Homework5-Macroeconomics

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1 Exercise

1.1 Creating a random sample

1.2 Firm output

1.3 Maximisation Problem

$max \sum_{i=0}^{n} s_i^{1-\gamma} k_i^{\gamma}$ st. $K = \sum_{i=0}^{n} k_i$ FOC for k_1: $\lambda = \gamma z_1 k_1^{\gamma-1}$

FOC for k_2:$\lambda = \gamma z_2 k_2^{\gamma-1}$

We get: $k_1^* = k_2(\frac{z_2}{z_1})^{\frac{1}{\gamma-1}}$

$K = \sum_{i=0}^{n} k_i = k_1 + (k_1 * (\frac{z1}{z2}^{\frac{1}{\gamma-1}} + \frac{z1}{z3}^{\frac{1}{\gamma-1}} + \frac{z1}{z4}^{\frac{1}{\gamma-1}} ...))$

$K = k_1 + (k_1 \sum_{i=0}^{n-1} \frac{z_1}{z_i}^{\frac{1}{\gamma-1}})$

$k_1^* = \frac{K}{\sum_{i=0}^{n} \frac{z_1}{z_i}^{\frac{1}{\gamma-1}}}$

1.4 Comparison between the random k and the efficient k

The optimal k shows larger inconsistencies in comparison to the random sample k. Although the mean for both capital accumulation paths remains the same (= 4.483052)

1.5 Output comparison

For zero co-variance between z and k we find an increase in output by optimal capital accumulation of 138.511. Exploiting efficient capital accumulation turns in the case of positive co-variance between $\ln z$ and $\ln k$ into a slightly smaller output maximisation of 137.377. For negative co-variance, the highest output of 138.918.

2 Change in $\gamma = 0.8$

For an increase in γ, we observe stronger effects in output gains for an implementation of an optimal k. From optimal capital accumulation for zero co-variance achieves output gains of 688.643. The positive co-variance case shows obvious lower output gains compared to the zero co-variance case. Referring to results of $\gamma = 0.6$, output gains are still immense with more than 442.227. The case with negative co-variance yields highest levels of output improvement 952.7106.

3 Random sample

3.1 Random sample: 10.000

The variance of a sample k of size 10.000 selected from the entire population is roughly 1.0022849. While the variance of z of the same sample is 0.996344680. Repeated selection shows, that both values stay close to one. The co-variance matrix in this selection is roughly close to the no-correlation case between z and k. For the sample selected, we observe correlation between z and k from the 4th position behind the comma.

$$co - variance = \begin{bmatrix} 1.00093186 & 1.73919326e^{-4} \\ 1.73919326e^{-4} & 1.00028107 \end{bmatrix}$$

3.2 Redo output maximisation with random sample selection: 10.000

Repeating the random sample selection case 1000 times shows on average comparable results to the observed population in exercise one. The histogram is normally distributed and peaks at 137.221. The random sample provides comparable output trajectories like the entire data set.

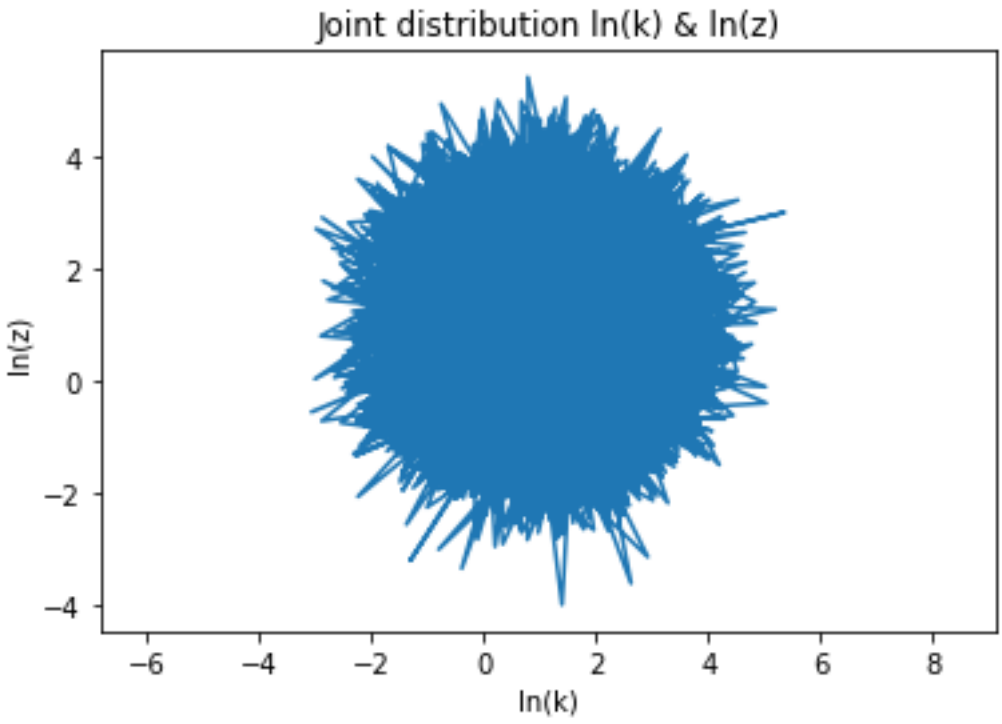
3.3 Probability of 10 percent to find optimal path of allocation

