Homework 8 - Macroeconomics

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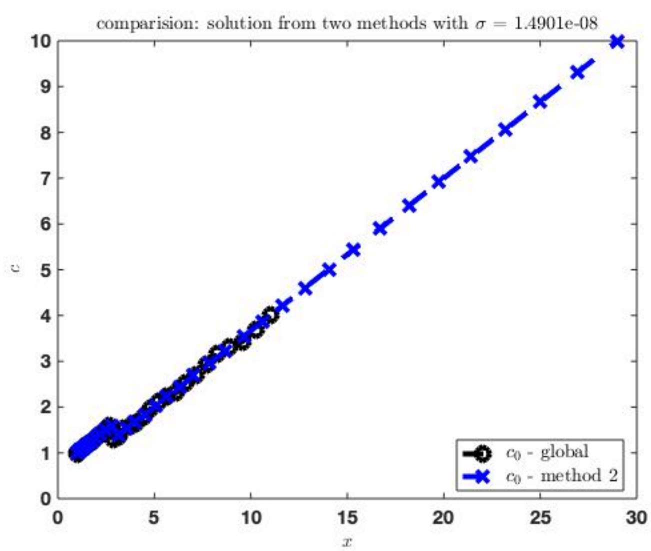
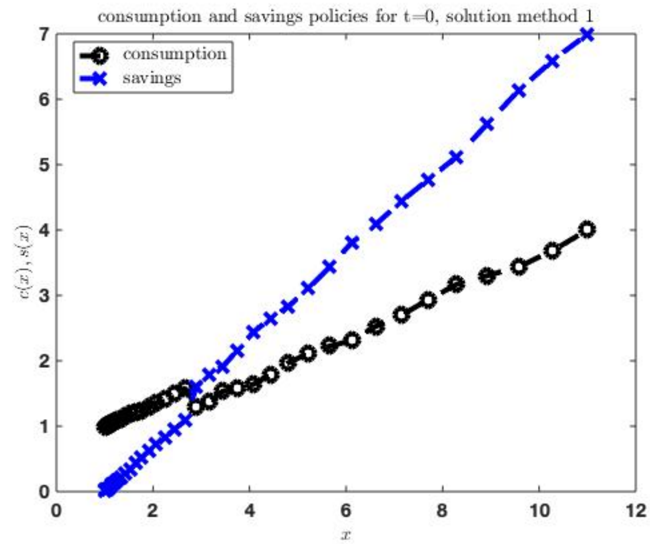
Part A - Discrete Continuous Choice Problem

Comparison of global solution, endogenous grid method exogenous root-finding method

For small variance of the taste and income shock, I find for all three solution paths a discontinuity in consumption around time period three. While the global solution shows more volatile consumption paths throughout time, the exogenous method shows extremest change in consumption pattern at the discontinuity.





Introduction to methods

The first method is the closed form solution. It solves the Value function on the created grid for cash on hand. Along golden search, which searches for a maximising point in iterative narrowing interval.

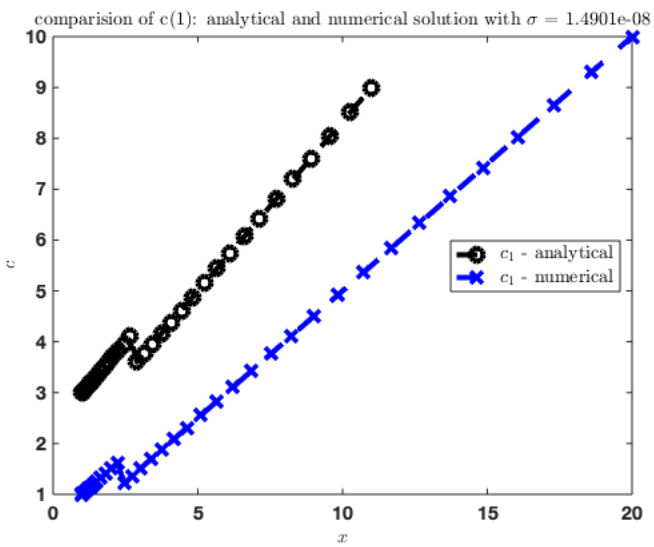
The second method is the endogenous discrete choice method. First we make a grid on savings. Given that we know $s = x - c$, we compute consumption based on a future grid point of assets. Subsequent we solve for today's assets and check if our solution is on the Grid, satisfying the border constraints. Thereby we assume assets of future period and solve for current period consumption and assets.

