reached for

1. Labour market: $H_demand = H_supply$

2. Money market: $M_o = M$
3. Goods market: $C = Y - G$

In python we can solve all unknowns as a system of differential equations. Therefore, we found conditions for all unknown parameters: r^a, w^a, h_l^l, h_h^l,c_l^a and c_h^a In total 6 parameters for each country.

Equations for country A:

1. $r^a = (1-\theta) * z(k_l^a + k_h^a)^{-\theta} * (h_l^a *_l^a + h_h^a *_h^a)^(\theta)$
2. $w^a = \theta * z(k_l^a + k_h^a)^{1-\theta} * (hl_a *_l^a + h_h^a * upeta_h^a)^(\theta - 1)$
3. $h_l^a = \frac{1}{\kappa} * c^{a-\sigma} * \lambda^a(1-\phi)(w^a *_l^a)^{1-phi})^{\frac{\nu}{phi}}$
4. $h_h^a = \frac{1}{\kappa} * c^{a-\sigma} * \lambda^a(1-\phi)(w^a *_h^a)^{1-phi})^{\frac{\nu}{phi}}$
5. $c_l^a = \lambda^a(w^a h_{l\,l}^{a\,a})^{1-\phi_l} + r^a(k_l^a)_l^a$
6. $c_h^a = \lambda^a(w^a h_{h\,h}^{a\,a})^{1-\phi_l} + r^a(k_h^a)_h^a$

The same equations apply to country B with changed subscript b

The **equilibrium for country A** is reached: beginlist

Country A, $interestrate : r^a = 0.40306953978018145$

Country A, $wages : w^a = 0.5969499565891875$

Country A, $labor - lowskill : h_l^a = 0.15102355534184986$

Country A, $labor - highskill : h_h^a = 0.35456965381278044$

Country A, $consumption - lowskill : c_l^a = 0.48266452397521814$

Country A, $consumption - highskill : c_h^a = 1.4758858005618243$ end list

The **equilibrium for country B** is reached: beginlist

Country B, $interestrate : r^a = 0.37844557644568017$

Country B, $wages : w^a = 0.6225710987842643$

Country B, $labor - lowskill : h_l^b = 0.2878110325124125$

Country B, $labor - highskill : h_h^b = 0.3154566393462799$

Country B, $consumption - lowskill : c_l^b = 0.8202890122624923$

Country B, $consumption - highskill : c_h^b = 1.0007983420627338$ endlist

2.2 Heterogeneous agents in two country case with cash mobility

In comparison to 2.1, we consider cash flows between country A and country B. This affects variables such as labour, consumption, wages and the interest rate mutually. The unknown variables we try to solve for in this case are: $r_l^a, r_l^b, w_l^a, w_l^b, h_l^a, h_h^a, h_l^b, h_h^b,\ c_l^a, c_h^a, c_l^b, c_h^b and k_l^a, k_h^a, k_l^b, k_h^b$. These are in total 16 parameters we try to solve for. We use the same equations as stated in 2.1 with respect to each country. Four additional equations are added to solve for k_l:

1. $r^a = (1-\theta)z(k_l^a + k_h^a)^{-\theta} * (h_l^a *_l^a + h_h^a *_h^a)^(\theta)$
2. $r^b = (1-\theta)z(k_l^b + k_h^b)^{-\theta} * (h_l^b *_l^b + h_h^b *_h^b)^(\theta)$
3. $w^a = \theta z(k_l^a + k_h^a)^{1-\theta} * (hl_a *_l^a + h_h^a *_h^a)^(\theta - 1)$
4. $w^b = \theta z(k_l^b + k_h^b)^{1-\theta} * (hl_b *_l^b + h_h^b *_h^b)^(\theta - 1)$
5. $h_l^a = \frac{1}{\kappa} * (c^a)^{-\sigma} * \lambda^a(1-\phi)(w^a *_l^a)^{1-phi})^{\frac{\nu}{phi}}$
6. $h_h^a = \frac{1}{\kappa} * (c^a)^{-\sigma} * \lambda^a(1-\phi)(w^a *_h^a)^{1-phi})^{\frac{\nu}{phi}}$
7. $h_l^b = \frac{1}{\kappa} * (c^b)^{-\sigma} * \lambda^b(1-\phi)(w^b *_l^b)^{1-phi})^{\frac{\nu}{phi}}$
8. $h_h^b = \frac{1}{\kappa} * (c^b)^{-\sigma} * \lambda^b(1-\phi)(w^b *_h^b)^{1-phi})^{\frac{\nu}{phi}}$
9. $k_l^a = \frac{r^b}{/} r^a *_l^a ** \frac{1}{_l^a - 1}$
10. $k_h^a = \frac{r^b}{/} r^a *_h^a ** \frac{1}{_h^a - 1}$
11. $k_l^a = \frac{r^a}{/} r^b *_l^b ** \frac{1}{_l^b - 1}$
12. $k_h^a = \frac{r^a}{/} r^b *_h^b ** \frac{1}{_h^b - 1}$
13. $c_l^a = \lambda^a(w^a h_{l\,l}^{a\,a})^{1-\phi_l} + r^a(k_l^a)_l^{a + r^b(\bar{k}^a - k^a)}$
14. $c_h^a = \lambda^a(w^a h_{h\,h}^{a\,a})^{1-\phi_l} + r^a(k_h^a)_h^{a + r^b(\bar{k}^a - k^a)}$
15. $c_l^b = \lambda^b(w^b h_{l\,l}^{b\,b})^{1-\phi_l} + r^b(k_l^b)_l^{b + r^a(\bar{k}^b - k^b)}$
16. $c_h^b = \lambda^b(w^b h_{h\,h}^{b\,b})^{1-\phi_l} + r^b(k_h^b)_h^{b + r^a(\bar{k}^b - k^b)}$

The **equilibrium for country A+B** is reached:

1. Country A+B, interest rate r in country A: $r^a = 0.2724447912344283$
2. Country A+B, interest rate r in country B: $r^b = 0.24488002793443495$
3. Country A+B, wages w in country A: $w^a = 1.3609150943991297$

4. Country A+B, wages rate w in country B: $w^b = 1.296282996466384$
5. Country A+B, labor - low skill in country A: $h_l^a = 0.14606175673965371$
6. Country A+B, labor - high skill in country A: $h_h^a = 0.3701888077449831$
7. Country A+B, labor - low skill in country B: $h_l^b = 0.2750347810826551$
8. Country A+B, labor - high skill in country B: $h_h^b = 0.30799206850950206$
9. Country A+B, consumption - low skill in country A: $c_l^a = 1.1569131048978745$
10. Country A+B, consumption - high skill in country A: $c_h^a = 3.1540115344386566$
11. Country A+B, consumption - low skill in country A: $c_l^b = 1.8283422499699646$
12. Country A+B, consumption - high skill in country B: $c_h^b = 2.160019408180131$
13. Country A+B, capital - low skill in country A: $k_l^a = 0.3094498627941823$
14. Country A+B, capital - high skill in country A: $k_h^a = 0.6686217219635883$
15. Country A+B, capital - low skill in country B: $k_l^b = 0.5828946771467958$
16. Country A+B, capital - high skill in country B: $k_h^b = 0.6322703836821464$

Results $h_l^a < h_l^b < h_h^b < h_h^a$ Due to higher dispersion in the difference in productivity, labor shares are more disperse in country A where productivity of low and high skill agents differs more than in country B.
Interest rate in country A is higher than in country B
Wage in country A is bigger than in country B.
Consumption for the low skill agents is smaller than for high skill agents. Ordered: agents from country A, low skill consumes lowest, from country B agent low skill, followed by country B agent high skill. Most consumes agent high-skill from country A. Based on higher wages in country A and higher labor output for high-skill agents, they consume most.
The model shows that in a country with high dispersion in productivity capital, consumption and labour shares are more unequally distributed trough out the country. Capital in country B is roughly equally distributed between the two types of agents, while in country A the low skill agent holds less than half of the amount of the high skill agent.