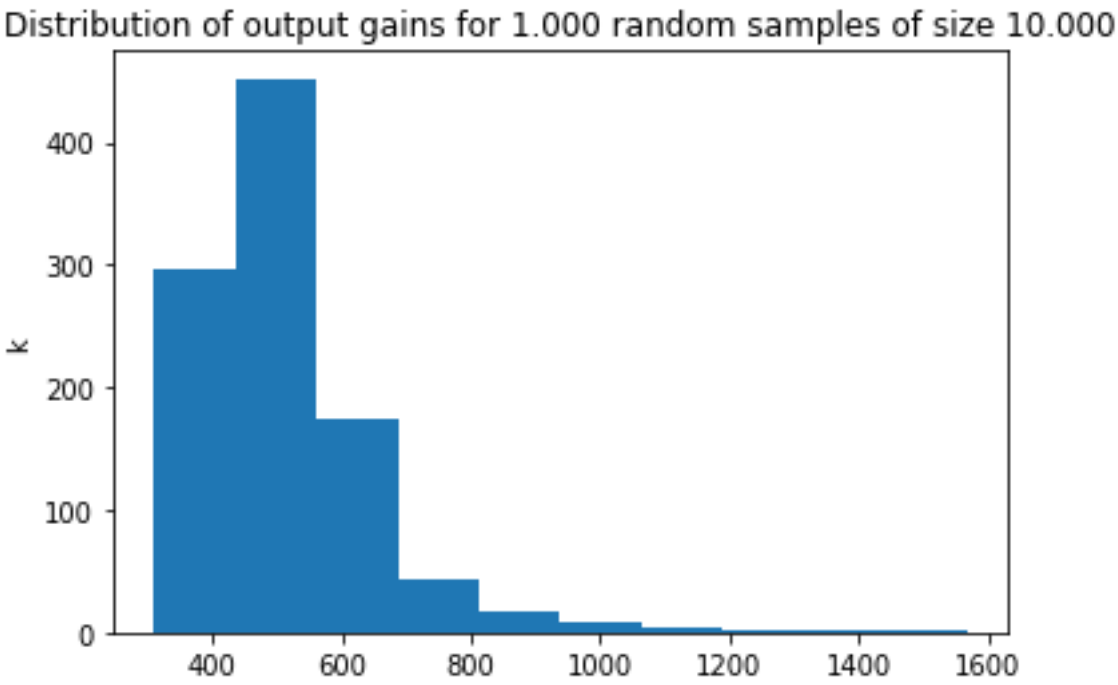
The probability to be close to differ up to 5percent from the optimum is 43%.While the probability for a sample of 10.000 observations achieves with a probability of 78.3% optimum output.

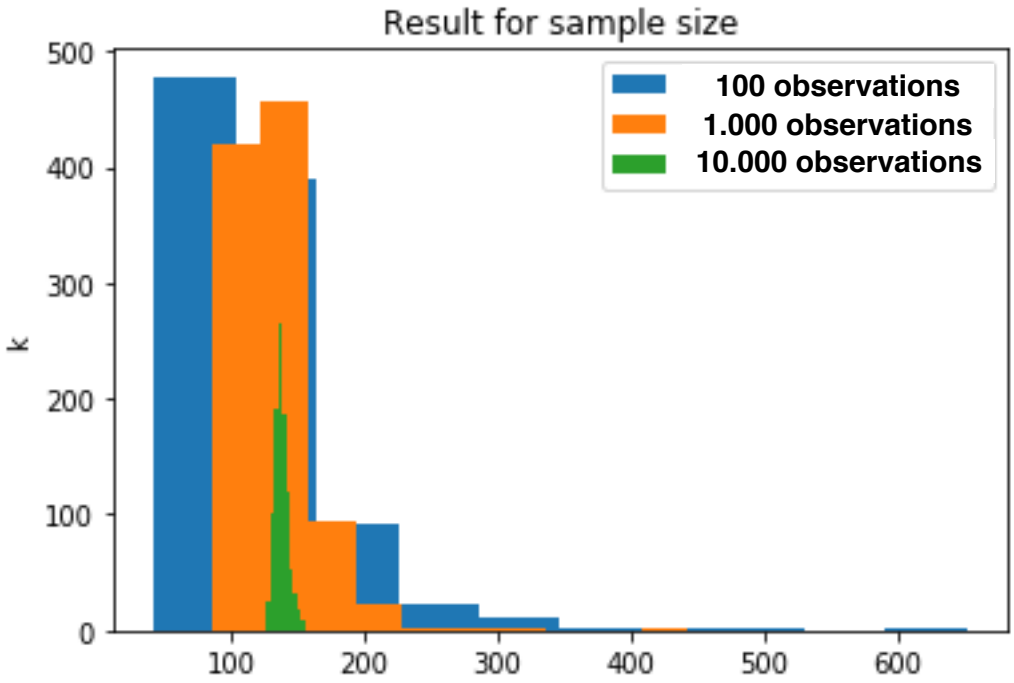
3.4 Comparison of sample size: $100, 1.000, 10.000$

This exercise clearly shows, that along the increase of sample size we are more likely to select optimal capital allocations that lead to maximised output.

For sample size 100 output gains is 105.8653815976956.
For sample size 1.000 output gains is 125.97410971663123.
For sample size 10.0000 output gains is 137.77507258238268.


Joint distribution ln(k) & ln(z)
ln(z)
ln(k)
4
2
0
−2
−4
−6
−4
−2
0
2
4
6
8


Distribution of output gains for 1.000 random samples of size 10.000
k
0
100
200
300
400
400
600
800
1000
1200
1400
1600


Result for sample size
100 observations
1.000 observations
10.000 observations
k
500
400
300
200
100
0
100
200
300
400
500
600