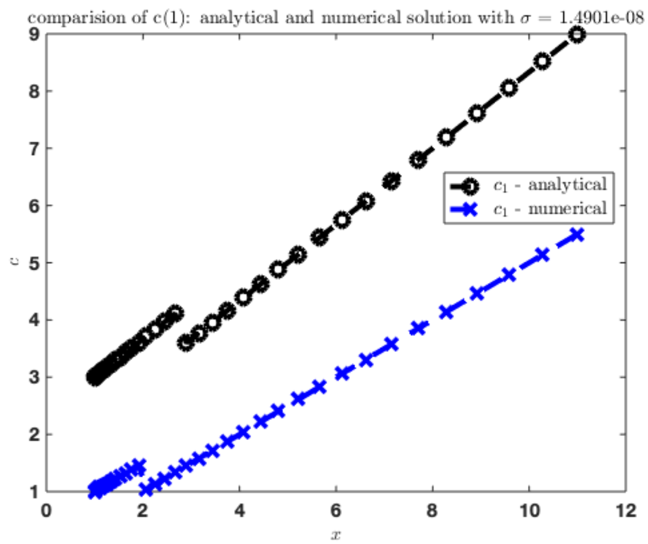
The third method considers a root-finding problem for consumption on an exogenous grid of cash on hand. We solve by FOC for consumption and check if consumption and savings satisfy our border constraints. If not we replace the solution by our border constraints of the cash on hand grid and solve for the value function.

Closed form solution for period 1

The code provides period 1 solutions apart from the case where we account for a taste shock. We know that $xt = a_{t+1} - ct$, rewriting the provided code line 207: $a_{t+1} = x_t + 1 - 2 * c_{t-1}$ we can solve for period 1.

My findings show distinct consumption patterns for method 2 and method 3 from method 1. Assuming the variance is equal to zero, the model solves deterministic assumptions for all three methods. The graphics presented show that we find higher consumption patterns for method 2 and 3 in comparison to the numerical solution. Comparing period $t = 10$, for method 2 and 3 consumption policy suggests 9 units, while in method 1 policy suggests 4 units of consumption.





Running time

Running time for the closed form solution is the longest. The shortest computation time takes method 2. This is due to the endogenous grid that simplifies the computation given the linear mapping between the state variables. Also, it isn't based on a maximisation routine which safes a lot time. Method 3 is based on such a routine. The exogenous method requires a lot of time, because of the root-finding which is time intensive in running time.

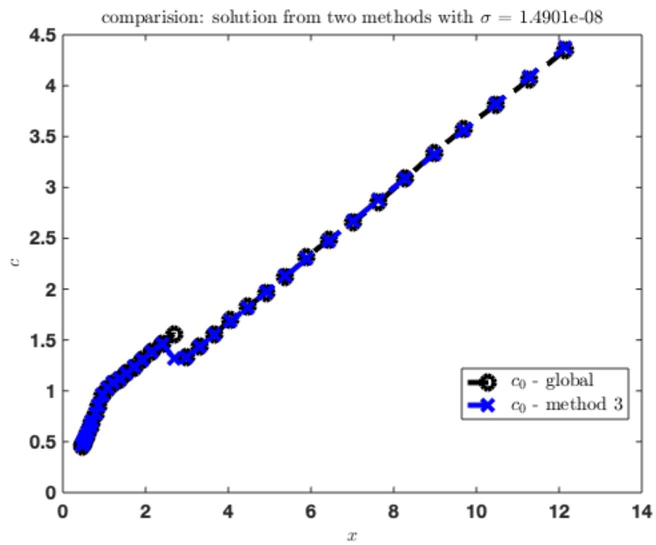
Running time comparison			
	method 1	method 2	method 3
Running time	24.61 sec	2.18 sec	17.64 sec