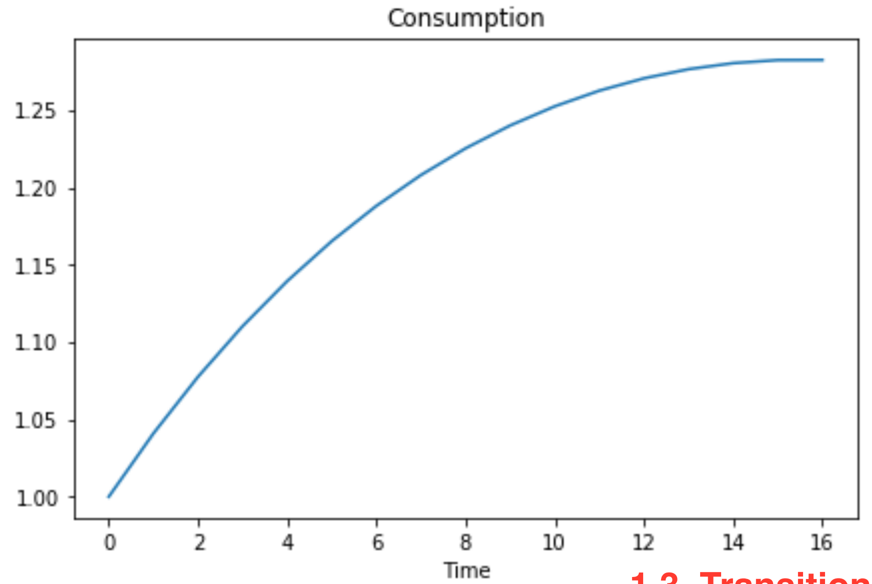
2.3 Progressive taxation

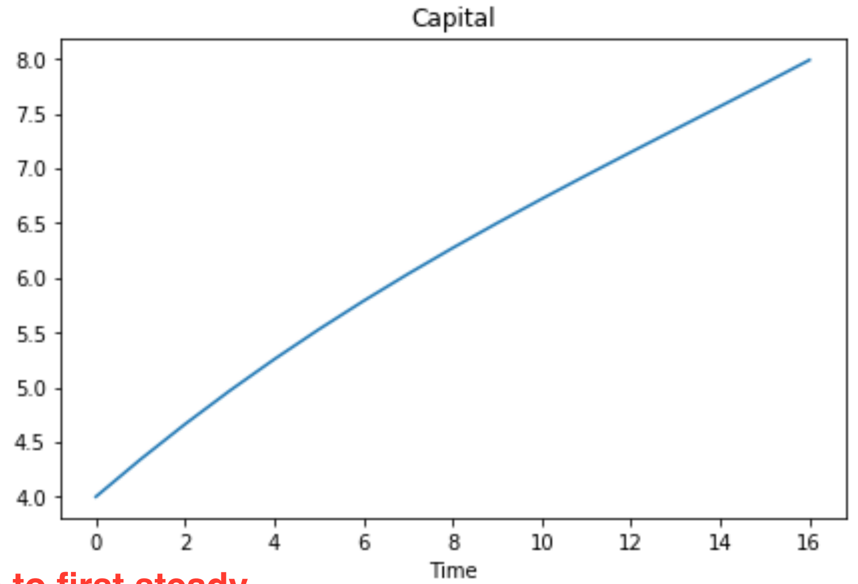
We are considering an income taxation that optimizes the welfare of the entire economy by the definition of the social planner. Hence, each welfare planner optimizes the progessive tax rate along its own economy. The maximizing problem might look alike: $\max \int G(u(c,h) * f(w)dw$ s.t. $\int T(w,h) * f(w)dw >= E$ $w(1 - T^{')*u_c+u_l=0}$

This is an Mirrlees (1971) approach to find optimal taxation. The problem maximizes a tax schedule based on agents utility along the distribution of wages $f(w)$. Where w is considered as an indicator for the heterogeneity in productivity. This is necessary as the government doesn't observe wages, only after-tax income. The first constraint is stating that the tax income must be greater or equal to the minimum expenditure E of the government. The second constraint optimizes the agents own utility maximisation problem, considering as budget constraint the reduced wealth due to the income tax.