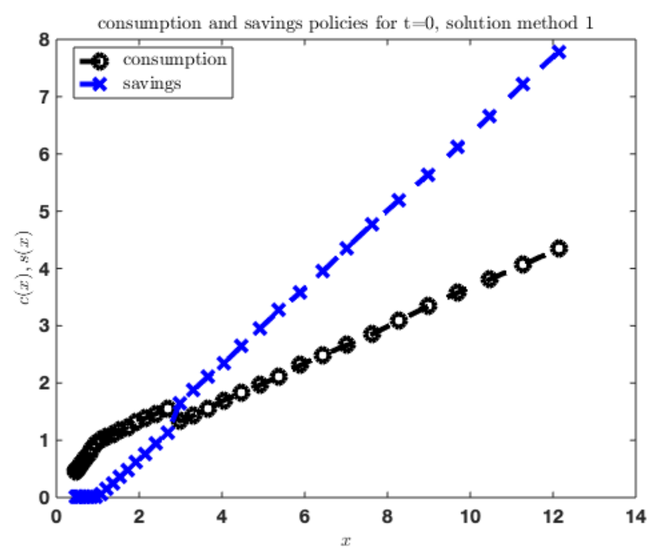
Difficulties for method 3

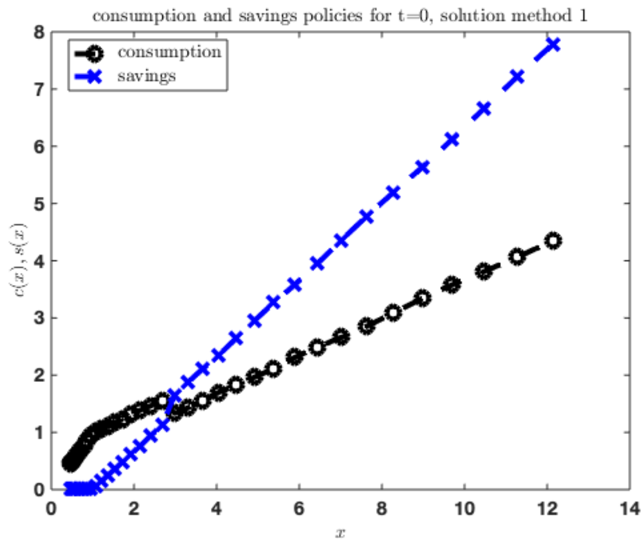
In the exogenous grid method we face a solution path along root-finding. Due to the consideration of shocks and assets the dimensions are quite high. This might lead in the exogenous grid method to complications as root-finding becomes more complex the bigger dimensions are. Thereby, it might encounter problems. We can support the root-finding method by setting an upper and lower bound for the considered grid method.

Variance of income shocks

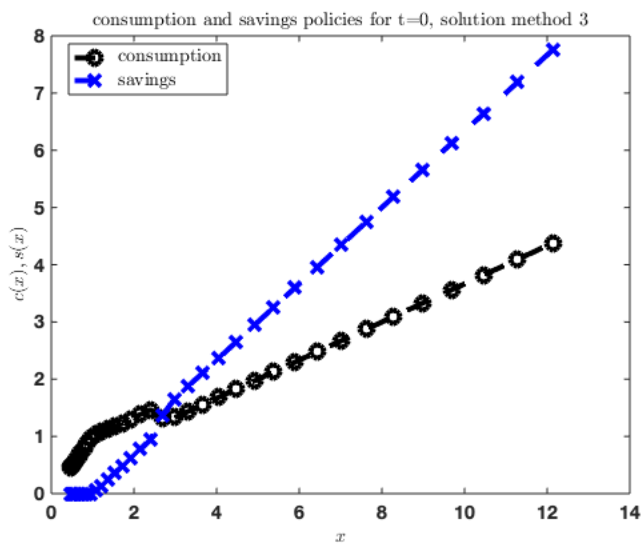
By increasing the variance of the income shock to $\omega_i = 0.01$, I find smoothed policy functions for consumption and savings. Due to the variance of the income shock, the discontinuity is partly smoothed out.







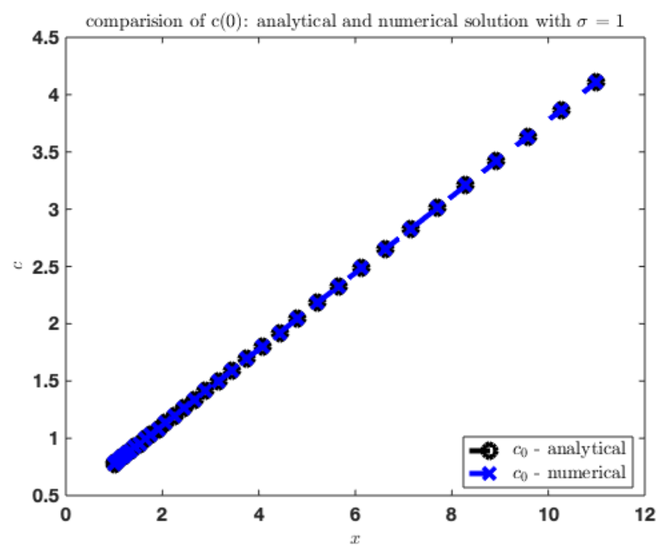
consumption and savings policies for t=0, solution method 1
consumption
savings
$c(x), s(x)$
x

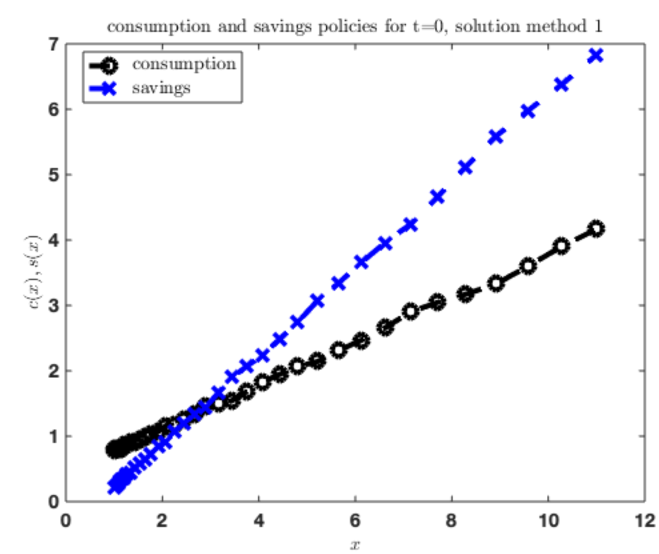


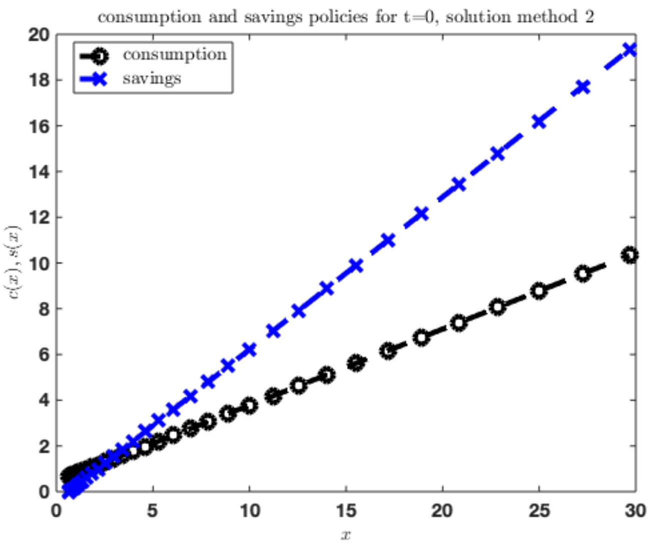
consumption and savings policies for t=0, solution method 3
consumption
savings
$c(x), s(x)$
x

Variance of taste shock

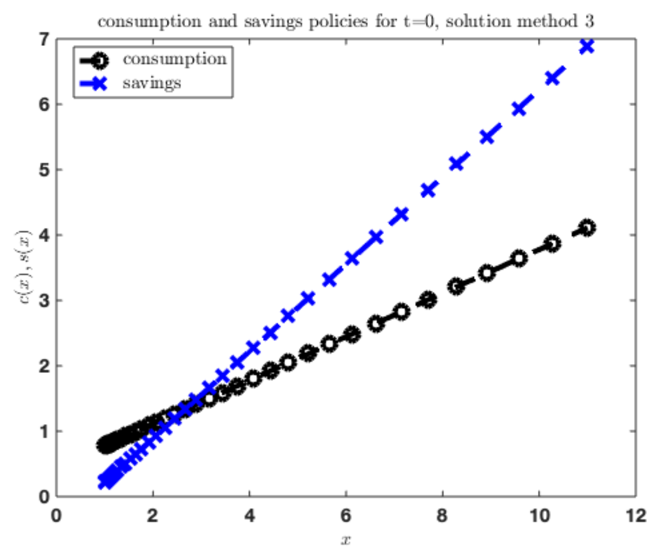
By increasing the variance of the taste shock to $\omega = 1.0$, takes the curvature of the value function. I find a completely concave function and almost linear policy functions for consumption and savings. The higher variance of a taste shock absorbs the discontinuity.