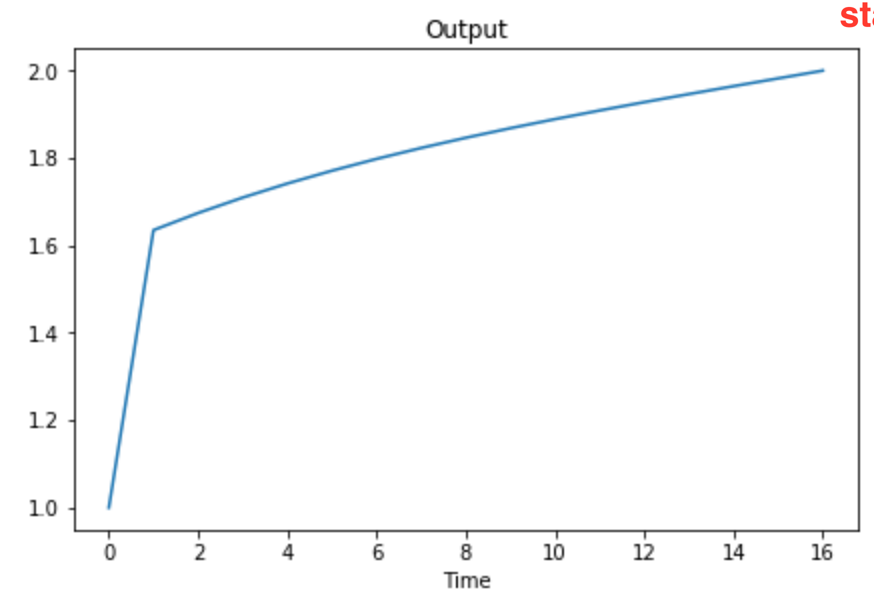
Considering results 2.2, a more progressive tax rate for country A compared to country B would equalize capital and consumption shares. Hence, low income agents would pay less taxes than high income agents. A more progressive tax rate in country A, could equalize the dispersion of productivity throughout the country and thereby distribute capital more equally. The same mechanism would support equalising capital, consumption in country B. In country B, I would recommend a less progressive tax given that the rates of productivity don't distinguish that much.

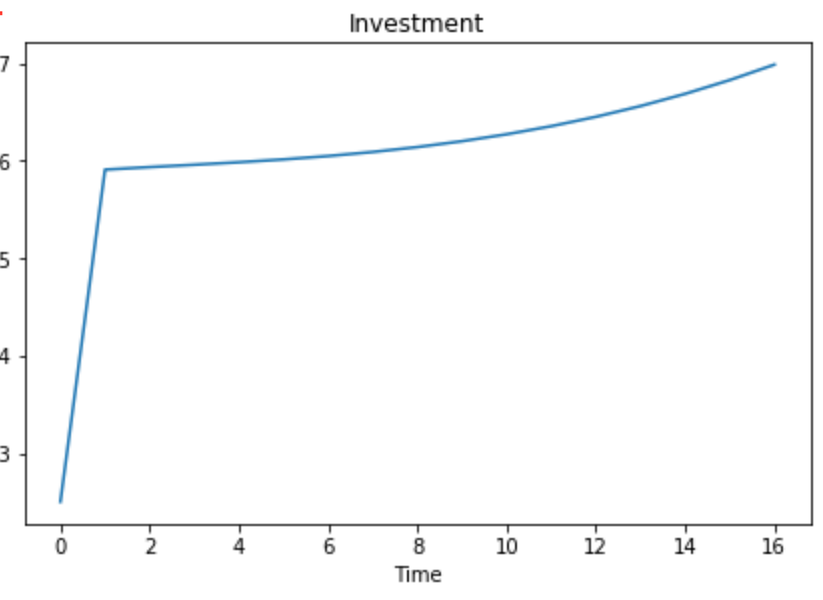
The question of implementing an income tax one should have in mind is, what is the major goal of the social planner. Maximising agents utility? Achieving equality in terms of capital? Or enlarging wealth disperion?





1.3. Transition path to first steady state k=4





1.4. Transition path with shock at t=10