consumption and savings policies for t=0, solution method 2
consumption
savings
$c(x), s(x)$
x



consumption and savings policies for t=0, solution method 3
consumption
savings
$c(x), s(x)$
x

Part B - Problem 1

Explaining the Code

The code allows for various settings. First, it allows to decide between a deterministic or stochastic setting, the consideration of a survival risk and last the computational method to account for uncertainty of states, namely by a Markov chain with 2 stated or 5 states.

1. The code starts by initialising parameters. 2. Secondly, it takes into consideration the selection of survival risk. In the case of survival risk, the samples demographics are affected by random shocks decreasing survival probability. Population is normalised to 1. 3. Finding the Household Solution: An iteration process starting at T, solves backwards by the exogenous on the cash on hand grid method based on a value function iteration.
In a second step optimal policies for consumption is obtained by the inverted euler function equation. Thereafter, optimal policy functions are computed for, coh, mpx, assets and finally gross and net savings and total wealth is computed in a forward loop. Thirdly, the code checks budget constraint boundaries for the variable: annuities. Before comparing the new interest rate with the applied interest rate, assets are derived as function of savings respective to ages. The code checks for convergence of the interest rate. Before plotting the optimal policy functions, aggregation accounts for distributional aspects over populations lifetime.
3. Before plotting the optimal policy functions, aggregation accounts for distributional aspects for the population w.r.t. shocks, assets and age. Where a transfer function accounts for the change in populations distribution going from state t to $t+1$. The sum of this exogenous law of motions Phisums up to one. Aggregated variables are consumption and assets.
4. Plotting results

Grid comparison

The advantage of working with a savings grid is the linear relation, the reduction of state variables and omitting the root-finding solution. This accelerates the running time, cuts precision slightly but maintains the contractions mapping. The disadvantage is that interpolation in an endogenous grid become more complicated. Implementation check code.

Endogenous grid method

Running time comparison shows that the endogenous grid method is faster. This result is driven through omitting the root-finding method.

Running time comparison		
	Exogenous Grid method	Endogenous Grid method
Running time	14.7781	3.122

Phi and TT

Phi presents the cross sectional distribution for each possible income shock, various ages and asset combinations. Where TT describes the law of motion over time. The transition matrix TT describes the change from state today to the future state tomorrow. It is also referred to as the exogenous law of motion. The code results in the aggregation of applicable distributions w.r.t. assets, consumption, labour and interest rate for a given population on relevant grid.

Comparison of cases

Now, we consider variations of the present code. We consider deterministic vs. stochastic model, survival vs. non survival risk and one could additionally consider risk in 2 vs. 5 states.
From a theoretical viewpoint one would expect for a situation for a deterministic setting a maximised consumption path as agents can optimally use income, assets to maximise their life-time consumption. In the case of stochastic elements in the model, expectations might lead to error that negatively impact the amount of consumption. Hence in case two and three I expect more volatility.

Please find attached respective graphs for all three considerations.
case: true, true: Considering a deterministic model without survival risk, provides a convex value function. Consumption stays out most constant, apart from radical drop shortly before retirement at age 65. Savings (rates) are positive, peaking at 0.4 at age 65 and constantly fall thereafter til -0.5.

case: false, true: Stochastic model without survival risk. In this case, the value function is more flattened and shows in early stages a slight concave form that twists into convex function later on. Assets peak at retirement age at a level of 14, as they need to save more given the income uncertainty they face. Hence, consumption is shifted to later periods in life. It increases steadily over life time. In comparison to the deterministic case, assets are more widely distributed throughout generations.

case: false, false: Here, a stochastic model with survival risk is computed. The value function displays almost linear characteristics. All policies occur more linear. This is due to the interaction of uncertainty introduced by the survival risk and counteracted by the stochastic characteristic of future states. Households behave more constant as the uncertain future doesn't grant fixed income and menance survival risk.

When agents risk aversion is decreased by θ, we find that agents tend to save less and hence consume more.

Problem 2

Extend to OLG

Please see code.

Inefficiencies in OLG economies with idiosyncratic risk

In an incomplete market with idiosyncratic risk we observe CRRA agents tend to over save. Hence, the capital stock is higher than in complete market. This yields from the uncertainty of market outcome that gives incentives to invest in personal security by hoarding assets. A government may counteract and introduce a PAYG security system. In this situation, rather then investing savings on the capital market, investing it in a social security system will be welfare improving for all generations.

PAYG System

In order to account for the inefficiency, a government could improve the wealth fare effect by implementing a social security system.
Therefore, we compute the consumption equivalent variation which is the comparison of the value function before and after the implementation of a social security scheme:
$g(a, \eta) = (\frac{V_o}{V_1})^{\frac{1}{1-\theta}} - 1$
V_0 is before policy change, V_1 after policy change. We assume to find, for risk averse household (high θ) a larger positive effect by the governmental security than for risk loving households. Overall, assets should peak at a lower level and support higher consumption levels throughout lifetime.

Problem 3

Transitional Dynamics for OLG

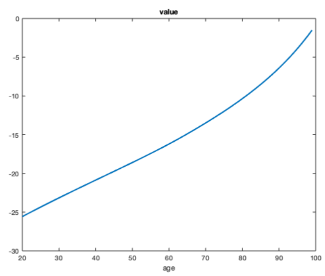
To compute the transitional dynamics for OLG with idiosyncratic risk we would follow following steps:

1. Define T, time horizon of transition dynamics
2. Solve for steady state v, c, a, ϕ, r, w, K
3. Solve for stationary equilibrium on the infinite time horizon. We assume that results after T achieved steady state equilibrium and intersect with those of the infinite time horizon.
4. Interpolate between r_0 and r_T to get an initial guess for the equilibrium time path.
5. Solve Household problem:
a Calculate wage respective to r_{guess}
b by backward iteration starting from T, we compute v, a_{t+1}, c for periods between zero and $T-1$.
c Given the policy functions we can iterate forward on the exogenous law of motion to calculate the transitional distribution path of the economy:
$\phi_{t+1}^m = H_t^m * \phi_t^m$
d Along $\phi_t^m(j)$ we can compute aggregate capital supply. We apply $K_t^m = A_t^m$ for all t and evaluate r_t^m for $t(1, T-1)$
6. Check for convergence in r
7. Check if $||A_T - K_T|| < \epsilon$ if this is not the case increase T.

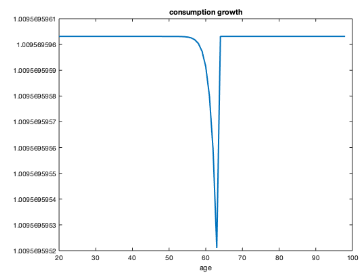
1 Problem C

Summary of Krueger and Ludwig (2016) *On the optimal provision of social insurance: Progressive taxation versus education subsidies in general equilibrium.* The paper examines human capital accumulation (education) decisions in the case of insurable idiosyncratic earning risk and a progressive tax scheme. Thereby, they point out to distinct policies are revealed under consideration of General Equilibrium transition costs. In the case, they account for a relative wage effect which is the substitution rate of skilled vs. unskilled workers ($= 1.4$), they find that larger educational subsidies, increase the overall welfare gains due to a larger share of graduate students (up to 67%). Subsidies are basically financed by debt (+20%), which allows at the same time for tax reductions of the progressive tax. This leads to indirect beneficial re-distributive effects. Modelling the case without relative wage effects, wages do not move. Redistribution is achieved due to tax deduction that overall reduced economic welfare (steady state). In a transitional optimal policy, through higher educative subsidies which leads to higher welfare gains.

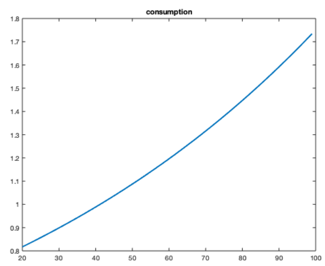
Abbildung (1)   Deterministic Model without Survival Risk



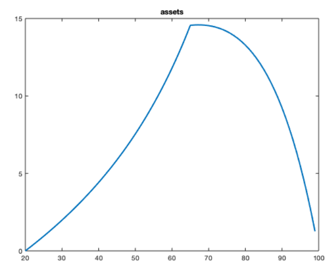
(a) Value Function



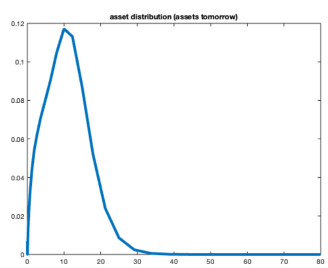
(b) Consumption Growth



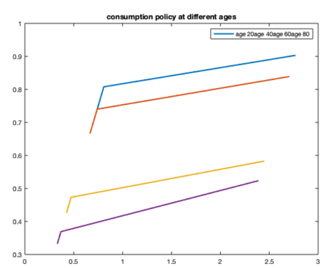
(c) Consumption Path



(d) Assets

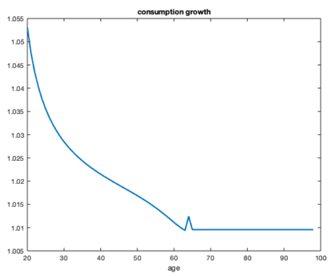


(e) Asset distribution

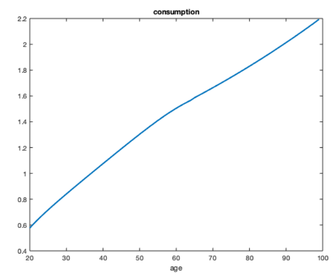


(f) Consumption policy at different age

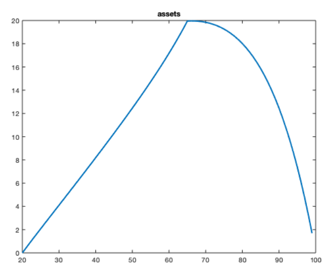
Abbildung (2) Stochastic model without survival risk



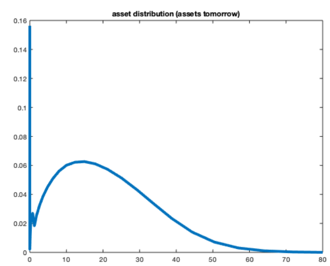
(a) Value Function



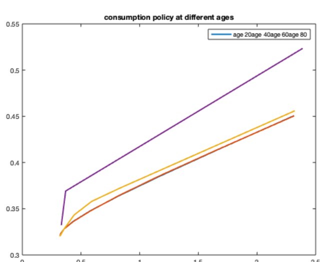
(b) Consumption



(c) Assets

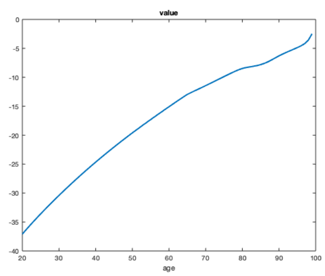


(d) Asset Distribution

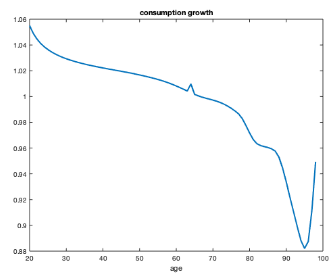


(e) Consumption policy at different age

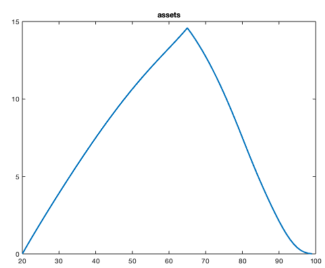
Abbildung (3) Stochastic model with survival risk



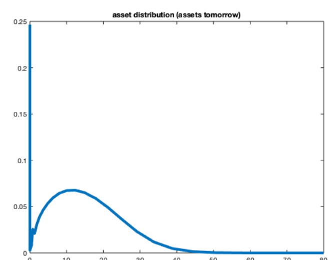
(a) Value Function



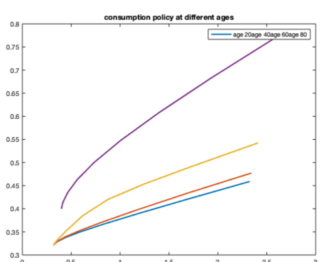
(b) Consumption Growth



(c) Assets



(d) Asset Distribution



(e) Consumption policy at different age